THE OPTIMUM DESIGN OF ELECTRODE AND INSULATOR CONTOURS BY NONLINEAR PROGRAMMING USING THE SURFACE CHARGE SIMULATION METHOD

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Abstract - A new method is presented to optimize electrode and insulator contours. In the method, electrode and insulator contours are modified by using iteration methods used in nonlinear programming so that desired electric field distribution is obtained. Gauss-Newton method, quasi-Newton method, conjugate gradient method or steepest descent method is used as the iteration method. The electric field distributions are computed by means of the surface charge simulation method. This paper shows that Gauss-Newton method gives very fast convergence.

INTRODUCTION

Recently, several useful papers concerning the optimization of electrode and insulator contour based on surface charge simulation method or charge simulation method were presented[1]-[3]. In such optimization, selection of the rule of contour modification is a important problem: we have to determine the direction to move the contour in and the distance to modify. Then modification of the contour is done so that desired electric field distribution is obtained.

The authors have developed an optimization method which determines optimal contour automatically. In the method, surface charge simulation method[4], by which electric field distribution is obtained with high accuracy, and iteration methods, which are techniques for nonlinear programming, are used for the computation of electric field and the modification of electrode and insulator contours, respectively. Gauss-Newton method, quasi-Newton method, conjugate gradient method or steepest descent method is used as the iteration method. Further, in order to demonstrate the validity of the proposed method, a Borda electrode model and a sphere electrode model are chosen as optimization examples.

SURFACE CHARGE SIMULATION METHOD

The electric field distributions are computed by means of the surface charge simulation method[4] using curved surface elements[5] and curved line elements for three-dimensional problems and two-dimensional problems, respectively. For performing the contour modification smoothly, the surface of each element is defined so that there is only one normal vector for each point on the boundary between two elements.

The potential, V, and the electric field, $\vec{E}$, induced at a specific point are given by

$$V = \frac{1}{\varepsilon_0}\iint_S \sigma\phi \, dS \qquad (1)$$

$$\vec{E} = -\frac{1}{\varepsilon_0}\iint_S \sigma \, \mathrm{grad}\phi \, dS \qquad (2)$$

where σ is a surface charge density, ϕ is a fundamental solution and S is the total area of electrode and insulator surfaces.

In the surface charge simulation method, the electrode and insulator surfaces are divided into surface elements, which are curved surface triangular elements in three-dimensional problems or curved line elements in two-dimensional problems, and nodes are defined on the the surface elements. For example, the node on the curved surface triangular element is defined at each vertex and the node on the curved line element is defined at each end. The surface charge density on the surface element is discretized by the magnitude at each node, and is approximated by a linear function of coordinates.

The flux continuity condition at a specific point on the insulator surface is given by

$$(\varepsilon_1\vec{E}_1 - \varepsilon_2\vec{E}_2)\cdot\vec{n} = 0 \qquad (3)$$

where ε_1 and ε_2 are permittivities of two dielectrics and $\vec{n}$ is a unit normal vector at a specific point.

When applying the potential condition given by Eq. (1) to the nodes on the electrode surfaces and the flux continuity condition given by Eq. (3) to the nodes on the insulator surfaces, a system of linear equations is obtained in the form:

$$[C]\{\sigma\} = \{v\} \qquad (4)$$

where [C] is a coefficient matrix, $\{\sigma\}$ is a discretized surface charge density vector and $\{v\}$ is a condition vector. An element of $\{v\}$ is a potential, which is given by the potential condition on the electrode surfaces, or zero, which is given by the flux continuity condition on the insulator surfaces.

OPTIMIZATION METHOD

In order to obtain the desired field distributions on any electrode and/or insulator surface, electrode and insulator contours are modified by using iteration methods used in nonlinear programming[6].

When applying the nonlinear programming, the object function, W, can be written as

$$W = \sum_{i=1}^{L} (E_i - E_{i0})^2 = \sum_{i=1}^{L} w_i^2 = \{w\}^T\{w\} \qquad (5)$$

where $w_i = E_i - E_{i0}$

E_i and E_{i0} are the computed electric field strength and the desired electric field strength at node, i, respectively, and L is the number of the nodes at which desired electric field strengths are given.

