

Mobile Robots

Perception: Sensors for Mobile Robots



Lecture 7 / **A**

Sensing and Estimation

- ▶ Sensing and estimation are fundamental aspects of the design of any autonomous mobile robot.
- ▶ Controlling a robot would be relatively simple if:
 - ▶ a complete model of the environment was available, and
 - ▶ the robot actuators could execute motion commands perfectly relative to this model.



- ▶ However, in most real-world situations:
 - ▶ a complete world model is not available, and
 - ▶ perfect control of mobile platforms is never a realistic assumption.
- ▶ Sensing and estimation are a means of compensating for this lack of complete information.
- ▶ To acquire a knowledge about the environment and the state of the robot system various sensors can be used.



- ▶ Sensing and estimation together constitute the process of transforming a physical quantity into a computer representation that can be used for further processing.
- ▶ The information provided by the sensors serve a basis for control, decision making, and interaction with other agents in the environment.
- ▶ Sensing is closely tied to transducers that transform some physical entity into a signal that can be processed by a computer.

Transduction principle

Transduction is a conversion of energy from one form to another.

- ▶ Perception is the process of representing the sensory information in an task-oriented model of the world.



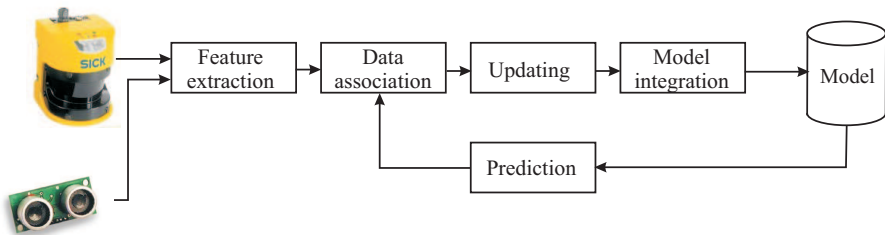
Remark:

Measurement in a real world environment is error prone.

- ▶ Sensor data is usually corrupted by:
 - ▶ statistical noise arises from the transducer,
 - ▶ discretization introduced in the digitization process,
 - ▶ ambiguity introduced by poor sensor selectivity.
- ▶ Therefore estimation methods are introduced:
 - ▶ to support appropriate integration of information into models of the environment,
 - ▶ and for improvement of the signal-to-noise ratio.



Perception Process



- ▶ The input to the perception process is typically twofold:
 - (1) data from a number of sensors/transducers,
 - (2) a partial model of the environment (a world model) that includes information about the state of the robot and other relevant entities in the external world.

Perception Process

- ▶ The initial step in sensory processing is data preprocessing and feature extraction.
- ▶ The role of preprocessing is to:
 - ▶ reduce noise from the transducer,
 - ▶ remove systematic errors, and
 - ▶ enhance relevant aspects of the data.
- ▶ Data association methods are used to estimate the relationship between sensor data and the model of the environment.
- ▶ This matching process typically is based on the optimization that maximizes the alignment of features to the model.
- ▶ The world model is updated with new information contained in the sensor data.
- ▶ Using a dynamical system model, it is possible to predict how the world changes over time until new sensory data is acquired.



Sensor

A **sensor** is a device that measures a physical quantity and converts it into a signal which can be read by an observer or by an instrument.

- ▶ *Object Proximity*
 - ▶ Presence/absence, distance, bearing, color, etc.
- ▶ *Physical orientation/attitude/position*
 - ▶ Magnitude, pitch, roll, yaw, coordinates, etc.
- ▶ *Light*
 - ▶ Presence, frequency, intensity, content (mod), direction
- ▶ *Sound*
 - ▶ Presence, frequency, intensity, content (mod), direction
- ▶ *Heat*
 - ▶ Temperature, wavelength, magnitude, direction
- ▶ *Magnetic and Electric Fields*
 - ▶ Presence, magnitude, orientation, content (mod)
- ▶ *Chemicals*
 - ▶ Presence, concentration, identity, etc.

Sensors can be classified depending on:

- ▶ **what** they measure, and
- ▶ **how** they measure it.

What is measured:

- ▶ **Proprioceptive** (PC) sensors
 - ▶ used to measure the **internal state** of a robot, e.g., motor speed, motor current, heading of the robot, battery voltage.
- ▶ **Exteroceptive** (EC) sensors
 - ▶ used to measure the external world – information from the robot environment, e.g., distances to objects, object images, light intensity, sound amplitude.



