EXERCISES 2

Example 2.1 Color normalization (registration)

Assume that in a computer program we represent all the colour coefficients in terms of **8 bits** – using unsigned **integer** values from the interval [0, 255].

Perform transformations between RGB and YUV colour spaces: for blue, green and red colours.

Solution 2.1

Transform the coefficients Y(U-128)(V-128) into RGB.

Is it possible to transform "pure" and "full" RGB colors into YUV ?

RGB = (255, 0, 0) → YUV?

RGB = (0, 255, 0) → YUV ?

RGB = (0, 0, 255) → YUV ?

The "pure" Red in RGB is: $pureRed_{RGB} = (255, 0, 0)$

- This gives an "out-of-range" V value: $pureRed_{YUV} = (76, 90, 284)$

Idea: reduce V to 255 and change others appropriately:

- YUV=(62, 97, 255) → RGB= (207.5 0 0).
- YUV=(110, 97, 255) → RGB= (255.3 48 48).

Transform the centers of basic colors in YUVIdealMacbeth into RGB:

- Red_{YUV} = (85, 111, 193) → RGB ?
- $Green_{YUV}$ = (117, 100, 95) → RGB ?
- $Blue_{YUV}$ = (66, 176, 113) → RGB ?

The "center" red color in "YUVIdealMacbeth" transforms as follows:

- $red_{YUV} = (85, 111, 193) \rightarrow red_{RGB} \cong (160, 53.5, 51.5)$

Conclusion: the thresholds in YUV for "red" color can be set as:

- Y= [32, 164],
- U= [73. 120],
- V=[150, 255]

Example 2.2

Suppose that a $64 \ x \ 64$, *8*-level image has the intensity-value distribution as shown in the table below.

Pixel value w	Histogram value $h(w)$	pmf value $p_w(w)$
0	790	0.19
1	1023	0.25
2	850	0.21
3	656	0.16
4	329	0.08
5	245	0.06
6	122	0.03
7	81	0.02

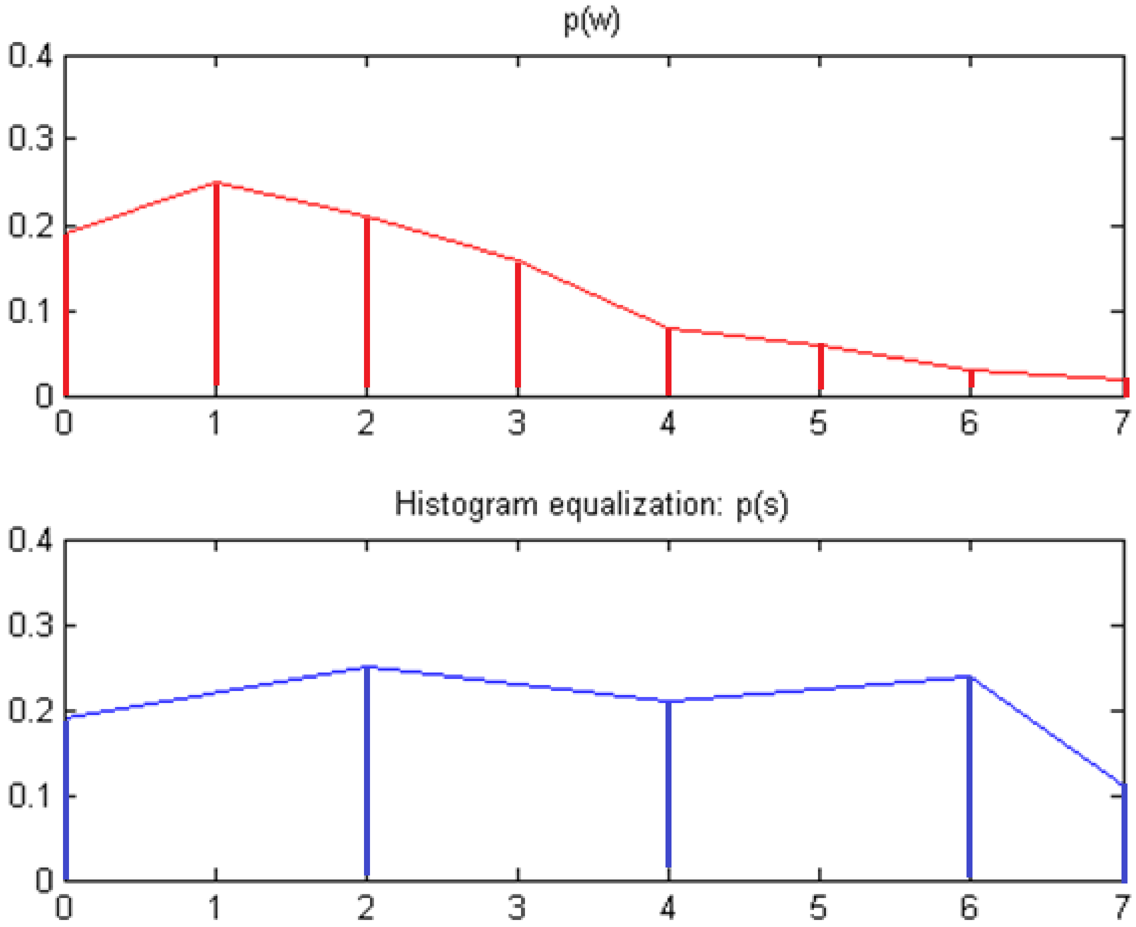
1. Perform *histogram equalization*.
2. Give graphical illustration of both histograms.
3. Determine the number of distinct levels in the output image.
4. Specify the pmf for values of the final output image.

