ANNA UNIVERSITY - III SEMESTER - REG 2021

DISCRETE MATHEMATICS

AIM

To extend student's Logical and Mathematical maturity and ability to deal with abstraction and to introduce most of the basic terminologies used in computer science courses and application of ideas to solve practical problems.

OBJECTIVES

At the end of the course, students would • Have knowledge of the concepts needed to test the logic of a program. • Have an understanding in identifying structures on many levels. • Be aware of the counting principles

UNIT I LOGIC AND PROOFS 9 + 3

Propositional Logic – Propositional equivalences-Predicates and quantifiers-Nested Quantifiers- Rules of inference-introduction to Proofs-Proof Methods and strategy

UNIT II COMBINATORICS 9 + 3

Mathematical inductions- Strong induction and well ordering-.The basics of counting-The pigeonhole principle –Permutations and combinations-Recurrence relations-Solving Linear recurrence relations-generating functions-inclusion and exclusion and applications.

UNIT III GRAPHS 9 + 3

Graphs and graph models-Graph terminology and special types of graphs-Representing graphs and graph isomorphism -connectivity-Euler and Hamilton paths

UNIT IV ALGEBRAIC STRUCTURES 9 + 3

Algebraic systems-Semi groups and monoids-Groups-Subgroups and homomorphisms-Cosets and Lagrange's theorem- Ring & Fields (Definitions and examples)

UNIT V LATTICES AND BOOLEAN ALGEBRA 9 + 3

Partial ordering-Posets-Lattices as Posets- Properties of lattices-Lattices as Algebraic systems –Sub lattices –direct product and Homomorphism-Some Special lattices- Boolean Algebra

L: 45, T: 15, TOTAL: 60 PERIODS

TEXT BOOKS:

1. Kenneth H.Rosen, "Discrete Mathematics and its Applications", Special Indian edition, Tata McGraw-Hill Pub. Co. Ltd., New Delhi, (2007). (For the units 1 to 3, Sections 1.1 to 1.7 , 4.1 & 4.2, 5.1 to 5.3, 6.1, 6.2, 6.4 to 6.6, 8.1 to 8.5)

2. Trembly J.P and Manohar R, "Discrete Mathematical Structures with Applications to Computer Science", Tata McGraw–Hill Pub. Co. Ltd, New Delhi, 30th Re-print (2007).(For units 4 & 5 , Sections 2-3.8 & 2-3.9,3-1,3-2 & 3-5, 4-1 & 4-2)

REFERENCES:

1. Ralph. P. Grimaldi, "Discrete and Combinatorial Mathematics: An Applied Introduction", Fourth Edition, Pearson Education Asia, Delhi, (2002).

2. Thomas Koshy, "Discrete Mathematics with Applications", Elsevier Publications, (2006).

3. Seymour Lipschutz and Mark Lipson, "Discrete Mathematics", Schaum's Outlines, Tata McGraw-Hill Pub. Co. Ltd., New Delhi, Second edition, (2007).

CONTENT

Unit I

LOGIC AND PROOFS

1.1 INTRODUCTION

PROPOSITION (OR) STATEMENT:

Proposition is a declarative statement that is either true or false but not both. The truth value of proposition is true or false.

Truth table

It displays the relationship between the truth values of proposition.

Negation of a proposition

If P is a proposition, then its negation is denoted by $\neg P$ or ~p and is defined by the following truth table.

P	$\neg P$
T	F
F	T

EXAMPLE

P - Ram is intelligent

$\neg P$ -Ram is not intelligent

proposition is a declarative sentence which is either true or false but not both.

COMPOUND PROPOSITION

It is a proposition consisting of two or more simple proposition using logical operators.

1.2 LOGICAL CONNECTIVES

(1) DISJUNCTION (OR)

The disjunction of two proposition P and Q is the proposition $P \vee Q$ [read as P or Q] and is defined by the following truth table.

