Mobile Robots

Wheeled Mobile Robots

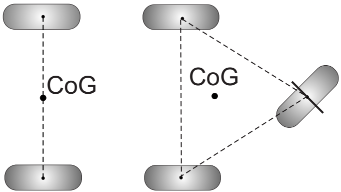
Lecture 3

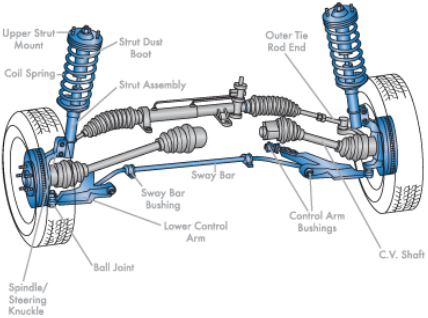
Wheeled Locomotion

- The most popular locomotion mechanism in mobile robotics (and in other vehicles).
- Simple to implement and highly efficient (wheeled robots consume less energy and move faster than other locomotion mechanisms).
- Bigger wheels allow overcoming higher obstacles, but they require higher torque or reductions in the gear box.

Wheeled Locomotion

- Wheeled mobile robots (WMRs) are almost always designed so that balance is not an issue.
- The minimum number of wheels required for stable balance:
  - two wheels – only if the Center of Gravity (CoG) is below the wheel axle,
  - three wheels – CoG is within the triangle formed by the ground contact point of the wheels.
- Stability is improved by four and more wheels, but flexible suspension system is required to allow all wheels to maintain ground contact on uneven terrain.





- The main problems in WMR design are the traction, maneuverability, stability, and control that depend on the wheel types and configurations (drives).
- The main disadvantage of WMRs is that they are not very good at navigating over obstacles, such as rocky terrain, sharp surfaces.

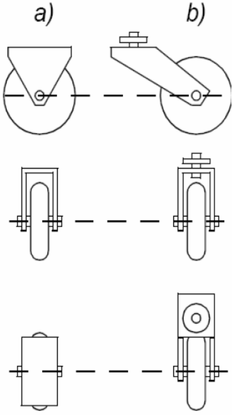
- WMRs are not stable on extremely smooth surfaces with low friction as they tend to slip and skid.
- Most WMRs arrangements are nonholonomic – they require high control effort.
- In robotics a car is often described as a non-holonomic vehicle.

Major Wheel Classes

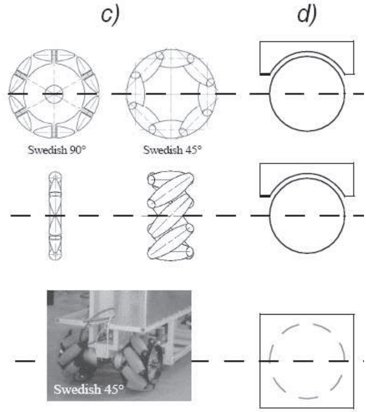
Two families of idealized wheels: **conventional** wheels and **special** wheels:

Conventional wheels:
a) Standard wheel fixed or steered
b) Castor wheel

Special wheels:
c) Swedish (or Mecanum) wheel
d) Spherical (or ball) wheel



[R. Siegwart, I. Nourbakhsh]



[R. Siegwart, I. Nourbakhsh]

Conventional Wheels

Standard wheel:

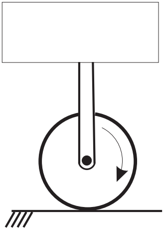
✓ Fixed

✓ Steered (centered orientable)

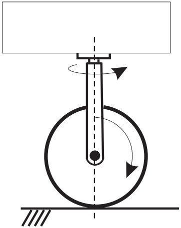
- rotation axes (two dof):
  - (Motorized) wheel axle
  - Normal (vertical) axis at contact point

✓ Castor (or caster) wheel:

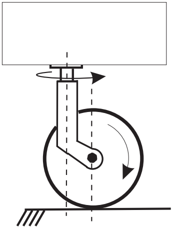
- off-centered orientable wheel
- rotation axes (three dof):
  - around the wheel axle
  - normal axis at contact point
  - castor axle



