Editor Report on
Fast Automatic Bayesian Cubature Using Lattice Sampling
by Jagadeeswaran and Hickernell

We have received two very careful reviews of this paper from experts in fields that should eventually be influenced by probabilistic numerics (PN).

Referee 1 (with the plain text review) raises numerous points. The most important one there is that the obtained speedup can be seen as an FFT because the critical matrix in it is a circulant matrix. The revision should at a minimum point out the connection to the FFT. It would be even better to replace any derivations that could be done by quoting FFT results. It is not surprising that when an $O(n^2)$ cost is reduced to $O(n\log(n))$ that an FFT is lurking somewhere. This referee does not seem to be very familiar with Gaussian process models that PN people make extensive use of. Because the goal is to reach people outside of PN some additional explanations are in order.

Referee 2 (with the PDF review) raises mostly points of clarification and motivation.

A revision should address the concerns of both reviewers, in all or most instances making the explanation in the text itself.

In my reading of it, I found a few things to be difficult, largely overlapping the concerns of the reviewers.

1. The results in Section 2.2.2 were confusing. The expression for $\rho_\mu(z|\boldsymbol{f}=\boldsymbol{y})$ took several readings to figure out. It is odd that the random variable μ takes typical values z. It is also odd that the expression doesn't involve z. After lots of flipping back and forth, it turns out that z is embedded in γ and β (though not α). It is not hard math to figure that out, just an unnecessary slowing down. It is not a surprise that a t distribution ends up as the answer. It would be better to just quote a result from the literature to replace all or most of that subsubsection.

2. It is weird to use complex numbers for the eigenvectors of a real symmetric matrix. It is valid but I think that it confuses people. So at the first use of complex numbers there should be an explanation that they are going to be useful later on. Better yet would be a hint about why they will be useful. Some complex conjugates tripped up one of the reviewers so that should be spelled out.

3. In (24), why is $V^H = nV^{-1}$ and not just V^{-1}? I might be missing something elementary here, but isn't V^H meant to be the Hermitian transpose of V? For instance if $V = I$ then for the Hermitian transpose $V^H = I$ too, and not nI. Maybe the n or $1/n$ could go into Λ more easily. At a minimum, a comment is in order. I looked in (37) for a $1/n$ but did not see it.

4. The argument at the end of section 4.3 involving is very slippery. I understand that it can be explained in terms of the summands having period n in both i and j.

Some minor points

1. The $\boldsymbol{x}_i$ in (4) would have to be distinct to force a strictly positive quadratic form.

2. The notation for the full Bayes treatment is not consistent. We have a conditional version of $\rho_\mu(z)$ that does not show the parameters along with $\boldsymbol{\rho}_{m,s^2}(\xi, \lambda)$ that does show the parameters ξ and λ. That seems to mix different conventions. One convention is to have $f_X(x;\theta)$ for a random variable X taking specific value x from distribution f depending on a parameter θ. Another is $f_X(x \mid \theta)$ and another is $f(x \mid \theta)$ (replacing f by p or π or here ρ). Maybe it is all logical but just using unconventional/inconsistent notation. It should be tidied up.

3. First paragraph of 2.3: for the $\mu \to$ for μ. Also intervals $\to$ interval.

4. End of Section 2: the the Matérn $\to$ the Matérn

5. In the big display near the middle of the left column on page 7 should $\boldsymbol{c}^T$ be $\boldsymbol{c}^H$? Otherwise how do we get $\tilde{c}_i^*$?

6. In equation (30), I think t should have $n-1$ df, not n_{j-1} df. Otherwise what is n_j?

7. 1st sentence of 4.3: shift-invariance $\to$ shift-invariant