COMPUTATION OF SHORT, MEDIUM AND LONG TRANSMISSION LINE

(i) To understand modeling and performance of Short transmission lines

(ii) To understand modeling and performance of medium transmission line Nominal –π methods

(iii) To understand modeling and performance of long transmission line using rigorous method.

Problem Statements:

An overhead 3-phase transmission line delivers 5000 kW at 22 kV at 0·8 p.f.lagging. The resistance and reactance of each conductor is 4 Ω and 6 Ω respectively. Determine :(i) sending end voltage (ii) percentage regulation (iii) transmission efficiency.

$$Z = R + j X_L$$

$$I = \frac{kW \times 10^3}{V_R \cos \phi_R}$$

Sending end voltage, $\vec{V_S} = \vec{V_R} + \vec{I}\,\vec{Z}$

Angle between VS and VR is α

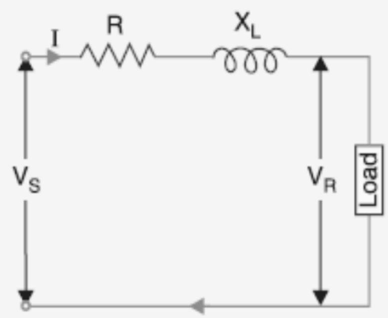
Sending end power factor angle is

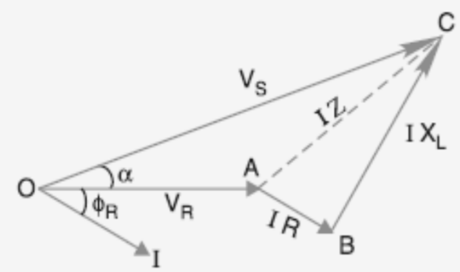
$$\phi_S = \phi_R + \alpha$$

Line losses = $3I^2R$

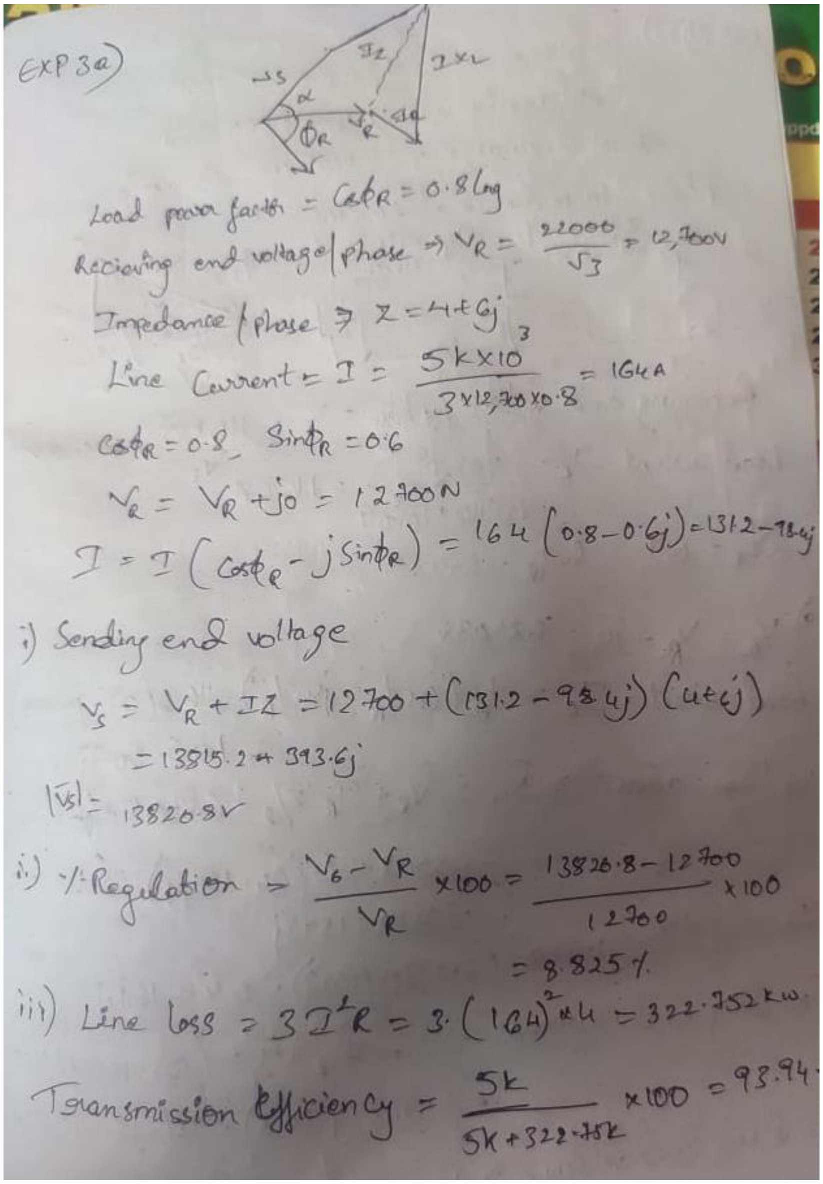
Power sent= power output+ losses

$$\text{Transmission efficiency} = \frac{\text{Power delivered}}{\text{Power sent}} \times 100$$





Manual Calculations:



