STEP1:

a) if the RHS of the recurrence relation is

$a_0 + a_1$ n .... a_r n^r, then substitute

c_0 $+c_1\, n + c_2$ n^2+..........c_r $(n-1)^r$ in place of $a_n - 1$ ....and so on ,in the LHS of the given recurrence relation

(b) if the RHS is a^n then we have

Case1:if the base a of the RHS is the characteristic root,then the solution is of the can^n .therefore substitute ca^n in place of a_n ,ca^{n-1} in place of c(n-1) a_{n-1} etc..

Case2: if the base a of RHS is not a root , then solution is of the form ca^n therefore substitute ca^n in place of a_n , ca^{n-1} in place of a_{n-1} etc..

STEP2:

At the end of step-1, we get a polynomial in 'n' with coefficient c_0,c_1......on LHS

Now, equating the LHS and compare the coefficients find the constants c_0,c_1,....

Example 1:

Solve $a_n = 3\, a_{n-1}$ +2n with a_1=3

Solution:

Give the non-homogeneous recurrence relation is

$a_n - 3\, a_{n-1}$ -2n=0

It's associated homogeneous equation is

$a_n - 3\, a_{n-1}$ =0 [omitting f(n) =2n]

It's characteristic equation is

r-3=0 => r=3

now, the solution of associated homogeneous equation is

$a_n\ (n) = \propto, 3^n$

To find particular solution

Since F(n) =2n is a polynomial of degree one,then the solution is of the from

$a_n = c_n$ +d (say) where c and d are constant

Now, the equation

$a_n = 3\ a_{n-1}$ +2n becomes

c_n +d =3(c(n-1)+d)+2n

[replace a_n by c_n +d a_{n-1} by c(n-1)+d]

- ⇨ c_n +d =3cn-3c+3d+2n
- ⇨ 2cn+2n-3c+2d=0
- ⇨ (2+2c)n+(2d-3c)=0
- ⇨ 2+2c=0 and 2d-3c=0
- ⇨ Saving we get c=-1 and d=-3/2 therefore cn+d is a solution if c=-1 and d=-3/2

a_n (p)=-n-3/2

Is a particular solution.

General solution

a_n=a_n(n)+ a_n(p)

$a_n = \propto 3^n$ -n-3/2 ………………..(A)

Given $a_1 = 3$ put n=1 in (A) we get

$a_1 = \propto\ 1(3)^1$ -1-3/2

3=3$\propto_1$-5/2

$3 \propto_1 = 11/2$

$\propto_1 = 11/6$

Substituting $\propto_1 = 11/6$ in (A) we get

General solution

$a_n = -n - 3/2 + (11/6)3^n$

Example:2

Solve s(k)-5s(k-1)+6s(k-2)=2

With s(0)=1 ,s(1)=-1

Solution:

Given non-homogeneous equation can be written as

$a_n = 5a_{n-1} + 6a_{n-2} - 2 = 0$

The characteristic equation is

$r^2 - 5r + 6 = 0$

roots are r=2,3

the general solution is

$3_n(n) = \propto_1 (2)^n + \propto_2 (3)^n$

To find particular solution

As RHS of the recurrence relation is constant ,the solution is of the form C , where C is a constant

Therefore the equation

$a_n - 5a_{n-1} - 6a_{n-2} - 2 = 2$

$c - 5c + 6c = 2$

$2c=2$

$c=2$

the particular solution is

$s_n(p)=1$

the general solution is

$s_n = s_n(n) + s_n(p)$

$s_n = \propto_1 (2)^n + \propto_2 (3)^n + 1$……….(A)

Given $s_0=1$ put n=0 in (A) we get

$s_0 = \propto_1 (2)^0 + \propto_2 (3)^0 + 1$

$s_0 = \propto_1 + \propto_2 + 1$

(A) => $s_0 = 1 = \propto_1 + \propto_2 + 1$

$\propto_1 + \propto_2 = 0$……………(1)

Given $a_1=-1$ put n=1 in(A)

⇨ $S_1 = \propto_1 (2)^1 + \propto_2 (3)^1 + 1$
⇨ (A) $-1 = \propto_1 (2) + \propto_2 (3) + 1$
⇨ $2 \propto_1 + 3 \propto_2 = -2$……….(1)

$\propto_1 + \propto_2 = 0$

$2 \propto_1 + 3 \propto_2 = -2$………………..(2)

By solving (1) and (2)

