ADVANCED DATA STRUCTURES & ALGORITHM ANALYSIS

UNIT 1

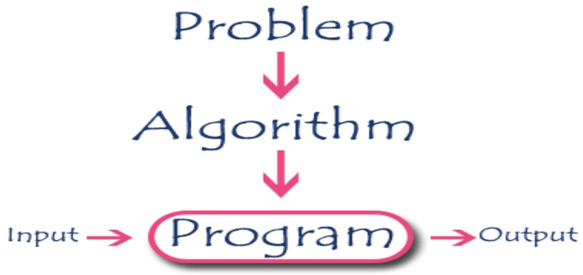
Introduction to Algorithm Analysis

What is an algorithm?

An algorithm is a step by step procedure to solve a problem. In normal language, the algorithm is defined as a sequence of statements which are used to perform a task. In computer science, an algorithm can be defined as follows...

An algorithm is a sequence of unambiguous instructions used for solving a problem, which can be implemented (as a program) on a computer.

Algorithms are used to convert our problem solution into step by step statements. These statements can be converted into computer programming instructions which form a program. This program is executed by a computer to produce a solution. Here, the program takes required data as input, processes data according to the program instructions and finally produces a result as shown in the following picture.



Specifications of Algorithms

Every algorithm must satisfy the following specifications...

Input - Every algorithm must take zero or more number of input values from external.

Output - Every algorithm must produce an output as result.

Definiteness - Every statement/instruction in an algorithm must be clear and unambiguous (only one interpretation).

Finiteness - For all different cases, the algorithm must produce result within a finite number of steps.

Effectiveness - Every instruction must be basic enough to be carried out and it also must be feasible.

Example for an Algorithm

Let us consider the following problem for finding the largest value in a given list of values.

Problem Statement : Find the largest number in the given list of numbers?

Input : A list of positive integer numbers. (List must contain at least one number).

Output : The largest number in the given list of positive integer numbers.

Consider the given list of numbers as 'L' (input), and the largest number as 'max' (Output).

Algorithm

1. **Step 1:** Define a variable 'max' and initialize with '0'.
2. **Step 2:** Compare first number (say 'x') in the list 'L' with 'max', if 'x' is larger than 'max', set 'max' to 'x'.
3. **Step 3:** Repeat step 2 for all numbers in the list 'L'.
4. **Step 4:** Display the value of 'max' as a result.

Algorithm Performance Analysis

What is Performance Analysis of an algorithm?

If we want to go from city "A" to city "B", there can be many ways of doing this. We can go by flight, by bus, by train and also by bicycle. Depending on the availability and convenience, we choose the one which suits us. Similarly, in computer science, there are multiple algorithms to solve a problem. When we have more than one algorithm to solve a problem, we need to select the best one. Performance analysis helps us to select the best algorithm from multiple algorithms to solve a problem.

When there are multiple alternative algorithms to solve a problem, we analyze them and pick the one which is best suitable for our requirements. The formal definition is as follows...

Performance of an algorithm is a process of making evaluative judgement about algorithms.

It can also be defined as follows...

Performance of an algorithm means predicting the resources which are required to an algorithm to perform its task.

That means when we have multiple algorithms to solve a problem, we need to select a suitable algorithm to solve that problem.

We compare algorithms with each other which are solving the same problem, to select the best algorithm. To compare algorithms, we use a set of parameters or set of elements like memory required by that algorithm, the execution speed of that algorithm, easy to understand, easy to implement, etc., Generally, the performance of an algorithm depends on the following elements...

1. Whether that algorithm is providing the exact solution for the problem?
2. Whether it is easy to understand?
3. Whether it is easy to implement?
4. How much space (memory) it requires to solve the problem?
5. How much time it takes to solve the problem? Etc.,

Space and Time Complexity analysis

When we want to analyse an algorithm, we consider only the space and time required by that particular algorithm and we ignore all the remaining elements.

Based on this information, performance analysis of an algorithm can also be defined as follows...

Performance analysis of an algorithm is the process of calculating space and time required by that algorithm.

Performance analysis of an algorithm is performed by using the following measures...

1. Space required to complete the task of that algorithm (**Space Complexity**). It includes program space and data space
2. Time required to complete the task of that algorithm (**Time Complexity**)

a) Space Complexity

What is Space complexity?

When we design an algorithm to solve a problem, it needs some computer memory to complete its execution. For any algorithm, memory is required for the following purposes...

1. To store program instructions.
2. To store constant values.
3. To store variable values.
4. And for few other things like funcion calls, jumping statements etc,.

Space complexity of an algorithm can be defined as follows...

Total amount of computer memory required by an algorithm to complete its execution is called as space complexity of that algorithm.

Generally, when a program is under execution it uses the computer memory for THREE reasons. They are as follows...

1. **Instruction Space:** It is the amount of memory used to store compiled version of instructions.
2. **Environmental Stack:** It is the amount of memory used to store information of partially executed functions at the time of function call.
3. **Data Space:** It is the amount of memory used to store all the variables and constants.

Note - When we want to perform analysis of an algorithm based on its Space complexity, we consider only Data Space and ignore Instruction Space as well as Environmental Stack. That means we calculate only the memory required to store Variables, Constants, Structures, etc., To calculate the space complexity, we must know the memory required to store different data type values (according to the compiler). For example, the C Programming Language compiler requires the following...

1. 2 bytes to store Integer value.
2. 4 bytes to store Floating Point value.
3. 1 byte to store Character value.
4. 6 (OR) 8 bytes to store double value.

Consider the following piece of code...

Example 1

```
int square(int a)
{
   return a*a;
}
```

In the above piece of code, it requires 2 bytes of memory to store variable **'a'** and another 2 bytes of memory is used for **return value**.

That means, totally it requires 4 bytes of memory to complete its execution. And this 4 bytes of memory is fixed for any input value of 'a'. This space complexity is said to be *Constant Space Complexity*.

If any algorithm requires a fixed amount of space for all input values then that space complexity is said to be Constant Space Complexity.

Consider the following piece of code...

Example 2

```
int sum(int A[ ], int n)
{
   int sum = 0, i;
   for(i = 0; i < n; i++)
      sum = sum + A[i];
   return sum;
}
```

In the above piece of code it requires

'n*2' bytes of memory to store array variable **'a[ ]'**

2 bytes of memory for integer parameter **'n'**

4 bytes of memory for local integer variables **'sum'** and **'i'** (2 bytes each)

2 bytes of memory for **return value**.

That means, totally it requires '2n+8' bytes of memory to complete its execution. Here, the total amount of memory required depends on the value of 'n'. As 'n' value increases the space required also increases proportionately. This type of space complexity is said to be *Linear Space Complexity*.

If the amount of space required by an algorithm is increased with the increase of input value, then that space complexity is said to be Linear Space Complexity.

b) Time Complexity

What is Time complexity?

Every algorithm requires some amount of computer time to execute its instruction to perform the task. This computer time required is called time complexity.

The time complexity of an algorithm can be defined as follows...

The time complexity of an algorithm is the total amount of time required by an algorithm to complete its execution.

Generally, the running time of an algorithm depends upon the following...