standard fixed wheel



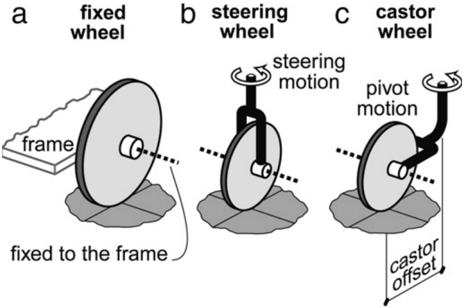
standard steered wheel



castor wheel

Conventional Wheels Characteristics

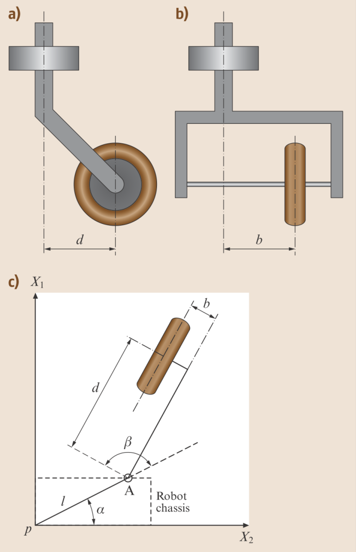
- The standard wheel and the castor wheel have a primary axis of rotation and are thus highly directional.
- To move in a different direction, the wheel must be steered first along a vertical axis.
- The standard wheel can accomplish the steering motion with no side effects, as the center of rotation passes through the contact of the ground.
- The castor wheel rotates around an offset axis, producing a force acting on the robot chassis during steering.



[Batlle, Barjau]

Three conditions should be defined for a conventional wheel design:

1. Determination of the two offsets d and b.
2. Mechanical design that allows steering motion or not – wheel orientation is fixed or not.
3. Determination of steering and driving actuation – active or passive drive.



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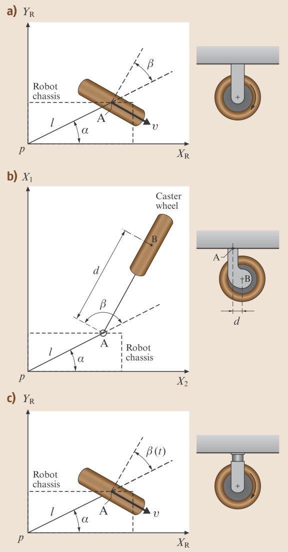
Types of Conventional Wheels

Four types of conventional wheels are commonly used:

1. A passively driven wheel with a fixed steering axis.
2. An active orientable wheel, where steering and driving motions are driven by actuators.
3. A passive caster wheel with offset d.
4. An active caster wheel with offset d, where the steering and driving motions are controlled by actuators.



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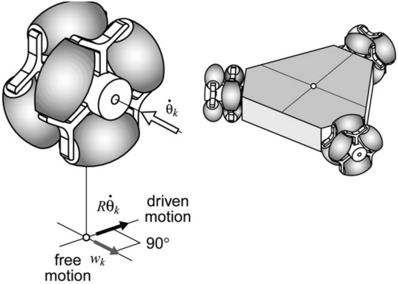
Swedish (or Mecanum) Wheels

- Swedish wheel consists of a fixed standard wheel with small rollers located along the outer rim of the wheel.
- The wheel's primary axis serves as the only actively powered joint, whereas the rollers are passive.
- An inventor, Bengt Ilon, came up with the idea in 1973 when he was an engineer with the Swedish company Mecanum AB.

Swedish wheel designs

Swedish Wheel Characteristics

- Swedish wheels are less constrained by directionality.
- The Swedish wheel acts as a normal wheel, but provides low resistance in a perpendicular direction (90°) or at an intermediate direction (45°) to the direction of the wheel.



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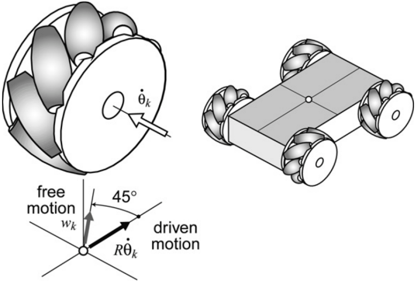
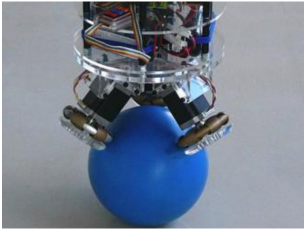


[Batlle, Barjau]

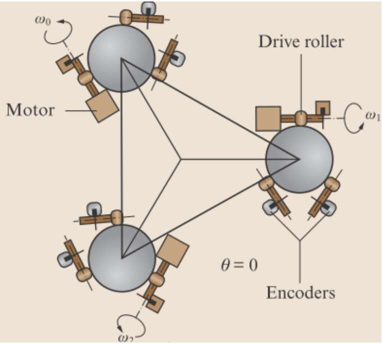
Spherical (or Ball) Wheels

- Smooth and continuous contact between the sphere and the ground is achieved.
- The payload must be quite low due to the point contact.
- The surface of the sphere can be polluted when traveling over dirty ground.
- It is difficult to overcome irregular ground conditions or obstacles.