EXP 3a)

Load power factor = $\cos\phi_R = 0.8$ lag

Recieving end voltage/phase ⇒ $V_R = \frac{22000}{\sqrt{3}} = 12,700V$

Impedance/phase ⇒ $Z = 4+6j$

Line Current = $I = \frac{5k\times10^3}{3\times12,700\times0.8} = 164A$

$\cos\phi_R = 0.8$, $\sin\phi_R = 0.6$

$V_R = V_R + j0 = 12700V$

$I = I(\cos\phi_R - j\sin\phi_R) = 164(0.8-0.6j) = 131.2-98.4j$

i) Sending end voltage

$V_s = V_R + IZ = 12700 + (131.2-98.4j)(4+6j)$

$= 13815.2 + 393.6j$

$|V_s| = 13820.8V$

ii) % Regulation = $\frac{V_s - V_R}{V_R}\times100 = \frac{13820.8-12700}{12700}\times100$

$= 8.825\%$

iii) Line loss = $3I^2R = 3\cdot(164)^2\times4 = 322.752$ kW

Transmission Efficiency = $\frac{5k}{5k+322.75k}\times100 = 93.94$

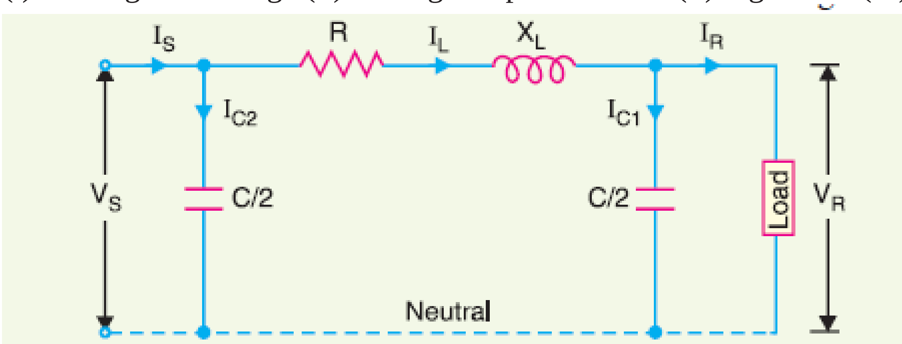
Details:

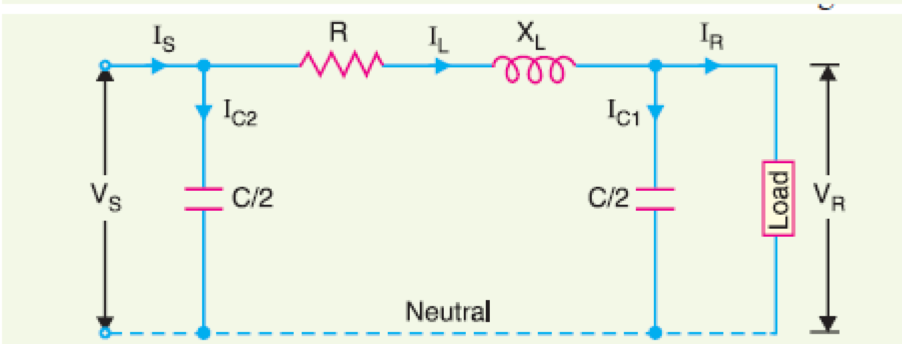
A **short transmission line** is defined as a transmission line with an effective length less than 80 km (50 miles), or with a voltage less than 69 kV. Unlike medium transmission lines and long transmission lines, the line charging current is negligible, and hence the shunt capacitance can be ignored.

