

Existence Proofs

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Outline

When One Example is Enough

Splitting an Octagon

Making Fun in Real Life

Know Your Rights

Nobody Can Win All The Times

Know What Are You Looking For



Proofs For Existential Statements

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- what does the proof look like?

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Proofs For Existential Statements

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- it depends
- claim: *object with given properties exists*
- proof: an example
- one example is enough

Cutting Figures

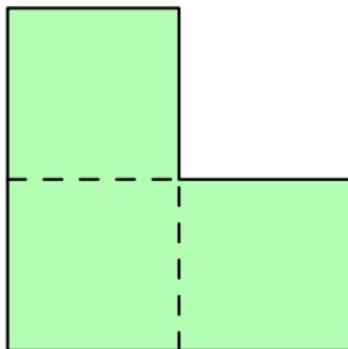
Cutting Figures

congruent pieces: of the same shape and size

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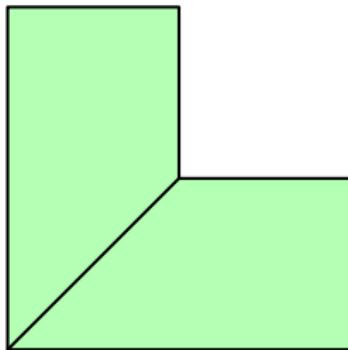
Prove that this figure can be cut into 2 congruent pieces



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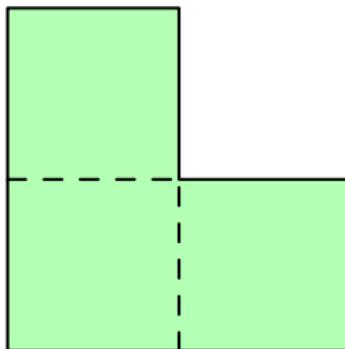
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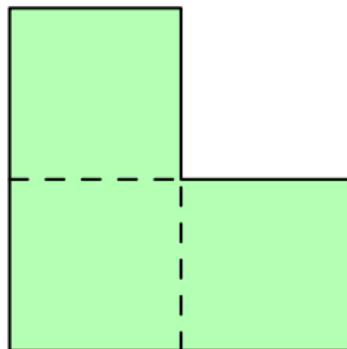
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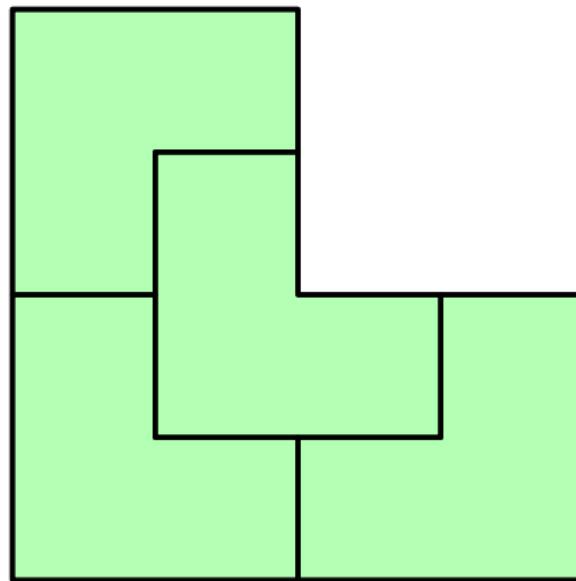
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what about 4 pieces?

Spoiler

Spoiler



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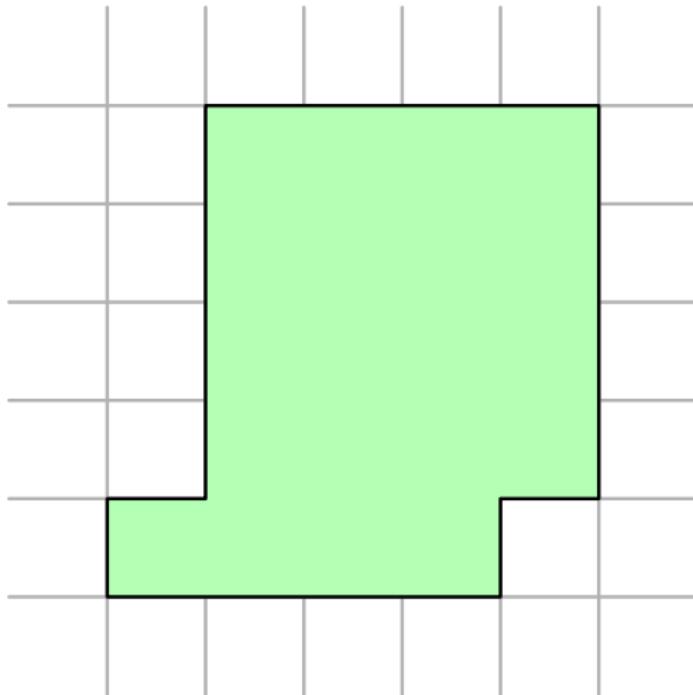
Making Fun in Real Life

