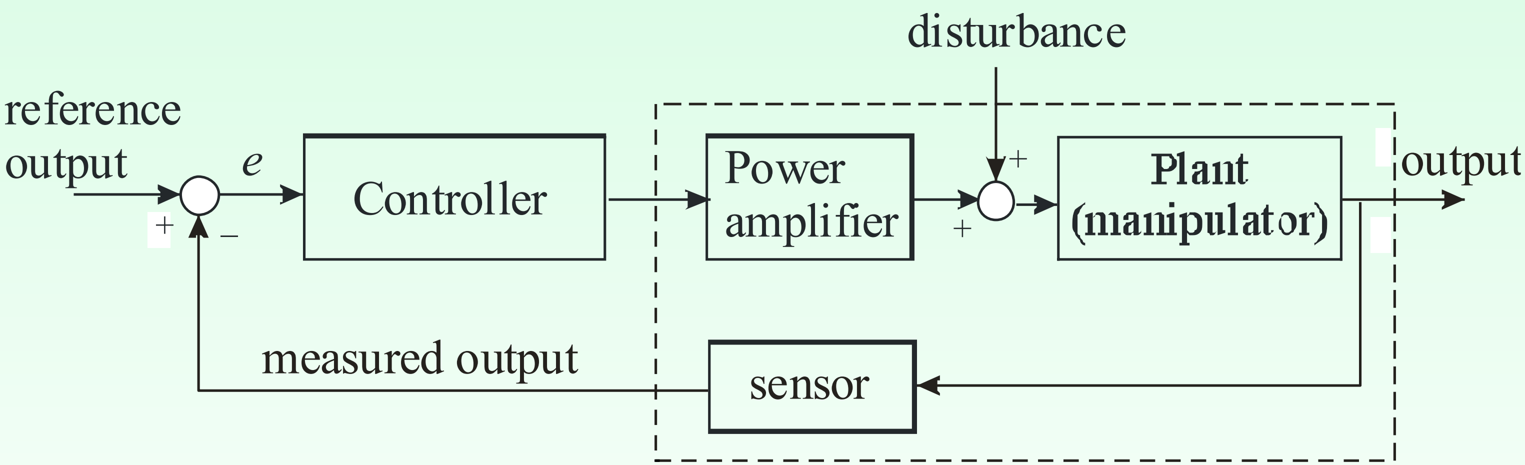
EMARO

Modeling and Control of Manipulators
Part II:
Control of Manipulators, *by P. Tatjewski*

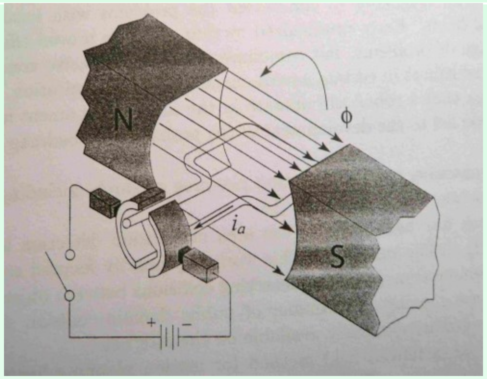
Single-Link Manipulator Control

Basic Feedback Control Structure



e - control error (reference output − measured output)

DC Motor Modeling (a reminder)



Permanent magnet DC motor

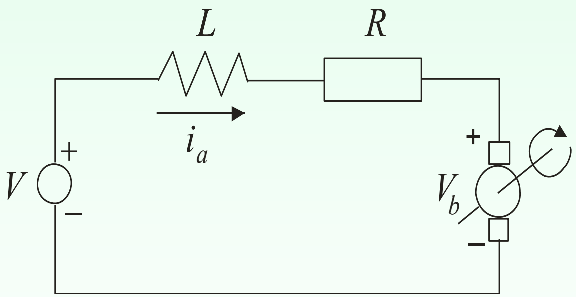
$$\tau_m = K_M \varphi i_a = K_m i_a$$

τ_m – generated torque

ϕ – magnetic flux

i_a – armature current

K_m – torque constant ([Nm/A])



$$L\frac{di_a}{dt} + R i_a = V - V_b$$

$$V_b = K_E \varphi \overset{\theta_m}{\omega_m} = K_b \frac{d\theta_m}{dt}$$

$$\Rightarrow \quad \boxed{L\frac{di_a}{dt} + R i_a = V - K_b \frac{d\theta_m}{dt}}$$

V – armature voltage

L – armature inductance

R – armature resistance

V_b – back emf

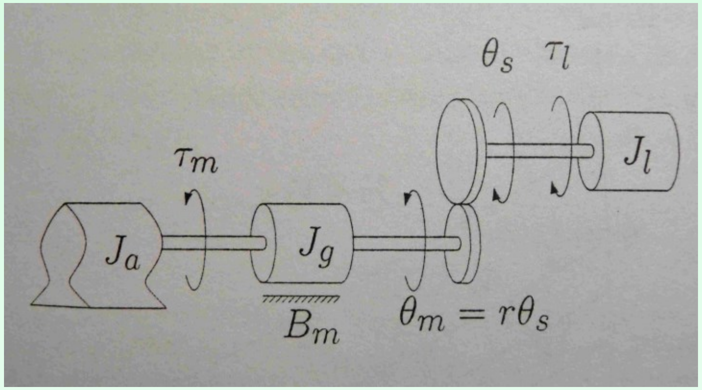
θ_m – rotor position (angle)

$\omega_m = d\theta_m / dt$ – rotor velocity

K_b – back emf constant ([V/(rad/s)], [V/rpm])

In ideal case: K_b [V/(rad/s)] = K_m [Nm/A]

Single Link with Actuator-Gear Train (a reminder)



$$(J_a + J_g)\frac{d^2\theta_m}{dt^2} + B_m\frac{d\theta_m}{dt} = \tau_m - \frac{\tau_l}{r} = K_m i_a - \frac{\tau_l}{r}$$

θ_s – arm position (angle [rd])

τ_l – load torque

r – gear ratio

Further: $J_m = J_a + J_g$

$$L\frac{di_a}{dt} + R i_a = V - K_b\frac{d\theta_m}{dt}$$

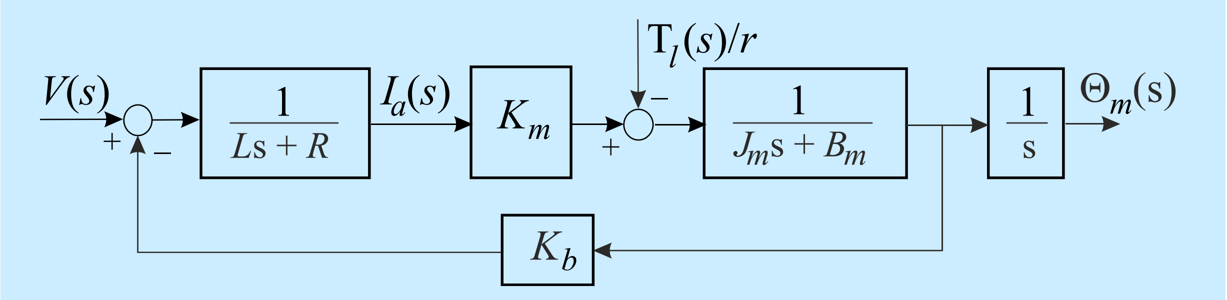
$$(Ls+R)I_a(s) = V(s) - K_b s\Theta_m(s)$$

$$I_a(s) = \frac{V(s)}{(Ls+R)} - \frac{K_b s}{(Ls+R)}\Theta_m(s)$$

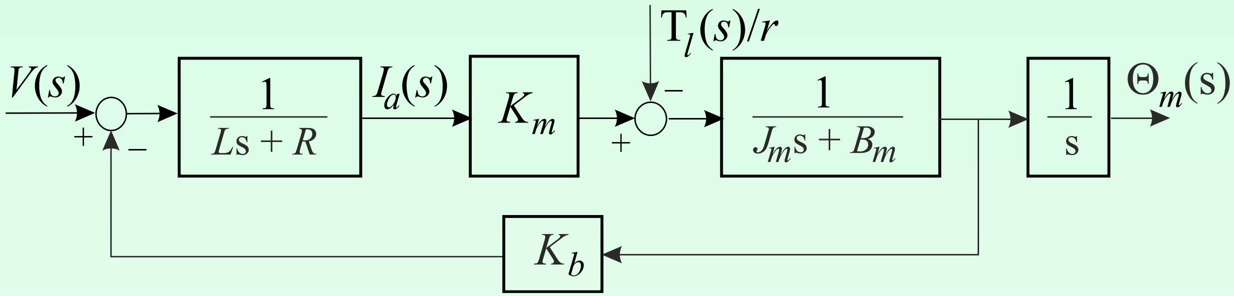
$$J_m\frac{d^2\theta_m}{dt^2} + B_m\frac{d\theta_m}{dt} = K_m i_a - \frac{\tau_l}{r}$$

$$(J_m s^2 + B_m s)\Theta_m(s) = K_m I_a(s) - \frac{\mathrm{T}_l(s)}{r}$$

$$\Theta_m(s) = \frac{1}{s(J_m s + B_m)}[K_m I_a(s) - \frac{\mathrm{T}_l(s)}{r}]$$



Single-Link Manipulator Modeling



$$\Theta_m(s) = \frac{1}{s(J_m s + B_m)}\left[K_m \frac{V(s)}{(Ls+R)} - \frac{K_m K_b s}{(Ls+R)}\Theta_m(s) - \mathrm{T}_{lm}(s)\right], \qquad \mathrm{T}_{lm}(s) = \frac{\mathrm{T}_l(s)}{r}$$

$$\Theta_m(s)\left(1 + \frac{K_m K_b s}{s(J_m s + B_m)(Ls+R)}\right) = \frac{1}{s(J_m s + B_m)}\left[K_m \frac{V(s)}{(Ls+R)} - \mathrm{T}_{lm}(s)\right]$$

$$\Theta_m(s)\left[s(J_m s + B_m)(Ls+R) + K_m K_b s\right] = K_m V(s) - (Ls+R)\mathrm{T}_{lm}(s)$$

$$\Theta_m(s) = \frac{K_m}{s(J_m s + B_m)(Ls+R) + K_m K_b s} V(s) - \frac{(Ls+R)}{s(J_m s + B_m)(Ls+R) + K_m K_b s}\mathrm{T}_{lm}(s)$$

Single-Link Manipulator Modeling

$$\Theta_m(s) = \frac{K_m}{s(J_m s + B_m)(Ls + R) + K_m K_b s} V(s) - \frac{(Ls + R)}{s(J_m s + B_m)(Ls + R) + K_m K_b s} \mathrm{T}_{lm}(s)$$