1. Whether it is running on **Single** processor machine or **Multi** processor machine.
2. Whether it is a **32 bit** machine or **64 bit** machine.
3. **Read** and **Write** speed of the machine.
4. The amount of time required by an algorithm to perform **Arithmetic** operations, **logical** operations, **return** value and **assignment** operations etc.,
5. **Input** data

Note - When we calculate time complexity of an algorithm, we consider only input data and ignore the remaining things, as they are machine dependent. We check only, how our program is behaving for the different input values to perform all the operations like Arithmetic, Logical, Return value and Assignment etc.,

Calculating Time Complexity of an algorithm based on the system configuration is a very difficult task because the configuration changes from one system to another system. To solve this problem, we must assume a model machine with a specific configuration. So that, we can able to calculate generalized time complexity according to that model machine.

To calculate the time complexity of an algorithm, we need to define a model machine. Let us assume a machine with following configuration...

1. It is a Single processor machine
2. It is a 32 bit Operating System machine

3. It performs sequential execution
4. It requires 1 unit of time for Arithmetic and Logical operations
5. It requires 1 unit of time for Assignment and Return value
6. It requires 1 unit of time for Read and Write operations

Now, we calculate the time complexity of following example code by using the above-defined model machine...

Consider the following piece of code...

Example 1

```
int sum(int a, int b)
{
  return a+b;
}
```

In the above sample code, it requires 1 unit of time to calculate a+b and 1 unit of time to return the value. That means, totally it takes 2 units of time to complete its execution. And it does not change based on the input values of a and b. That means for all input values, it requires the same amount of time i.e. 2 units.

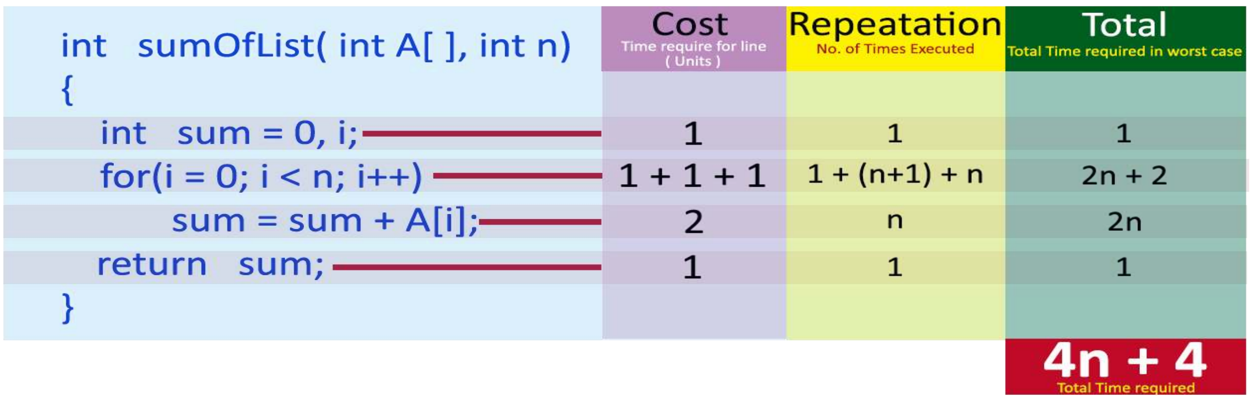
If any program requires a fixed amount of time for all input values then its time complexity is said to be Constant Time Complexity.

Consider the following piece of code...

Example 2

```
int sum(int A[], int n)
{
  int sum = 0, i;
  for(i = 0; i < n; i++)
    sum = sum + A[i];
  return sum;
}
```

For the above code, time complexity can be calculated as follows...



int sumOfList(int A[ ], int n) {	Cost Time require for line (Units)	Repeatation No. of Times Executed	Total Total Time required in worst case
int sum = 0, i;	1	1	1
for(i = 0; i < n; i++)	1 + 1 + 1	1 + (n+1) + n	2n + 2
sum = sum + A[i];	2	n	2n
return sum;	1	1	1
}			
			4n + 4 Total Time required

In above calculation

Cost is the amount of computer time required for a single operation in each line.

Repeatation is the amount of computer time required by each operation for all its repeatations.

Total is the amount of computer time required by each operation to execute.

So above code requires **'4n+4' Units** of computer time to complete the task. Here the exact time is not fixed. And it changes based on the **n** value. If we increase the **n** value then the time required also increases linearly.

Totally it takes '4n+4' units of time to complete its execution and it is *Linear Time Complexity*.

If the amount of time required by an algorithm is increased with the increase of input value then that time complexity is said to be Linear Time Complexity.

Asymptotic Notations

What is Asymptotic Notation?

Whenever we want to perform analysis of an algorithm, we need to calculate the complexity of that algorithm. But when we calculate the complexity of an algorithm it does not provide the exact amount of resource required. So instead of taking the exact amount of resource, we represent that complexity in a general form (Notation) which produces the basic nature of that algorithm. We use that general form (Notation) for analysis process.

Asymptotic notation of an algorithm is a mathematical representation of its complexity.

In asymptotic notation, when we want to represent the complexity of an algorithm, we use only the most significant terms in the complexity of that algorithm and ignore least significant terms in the complexity of that algorithm (Here complexity can be Space Complexity or Time Complexity).

For example, consider the following time complexities of two algorithms...

- **Algorithm 1 : $5n^2 + 2n + 1$**
- **Algorithm 2 : $10n^2 + 8n + 3$**

Generally, when we analyze an algorithm, we consider the time complexity for larger values of input data (i.e. **'n'** value). In above two time complexities, for larger value of **'n'** the term **'2n + 1'** in algorithm 1 has least significance than the term **'$5n^2$'**, and the term **'8n + 3'** in algorithm 2 has least significance than the term **'$10n^2$'**.

Here, for larger value of **'n'** the value of most significant terms (**$5n^2$** and **$10n^2$**) is very larger than the value of least significant terms (**2n + 1** and **8n + 3**). So for larger value of **'n'** we ignore the least significant terms to represent overall time required by an algorithm. In asymptotic notation, we use only the most significant terms to represent the time complexity of an algorithm.

Majorly, we use THREE types of Asymptotic Notations and those are as follows...

1. **Big - Oh (O)**
2. **Big - Omega (Ω)**
3. **Big - Theta (Θ)**

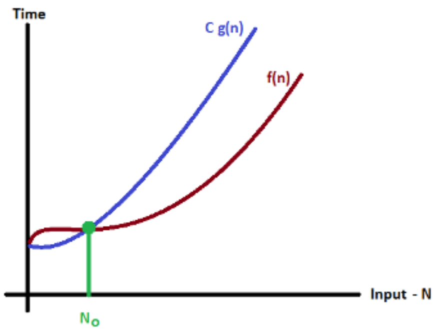
Big - Oh Notation (O)

Big - Oh notation is used to define the **upper bound** of an algorithm in terms of Time Complexity. That means Big - Oh notation always indicates the maximum time required by an algorithm for all input values. That means Big - Oh notation describes the worst case of an algorithm time complexity. Big - Oh Notation can be defined as follows...

Consider function f(n) as time complexity of an algorithm and g(n) is the most significant term. If f(n) <= C g(n) for all n >= n_0, C > 0 and n_0 >= 1. Then we can represent f(n) as O(g(n)).

f(n) = O(g(n))

Consider the following graph drawn for the values of f(n) and C g(n) for input (n) value on X-Axis and time required is on Y-Axis



In above graph after a particular input value n_0, always C g(n) is greater than f(n) which indicates the algorithm's upper bound.