$\propto_1 = 2, \propto_2 = -2$

Substituting $\propto_1 = 2, \propto_2 = -2$ in (A) we get

Solution is

⇨ $S_{(n)} = 2.(2)^n - 2.(3)^n + 1$

Example :3

Solve $\boldsymbol{a_n - 4\, a_{n-1} + 4\, a_{n-2} = 3n + 2^n}$

$\boldsymbol{a_0 = a_1 = 1}$

Solution:

The given recurrence relation is non-homogeneous

Now, its associated homogeneous equation is,

$a_n - 4\, a_{n-1} + 4\, a_{n-2} = 0$

Its characteristic equation is

$r^2 - 4r + 4 = 0$

$r = 2, 2$

solution , $a_n(n) = \propto_1 (2)^n + n \propto_2 (2)^n$

$a_n(n) = (\propto_1 + n \propto_2) 2^n$

To find particular solution

The first term in RHS of the given recurrence relation is 3n.therefore ,the solution is of the form $c_1 + c_2 n$

Replace a_n by $c_1 + c_2 n$, a_{n-1} by $c_1 + c_2(n-1)$

And a_{n-2} by $c_1 + c_2(n-2)$ we get

$(c_1 + c_2 n) - 4(c_1 + c_2(n-1)) + 4(c_1 + c_2(n-2)) = 3n$

⇨ $c_1 - 4c_1 + 4c_1 + c_2 n - 4c_2 n + 4c_2 n + 4c_2 - 8c_2 = 3n$

⇨ $c_1 + c_2 n - 4c_2 = 3n$

Equating the corresponding coefficient we have
c_1-4c_2=0 and c_2=3
c_1=12 and c_2=3

Given a_0=1 using in (2)
(2) => $\propto_1$+12=1
Given a_1=1 using in (2)
(2)=> ($\propto_1$+$\propto_2$)2+12+3+1/2 .2=1
=> (2 $\propto_1$+2 $\propto_2$)+16=1……………..(14)
(3) $\propto_1$=-11
Using in (4) we have $\propto_2$=7/2

Solution a_n=(-11+7/2n)2^n+12+3n+1/2$n^2 2^n$

Example:

HOW MANY INTEGERS BETWEEN 1 to 100 that are

i) not divisible by 7,11,or 13

ii) divisible by 3 but not by 7

Solution:

i) let A,B and C denote respectively the number of integer between 1 to 100 that are divisible by 7,11 and 13 respectively

now,

$|A| = [100/7] = 14$

$|B| = [100/11] = 9$

$|C| = [100/13] = 7$

$|A \wedge B| = [100/7] = 1$

$|A \wedge C| = [100/7*13] = 1$

$|B \wedge C| = [100/11*13] = 0$

$|A \wedge B \wedge C| = [100/7*11*13] = 0$

That are divisible by 7, 11 or 13 is $|A v B v C|$

By principle of inclusion and exclusion

$|A v B v C| = |A|+|B|+|C|-|A \wedge B|-|A \wedge C|-|B \wedge C|+|A \wedge B \wedge C|$

$= 14+9+7-(1+1+0)+0$

$= 30-2 = 28$

Now,

The number of integer not divisible by any of 7,11,and 13=total-|AvBvC|

=100-28=72

ii) let A and B denote the no. between 1 to 100 that are divisible by 3 and 7 respectively

|A| =[100/3]=33

|B|=[100/7]=14

| A^B |=[100/3*7]=14

The number of integer divisible by 3 but not by 7

=|A|-| A^B |

=33-4=29

Example:

There are 2500 student in a college of these 1700 have taken a course in C, 1000 have taken a course pascal and 550 have taken course in networking .further 750 have taken course in both C and pascal ,400 have taken courses in both C and Networking and 275 have taken courses in both pascal and networking. If 200 of these student have taken course in C pascal and Networking.

i)how many these 2500 students have taken a courses in any of these three courses C ,pascal and networking?

ii)How many of these 2500 students have not taken a courses in any of these three courses C,pascal and networking?

Solution:

Let A,B and C denotes student have taken a course in C,pascal and networking respectively

Given

|A|=1700

|B|=1000

|C|=550

| A^B | =750

| A^C|=40

| B^C =275

| A^B^C |=200

Number of student who have taken any one of these course=| A^B^C |

By principle of inclusion and exclusion

|AvBvC| =|A|+|B|+|C|-|A^B|-|A^C|-|B^C|+|A^B^C|

=(1700+1000+550)-(750+400+275)+200

=3450-1425=2025

The number between 1-100 that are divisible by 7 but not divisible by 2,3,5,7= } =|D|- | A^B^C ^CD|

=142-4=138

Example:

A survey of 500 television watches produced the following information.285 watch hockey games.195 watch football games 115 watch basketball games .70 watch football and hockey games.50 watch hockey and

basketball games and 30 watch football and hockey games.how many people watch exactly one of the three games?