Dividing numerators and denominators of the fractions by RB_m we get:

$$\Theta_m(s) = \frac{\dfrac{1}{B_m}}{s\left[(\dfrac{J_m}{B_m}s+1)(\dfrac{L}{R}s+1)+\dfrac{K_m K_b}{RB_m}\right]} \frac{K_m}{R} V(s) \;-\; \frac{(\dfrac{L}{R}s+1)\dfrac{1}{B_m}}{s\left[(\dfrac{J_m}{B_m}s+1)(\dfrac{L}{R}s+1)+\dfrac{K_m K_b}{RB_m}\right]} \mathrm{T}_{lm}(s)$$

In most cases $\dfrac{L}{R} << \dfrac{J_m}{B_m}$, then neglecting the electrical time constant $\dfrac{L}{R}$:

$$\Theta_m(s) = \frac{\dfrac{1}{B_m}}{s\left[(\dfrac{J_m}{B_m}s+1)+\dfrac{K_m K_b}{RB_m}\right]} \frac{K_m}{R} V(s) \;-\; \frac{\dfrac{1}{B_m}}{s\left[(\dfrac{J_m}{B_m}s+1)+\dfrac{K_m K_b}{RB_m}\right]} \mathrm{T}_{lm}(s)$$

Single-Link Manipulator - Simplified Model

Multiplying numerators and denominators of the fractions by B_m we get:

$$\Theta_m(s) = \frac{1}{s\left(J_m s + B_m + \frac{K_m K_b}{R}\right)}\left(\frac{K_m}{R}V(s) - \mathrm{T}_{lm}(s)\right)$$

Denoting $\boxed{B = B_m + \frac{K_b K_m}{R}}$ - effective damping

we get the model in simplified form:

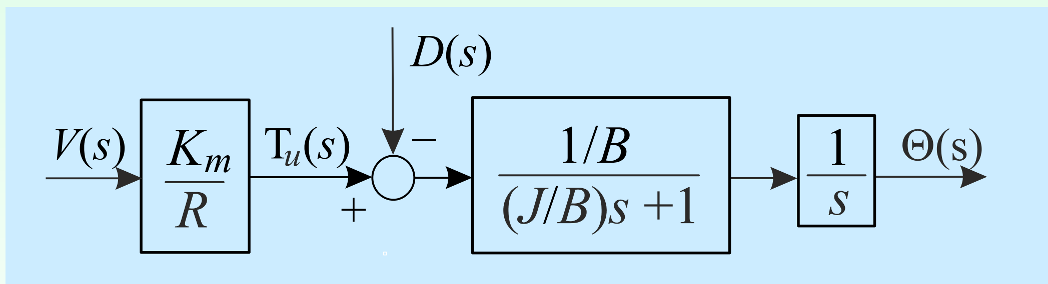
$$\Theta_m(s) = \frac{1}{s\left(J_m s + B\right)}\left(\frac{K_m}{R}V(s) - \mathrm{T}_{lm}(s)\right)$$

$$\left(\begin{array}{c} \left(J_m s^2 + Bs\right)\Theta_m(s) = \frac{K_m}{R}V(s) - \mathrm{T}_{lm}(s) \\ J_m \frac{d^2\Theta_m(t)}{dt^2} + B\frac{d\Theta_m(t)}{dt} = \frac{K_m}{R}V(t) - \tau_{lm}(t) \end{array}\right)$$

Single-Link Manipulator - Simplified Model

The equivalent final form:

$$\Theta_m(s) = \frac{1}{s}\cdot\frac{\dfrac{1}{B}}{\dfrac{J_m}{B}s+1}\left(\frac{K_m}{R}V(s) - \mathrm{T}_{lm}(s)\right)$$



where:

$$\tau_u(t) = \frac{K_m}{R}V(t) \;-\; \text{torque control input}$$

$$V(t) = u(t) \;-\; \text{voltage (motor) control input}$$

$$d(t) = \tau_{lm}(t) = \tau_l(t)/r \;-\; \text{load torque w.r.t. motor axis, a } \textbf{disturbance}$$

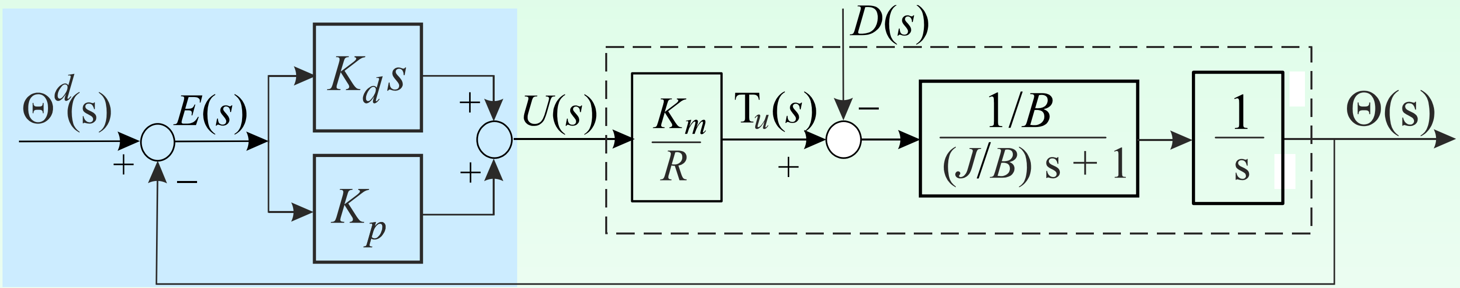
$$(\; J \equiv J_m, \quad \Theta \equiv \Theta_m, \quad \text{to simplify further notation})$$

PD Control

PD controller (ideal, parallel):

$$u(t) = K_p\left(e(t) + T_d\frac{de(t)}{dt}\right) = K_p e(t) + K_d\frac{de(t)}{dt}, \qquad e(t) = \theta^d(t) - \theta(t)$$

$$(K_d = K_p T_d)$$



Transfer function:

$$\Theta(s) = \frac{1/B}{(J/B)s^2 + s}\left[\frac{K_m}{R}\left(K_p + K_d s\right)\left(\Theta^d(s) - \Theta(s)\right) - D(s)\right]$$

$$\Theta(s)\left[1 + \frac{K_m}{RB}\cdot\frac{(K_p + K_d s)}{(J/B)s^2 + s}\right] = \frac{1/B}{(J/B)s^2 + s}\left[\frac{K_m}{R}\left(K_p + K_d s\right)\Theta^d(s) - D(s)\right]$$

$$\Theta(s)\left[Js^2 + Bs + \frac{K_m}{R}(K_p + K_d s)\right] = \left[\frac{K_m}{R}\left(K_p + K_d s\right)\Theta^d(s) - D(s)\right]$$

PD Control

$$\Theta(s) = \frac{1}{Js^2 + (B + K_d K_m / R)s + K_p K_m / R}\left(\frac{(K_p + K_d s)K_m}{R}\Theta^d(s) - D(s)\right)$$

$$E(s) = \Theta^d(s) - \Theta(s) =$$

$$= \frac{Js^2 + Bs}{Js^2 + (B + K_d K_m / R)s + K_p K_m / R}\Theta^d(s) + \frac{1}{Js^2 + (B + K_d K_m / R)s + K_p K_m / R}D(s)$$

Step inputs of set-point and disturbance: $\Theta^d(s) = \frac{a}{s}, \quad D(s) = \frac{b}{s}$

The error for step inputs:

$$E(s) = \frac{Js^2 + Bs}{Js^2 + (B + K_d K_m / R)s + K_p K_m / R}\cdot\frac{a}{s} + \frac{1}{Js^2 + (B + K_d K_m / R)s + K_p K_m / R}\cdot\frac{b}{s}$$

Therefore, the steady-state error:

$$\boxed{e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) = \frac{R}{K_p K_m} b}$$

PD Control – Parameter Tuning

$$\Theta(s) = \frac{1}{Js^2 + (B + K_d K_m / R)s + K_p K_m / R}\left(\frac{(K_p + K_d s)K_m}{R}\Theta^d(s) - D(s)\right)$$

$$= \frac{1}{s^2 + \frac{B + K_d K_m / R}{J}s + \frac{K_p K_m}{JR}}\left(\frac{(K_p + K_d s)K_m}{JR}\Theta^d(s) - \frac{1}{J}D(s)\right)$$

Closed-loop characterstic polynomial:

$$\Omega(s) = s^2 + \frac{B + K_d K_m / R}{J}s + \frac{K_p K_m}{JR} = s^2 + 2\zeta\omega s + \omega^2$$

Equating the coefficients in the right- and left-hand side polynomials, we get simple tuning rules by choosing values of ζ and ω:

$$K_p = \frac{R}{K_m}J\omega^2, \quad K_d = \frac{R}{K_m}(2J\zeta\omega - B)$$

PD Control – Parameter Tuning

$$K_p = \frac{R}{K_m} J\omega^2, \quad K_d = \frac{R}{K_m}(2J\zeta\omega - B)$$

Manipulator dynamics should usually be possibly fast, but often without overshoots. Therefore, **controller tuning**:

- start with $\zeta=1$, later adjusting ζ if needed,
- choose ω to determine speed of response, according to specifications: dynamic (errors in trajectory tracking), static: steady-state error due to disturbances.

The values ω and ζ uniquely define controller gains K_p and K_d.

High gains decrease tracking errors, high K_p decreases steady-state error, but increasing gains increases amplitudes of the control signal and decreases robustness of the control system.

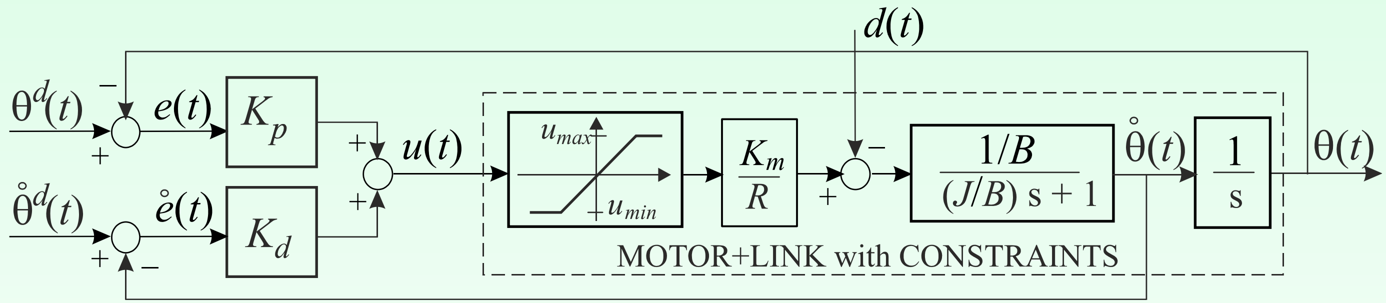
Upper limit for ω: a safe distance from structural resonance frequency ω_r (unmodeled dynamics), $\omega/r \le \omega_r \cdot \alpha$, (e.g., $\alpha = 0.1$).

