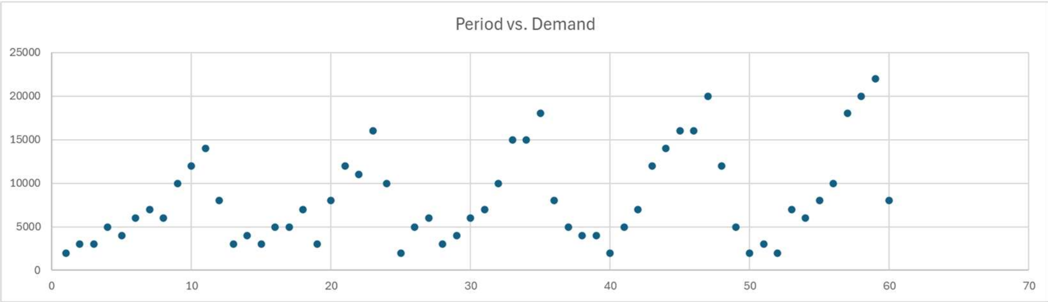
Question 1.

a. First, I analyzed the data trend by plotting sales over the given periods. The detailed calculations are available in the sheets named "Question1.a-sheet1" and "Question1.a-sheet2"



From this visualization, we can observe that the sales exhibit both yearly seasonality and an overall trend.

Next, the deseasonalized demand and seasonal factors were calculated by the following relations.

$$\overline{D}_t = \frac{1}{24}\left(\sum_{i=t-6}^{t+5} D_i + \sum_{i=t-5}^{t+6} D_i\right)$$

$$\bar{S}_t = \frac{D_t}{\overline{D}_t}$$

The table below summarizes the results.

Period	Demand	Deseasonalized Demand	Seasonal Factor
1	2000	-	-
2	3000	-	-
3	3000	-	-
4	5000	-	-
5	4000	-	-
6	6000	-	-
7	7000	6708.333333	1.043478261
8	6000	6791.666667	0.883435583
9	10000	6833.333333	1.463414634
10	12000	6833.333333	1.756097561
11	14000	6875	2.036363636
12	8000	6958.333333	1.149700599
13	3000	6833.333333	0.43902439
14	4000	6750	0.592592593
15	3000	6916.666667	0.43373494
16	5000	6958.333333	0.718562874
17	5000	7000	0.714285714
18	7000	7166.666667	0.976744186
19	3000	7208.333333	0.416184971
20	8000	7208.333333	1.10982659
21	12000	7375	1.627118644
22	11000	7416.666667	1.483146067
23	16000	7291.666667	2.194285714
24	10000	7208.333333	1.387283237
25	2000	7333.333333	0.272727273
26	5000	7583.333333	0.659340659
27	6000	7791.666667	0.770053476
28	3000	8083.333333	0.371134021
29	4000	8333.333333	0.48
30	6000	8333.333333	0.72
31	7000	8375	0.835820896
32	10000	8458.333333	1.18226601
33	15000	8333.333333	1.8
34	15000	8208.333333	1.827411168
35	18000	8208.333333	2.192893401
36	8000	8291.666667	0.964824121
37	5000	8541.666667	0.585365854
38	4000	8916.666667	0.448598131
39	4000	9125	0.438356164
40	2000	9208.333333	0.21719457
41	5000	9333.333333	0.535714286
42	7000	9583.333333	0.730434783
43	12000	9750	1.230769231
44	14000	9666.666667	1.448275862
45	16000	9541.666667	1.676855895
46	16000	9500	1.684210526
47	20000	9583.333333	2.086956522
48	12000	9625	1.246753247
49	5000	9416.666667	0.530973451
50	2000	9083.333333	0.220183486
51	3000	9000	0.333333333
52	2000	9250	0.216216216
53	7000	9500	0.736842105
54	6000	9416.666667	0.637168142
55	8000	-	-
56	10000	-	-
57	18000	-	-
58	20000	-	-
59	22000	-	-
60	8000	-	-

Next, to estimate future demand, I formulated the static model as follows.

$$F_t = (L + tT)S_t$$

To determine each parameter, I first performed linear regression on the deseasonalized demand.

