

# A simplified approximation of the four-parameter LF model of voice source

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A simplified approximation of the four-parameter, voice source model developed by Fant *et al.* [STL-QPSR 4, 1–12 (1985)] is described in this Letter. In our approximation, the computational complexity required to implement the four-parameter model is simplified without reducing model accuracy. The simplified approximation accommodates rapid, on-line adjustment of source parameters during speech synthesis.

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## INTRODUCTION

An accurate and versatile model of the human voice source is essential to the achievement of natural sounding synthetic speech (Fant *et al.*, 1985; Klatt and Klatt, 1990; Childers and Lee, 1991). Fant, Liljencrants, and Lin (1985) have developed a four-parameter model (hereafter referred to as the LF model) that has been widely used. The model has been used as an excitation input to a speech synthesizer and as a mathematical framework for analyzing different types of voices. The model is computationally complex. For example, it requires the solution of roots of two, nonlinear equations for each given set of model parameters. In addition, not all parameters in the model have direct physical correspondence with voice source events.

Our overall objective was to develop a simplified approximation of the LF model for use as excitation to a speech synthesizer. Specific goals were to reduce the computation needed to implement the LF model and to have all four parameters of the model correspond directly with physical events related to human voicing.

## I. LF MODEL AND ITS APPROXIMATION

In the LF model, the flow derivative of the voice source is specified by four temporal parameters  $t_c$ ,  $t_p$ ,  $t_e$ , and  $t_a$  (see Fig. 1). Three of the parameters have direct physical correspondence with human voicing events:  $t_c$  could be the fundamental period,  $t_p$  is the instant of maximum flow, and  $t_e$  is the instance of maximum glottal closing. Parameter  $t_a$  is the second (right) derivative of the volume flow at the minimum of the first derivative. It does not have apparent physical correspondence with human voicing events. These parameters are referred to as the analysis set of parameters and they may be estimated from inverse filtering of the speech signal.

Another set of parameters,  $E_0$ ,  $E_e$ ,  $\omega_g$ ,  $\alpha$ , and  $\epsilon$ , are used to synthesize the flow derivative in the LF model. These parameters can be derived from the analysis set and appear in the model as

$$E(t) = \begin{cases} E_1(t) = E_0 e^{\alpha t} \sin \omega_g t & (t \leq t_e), \\ E_2(t) = (-E_e / \epsilon t_a) [e^{-\epsilon(t-t_e)} - e^{-\epsilon(t_c-t_e)}] & (t_e < t \leq t_c), \end{cases} \quad (1)$$

where  $E(t)$  is the flow derivative.

All parameters in Eq. (1) need to be supplied explicitly in any numerical implementation of the LF model. Some parameters are relatively easy to obtain. For example,  $E_0$  is an arbitrary gain constant and  $\omega_g$  simply equals  $\pi/t_p$ . Others are not. For example, explicit values of  $\alpha$  and  $\epsilon$  are not available until roots of the following equations are solved:

$$1 - e^{-\epsilon(t_c-t_e)} = \epsilon t_a, \quad (2)$$

$$\int_0^{t_c} E(t) dt = 0. \quad (3)$$

Equation (2) results from setting  $E_2(t_e) = -E_e$ , where  $E_e > 0$  is the amplitude of flow derivative at  $t_e$ . Equation (3) ensures that the source flow function returns to baseline at the end of each period. Once  $\alpha$  is obtained,  $E_e$  can be computed as

$$E_e = -E_0 e^{\alpha t_e} \sin(\omega_g t_e). \quad (4)$$

To avoid solving Eq. (3), Fant *et al.* approximate the area under the second segment of their model using  $(E_e t_a / 2) K_a$ , where  $K_a$  is a function of  $t_c - t_e$  and  $t_a$  only. This substitution is referred to as the LF approximation. With this approximation,  $\alpha$  becomes the root of an ordinary nonlinear equation,

$$\frac{e^{\alpha t_e} [\alpha \sin(\omega_g t_e) - \omega_g \cos(\omega_g t_e)] + \omega_g}{\alpha^2 + \omega_g^2} - \frac{e^{\alpha t_e} \sin(\omega_g t_e) t_a}{2} K_a, \quad (5)$$

resulting from flow continuity at  $t_e$ . Here,  $\epsilon$  remains the root of Eq. (2). The roots of two, nonlinear equations for each given set of parameters also need to be solved to numerically compute source function by the LF approximation.