Remember: ω_r [rad/s] $= 2\pi f_r$, f_r [Hz].

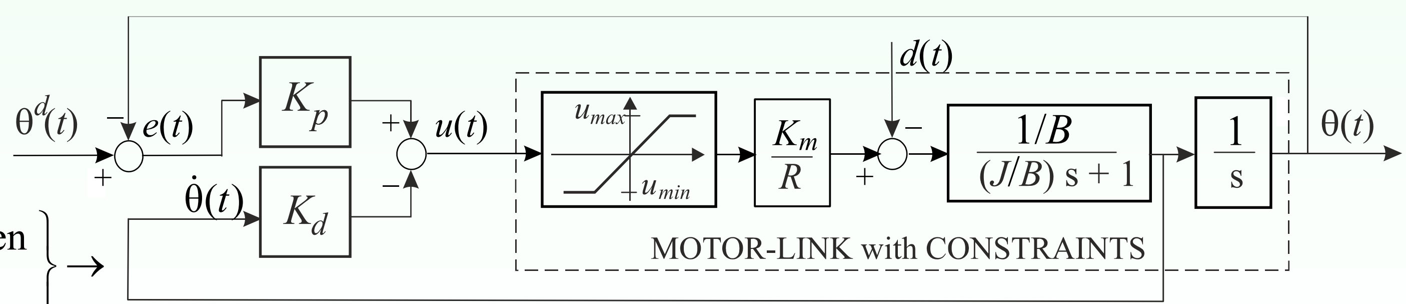
PD and P-D Control – Implementation Issues

Measurement of velocity is usually available in robotic applications. Then, the D action can be implemented as a derivative gain K_d :

a) **Standard PD structure**, for smooth trajectory tracking $(\dot{\theta}^d(t)$ available):



b) **P-D (modified) structure**, for piecewise constant $\theta^d(t)$ (position control):



$$\left.\begin{array}{r}\dot{\theta}(t) = -\dot{e}(t)\ \text{ when}\\ \theta^d(t) = \text{const.}\end{array}\right\} \rightarrow$$

PD (P-D) Control – Implementation Issues

P-D structure prevents from derivative kik-off after jumps in the reference (set-point) position (there is theoretically unbounded derivative at points of jumps)

Comments:

- when derivative of the controlled variable is not available, derivative action based on output measurement must be implemented,
- if output measurement is corrupted by high frequency noise, filtered derivative (limited bandwidth derivative) is recommended, a standard solution in process control applications:

$$K_d s = K_p T_d s \;\; \rightarrow \;\; K_p \frac{T_d s}{\frac{T_d}{\alpha} s + 1} \quad (\alpha \geq 5 \div 10,\;\; \alpha = 10 \;\; \text{is a good choice})$$

PD control suitable when high gear ratio r and high controller proportional gain decrease to acceptable values the steady-state position error, resulting from the influence of disturbances (including unmodeled dynamics), as we have

$$e_{ss} = \lim_{t \to \infty} e(t) = -\frac{R}{K_m K_p} \frac{\tau_l}{r}$$

If it is not the case, the PD controller should be augmented by **an integral action**.

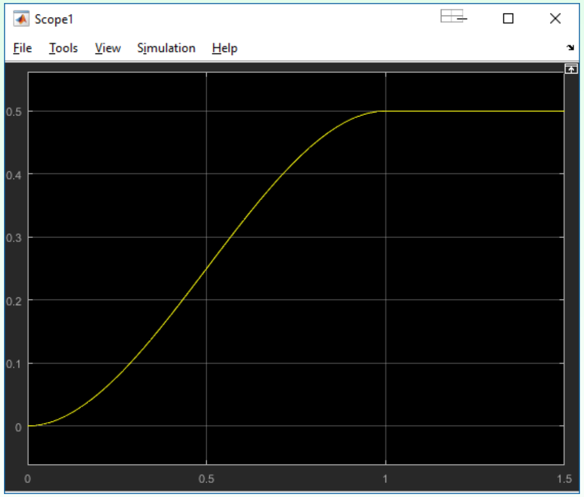
Example – PD Control

The data for a single link manipulator arm (revolute joint):

$$J = 8 \cdot 10^{-4}[\mathrm{kg \cdot m^2}],\quad K_b = 0.2[\mathrm{V/(rad/s)}],\quad K_m = 0.2[\mathrm{N \cdot m/A}],$$

$$R = 1[\Omega],\quad L = 0.001[H],\quad B_m = 2 \cdot 10^{-3}[\mathrm{N \cdot m/(rad/s)}],\quad r = 120,\quad V_{\max} = 35[V]$$

Design PD controller under the requirement: dynamic tracking error less then 0.01, for cubic reference trajectory from $\theta(0) = 0,\ \dot{\theta}(0) = 0$, to $\theta(1) = 0.5,\ \dot{\theta}(1) = 0$.

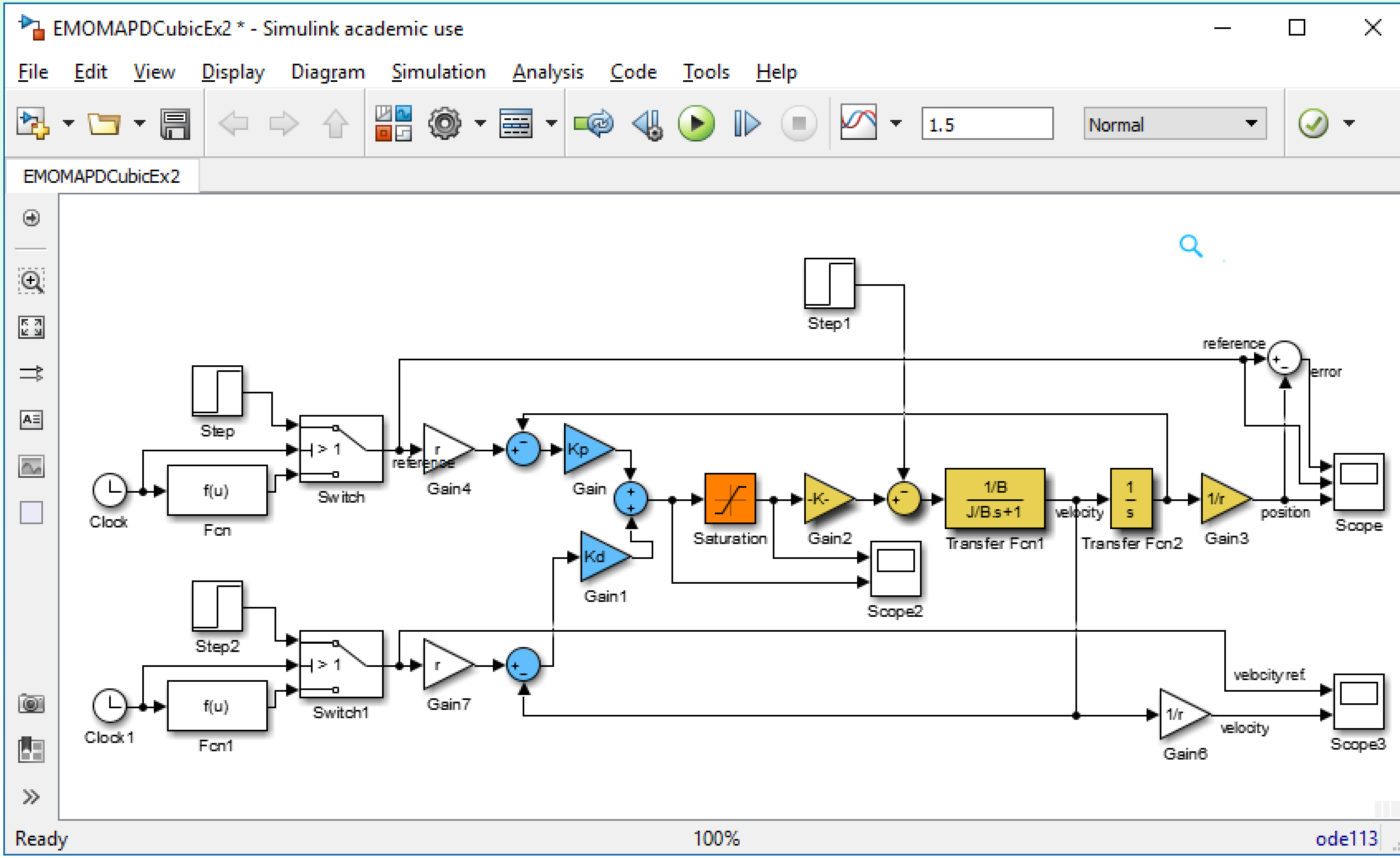


Reference trajectory is for arm position θ not for the motor shaft position θ_m, $\theta_m = r\theta$.

The effective damping coefficient:

$$B = B_m + K_b K_m / R = 0.002[\mathrm{Nms}] + 0.04[\mathrm{VsNm/(A\Omega)}] = 0.042[\mathrm{Nms}]$$