For short length, the shunt capacitance of this type of line is neglected and other parameters like electrical resistance and inductor of these short lines are lumped, hence the equivalent circuit is represented. The sending end and receiving end voltages make angle with that reference receiving end current, of φs and φr, respectively

MEDIUM TRANSMISION LINE

(ii) A 100-km long, 3-phase, 50-Hz transmission line has following line constants:
Resistance/phase/km = 0·1 Ω Reactance/phase/km = 0·5 Ω Susceptance/phase/km = 10 × 10− 6 S
If the line supplies load of 20 MW at 0·9 p.f. lagging at 66 kV at the receiving end, calculate by nominal π method:
(i) sending end voltage (ii)sending end power factor (ii) regulation (iii)





$$\vec{V_R} = V_R + j\,0$$

Load current, $\vec{I_R} = I_R\,(\cos\phi_R - j\sin\phi_R)$

Charging current at the load end is

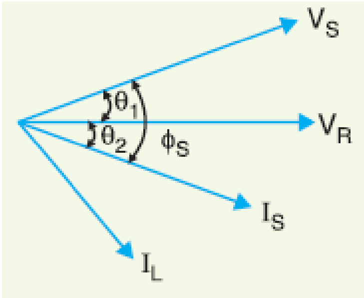
$$\vec{I_{C1}} = \vec{V_R}\, j\,\frac{Y}{2}$$

Line current, $\vec{I_L} = \vec{I_R} + \vec{I_{C1}}$

Sending end voltage, $\vec{V_S} = \vec{V_R} + \vec{I_L}\,\vec{Z}$

$$I_{C2} = j\vec{V_S}\,Y/2$$

Sending end current, $\vec{I_S} = \vec{I_L} + \vec{I_{C_2}}$



θ_1 = angle between $\overrightarrow{V_R}$ and $\overrightarrow{V_S}$

θ_2 = angle between $\overrightarrow{V_R}$ and $\overrightarrow{I_S}$

ϕ_S = angle between $\overrightarrow{V_S}$ and $\overrightarrow{I_S} = \theta_2 + \theta_1$

$$\text{Transmission efficiency} = \frac{\text{Power delivered}}{\text{Power sent}} \times 100$$

$$\%\ \text{Voltage regulation} = \frac{V_S - V_R}{V_R} \times 100$$

Manual Calculations:

3b) ii) Total Resistance = $R = 0.1 \times 100 = 10\Omega$

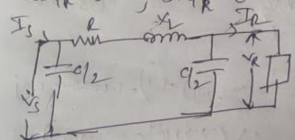
Total Reactance = $X_L = 0.5 \times 100 = 50\Omega$

Susceptance = $Y = 10 \times 10^{-6} \times 100 = 10 \times 10^{-4}$ S

Receiving End voltage / phase = $V_R = \frac{66 \times 10^3}{\sqrt{3}} = 38105$ V

Load current = $I_R = \frac{20 \times 10^6}{\sqrt{3} \times 66 \times 10^3 \times 0.9} = 195$ A

$\cos\phi_R = 0.9$; $\sin\phi_R = 0.435$



Taking receiving end voltage = $\bar{V}_R = V_R + j0 = 38105$ V

$\bar{I}_R = I_R(\cos\phi_R - j\sin\phi_R) = 195(0.9 - j0.435)$
$= 176 - j85$

$\bar{I}_{C_1} = \bar{V}_R \, jY/2 = \frac{38105 \times 10 \times 10^{-4}}{2} j = 19j$

$\bar{I}_L = \bar{I}_R + \bar{I}_{C_1} = (176 - 85j) + 19j = 176 - 66j$

Sending end voltage = $\bar{V}_S = \bar{V}_R + \bar{I}_L \bar{Z}$
$= 38105 + (176 - 66j)(10 + 50j)$
$= 43165 + 8140j$