Example

Consider the following f(n) and g(n)...

f(n) = 3n + 2

g(n) = n

If we want to represent **f(n)** as **O(g(n))** then it must satisfy **f(n) <= C g(n)** for all values of **C > 0** and **n_0>= 1**

f(n) <= C g(n)

⇒3n + 2 <= C n

Above condition is always TRUE for all values of **C = 4** and **n >= 2**.

By using Big - Oh notation we can represent the time complexity as follows...

3n + 2 = O(n)

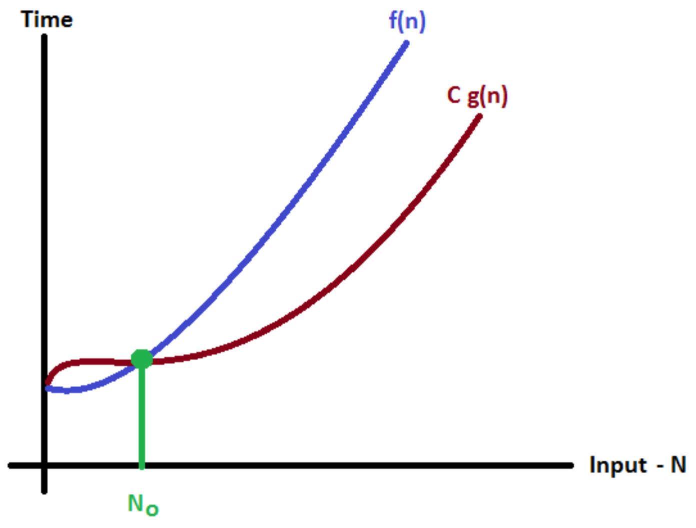
Big - Omege Notation (Ω)

Big - Omega notation is used to define the **lower bound** of an algorithm in terms of Time Complexity. That means Big-Omega notation always indicates the minimum time required by an algorithm for all input values. That means Big-Omega notation describes the best case of an algorithm time complexity. Big - Omega Notation can be defined as follows...

Consider function f(n) as time complexity of an algorithm and g(n) is the most significant term. If $f(n) >= C\ g(n)$ for all $n >= n_0$, $C > 0$ and $n_0 >= 1$. Then we can represent f(n) as Ω(g(n)).

f(n) = Ω(g(n))

Consider the following graph drawn for the values of f(n) and C g(n) for input (n) value on X-Axis and time required is on Y-Axis



In above graph after a particular input value n_0, always C g(n) is less than f(n) which indicates the algorithm's lower bound.

Example

Consider the following f(n) and g(n)...

f(n) = 3n + 2

g(n) = n

If we want to represent **f(n)** as **Ω(g(n))** then it must satisfy **f(n) >= C g(n)** for all values of **C > 0** and **n_0>= 1**

f(n) >= C g(n)

⇒3n + 2 >= C n

Above condition is always TRUE for all values of **C = 1** and **n >= 1**.

By using Big - Omega notation we can represent the time complexity as follows...

3n + 2 = Ω(n)

Big - Theta Notation (Θ)

Big - Theta notation is used to define the **average bound** of an algorithm in terms of Time Complexity.

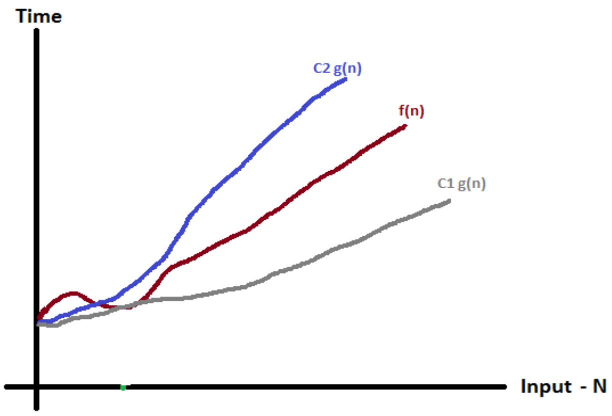
That means Big - Theta notation always indicates the average time required by an algorithm for all input values. That means Big - Theta notation describes the average case of an algorithm time complexity.

Big - Theta Notation can be defined as follows...

Consider function f(n) as time complexity of an algorithm and g(n) is the most significant term. If C_1 g(n) <= f(n) <= C_2 g(n) for all n >= n_0, $C_1 > 0$, $C_2 > 0$ and $n_0 >= 1$. Then we can represent f(n) as Θ(g(n)).

$$f(n) = \Theta(g(n))$$

Consider the following graph drawn for the values of f(n) and C g(n) for input (n) value on X-Axis and time required is on Y-Axis



In above graph after a particular input value n_0, always C_1 g(n) is less than f(n) and C_2 g(n) is greater than f(n) which indicates the algorithm's average bound.

Example

Consider the following f(n) and g(n)...

f(n) = 3n + 2

g(n) = n

If we want to represent **f(n)** as **Θ(g(n))** then it must satisfy **C_1 g(n) <= f(n) <= C_2 g(n)** for all values of **$C_1 > 0$, $C_2 > 0$** and **$n_0 >= 1$**

$C_1\, g(n) <= f(n) <= C_2\, g(n)$

$\Rightarrow C_1\, n <= 3n + 2 <= C_2\, n$

Above condition is always TRUE for all values of **$C_1 = 1$, $C_2 = 4$** and **n >= 2**.

By using Big - Theta notation we can represent the time compexity as follows...

3n + 2 = Θ(n)

AVL Trees

AVL Tree is invented by GM Adelson - Velsky and EM Landis in 1962. The tree is named AVL in honour of its inventors.

AVL Tree can be defined as height balanced binary search tree in which each node is associated with a balance factor which is calculated by subtracting the height of its right sub-tree from that of its left sub-tree.

Tree is said to be balanced if balance factor of each node is in between -1 to 1, otherwise, the tree will be unbalanced and need to be balanced.

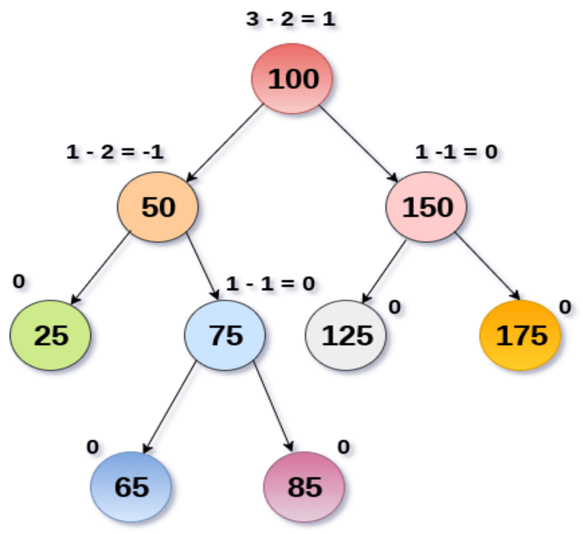
Balance Factor (k) = height (left(k)) - height (right(k))

If balance factor of any node is 1, it means that the left sub-tree is one level higher than the right sub-tree.