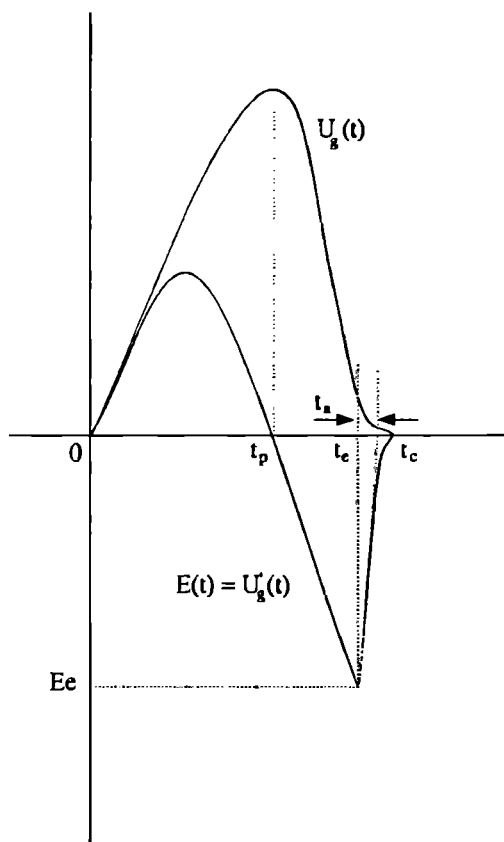


FIG. 1. The LF model.

## II. A SIMPLIFIED APPROXIMATION

The complicated relationships among parameters of the LF model are largely related to the constraint imposed by Eq. (3). This constraint ensures that the flow returns to the baseline at the end of each period. This constraint also forces all parameter adjustments to be made concurrently on both segments of the LF model and imposes complicated relationships among model parameters.

In our simplified approximation, we eliminate  $t_a$  in the second segment of the LF model and use  $E_e$  as an explicit model parameter (Ananthapadmanabha and Fant, 1982; Ananthapadmanabha, 1984). As a result, the four parameters of the voice source model now include  $t_c$ ,  $t_p$ ,  $t_e$ , and  $E_e$ . In our approximation, the first segment of the LF model remains unchanged. In addition, the second segment of the LF model remains an exponential function, but takes a slightly different form:

$$E_2(t) = -E_e e^{-\epsilon(t-t_e)}, \quad (6)$$

where  $E_e$  is a known constant. The antiderivative of this equation is

$$U_2(t) = \int_{t_e}^t E_2(t) dt = \frac{E_e}{\epsilon} e^{-\epsilon(t-t_e)} - \frac{E_e}{\epsilon} e^{-\epsilon(t_c-t_e)}. \quad (7)$$

Using Eqs. (4) and (6),  $\alpha$  is simply

$$\alpha = \frac{1}{t_e} \ln \left( \frac{-E_e}{E_0 \sin(\omega_g t_e)} \right) \quad (8)$$

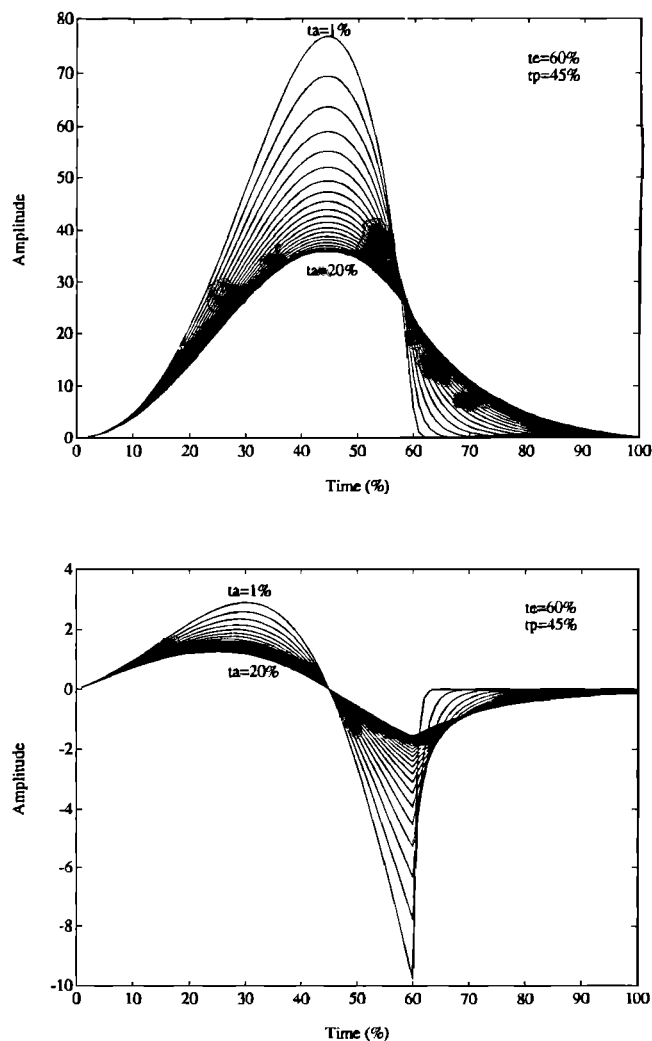


FIG. 2. (a) Flow of approximation I. (b) Flow derivative of approximation I.

