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PREFACE

''ڈوگر پبلشرز'' کا ادارہ کسی تعارف کا محتاج نہیں۔ یہ اشاعتی دنیا کا ایک معتبر نام تصور کیا جاتا ہے۔ تعلیمی میدان میں اس ادارے کی خدمات لازوال ہیں۔ ہمارے ادارے کے تحت شائع ہونے والی ''ڈوگرز یونیک'' کتب کو اساتذہ اور طلباء و طالبات میں جو پذیرائی نصیب ہوئی' وہ بہت کم اداروں کا مقدر ہوتی ہے۔ خداوندِ کریم کے فضل و کرم سے ہماری کتب نہ صرف ملک کے طول و عرض میں پڑھی اور پڑھائی جاتی ہیں بلکہ بیرونِ ملک بھی ان کی طلب میں روز افزوں اضافہ ہو رہا ہے۔ شاید یہی وجہ ہے کہ طلبہ کو جب کبھی کوئی دشواری پیش آتی ہے تو وہ راہنمائی کے لیے ہمارے ادارے کا رخ کرتے ہیں اور ہماری یہ کوشش ہوتی ہے کہ اپنے معزز قارئین کو کبھی مایوس نہ کریں۔ اس مقصد کیلئے ہم نے ملک کے نامور ماہرینِ تعلیم' ماہرینِ تعلیمی نفسیات' سینئر پروفیسرز' اساتذہ کرام' ماہرینِ نصاب اور امتحانات پر مشتمل ایک ایڈیٹوریل بورڈ تشکیل دیا ہے جو کہ طلبہ کی مشکلات' دشواریوں اور ضروریات کو مدنظر رکھتے ہوئے مختلف عنوانات اور مضامین پر کتب مرتب کرتا ہے۔

ہمارے ملک میں جب بیرونی ممالک میں رائج ETS کی طرز پر NTS کا آغاز کیا گیا تو طلبہ کی کثیر تعداد نے راہنمائی کے لیے ہمارے ادارے کا رخ کیا۔ شروع شروع میں NTS کا دائرہ کار محدود تھا لیکن وقت گزرنے کے ساتھ ساتھ اس کی اہمیت اور افادیت کو مدنظر رکھتے ہوئے اس نظام کو وسعت دی گئی' لہٰذا جب ہم نے محسوس کیا کہ طلبہ کو گائیڈ کرنے کے لیے ایک کتاب کی اشد ضرورت ہے تو ہم نے قومی خدمت کو مدنظر رکھتے ہوئے یہ کتاب تحریر کرنے کا ارادہ کیا۔ حد شکر ہے اس ذاتِ باری تعالیٰ کا جس نے مجھے ہمت اور حوصلہ عطا فرمایا کہ میں اس کام کو پایۂ تکمیل تک پہنچا سکا۔ یہ ایک معیاری' منفرد اور جامع کتاب ہے' جس میں NTS کا بطور احسن احاطہ کیا گیا ہے اور کسی بھی پہلو سے صرف نظر نہیں کیا گیا۔ یہ کتاب NTS کے حوالے سے طلبہ کی بہترین معاون ثابت ہوگی۔ اس میں غیر ضروری چیزوں کو شامل نہیں کیا گیا۔ اس گائیڈ کو پڑھنے کے بعد جب طلبہ کمرہ امتحان میں بیٹھیں گے تو انہیں پیپر کے حوالے سے اجنبیت کا احساس نہیں ہوگا۔ گائیڈ میں ماڈل پرچہ جات اسی نقطۂ نظر کو مدنظر رکھ کر شامل کیے گئے ہیں۔

NTS کے امتحانات میں شرکت کرنے والے معزز طلباء و طالبات اس بات کو پیش نظر رکھیں کہ یہ کتاب محض ایک گائیڈ ہے جو کہ آپ کی راہنمائی اور مشکلات کا ازالہ کرنے کے لیے تحریر کی گئی ہے۔ یہ کسی بھی صورت میں امتحانی پرچہ جات کا مکمل احاطہ نہیں کرتی۔ امتحانات کی مکمل تیاری کے لیے آپ کو امتحان کی نوعیت کے اعتبار سے تجویز کردہ کتب کا مطالعہ کرنا ضروری ہے۔ عام طور پر GRE(Local)' GAT(General)' SAT یا GMAT کی کتب تجویز کی جاتی ہیں۔ یہ تمام کتب بین الاقوامی طور پر منعقد ہونے والے مختلف امتحانات کی ضروریات کو مدنظر رکھ کر تحریر کی جاتی ہیں۔ آپ کو ان عنوانات پر بیشمار غیر ملکی کتب مارکیٹ میں دستیاب ہو سکتی ہیں۔ ہمارا ادارہ اللہ تعالیٰ کے فضل و کرم سے GMAT' SAT اور GRE پر پہلی مرتبہ پاکستان کے تعلیمی پس منظر کو مدنظر رکھتے ہوئے یونیک کتب شائع کر رہا ہے جو کہ بہت جلد مارکیٹ میں دستیاب ہوں گی اور ہمیں اُمید کامل ہے کہ یہ کتب ایک سنگِ میل ثابت ہوں گی۔