Measurement principle (how it is measured):

▶ **Passive (P)** sensors

- ▶ sensors measure ambient energy entering the sensor (energy coming from the environment): temperature probes, CCD and CMOS cameras, microphones, etc.

▶ **Active (A)** sensors

- ▶ emit the required energy and measure the environmental reaction, examples are: wheel encoders, ultrasonic sensors, laser range finders,
- ▶ they have better performance, but active sensing has some influence on the environment.



Functional Classification of Sensors

General classification (typical use)	Sensor type	PC/EC	A/P
Tactile, proximity sensors (detection of physical contact or closeness)	Switches, whiskers, bumpers	EC	P
	Optical barriers	EC	A
	Non-contact proximity sensors	EC	A
Wheel/motor sensors (wheel/motor speed and position)	Brush encoders	PC	P
	Potentiometers	PC	P
	Resolvers	PC	A
	Optical encoders	PC	A
	Magnetic encoders	PC	A
	Inductive encoders	PC	A
	Capacitive encoders	PC	A
Heading sensors (orientation of the robot with respect to a fixed reference frame)	Compass	EC	P
	Gyroscopes	PC	P
	Inclinometers	EC	A/P
Beacons (Localization in an inertial reference frame)	GPS	EC	A
	Active optical or RF beacons	EC	A
	Active ultrasonic beacons	EC	A
	Reflective beacons	EC	A
Active ranging (reflectivity, time-of-flight and geometric triangulation)	Reflectivity sensors	EC	A
	Ultrasonic sensor	EC	A
	Laser rangefinder	EC	A
	Optical triangulation (1D)	EC	A
	Structured light	EC	A
	Capacitive/magnetic sensors	EC	A/P

A-active; P-passive; A/P-active/passive; PC-proprioceptive; EC-exteroceptive



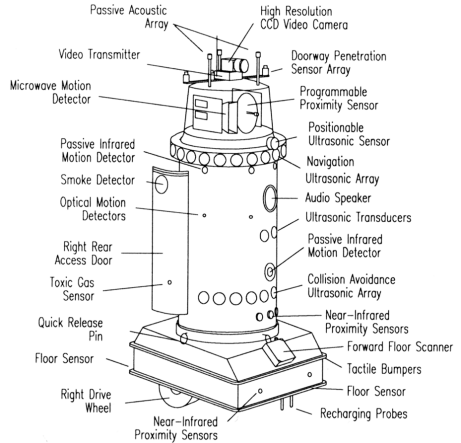
Functional Classification of Sensors

General classification (typical use)	Sensor type	PC/EC	A/P
Motion/speed sensors (speed relative to fixed or moving objects)	Doppler radar	EC	A
	Doppler sound	EC	A
	Camera	EC	P
	Accelerometer	EC	P
Identification	Cameras	EC	P
	Radio frequency identification RFID	EC	A
	Laser ranging	EC	A
	Radar	EC	A
	Ultrasound	EC	A
	Sound	EC	P
Vision-based sensors (visual ranging, image analysis)	CCD/CMOS cameras	EC	P

