IE426: Supply Chain Management **Lecturer:** Prof. Woonam Hwang
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Question 1

(a)

The economic order quantity (EOQ) can be obtained as follows:

$$Q^* = \sqrt{\frac{2DS}{hC}} = \sqrt{\frac{2\times(300\times365)\times1500}{0.2\times450}} = 1,910 \text{ motorcycles} \tag{1}$$

Following this result, the cycle inventory is $Q^*/2 = 955$ motorcycles.

(b)

With the baseline setting, the total annual cost is as follows:

$$\text{Total Annual Cost} = \frac{D}{Q^*}S + \frac{Q^*}{2}hC + CD = \frac{(300\times365)}{1910}\times1500 + 955\times0.2\times450 + 450\times(300\times365) = \$49,446,945 \tag{2}$$

Since 1,910 engines are transported per order, 20 trucks are required for each order, implying 20 times the transportation cost. As a result, the total cost becomes:

$$\text{Total Annual Cost} = 20\times\frac{(300\times365)}{1910}\times1500 + 955\times0.2\times450 + 450\times(300\times365) = \$51,080,845 \tag{3}$$

Even though the transportation cost increases significantly compared to the baseline, the annual cost does not change much (only 3%) because transportation cost constitutes only a small portion of the total cost.

To make $Q^* = 100$, S should be:

$$S = \frac{(Q^*)^2hC}{2D} = \frac{100^2\times0.2\times450}{2\times(300\times365)} = \$4.1. \tag{4}$$

(c)

For the Ford, the EOQ (Q_F^*) is as follows:

$$Q_F^* = \sqrt{\frac{2D_FS_F}{h_FC_F}} = \sqrt{\frac{2\times(100\times12)\times500}{0.2\times100}} = 245 \text{ parts} \tag{5}$$

Following this, the optimal order frequency (n_F^*)is as follows:

$$n_F^* = \frac{D_F}{Q_F^*} = \frac{100\times12}{245} = 4.90/\text{year} \tag{6}$$

As a result, the annual ordering and holding cost of Ford can be obtained as follows:

$$\text{Ford's Annual Ordering Cost} = n_F^*\times S_F = 4.90\times500 = \$2,450 \tag{7}$$

$$\text{Ford's Annual Holding Cost} = \frac{Q_F^*}{2} \times h_F C_F = \frac{245}{2} \times 0.2 \times 100 = \$2,450 \tag{8}$$

Similarly, we can obtain the EOQ (Q_G^*), the optimal order frequency (n_G^*), the annual ordering cost, and the annual holding cost of GM as follows:

$$Q_G^* = \sqrt{\frac{2D_G S_G}{h_G C_G}} = \sqrt{\frac{2 \times (120 \times 12) \times 500}{0.2 \times 100}} = 268 \text{ parts} \tag{9}$$

$$n_G^* = \frac{D_G}{Q_G^*} = \frac{120 \times 12}{268} = 5.37/\text{year} \tag{10}$$

$$\text{GM's Annual Ordering Cost} = n_G^* \times S_G = 5.37 \times 500 = \$2,685 \tag{11}$$

$$\text{GM's Annual Holding Cost} = \frac{Q_G^*}{2} \times h_G C_G = \frac{268}{2} \times 0.2 \times 100 = \$2,680 \tag{12}$$

With the joint order strategy, the optimal order frequency (n_j^*) can be obtained as follows:

$$n_j^* = \sqrt{\frac{D_F h_F C_F + D_G h_G C_G}{2S_j}} = \sqrt{\frac{(100 \times 12) \times 0.2 \times 100 + (120 \times 12) \times 0.2 \times 100}{2 \times 600}} = 6.63/\text{year} \tag{13}$$

Following this, the order quantity of each company becomes as follows:

$$Q_{F,j}^* = \frac{D_F}{n_j^*} = \frac{100 \times 12}{6.63} = 181 \text{ parts} \tag{14}$$

$$Q_{G,j}^* = \frac{D_G}{n_j^*} = \frac{120 \times 12}{6.63} = 217 \text{ parts} \tag{15}$$