If balance factor of any node is 0, it means that the left sub-tree and right sub-tree contain equal height.

If balance factor of any node is -1, it means that the left sub-tree is one level lower than the right sub-tree.

An AVL tree is given in the following figure. We can see that, balance factor associated with each node is in between -1 and +1. therefore, it is an example of AVL tree.



AVL Tree

Complexity

Algorithm	Average case	Worst case
Space	o(n)	o(n)
Search	o(log n)	o(log n)
Insert	o(log n)	o(log n)
Delete	o(log n)	o(log n)

Operations on AVL tree

Due to the fact that, AVL tree is also a binary search tree therefore, all the operations are performed in the same way as they are performed in a binary search tree. Searching and traversing do not lead to the violation in property of AVL tree. However, insertion and deletion are the operations which can violate this property and therefore, they need to be revisited.

SNo	Operation	Description
1	Insertion	Insertion in AVL tree is performed in the same way as it is performed in a binary search tree. However, it may lead to violation in the AVL tree property and therefore the tree may need balancing. The tree can be balanced by applying rotations.
2	Deletion	Deletion can also be performed in the same way as it is performed in a binary search tree. Deletion may also disturb the balance of the tree therefore, various types of rotations are used to rebalance the tree.

→Why AVL Tree?

AVL tree controls the height of the binary search tree by not letting it to be skewed. The time taken for all operations in a binary search tree of height h is **O(h)**. However, it can be extended to **O(n)** if the BST becomes skewed (i.e. worst case). By limiting this height to log n, AVL tree imposes an upper bound on each operation to be **O(log n)** where n is the number of nodes.

→AVL Rotations

We perform rotation in AVL tree only in case if Balance Factor is other than **-1, 0, and 1**. There are basically four types of rotations which are as follows:

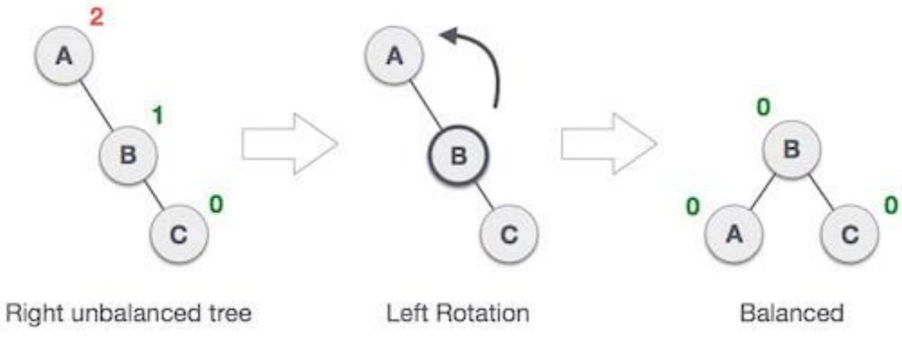
1. L L rotation: Inserted node is in the left subtree of left subtree of A
2. R R rotation : Inserted node is in the right subtree of right subtree of A
3. L R rotation : Inserted node is in the right subtree of left subtree of A
4. R L rotation : Inserted node is in the left subtree of right subtree of A

Where node A is the node whose balance Factor is other than -1, 0, 1.

The first two rotations LL and RR are single rotations and the next two rotations LR and RL are double rotations. For a tree to be unbalanced, minimum height must be at least 2, Let us understand each rotation

1. RR Rotation

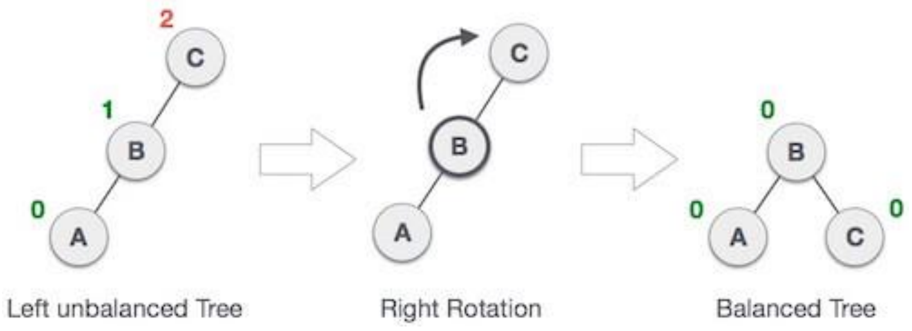
When BST becomes unbalanced, due to a node is inserted into the right subtree of the right subtree of A, then we perform RR rotation, RR rotation is an anticlockwise rotation, which is applied on the edge below a node having balance factor -2



In above example, node A has balance factor -2 because a node C is inserted in the right subtree of A right subtree. We perform the RR rotation on the edge below A.

2. LL Rotation

When BST becomes unbalanced, due to a node is inserted into the left subtree of the left subtree of C, then we perform LL rotation, LL rotation is clockwise rotation, which is applied on the edge below a node having balance factor 2.

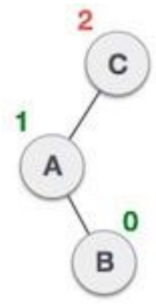
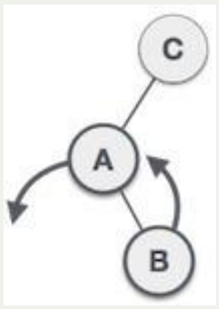
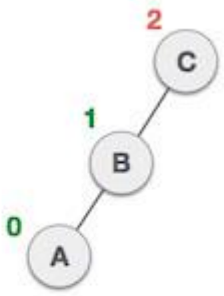
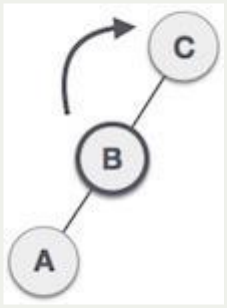
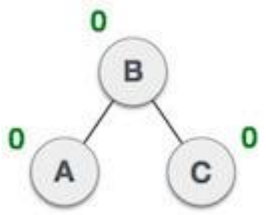


In above example, node C has balance factor 2 because a node A is inserted in the left subtree of C left subtree. We perform the LL rotation on the edge below A.

3. LR Rotation

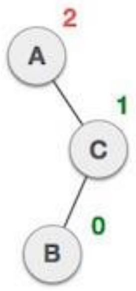
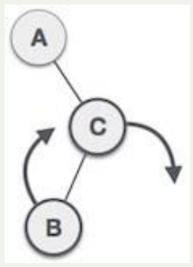
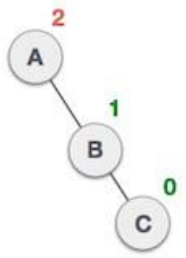
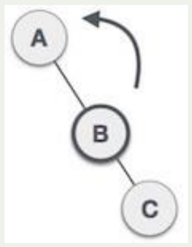
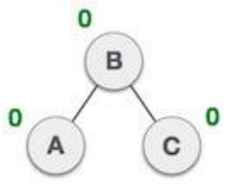
Double rotations are bit tougher than single rotation which has already explained above. LR rotation = RR rotation + LL rotation, i.e., first RR rotation is performed on subtree and then LL rotation is performed on full tree, by full tree we mean the first node from the path of inserted node whose balance factor is other than -1, 0, or 1.