T	T	T
T	F	T
F	T	T
F	F	F

(1) CONJUNCTION (AND)

If P and Q are two propositions , then the conjunction of P and Q is denoted by P∧Q (read as P and Q) and is defined by following truth table.

P	Q	P∧Q
T	T	T
T	F	F
F	T	F
F	F	F

CONDITIONAL AND BI- CONDITIONAL PROPOSITION

(1) Conditional proposition

If p and q are propositions, then the implication "If p then q " denoted by p→q , called the conditional statement of p and q , is defined by following truth table.

p	q	p→q
T	T	T
T	F	F
F	T	T
F	F	T

NOTE

p→q is false when p is true and q is false. Otherwise it is true.

The different situations where the conditional statements applied are listed below.

(1) If p then q
(2) p only if q
(3) q whenever p
(4) q is necessary for p
(5) q follows from p
(6) q when p
(7) p is sufficient for q
(8) p implies q

Converse, contrapositive and Inverse statement

If p→q is a conditional statement, then

(1) q→p is called converse of p→q
(2) ¬q→¬p is called contrapositive of p→q
(3) ¬p→¬q is called inverse of p→q

EXAMPLE

q : Ram study DBMS

p→q: If Ram is a computer science student, then the will study DBMS.

(2) Bi-conditional proposition

If p and q are proposition, then the proposition p if and only if q, denoted by $p \leftrightarrow q$is called the bi-conditional statement and is defined by the following truth table.

p	q	
T	T	T
T	F	F
F	T	F
F	F	T

NOTE

$P \leftrightarrow Q$is true if both p and q have same truth values. Otherwise $P \leftrightarrow Q$ is false.

EXAMPLE

P: You can take the flight

q: You buy a ticket

p↔q: You can take the flight if and only if buy a ticket.

Symbolize the statements using Logical Connectives

Example: 1

The automated reply can be sent when the file system is full.

P: The automated reply can be sent

Q: The file system is full

Solution:

Symbolic form :q→¬ p

EXAMPLE: 2

Write the symbolized form of the statement. If either Ram takes C++ or Kumar takes pascal, then Latha will take Lotus.

R:Ram takes C++

K:Kumar takes Pascal

L:Latha takes Lotus

Symbolic form: $(R \vee K) \to L$

Example 3

Let p,q,r represent the following propositions,

P:It is raining

q: The sun is shining

r: There are clouds in the sky

Symbolize the following statements.

(1) If it is raining, then there are clouds in the sky
(2) If it is not raining, then the sun is not shining and there are clouds in the sky.
(3) The sun is shining if and only if it is not raining.

Solution:

Symbolic form:

(1) $p \to r$
(2) $\neg p \to (\neg q \wedge r)$
(3) $q \leftrightarrow \neg r$

Example: 4

Symbolize the following statements:

(1) If the moon is out and it is not snowing, then Ram goes out for a walk.
(2) If the moon is out, then if it is not snowing,Ram goes out for a walk.
(3) It is not the case that Ram goes out for a walk if and only if it is not snowing or the moon is out.

Solution:

Let the propositions be,

P: The moon is out

Q: It is snowing

R: Ram goes out for a walk.

Symbolic form:

(1) $(p \wedge \neg q) \to r$
(2) $p \to (\neg q \to r)$
(3) $\neg(r \leftrightarrow (\neg q \vee p))$

Example: 5

propositions.

P:I finish writing my computer program before lunch

q: I shall play Tennis in afternoon.

r: The sun is shining

s: The boundary is low.

(1) If the sun is shining, I shall play tennis in the afternoon.
(2) Finishing the writing of my computer program before lunch is necessary for playing tennis in this afternoon.
(3) Low boundary and sunshine are sufficient to play Tennis in this afternoon.

Solution:

Symbolic form:

(1) $r \to q$
(2) $q \to p$
(3) $(s \wedge r) \to q$

<u>Construction of Truth Tables</u>

<u>EXAMPLE: 1</u>

Show that the truth values of the formula $P \wedge (P \to Q) \to Q$ are independent of their components.