With this ordering policy, the annual holding cost of each company is as follows:

$$\text{Ford's Annual Holding Cost with Joint Ordering} = \frac{Q_{F,j}^*}{2} h_F C_F = \frac{181}{2} \times 0.2 \times 100 = \$1,810 \tag{16}$$

$$\text{GM's Annual Holding Cost with Joint Ordering} = \frac{Q_{G,j}^*}{2} h_G C_G = \frac{217}{2} \times 0.2 \times 100 = \$2,170 \tag{17}$$

Ford's and GM's annual holding costs decrease compared to the independent ordering strategy. In addition, the joint ordering cost is as follows:

$$\text{Annual Joint Ordering Cost} = n_j^* \times S_j = 6.63 \times 600 = \$3,978 \tag{18}$$

This value is less than the sum of the ordering costs with the independent ordering strategy, \$5,135.

In the independent ordering strategy, the sum of Ford's annual ordering and holding costs is \$4,900. Thus, Ford will be satisfied if they pay a fraction of the ordering cost less than (4,900-1,810)/5,135=60.2%. Similarly, GM will be satisfied if they pay a fraction of the ordering cost less than (5,365-2,170)/5,135=62.2%. Within this range, depending on the bargaining power of these two companies, a mutually beneficial deal should be negotiated.

(d)

Let's assume that $Q^* \leq 30,000$. Then, the EOQ can be obtained as follows:

$$Q^* = \sqrt{\frac{2DS}{hC}} = \sqrt{\frac{2 \times (20000 \times 12) \times 400}{0.2 \times 5}} = 13,856 \tag{19}$$

Since this solution satisfies the assumption, it can be a candidate for the optimal solution.

Next, assume that $Q^* > 30,000$. Then, the EOQ can be obtained as follows:

$$Q^* = \sqrt{\frac{2DS}{hC}} = \sqrt{\frac{2 \times (20000 \times 12) \times 400}{0.2 \times 4.5}} = 14,605 \tag{20}$$

Since this solution violates the assumption, it cannot be a candidate for the optimal solution. Therefore, Grainger should order 13,856 boxes per order. From the supplier's perspective, they should offer a larger discount to encourage sales exceeding 30,000.

Question 2

(a)

The safety stock (ss) can be obtained by the following relation:

$$D \times L + ss = ROP \tag{21}$$

Here, $L = 2$ weeks, $D = 170$/week, and $ROP = 400$, thus $ss = 60$. Here, CSL is obtained as follows:

$$CSL = P[D_L \leq 400] = F\left(400, D \times L, \sigma_D \times \sqrt{L}\right) = F\left(400, 340, 85\sqrt{2}\right) = 0.69 \tag{22}$$

Here, $F(x, \mu, \sigma)$ is the CDF of the normal distribution with a mean of μ and a standard deviation of σ.

To make CSL be 0.93, ss should be satisfies the following relation:

$$CSL = P\left[340 + ss, 340, 85\sqrt{2}\right] = 0.93 \tag{23}$$

This means

$$ss = \Phi^{-1}(0.93) \times 85\sqrt{2} = 177 \tag{24}$$

Following this, the ROP should be 517 $(= D + ss = 340 + 177)$

(b)

To maintain CSL to be 95%, the ss of each store should be

$$ss = \Phi^{-1}(0.95) \times 600 = 987 \tag{25}$$

As a result, the total safety stock of 25 stores is 24,675 $(= 987 \times 25)$.

When one online channel responds to all the demands, the mean demand during the lead time is 30,000$(= 1200 \times 25)$. If the demand across stores is independent, the standard deviation of the aggregate demand is 3,000 $(= \sqrt{25} \times 600)$, thus ss becomes

$$ss = F^{-1}(0.95) \times 3000 = 4,935 \tag{26}$$

In this case, Croma can save 19,740 $(= 24675 - 4935)$ in safety inventory.

On the other hand, if the demand across stores has a correlation coefficient of $\rho = 0.5$, the standard deviation of the aggregate demand is 10,817$(= \sqrt{600^2 \times 25 + 600^2 \times 25 \times 24 \times 0.5})$, thus ss becomes

$$ss = F^{-1}(0.95) \times 10817 = 17,792 \tag{27}$$

In this case, Croma can save 6,883 $(= 24675 - 17792)$ in safety inventory.