based on Eq. (1). Here,  $\epsilon$  is obtained by the flow continuity constraint,

$$U_1(t_e) = U_2(t_e), \quad (9)$$

where

$$U_1(t) = \int_0^t E_1(t) dt. \quad (10)$$

Because  $U_1(t)$  is not a function of  $\epsilon$  [see the left-hand side of Eq. (5)], the root of only one, nonlinear equation needs to be solved to compute the source function. If it is further assumed that the closed phase is relatively long and the return to flow baseline is relatively fast following glottal closing [i.e.,  $\epsilon(t_c - t_e) \gg 1$ ], the term  $e^{-\epsilon(t_c - t_e)}$  would be near zero. Under these assumptions,  $\epsilon$  can be directly obtained as

$$\epsilon = E_e / U_1(t_e) \quad (11)$$

and no root solving is necessary.

## III. EVALUATION OF THE APPROXIMATION

For purposes of comparison, the signal-to-noise (S/N) ratios of flow and flow derivative were computed, where the

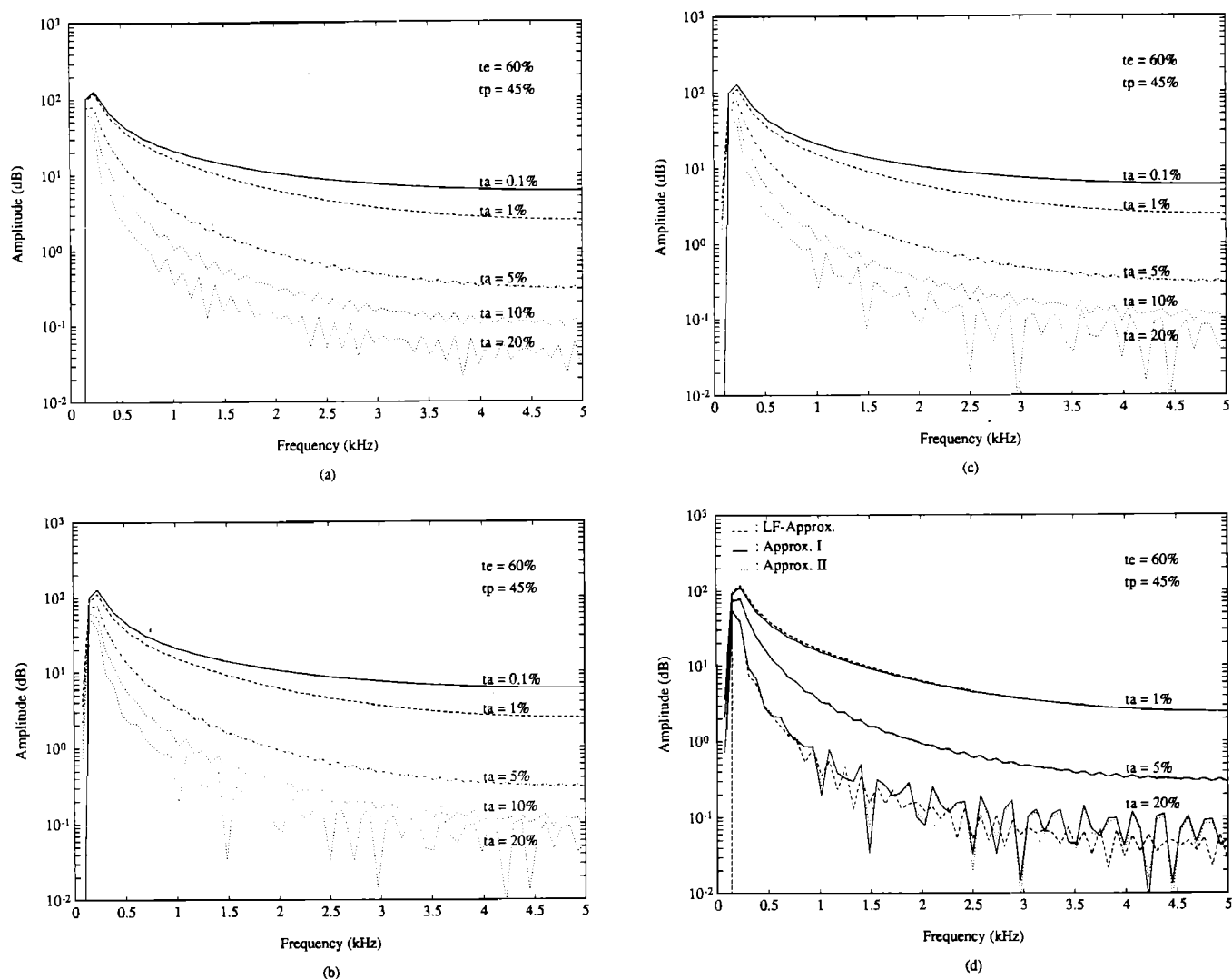


FIG. 3. Spectra of selected flow derivatives of (a) LF model, (b) approximation I, (c) approximation II, and (d) LF model and its approximations.

S/N ratio is defined as the ratio between the rms energy of the original LF model and the rms energy of the amplitude difference between the LF model and our approximation. The S/N ratios were computed for the approximations described in Sec. II.

In these evaluations,  $\alpha$  and  $\epsilon$  were obtained using Eqs. (2) and (3) for the LF model. The integration required in Eq. (3) was accomplished using forward Euler's method. For our simplified approximation,  $\alpha$  was obtained using Eq. (8). Here,  $\epsilon$  was obtained using Eq. (9) or (11) depending on the desired level of approximation. When Eq. (9) was used, we refer to the result as approximation I. When Eq. (11) was used, we refer to the result as approximation II. A bi-section method was used to solve all necessary roots in the evaluation process (Dahlquist *et al.