Solution:

The truth table for the formula is,

P	Q	$P \to Q$	$P \wedge (P \to Q)$	$(P \wedge (P \to Q)) \to Q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

The truth values of the given formula are all true for every possible truth values of P and Q. Therefore, the truth value of the given formula is independent of their components.

Example 1. Without constructing the truth table show that

$p \to (q \to p) \equiv \neg p(p \to q)$

Solution

$p \to (q \to p) \equiv p \to (\neg q \vee p)$

$\equiv \neg p \vee (\neg q \vee p)$

$\neg$ q)

$\equiv (\neg p \vee p) \vee \neg q$

$\equiv T \vee \neg q$

$\equiv T.$

Example 2. Prove that $p \rightarrow q$ is logically prove that $(\neg p \vee q)$

Solution:

p	q	$p \rightarrow q$	$\neg p \vee \vee\ q$
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

EXAMPLE: 2

Write the symbolized form of the statement. If either Ram takes C++ or Kumar takes pascal, then Latha will take Lotus.

R:Ram takes C++

K:Kumar takes Pascal

L:Latha takes Lotus

Solution:

Symbolic form: $(R \vee K) \rightarrow L$

Tautology.

A statement that is true for all possible values of its propositional variables is called
a tautology universely valid formula or a logical truth.

Example:1. Write the converse, inverse, contra positive of 'If you work hard then you will be rewarded'

Solution:

p: you will be work hard.

q: you will be rewarded.

$\neg$p: You will not be work hard.

$\neg$ q: You will no tbe rewarded.

Converse: $q \rightarrow p$, If you will be rewarded then you will be work hard

Contrapositive: $\neg q \rightarrow p$,if You will not be rewarded then You will not be work hard.

Inverse: $\neg p \rightarrow \neg q$, if You will not be work hard then You will no tbe rewarded.

Example:2. Write the converse, inverse, contra positive of 'If you work hard then you will be rewarded'

Solution:

p: you will be work hard.

q: you will be rewarded.

¬p: You will not be work hard.

¬ q: You will no tbe rewarded.

Converse: q→ p, If you will be rewarded then you will be work hard

Contrapositive: ¬ q→ p,if You will not be rewarded then You will not be work hard.

Inverse: ¬ p→ ¬ q, if You will not be work hard then You will no tbe rewarded.

Example 4.Prove that $(P \to Q) \wedge (Q \to R) \to (P \to R)$

Proof:

Let S: $(P \to Q) \wedge (Q \to R) \to (P \to R)$

To prove: S is a tautology

P	Q	R	$(P \to Q)$	$(Q \to R)$	$(P \to R)$	$(P \to Q) \wedge (Q \to R)$	S
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	T	F	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	T	F	T	F	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

The last column shows that S is a tautology

1.3 PROPOSITIONAL EQUIVALENCE:

Logical Equivalence:

Let p and q be two statements formulas, p is said to be logically equivalent to q if p & q have the same set of truth values or equivalently p & q are logically equivalent if $p \leftrightarrow q$ is tautology.

Hence, $p \Leftrightarrow q$ if and only if $p \leftrightarrow q$ is a tautology.

Logical Implication or Tautological Implication

A statement formula A logically implies another, statement formula B if and only if $A \to B$ is a tautology.

$\therefore A \Rightarrow B$ [A logically iff $A \to B$ is tautology, implies B]

If $A \Rightarrow B$, then

B is called consequent.

Further $A \Rightarrow B$ guarantees that B has the truth value T whenever A has the truth value T.

$\therefore$ In order to show any of the given implications, it is sufficient to show that an assignment of the truth value T to the antecedent of the given conditional leads to the truth value T for the consequent.

1. Prove without using truth table $(P \to Q) \wedge \neg Q \Rightarrow \neg P$

Proof:

Antecedent: $(P \to Q) \wedge \neg Q$

Consequent: $\neg P$

Assume that, the antecedent has the truth value T.

$\therefore \neg Q$ And $(P \to Q)$ both are true.