A-active; P-passive; A/P-active/passive; PC-proprioceptive; EC-exteroceptive

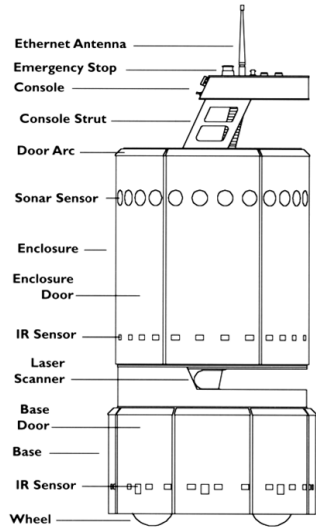


Sensors for Mobile Robots



Robart II robot, H.R. Everett

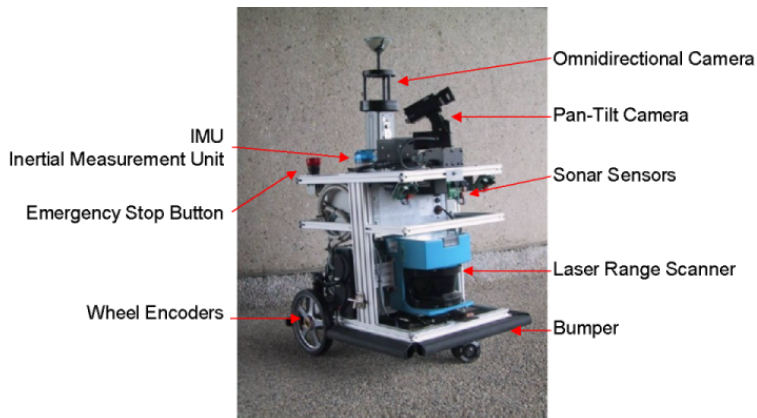
Sensors for Mobile Robots



B21 robot, Real World Interface



Sensors for Mobile Robots



BibaBot robot, BlueBotics SA, Switzerland

Sensors for Mobile Robots

Electron robots, IC&CE WUT



laser scanner on the pan mechanism



laser scanner + omnicamera



stereovision system

Sensors for Mobile Robots

► Features-front



Aibo, Sony, Japan



Basic sensor response ratings:

- ▶ **Dynamic range:** Ratio of maximum to minimum input values, usually measured in decibels [dB].
 - ▶ When referring to measurements of power or intensity (eg., from 1 Milliwatt to 100 Watts), a ratio can be expressed as

$$L_{dB} = 10 \cdot \log_{10} \left(\frac{100}{0.001} \right) = 50dB$$

- ▶ When referring to measurements of voltage, current (e.g., for 1 Millivolt (mV) to 100 Volt), a ratio can be expressed as

$$L_{dB} = 20 \cdot \log_{10} \left(\frac{100}{0.001} \right) = 100dB$$

- ▶ In mobile robots sensors frequently exposed to input values beyond their working range, e.g. laser rangefinder will have a minimum operating range and can thus provide spurious data when measurements are taken with the object closer than that minimum.



Sensor Characteristics

- ▶ **Range:** The minimum and maximum input values.
- ▶ **Resolution:** Minimum detectable difference between two values
 - ▶ usually it is the lower limit of the dynamic range of a sensor,
 - ▶ but for digital sensors, it is usually the A/D resolution: e.g. $0-5V / 255$ (8 bit) $\approx 20mV$.
- ▶ **Linearity:** Variation of output as a function of input.
 - ▶ Sensor is linear if two different inputs, x and y , yield outputs $f(x)$ and $f(y)$, and for any values a and b , $f(ax + by) = af(x) + bf(y)$.
- ▶ **Bandwidth or Frequency:** the speed with which a sensor can provide a stream of readings
 - ▶ usually there is an upper limit depending on the sensor and the sampling rate
 - ▶ a lower limit is also possible, e.g. for an acceleration sensor

Remark

The above sensor characteristics can be reasonably measured in a laboratory environment with confident extrapolation to performance in real-world deployment.



Remark:

For the most sophisticated sensors (e.g. visual sensors, active ranging sensors) the measurement process cannot be reliable without deep understanding of the complex interactions between all environmental characteristics and the sensor itself.

Characteristics that are especially relevant for real-world environments:

- ▶ **Sensitivity:** a ratio of output change to input change (this is a measure of degree to which an incremental change in the target input signal changes the output signal)
 - ▶ unfortunately, in real-world environment, the sensor has very often high sensitivity to other environmental parameters, e.g. illumination, temperature
- ▶ **Cross-sensitivity:** sensitivity to environmental parameters that are orthogonal to the target parameters for the sensor
 - ▶ the magnetic compass demonstrates high sensitivity to ferrous building materials, it often makes the sensor useless in some indoor environments.



- ▶ **Error:** the difference between the sensor's output and the true value

$$\varepsilon = m - v,$$

where m – measured value, v – true value

- ▶ in fact, obtaining the ground truth, v , can be difficult or impossible
- ▶ **Systematic errors:** deterministic errors
 - ▶ caused by factors that can (in theory) be modeled: prediction
 - ▶ can sometimes be compensated for by means of some kind of calibration strategy
- ▶ **Random errors:** non-deterministic errors
 - ▶ cannot be predicted,
 - ▶ however, they can be described probabilistically (stochastically).
 - ▶ Noise is a random error that can be reduced by signal processing, such as filtering, usually at the expense of the dynamic behaviour of the sensor.



In Situ Sensor Performance

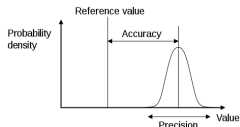
- ▶ **Accuracy:** the degree of closeness of measurements of a quantity to its actual (true) value.

$$accuracy = 1 - \frac{\varepsilon}{v},$$

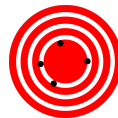
- ▶ **Precision:** reproducibility (also called repeatability) of sensor measurements, it is the ratio of sensor's output range to the standard deviation σ of the random error of a sensor

$$precision = \frac{range}{\sigma}$$

- ▶ sensor taking multiple readings of the same environmental state has high precision if it generates the same output,
- ▶ precision does not have any bearing on the accuracy of the sensor's output with respect to the true value.