Know Your Rights

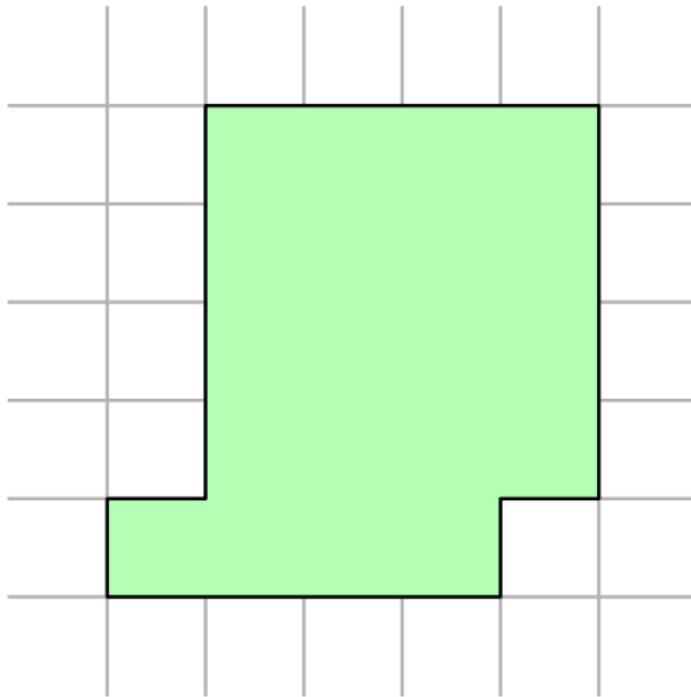
Nobody Can Win All The Times

The Octagon

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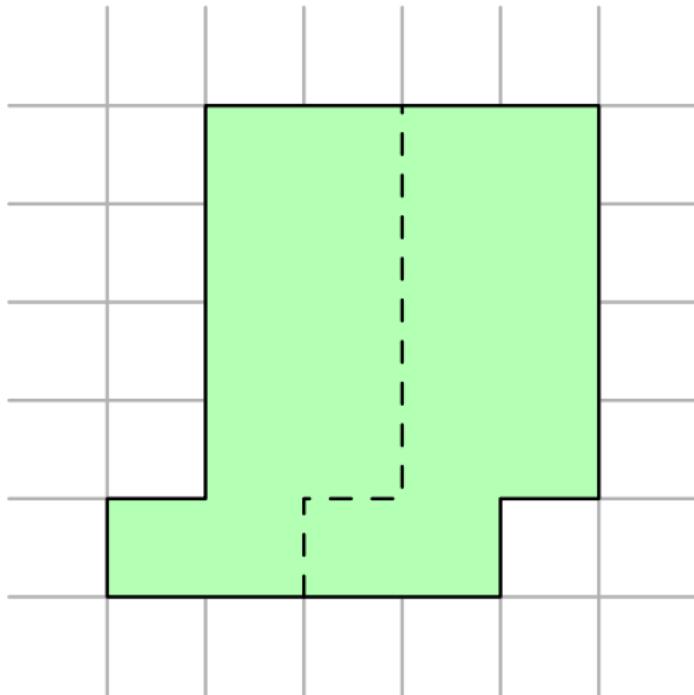
The Octagon