$\Rightarrow$ Truth value of Q is F and the truth value of P is also F.

$\therefore$ Consequent $\neg P$ is true.

$\therefore$ The truth of the antecedent implies the truth of the consequent.

$$\therefore (P \to Q) \wedge \neg Q \Rightarrow \neg P$$

Example:1Without constructing the truth table show that $p \to (q \to p) \equiv \neg p (p \to q)$

Solution

$p \to (q \to p) \equiv p \to (\neg q \vee p)$

$\equiv \neg p \vee (\neg q \vee p)$

$\equiv \neg p \vee (p \vee \neg q)$

$\equiv (\neg p \vee p) \vee \neg q$

$\equiv T \vee \neg q$

$\equiv T.$

Example 2:Show that $\neg (p \leftrightarrow q) \equiv (p \vee q) \wedge \neg (p \wedge q)$ without constructing the truth table

Solution :

$\neg (p \leftrightarrow q) \equiv (p \vee q) \wedge \neg (p \wedge q)$

$\neg (p \leftrightarrow q) \equiv \neg (p \to q) \wedge (q \to p)$

$\equiv \neg (\neg p \vee q) \wedge (\neg q \vee p)$

$\neg q) \vee ((\neg p \vee q) \wedge p)$

$\equiv \neg(\neg p \wedge \neg q) \vee (q \wedge \neg q) \vee ((\neg p \wedge p) \vee (q \wedge p)$

$\equiv \neg(\neg p \vee q) \vee F \vee F \vee (q \wedge p)$

$\equiv \neg(\neg p \vee q) \vee (q \wedge p)$

$\equiv (p \vee q) \wedge (q \wedge p).$

Consider $(\neg P \wedge \neg Q) \vee (\neg P \wedge \neg R) \Rightarrow \neg(P \vee Q) \vee \neg(P \vee R) \Rightarrow \neg((P \vee Q) \wedge (P \vee R)) \quad (2)$

Using (1) and (2)

$((P \vee Q) \wedge (P \vee Q) \wedge (P \vee R)) \vee \neg((P \vee Q) \wedge (P \vee R))$

$\Rightarrow [(P \vee Q) \wedge (P \vee R)] \vee \neg[(P \vee Q) \wedge (P \vee R)] \Rightarrow T$

Prove the following equivalences by proving the equivalences of the dual

$\neg((\neg P \wedge Q) \vee (\neg P \wedge \neg Q)) \vee (P \wedge Q) \equiv P$

Solution: It‟s dual is

$\neg((\neg P \vee Q) \wedge (\neg P \vee \neg Q)) \wedge (P \vee Q) \equiv P$

Consider,

$\neg((\neg P \vee Q) \wedge (\neg P \vee \neg Q)) \wedge (P \vee Q) \equiv P$	**Reasons**
$\Rightarrow ((P \wedge \neg Q) \vee (P \wedge Q)) \wedge (P \vee Q)$	(Demorgan‟s law)
	(Commutative law)
$\Rightarrow ((Q \wedge P) \vee (\neg Q \wedge P)) \wedge (P \vee Q)$	(Distributive law)
	$(P \vee \neg P \Rightarrow T)$
$\Rightarrow ((Q \vee \neg Q) \wedge P) \wedge (P \vee Q)$	
	$(P \wedge T = P)$
$\Rightarrow (T \wedge P) \wedge (P \vee Q)$	(Absorption law)

Obtain DNF of $Q \vee (P \wedge R) \wedge \neg((P \vee R) \wedge Q)$.