Accuracy indicates proximity of measurement results to the true value,
precision to the repeatability of the measurement.



High precision, but low accuracy. High accuracy, but low precision.



The Challenges in Mobile Robotics

- ▶ Mobile robots depend heavily on exteroceptive sensors.
- ▶ A mobile robot must perceive, analyze and interpret the state of the surrounding.
- ▶ Measurements in real world environment are dynamically changing and error prone.
- ▶ Sources of errors are: specular reflections, changing illuminations, light or sound absorbing surfaces, cross-sensitivity of robot sensors to robot pose and robot-environment dynamics (deviations appear as random errors as they are hard to model).
- ▶ Behavior of sensor's random error is modeled by probability distribution over various output values.
 - ▶ usually very little knowledge about the causes of random errors is available
 - ▶ often a probability distribution is assumed to be symmetric or even Gaussian, these are strong assumptions that enable powerful mathematical tools to be applied to mobile robot problems, but it is important to realize how wrong these assumptions usually can be.



Tactile Sensors

▶ Bumpers and Guards:

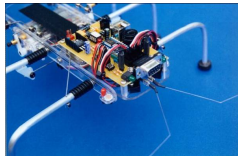
- ▶ micro-switches and wires
- ▶ impact/collision sensor, senses pressure/contact
- ▶ devices that control a binary state: they are either on or off, closed or open,
- ▶ simple, cheap, and easy to read

▶ Whiskers

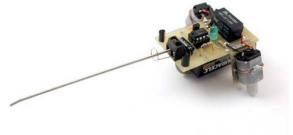
- ▶ typically a wire suspended through conductive loop
- ▶ deflection causes contact with the loop
- ▶ reaches beyond robot a 15-20 cm
- ▶ simple and cheap device with binary output



Lego touch sensor

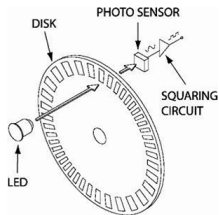


Whiskers

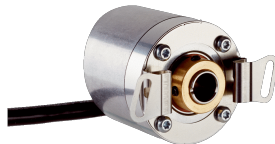


Wheel/Motor Encoders I

- ▶ Estimation of rotational motion is fundamental for estimation of ego-motion for mobile robots.
- ▶ Optical encoders are proprioceptive sensors thus the position estimation in relation to a fixed reference frame is only valuable for short movements.
- ▶ Encoders measure angular position or speed of the wheels or steering.
- ▶ A code disc rotates with the wheel, and a photo-emitter/detector pair senses the light being blocked and unblocked.

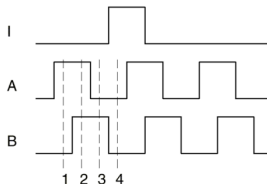
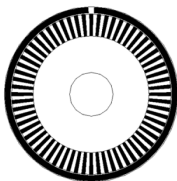


Incremental encoder



Wheel/Motor Encoders II

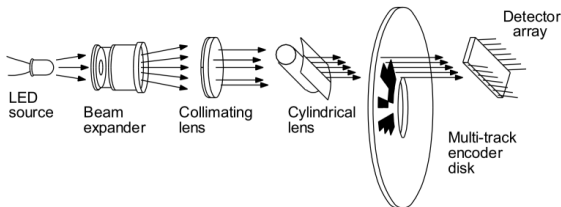
- ▶ The most common sensor is the quadrature encoder, which is composed of a transparent disc, with two periodic patterns that are out of phase.
- ▶ Wheel rotations can be integrated over time to estimate position, but positional estimate is subject to cumulative error.
- ▶ Direction of rotation given by the phase difference between the emitter signals (i.e. by which one is 'leading').
- ▶ Resolution is measured in cycles per revolution (CPR).