Solution:

H=> let television watches who watch hockey

F=> let television watches who watch football

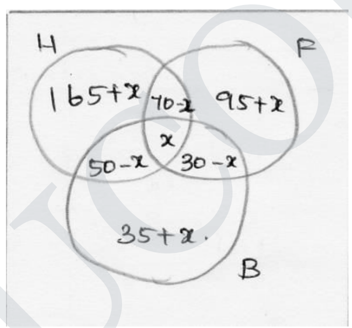
B=> let television watches who watch basketball

Given

n(H)=285,n(F)=195,n(B)=115,n(H^F)=70,n(H^B),n(F^B)=30

let x be the number television watches who watch all three games

now, we have



Given 50 members does not watch any of the three games.

Hence (165+x)+(95+x)+(35+x)+(70+x)+(50+x)+(30+x)+x=500

=445+x=500

X=55

Number of students who watches exactly one game is=165+x+95+x+35+x

=295+3*55

=460

2.5 .Generating function:

Of real numbers is the infinite sum.

G(x)=G(s,x)= $a_0+a_1x+,\ldots.. a_nx^n+\ldots..= \sum_{n=0}^{\infty} a^n x^n$

For example,

i) the generating function for the sequence 'S' with the terms 1,1,1,1…..i.s given by,

$$G(x)=G(s,x)= \sum_{n=0}^{\infty} x^n = 1/1-x$$

ii)the generation function for the sequence 'S' with terms 1,2,3,4…..is given by

$$G(x)=G(s,x)= \sum_{n=0}^{\infty}(n+1)x^n$$

$$=1+2x+3x^2+\ldots\ldots..$$

$$=(1-x)^{-2}=1/(1-x)^2$$

2.Solution of recurrence relation using generating function

Procedure:

Step1:rewrite the given recurrence relation as an equation with 0 as RHS

Step2:multiply the equation obtained in step(1) by x^n and summing if form 1 to ∞ (or 0 to ∞) or (2 to ∞).

Step3:put G(x)= $\sum_{n=0}^{\infty} a^n x^n$ and write G(x) as a function of x

Step 4:decompose G(x) into partial fraction

Step5:express G(x) as a sum of familiar series

Step6:Express a_n as the coefficient of x^n in G(x)

The following table represent some sequence and their generating functions

step1	**sequence**	**generating function**
1	1	1/1-z
2	$(-1)^n$	1/1+z
3	a^n	1/1-az
4	$(-a)^n$	1/1+az
5	n+1	$1/1-(z)^2$
6	n	$1/(1-z)^2$
7	n^2	$z(1+z)/(1-z)^3$
8	na^n	$az/(1-az)^2$

Eg:use method of generating function to solve the recurrence relation

$a_n=3a_{n-1}+1$; $n\geq 1$ given that $a_0=1$

solution:

let the generating function of $\{a_n\}$ be

$G(x)=\sum_{n=0}^{\infty} a_n x^n$

$a_n=3a_{n-1}+1$

multiplying by x^n and summing from 1 to ∞,

$\sum_{n=0}^{\infty} a_n x^n = 3\sum_{n=1}^{\infty}(a_{n-1}x^n)+\sum_{n=1}^{\infty}(x^n)$

$\sum_{n=0}^{\infty} a_n x^n = 3\sum_{n=1}^{\infty}(a_{n-1}x^{n-1})+\sum_{n=1}^{\infty}(x^n)$

$G(x)-a_0=3xG(x)+x/1-x$

$G(x)(1-3x)=a_0+x/1-x$

=1+x/1-x

G(x)(1-3x)=1=x+x/1-x

G(x)=1/(1-x)(1-3x)

By applying partial fraction

G(x)=-1/2/1-x+3/2/1-3x

$G(x)=-1/2(1-x)^{-1}+3/2(1-3x)^{-1}$

$G(x)[1-x-x^2]=a_0-a_1x-a_0x$

$G(x)[1-x-x^2]= a_0-a_0x+a_1x$

$G(x)=1/1-x-x^2$ $[a_0=1, a_1=1]$

$$=\frac{1}{(1-1+\sqrt{5}\ \ x/2)(1-1-\sqrt{5}\ \ x/2)}$$

$$=\frac{A}{(1-(\frac{1+\sqrt{5}}{2})x)}+\frac{B}{(1-(\frac{1-\sqrt{5}}{2})x)}$$

Now,

$$1/1-x-x^2=\frac{A}{(1-(\frac{1+\sqrt{5}}{2})x)}+\frac{B}{(1-(\frac{1-\sqrt{5}}{2})x)}\text{............}(1)$$

$$1=A[1-(\frac{1+\sqrt{5}}{2})x)]+B[1-(\frac{1-\sqrt{5}}{2})x)]\text{......}(2)$$