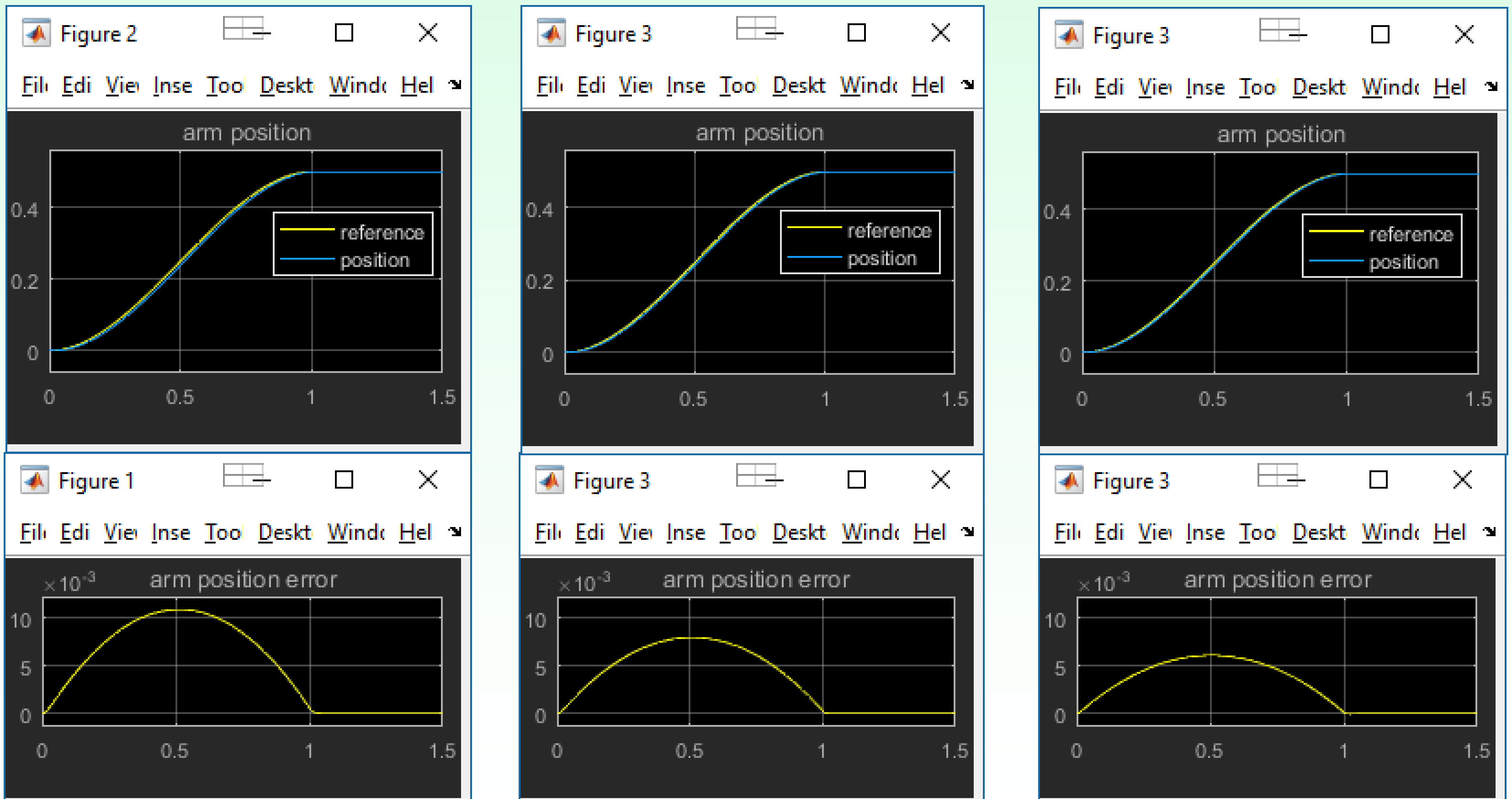
Example – PD Control (2)



Example – PD Control (3)

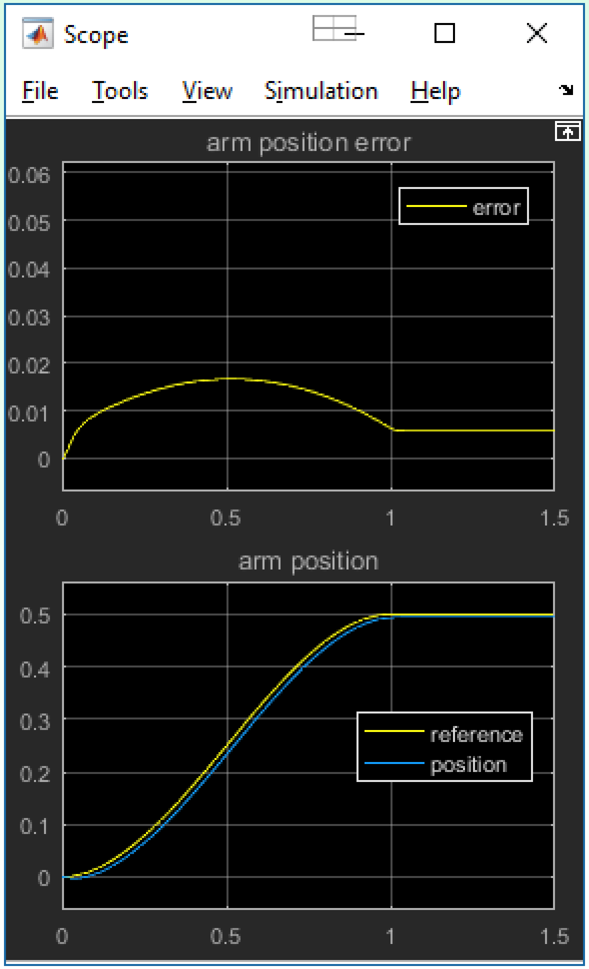
Choosing different values of ω we get ($\zeta=1$):

1. For ω = 60: K_p= 14.4 [V], K_d= 0.27 [V/(rad/s)],
2. For ω = 70: K_p= 19.6 [V], K_d= 0.35 [V/(rad/s)],
3. For ω = 80: K_p= 25.6 [V], K_d= 0.43 [V/(rad/s)],



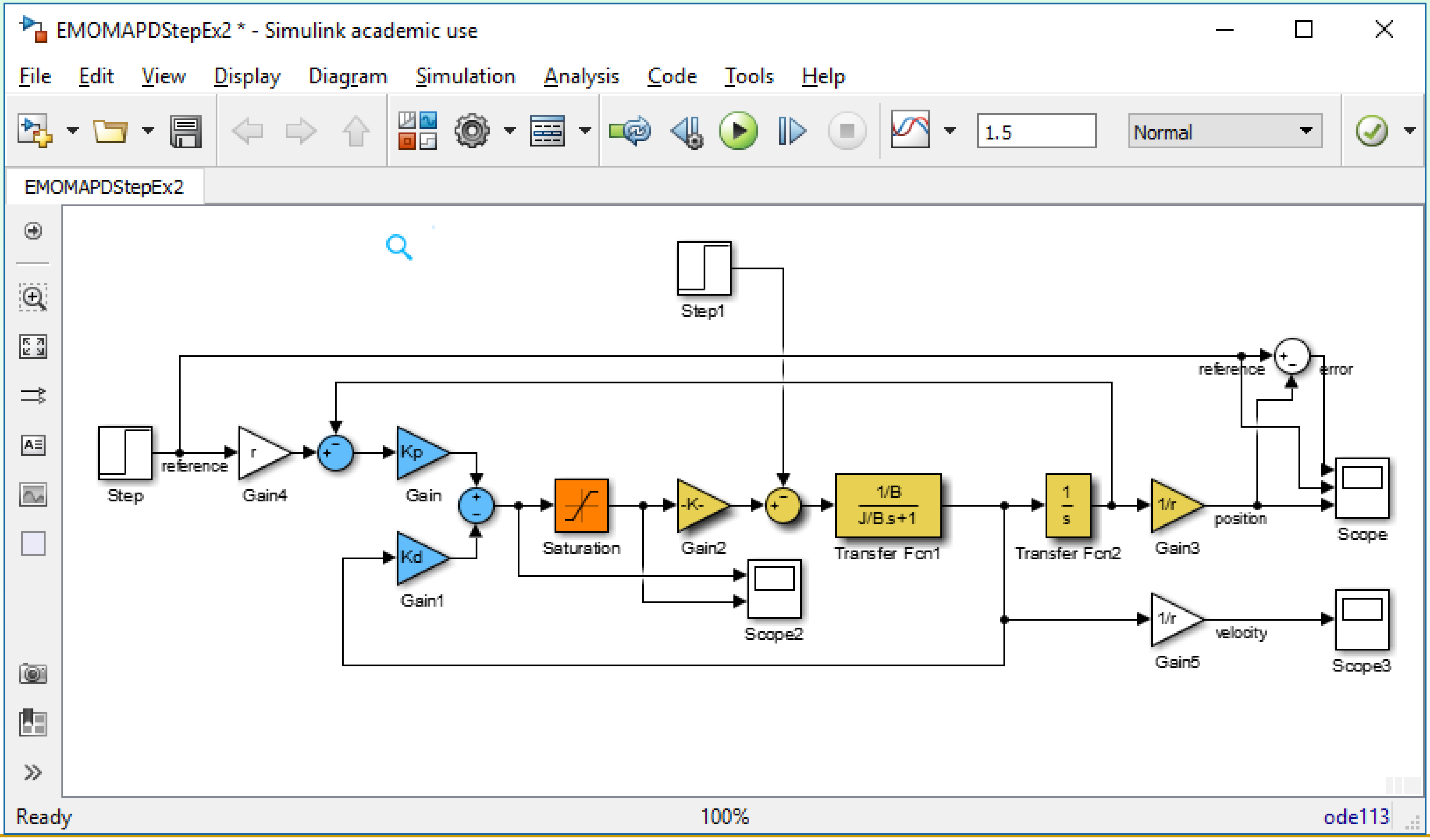
Example – PD Control (4)

Simulation results for the case with constant distirbance equal to 2 (PD controller tuned with ω = 70), notice the steady-state error:

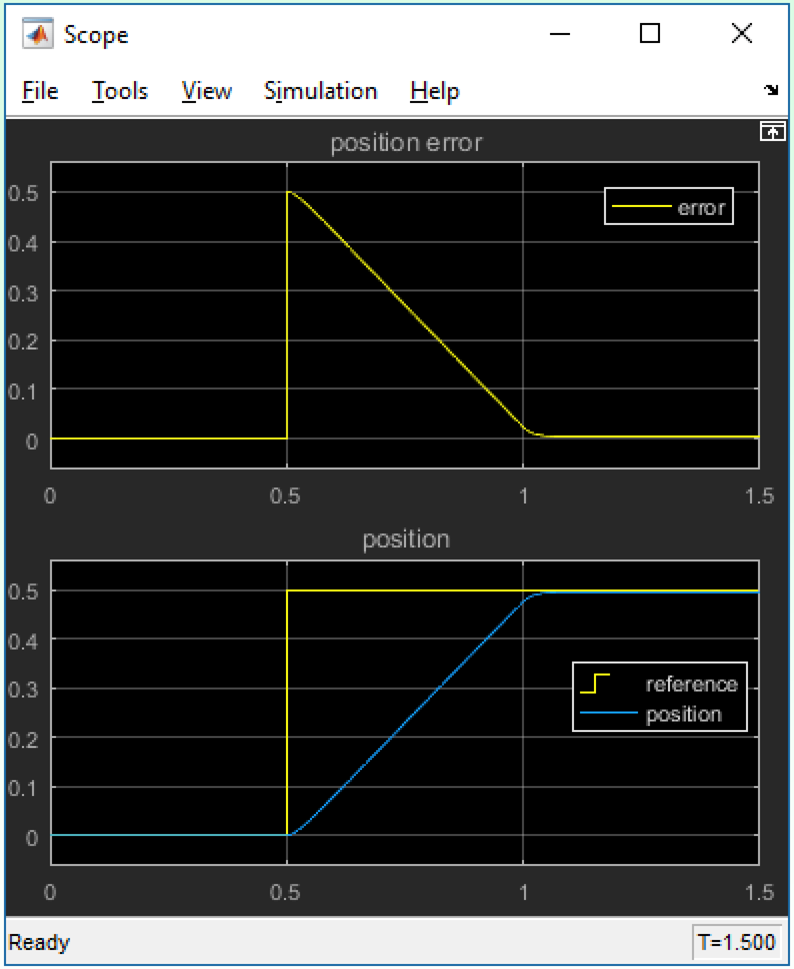


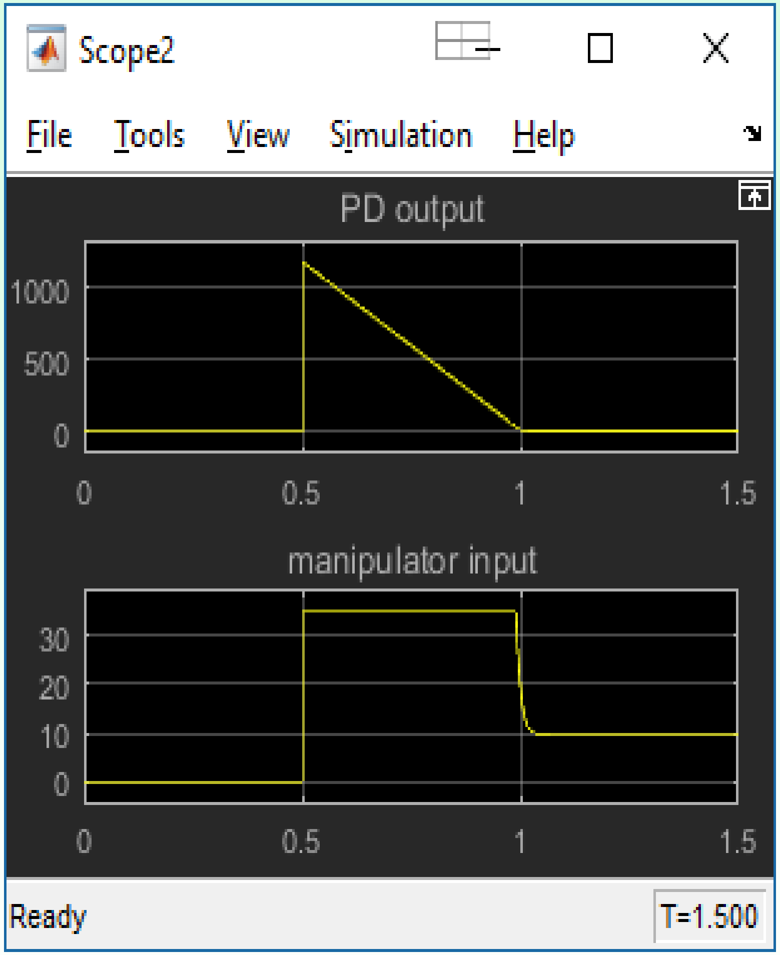
Example – P-D Control

The P-D control structure: the step-change of the output angle reference value from 0 to 0.5 was simulated (P-D controller tuned with ω = 70):



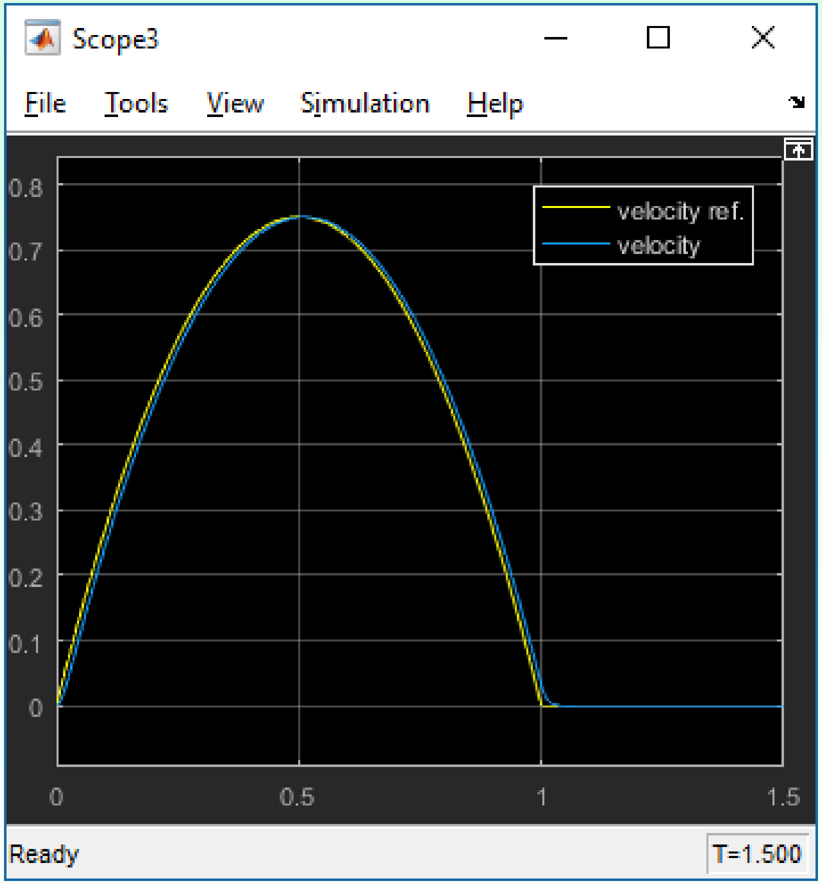
Example – P-D Control (2)

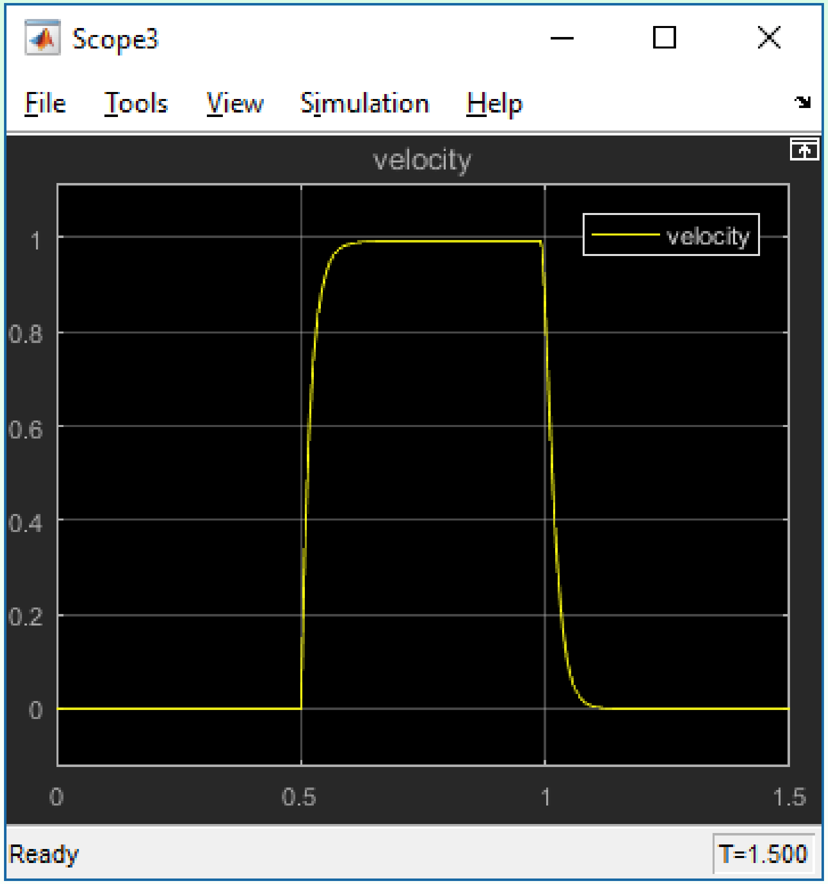




Trajectories for the step change of the reference value with saturation limit *u(t)* ≤ 35V (constraint on the control signal $u(t)$ limits speed of response).

Example – P-D Control (3)





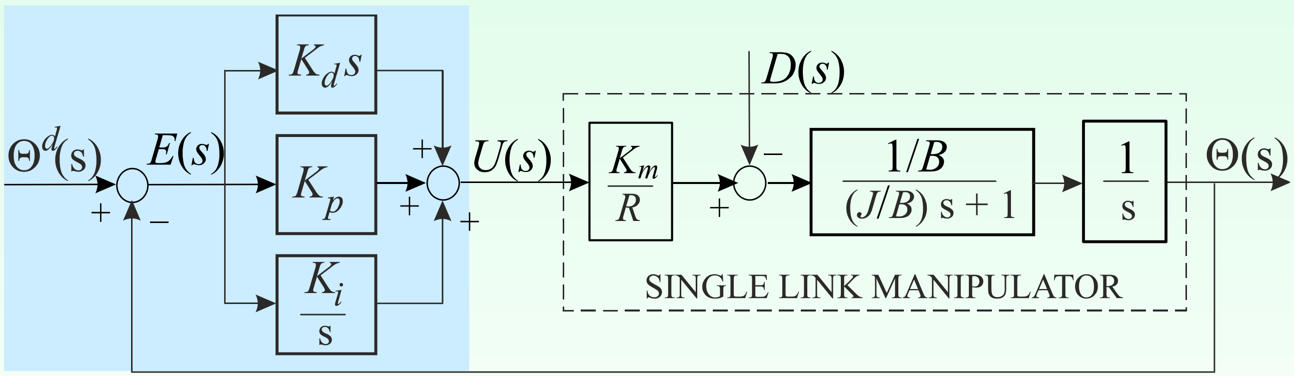
Trajectories of velocities for cubic trajectory of the reference output (left) and step-change of the reference output (right)

PID Control

PID controller structure (ideal, parallel):

$$u(t) = K_p\left(e(t) + T_d\frac{de(t)}{dt} + \frac{1}{T_i}\int_0^t e(\tau)d\tau\right) = K_p e(t) + K_d\frac{de(t)}{dt} + K_i\int_0^t e(\tau)d\tau \qquad (K_i = K_p / T_i)$$

$$U(s) = (K_p + K_d s + \frac{K_i}{s})E(s)$$



Transfer function:

$$\Theta(s) = \frac{1/B}{(J/B)s^2 + s}\left[-D(s) + \frac{K_m}{R}\left(K_p + K_d s + \frac{K_i}{s}\right)\left(\Theta^d(s) - \Theta(s)\right)\right]$$

$$\Theta(s)\left[1 + \frac{\frac{K_m}{RB}(K_p + K_d s + \frac{K_i}{s})}{(J/B)s^2 + s}\right] = \frac{1/B}{(J/B)s^2 + s}\left[-D(s) + \frac{K_m}{R}\left(K_p + K_d s + \frac{K_i}{s}\right)\Theta^d(s)\right]$$