State	Ch A	Ch B
S ₁	High	Low
S ₂	High	High
S ₃	Low	High
S ₄	Low	Low

Phase-quadrature encoder

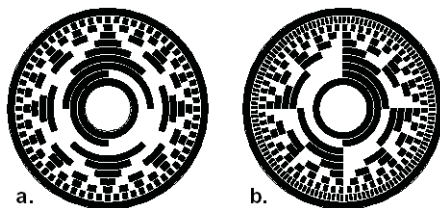
Absolute Encoders I



- ▶ Best suited for slow and/or infrequent rotations such as steering angle encoding.
- ▶ Used when potential loss of reference from power interruption cannot be tolerated.
- ▶ Discrete detector elements in a photovoltaic array are individually aligned in break-beam fashion with concentric encoder tracks.
- ▶ A line source of light passing through a coded pattern of opaque and transparent segments on the rotating encoder disk results in a parallel output that uniquely specifies the absolute angular position.



- ▶ The most common coding schemes are Gray code, natural binary, and binary-coded decimal.

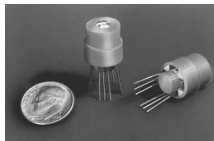


Code discs: a. Gray code; b. Natural binary code

- ▶ Heading sensors are used to determine the robot's orientation and inclination.
- ▶ Together with an appropriate velocity information, they allow to integrate the movement to a pose estimate: **dead reckoning**.
- ▶ Heading sensors can help compensate for the foremost weakness of odometry: in an odometry-based positioning method, any small momentary orientation error will cause a constantly growing lateral position error.
- ▶ Types of heading sensors:
 - ▶ Compass (terrestrial magnetic field): *exteroceptive*
 - ▶ Inclinometer (measurement of angles in reference to gravity): *exteroceptive*
 - ▶ Gyroscope (orientation to a fixed frame): *proprioceptive*
 - ▶ Mechanical Gyroscopes
 - ▶ Optical Gyroscopes

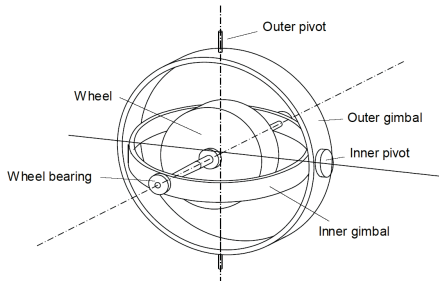
Compasses

- ▶ A navigational instrument for determining direction relative to the earth's magnetic poles.
- ▶ Compasses are exteroceptive sensors.
- ▶ Large variety of solutions to measure the earth magnetic field: mechanical magnetic compass, direct measure of the magnetic field (Hall-effect, magnetoresistive sensors).
- ▶ Hall effect compasses measure the voltage difference induced by the magnetic field in two orthogonal directions:
 - ▶ cheap, but resolution and accuracy are rather poor
 - ▶ filtering circuits can improve performance (e.g. by averaging values over time), but reduce bandwidth.
- ▶ Major drawbacks: weakness of the earth field, disturbance by magnetic objects or other sources, not feasible for indoor environments.



Hall-effect compass

Mechanical Gyroscopes I



Two-axis mechanical gyroscope

- ▶ Gyroscopes rely on the principle of the conservation of angular momentum.
- ▶ Angular momentum is the tendency of a rotating object to keep rotating at the same angular speed about the same axis of rotation in the absence of an external torque.

Mechanical Gyroscopes II

- ▶ Reactive torque τ (tracking stability) is proportional to the spinning speed ω , the precession speed Ω and the wheel's inertia I .

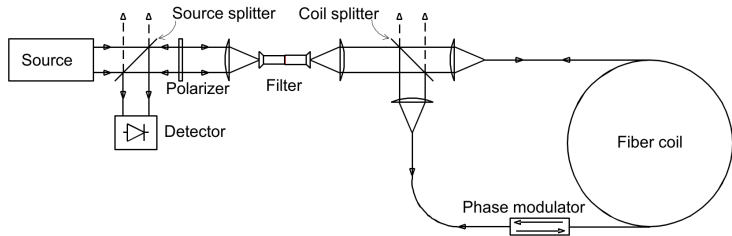
$$\tau = I\omega\Omega; \quad L = I \times \omega$$

where L is the angular momentum of the spinning wheel.

- ▶ No torque can be transmitted from the outer pivot to the wheel axis, and spinning axis will therefore be space-stable.
- ▶ Quality: 0.1° in 6 hours.
- ▶ If the spinning axis is aligned with the north-south meridian, the earth's rotation has no effect on the gyro's horizontal axis.
- ▶ If it points east-west, the horizontal axis reads the earth rotation.
- ▶ Rate gyros measure a vehicle's rotation rate (its angular rate of rotation).
- ▶ They have the same basic arrangement shown as regular mechanical gyros, but gimbals are restrained by a torsional spring that enables to measure angular speeds instead of the orientation.