Solution 2.2

1.

w	$p_w(w)$	$F_w(w)$	$z = T(w)$	4. new pmf: $p_z(z)$
0	0.19	0.19	$0.19 \cdot 7 = 1.33 \rightarrow 1$	0.19
1	0.25	0.44	$3.08 \rightarrow 3$	0.25
2	0.21	0.65	$4.55 \rightarrow 5$	0.21
3	0.16	0.81	$5.67 \rightarrow 6$	0.16 + 0.08 = 0.24
4	0.08	0.89	$6.23 \rightarrow 6$	
5	0.06	0.95	$6.65 \rightarrow 7$	0.06+0.03+0.02 = 0.11
6	0.03	0.98	$6.86 \rightarrow 7$	
7	0.02	1.00	7	

2. Two histograms:



1. Reduction of the level number from 8 to 5.

Example 2.3

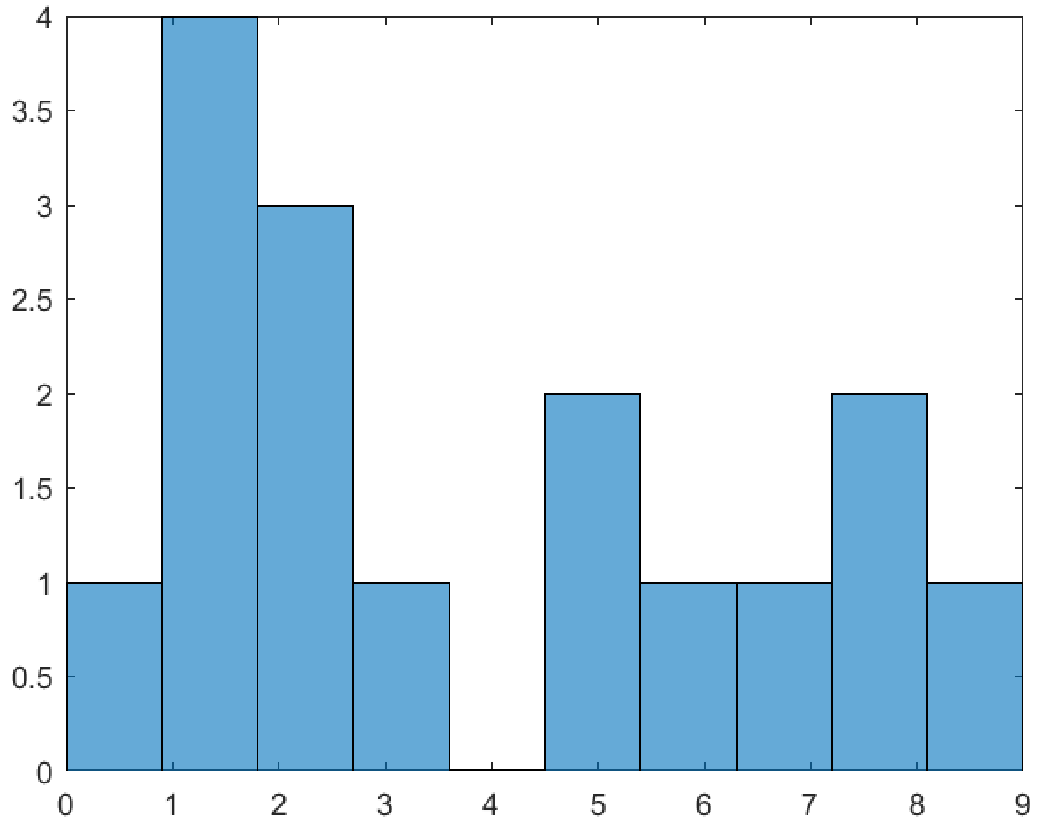
Let the following image be given.

