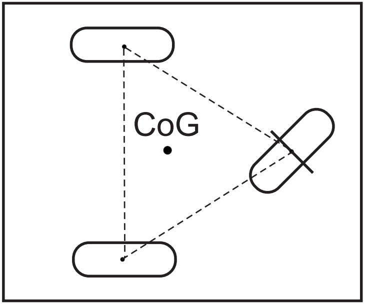
Mobile Robots

Wheeled Robot Structures

Lecture 4

- The mobile base consists of two parts: a platform and several wheel modules.
- The choice of wheel types and the choice of wheel arrangement (or wheel geometry) must be considered simultaneously when designing the locomotion mechanism.
- Three fundamental characteristics of a robot are governed by these choices: **maneuverability**, **controllability** and **stability**.
- There is no single wheel configuration that maximizes all these qualities for the variety of environments faced by different mobile robots.
- The highest level of maneuverability requires wheels that can move in more than just one direction, and so omnidirectional robots usually employ Swedish or spherical wheels that are powered.
- Generally, there is an inverse correlation between *controllability* and *maneuverability*, e.g. four-castor wheel configuration requires significant processing to convert desired rotational and translational velocities to individual wheel commands.

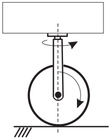
- Conventionally, static stability requires a minimum of three wheels, and the center of gravity (CoG) must be contained within the triangle formed by the ground contact points of the wheels. However, the two wheeled robot can be stable. (When?)

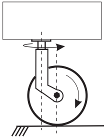


Steering Action

There are different results from a steering action of different wheel types:

- Steered standard wheel: the steering action does not by itself cause a movement of the robot platform (chassis).
- Castor wheel: the steering action itself moves the robot platform because of the offset between the ground contact point and the vertical axis of rotation.

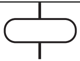
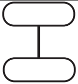




Omnidirectional motion of the castor wheel:

- Given any robot platform motion $\dot{\xi}_I$, there exists some value for spin speed $\dot{\varphi}$ and steering speed $\dot{\beta}$ such that the constraints are fulfilled.
- A robot with only castor wheels can move with any velocity in the space of possible robot motions.
- Kinematics of castor wheels is complex, but such wheels do not impose any real constraints on the kinematics of a robot platform.

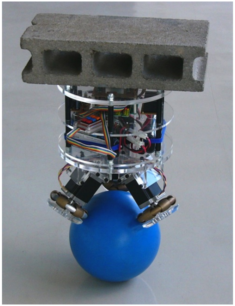
Wheel configurations: Legend

	unpowered omnidirectional wheel (spherical, Swedish, castor)
	motorized Swedish wheel (Stanford wheel)
	unpowered standard wheel
	steered standard wheel
	motorized standard wheel
	motorized and steered standard wheel
	motorized and steered castor wheel
	connected wheels

One-wheel configurations

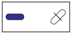
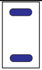
Murata unicycle robot

Star Wars BB-8 robot



Spherical robot

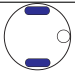
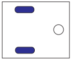
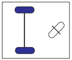
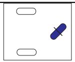
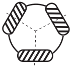
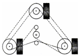
Two-wheel configurations

Arrange-ment	Description	Typical examples
	One steering wheel in the front, one traction wheel in the rear	Bicycle, motorcycle
	Two-wheel differential drive with the center of mass below the axle	Cye personal robot

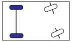
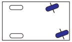
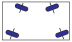
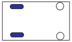
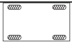
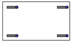
Ghostrider, a motorcycle robot

Cye, a domestic differential-drive robot

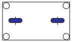
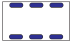
Three-wheel configurations

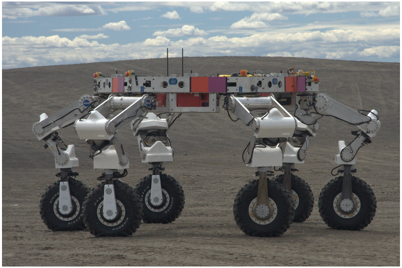
Arrange-ment	Description	Typical examples
	Two-wheel centered differential drive with a third point of contact	Nomad Scout, smartRob EPFL
	Two independently driven wheels in the front/rear + one unpowered omni-wheel in the rear/front	Many indoor robots
	Two connected traction wheels in the rear + one steered free wheel in the front	Piaggio mintruck
	Two free wheels in the rear + one steered wheel in the front	Neptune (CMU), Hero-1
	Three motorized Swedish or spherical wheels arranged in a triangle	Stanford wheel, Tribolo EPFL
	Three synchronously motorized and steered wheels; the orientation is not controllable	Denning MRV-2, I-Robot B24, Nomad 200