In the proposed optimization method, the nodes on the electrode and insulator to be optimized move only in the direction of the normal vector in order to reduce the number of design variables. The design variable vector consists of the displacements of the nodes and is given as

$$\{x\} = \{x_1\, x_2 \cdots x_j \cdots x_M\}^T \qquad (6)$$

where x_j is the displacement of node, j, in the normal direction and M is the number of the nodes to be modified.

The design variable vector, $\{x\}$, is determined so that the object function is minimized. The approximate solution of $\{x\}$ at the kth iteration step is given by

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$$\{x\}^{k+1} = \{x\}^{k} + \alpha^{k}\{d\}^{k} \quad (7)$$

where $\{d\}^k$ is the search direction vector and α^k is a coefficient to be determined by a linear-search method.

$\{d\}^k$ is given as follows:

$$\{d\}^k = -([G]^k)^{-1}\nabla W^k \quad \text{(Gauss-Newton method)} \quad (8)$$

$$\{d\}^k = -[H]^k \nabla W^k \quad \text{(Quasi-Newton method)} \quad (9)$$

$$\{d\}^k = -\nabla W^k + \{d\}^{k-1}\frac{[(\nabla W^k)^T \nabla W^k]}{[(\nabla W^{k-1})^T \nabla W^{k-1}]} \quad \text{(Conjugate gradient method)} \quad (10)$$

$$\{d\}^k = -\nabla W^k \quad \text{(Steepest descent method)} \quad (11)$$

where

$$\nabla W = 2[J]^T\{w\}$$

$$[G] \simeq 2[J]^T[J]$$

[G] is the Hessian matrix and [J] is the Jacobi matrix. [H] is computed by the formula of the DFP method [6].

We have to compute the Jacobi matrix in order to evaluate Eqs. (8)-(11). Using Eq. (6), the Jacobi matrix is written as

$$[J] = \left[\frac{\partial\{w\}}{\partial x_1}\,\frac{\partial\{w\}}{\partial x_2}\cdots\frac{\partial\{w\}}{\partial x_M}\right]$$

$$= \left[\frac{\partial\{E\}}{\partial x_1}\,\frac{\partial\{E\}}{\partial x_2}\cdots\frac{\partial\{E\}}{\partial x_M}\right] \quad (12)$$

From Eq. (2), electric field vector {E} is given by

$$\{E\} = [F]\{\sigma\} \quad (13)$$

where [F] is a coefficient matrix. Using Eqs. (5) and (13), we can obtain

$$\frac{\partial\{E\}}{\partial x_m} = \left(\frac{\partial[F]}{\partial x_m} + [F][C]^{-1}\frac{\partial[C]}{\partial x_m}\right)\{\sigma\} \quad (14)$$

For computing the Jacobi matrix by the use of the difference approximations of Eq. (12), field computation is required M times. But by the use of the difference approximates of Eq. (14), the number of field computation times can be reduced to one.

The optimization is carried out automatically by the program which consists of two parts. One is used to determine the search direction, and another is used to determine the design variable vector and modify the electrode and insulator contours. The computations are iterated until the object function reaches the minimum value.

COMPUTATION RESULTS

In order to verify the effectiveness of the proposed optimization method, two models were investigated: a Borda electrode model and a sphere model in a cube. In the two models, the desired field strength is the uniform strength which is defined by dividing the potential difference by the shortest distance between electrodes.

Borda Electrode Model

The Borda electrode model, which is the two-dimensional model called $\pi/2$ Borda Electrode, is shown in Fig. 1. Gauss-Newton method, quasi-Newton method, conjugate gradient method and steepest descent method are used for the Borda electrode model. In this model, contour A-B without nodes A and B is modified and

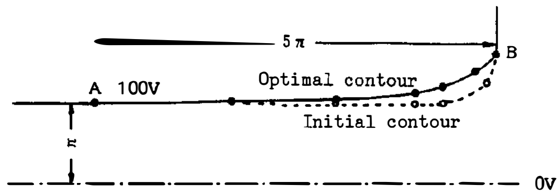

Fig. 1 A Borda electrode model(L=7, M=5).