*, 1974). For easy comparison, the gain constant,  $E_0$ , was set to unity and  $t_a$  was always used to denote the evaluation results in place of  $E_e$ , because  $t_a$  and  $E_e$  are equivalent independent parameters for our simplified approximations.

The flow and flow derivative at various parameter values for approximation I are illustrated in Fig. 2. Here,  $t_c$ ,  $t_e$ , and  $t_p$  were held constant, and  $t_a$  was varied from 1% to 20%<sup>1</sup> of

the fundamental period ( $t_c$ ). Plots for the LF model and approximation II are not illustrated because they were nearly identical to the plots for approximation I. Spectra of selected flow derivatives are shown in Fig. 3(a), (b), and (c) for the LF model, approximation I, and approximation II, respectively. For convenience of comparison, these spectra are merged in Fig. 3(d). These results demonstrate that our simplified approximations are capable of producing a wide range of source functions characterized by temporal and spectral content that is comparable to that produced by the LF model.

Some S/N ratios of the approximated flow and flow derivative are shown in Fig. 4 for various parameter models. Here,  $t_a$  was 1%–20% and  $t_p$  was 40%–50% of the fundamental period. Additional S/N ratios of the approximated flow and flow derivative are shown in Fig. 5. Here,  $t_a$  was 1%–20%, while  $t_e$  was 55%–65% of the fundamental period.

Evaluation results summarized in Figs. 2–5 show that the differences between approximation I and II are relatively small when  $t_a$  is small. Observable differences exist between approximation I and II when  $t_a$  is relatively large (see Figs. 4 and 5). The differences between approximation I and II at

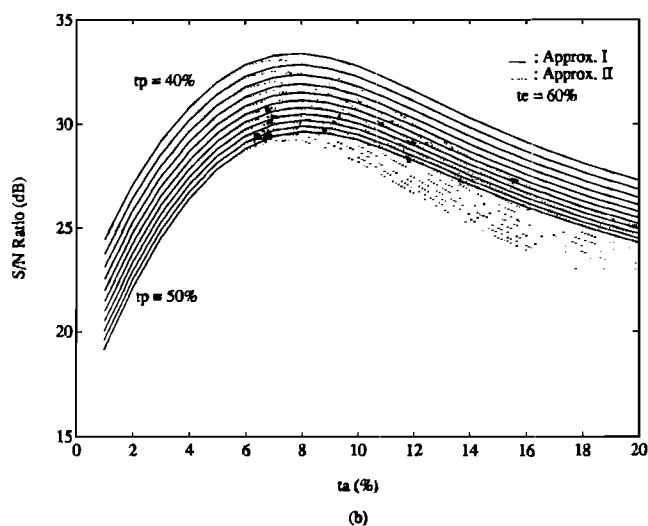
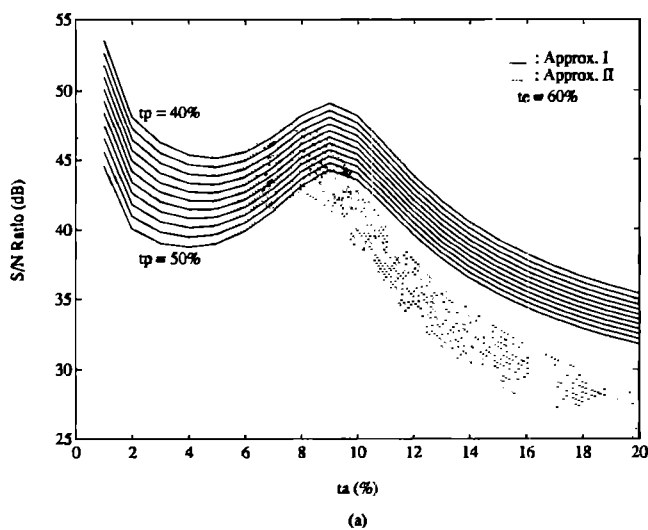