Sending End Current = $I_S = I_L + I_{C2}$
$= (176 - 66j) + (-4 + 21.6j)$
$= 172 - 44.4j$

i) θ_1 = Angle b/w V_R & V_S = 10.65

θ_2 = Angle b/w V_R & V_S = -14.5

ϕ_S = Angle b/w V_S & I_S = $\theta_2 + \theta_1$ = 14.5 + 10.65 = 25.15

Sending end p.f = Cos 25.15 = 0.905 lag

ii) $\% VR = \frac{V_S - V_R}{V_R} \times 100 = \frac{43925 - 38105}{38105} \times 100 = 15.27\%$

ii) Send End power $= 3 V_S I_S \cos\phi = 3 \times 43925 \times 177.6 \times 0.905$ = 21.18 MW

Transmission Efficiency $= \frac{20}{21.18} \times 100 = 94\%$

Details:

A medium transmission line is defined as a transmission line with an effective length more than 80 km (50 miles) but less than 250 km (150 miles). Unlike a short transmission line, the line charging current of a medium transmission line is appreciable and hence the shunt capacitance must be considered (this is also the case for long transmission lines). This shunt capacitance is captured within the admittance (“Y”) of the ABCD circuit parameters.

The ABCD parameters of a medium length transmission line is calculated using a lumped shunt admittance, along with the lumped impedance in series to the circuit. These lumped parameters of a medium length transmission line can be represented using three different models, namely:

Nominal Π representation (nominal pi model)

Nominal T representation (nominal T model)

End Condenser Method

LONG TRANSMISSION LINE

1. **A three-phase transmission line 200 km long has the following constants, Resistance/phase/km = 0·16 Ω, Reactance/phase/km = 0·25 Ω, Shunt admittance/phase/km = 1·5 × 10–6 S, calculate by rigorous method the sending end voltage and current when the line is delivering a load of 20 MW at 0·8 p.f. lagging. The receiving end voltage is kept constant at 110 kV.**

$$V_S = V_R \cosh\sqrt{YZ} + I_R\sqrt{\frac{Z}{Y}}\sinh\sqrt{YZ}$$

$$I_S = V_R\sqrt{\frac{Y}{Z}}\sinh\sqrt{YZ} + I_R\cosh\sqrt{YZ}$$

Comparing these equations with those of (*i*) and (*ii*), we get,

$$\vec{A} = \vec{D} = \cosh\sqrt{YZ}\,; \quad \vec{B} = \sqrt{\frac{Z}{Y}}\sinh\sqrt{YZ}\,; \quad \vec{C} = \sqrt{\frac{Y}{Z}}\sinh\sqrt{YZ}$$

$$\sinh\sqrt{YZ} = \left(\sqrt{YZ} + \frac{(YZ)^{3/2}}{6} + \ldots\ldots\right)$$

$$\cosh\sqrt{YZ} = \left(1 + \frac{ZY}{2} + \frac{Z^2Y^2}{24} + \ldots\ldots\right)$$

Y = total shunt admittance of the line

Z = total series impedance of the line

$$\text{Line current,} \quad I = \frac{\text{Power delivered}}{\sqrt{3}\times\text{line voltage}\times\text{power factor}}$$

$$\text{Transmission efficiency} = \frac{\text{Power delivered}}{\text{Power sent}}\times 100$$

$$\%\,\text{Regulation} = \frac{(V_S/A - V_R)}{V_R}\times 100$$

Manual Calculations:

4) $R = 0.16\,\Omega/km$, $Y = 1.5\times10^{-6}\,\mho/km$

$X_L = 0.25\,\Omega/km$, $L = 200km$

$V_{Rline} = 110kV$, $\cos\phi_R = 0.8$ Pf lag, $P_R = 20MW$

Total Resistance/phase $R = 0.16 \times 200 = 32\Omega$

Total Reactance/phase $X_L = 0.25\times 200 = 50\Omega$

Total Shunt admittance/phase $Y = j\,1.5\times10^{-6} \times 200$

$= 0.0003\angle 90^\circ$

$Z = R + j\cdot X_L = 32 + j50 = 59.4\angle 58^\circ$

$\sqrt{\frac{Z}{Y}} = \sqrt{\frac{59.4\angle 58^\circ}{0.0003\angle 90^\circ}} = 445\angle -16^\circ$; $\sqrt{\frac{Y}{Z}} = \sqrt{\frac{0.0003\angle 90^\circ}{59.4\angle 58^\circ}} = 0.00224\angle 16^\circ$