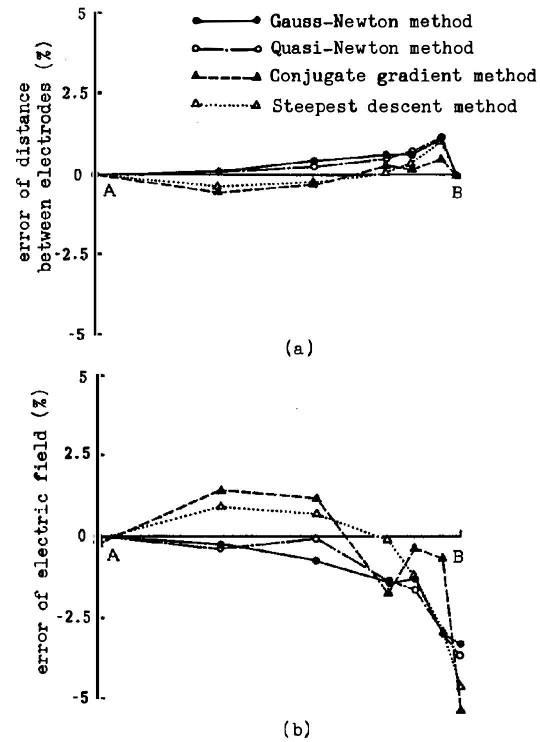

Fig. 2 Computation errors of distance between electrodes for the Borda electrode model, (a) errors of distance, (b) errors of electric field.

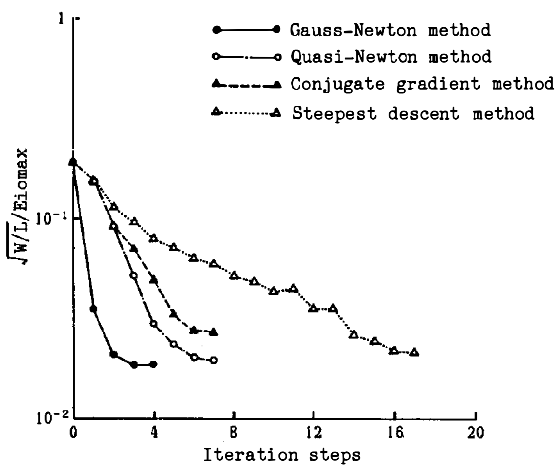

Fig. 3 Changes in the normalized value of the object function for the Borda electrode model.

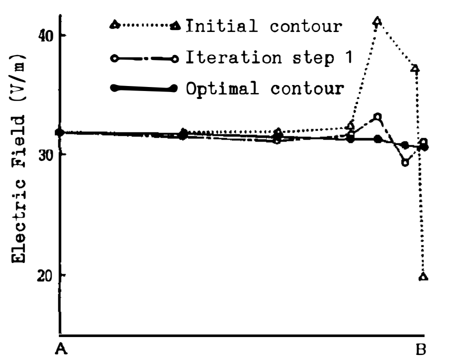

Fig. 4 Changes of electric field distributions along contour A-B of the Borda electrode.

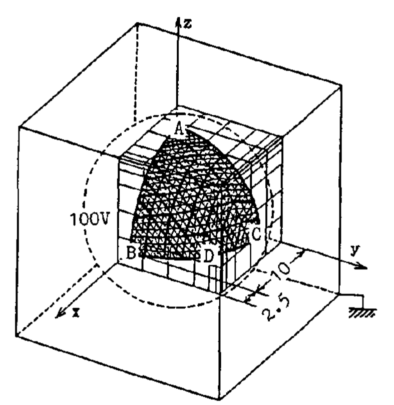

Fig. 5 A sphere electrode model in a cube (L=45, M=42).

