4.1. Quasi Likelihood Approach

In GLM, $g(\mu_i) = \sum_j \beta_j x_{ij}$ and likelihood equations are

$$\sum_i \frac{(y_i - \mu_i)x_{ij}}{\mathrm{Var}(Y_i)} \frac{d\mu_i}{d\eta_i} = 0, \quad j = 0, 1, \ldots, p.$$

Let the score function β be

$$S = (S_0(\beta), S_1, (\beta), \ldots, S_p(\beta)).$$

Note that ML estimates depend on the distribution of Y_i only through μ_i and $\mathrm{Var}(Y_i) = V(\mu_i)$.

Remark(Quasi likelihood approach)

1. Use model $g(\mu_i) = \sum_j \beta_j x_{ij}$ and variance function $V(\mu_i)$ but do not assume distribution for Y_i.
2. Use estimating equations $S(\beta) = 0$ even if they do not correspont to likelihood equations for distribution in exponential family.
3. To allow overdispersion, take $V(\mu_i) = \phi V^*(\mu_i)$, where $V^*(\mu_i)$ is variance function for common model such as $V^*(\mu_i) = \mu_i$ for count data.

Definition

A function $h(Y, \beta)$ is an unbiased *estimating function* if

$$E[h(Y; \beta)] = 0 \;\text{ for all } \beta.$$

$S_j(\beta)$ is unbiased estimating function and $S(\beta) = 0$ are estimating equations.

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