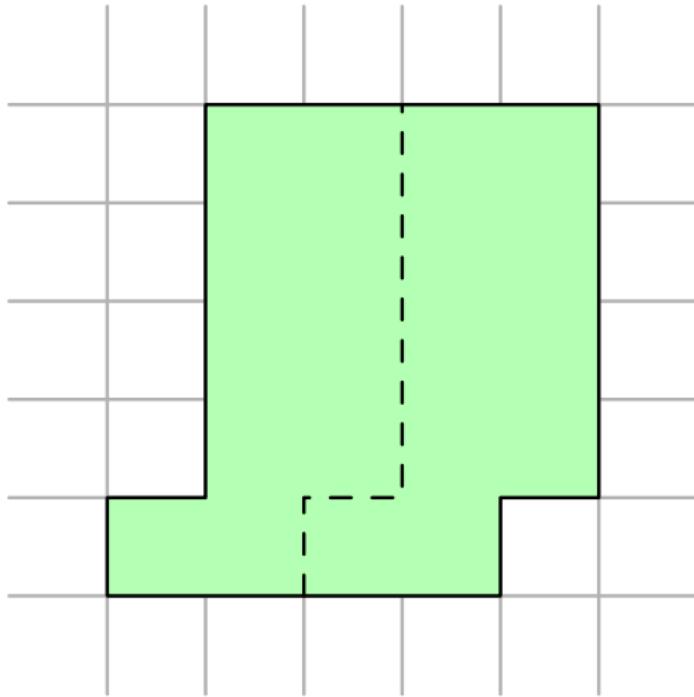
split into two congruent pieces

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what about three congruent pieces?

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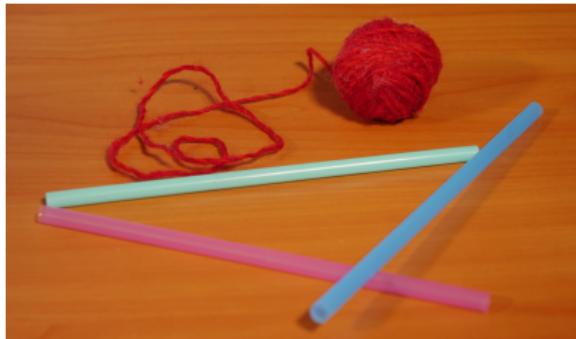
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Tensegrities

