Southern and Volga Russian Regional Contest

Saratov State University

18 ноября 2024 г.

J. Waiting for...

Events of two types occur: a bus arrives with b free seats and p people come to the stop. It is required to determine whether there will be free seats on each bus.

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- If p people arrive, we will do $s += p$
- If a bus arrives with b seats, we will compare b and s
- If $b > s$, the answer is `YES`, otherwise `NO`
- Then we will do $s -= \min(b, s)$

N. Fixing the Expression

There is a logical expression of the form “one digit is less/greater/equal to another”, change the minimum number of symbols in it to make it true.

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- You can change the sign between the digits instead of changing the digits

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- You can change the sign between the digits instead of changing the digits
- If the expression is already correct, the sign will change to the same one, meaning there will be no changes

L. Bridge Renovation

Break planks of length 60 into 18, 21, and 25, n planks of each length.

- Greedy ideas

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- Creating pairs $21 + 21 + 18$ is profitable

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- Creating pairs $21 + 21 + 18$ is profitable
- Need $\left\lfloor \frac{n}{2} \right\rfloor$ pairs of $21 + 21 + 18$
- Additionally, need (n mod 2) pairs of $25 + 21$

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- Creating pairs $21 + 21 + 18$ is profitable
- Need $\lfloor \frac{n}{2} \rfloor$ pairs of $21 + 21 + 18$
- Additionally, need $(n \bmod 2)$ pairs of $25 + 21$
- **For boards 25 and 21, exactly n planks of 60 are needed**

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Break planks of length 60 into 18, 21, and 25, n planks of each length.

- There are $\lceil \frac{n}{2} \rceil$ planks of 18 left
- Creating pairs $18 + 18 + 18$ is profitable
- Need $\left\lceil \frac{\lceil \frac{n}{2} \rceil}{3} \right\rceil = \lceil \frac{n}{6} \rceil$ pairs of $18 + 18 + 18$

L. Bridge Renovation

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- There are $\left\lceil \frac{n}{2} \right\rceil$ planks of 18 left
- Creating pairs $18 + 18 + 18$ is profitable
- Need $\left\lceil \frac{\left\lceil \frac{n}{2} \right\rceil}{3} \right\rceil = \left\lceil \frac{n}{6} \right\rceil$ pairs of $18 + 18 + 18$

Answer to the problem: $n + \left\lceil \frac{n}{6} \right\rceil$

C. DIY

There is a list of numbers. You need to select 8 of them and use them as coordinates for the corners of a rectangle with sides parallel to the coordinate axes. The rectangle should have the maximum possible area.

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- If there are no 4 pairs of identical elements, the answer is NO
- Otherwise, let's "compress" the list so that if there were x occurrences in the original, it will become $\lfloor \frac{x}{2} \rfloor$ occurrences in the compressed list

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- Sort the compressed list. Let's call the new list b, and its length m
- On each axis, the smaller the “left” coordinate and the larger the “right”, the larger the size of the rectangle
- You can always take elements b_1, b_2, b_{m-1}, b_m
- It can be proven that it is profitable to take pairs (b_1, b_{m-1}) and (b_2, b_m)

A. Bonus Project

n developers are working on the project. To complete it, they must spend a total of k hours. The developers take turns naming an integer number of hours c_i that each will work on the project. If they complete the project, the i-th will receive $a_i - c_i \cdot b_i$ burles. How many hours will each name if each developer wants to maximize their profit?

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- Let's solve the problem from the last developer to the first
- The last developer knows how many hours are left to work on the project
- If their limit is less, they will not work, and no one will receive profit

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Time complexity: $O(n)$

G. Guess One Character

The jury has a binary string, and you need to guess one character in it. You can ask no more than 3 queries of the form “how many times some string occurs in the jury string as a substring?”

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- Based on this, we can ask the following queries — 1, 11, and 10
- After each 1, except the last, there will be a character, so if the answer to the first query equals the sum of the other two, then the last character is not 1

B. Make It Equal

There is an integer array. Make the array consist of identical non-negative numbers, using the minimum number of operations of the type “decrease an element by 2 and increase the next (cyclically) by 1”.

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- You can use binary search on the final value of the array

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- To check if a value is reachable in binary search, we will cyclically fix (apply operations to make the element no more than k) elements until the sum of the array becomes less than or equal to $k \cdot n$
- With such fixes, we perform operations that definitely need to be done
- Such a series of fixes converges in $O(n + \log A)$, because after the first round of fixes we will have only one element greater than the required value

K. Grid Walk

Find the cheapest path from $(1, 1)$ to (n, n), where $c(i, j) = \gcd(i, a) + \gcd(j, b)$.