(c)

With the periodic review, the CSL can be obtained by the following relation.

$$CSL = P\left[D_{L+T} \leq OUL\right] = F\left(OUL, D \times (L+T), \sigma_D \times \sqrt{L+T}\right) = F\left(OUL, 12000, 800\sqrt{3}\right) = 0.95 \tag{28}$$

Here, L and T are the lead time and the review period, respectively. As a result, the order-up-to level (OUL) can be obtained as follows:

$$OUL = F^{-1}\left(0.95, 12000, 800\sqrt{3}\right) = 14,279 \tag{29}$$

Then, the average order size is 8000 ($= D \times T$), and the safety stock (ss) is 2,279 ($= OUL - D \times (L+T)$)

If Liverpool switched to a continuous review, the required safety inventory would be

$$ss = \Phi^{-1}(0.95) \times \left(\sigma_D \sqrt{L}\right) = \Phi^{-1}(0.95) \times 800 = 1,316 \tag{30}$$

Therefore, Liverpool could save the safety inventory by 963 ($= 2279 - 1316$).

Question 3

(a)

The profit margin per unit is the difference between the unit price and the unit cost, $p - c = \$50$, and the cost of leftover inventory per unit is the difference between the unit cost and the unit salvage value plus unit holding cost, $c - s + h = \$220$. The optimal customer service level (CSL^*) that maximizes the profit satisfying the following condition:

$$(1 - CSL^*)(p - c) - CSL^*(c - s + h) = 0 \tag{31}$$

Therefore,

$$CSL^* = \frac{p - c}{p - s + h} = \frac{50}{270} = 0.19 \tag{32}$$

With this CSL^* value, the optimal order quantity (O^*) satisfies the following relation:

$$CSL^* = P[D \leq O^*] = F(O^*, 100, 40) = 0.19 \tag{33}$$

Here, D follows the normal distribution with a mean of 100 and a standard deviation of 40. We can obtain O^* as follows.

$$O^* = F^{-1}(0.19, 100, 40) = 65 \tag{34}$$

From this policy, the expected profit is as follows.

$$P(O^*)$$

$$= \int_{-\infty}^{O^*} \left((p-c)x - (c-s+h)(O^* - x)\right) f(x, 100, 40)\, dx + \int_{O^*}^{\infty} (p-c)\, O^* f(x, 100, 40)\, dx \tag{35}$$

$$= \int_{-\infty}^{O^*} (p-s+h)\, x f(x, 100, 40)\, dx - \int_{-\infty}^{O^*} (c-s+h)\, O^* f(x, 100, 40)\, dx + \int_{O^*}^{\infty} (p-c)\, O^* f(x, 100, 40)\, dx \tag{36}$$

$$\underset{z=\frac{x-100}{40}}{=} \int_{-\infty}^{\frac{O^*-100}{40}} (p-s+h)(40z+100)\,\phi(z)\, dz - (c-s+h)\, O^* F(O^*, 100, 40) + (p-c)\left(1 - F(O^*, 100, 40)\right) \tag{37}$$

$$= (p-s+h)\left(100\Phi\left(\frac{O^*-100}{40}\right) - 40\phi\left(\frac{O^*-100}{40}\right)\right) - (c-s+h)\, O^* \Phi\left(\frac{O^*-100}{40}\right) + (p-c)\, O^* \Phi\left(\frac{100-O^*}{40}\right) \tag{38}$$

$$= \$2,114 \tag{39}$$

Here, $CSL^* = 0.19$ means 81 out of 100 customers turn away because of stocking out. This low service level is attributed to the low profit from selling a unit compared to the loss by overstocking.