Let us understand each and every step very clearly:

State	Action
	A node B has been inserted into the right subtree of A the left subtree of C, because of which C has become an unbalanced node having balance factor 2. This case is L R rotation where: Inserted node is in the right subtree of left subtree of C
	As LR rotation = RR + LL rotation, hence RR (anticlockwise) on subtree rooted at A is performed first. By doing RR rotation, node **A**, has become the left subtree of **B**.
	After performing RR rotation, node C is still unbalanced, i.e., having balance factor 2, as inserted node A is in the left of left of **C**
	Now we perform LL clockwise rotation on full tree, i.e. on node C. node **C** has now become the right subtree of node B, A is left subtree of B
	Balance factor of each node is now either -1, 0, or 1, i.e. BST is balanced now.

4. RL Rotation

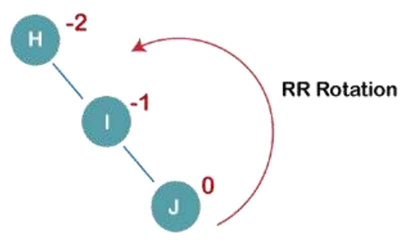
As already discussed, that double rotations are bit tougher than single rotation which has already explained above. R L rotation = LL rotation + RR rotation, i.e., first LL rotation is performed on subtree and then RR rotation is performed on full tree, by full tree we mean the first node from the path of inserted node whose balance factor is other than -1, 0, or 1.

State	**Action**
	A node **B** has been inserted into the left subtree of **C** the right subtree of **A**, because of which A has become an unbalanced node having balance factor - 2. This case is RL rotation where: Inserted node is in the left subtree of right subtree of A
	As RL rotation = LL rotation + RR rotation, hence, LL (clockwise) on subtree rooted at **C** is performed first. By doing RR rotation, node **C** has become the right subtree of **B**.
	After performing LL rotation, node **A** is still unbalanced, i.e. having balance factor -2, which is because of the right-subtree of the right-subtree node A.
	Now we perform RR rotation (anticlockwise rotation) on full tree, i.e. on node A. node **C** has now become the right subtree of node B, and node A has become the left subtree of B.
	Balance factor of each node is now either -1, 0, or 1, i.e., BST is balanced now.

→Example Construct an AVL tree having the following elements

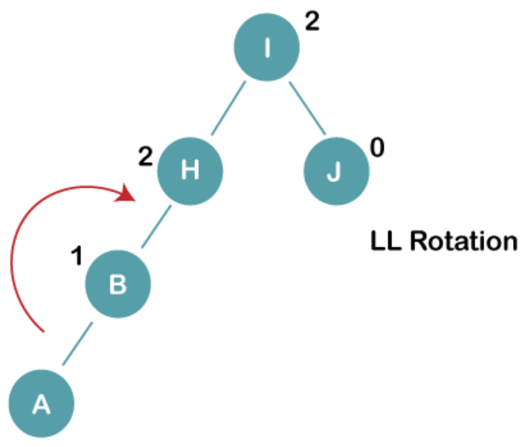
H, I, J, B, A, E, C, F, D, G, K, L

1. Insert H, I, J



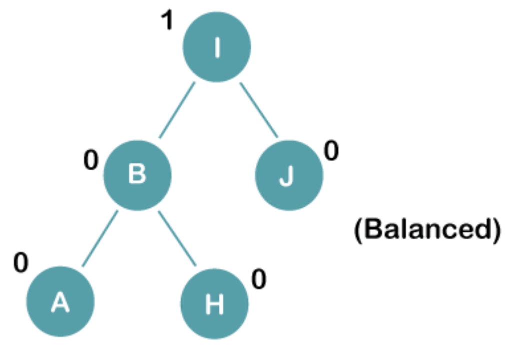
On inserting the above elements, especially in the case of H, the BST becomes unbalanced as the Balance Factor of H is -2. Since the BST is right-skewed, we will perform RR Rotation on node H.

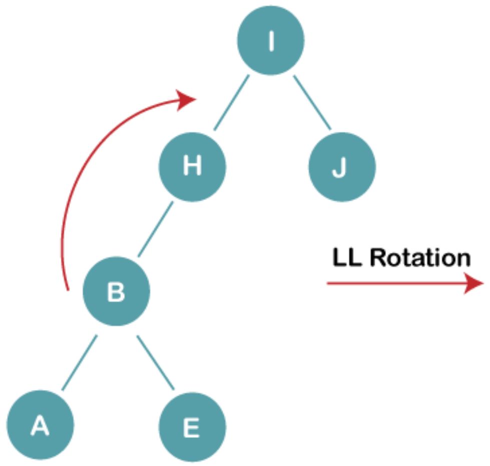
2. Insert B, A



On inserting the above elements, especially in case of A, the BST becomes unbalanced as the Balance Factor of H and I is 2, we consider the first node from the last inserted node i.e. H. Since the BST from H is left-skewed, we will perform LL Rotation on node H.

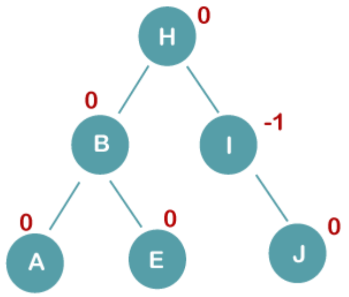
The resultant balance tree is:





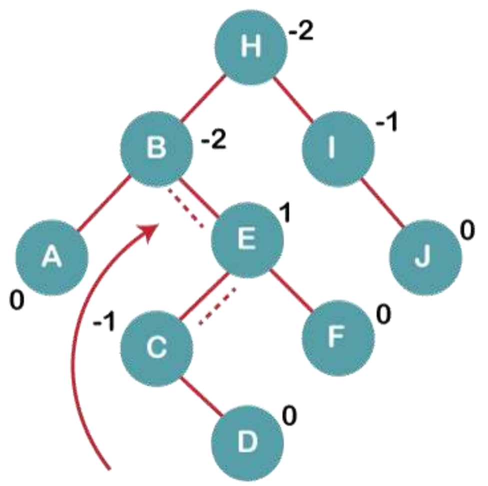
3b) We first perform LL rotation on the node I

The resultant balanced tree after LL rotation is:



(Balanced)

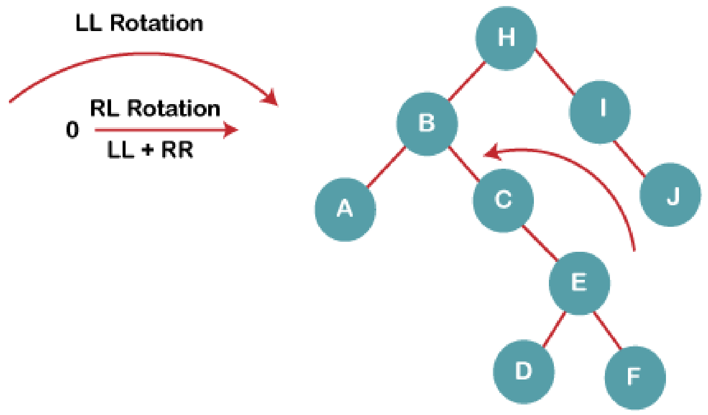
4. Insert C, F, D



On inserting C, F, D, BST becomes unbalanced as the Balance Factor of B and H is -2, since if we travel from D to B we find that it is inserted in the right subtree of left subtree of B, we will perform RL Rotation on node I. RL = LL + RR rotation.