Put x=0 in (2)

(2)=> A+B=1

Put $x= 2/1-\sqrt{5}$ in (2)

(2)=> $1=B[1-\frac{1+\sqrt{5}}{1-\sqrt{5}}]$

$1=B[\frac{1-\sqrt{5}-1-\sqrt{5}}{1-\sqrt{5}}\ ]$

$$1=B[\frac{-2\sqrt{5}}{1-\sqrt{5}}]$$

$$B=\frac{1-\sqrt{5}}{-2\sqrt{5}}$$

(3) => $A=\frac{1+\sqrt{5}}{2\sqrt{5}}$

Sub A and B in (1)

$$G(x)=\frac{1}{\sqrt{5}}(\frac{1+\sqrt{5}}{2})[1-(\frac{1+\sqrt{5}}{2})x]^{-1}-\frac{1}{\sqrt{5}}(\frac{1-\sqrt{5}}{2})[1-(\frac{1-\sqrt{5}}{2})x]^{-1}$$

$$=\frac{1}{\sqrt{5}}(\frac{1+\sqrt{5}}{2})[1+(\frac{1+\sqrt{5}}{2})x+(\frac{1-\sqrt{5}}{2}x)]^{2}+\ldots\ldots.$$

$$=\frac{-1}{\sqrt{5}}(\frac{1-\sqrt{5}}{2})[1+(\frac{1-\sqrt{5}}{2})x+(\frac{1-\sqrt{5}}{2}x)]^{2}+\ldots\ldots.$$

a_n=coefficient of x^n in G(x)

solving we get

$$a_n=\frac{1}{\sqrt{5}}(\frac{1+\sqrt{5}}{2})^{n+1}-\frac{1}{\sqrt{5}}(\frac{1-\sqrt{5}}{2})^{n+1}$$

2.6THE PRINCIPLE OF INCLUSION –EXCLUSION

Assume two tasksT
time(simultaneously) now to find the number of ways to do one of the two tasks $T_1$and T_2, if we add number ways to do each task then it leads to an over count. since the ways to do both tasks are counted twice. To correctly count the number of ways to do each of the two tasks and then number of ways to do both tasks

i.e $\wedge(T_1 v T_2)=\wedge(T_1)+\wedge(T_2)-\wedge(T_1\wedge T_2)$

this technique is called the principle of Inclusion –exclusion

FORMULA:$_4$

1) $|A_1 v A_2 v A_3|=|A_1|+|A_2|+|A_3|-|A_1\wedge A_2|-|A_1\wedge A_3|-|A_2\wedge A_3|+|A_1\wedge A_2\wedge A_3|$

2) $|A_1 v A_2 v A_3 v A_4|=|A_1|+|A_2|+|A_3|+|A_4|-|A_1\wedge A_2|-|A_1\wedge A_3|-|A_1\wedge A_4|-|A_2\wedge A_3|-|A_2\wedge A_4|-|A_3\wedge A_4|+|A_1\wedge A_2\wedge A_3|+|A_1\wedge A_2\wedge A_4|+|A_1\wedge A_3\wedge A_4|+|A_2\wedge A_3\wedge A_4|+|A_1\wedge A_2\wedge A_3\wedge A_4|$

Example1:

A survey of 500 from a school produced the following information.200 play volleyball,120 play hockey,60 play both volleyball and hockey. How many are not playing either volleyball or hockey?

Solution:

Let A denote the students who volleyball

Let B denote the students who play hockey

It is given that

n=500

|A|=200

|B|=120

$|A \wedge B| = 60$

Bt the principle of inclusion-exclusion, the number of students playing either volleyball or hockey

$|A \vee B| = |A| + |B| - |A \wedge B|$

$|A \vee B| = 200 + 120 - 60 = 260$

The number of students not playing either volleyball or hockey=500-260

=240

Example2:

In a survey of 100 students it was found that 30 studied mathematics,54 studied statistics,25 studied operation research,1 studied all the three subjects.20 studied mathematics and statistic,3 studied mathematics and operation research And 15 studied statistics and operation research

1.how many students studied none of these subjects?

2.how many students studied only mathematics?