(b)

In the current production facility, the optimal CSL for Reguplo (CSL_R^*) and the other products (CSL_O^*) are as follows:

$$CSL_R^* = \frac{200-100}{200-80} = 0.83 \tag{40}$$

$$CSL_O^* = \frac{220-110}{220-80} = 0.79 \tag{41}$$

Following this, The optimal manufacturing quantities of Reguplo (Q_R^*) and the other products (Q_O^*) are determined as follows:

$$Q_R^* = F^{-1}(0.83, 10000, 1000) = 10,954 \tag{42}$$

$$Q_O^* = F^{-1}(0.79, 1000, 700) = 1,564 \tag{43}$$

Based on these calculations, the expected profits by selling Reguplo (P_R^*) and other products (P_O^*) are obtained as follows:

$$\begin{aligned} P_R^* = {} & (200-80) \times 10000 \times \Phi\left(\frac{10954-10000}{1000}\right) - (200-80) \times 1000 \times \phi\left(\frac{10954-10000}{1000}\right) \\ & - 10954 \times (100-80) \times F(10954, 10000, 1000) + 10954 \times (200-100) \times (1 - F(10954, 10000, 1000)) && (44) \\ = {} & \$970,015 && (45) \end{aligned}$$

$$\begin{aligned} P_O^* = {} & (220-80) \times 1000 \times \Phi\left(\frac{1564-1000}{700}\right) - (220-80) \times 700 \times \phi\left(\frac{1564-1000}{700}\right) \\ & - 1564 \times (110-80) \times F(1564, 1000, 700) + 1564 \times (220-110) \times (1 - F(1564, 1000, 700)) && (46) \\ = {} & \$81,418 && (47) \end{aligned}$$

As a result, the total profit is $P_R^* + 3P_O^* = \$1,214,269$.

The impact analysis of introducing tailored production can be performed similarly.

$$CSL_R^* = \frac{200-90}{200-80} = 0.92 \tag{48}$$

$$CSL_O^* = \frac{220-120}{220-80} = 0.71 \tag{49}$$

$$Q_R^* = F^{-1}(0.92, 10000, 1000) = 11,405 \tag{50}$$

$$Q_O^* = F^{-1}(0.71, 1000, 700) = 1,387 \tag{51}$$

$$\begin{aligned} P_R^* = {} & (200-80) \times 10000 \times \Phi\left(\frac{11405-10000}{1000}\right) - (200-80) \times 1000 \times \phi\left(\frac{11405-10000}{1000}\right) \\ & - 11405 \times (90-80) \times F(11405, 10000, 1000) + 11405 \times (200-90) \times (1 - F(11405, 10000, 1000)) && (52) \\ = {} & \$1,081,598 && (53) \end{aligned}$$

$$\begin{aligned} P_O^* = {} & (220-80) \times 1000 \times \Phi\left(\frac{1387-1000}{700}\right) - (220-80) \times 700 \times \phi\left(\frac{1387-1000}{700}\right) \\ & - 1387 \times (120-80) \times F(1387, 1000, 700) + 1387 \times (220-120) \times (1 - F(1387, 1000, 700)) && (54) \\ = {} & \$66,686 && (55) \end{aligned}$$

As a result, the expected profit increases to $P_R^* + 3P_O^* = \$1,281,656$.

This result is attributed to the fact that the increase in profit from selling Reguplo outweighs the decrease in profit from the other three products. Since the overall expected profit increases, I recommend implementing a tailored production strategy.

(c)

The optimal customer service level (CSL^*) is obtained as follows:

$$CSL^* = \frac{p-c}{p-s+h} = \frac{125-80}{125-70+10} = 0.69 \tag{56}$$

Then, the optimal order quantity (Q^*) is as follows:

$$Q^* = F^{-1}(0.69, 4000, 1750) = 4,868 \tag{57}$$

With this order quantity, the expected profit (P^*) is as follows:

$$P^* = (125-60) \times 4000 \times \Phi\left(\frac{4868-4000}{1750}\right) - (125-60) \times 1750 \times \phi\left(\frac{4868-4000}{1750}\right)$$

$$-4868 \times (80-60) \times F(4868, 4000, 1750) + 4868 \times (125-80) \times (1 - F(4868, 4000, 1750)) \tag{58}$$

$$= \$140,000 \tag{59}$$

Also, the expected overstock (S_o^*) is as follows:

$$S_o^* = (4868-4000) \times \Phi\left(\frac{4868-4000}{1750}\right) + 1750 \times \phi\left(\frac{4868-4000}{1750}\right) = 1,216 \tag{60}$$

Sending surplus jackets to the Southern Hemisphere increases the salvage cost. This change increases the ordering quantity, expected profit, and expected overstock.