Referee report on the manuscript

"Fast automatic Bayesian cubature using Sobol' Sampling"
by R. Jagadeeswaran and F.J. Hickernell

Reviewer 1:

Below is a list of minor comments:

page 2, line 5: "analytic expression exists" -> add "or is easily available"
("Nonexistence" is not equivalent to "not (yet) known" or "cumbersome to obtain".)
Rephrased it. (no closed-form analytic expression is easily available)

page 3, equations (1.5)-(1.7): please make it clear where the integrand f is hidden and
where the evaluation points x_i are used. (It's in the y_i.)
Added clarification (where the integrand values are evaluated as $y_i = f(\bsx_i)$.)

page 5, line 1: typo's "digital sequence nets" and missing period before "The"
Actually, it becomes confusing later on. Section 1.3.1 is about digital sequences
and from section 1.3.2 on its about digital nets. That should be cleared out.
Fixed (period '.' added) Clarified

page 5, last line - 8: typo "represntation" -> "representation"
Fixed the typo

page 6, line 1: add word -> "from Definition 1 and can be found"
Fixed ('and' added)

page 6, displayed equation in the middle: specify range for r and eta
added r \in {\N}, \eta \in (0, 1)

Figures, all: adjust point size of axis labels and legends to make them of the same size
as the text;
adjust the horizontal axis to provide more space for the information. No need to go
down to 10^(-4) or even 10^(-9) in Fig 1.7, if there is (almost) nothing below 10^(-4)
Done: changed the range to 10^(-5) to 10^(1). Two plots have few instances smaller than .
10^(-4), thus the choice 10^(-5).

page 7, line before section 1.3.3: Why "may"?

If \eta is same for all the dimensions, $\eta_1 == \eta_2 == … == \eta_d$ so $\theta = (\eta_1, \eta) then dimension of \theta is just 2. But if we have unique \eta_i != \eta_j then dimension of \theta is d+1. Rephrased (The parameter vector $\bstheta$ now is of dimension $d+1$.)

page 8, rephrase first sentence of proof. "Define the matrices that by Lemma 1" sounds strange. There is too much in that first sentence
This has been reworded.

page 10, first para of section 1.4.1: OK the timings should be read comparatively, but normally one specifies the platform when giving timings
Added my CPU details (i7-7700HQ).

page 10, second para of section 1.4.1: such a comparison is problematic. Here too much depends on the platform plus implementation details. Most of the time in the cubature context one says the cost is in the function evaluations, but this example is probably too simple to state just that.
We do not want to list in terms of function evaluations because there is some overhead in taking the fast transform and constructing the credible intervals. We think that time comparisons are more fair. We are comparing across the same hardware platform.

page 10, please mention the dimension of the problem explicitly (as is done for the following 2 examples).
Added (d = 2)

page 14, line 2, "efficient" in what way?
Added clarification (with number of samples used in integration)

Figures 1.8, 1.9, and 1.10: I first missed the hollow stars. I guess many became effectively invisible because they are overprinted by blue dots
We mention the explicit number of hollow stars in the caption?

page 16, section 1.4.4: it is unclear what was one in the mentioned 40 or 100 seconds. The whole test run to produce the figures?
That time is for one run. Maximum 40 seconds to run a test with Keister and multivariate Gaussian. Maximum 100 seconds for option pricing.
Also, added some clarification (max value in Y-axis)

The log(eps) value leading to the 400 dots comes from a uniform distribution. Comment on why the log(time) is clear not uniformly distributed
It's because we double the number of samples in every iteration till the error threshold is met. Compute time also increases proportionally.

page 16, line 3: Can you quantify what "few exceptions" means? The dimension of the problem

is not that high here. It's 13. For me the examples show that eps = 10^-4 seems to be the limit for this approach, a message that deserves to be mentioned in the Conclusion.
It may not be so much the case of a small epsilon, but the limit on the sample size that we imposed. Wording added.

page 16, section 1.5, line 6: integrand -> integrands
Fixed
page 16, section 1.5, line 17: remove "face"
Fixed
----------the-end----------

Reviewer 2:

1. p.2, line 18: describe the parameters in the notation $\mathcal{GP}(m, s^2 C_\theta)$ already here.

Introduced the parameters here

2. p.2, item i): explain V and Λ_θ! What do you mean by V^H?

Complex conjugate transpose, defined in a footnote

3. p.3, line -11 (and other places): there is no assumption (1.4). Write "assumptions (1.4a)–(1.4d)"

Fixed

4. p.3–4, explain the formulas (1.5) to (1.8b) in more detail. What are the y_i, the $\widetilde{y}_i$, what the $\lambda_{\theta,i}$?

Fixed

5. p.5, line 1: what do you mean by "digital sequences nets"? I guess you mean just "digital sequences". Close the sentence with a dot.

Fixed

6. p.5, line 3ff: It is really strange to formulate Theorem 1 here at this place. Give first all necessary definitions (in particular (1.14)), then present the theorem and then it's proof.

moved the Theorem to the end

7. p.5, Definition 1: the notation ${}_20.z_1z_2\cdots$ is very unusual. I suggest to write $(0.z_1z_2\ldots)_2$ and explain that the subscript 2 indicates that the expansion is considered in base 2.

changed the notation

8. p.5, line –9: read "representation"

Fixed

9. p.6, line 3: where is the corresponding closing bracket?

Fixed

10. p.6, line -9: write "$\ldots r = 1$ (see [17]):"

Fixed

11. p.6, line -6: what is the η_ℓ here?

Explained it.

12. p.7, line -3 and -4: how can an eigenvector be a square-matrix? I do not understand what you mean here. Maybe that the eigenvectors can be expressed by means of Walsh-Hadamard matrices? Please give a better explanation.

Fred reworded and explained

13. p.8, Lemma 2: what is meant by "2×2 block-Toepliz matrix"?

This has been corrected. They are block-Hankel and a reference is given.

14. p.8, line -10: write "The proof of (1.16) follows by induction."

Fixed

15. p.8, line -5: there appears a term "$k + 2$" that has to be deleted

Fixed

16. p.8, line -4: **I cannot see this equality, but maybe I do not understand your notation. Please explain! The same appears on p.9, line 6.**

Thank you, There were some errors in the proof that have been corrected.

17. p.8, line -15: write "$\mathcal{O}(n \log n)$"

Fixed

18. p.10, line 9: an "exact" answer to which question?

exact == True integral value. For all three examples. This has been reworded.