PID Control

Multiplying by Js^2+Bs we get

$$\Theta(s)\left[Js^2+Bs+\frac{K_m}{R}(K_p+K_d s+\frac{K_i}{s})\right]=\left[-D(s)+\frac{K_m}{R}\left(K_p+K_d s+\frac{K_i}{s}\right)\Theta^d(s)\right]$$

$$\Theta(s)\left[Js^3+Bs^2+\frac{K_m}{R}(K_p s+K_d s^2+K_i)\right]=\left[-D(s)s+\frac{K_m}{R}\left(K_p s+K_d s^2+K_i\right)\Theta^d(s)\right]$$

$$\Theta(s)=\frac{\frac{K_m}{R}(K_d s^2+K_p s+K_i)}{Js^3+(B+\frac{K_m K_d}{R})s^2+\frac{K_m K_p}{R}s+\frac{K_m K_i}{R}}\Theta^d(s)-\frac{s}{Js^3+(B+\frac{K_m K_d}{R})s^2+\frac{K_m K_p}{R}s+\frac{K_m K_i}{R}}D(s)$$

For step inputs $(\Theta^d(s)=\frac{a}{s},\ D(s)=\frac{b}{s})$:

$$\lim_{t\to\infty}\theta(t)=\lim_{s\to 0}s\Theta(s)=a \quad\Rightarrow\quad e_{ss}=\lim_{t\to\infty}e(t)=0$$

PID Control – Parameter Tuning

Closed-loop characteristic polynomial:

$$Js^3 + \left(B + \frac{K_m K_d}{R}\right) s^2 + \frac{K_m K_p}{R} s + \frac{K_m K_i}{R}$$

Stability condition (from Hurwitz criterion):

$$\begin{vmatrix} B + \dfrac{K_m K_d}{R} & J \\ \dfrac{K_m K_i}{R} & \dfrac{K_m K_p}{R} \end{vmatrix} > 0$$

$$(B + \frac{K_m K_d}{R}) K_p - K_i J > 0$$

$$K_i < \frac{(B + \frac{K_m K_d}{R}) K_p}{J}$$

A simple way to tune the controller is to assume location of poles of the closed-loop characteristic polynomial.

PID Control – Parameter Tuning

Assuming one triple real pole $-\alpha$, i.e., $\Omega(s) = J(s+\alpha)^3$,

$$Js^3 + (B + \frac{K_m K_d}{R})s^2 + \frac{K_m K_p}{R} s + \frac{K_m K_i}{R} = J(s^3 + 3\alpha\, s^2 + 3\alpha^2 s + \alpha^3)$$

we get:

$$K_p = \frac{R}{K_m} 3J\alpha^2, \qquad K_d = \frac{R}{K_m}(3J\alpha - B), \qquad K_i = \frac{R}{K_m} J\alpha^3$$

Assuming a single pole $-\alpha$ and a second order dynamic term:

$$\Omega(s) = J(s+\alpha)(s^2 + 2\zeta\omega s + (\omega)^2)$$
$$= Js^3 + J(2\zeta\omega + \alpha)s^2 + J((\omega)^2 + 2\zeta\omega\alpha)s + J\alpha(\omega)^2$$

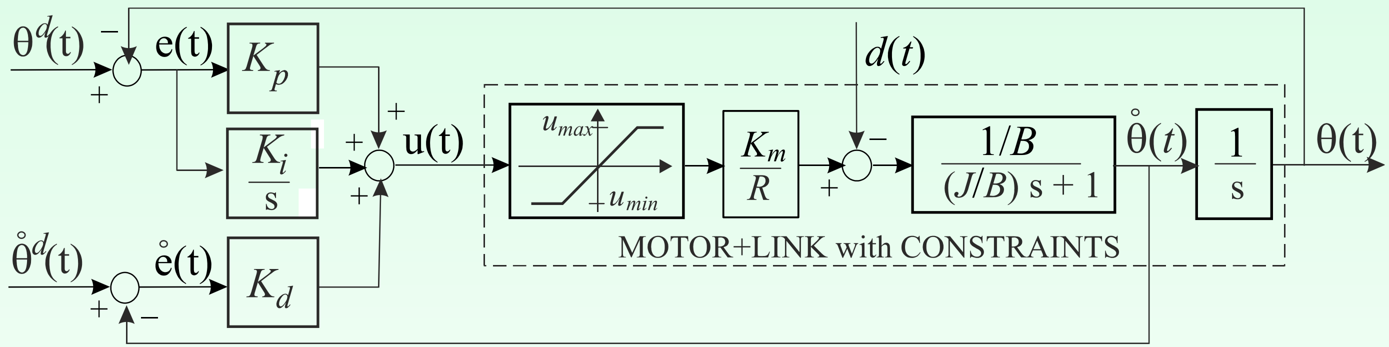
$$K_p = \frac{R}{K_m} J\omega(\omega + 2\zeta\alpha), \quad K_d = \frac{R}{K_m}(2J\zeta\omega + J\alpha - B), \quad K_i = \frac{R}{K_m} J\alpha(\omega)^2$$

Comments made for PD control, concerning limitations on controller design, apply.

PID and PI-D Control – Implementation Issues

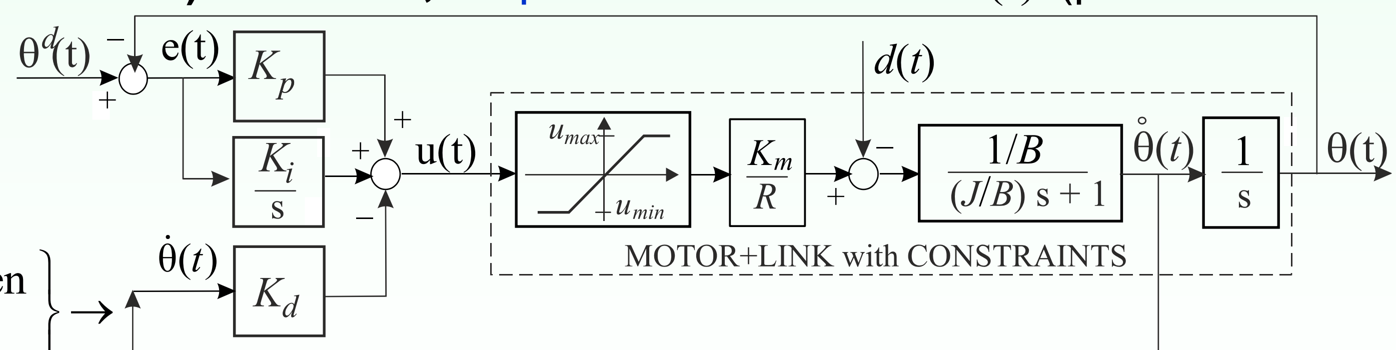
Similarly as it was for PD controller, assuming measurement of velocity leads to the PID structures:

a) **Standard PID structure**, for smooth trajectory tracking, ($\dot{\theta}^d(t)$ available):



b) **PI-D (modified) structure,** for piecewise constant $\theta^d(t)$ (position control):

$$\left.\begin{aligned}\dot{\theta}(t) &= -\dot{e}(t) \quad \text{when} \\ \theta^d(t) &= \text{const.}\end{aligned}\right\} \rightarrow$$



PID Control – discretization (a reminder)

Discrete- time version of the PID controller

For the standard continuous-time PID structure (with ideal differentiation):

$$R(s) = \frac{U(s)}{E(s)} = K_p + K_i \cdot \frac{1}{s} + K_d s = K_p + K_i \cdot \frac{1}{s} + \frac{K_d}{\frac{1}{s}}$$

using the backward-Euler scheme:

$$\frac{1}{s} \sim T_c \frac{z}{z-1} \qquad \left(u(k) = u(k-1) + T_c \cdot e(k)\right)$$

we get

$$\frac{U_I}{E} = K_i \cdot \frac{T_c z}{z-1} = K_i \cdot \frac{T_c}{1-z^{-1}} \quad \Rightarrow \quad u_I(k) = u_I(k-1) + K_i \cdot T_c e(k),$$

$$\frac{U_D}{E} = K_d \frac{1}{\frac{T_c z}{z-1}} = K_d \cdot \frac{z-1}{T_c z} \quad \Rightarrow \quad u_D(k) = \frac{K_d}{T_c}(e(k) - e(k-1))$$

$$u(k) = K_p e(k) + u_I(k) + u_D(k) = K_p e(k) + u_I(k) + \frac{K_d}{T_c}(e(k) - e(k-1))$$

or, with the derivatives measured:

$$u(k) = K_p e(k) + u_I(k) + u_D(k) = K_p e(k) + u_I(k) + K_d \dot{e}(k)$$

PID Control – discretization

Therefore, for **the discrete-time controller** (standard form):

$$u(k) = P \cdot e(k) + u_I(k) + D \cdot (e(k) - e(k-1)), \quad u_I(k) = u_I(k-1) + I \cdot e(k),$$

we have relations between discrete-time and continuous-time gains:

$$P = K_p, \quad D = K_d / T_c, \quad I = K_i \cdot T_c$$

or, when the derivatives are measured: $u(k) = P \cdot e(k) + u_I(k) + D \cdot \dot{e}(k)$

$$P = K_p, \quad D = K_d, \quad I = K_i \cdot T_c$$

ATTENTION: when saturation of control signals is possible, PID or PI-D controllers should be implemented with **anti-windup** (back-substitution structure or conditional integration)

AW by conditional integration, at k-th time instant (velocity measured):

$$u_{I0} = u_I(k-1) + I \cdot e(k);$$

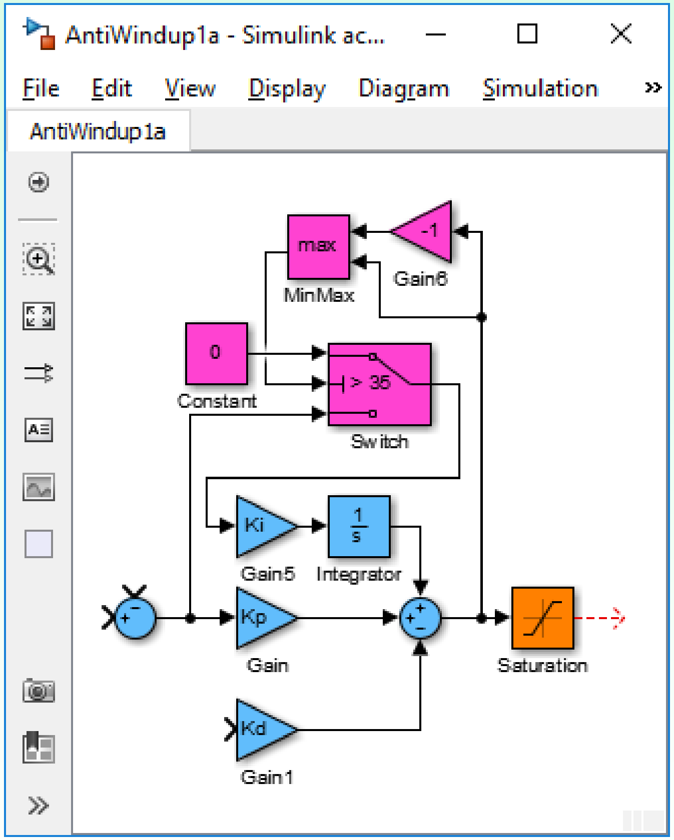
$$u(k) = P \cdot e(k) + u_{I0} + D \cdot \dot{e}(k);$$

$$\text{IF } (u(k) > u_{max} \text{ OR } u(k) < u_{min}) \text{ AND } \operatorname{sign}(u(k)) = \operatorname{sign}(u_{I0}), \quad u_I(k) = u_I(k-1);$$