Four-wheel configurations

Arrange-ment	Description	Typical examples
	Two motorized wheels in the rear, 2 steered wheels in the front; steering must be different for two wheels to avoid slipping	Car-like robot with rear-wheel drive
	Two motorized and steered wheels in the front, two free wheels in the rear; steering must be different for two wheels to avoid slipping	Car-like robot with front-wheel drive
	Four motorized and steered wheels	Seekur, Hyperion (CMU)
	Two traction wheels (differential) in rear/front, two omnidirectional wheels in the front/rear	Charlie (DMT_EPFL)
	Four omnidirectional wheels	Uranus (CMU)
	Four motorized and steered cas-tor wheels	Nomad XR4000

Six-wheel configurations

Arrange-ment	Description	Typical examples
	Two motorized and steered wheels aligned in center, one omnidirectional wheel at each corner	?
	Six-wheel drive (can be treated as a differential)	Electron robot

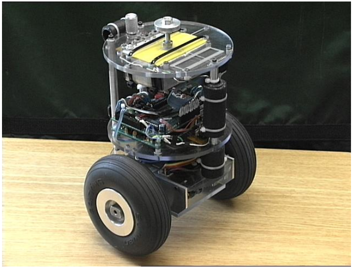


ATHLETE rover (JPL)

Electron robot

Differential Drive Robot

- Differential-drive robot has two driving wheels (plus omnidirectional wheel or roller-ball for balance).
- When both wheels turn at the same speed in the same direction, the robot moves straight in that direction.
- When one wheel turns faster than the other, the robot turns in an arc toward the slower wheel.
- When the wheels turn in opposite directions, the robot turns in place.

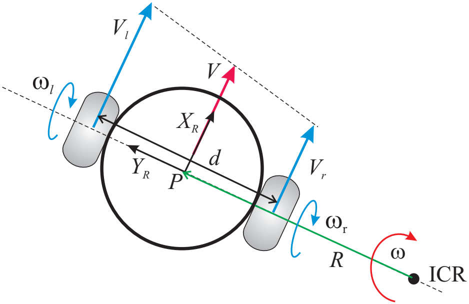


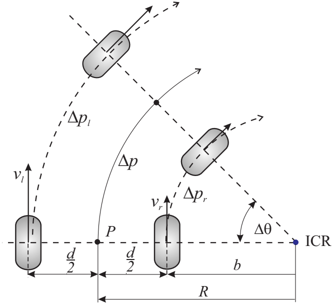
Two-wheel robot

Two fixed wheels + castor wheel

Differential Drive Robot

- The robot must rotate around a point that lies on the common axis – this point is known as the Instantaneous Center of Curvature (ICC) or the Instantaneous Center of Rotation (ICR).





v_r, v_l	–	right/left wheel linear velocity
ω_r, ω_l	–	right/left wheel angular velocity
r	–	wheel radius
v	–	robot linear velocity
ω	–	robot angular velocity
R	–	instantaneous radius of rotation

Wheel linear velocity:

$$v_r = r\,\omega_r = \omega(R - \frac{d}{2}) = \omega R - \omega\frac{d}{2} \tag{1}$$

$$v_l = r\,\omega_l = \omega(R + \frac{d}{2}) = \omega R + \omega\frac{d}{2} \tag{2}$$

Subtracting (1)-(2)

$$v_r - v_l = -\frac{2\omega d}{2}$$

$$\omega = \frac{v_r - v_l}{d}$$

Adding (1)+(2)

$$2\omega R = v_l + v_r$$

$$R = \frac{d\,(v_l + v_r)}{2\,(v_r - v_l)} = \frac{v}{\omega}$$

Robot linear v and angular ω velocity:

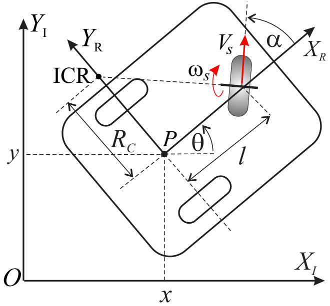
$$v = \omega\,R = \frac{v_l + v_r}{2} = \frac{r\,(\omega_l + \omega_r)}{2}, \qquad \omega = \frac{d\theta}{dt} = \frac{v}{R} = \frac{v_l - v_r}{d} = \frac{r\,(\omega_l - \omega_r)}{d}$$

Tricycle Robot

- Steerable and powered front wheel.
- Free-spinning two rear wheels (passive wheels).
- Limited radius of curvature.
- ICR must lie on the line that passes through the fixed wheels.

$$R = l\tan(\frac{\pi}{2} - \alpha); \qquad \omega = \frac{\omega_s \cdot r}{\sqrt{l^2 + R^2}} = \frac{v_s}{l}\sin\alpha; \qquad v = v_s\cos\alpha$$

r – steering wheel radius, ω_s, v_s – steering wheel angular, linear velocity.