Spherical Wheel Characteristics

- The spherical wheel is a truly omnidirectional wheel, typically it may be actively powered to spin along any direction.
- The design of the sphere-supporting mechanism is difficult.
- One mechanism for implementing this spherical design imitates the computer ball mouse, providing actively powered rollers that rest against the top surface of the sphere and impart rotational force.



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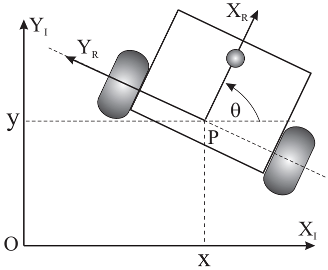
Representing Robot Pose

The robot is modeled as **a rigid body** on wheels, operating on a horizontal plane:



- The total dimensionality of the robot chassis on the plane is three:
  - Two for position in the plane.
  - One for orientation along the vertical axis – orthogonal to the plane.
- The global reference frame is given by the inertial basis $\{O; X_I, Y_I\}$ and the robot local reference frame is given by $\{P; X_R, Y_R\}$.
- The position of the point P in the global reference frame is given by coordinates x and y, and the angular difference between the global and local frames is given by θ.

The pose (or posture) and velocity of the robot can be described by

$$\boldsymbol{\xi}_I \triangleq \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}_I, \qquad \dot{\boldsymbol{\xi}}_I \triangleq \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix}_I$$

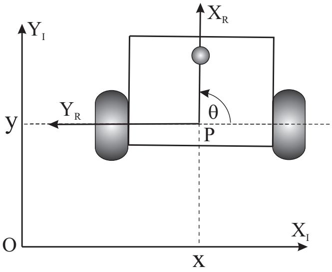
respectively.
The mapping between the velocity in the global reference frame $\{O; X_I, Y_I\}$ and the velocity in the robot local reference frame $\{P; X_R, Y_R\}$ can be accomplished using the orthogonal rotation matrix:

$$R(\theta) \triangleq \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad R^{\mathsf{T}} R = R R^{\mathsf{T}} = I_{3\times 3} \longrightarrow R^{-1} = R^{\mathsf{T}}$$

This matrix can be used to map motion in the frame $\{O; X_I, Y_I\}$ to motion in terms of frame $\{P; X_R, Y_R\}$:

$$\dot{\boldsymbol{\xi}}_R = R^{\mathsf{T}}(\theta)\dot{\boldsymbol{\xi}}_I = R^{\mathsf{T}}(\theta) \cdot [\dot{x}\ \dot{y}\ \dot{\theta}]_I^{\mathsf{T}}$$

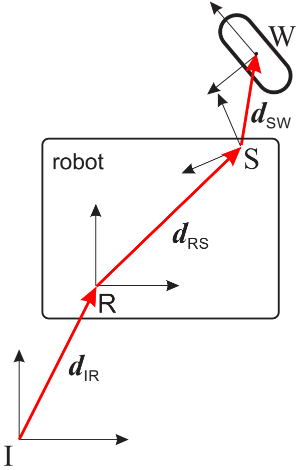
An Example: Motion Mapping



$$R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\dot{\xi}_R = R^{\mathsf{T}}(\frac{\pi}{2})\dot{\xi}_I = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \dot{y} \\ -\dot{x} \\ \dot{\theta} \end{bmatrix}$$

Deriving a general standard wheel linear velocity equation:



robot angular velocity

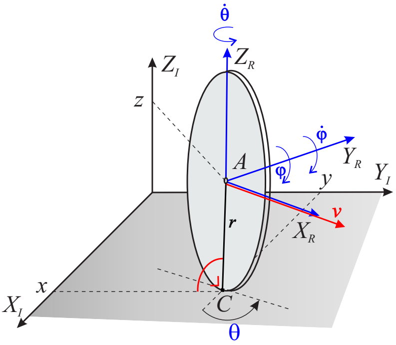
steering rate

$$v_{IW} = \underbrace{v_{IR}}_{\text{robot linear velocity}} + \omega_{IR} \times \underbrace{d_{RS}}_{\text{steering offset}} + \omega_{IR} \times d_{SW} + \overbrace{\omega_{RS}}^{\text{steering rate}} \times d_{SW}$$

wheel offset

Simplifying assumptions:

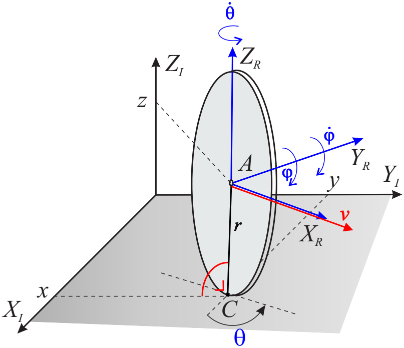
- Movement on a horizontal plane.
- The plane of the wheel always remains vertical.
- One single point of contact C between the wheel and the ground plane.
- There is no slipping, skidding or sliding at the single point of contact. The wheel undergoes motion only under conditions of pure rolling and rotation about the vertical axis through the contact point.
- No friction for rotation around the contact point.
- Steering axes are orthogonal to the surface.