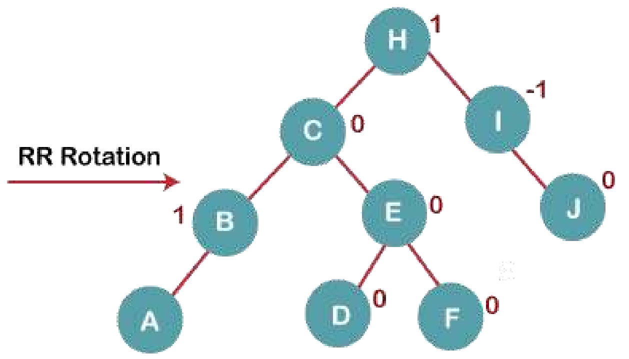
4a) We first perform LL rotation on node E

The resultant tree after LL rotation is:



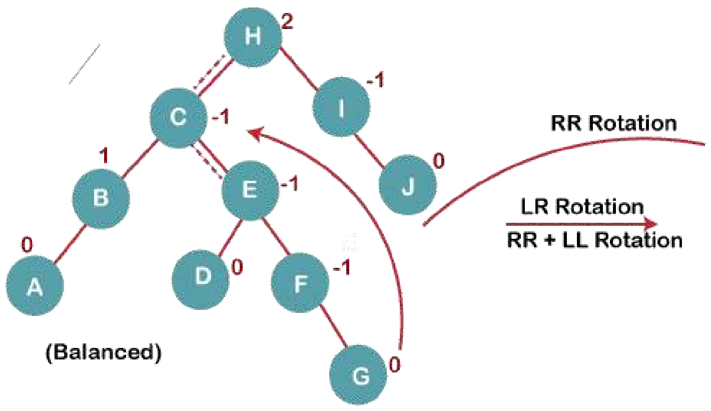
4b) We then perform RR rotation on node B

The resultant balanced tree after RR rotation is:



(Balanced)

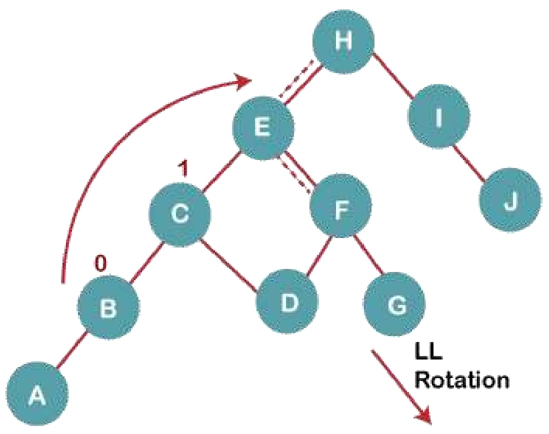
5. Insert G



On inserting G, BST become unbalanced as the Balance Factor of H is 2, since if we travel from G to H, we find that it is inserted in the left subtree of right subtree of H, we will perform LR Rotation on node I. LR = RR + LL rotation.

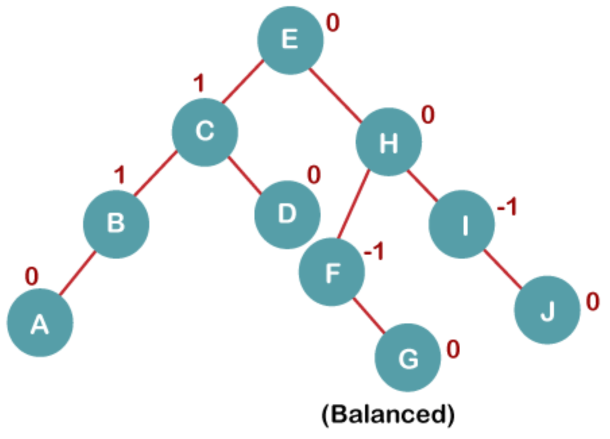
5 a) We first perform RR rotation on node C

The resultant tree after RR rotation is:

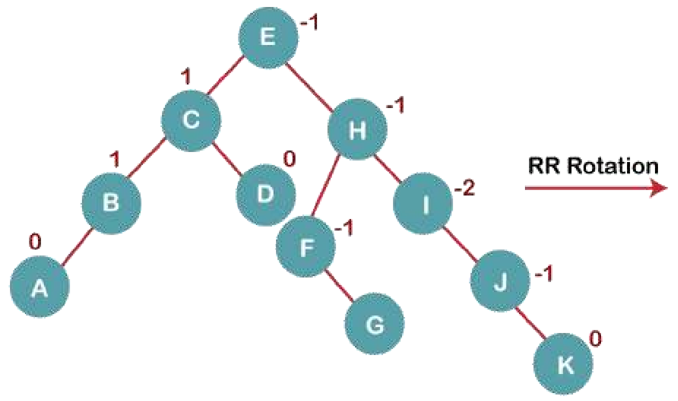


5 b) We then perform LL rotation on node H

The resultant balanced tree after LL rotation is:

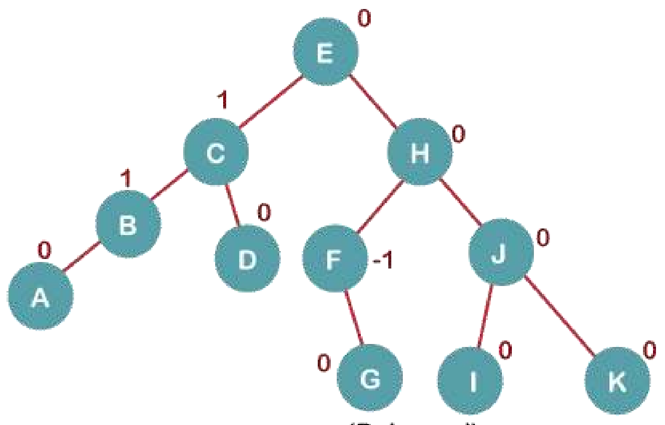


6. Insert K



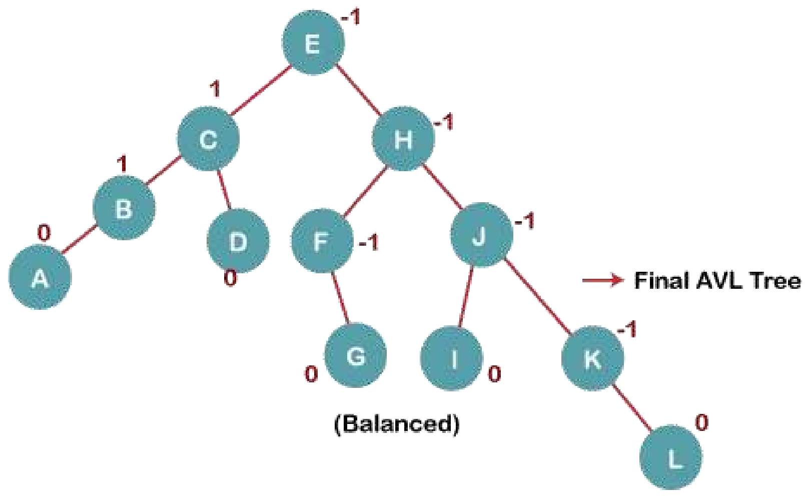
On inserting K, BST becomes unbalanced as the Balance Factor of I is -2. Since the BST is right-skewed from I to K, hence we will perform RR Rotation on the node I.