Ackermann Steering

- Used almost exclusively in the automotive industry.
- Wheels have limited turning angles (no in-place rotation).
- The extended axes for the two front wheels intersect in a common point that lies on the extended axis of the rear axle.
- Such a steering geometry satisfies the Ackerman equation:

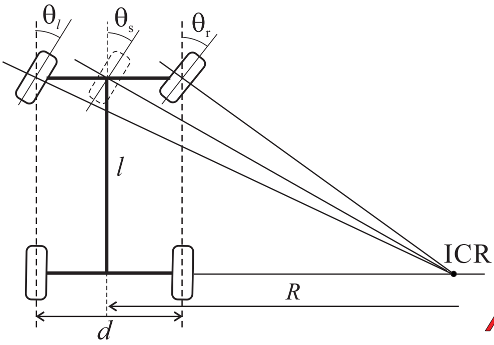
$$\cot\theta_l - \cot\theta_r = \frac{d}{l}$$

Vehicle steering angle can be expressed:

$$\cot\theta_s = \frac{d}{2l} - \cot\theta_r$$

or, alternatively,

$$\cot\theta_s = \cot\theta_l - \frac{d}{2l}$$



Instantaneous radius:

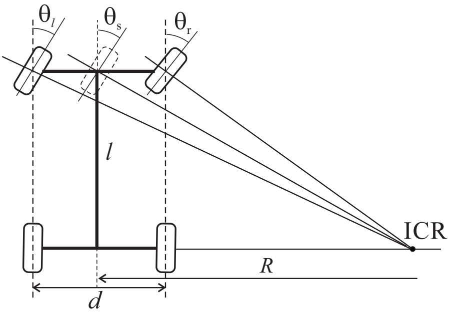
$$R = \frac{l}{\tan\theta_r} + \frac{d}{2}$$

Front wheels linear velocity:

$$v_{fr} = \frac{\omega \cdot l}{\sin\theta_r}; \quad v_{fl} = \frac{\omega \cdot l}{\sin\theta_l}$$

Rear wheels linear velocity:

$$v_{rr} = \omega(R - \frac{d}{2}); \quad v_{rl} = \omega(R + \frac{d}{2})$$

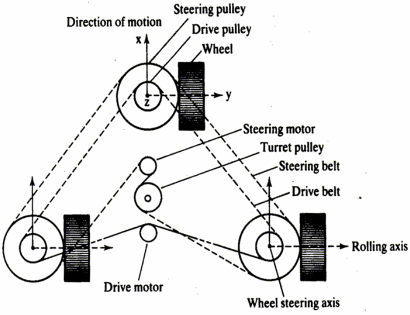


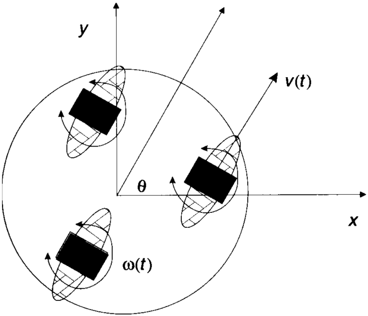
Front wheels steering angle:

$$\theta_l = \tan^{-1}\left(\frac{l}{R + d/2}\right); \qquad \theta_r = \tan^{-1}\left(\frac{l}{R - d/2}\right)$$

Synchro Drive

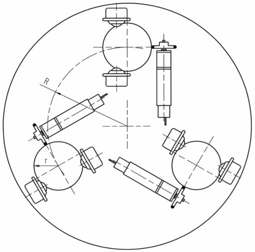
- All wheels are actuated synchronously by one motor – defines the speed of the vehicle.
- All wheels steered synchronously by a second motor – sets the heading of the vehicle.
- The orientation of the robot frame remains the same – there is no direct way of reorienting the robot platform.

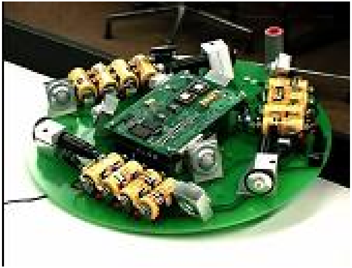




Omnidirectional drive with three spherical wheels

- The omnidirectional robot with three spherical wheels each actuated by one motor.
- The spherical wheels are suspended by three contact points, two given by spherical bearings, and one by a wheel connected to the motor axle.
- The robot has good maneuverability and simple design, but it is limited to flat surfaces and small loads.

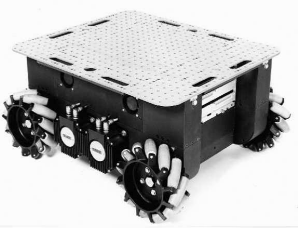




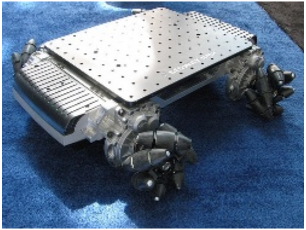
Arrangement of spherical bearings and motors – Tribolo Robot EPFL

Omnidirectional drive with four Swedish wheels

- Four Swedish 45-degree wheels each driven by separated motor.