Wheel Kinematic Constraints

Under the above simplifying assumptions, there are two differential kinematic constraints for each wheel type:

1. **Pure rolling contact** – the wheel **must roll** when motion takes place in the appropriate direction.
2. **No lateral slippage** – the wheel **must not slide** orthogonal (skid sideways) to the wheel plane.



Generalised coordinates of the wheel:

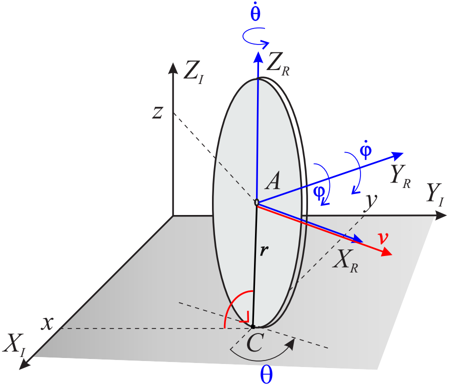
$$\boldsymbol{q} = [x, y, \theta, \varphi]^{\mathsf{T}} \in \mathbb{R}^2 \times \mathbb{S}^1 \times \mathbb{S}^1$$

Generalised velocities:

$$\dot{\boldsymbol{q}} = [\dot{x}, \dot{y}, \dot{\theta}, \dot{\varphi}]^{\mathsf{T}} \in \mathbb{R}^4$$

where

- $\dot{x}, \dot{y}$ – linear velocity coordinates in the inertial coordinate frame
- $\dot{\theta}, \dot{\varphi}$ – angular velocities



A wheel rolling on the plane

Single Wheel Kinematic Constraints

Linear velocity expressed in the wheel coordinate frame

$$\boldsymbol{v} = [v_x, v_y]^\mathsf{T} \in \mathbb{R}^2$$

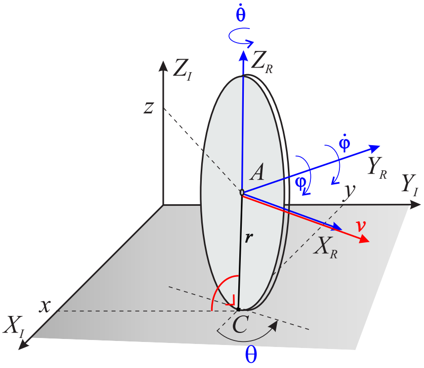
Velocity transformed to the inertial coordinate frame

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = R(\theta)\boldsymbol{v}, \qquad (1)$$

where

$$R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \in SO(2)$$

is the rotation matrix (an element of the special orthogonal group $SO(2)$).



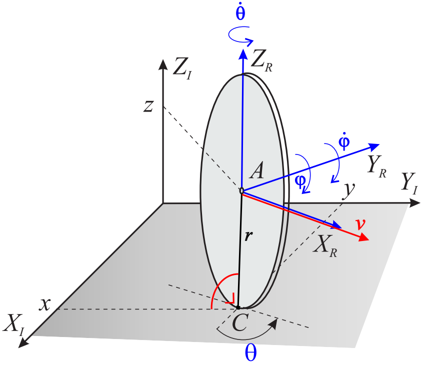
Calculating $\boldsymbol{v}$ from (1) we get

$$\boldsymbol{v} = R^{\mathsf{T}}(\theta)\ [\dot{x}, \dot{y}]^{\mathsf{T}},$$

where elements of $\boldsymbol{v}$ can be given as

$$v_x = \dot{x}\cos\theta + \dot{y}\sin\theta \qquad (2)$$

$$v_y = -\dot{x}\sin\theta + \dot{y}\cos\theta \qquad (3)$$



Assuming pure rolling the longitudinal velocity v_x (along axis X_R) and orthogonal velocity v_y (along axis Y_R) are dependent on the generalised velocity vector $\dot{\boldsymbol{q}}$.