The resultant balanced tree after RR rotation is:


(Balanced)

7. Insert L

On inserting the L tree is still balanced as the Balance Factor of each node is now either, -1, 0, +1. Hence the tree is a Balanced AVL tree



Applications of AVL Tree:

1. It is used to index huge records in a database and also to efficiently search in that.
2. For all types of in-memory collections, including sets and dictionaries, AVL Trees are used.
3. Database applications, where insertions and deletions are less common but frequent data lookups are necessary
4. Software that needs optimized search.
5. It is applied in corporate areas and storyline games.

B Trees

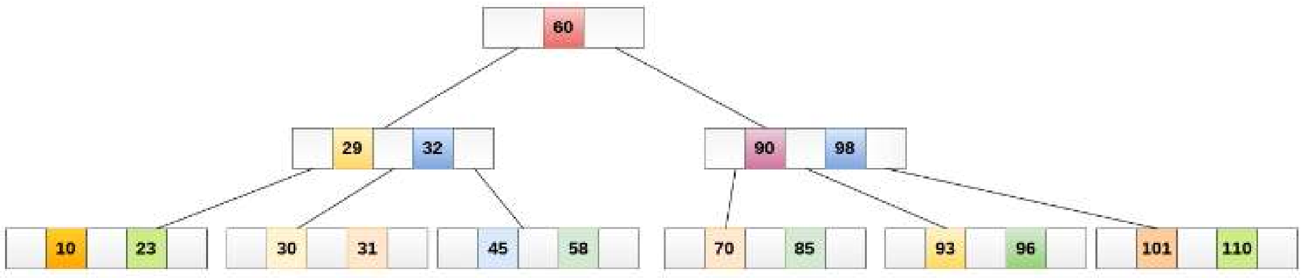
B Tree is a specialized m-way tree that can be widely used for disk access. A B-Tree of order m can have at most m-1 keys and m children. One of the main reason of using B tree is its capability to store large number of keys in a single node and large key values by keeping the height of the tree relatively small.

A B tree of order m contains all the properties of an M way tree. In addition, it contains the following properties.

- Every node in a B-Tree contains at most m children.
- Every node in a B-Tree except the root node and the leaf node contain at least m/2 children.
- The root nodes must have at least 2 nodes.
- All leaf nodes must be at the same level.

It is not necessary that, all the nodes contain the same number of children but, each node must have m/2 number of nodes.

A B tree of order 4 is shown in the following image.



While performing some operations on B Tree, any property of B Tree may violate such as number of minimum children a node can have. To maintain the properties of B Tree, the tree may split or join.

B Tree Operations

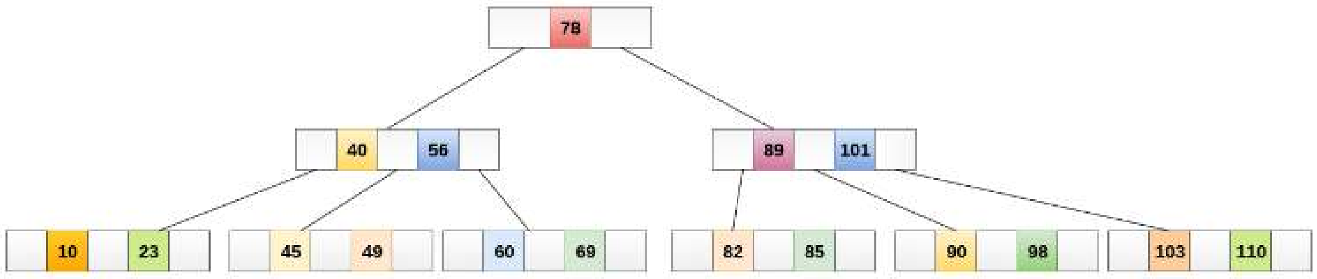
Searching:

Searching in B Trees is similar to that in Binary search tree. For example, if we search for an item 49 in the following B Tree. The process will something like following :

- Compare item 49 with root node 78. since 49 < 78 hence, move to its left sub-tree.

- Since, 40<49<56, traverse right sub-tree of 40.
- 49>45, move to right. Compare 49.
- match found, return.

Searching in a B tree depends upon the height of the tree. The search algorithm takes O(log n) time to search any element in a B tree.



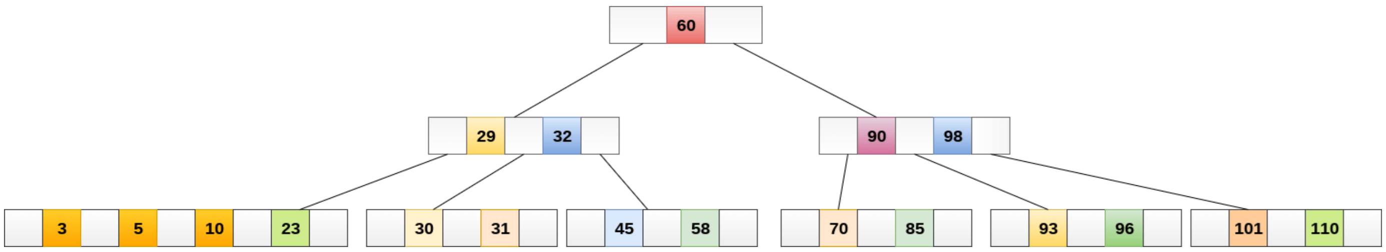
Inserting:

Insertions are done at the leaf node level. The following algorithm needs to be followed in order to insert an item into B Tree.

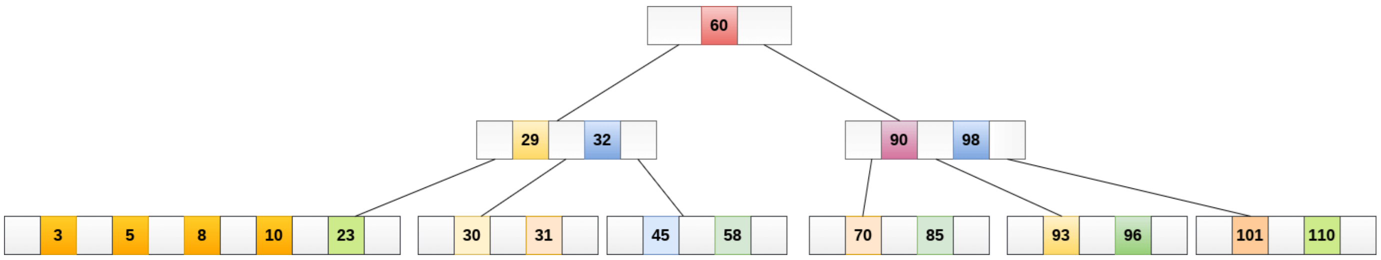
1. Traverse the B Tree in order to find the appropriate leaf node at which the node can be inserted.
2. If the leaf node contain less than m-1 keys then insert the element in the increasing order.
3. Else, if the leaf node contains m-1 keys, then follow the following steps.
   - Insert the new element in the increasing order of elements.
   - Split the node into the two nodes at the median.
   - Push the median element upto its parent node.
   - If the parent node also contain m-1 number of keys, then split it too by following the same steps.