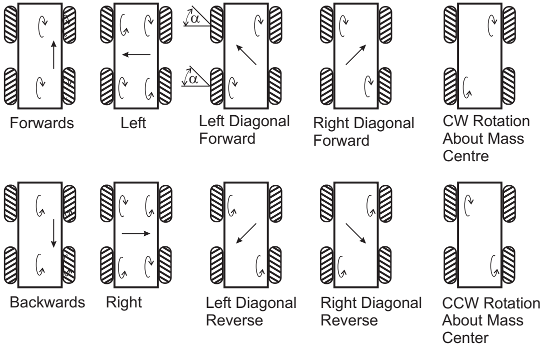


The CMU Uranus robot



Omnidirectional drive with four Swedish wheels

- By varying the direction of rotation and relative speeds of the four wheels, the robot can move along any trajectory in the plane.
- When all four wheels spin “forwards” or “backwards” the robot moves in a straight line forward or backward, respectively.
- When one diagonal pair of wheels spin in the same direction and other diagonal pair spin in the opposite direction, the vehicle moves laterally.

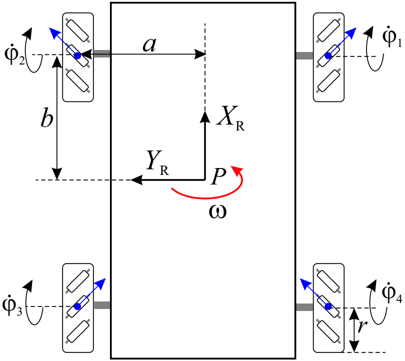


Omnidirectional drive with four Swedish wheels

Wheels configuration:

i	α_i	γ_i
1	$-\arctan(\frac{a}{b})$	$+\frac{\pi}{4}$
2	$+\arctan(\frac{a}{b})$	$-\frac{\pi}{4}$
3	$\frac{\pi}{2}+\arctan(\frac{a}{b})$	$+\frac{\pi}{4}$
4	$-\frac{\pi}{2}-\arctan(\frac{a}{b})$	$-\frac{\pi}{4}$

$$l_i = \sqrt{a^2 + b^2}$$



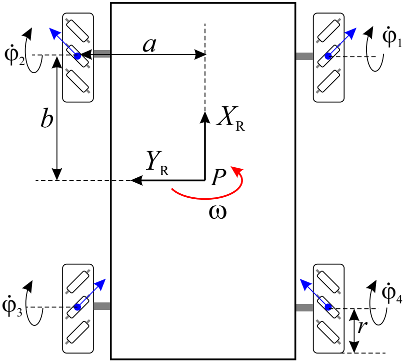
Omnidirectional drive with four Swedish wheels

Relationship between wheel velocities and platform velocity:

$$\begin{bmatrix} \dot{\varphi}_1 \\ \dot{\varphi}_2 \\ \dot{\varphi}_3 \\ \dot{\varphi}_4 \end{bmatrix} = J \begin{bmatrix} \dot{x}_R \\ \dot{y}_R \\ \dot{\theta}_R \end{bmatrix},$$

gdzie

$$J = \frac{1}{r} \begin{bmatrix} 1 & 1 & (a+b) \\ 1 & -1 & -(a+b) \\ 1 & 1 & -(a+b) \\ 1 & -1 & (a+b) \end{bmatrix}$$

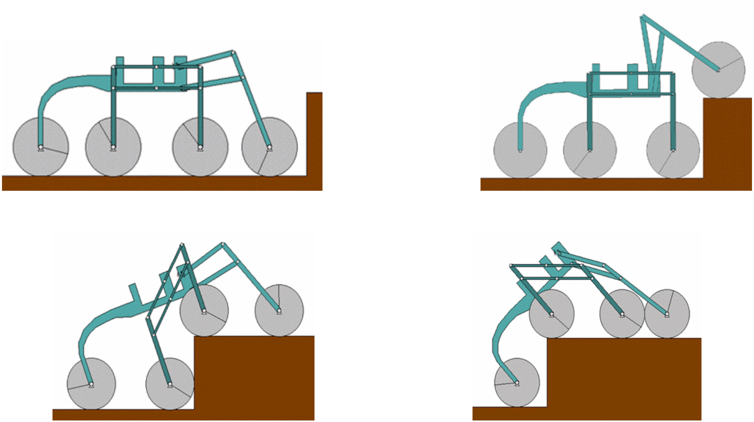


Robot Shrimp – walking wheels

- The Shrimp robot has six wheels: one fixed wheel in the rear; two bogies on each side; one front wheel with spring suspension.
- Robot is sizing around 60 cm in length and 20 cm in height.
- Highly stable in rough terrain.
- Overcomes obstacles up to 2 times its wheel diameter

Shrimp rover EPFL Lausanne, Switzerland

Robot Shrimp – walking wheels



Given a mobile robot with N wheels, how to compute the kinematic constraints of the robot platform?