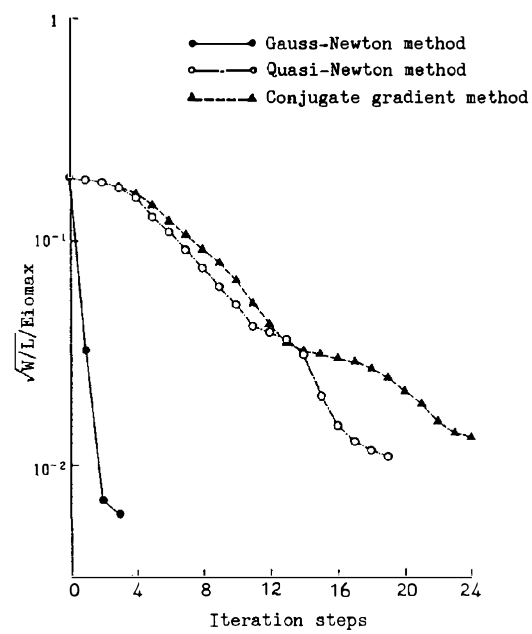

Fig. 6 Changes in the normalized value of the object function for the sphere electrode model.

electric field at each node on contour A-B is evaluated. Therefore, the number of nodes to be modified and the number of nodes, at which desired electric field strengths are given, are 5 and 7, respectively: M=5 and L=7. And 100/π (V/unit length) is given uniformly as desired electric field strength.

In Fig. 1, broken line and solid line are the initial contour and the optimal contour which was obtained by Gauss-Newton method, respectively. The maximum error of the distance between electrodes, which were determined by each method, was approximately less than one percent as shown in Fig. 2(a). And errors of electric field are shown in Fig. 2(b). The computed electric field strengths are not over 102 percent of the desired electric field strength. Figure 3 shows the changes in the normalized value of object function. It was verified from Fig. 3 that Gauss-Newton method gives very fast convergence. The numbers of iteration steps were 4, 7, 7 and 17 for Gauss-Newton method, quasi-Newton method, conjugate gradient method and steepest descent method, respectively. Figure 4 shows the changes of the electric field distribution along contour A-B in the case of Gauss-Newton method.

Computation results of the Borda electrode model show that the optimal contour which provides desired electric field distribution can be determined automatically and stably by the proposed method.

Sphere Electrode Model

A sphere electrode model in a cube is an example of three-dimensional problems. Figure 5 shows 64 triangular elements and 108 rectangular elements on the sphere electrode surface and the cube surface, respectively. The rectangular element is a special element for approximating rectangular surface and the surface charge density on the rectangular element is approximated by a linear function of coordinates using four nodes. The number of unknown surface charge densities is 192. Three-dimensional shapes of the electrode are displayed by using small triangular mesh as shown in Fig. 5 and Fig. 7. In this case, triangular region A-B-C-A without nodes A, B and C is modified and electric field at each node on the triangular region A-B-C-A is evaluated. The number of nodes to be modified and the number of nodes, at which desired electric field strengths are given, are 42 and 45, respectively: M=42 and L=45. And 40 (V/unit length) is given uniformly as desired electric field strength.

Figure 6 shows the changes in the normalized value of object function for Gauss-Newton method, quasi-Newton method and conjugate gradient method. The same characteristics of the convergence for the three methods as the case of the Borda electrode model were obtained. Then the optimal electrode shape determined

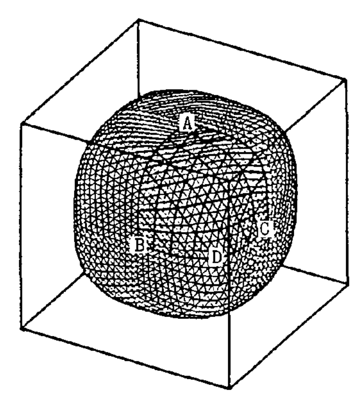

Fig. 7 Optimal shape of the sphere electrode.