Tensegrities

- drinking straws and thread



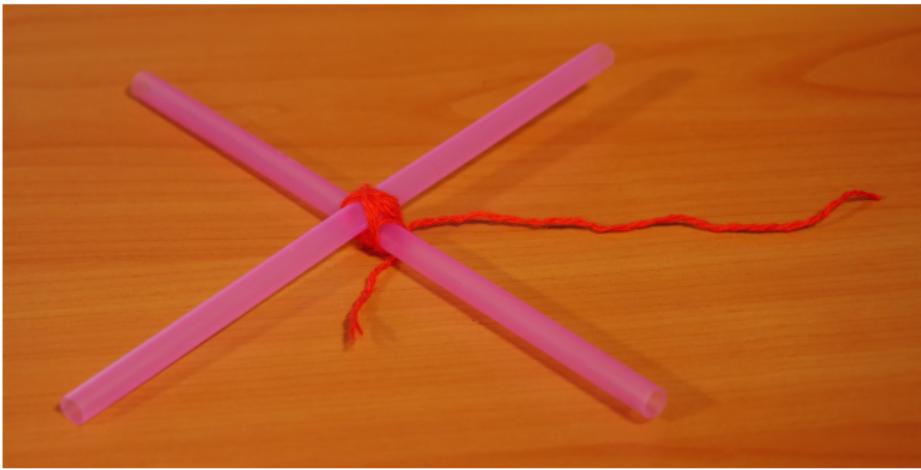
Tensegrities

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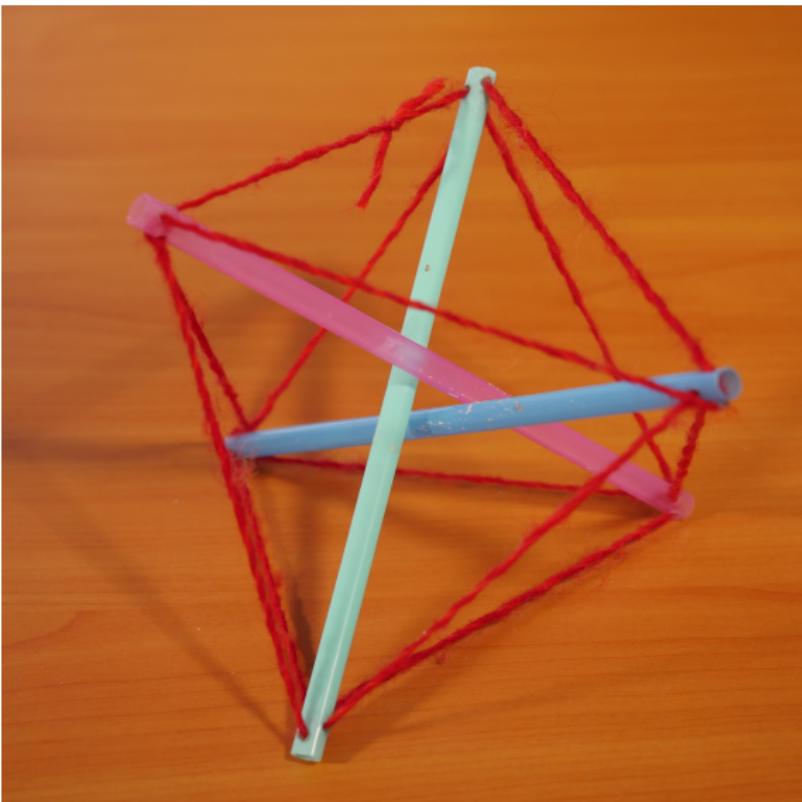


- there exists a “tensegrity”: a solid construction; straws do not touch each other; connected by threads

Not Allowed



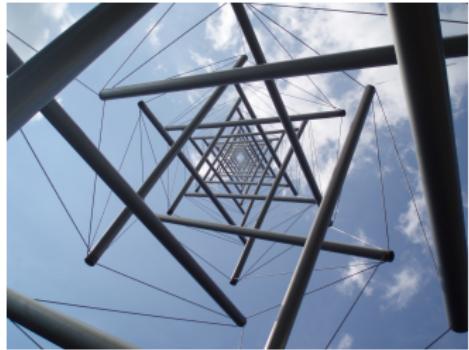
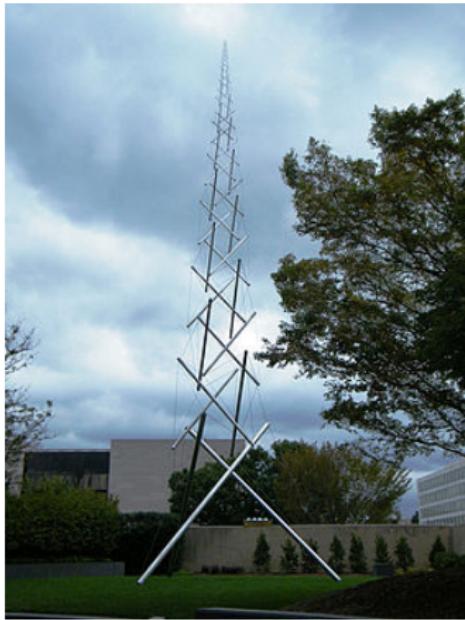
Tensegrity Finished



A Tensegrity: Animation

[Source: https://commons.wikimedia.org/wiki/File:Tensegrity_simple_3.gif]

Tensegrities in the Real Life



[Source:https://en.wikipedia.org/wiki/Needle_Tower]

made by Kenneth Snelson, a student of Buckminster Fuller (who invented the word and made many of them)

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- you say: $7125 = 57 \cdot 125$

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- not difficult to find: not so many choices
- divisible by 7: 14, 21, 28, 35, 42, 49, 56, 63
- but what if we asked for a number that becomes 57 times smaller?
- you say: $7125 = 57 \cdot 125$
- no need to explain how you found it

No Rights for Teachers

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- a divisible by 7: only $a = 7$ works
- $10^k = 8 \times X$; 10 and 100 not multiples of 8
- 1000 works, $X = 125$
- also $71250 = 57 \times 1250$, etc.

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- $+1 + 2 - 3 = 0$

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- easy: $4 + 7$ (also $1 + 2 + 3 + 5 = 11$)

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- what about weights 2, 4, 6, 8, 10, 12?

Obstacles

- if the weights are 1, 2, 3, 4, 5, 6?
- total weight 21: not a multiple of 2
- mission impossible
- what about weights 2, 4, 6, 8, 10, 12?
- hint: just changing the units

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- NP-complete \approx infeasible

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- claim: *an object with some property exists*
- proof: *an example*
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- no need to disclose the sources
- beware: claim may be false!