- Each wheel imposes zero or more constraints on robot motion.
- Combination of the wheel constraints imposes the overall constraints for the vehicle.
- Only fixed and steerable standard wheels impose constraints.
- The castor wheel, Swedish wheel, and spherical wheel impose no kinematic constraints on the robot platform.

Subscripts used to identify the quantities related to four types of wheels:

- 'f' for fixed,
- 's' for steering wheels,
- 'c' for castor wheels,
- 'sw' for Swedish wheels.

Suppose we have a total of $N = N_f + N_s + N_c + N_{sw}$ wheels.

The configuration of the robot is described by the generalized coordinate vector:

- *posture coordinates*: the posture vector $\boldsymbol{\xi}_I(t) = [x(t), y(t), \theta(t)]^\mathsf{T}$
- *orientation coordinates*: the $N_s + N_c$ orientation angles of the steering and caster wheels $\boldsymbol{\beta}(t) = [\boldsymbol{\beta}_s, \boldsymbol{\beta}_c]^\mathsf{T}$
- *rotation coordinates*: the N rotation angles of the wheels, $\boldsymbol{\varphi}(t) = [\boldsymbol{\varphi}_f(t), \boldsymbol{\varphi}_s(t), \boldsymbol{\varphi}_c(t), \boldsymbol{\varphi}_{sw}(t)]^\mathsf{T}$.

Rolling Constraints

Equations for the kinematic constraints in the matrix form:

$$J_1(\beta_s)R^{\mathsf{T}}(\theta)\dot{\xi}_I + J_2\dot{\varphi} = 0, \tag{3}$$

where $R(\theta)$ is a rotation matrix, and $J_1(\beta_s)$ is a matrix with projections for all wheels to their motions along their individual wheel planes:

$$J_1(\beta_s) \triangleq \begin{bmatrix} J_{1f} \\ J_{1s}(\beta_s) \end{bmatrix},$$

where

- J_{1f} a constant matrix of size $(N_f \times 3)$
- $J_{1s}(\beta_s)$ matrix of size $(N_s \times 3)$, is only a function of β_s, not β_f.
- J_2 is a constant diagonal matrix $(N \times N)$ whose entries are radii r of all standard wheels

Sliding Constraints

The sliding constraints of the standard wheels (fixed and steerable) can be combined into a single expression:

$$C_1^*(\beta_s)R^\mathsf{T}(\theta)\dot{\xi}_I = 0, \tag{4}$$

where

$$C_1^*(\beta_s) \triangleq \begin{bmatrix} C_{1f} \\ C_{1s}(\beta_s) \end{bmatrix},$$

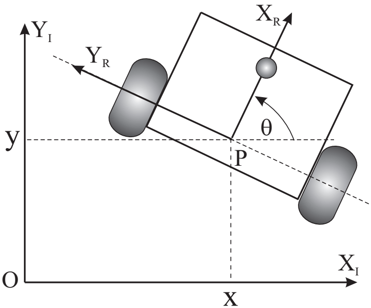
C_{1f} and C_{1s} are $(N_f \times 3)$ and $(N_s \times 3)$ matrices whose rows are the three terms in the (1×3) row vector of for all fixed and steerable standard wheels.

- The sliding constraint describes that, for all standard wheels, the components of motion orthogonal to their wheel planes must be zero.
- The sliding constraint over all standard wheels has the most significant impact on defining the overall **maneuverability** of the robot platform.

Constraints for Differential Drive Robot

Differential drive robot with three wheels.
The passive spherical wheel is omnidirectional and does not enforce any constraints.
Since the remaining two fixed drive wheels are not steerable, $J_1(\beta_s)$ and $C_1^*(\beta_s)$ simplify to J_{1f} and C_{1f},respectively.

- For the right wheel: $\alpha = -\frac{\pi}{2}$, $\beta = \pi$.
- For the left wheel: $\alpha = \frac{\pi}{2}$, $\beta = 0$.



The rolling constraint for fixed wheel is expressed as:

$$[\sin(\alpha+\beta) \;\; -\cos(\alpha+\beta) \;\; (-l)\cos\beta] R^{\mathsf{T}}(\theta)\dot{\xi}_I - r\dot{\varphi} = 0,$$

hence, the rolling constraints for both fixed standard wheels can be written as

$$\begin{bmatrix} 1 & 0 & l \\ 1 & 0 & -l \end{bmatrix} R^{\mathsf{T}}(\theta)\dot{\xi}_I - \begin{bmatrix} r & 0 \\ 0 & r \end{bmatrix} \begin{bmatrix} \dot{\varphi}_r \\ \dot{\varphi}_l \end{bmatrix} = 0$$

The sliding constraint for this wheel:

$$\left[\cos(\underbrace{\alpha+\beta}_{\frac{\pi}{2}}) \;\; \sin(\underbrace{\alpha+\beta}_{\frac{\pi}{2}}) \;\; l\sin(\underbrace{\beta}_{0}) \right] R^{\mathsf{T}}(\theta)\dot{\xi}_I(t) = 0$$

after substituting we obtain the identical sliding constraints for both fixed standard wheels

$$[0 \; 1 \; 0] R^{\mathsf{T}}(\theta)\dot{\xi}_I = 0$$

Mobile Robot Maneuverability

Kinematic mobility of a robot platform:

- The ability to directly move in the environment.
- The basic constraint is that every wheel must satisfy its sliding constraint.