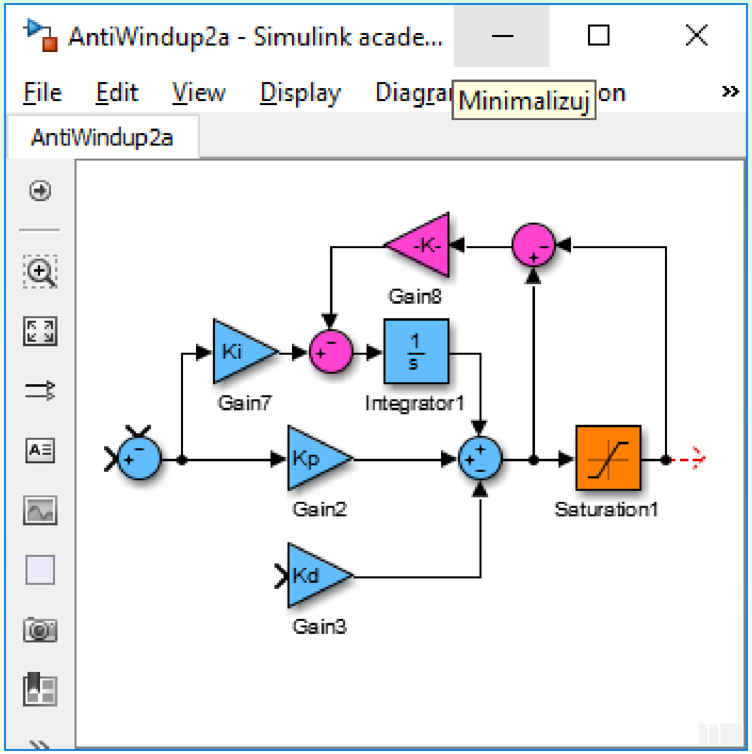
$$\text{ELSE} \quad u_I(k) = u_{I0};$$

PID/PI-D Control – anti-windup

Anti-windup loops, implemented in Simulink:



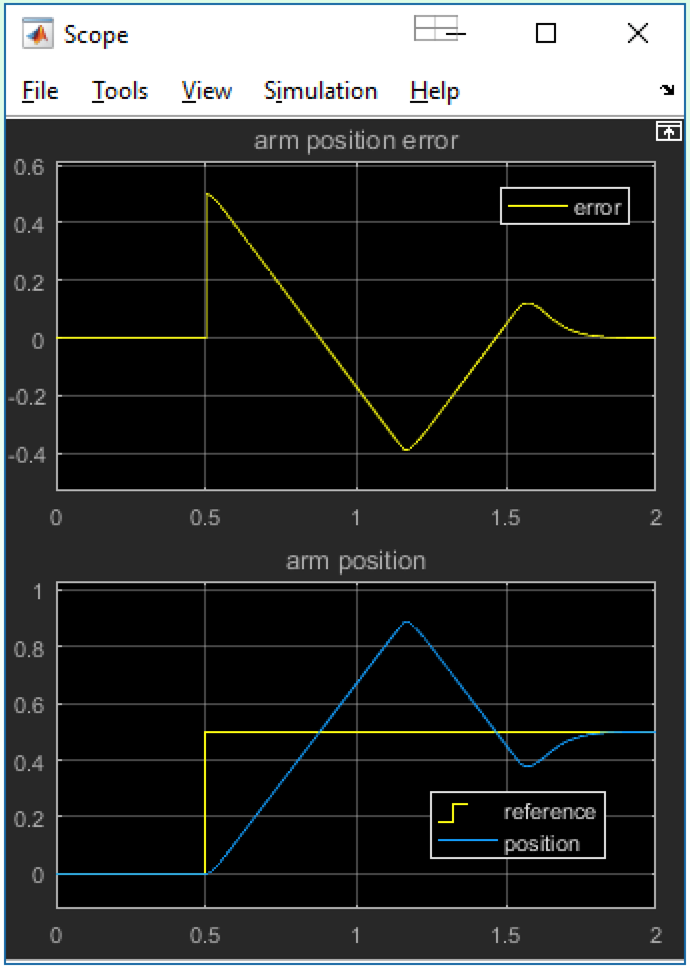
Anti-windup by conditional integration (simplified)

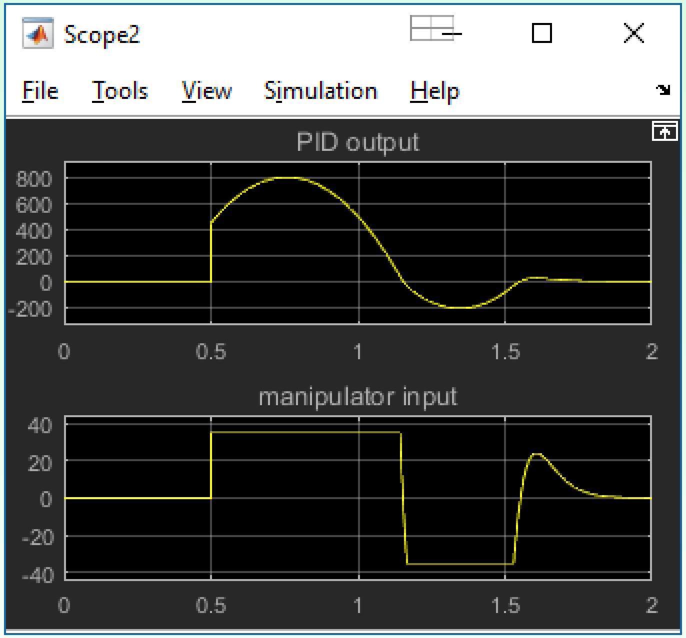


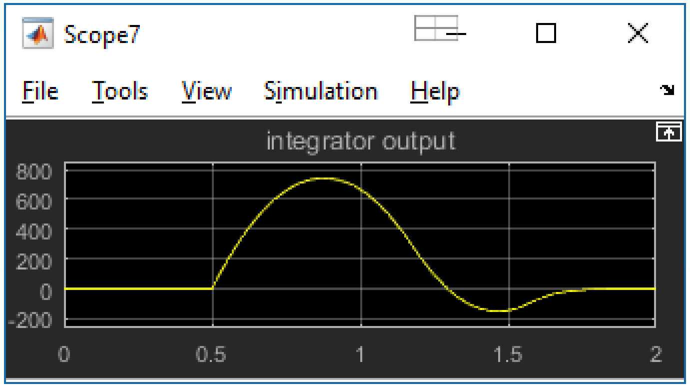
Classical anti-windup loop called „back-calculation" - recommended initial feedback gain value: $K \cong 1/T_i = K_i / K_p$

Example – PI-D Control without anti-windup

PI-D control of the Example manipulator, for step reference input, without anti-windup (PID tuning for α=18)