3	2	8	2
5	1	7	1
9	1	6	1
8	0	5	2

1. Determine the threshold for image **binarization** using the **Otsu method**. Show intermediate results.
2. Perform **image thresholding**.

Solution 2-3

1. Create an image histogram



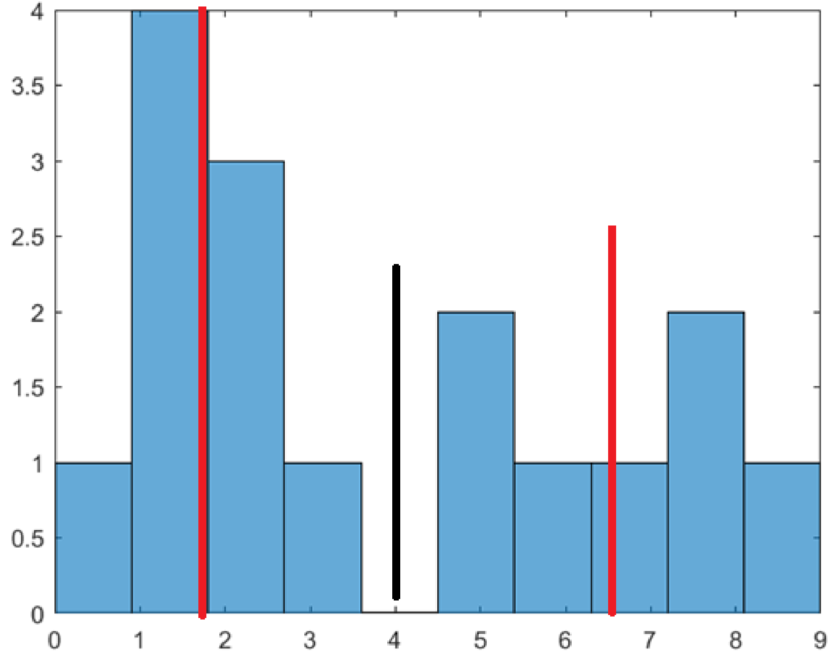
2. Iterate for $\sigma^2(\theta)$

θ	P_1	P_2	m_1	m_2	$P_1 \cdot P_2$	$(m_1\text{-}m_2)^2$	$\sigma^2(\theta)$

0	0.0625	0.9375	0	4.0667	0.0586	16.5378	0.9690
1	0.3125	0.6875	0.8000	5.1818	0.2148	19.2003	4.1251
2	0.5000	0.5000	1.2500	6.3750	0.2500	26.2656	6.5664
3	0.5625	0.4375	1.4444	6.8571	0.2461	29.2973	7.2099
4	0.5625	0.4375	1.4444	6.8571	0.2461	29.2973	7.2099
5	0.6875	0.3125	2.0909	7.6000	0.2148	30.3501	6.5205
6	0.7500	0.2500	2.4167	8.0000	0.1875	31.1736	5.8451
7	0.8125	0.1875	2.7692	8.3333	0.1523	30.9592	4.7164
8	0.9375	0.0625	3.4667	9.0000	0.0586	30.6178	1.7940

3. Select

$\theta = \operatorname{argmax} \sigma^2(\theta) = 3 \text{ or } 4 \rightarrow 4$



Example 2.4

Apply the Sobel and Scharr *edge operators* to the two images given below. Present the edge strengths and orientations. Provide comments to these results.