In addition to instantaneous kinematic motion, a mobile robot is able to further manipulate its pose over time by steering steerable wheels.

Overall maneuverability of a robot:

- The mobility available based on the kinematic sliding constraints of the standard wheels.
- The additional freedom contributed by steering and spinning the steerable standard wheels.

We introduce three characteristics:

- Degree of mobility – δ_m
- Degree of steerability – δ_s
- Degree of maneuverability – $\delta_M = \delta_m + \delta_s$

- To avoid any lateral slip the motion vector $R(\theta)\dot{\xi}_I$ has to satisfy the following constraints:

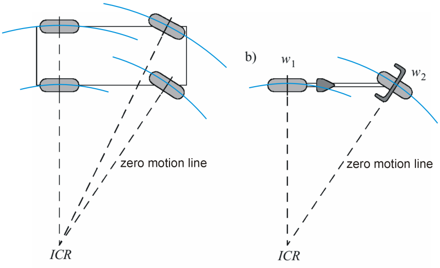
$$C_1^*(\beta_s)R^\mathsf{T}(\theta)\dot{\xi}_I = 0, \text{ where } C_1^*(\beta_s) = \begin{bmatrix} C_{1f} \\ C_{1s}(\beta_s) \end{bmatrix}$$

- The motion vector $R^\mathsf{T}(\theta)\dot{\xi}_I$ must belong to the null space of the projection matrix $C_1^*(\beta_s)$.
- **Null space** of $C_1^*(\beta_s)$ is the space $\mathcal{N}$ such that for any vector $n \in \mathcal{N}$

$$C_1^*(\beta_s) \cdot n = 0$$

- Geometrically this can be shown by the Instantaneous Center of Rotation (ICR).

- At any given instant, wheel motion along the zero motion line must be zero. The wheel is moving instantaneously along some circle of radius R, such that the center of that circle is located on the zero motion line.
- This center is called the Instantaneous Center of Rotation (ICR).
- The ICR geometric construction demonstrates how robot mobility is a function of the number of constraints on the robot's motion, not the number of wheels.



ICR: a) a four wheel car-like Ackerman steering; b) a bicycle

A bicycle and differential-drive platform have the same number of wheels, but the bicycle has two independent kinematic constraints while the differential-drive has only one:

- For a bicycle, the two constraints are independent and together constrain the overall robot motion, thus, there is only a single point for the ICR.
- For a differential-drive robot, the second wheel imposes no additional kinematic constraints since its zero motion line is identical to that of the first wheel. Thus, the ICR is constrained to lie along a line, not at a specific point.

A wheel may not be able to contribute an independent constraint to the robot kinematics, for example, the second steerable standard wheel of the Ackerman vehicle. (Why?)

The degree of mobility δ_m for a robot is defined as

$$\delta_m = \dim \mathcal{N}(C_1^*(\beta_s)) = 3 - \operatorname{rank}(C_1^*(\beta_s)), \quad 0 \leq \operatorname{rank} C_1^*(\beta_s) \leq 3$$

- The degree of mobility δ_m is a measure of the number of DOF of the robot that can be **immediately** manipulated through changes in wheel velocity.
- The range of δ_m is:

$$1 \leq \delta_m \leq 3$$

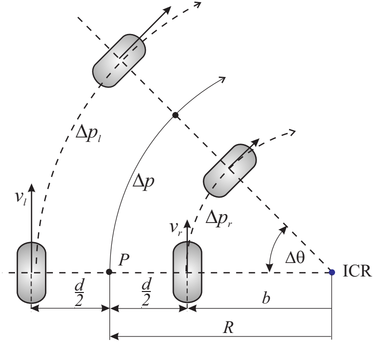
Note: The degree of mobility quantifies the degrees of controllable freedom based on changes to wheel velocity.

Degree of Mobility

- Robot platform kinematics is a function of the set of independent constraints: the greater the rank of $C_1^*(\beta_s)$ the more constrained is the mobility.
- $\text{rank } C_1^*(\beta_s) = 0$:
  - It is only possible if there are zero independent kinematic constraints in $C_1^*(\beta_s)$.
  - There are neither fixed nor steerable standard wheels attached to the robot frame.
- $\text{rank } C_1^*(\beta_s) = 3$:
  - The robot is completely constrained in all directions and is degenerate.
- If $\text{rank} C_{1f} > 1$ then the vehicle can, at best, travel along a circle or along a straight line.
  - The robot has two or more independent constraints due to fixed standard wheels that do not share the same horizontal axis of rotation.