- $P(x, y)$ is some path from $(1, 1)$ to (x, y)

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- If $\gcd(x,a) = 1$ or $\gcd(y,b) = 1$, then the lower bound **is achieved**

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- For $n \le 10^6$, $n - x \le 25$ and $n - y \le 25$

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- $dp[26][26]$

Time complexity: $O(n \log n)$

I. Polyathlon

A competition of m sports is taking place, in which n athletes participate. A binary matrix is given — is the i-th participant strong in the j-th sport? Sports $x, x+1, \ldots$ are played **cyclically**, after each sport, weak participants are eliminated if there is at least one strong participant. For each starting x, output the winner.

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- For each cyclic shift, find the maximum

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- We will maintain the entire decreasing order of rows
- Having the order for $x + 1$, we will recalculate for x
- Athletes with 1 in position x move to the front in the same order
- Athletes with 0 in position x remain at the back in the same order

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Time complexity: $O(nm)$

D. Divide OR Conquer

Given an array of n integers, you have to count the number of ways to cut it into segments such that the bitwise OR on the segments forms a non-decreasing sequence.

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- There are $\log_2 10^9$ different ORs among such segments
- Use dynamic programming $dp[i][x]$
- “How many ways to cut a prefix of length i into segments so that the OR of the last segment is exactly x?”

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- For each l, find all pairs (r, x) — the minimum r for which the OR on $[l, r)$ equals x

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- For each l, find all pairs (r, x) — the minimum r for which the OR on $[l, r)$ equals x
- Pairs for l can be obtained from pairs for $l + 1$
- Sort pairs (r, x) in descending order of x

D. Divide OR Conquer

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- We will do delayed addition with sweep line technique
- At position r_j, we will add, at position r_{j+1} we will subtract the same value

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- At each position, we will maintain a list of the form (OR value, sum of dp values)
- We will update the list with events from the sweep line
- With two pointers, we will calculate the values of all dynamics for the position
- Solution complexity: $O(n \log n \log \log n)$

M. Royal Flush

There is a deck of cards from 1 to 4 suits. We take the top 5 cards from it and on each turn can choose which cards to discard and which to keep. After each discard, we take the next cards until we have 5 cards in hand. The task is to obtain a royal flush in the minimum expected number of moves.

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- There are $n + 1$ types of cards: cards of each of the n suits involved in constructing the royal flush, and all others
- Cards of types not required in the royal flush can be discarded immediately, but for other types, it is more complicated
- For each state, we need to optimally choose which cards to discard — let's try to choose it with dynamic programming

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M. Royal Flush

- The state can be described by the following parameters: how many cards are in the deck, how many cards of each type we have in hand
- If we discard a card required for the royal flush, then the royal flush for that suit can no longer be collected; for each suit, we need to keep track of whether it can still be collected
- For example, we can store -1 for the number of cards of a type that can no longer be collected, but for this, we will need to maintain the total number of cards in hand as an additional state

M. Royal Flush

- Transitions: if the number of cards in hand is less than 5, we take a random card (the dp value is equal to the weighted sum of dp values from the states we transition to)

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- If it works too slowly, we can precalculate the answers locally

There are n barrels. We throw clay at the bottom of the barrels to maximize the volume of water in the first barrel.

- Greedy ideas

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- Simplifying the procedure:
  - throw $h[i-1]$ units of clay immediately
  - recalculate the balance
  - seal the barrel with one additional unit of clay

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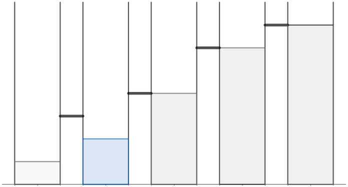
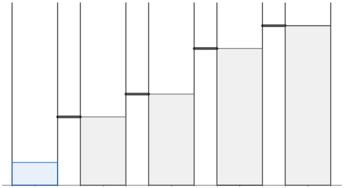
- In the general case, on the suffix, we store a “stack” of filled segments
- Water is collected in the last unfilled segment.
- $2 + 1$ cases:

E. Barrels

First case:

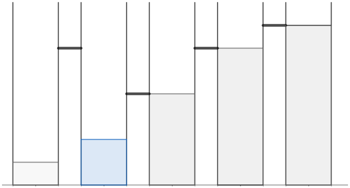


“The stack” expands

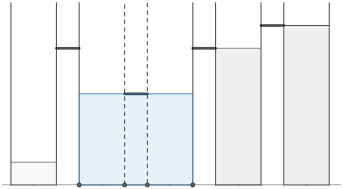


E. Barrels

Second case:



The segments merge



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- Complexity: $O(n)$ in total.

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Time complexity: $O(n \log n)$

H. Galactic Council

The formal statement: there is an array where initially all values are equal to 0. We perform m operations: increase by 1 an element that is not the leftmost maximum. After each operation, there is a requirement for which position the leftmost maximum should be.

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- There are costs for performing operations, the constraints are small — let's try to build a minimum cost flow

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- The flow size in such a network is m, the number of vertices is $O(nm)$, the number of edges is $O(nm)$, so even without potentials it works in $O(n^2 m^3)$