Optical Gyroscopes

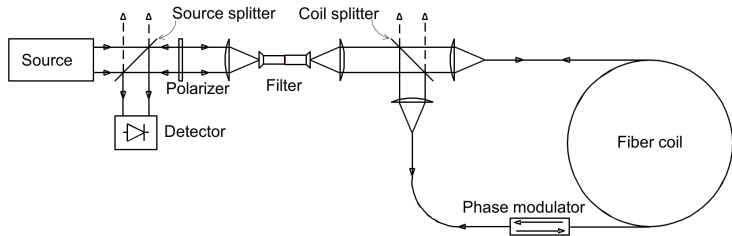


Fiber-optic gyroscope

- ▶ Optical gyroscopes rely on the *Sagnac effect* rather than rotational inertia in order to measure (relative) heading.
- ▶ They are angular speed sensors that use two monochromatic light (or laser) beams from the same source.
- ▶ One is traveling in a fiber clockwise, the other counterclockwise around a cylinder.



Optical Gyroscopes

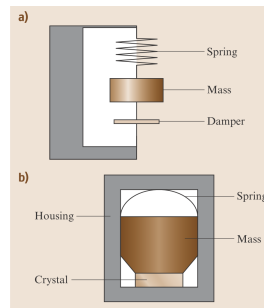


Fiber-optic gyroscope

- ▶ Laser beam traveling in direction of rotation has a slightly shorter path and will have a higher frequency.
- ▶ The difference in frequency Δf of the two beams is proportional to the angular velocity Ω of the cylinder.
- ▶ New solid-state optical gyroscopes based on the same principle are built using micro-fabrication technology.

Accelerometers

- ▶ Accelerometers can be used to measure external forces acting on the vehicle.
- ▶ They are sensitive to all external forces acting upon them – including gravity.
- ▶ A mechanical accelerometer is a spring-mass-damper system with some mechanism for external monitoring.
- ▶ A piezoelectric accelerometer is based on a property exhibited by certain crystals, across which a voltage is generated when they are stressed.



a) Mechanical accelerometer

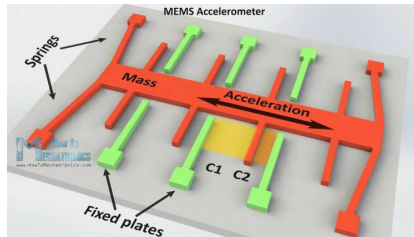
b) Piezoelectric accelerometer

Assuming an ideal spring with a force proportional to its displacement, the external forces balance the internal ones:

$$F_{applied} = F_{inertial} + F_{damping} + F_{spring} = m\ddot{x} + c\dot{x} + kx,$$

Accelerometers

- ▶ MEMS, or Micro Electro-Mechanical System accelerometers measure the linear acceleration of whatever they are attached to.
- ▶ MEMS accelerometers implement capacitive sensing output a voltage dependent on the distance between moving and fixed plates.
- ▶ They sense the displacement of the plates proportional to the applied acceleration.



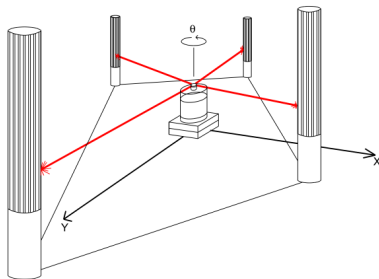
MEMS accelerometer working principle

Ground-Based Active and Passive Beacons

- ▶ Beacons are signaling guiding devices with a precisely known position.
- ▶ Beacon-based navigation is used since the humans started to travel:
 - ▶ natural beacons (landmarks) like stars, mountains or the sun
 - ▶ artificial beacons like lighthouses
- ▶ The Global Positioning System (GPS) revolutionized modern navigation technology (also in outdoor robotics, but for indoor robots, GPS is not applicable).

Major drawback with the use of beacons in indoor environment:

- ▶ Beacons require costly changes in the environment.
- ▶ Limit flexibility and adaptability to changing environments.



- ▶ Active range sensors are the most popular sensors in mobile robotics.
- ▶ They are widely used for obstacle detection and avoidance, and also used for the purpose of map building and robot localization.
- ▶ There are three basic approaches to measuring range:
 - ▶ Sensors based on measuring the *time of flight* (TOF) of a pulse of emitted energy traveling to a reflecting object, then echoing back to a receiver.
 - ▶ The phase-shift measurement (or phase-detection) ranging technique involves continuous wave transmission as opposed to the short pulsed outputs used in TOF systems.
 - ▶ Radars based on measuring the beat frequency between a frequency-modulated continuous wave (FMCW) and its received reflection.
- ▶ There are also geometric active ranging sensors: the optical triangulation sensor and structured light sensor.