For a differential-drive base, $\operatorname{rank} C_1^*(\beta_s) = 1$ and $\delta_m = 2$:

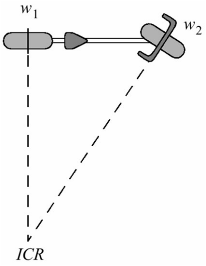
- A differential-drive robot can control the change rates in both orientation and its forward/reverse speed by manipulating wheel velocities.
- Its ICR is constrained to lie on the infinite line extending from its wheels' horizontal axles.



Degree of Mobility: Examples

For a bicycle platform, $\operatorname{rank} C_1^*(\beta_s) = 2$ and $\delta_m = 1$:

- There is one less degree of mobility compared to differential-drive robot.
- A bicycle only has control over its forward/reverse speed by direct manipulation of wheel velocities.
- Only by steering can the bicycle change its ICR.



- Any robot consisting only of omnidirectional wheels such as castor, Swedish or spherical wheels will have the maximum mobility, $\delta_m = 3$.
- Such a robot can directly manipulate all three degrees of freedom.

Degree of Steerability

The degree of steerability δ_s is defined as

$$\delta_s = \operatorname{rank} C_{1s}(\beta_s)$$

The degree of steerability δ_s **corresponds to a number of independent steerable wheels**.
An increase in the rank of $C_{1s}(\beta_s)$ implies more degrees of steering freedom and thus greater eventual maneuverability.

Since $C_1^*(\beta_s)$ includes $C_{1s}(\beta_s)$, a steered standard wheel can both decrease mobility and increase steerability:

- Its particular orientation at any instant imposes a kinematic constraint.
- Its ability to change that orientation can lead to additional trajectories.

The impact of steering is indirect since the robot must move for the change in steering angle of the steerable standard wheel to have impact on robot pose.

Degree of Steerability

The **range** of δ_s is given by $0 \leq \delta_s \leq 2$.

1. For $\delta_s = 0$: The robot has no steerable standard wheels, $N_s = 0$.
2. For $\delta_s = 1$: The most common case when a robot has one or more steerable standard wheels.
   - For an ordinary vehicle (like automobile) with $N_s = 2$ and $N_f = 2$, the fixed wheels share a common axle and thus $\mathrm{rank} C_{1f} = 1$.
   - The fixed wheels and one of the steerable wheels constrain the ICR.
   - The second steerable wheel cannot impose any independent kinematic constraint and thus $\mathrm{rank} C_{1s}(\beta_s) = 1$.
   - In this case, $\delta_m = 1$ and $\delta_s = 1$.
3. For $\delta_s = 2$: Only possible in robots with no fixed standard wheels: $N_f = 0$.
   - The pseudo-bicycle with two separate steerable standard wheels.
   - Its ICR can be placed anywhere on the ground plane, which is also the meaning of $\delta_s = 2$.

The overall DOFs that a robot can manipulate is called the **degree of maneuverability** δ_M:

$$\delta_M = \delta_m + \delta_s \tag{5}$$

It is defined in terms of mobility and steerability.
Thus, the maneuverability consists of:

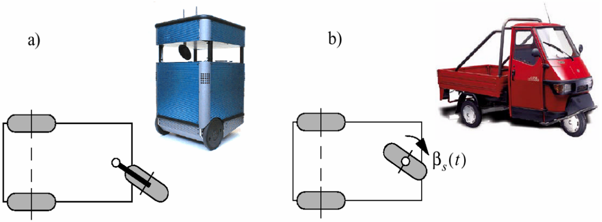
1. The degrees of freedom that the robot manipulates directly through wheel velocity $= \delta_m$.
2. The degrees of freedom that it indirectly manipulates by changing the steering configuration and moving $= \delta_s$.

Two robots with the same δ_M are not necessarily equivalent.

- For any robot with $\delta_M = 2$ the ICR is always constrained to lie on a line.
- For any robot with $\delta_M = 3$ the ICR can be set to any point on the plane.
- The case $\delta_M = 1$ is not acceptable because it corresponds to the rotation of the robot around a fixed ICR.

Differential-drive and tricycle geometries have equal maneuverability $\delta_M = 2$:

- In differential drive all maneuverability is the result of direct mobility $(2, 0)$.
- In the case of a tricycle the maneuverability results from steering also $(1, 1)$.
- Neither configuration allows the ICR to range anywhere on the plane.
- In both cases, the ICR must lie on a predefined line w.r.t. the robot reference frame.



