Andy's math/science background information

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Contents

1 Introduction

This document is a place to write up notes on background in math that help in learning science.

2 Notation

$\mathbb{R}^3$ - The set of all points in 3-dimensional space, i.e. where each point is specified by a sequence of 3 real-valued coordinates.

∇ - Called "nabla", and often called "del". Some history of it can be found on the Nabla Symbol page [3].

$\boldsymbol{\nabla} f$ - Gradient of a scalar function $f : \mathbb{R}^3 \to \mathbb{R}$.

grad$f$ - another way to write the gradient of f, $\boldsymbol{\nabla} f$

TODO: Any 3Blue1Brown-quality YouTube videos showing examples and definition of gradient, divergence, and curl? If so, give links to them below in appropriate sections.

2.1 Maxwell's Equations

In partial differentiatal form:

$$\boldsymbol{\nabla} \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} \qquad \text{Gauss's Law}$$

$$\boldsymbol{\nabla} \cdot \mathbf{B} = 0 \qquad \text{Gauss's Law for magnetism}$$

$$\boldsymbol{\nabla} \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad \text{Maxwell-Faraday Equation (Faraday's law of induction)}$$

$$\boldsymbol{\nabla} \times \mathbf{B} = \mu_0(\mathbf{J} + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}) \qquad \text{Ampère-Maxwell law}$$

where:

- $\mathbf{E}$ is the electric field
- $\mathbf{B}$ is the magnetic field
- ρ is the electric charge density
- $\mathbf{J}$ is the current density
- ϵ_0 is the vacuum permittivity
- μ_0 is the vacuum permeability

3 Gradient

Gradient has been generalized to many coordinate systems other than the 3-dimensional Cartesian coordinate system $\mathbb{R}^3$, but I will focus on $\mathbb{R}^3$. The Wikipedia page on Gradient [2] is not too bad for me, as long as I skim over the parts that generalize it to other coordinate systems.

Griffiths [1] Section 1.2.2 "Gradient" is good at giving the definition and some useful examples and properties of the gradient. He defines $\boldsymbol{\nabla} T$ this way, where $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$, and $\hat{\mathbf{z}}$ are the unit vectors in the direction of the three coordinate axes:

$$\boldsymbol{\nabla} T \equiv \frac{\partial T}{\partial x}\hat{\mathbf{x}} + \frac{\partial T}{\partial y}\hat{\mathbf{y}} + \frac{\partial T}{\partial z}\hat{\mathbf{z}}$$

where:

- $T : \mathbb{R}^3 \to \mathbb{R}$ is a scalar function that is continuous and differentiable

Some properties of the gradient:

- $\boldsymbol{\nabla} T(x, y, z)$ is the gradient evaluated at a position given by (x, y, z). The vector points in a direction that function T increases most quickly.
- The function T can be approximated at points $\mathbf{r}$ near $\mathbf{r}_0 = (x, y, z)$ by the linear function $T(\mathbf{r}_0) + (\mathbf{r} - \mathbf{r}_0) \cdot \boldsymbol{\nabla} T(\mathbf{r}_0)$
- The instantaneous rate of change of T in direction $\mathbf{u}$ (a unit vector) from $\mathbf{r}_0$ is $\mathbf{u} \cdot \boldsymbol{\nabla} T(\mathbf{r}_0)$. Note that it is always 0 in a direction perpendicular to $\boldsymbol{\nabla} T(\mathbf{r}_0)$, and the negative of the magnitude of $\boldsymbol{\nabla} T(\mathbf{r}_0)$ in the opposite direction.
- Consider a "level set", i.e. a surface defined by all of the points $\mathbf{r}$ where $T(\mathbf{r}) = c$ for some constant c. Then $\boldsymbol{\nabla} T$ evaluated at any point on that surface, is normal to the surface.

References

[1] David J. Griffiths. *Introduction to Electrodynamics*. Pearson Education, 4th edition, 2013.

[2] Wikipedia. Gradient, 2025. URL `https://en.wikipedia.org/wiki/Gradient`.

[3] Wikipedia. Nabla Symbol, 2025. URL `https://en.wikipedia.org/wiki/Nabla_symbol`.