From matlab the values of $\cosh(\sqrt{ZY}) = 0.9925 + j0.0048$

$= 0.9925\angle 0.277$

$\sinh(\sqrt{ZY}) = 0.0377 + j0.128$

$= 0.133\angle 73.77$

Therefore the values of ABCD parameter are

$A = D = \cosh(\sqrt{ZY}) = 0.9925\angle 0.277 = 0.9925 + j0.0048$

$B = \sqrt{\frac{Z}{Y}}\sinh(\sqrt{ZY}) = (445\angle -16)(0.133\angle 73.77^\circ)$

$= 59.185\angle 57.77 = 31.564 + j50.068$

$C = \sqrt{\frac{Y}{Z}}\sinh(\sqrt{ZY}) = (0.00224\angle 16^\circ)(0.133\angle 73.77^\circ)$

$= 1.195\times10^{-6} + j\,2.979\times10^{-4}$

$$\bar{V}_s = \bar{A}\bar{V}_r + \bar{B}\bar{I}_r \qquad V_r = V_{r\,line} / \sqrt{3}$$

$$\bar{I}_s = \bar{C}\bar{V}_r + \bar{D}\bar{I}_r \qquad \bar{V}_r = \frac{110\times10^3}{\sqrt{3}} = 66508\angle 0^\circ\ V$$

$$I_r = \frac{P_r}{\sqrt{3}\times V_r\times \cos\phi_r} = \frac{20\times10^6}{\sqrt{3}\times 110\times10^3\times 0.8} = 131\angle -36.86^\circ\ A$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 0.9925+j0.0048 & 31.564+j50.065 \\ 1.195\times10^{-6}+j7.97\times10^{-4} & 0.09925+j0.0048 \end{bmatrix}$$

$$\bar{V}_s = \left[(0.9925\angle 0.277)(63508\angle 0^\circ)\right] + \left[(59.18\angle 57.7)(131\angle -36.86)\right]$$

$$= 70262.411 + j3100.918$$

$$\bar{V}_s = 70330.8\angle 2.527, \quad |\bar{V}_s| = 70.33\,kV$$

$$|V_{s\,line}| = 121.898\,kV \Rightarrow V_{s\,line} = (121.816\angle 2.52)\,kV$$

$$I_s = \left[(2.997\times10^{-4}\angle 89.77^\circ)(63508\angle 0^\circ)\right] + \left[(0.9925\angle 0.272)(131\angle -36.86)\right]$$

$$= 104.479 - j58.569$$

$$\bar{I}_s = 119.776\angle -29.274^\circ\ A; \quad I_s = 119.776\,A$$

$$\%\,V_R = \frac{\left(\left|\frac{V_s}{A}\right| - V_r\right)}{V_r}\times100 = \frac{(70862-63508)}{63508}\times100 = 11.579\,\%$$

$$\theta_1 = \angle V_s = 2.527, \quad \theta_2 = \angle I_s = -29.274$$

$$\phi_s = |\theta_1| + |\theta_2| = 31.801^\circ$$

$$P_s = 3|V_s||I_s|\cos\phi_s = 3\times70330.80\times119.776\times\cos(31.801)$$

$$= 21.478\,MW$$

Details:

A **long transmission line** is defined as a transmission line with an effective length more than 250 km (150 miles). Unlike short transmission lines and medium transmission lines, it is no longer reasonable to assume that the line parameters are lumped. To accurately model a long transmission line we must consider the exact effect of the distributed parameters over the entire length of the line. Although this makes the calculation of ABCD parameters of transmission line more complex, it also allows us to derive expressions for the voltage and current at any point along the line.

In a long transmission line the line constants are uniformly distributed over the entire length of line. This is because the effective circuit length is much higher than what it was for the former models (long and medium line) and hence we can no longer make the following approximations:

- Ignoring the shunt admittance of the network, like in a small transmission line model.
- Considering the circuit impedance and admittance to be lumped and concentrated at a point as was the case for the medium line model.

Rather, for all practical reasons, we should consider the circuit impedance and admittance being distributed over the entire circuit length as shown in the figure below. The calculations of circuit parameters, for this reason, are going to be slightly more rigorous