SUMMARY OUTPUT								
Regression Statistics								
Multiple R	0.956756465							
R Square	0.915382933							
Adjusted R Square	0.913543431							
Standard Error	309.9811871							
Observations	48							
ANOVA								
	df	SS	MS	F	Significance F			
Regression	1	47816011.47	47816011.47	497.6255525	2.61879E-26			
Residual	46	4420063.473	96088.33636					
Total	47	52236074.94						
	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%	Lower 95.0%	Upper 95.0%
Intercept	5963.188112	108.1899835	55.11774676	1.16208E-43	5745.412971	6180.963254	5745.412971	6180.963254
X Variable 1	72.04597264	3.229671659	22.30752233	2.61879E-26	65.54498078	78.54696451	65.54498078	78.54696451

Additionally, I calculated the seasonal factor for each month by averaging the corresponding seasonal factors derived from the data.

The resultant parameters were obtained as follows.

Model Parameters	
L	5963.188
T	72.04597
S1	0.457023
S2	0.480179
S3	0.493869
S4	0.380777
S5	0.616711
S6	0.766087
S7	0.881563
S8	1.155951
S9	1.641847
S10	1.687716
S11	2.127625
S12	1.18714

Here, $S_{12i+j} = S_j$, for $i = 1,2,3,4,5$ and $j = 1,2, \dots ,12$

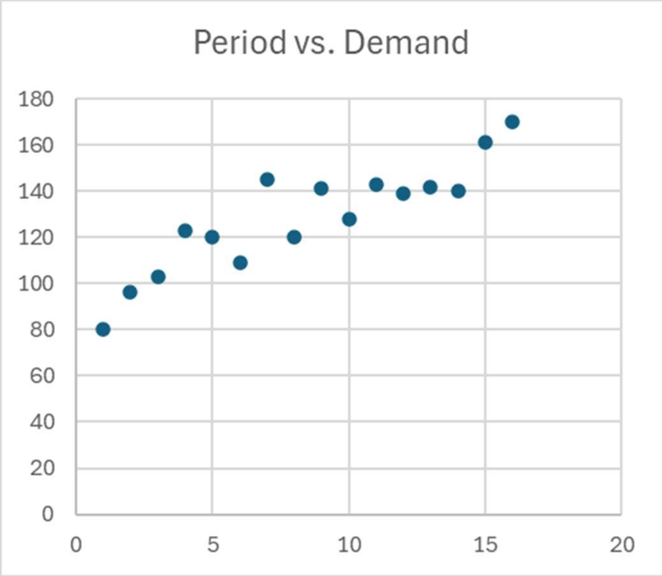
Finally, the Year 6 forecast is as follows.

Sales	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
JAN	2000	3000	2000	5000	5000	4733.838108
FEB	3000	4000	5000	4000	2000	5008.282468
MAR	3000	3000	6000	4000	3000	5186.658939
APR	5000	5000	3000	2000	2000	4026.384794
MAY	4000	5000	4000	5000	7000	6565.60901
JUN	6000	7000	6000	7000	6000	8211.088388
JUL	7000	3000	7000	12000	8000	9512.304939
AUG	6000	8000	10000	14000	10000	12556.30314
SEP	10000	12000	15000	16000	18000	17952.54974
OCT	12000	11000	15000	16000	20000	18575.69148
NOV	14000	16000	18000	20000	22000	23570.78979
DEC	8000	10000	8000	12000	8000	13237.20572
Total	80000	87000	99000	117000	111000	129136.7065

With this model, TS, MAD, MAPE, MSE are calculated as follows.