Solution:

1) Let A denote the students who studied mathematics

Let B denote the students who studied statistics

Let C denote the student who studied operation research

Thus $|A|=30$,$|B|=54$,$|C|=25$,$|A \wedge B|=20$,$|A \wedge C|=3$,$|B \wedge C|=15$,and $|A \wedge B \wedge C|=1$

By the principle of inclusion-exclusion students who studied any one of the subject is

$|A \vee B \vee C| = |A| + |B| + |C| = |A \wedge B| - |A \wedge C| - |B \wedge C| + |A \wedge B \wedge C|$

$= 30 + 54 + 25 - 20 - 3 - 15 + 1$

$= 110 - 38 = 72$

Students who studied none of these 3 subjects=100-72=28

2) now ,

The number of students studied only mathematics and statistics=n(A^B)-n(A^B^C)

=20-1=19

The number of students studied only mathematics and operation research=n(A^C)-n(A^B^C)

=3-1=2

Then The number of students studied only mathematics =30-19-2=9

Example3:

How many positive integers not exceeding 1000 are divisible by 7 or 11?

Solution:

Let A denote the set of positive integers not exceeding 1000 are divisible by 7

Let B denote the set of positive integers not exceeding 1000 that are divisible by 11

Then |A|=[1000/7]=[142.8]=142

|B|=[1000/11]=[90.9]=90

|A^B|=[1000/7*11]=[12.9]=12

The number of positive integers not exceeding 1000 that are divisible either 7 or 11 is |AvB|

By the principle of inclusion –exclusion

|AvB|=|A|+|B|-|A^B|

=142+90-12=220

There are 220 positive integers not exceeding 1000 divisible by either 7 or 11

Example:

A survey among 100 students shows that of the three ice cream flavours vanilla,chocolate,and strawberry ,50 students like vanilla,43 like chocalate ,28 like strawberry,13 like vanilla, and chocolate,11like chocalets and strawberry,12 like strawberry and vanilla and 5 like all of them.

Find the number of students surveyed who like each of the following flavours

1.chocalate but not strawberry

2.chocalate and strawberry ,but not vanilla

3.vanilla or chocolate, but not strawberry

Solution:

Let A denote the set of students who like vanilla

Let B denote the set of students who like chocalate

Let C denote the set of students who like strawberry

Since 5 students like all flavours

|A^B^C|=5

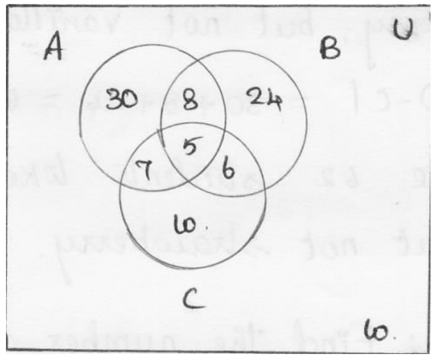
12 students like both strawberry and vanilla

|A^C|=12

But 5 of them like chocolate also, therefore

|A^C-B|=7

Similarly |B^C-A|=6



Of the 28 students who like strawberry we have already accounted for

7+5+6=18

So, the remaining 10 students belong to the set C-|AvB| similarly

|A-BvC|=30 and |B-AvC|=24

Thus for we have accounted for 90 of the 100 students the remaining 10 students like outside the region AvBvC

Now,

1.|B-C|=24+8=32

So 32 students like chocolate but not strawberry

2.|B^C-A|=6

Therefore 6 students like both chocolate and strawberry but not vanilla

3.|AvB-C|=30+8+24=62

Therefore 62 students like vanilla or chocolate but not strawberry

Example 5: find the number of integers between 1 to 250 that are not divisible by any of the integers 2,3,5 and 7

Solution:

Let A denote the integer from 1 to 250 that are divisible by 2

Let B denote the integer from 1 to 250 that are divisible by 3

Let C denote the integer from 1 to 250 that are divisible by 5

Let D denote the integer from 1 to 250 that are divisible by 7

|A|=[250/2]=125

|B|=[250/3]=83

|C|=[250/5]=50

|D|=[250/7]=35

Now, the number of integer between 1-250 that are divisible by 2 and 3=|A^B|=[250/2*3]=41

The number of integer divisible by 2 and 5=|A^C|=[250/2*5]=25

Similarly

|A^D|=[250/2*7]=17

|B^C|=[250/3*5]=16

|B^D|=[250/3*7]=11

|C^D| =[250/5*7]=7

The number of integer divisible by 2,3,5=|A^B^C|=[250/2*3*5]=8.