معزز قارئین! ہم نے آپ کی آسانی کیلئے اس کتاب کو مختلف حصوں میں تقسیم کیا تاکہ آپ کو امتحانات کی تیاری میں کوئی دشواری پیش نہ آئے۔ English اور Arithmetic کے مضامین پر خصوصی توجہ مرکوز کی گئی ہے جو کہ تمام اقسام کے امتحانات میں لازمی طور پر شامل ہوں گے۔ آپ کی سہولت کی خاطر ماڈل پیپرز کا اضافہ بھی کیا گیا ہے۔ یہ کتاب ایک انسانی کاوش ہے جو کہ یقیناً غلطیوں سے مبرا نہیں ہوگی۔ اس سلسلے میں ہمیں آپ کا تعاون درکار ہوگا۔ آپ اگر اس کتاب میں کوئی کمی یا غلطی پائیں تو ہمیں ضرور مطلع فرمائیں۔ اس کے علاوہ اس کی بہتری کے لیے آپ کے پاس کوئی تجاویز ہوں تو وہ بھی ضرور ارسال کریں' ہم انہیں ضرور مدنظر رکھیں گے۔ آخر میں' میں اس کتاب کی تکمیل کے حوالے سے ڈوگر پبلشرز ایڈیٹوریل بورڈ کے اراکین کا تہہ دل سے شکریہ ادا کرتا ہوں جنہوں نے اس سلسلے میں اپنا بھرپور کردار ادا کیا۔

طالب دعا

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چیف ایگزیکٹو/مینجنگ ایڈیٹر

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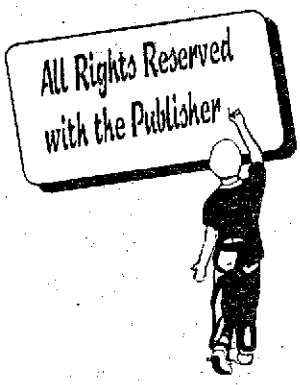
Publisher and Chief Executive:
SHAHID MAHMOOD DOGAR
M.A., DIA, ELT, CIT (B.C.), MBA(s)



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2007-2008

Printed at :
NAQVI PRINTERS
Darbar Market, Lahore.

Type Set by :
SAYYED
Urdu Bazar, Lahore.

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تعارف (Introduction)

نظامِ تعلیم ایک جسم کی مانند ہوتا ہے جو کہ مختلف حصوں پر مشتمل ہوتا ہے اور یہ تمام حصے باہم مربوط ہو کر اپنے اپنے افعال سرانجام دیتے ہیں۔ نصاب، معلم، متعلم، تعلم، غیر نصابی مشاغل اور امتحانات وغیرہ اس کے چند بنیادی حصے ہیں۔ امتحانات، آزمائشیں اور ٹیسٹ وغیرہ نظامِ تعلیم میں ریڑھ کی ہڈی کی حیثیت رکھتے ہیں۔ کوئی بھی نظامِ تعلیم ایک مؤثر امتحانی سسٹم کے بغیر نامکمل ہوتا ہے۔ زمانہ قدیم سے لیکر آج تک ہر دور میں امتحانات کا کوئی نہ کوئی مروجہ طریقہ کار رہا ہے۔ ان کا وجود ہمیشہ رہا اور کسی نہ کسی صورت میں آئندہ آنے والے ادوار میں بھی رہے گا۔ یہ مختلف قابلیتوں اور صلاحیتوں کو ماپنے کا واحد پیمانہ ہے۔ پاکستان بھی اس سے مستثنیٰ نہیں ہے اور ہمارے ماہرینِ تعلیم اس کو بہتر سے بہتر بنانے کے لیے ہمیشہ کوشاں رہتے ہیں۔ حکومتِ پاکستان کی جانب سے بھی یہ اوّلین ترجیحات میں شامل ہے۔ اسی سلسلے میں NTS کو متعارف کروایا گیا۔

بین الاقوامی طور پر NTS کئی چیزوں کے ناموں کا مخفف ہے لیکن یہ ایک نیا امتحانی طریقہ کار (New Testing Scheme) ہے۔ پاکستان میں اسے نیشنل ٹیسٹنگ سروس کے نام سے موسوم کیا گیا ہے اور یہ ٹیسٹ منعقد کروانے کے لیے نجی طور پر ایک ادارے کو ذمہ داری سونپی گئی ہے۔ ابتدا میں یہ ٹیسٹ پی۔ ایچ۔ ڈی اور ایم۔ فل کے انٹرنیشنل لیول پر داخلوں کیلئے لئے جاتے تھے لیکن وقت کے ساتھ ساتھ اس کے دائرہ کار میں توسیع کی جارہی ہے اور ملکی سطح پر مختلف یونیورسٹیوں، کالجوں اور اداروں میں داخلہ ٹیسٹوں کے انعقاد کی ذمہ داری بھی اسی ادارے کو سونپی جارہی ہے۔

