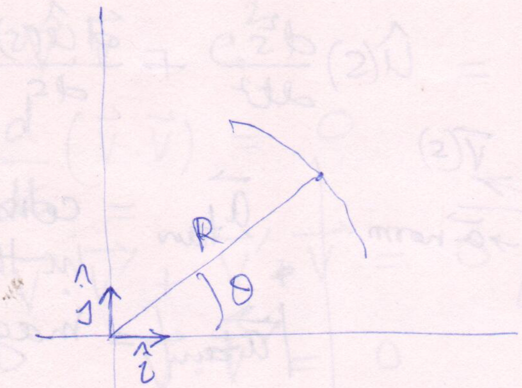
Example: uniform circular motion of a particle.



$\omega = \dfrac{d\theta}{dt} = $ const.

R: const.

$\theta = \omega t$

$$\vec{r}(t) = R\cos\omega t\,\hat{i} + R\sin\omega t\,\hat{j}$$

Note: $|\vec{r}(t)| = R$: constant.

$$\vec{V}(t) = \frac{d\vec{r}(t)}{dt} = -R\omega\sin\omega t\,\hat{i} + R\omega\cos\omega t\,\hat{j}$$

Note: $|\vec{V}(t)| = R\omega$: constant : speed.

$\vec{r}(t)\cdot\vec{V}(t) = 0 \Rightarrow \vec{r}$ and $\vec{V}$ are $\perp$ to each other at any t

$$\vec{a}(t) = \frac{d\vec{V}(t)}{dt} = -R\omega^2\cos\omega t\,\hat{i} - R\omega^2\sin\omega t\,\hat{j}$$

$$= -\cancel{R}\omega^2(R\cos\omega t\,\hat{i} + R\sin\omega t\,\hat{j})$$

$$= -\omega^2\vec{R}$$ (opposite to $\vec{R}$ $\Rightarrow$ centripetal)

Also, $\vec{R} \perp \vec{V}$ at any t

Note: ~~to~~ If we consider $\vec{\Omega} = \omega\hat{k}$ as the angular velocity vector (coming out of the board), then

$$\vec{V}(t) = \cancel{to}\ \vec{\Omega}\times\vec{r}(t)$$

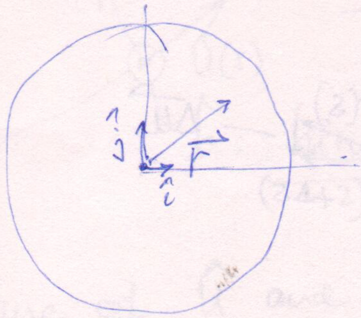
and

$$\vec{a}(t) = \vec{\Omega}\times\vec{V}(t) = \vec{\Omega}\times(\vec{\Omega}\times\vec{r}(t))$$

[Note the restrictions!]

Other cases:

Constant angular speed/velocity Ω



Uniform radial movement.

$$R = \alpha t, \qquad \theta = \omega t.$$

$$\vec{r}(t) = \alpha t\cos\omega t\,\hat{\imath} + \alpha t\sin\omega t\,\hat{\jmath}$$

$$\vec{V}(t) = \frac{d}{dt}\vec{r}(t) = (\alpha\cos\omega t\,\cancel{\hat{\imath}} - \alpha\omega t\sin\omega t)\,\hat{\imath}$$

$$+ (\alpha\sin\omega t + \alpha\omega t\cos\omega t)\,\hat{\jmath}$$

$$= (\alpha\cos\omega t\,\hat{\imath} + \alpha\sin\omega t\,\hat{\jmath}) \qquad (A)$$

$$+ (-\alpha\omega t\sin\omega t\,\hat{\imath} + \alpha\omega t\cos\omega t\,\hat{\jmath}) \qquad (B)$$

(B) is the tangential component, $= \vec{\Omega}\times\vec{r}(t)$

(A) is the radial component $= \frac{dR}{dt}\,\hat{u}_r(t)$

[check both]

$$\vec{a}(t) = \frac{d}{dt}\vec{V}(t) = (-\alpha\omega\sin\omega t - \alpha\omega\sin\omega t - \alpha\omega^2 t\cos\omega t)\,\hat{\imath}$$

$$+ (\alpha\omega\cos\omega t + \alpha\omega\cos\omega t - \alpha\omega^2 t\sin\omega t)\,\hat{\jmath}$$

$$= (-2\alpha\omega\sin\omega t\,\hat{\imath} + 2\alpha\omega\cos\omega t\,\hat{\jmath}) \qquad (C)$$

$$- \alpha\omega^2 t(\cos\omega t\,\hat{\imath} + \sin\omega t\,\hat{\jmath}) \qquad (D)$$

(D) is the radial component (centripetal) $= -\omega^2\vec{r}(t)$

$$= \vec{\Omega}\times(\vec{\Omega}\times\vec{r}(t))$$

(C) is the tangential component

$$= -2\vec{\Omega}\times\left(\frac{dR}{dt}\,\hat{u}_r(t)\right)$$

→ This is known as the Coriolis Component.