- Pure rolling (without longitudinal slipping) constraint in the longitudinal direction comes from perfect transformation of the rotational motion to linear motion (there is pure rolling at the contact point):

$$v_x - r\,\dot{\varphi} = 0 \tag{4}$$

- Substituting (2) to (4) yields

$$\dot{x}\cos\theta - \dot{y}\sin\theta - r\dot{\varphi} = 0, \tag{5}$$

- Equation (5) can be written in the matrix form

$$\underbrace{\begin{bmatrix}\cos\theta & -\sin\theta & 0 & -r\end{bmatrix}}_{A_1(\boldsymbol{q})}\dot{\boldsymbol{q}} = 0, \tag{6}$$

where $A_1(\boldsymbol{q}) \in \mathbb{R}^{1\times 4}$ denotes constraint matrix.

Single Wheel Differential Constraints

- In the lateral direction (orthogonal to the wheel plane) the wheel's motion must be zero:

$$v_y = 0 \tag{7}$$

- Substituting from eq. (3) for v_y yields

$$\dot{x}\sin\theta + \dot{y}\cos\theta = 0 \tag{8}$$

- Also (8) can be expressed in the matrix form

$$\underbrace{\begin{bmatrix}\sin\theta & \cos\theta & 0 & 0\end{bmatrix}}_{A_2(\boldsymbol{q})}\dot{\boldsymbol{q}} = 0, \tag{9}$$

where $A_2(\boldsymbol{q}) \in \mathbb{R}^{1\times 4}$ is a constraint matrix.

Single Wheel Differential Constraints

- Differential kinematic (or velocity) constraints can be written in the matrix form (called Pfaffian form):

$$\underbrace{\begin{bmatrix} \cos\theta & -\sin\theta & 0 & -r \\ \sin\theta & \cos\theta & 0 & 0 \end{bmatrix}}_{A(\boldsymbol{q})} \dot{\boldsymbol{q}} = 0, \tag{10}$$

  where

$$A(\boldsymbol{q}) = \begin{bmatrix} A_1(\boldsymbol{q}) \\ A_2(\boldsymbol{q}) \end{bmatrix} \in \mathbb{R}^{2\times 4}$$

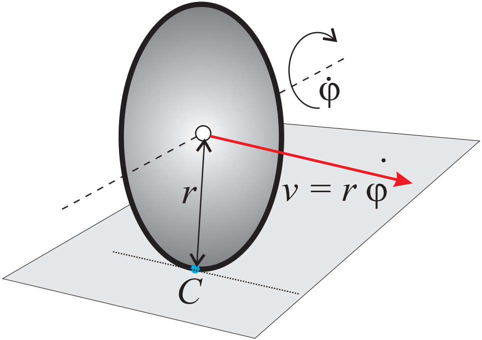
- Constraints (10) are linear. Formally, the generalised velocity vector $\dot{\boldsymbol{q}}$ lies in the null space of the matrix $A(\boldsymbol{q})$ (or kernel of $A(\boldsymbol{q})$):

$$\dot{\boldsymbol{q}} \in \mathrm{Ker}(A(\boldsymbol{q}))$$

For a conventional wheel the velocity v of the center of the wheel is:

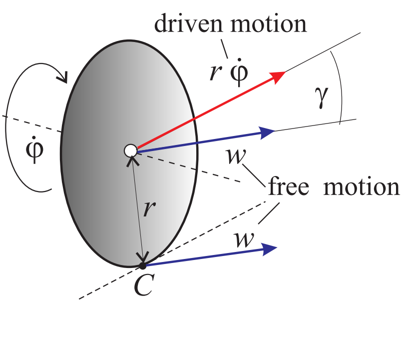
- parallel to the wheel plane (nonslip condition), and
- proportional to the wheel angular velocity (pure rolling condition).

The velocity of the contact point C is equal to zero.



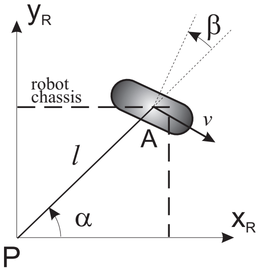
For a Swedish wheel:

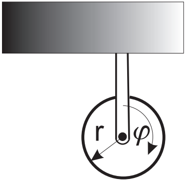
- Only one of the velocity components of the wheel contact point is zero (due to the relative rotation of the rollers with respect to the wheel).
- The direction of this zero component is fixed with respect to the wheel plane and depends on the wheel construction.



