Frame Transformation: Vehicle to Earth

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Some of the terminology used in this document may be used incorrectly as the author is not 100% certain of what she is doing, quite yet.

1 Introduction

The following information is meant to illustrate the process of computing the reference frame transformation, from vehicle (V) frame to earth (E) frame, of a two-wheel ground vehicle moving on the ground plane (D = 0). This is done through a series of equations to be enumerated in Section 3.

2 Variables

Symbol	Units	Description
e_L	$pulse$	pulses of left wheel encoder
e_R	$pulse$	pulses of right wheel encoder
T	s	time step size
E_L	$\frac{pulse}{s}$	pulses of left wheel encoder over the time step
E_R	$\frac{pulse}{s}$	pulses of right wheel encoder over the time step
ω_L	$\frac{rad}{s}$	rate of rotation of left wheel
ω_R	$\frac{rad}{s}$	rate of rotation of right wheel
v_L	$\frac{m}{s}$	velocity of left wheel
v_R	$\frac{m}{s}$	velocity of right wheel
u_v	$\frac{m}{s}$	velocity of vehicle
ω_v	$\frac{rad}{s}$	rate of rotation of vehicle
R_L	m	radius of left wheel
R_R	m	radius of right wheel
L	m	length of vehicle axle
ψ_v	rad	yaw angle of vehicle
$\dot{\psi}_v$	$\frac{rad}{s}$	rate of rotation of vehicle
T_{EV}	m	position vector from origin of E-frame to origin of V-frame
$\dot{T}_{EV}$	$\frac{m}{s}$	velocity vector of vehicle in E-frame

3 Equations

$$\begin{aligned}
\omega_L &= \frac{2\pi}{N} E_L && (1)\\
\omega_R &= \frac{2\pi}{N} E_R && (2)\\
v_L &= R_L \cdot \omega_L && (3)\\
v_R &= R_R \cdot \omega_R && (4)\\
u_v &= \frac{v_L + v_R}{2} && (5)\\
\omega_v &= \frac{1}{L}(v_L - v_R) && (6)\\
\dot{\psi}_v &= \omega_v && (7)\\
\dot{T}_{EV} &= \begin{bmatrix} \cos\psi_k \\ \sin\psi_k \\ 0 \end{bmatrix} u_v && (8)\\
E_L &= \frac{e_L}{T} && (9)\\
E_R &= \frac{e_R}{T} && (10)
\end{aligned}$$

Using the equations above as a foundation, we can progress to the first step of the transformation process. We need to find ψ_k and T_{EV}, we will do this using Euler's Method.

$$\begin{aligned}
\psi_k &= \psi_{k-1} + \omega_v \cdot T \\
&= \psi_{k-1} + \frac{1}{L}(v_L - v_R) \cdot T \\
&= \psi_{k-1} + \frac{1}{L}(R_L \cdot \omega_L - R_R \cdot \omega_R) \cdot T \\
&= \psi_{k-1} + \frac{1}{L}\left(R_L\left(\frac{2\pi}{N}E_L\right) - R_R\left(\frac{2\pi}{N}E_R\right)\right) \cdot T \\
&= \psi_{k-1} + \frac{1}{L}\left(R_L\left(\frac{2\pi}{N} \cdot \frac{e_L}{\cancel{T}}\right) - R_R\left(\frac{2\pi}{N} \cdot \frac{e_R}{\cancel{T}}\right)\right) \cdot \cancel{T} \\
&= \psi_{k-1} + \frac{1}{L}\left(R_L\left(\frac{2\pi \cdot e_L}{N}\right) - R_R\left(\frac{2\pi \cdot e_R}{N}\right)\right) \\
&= \psi_{k-1} + \frac{2\pi}{N \cdot L}(R_L \cdot e_L - R_R \cdot e_R) && (11)
\end{aligned}$$

We have now found ψ_k in terms of only estimable and known values: a previous yaw angle (ψ_{k-1}), the pulse value of both encoders ($e_{L,R}$), the radius of both wheels ($R_{L,R}$) and the length of the axle (L). Using (1)-(5) and (8)-(10) we will complete the same process for $T_{EV}(k)$, the position vector.

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\begin{aligned}
T_{EV}(k) &= T_{EV}(k-1) + \begin{bmatrix} \cos\psi_k \\ \sin\psi_k \\ 0 \end{bmatrix} u_v \cdot T \\
&= T_{EV}(k-1) + \begin{bmatrix} \cos\psi_k \\ \sin\psi_k \\ 0 \end{bmatrix} \left(\frac{v_L + v_R}{2}\right) \cdot T \\
&= T_{EV}(k-1) + \begin{bmatrix} \cos\psi_k \\ \sin\psi_k \\ 0 \end{bmatrix} \frac{1}{2}(R_L \cdot \omega_L + R_R \cdot \omega_R) \cdot T \\
&\quad \text{As seen in the derivation of } \psi_k \text{ this can be simplified to:} \\
&= T_{EV}(k-1) + \begin{bmatrix} \cos\psi_k \\ \sin\psi_k \\ 0 \end{bmatrix} \frac{1}{2}\left(R_L\left(\frac{2\pi \cdot e_L}{N}\right) + R_R\left(\frac{2\pi \cdot e_R}{N}\right)\right) \\
&= T_{EV}(k-1) + \begin{bmatrix} \cos\psi_k \\ \sin\psi_k \\ 0 \end{bmatrix} \frac{\pi}{N}(R_L \cdot e_L + R_R \cdot e_R) \qquad (12)
\end{aligned}
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