Solution:

$Q \vee (P \wedge R) \wedge \neg((P \vee R) \wedge Q)$

$\Leftrightarrow (Q \vee (P \wedge R)) \wedge ((\neg P \wedge \neg R) \vee \neg Q)$ (Demorgan law)

$\Leftrightarrow (Q \wedge (\neg P \wedge \neg R)) \vee (Q \wedge \neg Q) \vee ((P \wedge R) \wedge \neg P \wedge \neg R) \vee ((P \wedge R) \wedge \neg Q)$

(Extended distributed law)

$\Leftrightarrow (\neg P \wedge Q \wedge \neg R) \vee F \vee (F \wedge R \wedge \neg R) \vee (P \wedge \neg Q \wedge R)$ (Negation law)

$\Leftrightarrow (\neg P \wedge Q \wedge \neg R) \vee (P \wedge \neg Q \wedge R)$ (Negation law)

Obtain Pcnf and Pdnf of the formula $(\neg P \vee \neg Q) \rightarrow (P \leftrightarrow \neg Q)$

Solution:

Let S = $(\neg P \vee \neg Q) \rightarrow (P \leftrightarrow \neg Q)$

P	Q	$\neg$ P	$\neg$ Q	$\neg P \vee \neg Q$	$P \leftrightarrow \neg Q$	S	Minterm	Maxterm
T	T	F	F	F	F	T	$P \wedge Q$	
T	F	F	T	T	T	T	$P \wedge \neg Q$	
F	T	T	F	T	T	T	$\neg P \wedge Q$	
F	F	T	T	T	F	F		$P \vee Q$

PCNF: $P \vee Q$ and PDNF: $(P \wedge Q) \vee (P \wedge \neg Q) \vee (\neg P \wedge Q)$

Obtain PDNF of $P \rightarrow (P \wedge (Q \rightarrow P))$.

Solution:

$P \rightarrow (P \wedge (Q \rightarrow P)) \Leftrightarrow \sim P \vee (P \wedge (\sim Q \vee P))$

$\Leftrightarrow \sim P \vee (P \wedge \sim Q) \vee (P \wedge P)$

$\Leftrightarrow (\sim P \wedge T) \vee (P \wedge \sim Q) \vee (P \wedge P)$

$\Leftrightarrow (\sim P \wedge (Q \vee \sim Q)) \vee (P \wedge \sim Q)) \vee (P \wedge (Q \vee \sim Q))$

$\Leftrightarrow ($ ~

$\Leftrightarrow (\sim$

$P \wedge Q) \vee ($~

$P \wedge \sim Q) \vee$
$(P \wedge \sim Q)$
$\vee (P \wedge Q)$
$\vee (P \wedge \sim Q$
$)$

$P \wedge Q) \vee ($~

$P \wedge \sim Q) \vee$
$(P \wedge \sim Q) \vee$
$(P \wedge Q)$

1.4 PREDICATES & QUANTIFIERS:

Quantifiers.

Universal Quantifiers:

The universal Quantification of P(x) is the proposition.”P(x) is true for all values of x in the universe of discourse”.

The notation ¥x P(x) denotes the universal quantification of P(x).here ¥ is called the universal quantifier.

Existential Quantifier:

The existential Quantification of P(x) is the proposition.” There exists an element x in the universe of discourse such that P(x) is true”.

We use the notation Ǝx P(x) for the existential quantification of p(x).here Ǝ is called the existential quantifier.

Normal Forms:

DNF:

A formula which is equivalent to a given formula and which consists of sum of elementary products is called a disjunctive normal form of the given formula

PDNF: a formula which is equivalent to a given formula which is consists of sum its minterms is called PDNF.

PCNF: a formula which is equivalent to a given formula which consists of product of maxterms is called PCNF.

Obtain PCNF of (¬ p→ r) ∧(q↔ p). and hence obtain its PDNF.

Solution:

PCNF:

$S \Leftrightarrow (\neg p \rightarrow r) \wedge (q \leftrightarrow p).$

→ p). ∧ (p → q)

$\Leftrightarrow (p\vee r) \wedge((\neg q\vee p). \wedge (\neg p \vee q)$

$\Leftrightarrow ((p\vee r) \vee F) \wedge((\neg q\vee p).\vee F) \quad \wedge ((\neg p \vee q) \vee F)$

$\Leftrightarrow ((p \vee r) \vee (q \wedge \neg q))\wedge((\neg q\vee p).\vee (r \wedge \neg r)) \quad \wedge ((\neg p \vee q) \vee (p \wedge \neg p))$.