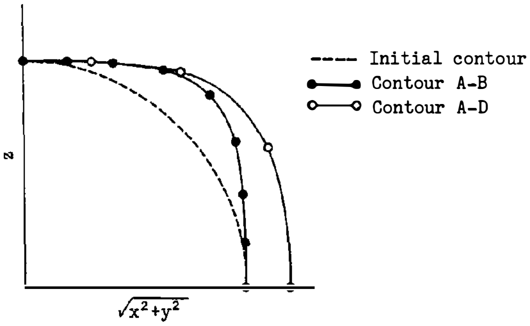

Fig. 8 Optimal contours A-B and A-D.

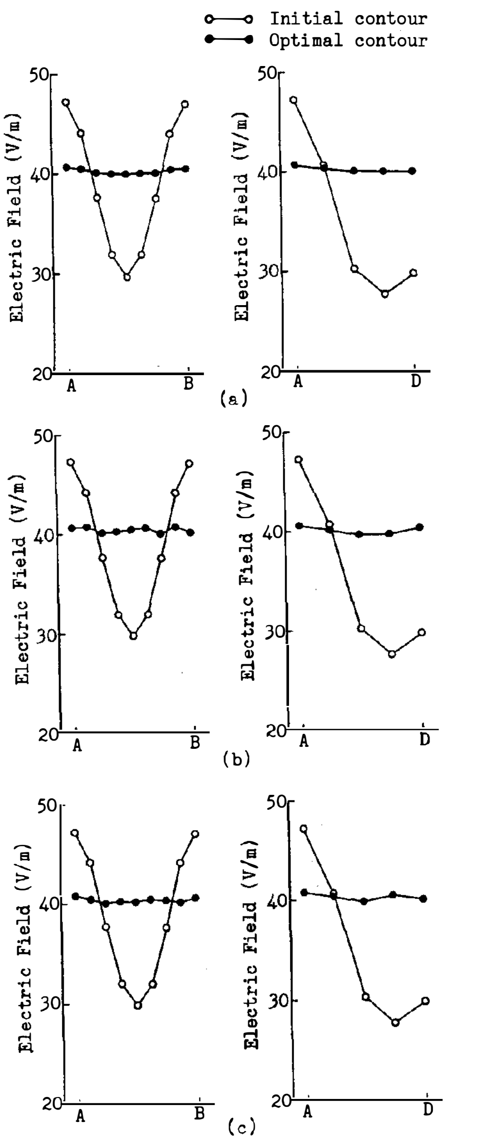

Fig. 9 Obtained electric field distributions on the electrode surfaces, (a) by Gauss-Newton method, (b) by quasi-Newton method, (c) by conjugate gradient method.

by Gauss-Newton method after three iterations is shown in Fig. 7. And the optimal contours A-B and A-D in Fig. 7 are shown in Fig. 8. Figure 9 shows that the electric field distribution can be determined by each method so that the electric field distribution become approximately uniform. The numbers of iteration steps were 3, 19 and 24 for Gauss-Newton method, quasi-Newton method and conjugate gradient method, respectively. By the sphere electrode model, validity of the proposed method was verified for three-dimensional problem, too.

CONCLUSION

This paper proposed a new method which optimizes the electrode and insulator contour using the iteration methods of nonlinear programming on the basis of the surface charge simulation method.

Computation results of two-dimensional and three-dimensional models showed that the optimal contour which provides desired electric field distribution is determined automatically and stably by the proposed method. Then it was verified that Gauss-Newton method, which is one of the iteration methods used in nonlinear programming, gives very fast convergence, and the number of iteration steps of quasi-Newton method and conjugate gradient method increases in proportion to the number of points to be modified. Quasi-Newton method and conjugate gradient method may be used for a model in which the optimal contour can not be obtained by Gauss-Newton method because ill condition arises[6].

We feel that the proposed optimization method is practicable for the design of electrical machinery and apparatus including electric field and/or magnetic field problems.

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