Referee's report on

Fast Automatic Bayesian Cubature Using Sobol' Sampling

by R. Jagadeeswaran and F.J. Hickernell

This paper is devoted to automatic cubatures that approximate integrals to user-specified error thresholds. In the recent paper [11] the authors tackled this question by means of lattice sampling. The present paper deals with digital sequence sampling, in particular Sobol' sequences are employed. The proposed algorithm is implemented and tested on three concrete examples. A discussion of the numerical results and an outlook on future work closes the paper.

The paper is based on material presented in [11]. It is reasonable to extend the method from there based on lattice sampling to digital sequence sampling. The presentation is very terse and at several places the reader is referred to [11] for related information or a definition. For a reader who does not have publication [11] at hand it would be of great help when the paper could be slightly more self-contained.

It seems there is also a bug in the proof of Lemma 2 that will be addressed in the comments below.

The paper can be accepted but not before the comments below are taken into account.

Comments:

1. p.2, line 18: describe the parameters in the notation $\mathcal{GP}(m, s^2 C_\theta)$ already here.

2. p.2, item i): explain V and Λ_θ! What do you mean by V^H?

3. p.3, line -11 (and other places): there is no assumption (1.4). Write "assumptions (1.4a)–(1.4d)"

4. p.3–4, explain the formulas (1.5) to (1.8b) in more detail. What are the y_i, the $\widetilde{y}_i$, what the $\lambda_{\theta,i}$?

5. p.5, line 1: what do you mean by "digital sequences nets"? I guess you mean just "digital sequences". Close the sentence with a dot.

6. p.5, line 3ff: It is really strange to formulate Theorem 1 here at this place. Give first all necessary definitions (in particular (1.14)), then present the theorem and then it's proof.

7. p.5, Definition 1: the notation ${}_20.z_1z_2\cdots$ is very unusual. I suggest to write $(0.z_1z_2\ldots)_2$ and explain that the subscript 2 indicates that the expansion is considered in base 2.

8. p.5, line –9: read "representation"

9. p.6, line 3: where is the corresponding closing bracket?

10. p.6, line -9: write "$\ldots r = 1$ (see [17]):"

11. p.6, line -6: what is the η_ℓ here?

12. p.7, line -3 and -4: how can an eigenvector be a square-matrix? I do not understand what you mean here. Maybe that the eigenvectors can be expressed by means of Walsh-Hadamard matrices? Please give a better explanation.

13. p.8, Lemma 2: what is meant by "2×2 block-Toepliz matrix"?

14. p.8, line -10: write "The proof of (1.16) follows by induction."

15. p.8, line -5: there appears a term "$k + 2$" that has to be deleted

16. p.8, line -4: **I cannot see this equality, but maybe I do not understand your notation. Please explain! The same appears on p.9, line 6.**

17. p.8, line -15: write "$\mathcal{O}(n \log n)$"

18. p.10, line 9: an "exact" answer to which question?

That's all!