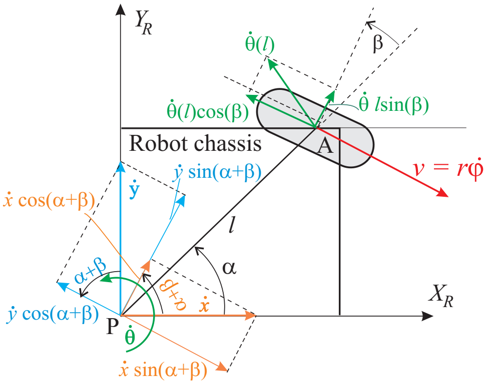
Fixed Standard Wheel

- Has no vertical axis of rotation for steering (its angle to the chassis is fixed).
- Possible motions:
  - Move back and forth along the wheel plane.
  - Rotate around its contact point with the ground plane.
- The position of the wheel center A is expressed in polar coordinates by distance l and angle α.
- The angle β of the wheel plane relative to the chassis is fixed.
- The radius of the wheel is r, and its angle of rotation around horizontal axle is a function of time $t : \varphi(t)$.





Constraints for Fixed Standard Wheel



1. $\dot{x}_R \sin(\alpha+\beta) - \dot{y}_R \cos(\alpha+\beta) - \dot{\theta}_R l \cos\beta = v$
2. $\dot{x}_R \cos(\alpha+\beta) + \dot{y}_R \sin(\alpha+\beta) + \dot{\theta}_R l \sin\beta = 0$

1. Pure rolling constraint:
   All motion along the direction of the wheel plane must be accompanied by the appropriate amount of wheel spin so that there is pure rolling at the contact point.
2. Sliding constraint:
   The component of the wheel's motion orthogonal the wheel plane must be zero.

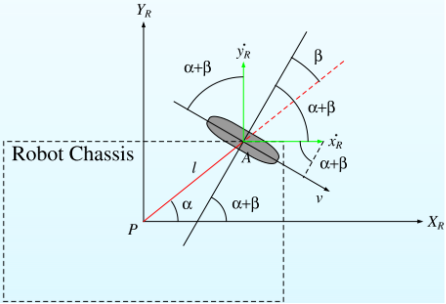
Constraints for Fixed Standard Wheel

From velocity along the motion direction in the local reference frame:

$$\dot{x}_R \sin(\alpha + \beta) - \dot{y}_R \cos(\alpha + \beta) - \dot{\theta}_R l \cos\beta = v = r\dot{\varphi}$$

This equation can be written as

$$[\sin(\alpha + \beta) \;\; -\cos(\alpha + \beta) \;\; (-l)\cos\beta] \begin{bmatrix} \dot{x}_R \\ \dot{y}_R \\ \dot{\theta}_R \end{bmatrix} = r\dot{\varphi}$$



Constraints for Fixed Standard Wheel

- The velocity vector $\dot{\xi}_I$ is given in the global reference frame, but all other parameters are in terms of the robot's local frame.
- Therefore the motion from the coordinate frame $\{O; X_I, Y_I\}$ must be transform into motion in the local frame $\{P; X_R, Y_R\}$.
- The motion along the wheel plane must be equal to the motion accomplished by spinning the wheel, $r\dot{\varphi}$.

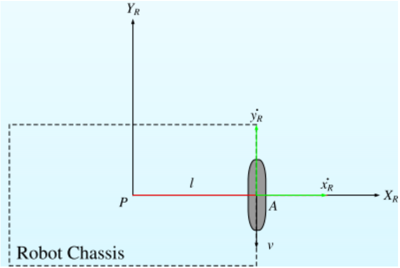
Pure rolling condition at the contact point can be expressed as:

$$[\sin(\alpha+\beta) \;\; -\cos(\alpha+\beta) \;\; -l\cos\beta] \underbrace{R^{\mathsf{T}}(\theta)\dot{\xi}_I}_{\dot{\xi}_R} - r\dot{\varphi} = 0$$

The sliding constraint for this wheel enforces that the component of the motion orthogonal to the wheel plane must be zero:

$$[\cos(\alpha+\beta) \;\; \sin(\alpha+\beta) \;\; l\sin(\beta)] \; R^{\mathsf{T}}(\theta)\dot{\xi}_I = 0$$

Constraints for Fixed Standard Wheel – Example



For the case shown in the figure, the center A of the wheel is in a position with $\alpha = 0, \beta = 0$. If orientation $\theta = 0$, then the sliding constraint reduces

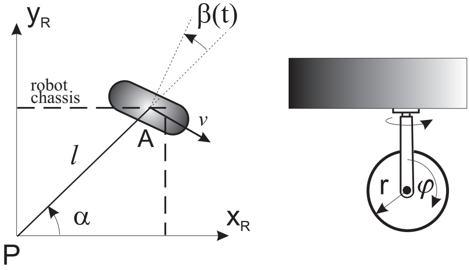
$$[1\ 0\ 0] \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = [1\ 0\ 0] \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = 0$$

This constraints the component of motion along X_I axis to be zero, the wheel is constrained from sliding sideways.