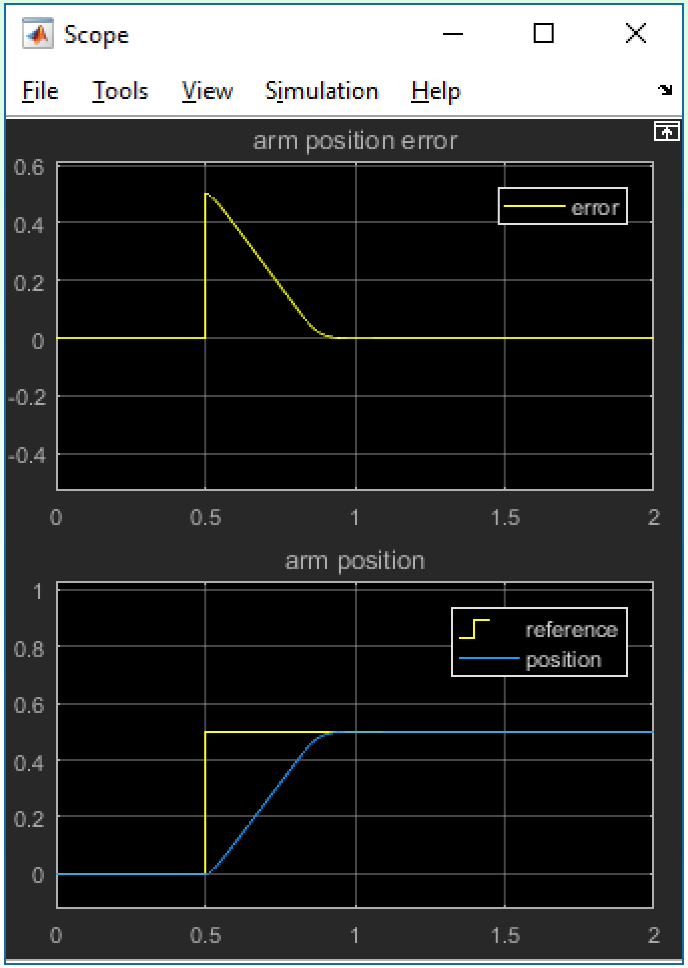


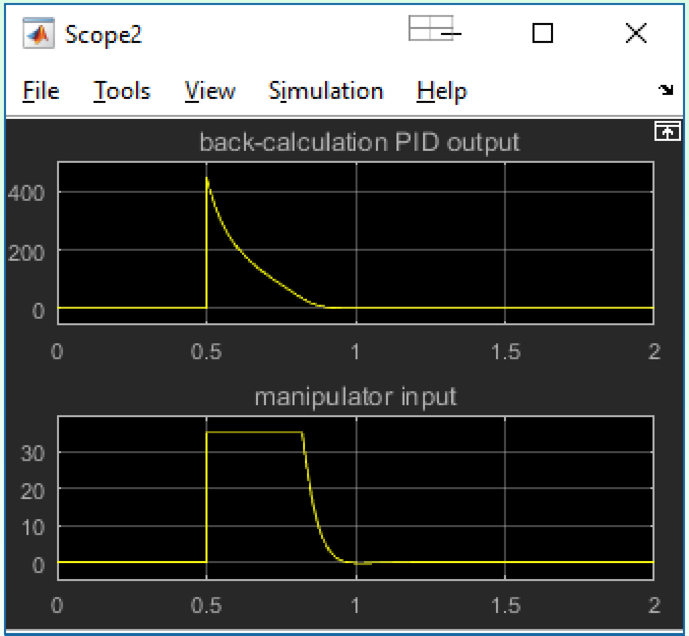


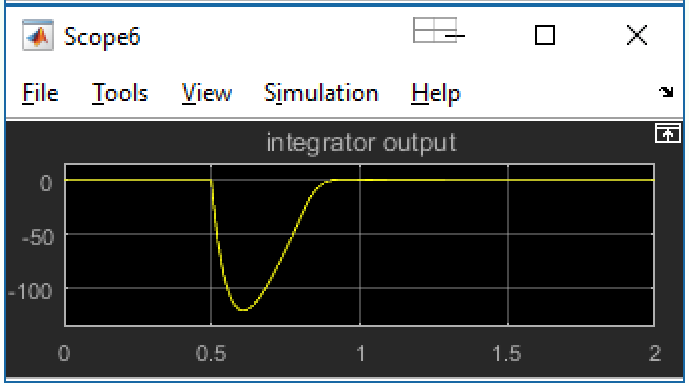


Example – PI-D Control with anti-windup

PI-D control of the Example manipulator, for step reference input and with anti-windup (back-calculation, with $K=2/T_i$):

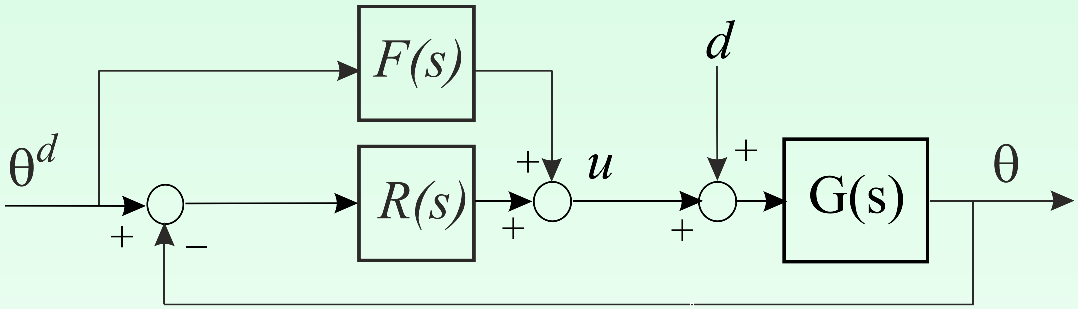






Feedback-Feedforward Control - General

If smooth reference trajectory is known in advance, then a sound solution is the feedback-feedforward control structure, of the general form



Ideal feedforward action would be if

$$F(s) = \frac{1}{G(s)} = \frac{g_{den}(s)}{g_{num}(s)}$$

which is realizable if $G(s)$ is minimum-phase. i.e., $g_{num}(s)$ has no unstable zeros and is without delay.

If the process model were ideal and without disturbances, the feedforward control only would suffice. In practice, feedback control is necessary to cope with model inaccuracies (errors in parameters, unmodeled dynamics) and with disturbances.

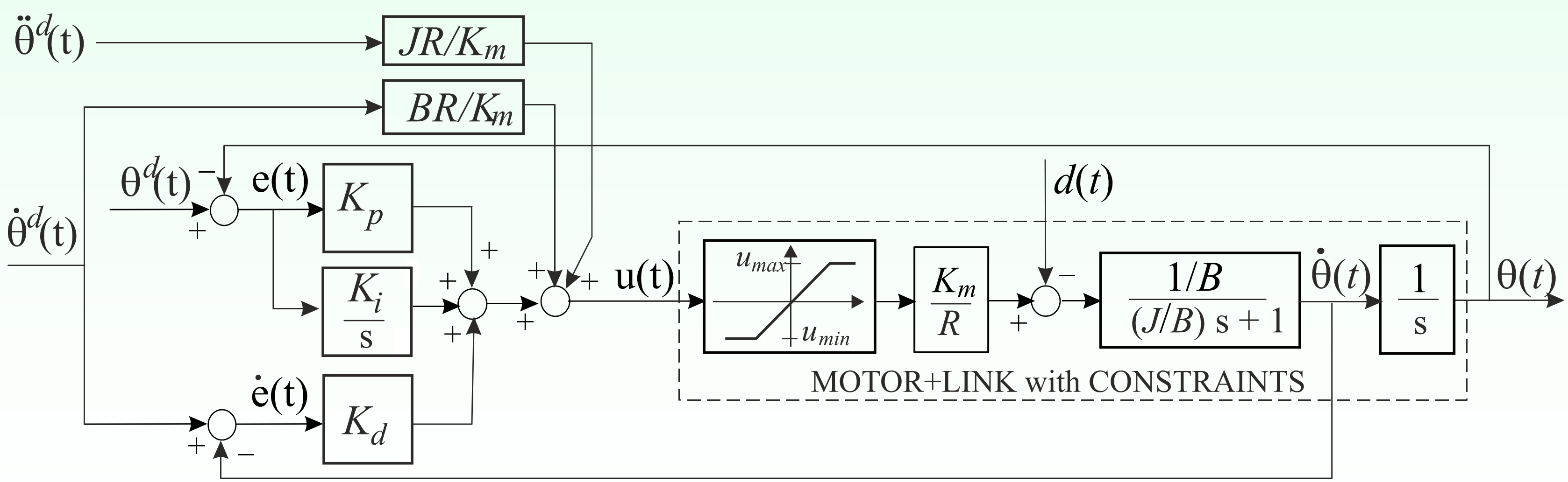
Feedb.-Feedf. Control – Single-Link Manipulator

For a single link manipulator:

$$G(s) = \frac{K_m}{RB}\frac{1}{((J/B)s+1)s} = \frac{K_m}{R}\frac{1}{(Js+B)s}$$

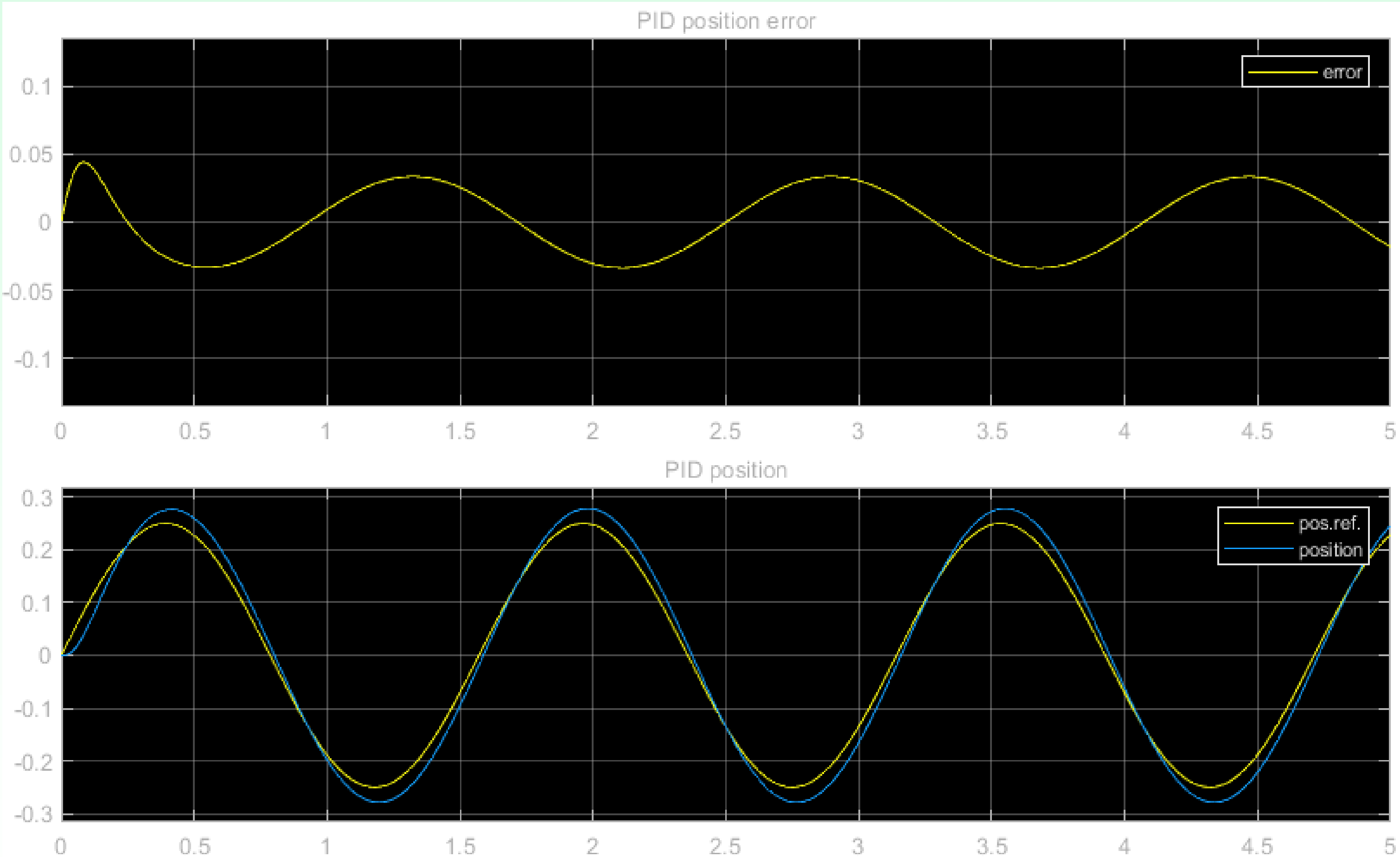
$$F(s) = \frac{1}{G(s)} = \frac{JR}{K_m}s^2 + \frac{BR}{K_m}s$$

and $F(s)$ is **realizable** for trajectory tracking with **precomputed** reference velocity and acceleration profiles. The resulting PID control structure:



Example – PID Control (sinusoidal reference input)

PID control of the Example manipulator, for $\sin(4t)$ reference input (PID tuning for α = 18), without feedforward



Example – PID Control (sinusoidal reference input)

PID control of the Example manipulator, for $\sin(4t)$ reference input (PID tuning for α = 18), with feedforward