Range Sensors (Time-of-Flight)

- ▶ Principle of operation of most radar, laser and active acoustic devices.
- ▶ TOF ultrasonic sensors as well as laser range sensors make use of propagation speed of sound or electromagnetic waves, respectively.
- ▶ The traveled distance of a sound or electromagnetic wave is given by

$$d = c \cdot t,$$

where

d – distance traveled (usually round-trip)

c – speed of wave propagation

t – time of flight

- ▶ The range is calculated as $r = \frac{d}{2}$.
- ▶ For efficient operation a narrow beam must be formed to concentrate the energy and to constrain the angular uncertainty of the target direction.
- ▶ The relevant parameters involved in range calculation are the speed of sound in air (roughly 0.3 m/ms), and the speed of light (0.3 m/ns).

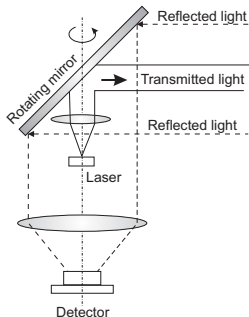


Potential error sources for TOF systems:

- ▶ Uncertainties about the exact time of arrival of the reflected signal.
- ▶ Inaccuracies in the time of flight measure (laser range sensors).
- ▶ Opening angle of transmitted beam (ultrasonic range sensors).
- ▶ Interaction with the target (surface, specular reflections).
- ▶ Variations in the speed of propagation, particularly in the case of acoustical systems.
- ▶ Speed of mobile robot and (dynamic) target.



Laser Rangefinder



Laser scanners



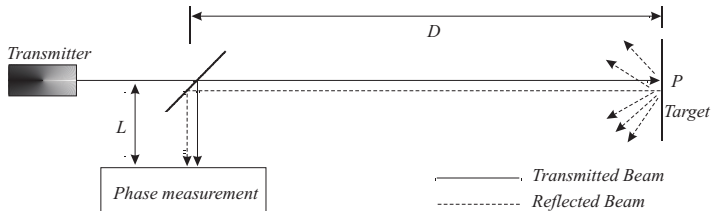
Laser range sensor with rotating mirror; SICK Inc., Germany; URG-04LX Hokuyo Co., Japan

- ▶ The laser rangefinder is a TOF sensor.
- ▶ Transmitted and received beams are coaxial.
- ▶ Transmitter illuminates a target with a collimated beam.
- ▶ Receiver detects the time needed for round-trip.



Phase-Shift Laser Rangefinder I

The sensor transmits amplitude modulated light at a known frequency and measures the phase shift between transmitted and reflected signals.



$$\lambda = \frac{c}{f}; \quad D' = L + 2D = L + \frac{\theta}{2\pi} \lambda$$

where:

λ – the wavelength of the modulating signal,

c – the speed of light,

f – the modulating frequency,

D' – total distance covered by the emitted light,

θ – phase difference between the transmitted and reflected beam.

Phase-Shift Laser Rangefinder II

The distance D between the beam splitter and the target is given by

$$D = \frac{\lambda}{4\pi} \theta,$$

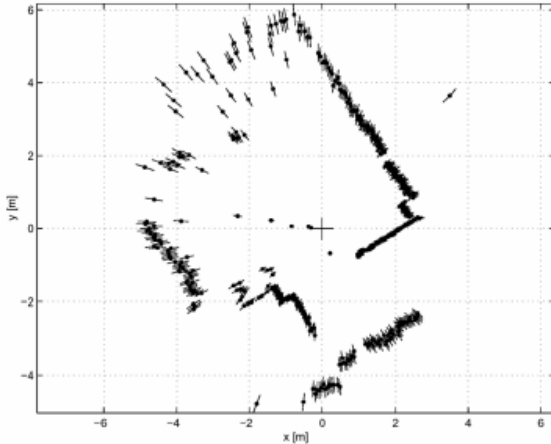
where θ is the measured phase difference between transmitted and reflected light beams.

