

Frame Transformation: Vehicle to Earth

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Some of the terminology used in this document may be used incorrectly as the author is not 100% certain of what she is doing, quite yet.

1 Introduction

The following information is meant to illustrate the process of computing the reference frame transformation, from vehicle (V) frame to earth (E) frame, of a two-wheel ground vehicle moving on the ground plane ($D = 0$). This is done through a series of equations to be enumerated in [Section 3](#).

2 Variables

Symbol	Units	Description
e_L	<i>pulse</i>	pulses of left wheel encoder
e_R	<i>pulse</i>	pulses of right wheel encoder
T	<i>s</i>	time step size
E_L	$\frac{pulse}{s}$	pulses of left wheel encoder over the time step
E_R	$\frac{pulse}{s}$	pulses of right wheel encoder over the time step
ω_L	$\frac{rad}{s}$	rate of rotation of left wheel
ω_R	$\frac{rad}{s}$	rate of rotation of right wheel
v_L	$\frac{m}{s}$	velocity of left wheel
v_R	$\frac{m}{s}$	velocity of right wheel
u_v	$\frac{m}{s}$	velocity of vehicle
ω_v	$\frac{rad}{s}$	rate of rotation of vehicle
R_L	<i>m</i>	radius of left wheel
R_R	<i>m</i>	radius of right wheel
L	<i>m</i>	length of vehicle axle
ψ_v	<i>rad</i>	yaw angle of vehicle
$\dot{\psi}_v$	$\frac{rad}{s}$	rate of rotation of vehicle
T_{EV}	<i>m?</i>	position vector from origin of E-frame to origin of V-frame
\dot{T}_{EV}	$\frac{m}{s}$	velocity vector of vehicle in E-frame

3 Equations

$$\omega_L = \frac{2\pi}{N} E_L \quad (1)$$

$$\omega_R = \frac{2\pi}{N} E_R \quad (2)$$

$$v_L = R_L \cdot \omega_L \quad (3)$$

$$v_R = R_R \cdot \omega_R \quad (4)$$

$$u_v = \frac{v_L + v_R}{2} \quad (5)$$

$$\omega_v = \frac{1}{L}(v_L - v_R) \quad (6)$$

$$\dot{\psi}_v = \omega_v \quad (7)$$

$$\dot{T}_{EV} = \begin{bmatrix} \cos \psi_k \\ \sin \psi_k \\ 0 \end{bmatrix} u_v \quad (8)$$

$$E_L = \frac{e_L}{T} \quad (9)$$

$$E_R = \frac{e_R}{T} \quad (10)$$

Using the equations above as a foundation, we can progress to the first step of the transformation process. We want to find ψ_k and T_{EV} , we will do this using Euler's Method.

$$\begin{aligned} \psi_k &= \psi_{k-1} + \omega_v \cdot T \\ &= \psi_{k-1} + \frac{1}{L}(v_L - v_R) \cdot T \\ &= \psi_{k-1} + \frac{1}{L}(R_L \cdot \omega_L - R_R \cdot \omega_R) \cdot T \\ &= \psi_{k-1} + \frac{1}{L}(R_L(\frac{2\pi}{N}E_L) - R_R(\frac{2\pi}{N}E_R)) \cdot T \\ &= \psi_{k-1} + \frac{1}{L}(R_L(\frac{2\pi}{N} \cdot \frac{e_L}{T}) - R_R(\frac{2\pi}{N} \cdot \frac{e_R}{T})) \cdot T \\ &= \psi_{k-1} + \frac{1}{L}(R_L(\frac{2\pi \cdot e_L}{N}) - R_R(\frac{2\pi \cdot e_R}{N})) \\ &= \psi_{k-1} + \frac{2\pi}{N \cdot L}(R_L \cdot e_L - R_R \cdot e_R) \end{aligned} \quad (11)$$

ψ_k is the yaw angle at $t=kT$.
 ψ_{k-1} is the yaw angle at the previous time $t=(k-1)T$.
 We can talk about how we estimate these quantities in the future.

We have now found ψ_k in terms of only known ((what is ψ_{k-1} , do we know it?)) values: ψ_{k-1} , the pulse value of both encoders ($e_{L,R}$), the radius of both wheels ($R_{L,R}$) and the length of the axle (L). Using (1)-(5) and (8)-(10) we will complete the same process for $T_{EV}(k)$, the position vector.

$$\begin{aligned}
T_{EV}(k) &= T_{EV}(k-1) + \begin{bmatrix} \cos \psi_k \\ \sin \psi_k \\ 0 \end{bmatrix} u_v \cdot T \\
&= T_{EV}(k-1) + \begin{bmatrix} \cos \psi_k \\ \sin \psi_k \\ 0 \end{bmatrix} \left(\frac{v_L + v_R}{2} \right) \cdot T \\
&= T_{EV}(k-1) + \begin{bmatrix} \cos \psi_k \\ \sin \psi_k \\ 0 \end{bmatrix} \frac{1}{2} (R_L \cdot \omega_L + R_R \cdot \omega_R) \cdot T
\end{aligned}$$

As seen in the derivation of ψ_k this can be simplified to:

$$\begin{aligned}
&= T_{EV}(k-1) + \begin{bmatrix} \cos \psi_k \\ \sin \psi_k \\ 0 \end{bmatrix} \frac{1}{2} \left(R_L \left(\frac{2\pi \cdot e_L}{N} \right) + R_R \left(\frac{2\pi \cdot e_R}{N} \right) \right) \\
&= T_{EV}(k-1) + \begin{bmatrix} \cos \psi_k \\ \sin \psi_k \\ 0 \end{bmatrix} \frac{\pi}{N} (R_L \cdot e_L + R_R \cdot e_R) \tag{12}
\end{aligned}$$

I am thinking that the previous equation we found for ψ_k must be plugged into the vector but I fail to see how this will all work out nicely.

No. We do not do that substitution.