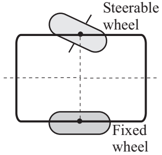
a) differential-drive b) tricycle

Non-degenerate Wheeled Robots

A mobile robot is called **non-degenerate** if:

1. If the robot has more than one conventional fixed wheel ($N_f > 1$), then they are all on a single common axle. Mathematically: $\operatorname{rank} C_{1f} \leq 1$.
2. The centers of the conventional steerable wheels do not belong to this common axle of the fixed wheels.
3. The number $\operatorname{rank} C_{1s}(\beta_s) \leq 2$ is equal to degree of steerability δ_s, i.e. the number of steering wheels that can be oriented independently in order to steer the robot.

If a mobile robot is equipped with more than δ_s conventional centered-orientable wheels (i.e. $N_c > \delta_s$), the motion of the extra wheels must be coordinated to guarantee the existence of ICR at each time instant.

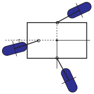


Degenerate robot

Basic Types of Three-Wheel Configurations

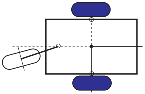
There are five structures of practical interest corresponding to the five pairs (δ_m, δ_s):

δ_m	3	2	2	1	1
δ_s	0	0	1	1	2
δ_M	3	2	3	2	3



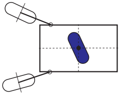
Omnidirectional

$$\delta_M = 3 \quad \delta_m = 3 \quad \delta_s = 0$$



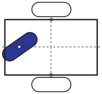
Differential

$$\delta_M = 2 \quad \delta_m = 2 \quad \delta_s = 0$$



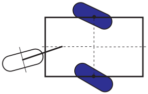
Omni-Steer

$$\delta_M = 3 \quad \delta_m = 2 \quad \delta_s = 1$$



Tricycle

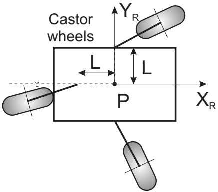
$$\delta_M = 2 \quad \delta_m = 1 \quad \delta_s = 1$$



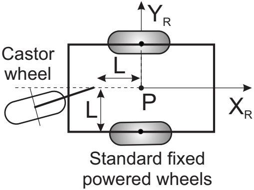
Two-Steer

$$\delta_M = 3 \quad \delta_m = 1 \quad \delta_s = 2$$

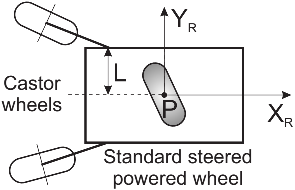
Five Basic Types of Three-Wheel Configurations



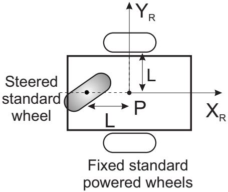
(3, 0)



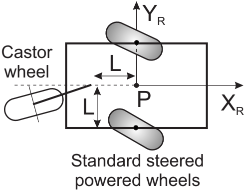
(2, 0)



(2, 1)



(1, 1)



(1, 2)

Mobile Robot Workspace: Degrees of Freedom

- Maneuverability is equivalent to the vehicle's degree of freedom (DOF) that can be controlled directly and indirectly.
- We care about the ways in which the robot can use its control degrees of freedom to position itself in the environment.
- For the car-like robot (Ackerman steering vehicle):
  - The total number of control degrees of freedom is $\delta_M = 2$, one for steering and the other for actuation of the drive wheel.
  - The total degrees of freedom of the vehicle is 3: the car can position itself on the plane at any (x, y) point and with any angle θ.
  - Note: The dimension of a robot's configuration space can exceed δ_M.

We also care about how the robot is able to move between various configurations:

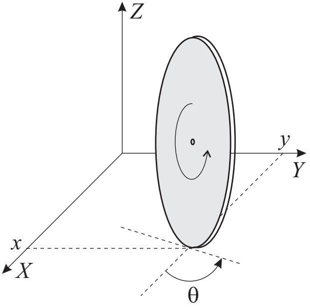
- What are the types of paths it can follow?
- What are its possible trajectories through this configuration space?

Admissible Velocity Space

Given the kinematic constraints of a robot, its velocity space describes the independent components of robot motion that the robot can control.

The velocity space of a unicycle can be represented by $(\dot{\varphi}, \dot{\theta})$:

- $\dot{\varphi}$ represents the instantaneous forward speed of the unicycle.
- $\dot{\theta}$ represents the instantaneous change in orientation.



Unicycle

Admissible Velocity Space

The number of dimensions in the velocity space of a robot is the number of independently achievable velocities.
It is called the **differential degrees of freedom** ($DDOF$).

A robot's $DDOF$ is always equal to its degree of mobility δ_m.

- The $DDOF$ of a bicycle is 1 ($\dot{\varphi}$).
- The $DDOF$ of a omnibot is 3 ($\dot{x}, \dot{y}, \dot{\theta}$).

The relationship between DOF and $DDOF$:

$$DDOF \leq \delta_M \leq DOF$$

The workspace DOF governs the robot's ability to achieve various poses, and the robot's $DDOF$ governs its ability to achieve various paths.