$\Leftrightarrow ((p \vee r \vee q)\wedge(p \vee r \vee \neg q)) \wedge ((\neg q\vee p\vee r) \wedge.(\neg q \vee p \vee \neg r) \wedge$
$((\neg p \vee q \vee r) \vee (\neg p \vee q \vee \neg r)$

$\Leftrightarrow ((p \vee r \vee q) \wedge ((\neg q\vee p\vee r) \wedge.(\neg q \vee p \vee \neg r) \wedge ((\neg p \vee q \vee r) \vee (\neg p \vee$
$q \vee \neg r)$

PCNF of S: $((p \vee r \vee q) \wedge ((\neg q\vee p\vee r) \wedge.(\neg q \vee p \vee \neg r) \wedge ((\neg p \vee q \vee r) \vee$
$(\neg p \vee q \vee \neg r)$

PCNF of $\neg$S: $(p\vee q\vee r) \wedge (\neg p \vee \neg q \vee r| \quad) \wedge (\neg p \vee \neg q \vee \neg r)$

PDNF of S: $(p\wedge q\wedge r) \vee (\neg p \wedge \neg q \wedge r) \vee (\neg p \vee\wedge \neg q \wedge \neg r)$.

1.5 RULES OF INFERENCE:

EXAMPLE:1 Verify that validating of the following inference. If one person is more successful than another, then he has worked harder to deserve success. Ram has not worked harder than Siva. Therefore, Ram is not more successful than Siva.

Solution:

Let the universe consists of all persons.

Let S(x,y): x is more successful than y.

H(x,y): x has worked harder than y to deserve success.

a: Ram

b: Siva

Then, given set of premises are

1) $(x)\ (y)\ [S(x,y) \rightarrow H(x,y)]$

2) ¬ H(a,b)

3) Conslution is ¬ S(a,b).

{1}	1) (x) (y) [S(x,y) → H(x,y)]	Rule P
{2}	2) (y) [S(a,y) → H(a,y)]	Rule US
{3}	3) [S(a,b) → H(a,b)]	Rule US
{4}	4) ¬ H(a,b)	Rule P
{5}	5) ¬ S(a,b)	Rule T (¬ P, P →Q ⇒ ¬ Q)

EXAMPLE: 2Show that (x) (H(x) →M(x)) ∧ H$(s) \Rightarrow M(s)$

Solution :

Steps	Premises	Rule	Reason
1	(x) (H(x) →M(x))	P	Given premise
2	H(s) →M(s)	US (1)	(Vx) p (x) ⇒ $p(y)$
3	H(s)	P	Given premise
4	M(s)	T	(2) (3) (p → q, p ⇒ q)

EXAMPLE: 3 Show that ¬ p(a,b) follows logically from (x) (y) (p(x,y) →w(x,y)) $and\ \neg w(a,b)$

Solution :

1. (x) (y) (p(x,y) → w(x,y)   p
2. (y), p(a,y) → w(a,y)   US, (1)
3. P(a,b) → w(a,b)   US (2)
4. $\neg w(a,b)$ p Given
5. $\neg p(a,b)$ T (3),(4), $(p\rightarrow Q) \wedge \neg Q \Rightarrow \neg p$

EXAMPLE:4.

Symbolise: For every x, these exixts a y such that $x^2+y^2 \geq 100$

Solution :

$(\forall x)(\exists y)(x^2+y^2 \geq 100)$

Example:Let p, q, r be the following statements:

p: I will study discrete mathematics q: I will watch T.V.

r: I am in a good mood.

Write the following statements in terms of p, q, r and logical connectives. (1) If I do not study and I watch T.V., then I am in good mood.

(2) If I am in good mood, then I will study or I will watch T.V.

(3) If I am not in good mood, then I will not watch T.V. or I will study.