1)

1	3	5	4
1	2	5	5
2	3	5	5
1	2	4	5
0	3	5	5

2)

1	2	1	2
2	2	3	6
3	3	6	6
3	6	6	5
5	6	6	6

Solution 2.4

1) Sobel operator
Gx

	15	9	
	13	10	
	14	10	

2) Sobel operator
Gx

	5	11	
	10	9	
	10	1	

1) Sobel operator
Gy

	1	1	
	-1	-2	
	-2	0	

2) Sobel operator
Gy

	9	15	
	12	9	
	8	3	

1) Sobel operator

S' strength = |*Gx*| + |*Gy*|

	16	16	
	14	12	
	16	10	

2) Sobel operator

S' strength

	14	26	
	22	18	
	18	4	

1) Sobel operator

θ = atan (Gy/Gx) → 0°

	3.8°	6.3°	
	-4.4°	-11.3°	
	-8.1°	0°	

2) Sobel operator

θ → ca. 48°

	60.9°	53.7°	
	50.2°	45°	
	38.7°	71.6°	

Red – edge strength is low, direction can be unreliable (noisy)

With number of directions = 32 :

1) Sobel operator

r = direction_index(θ) → 0

	0	1	
	0	31	
	31	0	

2) Sobel operator

θ → 4 - 5

	5	5	
	4	4	
	3	6	

Scharr operator

1) Gx

	61	39	
	51	38	
	54	42	

2) Gx

1) *Gy*

	3	3	
	-3	-10	
	-6	0	

2) *Gy*

1) *S' strength = |Gx| + |Gy|*

	64	42	
	54	48	
	60	42	

2) *S' strength*

	50	114	
	94	78	
	78	10	

1) θ = atan (Gy/Gx) → 0°

	2.8°	4.4°	
	-3.4°	-14.7°	
	-6.3°	0°	

2) θ → ca. 48°

	58.5°	53.0°	
	51.1°	45°	
	40.6°	96.3°	

With number of directions = 32 :

1) r = direction_index(θ) → 0

	0	0	
	0	31	
	31	0	

2) θ → 4 - 5

	5	5	
	5	4	
	4	9	

Comments. The Sobel operator and the Scharr operator perform equally good in both image cases. The Scharr operator allows an easy thresholding of the strength, in order to decide “edge or no edge”.

Example 2.5

Simulate 3 different *edge thinning* procedures as applied to the results of the Sobel operator for two above images (given in example 2.4).

1) S' strength = |*Gx*| + |*Gy*|

	16	16	
	14	12	
	16	10	

2) S' strength

	14	26	
	22	18	
	18	4	

Solution 2.5

Use a simple thresholding

1) S' strength = |Gx| + |Gy|

	16	16	
	14	12	
	16	10	

2) S' strength

	14	26	
	22	18	
	18	4	

Θ=13

	16	16	
	14	0	
	16	0	

Θ=21

	0	26	
	22	0	
	0	0	

Use non-maximum suppression (taking account edge directions)

1) *S' strength* = $|Gx| + |Gy|$

	16	16	
	14	12	
	16	10	

2) S' strength

	14	26	
	22	18	
	18	4	

1) θ = atan (Gy/Gx) → 0°

	2.8°	4.4°	
	-3.4°	-14.7°	
	-6.3°	0°	

2) θ → ca. 48°

	58.5°	53.0°	
	51.1°	45°	
	40.6°	96.3°	

Non-parametric!

	16	16	
	14 ok	0 (left)	
	16	0 (left)	

Non-parametric!

	0	26	
	22	18	
	18	0 or 4?	

Use hysteresis thresholding (Canny)

2) S' strength = |*Gx*| + |*Gy*|

	16	16	
	14	12	
	16	10	

2) S' strength

	14	26	
	22	18	
	18	4	

2) θ = atan (Gy/Gx) → 0°

	2.8°	4.4°	
	-3.4°	-14.7°	
	-6.3°	0°	

2) θ → ca. 48°

	58.5°	53.0°	
	51.1°	45°	
	40.6°	96.3°	

$\Theta_L=13,\ \Theta_H=15$

	16	16	
	14	0	
	16	0	

$\Theta_L=20,\ \Theta_H=25$

	0	26	
	22	0	
	0	0	