FIG. 4. (a) Signal-to-noise ratios of flow approximations. (b) Signal-to-noise ratios of flow derivative approximations.

large  $t_a$  values are due to the need for the flow function to return to the baseline at  $t_e$  in approximation I [see Eq. (7)], while the flow function approaches the baseline asymptotically in approximation II. These evaluation results also indicate that the S/N ratios are generally higher for flow than for flow derivative. The approximation of the flow is most accurate when  $t_a$  is relatively small ( $<10\%$  of the pitch period). The approximations of flow derivative are most accurate when  $t_a$  is in the middle range and not in an extreme condition.

Approximations I and II are based on flow rather than flow derivative. Use of the integrated forms of these approximations is recommended whenever possible. For example, during speech synthesis, the derivation of the synthetic signal should be made in the final operation of the synthesizer. On the basis of informal listening, we could not detect any differences between speech synthesized using approximations I and II and speech synthesized using the LF model. The simplifications of approximations I and II permit rapid, on-line adjustment of source parameters during speech synthesis.

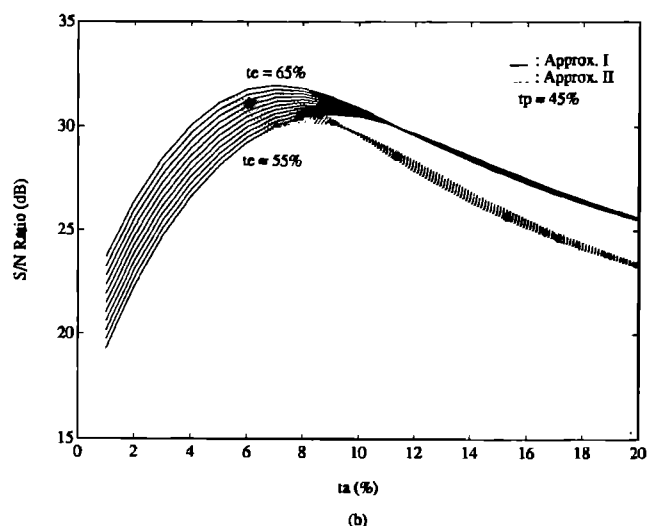
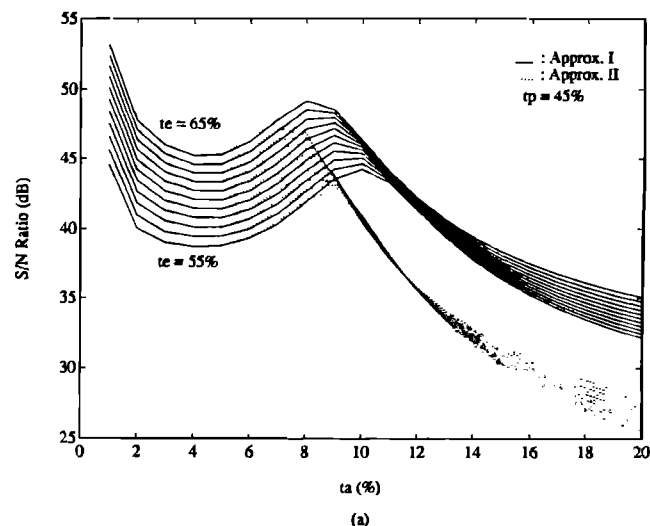


FIG. 5. Signal-to-noise ratios of flow approximations. (b) Signal-to-noise ratios of flow derivative approximations.

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<sup>1</sup>Note that the corresponding  $E_e$  value of each  $t_a$  is available from the flow derivative plots.

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