Performance Metrics	
TS	-2.09262
MAD	1115.362
MAPE	21.22319
MSE	2265146

b. To analyze the overall trend, I plotted the demand against the period.



The demand shows a clear trend that cannot be adequately captured by just a single parameter (i.e., level). Therefore, I initially hypothesized that Holt's model would provide a better fit.

Exponential smoothing, detailed calculations are in "Question1.b-sheet1"

Year	Quarter	Period	Demand ($1000)	Level ($1000)	Estimated Demand ($1000)	Errors ($1000)
		0		128.75		
1	I	1	80	123.875	128.75	48.75
	II	2	96	121.0875	123.875	27.875
	III	3	103	119.27875	121.0875	18.0875
	IV	4	123	119.650875	119.27875	-3.72125
2	I	5	120	119.6857875	119.650875	-0.349125
	II	6	109	118.6172088	119.6857875	10.6857875
	III	7	145	121.2554879	118.6172088	-26.38279125
	IV	8	120	121.1299391	121.2554879	1.255487875
3	I	9	141	123.1169452	121.1299391	-19.87006091
	II	10	128	123.6052507	123.1169452	-4.883054821
	III	11	143	125.5447256	123.6052507	-19.39474934
	IV	12	139	126.890253	125.5447256	-13.45527441
4	I	13	142	128.4012277	126.890253	-15.10974696
	II	14	140	129.561105	128.4012277	-11.59877227
	III	15	161	132.7049945	129.561105	-31.43889504
	IV	16	170	136.434495	132.7049945	-37.29500554
5	I	17	-		136.434495	
	II	18	-		136.434495	
	III	19	-		136.434495	
	IV	20	-		136.434495	

Metrics	
TS (-)	-4.237493038
MAD ($K)	18.13453131
MAPE (-)	14.43813353
MSE ($M)	501.533821

Holt's method, detailed calculations are in "Question1.b-sheet2" and "Question1.b-sheet3." Note that linear regression was performed to obtain L_0 and T_0, with its result in "Question1.b-sheet3."

Year	Quarter	Period	Demand ($K)	Level ($K)	Trend ($K)	Estimated Demand ($K)	Errors ($K)
		0		90.675	4.479411765		
1	I	1	80	93.63897059	4.327867647	95.15441176	15.15441176
	II	2	96	97.77015441	4.308199265	97.96683824	1.966838235
	III	3	103	102.1705183	4.317415728	102.0783537	-0.921646324
	IV	4	123	108.1391406	4.482536388	106.487934	-16.51206596
2	I	5	120	113.3595093	4.556319617	112.621677	-7.378322979
	II	6	109	117.024246	4.467161328	117.9158289	8.915828936
	III	7	145	123.8422666	4.702247254	121.4914074	-23.50859263
	IV	8	120	127.6900625	4.616802115	128.5445139	8.544513888
3	I	9	141	133.1761782	4.703733469	132.3068646	-8.693135386
	II	10	128	136.8919205	4.604934353	137.8799116	9.879911622
	III	11	143	141.6471693	4.619965805	141.4968548	-1.503145187
	IV	12	139	145.5404216	4.547294454	146.2671351	7.267135137
4	I	13	142	149.2789445	4.466417293	150.0877161	8.087716077
	II	14	140	152.3708256	4.328963675	153.7453618	13.74536176
	III	15	161	157.1298103	4.371965783	156.6997893	-4.300210739
	IV	16	170	162.3515985	4.456948021	161.5017761	-8.498223883
5	I	17	-			166.8085465	
	II	18	-			171.2654945	
	III	19	-			175.7224426	
	IV	20	-			180.1793906	

Metrics	
TS (-)	0.24808613
MAD ($K)	9.054816282
MAPE (-)	7.284651333
MSE ($M)	114.9955251

All the metrics of Holt's model are smaller than exponential smoothing's, implying that the Holt's model fits better than the other. Therefore, the initial guess I had seems correct.

Question 2.

a.

b.