(1) $(\neg p \wedge q) \rightarrow r$

(2) $r \rightarrow (p \vee q)$

(3) $\neg r \rightarrow (\neg q \vee p)$

1.6 Introduction to proofs & statergy

Method of proofs :

Trival proof:

In an implication $p \rightarrow q$, if we can establish that q is true , then regardless of the truth value of p, the implication $p \rightarrow q$ So the construction of a trivial proof of $p \rightarrow q$ needs to show that the truth value of q is true.

Vacous proof:

If the hypothesis p of an implication $p \rightarrow q$ is false , then $p \rightarrow q$ is true for any proposition q.

Prove that $\sqrt{2}$ is irrational.

Solution :

Suppose $\sqrt{2}$ is irrational.

$\therefore \sqrt{2} = \frac{p}{q}$ for p,q $\in z, q \neq 0, p \,\&\, q$ have no common divisor.

$\therefore \frac{p^2}{q^2} = 2 \Rightarrow p^2 = 2q^2$.

Since p^2 is an even integer, p is an even integer.

$\therefore$ p= 2m for some integer m.

$\therefore$ $(2m)^2 = 2q^2 \Rightarrow q^2 = 2m^2$

Since q^2 is an even integer, q is an even integer.

$\therefore$ q= 2k f or some integer k.

Thus p & q are even . Hence they have a common factor 2. Which is a contradiction to our assumption.

$\therefore \sqrt{2}$ is irrational.

UNIT II COMBINATORICS

Pigeon Hole Principle:

If (n=1) pigeon occupies 'n' holes then atleast one hole has more than 1 pigeon.
Proof:
Assume (n+1) pigeon occupies 'n' holes.
Claim: Atleast one hole has more than one pigeon.
Suppose not, ie. Atleast one hole has not more than one pigeon.
Therefore, each and every hole has exactly one pigeon.
Since, there are 'n' holes, which implies, we have totally 'n' pigeon.
Which is a $\Rightarrow \Leftarrow$ to our assumption that there are (n+1) pigeon.
Therefore, atleast one hole has more than 1 pigeon.

2.1 MATHEMATICAL INDUCTION

EXAMPLE 1:show that

SOLUTION: $\frac{1}{1.2} + \frac{1}{2.3} + \ldots\ldots + \frac{1}{n(n+1)} = \frac{n}{n+1}$

Let P(n) : $\frac{1}{1.2} + \frac{1}{2.3} + \ldots\ldots + \frac{1}{n(n+1)}$

1.P(1): $\frac{1}{1.2} = \frac{1}{1(1+1)}$ is true.

2.ASSUME

P(k): $\frac{1}{1.2} + \frac{1}{2.3} + \ldots\ldots + \frac{1}{k(k+1)}$

$= \frac{k}{k+1}$ is true. -> (1)

CLAIM : P(k+1) is true.

P(k+1) = $\frac{1}{1.2} + \frac{1}{2.3} + \ldots\ldots + \frac{1}{k(k+1)} + \frac{1}{(k+1)(k+2)}$

$= \frac{k}{k+1} \; \frac{1}{(k+1)(k+2)}$ using (1)

$= \frac{k(k+2)+1}{(k+1)(k+2)}$

$= \frac{(k.k)+2k+1}{(k+1)(k+2)}$

$= \frac{(k+1)(k+1)}{(k+1)(k+2)}$

$= \frac{(k+1)}{(k+2)}$

$= \frac{k+1}{(k+1)+1}$

P(k+1) is true.

BY THE PRINCIPLE OF MATHEMATICAL INDUCTION

$\frac{1}{1.2} + \frac{1}{2.3} + \ldots\ldots + \frac{1}{n(n+1)} = \frac{n}{n+1}$ Is true for all n .

EXAMPLE 2 : Using mathematical induction prove that if

n>=1, then 1.1! + 2.2! +3.3! +......+n.n! = (n+1)! - 1

SOLUTION:

Let p(n) : 1.1! + 2.2! +3.3! +......+n.n! = (n+1)! - 1

1.P(1) : 1.1! = (1+1)! – 1 is true