- ▶ The transmission of a single frequency modulated wave can result in ambiguous range estimates.
- ▶ For example, if $\lambda = 60$ m, a target at a range of 5 m would give indistinguishable phase measurement from a target at 65 m, since each phase angle would be 2π radians apart.
- ▶ We have “ambiguity interval” of λ , but in practice the range of the sensor is much lower than λ due to attenuation of the signal in air.
- ▶ The confidence in the range (phase estimate) is inversely proportional to the square of the received signal amplitude, directly affecting the sensor's accuracy. Dark, distant objects will not produce as good range estimates as close, bright objects.

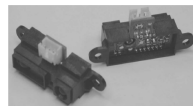
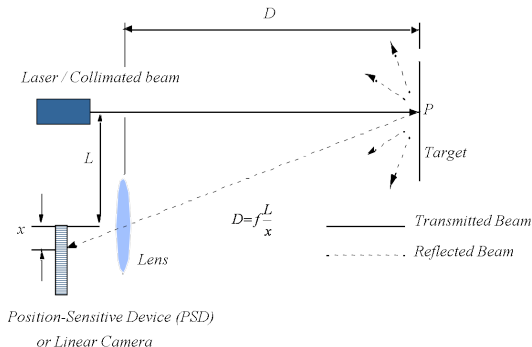


Laser Rangefinder Measurements

Typical range image of a 2D laser range sensor with a rotating mirror. The length of the lines through the measurement points indicate the uncertainties.



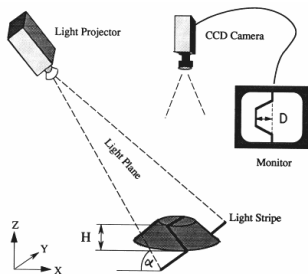
Laser Triangulation (1D sensor)



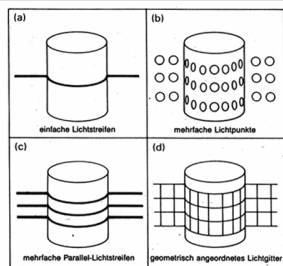
- ▶ A collimated beam (focused LED or laser beam) is transmitted toward the target.
- ▶ The reflected light is collected by a lens and projected onto a PSD or linear camera.
- ▶ The distance is proportional to $1/x$, therefore the resolution is best for close objects and becomes poor at a distance.



Structured Light (2D or 3D)



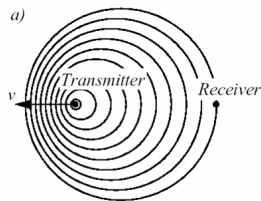
$$H = D \tan \alpha$$



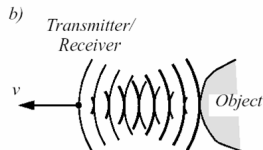
- ▶ The sensor can recover distance to a large set of points instead of to only one point.
- ▶ The emitter projects a known pattern, or structured light, onto the environment.
- ▶ The projected light has a known structure, and the image taken by CDD or CMOS camera can be filtered to identify pattern's reflection.
- ▶ The sensor can work in dark environments as well as environments in which objects are featureless (e.g. uniformly colored and edgeless).



Motion/Speed Sensor: Doppler Effect Based (Radar or Sound)



a) between two moving objects



b) between a moving and a stationary object

$$f_r = f_t \left(1 + \frac{v}{c}\right), \text{ if transmitter is moving; } f_r = f_t \frac{1}{1 + \frac{v}{c}}, \text{ if receiver is moving}$$

$$\Delta f = f_t - f_r = \frac{2f_t v \cos \theta}{c} - \text{Doppler frequency shift; } v = \frac{\Delta f \cdot c}{2f_t \cos \theta} - \text{relative speed,}$$

where: θ – relative angle between direction of motion and axis

- ▶ Well-known example of Doppler-effect: the change in siren pitch when an approaching fire engine passes by and recedes.
- ▶ The Doppler effect applies to sound and electromagnetic waves.



- ▶ One sensor is usually not enough: noise, limited accuracy, unreliable – failure/redundancy, limited point of view of the environment.
- ▶ Combine readings from several sensors into an uniform data structure – produces a merged data set (as though there was one ‘virtual sensor’).
- ▶ Multisensor data fusion is the process of combining observations from a number of different sensors to provide a robust and complete description of an environment or process of interest.
- ▶ Most current data fusion methods employ probabilistic descriptions of observations and processes and use Bayes’ rule to combine this information.
- ▶ Bayes’ rule provides a means to make inferences about an object or environment described by a state x , given an observation z .
- ▶ It may be implemented in a number of ways:
 - ▶ Grid-based models,
 - ▶ Kalman filters,
 - ▶ Extended Kalman Filters
 - ▶ Sequential Monte Carlo techniques