Steered Standard Wheel

The only difference between the steered standard wheel and fixed standard wheel:

- The steered standard wheel may rotate around a vertical axis of rotation passing through the center of the wheel and the ground contact point.
- The angle β of the wheel plane relative to the chassis varies as a function of time $\beta(t)$.
- Other parameters are identical to that of the fixed standard wheel.



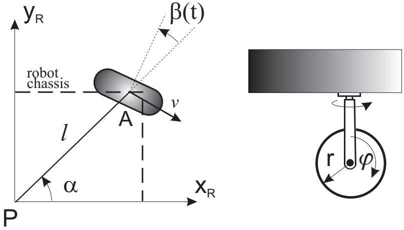
Constraints for Steered Standard Wheel

The rolling and sliding constraints are

$$[\sin(\alpha+\beta(t)) \;\; -\cos(\alpha+\beta(t)) \;\; (-l)\cos(\beta(t))]\, R^{\mathsf{T}}(\theta)\dot{\boldsymbol{\xi}}_I(t) - r\dot{\varphi}(t) = 0$$

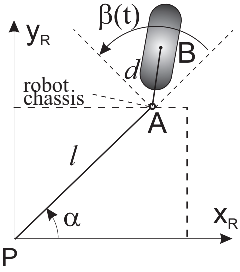
$$[\cos(\alpha+\beta(t)) \;\; \sin(\alpha+\beta(t)) \;\; l\sin(\beta(t))]\, R^{\mathsf{T}}(\theta)\dot{\boldsymbol{\xi}}_I(t) = 0$$

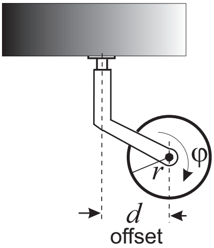
- These constraints are identical to those of the fixed standard wheel.
- Unlike $\dot{\varphi}$, the change of steering angle, $\dot{\beta}$, does not have a direct impact on the instantaneous motion constraints of the robot.
- It is only by integrating over time that changes in steering angle can affect the mobility of a vehicle.



Castor Wheel

- The vertical rotation axis in a castor wheel does not pass through the ground contact point.
- The wheel contact point is at position B, connected by a rigid rod AB of fixed length d to point A.
- The point A fixes the location of the vertical axis around which B steers.
- The castor wheel has two parameters that vary:
  - $\varphi(t)$ represents the wheel spin over time,
  - $\beta(t)$ the steering angle and orientation of AB over time.



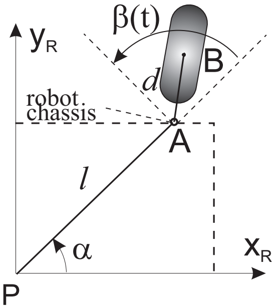


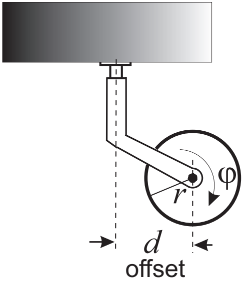
Constraints for Castor Wheel

Rolling Constraint:
The rolling constraint of the castor wheel is identical to that of the steered standard wheel

$$[\sin(\alpha+\beta(t)) \quad -\cos(\alpha+\beta(t)) \quad (-l)\cos(\beta(t))]\, R^{\mathsf{T}}(\theta)\dot{\boldsymbol{\xi}}(t) - r\dot{\varphi}(t) = 0$$

since the offset axis plays no role during motion that is aligned with the wheel plane.





Constraints for Castor Wheel

The lateral force on the wheel occurs at point A because this is the attachment point of the wheel to the chassis.

- Due to the offset ground contact point relative to A, the constraint that there be zero lateral movement could be wrong.
- Instead, the constraint is much like a rolling constraint – an appropriate rotation of the vertical axis must take place.

Compared to steered standard wheel, two more terms should be added to the sliding constraint:

- The line velocity due to the angle β of the castor wheel rotation: $d\dot{\beta}$
- The line velocity due to the angle θ of the chassis rotation: $d\dot{\theta}$

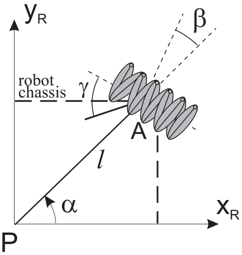
The sliding constraint of the castor wheel is

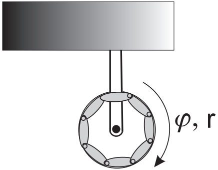
$$[\cos(\alpha + \beta(t)) \;\; \sin(\alpha + \beta(t)) \;\; d + l\sin(\beta(t))]\, R^{\mathsf{T}}(\theta)\dot{\boldsymbol{\xi}}(t) + d\,\dot{\beta}(t) = 0$$

It means that:

- Any motion orthogonal to the wheel plane must be balanced by an equivalent and opposite amount of castor steering motion.
- By setting the value of $\dot{\beta}$, any arbitrary lateral motion can be acceptable.