Example:

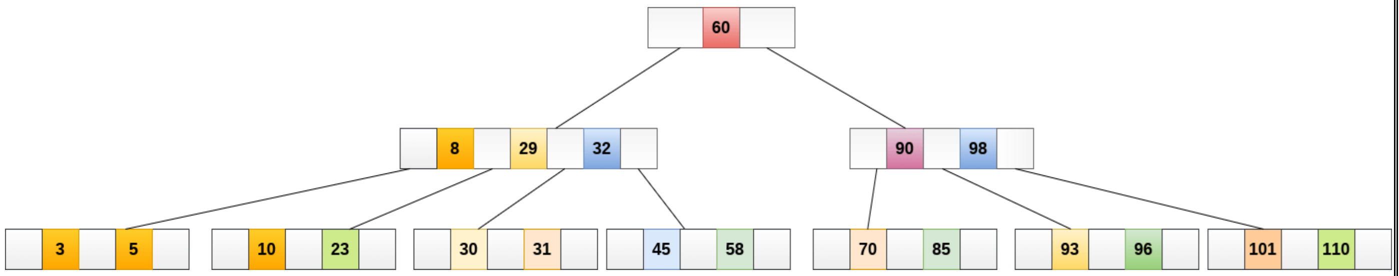
Insert the node 8 into the B Tree of order 5 shown in the following image.



8 will be inserted to the right of 5, therefore insert 8.



The node, now contain 5 keys which is greater than (5 -1 = 4) keys. Therefore split the node from the median i.e. 8 and push it up to its parent node shown as follows.



Deletion:

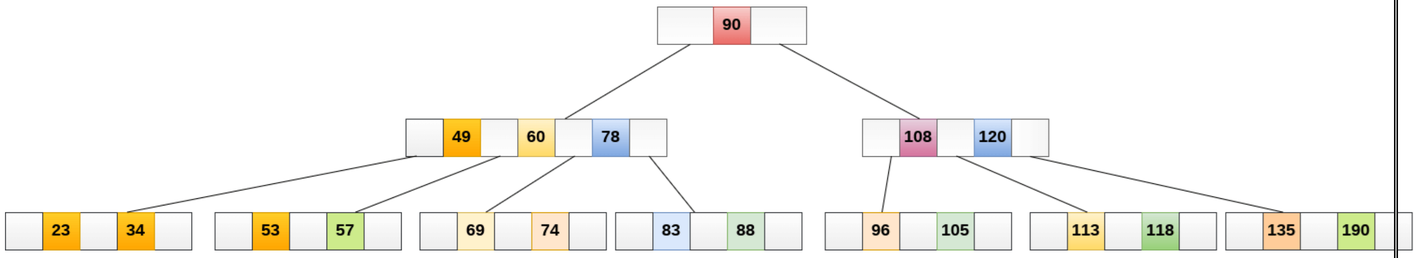
Deletion is also performed at the leaf nodes. The node which is to be deleted can either be a leaf node or an internal node. Following algorithm needs to be followed in order to delete a node from a B tree.

1. Locate the leaf node.
2. If there are more than m/2 keys in the leaf node then delete the desired key from the node.
3. If the leaf node doesn't contain m/2 keys then complete the keys by taking the element from eight or left sibling.
    - If the left sibling contains more than m/2 elements then push its largest element up to its parent and move the intervening element down to the node where the key is deleted.
    - If the right sibling contains more than m/2 elements then push its smallest element up to the parent and move intervening element down to the node where the key is deleted.
4. If neither of the sibling contain more than m/2 elements then create a new leaf node by joining two leaf nodes and the intervening element of the parent node.
5. If parent is left with less than m/2 nodes then, apply the above process on the parent too.

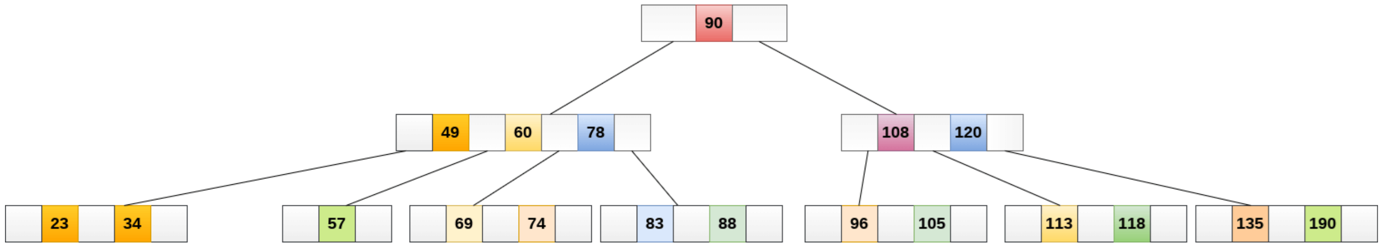
If the the node which is to be deleted is an internal node, then replace the node with its in-order successor or predecessor. Since, successor or predecessor will always be on the leaf node hence, the process will be similar as the node is being deleted from the leaf node.

Example 1

Delete the node 53 from the B Tree of order 5 shown in the following figure.

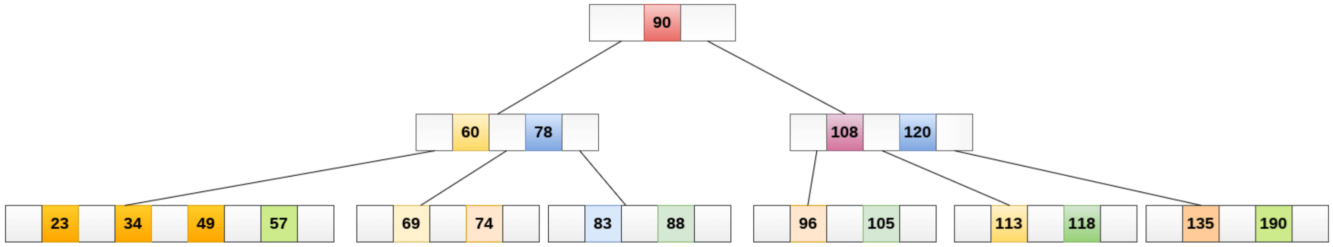


53 is present in the right child of element 49. Delete it.



Now, 57 is the only element which is left in the node, the minimum number of elements that must be present in a B tree of order 5, is 2. it is less than that, the elements in its left and right sub-tree are also not sufficient therefore, merge it with the left sibling and intervening element of parent i.e. 49.

The final B tree is shown as follows.



Applications of B Tree:

- Using the B-Tree, finding data in a data set can be done in a great deal less time.
- Multilevel indexing is possible with the indexing feature.
- Other applications of B-Trees include encryption, computer networks, and natural language processing.
- Since accessing values stored in a large database that is stored on a disc takes a long time, B trees are used to index the data and provide quick access to the actual data stored on the disks.
- In the worst case, it takes O(n) running time to search a database with n key values that is not sorted or indexed. However, if we use B Tree to index this database, it will be searched in O (log n) time in worst case.