- Swedish wheels have no vertical axis of rotation, yet are able to perform omnidirectional motion like the castor wheel.
- One more degree of freedom is provided by the passive rollers.
- The angle γ between the roller axes and the wheel plane can vary.
- Since each axis can spin clockwise or counterclockwise, any vector along one axis can be combined with any vector along the other axis.
- These two axes are not necessarily independent (except in the case of the Swedish 90° wheel).





Swedish Wheel: Sliding Constraint

The instantaneous constraint is due to the specific orientation of the small rollers.

- The axis around which those rollers spin is a zero component of velocity at the contact point.
- Moving in that direction without spinning the main axis is not possible without sliding.

Compared to fixed standard wheel, the sliding constraint should be modified by adding the angle γ such that the motion along the roller axis is zero:

$$[\sin(\alpha+\beta+\gamma) \;\; -\cos(\alpha+\beta+\gamma) \;\; (-l)\cos(\beta+\gamma)]\, R^{\mathsf{T}}(\theta)\dot{\boldsymbol{\xi}} - r\dot{\varphi}\cos\gamma = 0$$

Orthogonal to the roller axis the motion is not constraint because of the free rotation $\dot{\varphi}_{sw}$ of the small rollers.

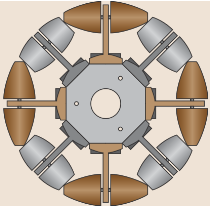
$$[\cos(\alpha+\beta+\gamma) \;\; \sin(\alpha+\beta+\gamma) \;\; l\sin(\beta+\gamma)]\, R^{\mathsf{T}}(\theta)\dot{\boldsymbol{\xi}} - r\sin\gamma\dot{\varphi} - r_{sw}\dot{\varphi}_{sw} = 0$$

90-Degree Swedish Wheel

The behavior of the sliding and rolling constraints and thereby the Swedish wheel changes dramatically as the angle γ varies.

The angle $\gamma = 0$ represents the Swedish 90-degree wheel:

- The zero component of velocity is in line with the wheel plane.
- The sliding constraint reduces to the fixed standard wheel rolling constraint.
- There is no sliding constraint orthogonal to the wheel plane because of the rollers.
- Any desired motion vector can be made to satisfy sliding constraint varying the value of $\dot{\varphi}$.
- The wheel is omnidirectional. It results in decoupled motion: the rollers and the main wheel provide orthogonal directions of motion.



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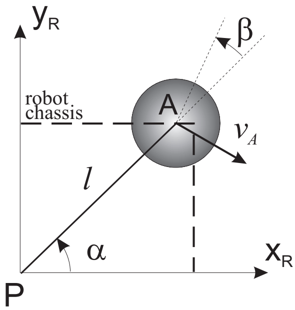
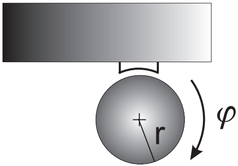
The spherical wheel is omnidirectional and places on constraints on the robot chassis kinematics.

- The spherical wheel places no direct constraints on motion.
- Such a mechanism has no principle axis of rotation, thus no appropriate rolling or sliding constraints exist.
- The spherical wheel is omnidirectional and places *no constraints* on the robot kinematics.





Kinematic Constraints for Spherical Wheel

The roll rate of the ball in the direction of motion v_A of point A is given by

$$[\sin(\alpha + \beta(t)) \;\; -\cos(\alpha + \beta(t)) \;\; (-l)\cos(\beta(t))]\, R^{\mathsf{T}}(\theta)\dot{\boldsymbol{\xi}}(t) - r\dot{\varphi}(t) = 0$$

By definition the wheel rotation orthogonal to the direction of motion is zero:

$$[\cos(\alpha + \beta(t)) \;\; \sin(\alpha + \beta(t)) \;\; (-l)\sin(\beta(t))]\, R^{\mathsf{T}}(\theta)\dot{\boldsymbol{\xi}}(t) = 0$$

- The above equations for spherical wheel are exactly the same as for the fixed standard wheel, but interpretation is quite different.
- The omnidirectional spherical wheel can have any arbitrary direction of movement, because the motion direction given by the angle β is a free variable.