ڈوگر پبلشرز نے شعبہ تعلیم کے تمام میدانوں میں خدمت سرانجام دینے کا عزم کر رکھا ہے، اسی لیے NTS کے حوالے سے طلبہ کی مشکلات کو مدنظر رکھتے ہوئے ان کی مناسب رہنمائی کے لیے یہ کتاب تیار کی گئی اور یہ کام جذبۂ خدمت کو مدنظر رکھ کر کیا گیا ہے۔ دنیا کے کچھ ممالک میں NTS کے بجائے ETS رائج ہے۔ ہمارے کچھ ہمسایہ ممالک میں بھی امتحانات کا یہ طریقہ کار رائج ہے۔ پاکستان میں اس ادارے کا قیام جولائی 2002ء میں عمل میں لایا گیا جس کا مقصد طلبہ کے علم، ذہانت اور معیارِ تعلیم کی آزمائش اور پیمائش کرنا تھا۔ ہمارے ملک کے تعلیمی اداروں میں پڑھائے جانے والے نصاب میں بڑا تضاد پایا جاتا ہے اور مختلف ادارے مختلف سطحوں پر علیحدہ علیحدہ نصاب پڑھاتے ہیں، اس لیے طلبہ کا علیحدہ علیحدہ معیار متعین کرنا نہایت ہی مشکل تھا، بالکل اسی طرح معیارِ تعلیم میں بھی بڑا فرق ہے۔ اسی وجہ سے بڑے اداروں کے طلبہ مستفید ہو جاتے تھے اور چھوٹے اداروں کے طلبہ احساسِ محرومی کا شکار رہتے تھے لیکن NTS کے قیام سے اس چیز پر کافی حد تک قابو پانے کی کوشش کی گئی ہے۔

مقاصد (Objectives): جیسا کہ اوپر بیان کیا گیا ہے، NTS کا بنیادی مقصد طلبہ کی صلاحیتوں، قابلیتوں اور میلانات کی پیمائش کرنا ہے اور پاکستان میں معیارِ تعلیم میں بہتری لانا ہے۔ اس کا مقصد بین الاقوامی اور ملکی سطح پر یونیورسٹیوں، کالجوں اور اداروں میں داخلہ لینے کے خواہش مند امیدواروں کے داخلہ امتحانات منعقد کروانا ہے۔ اس کے چند چیدہ چیدہ مقاصد درج ذیل ہیں:

1- ہمارے موجودہ طریقۂ امتحانات میں پائی جانے والی خامیوں کا ازالہ کرنا اور طلبہ کی صلاحیتوں کا بطور احسن جائزہ لینا اور پیمائش کرنا۔
2- ایک مناسب، درست اور سائنٹیفک طریقۂ کار متعارف کروانا۔ اس مؤثر طریقۂ کار کے ذریعہ مختلف اقسام کے داخلہ ٹیسٹ منعقد کروانا۔
3- بین الاقوامی اور ملکی سطح پر یونیورسٹیوں، کالجز اور اداروں میں داخلے کے متمنی طلباء و طالبات کے لیے ایک معیاری اور یکساں امتحانی طریقۂ کار روشناس کروانا تاکہ کسی کی حق تلفی نہ ہو۔
4- بین الاقوامی سطح پر جو ایجنسیاں ٹیسٹ لیتی ہیں، ان کے لیے داخلہ ٹیسٹوں کی تیاری کروانا تاکہ جب طلبہ امتحانات میں بیٹھیں تو انہیں اجنبیت کا احساس نہ ہو۔
5- کسی ایک مخصوص مضمون میں طلبہ کو نہ پرکھنا بلکہ ان کی کماحقہ اور بنیادی صلاحیتوں کو پرکھنا۔ طلبہ میں پائی جانے والی خامیوں کا ازالہ کرنا۔
6- بشمول ملازمت پیشہ اور بیروزگار امیدواروں کے علم اور مہارتوں کی بنیاد پر پیمائش کرنے اور صلاحیتوں کے جائزہ لینے کے لیے مؤثر اور قابل بھروسہ نظام مہیا کرنا۔
7- تعلیمی تیاری اور معاشرے میں عملی طور پر ضرورت کے اعتبار کے درمیان موجود خلا کو پُر کرنا، یعنی اتنے ہی پیشہ ور افراد تیار کیے جائیں جن کی مارکیٹ میں کھپت ہو سکے۔

NTS ایک غیر منافع بخش خود مختار ادارہ ہے جو کہ پوری محنت، لگن اور جانفشانی سے اپنے مقاصد کو بروئے کار لانے میں مصروفِ عمل ہے۔ طلبہ کے حوالے سے درج ذیل چند بنیادی امور پر اس کی خصوصی توجہ ہے:

☆ وسیع پیمانے پر اعلیٰ معیارِ تعلیم کو فروغ دینا۔
☆ آزمائشوں، امتحانات اور پیمائش کو درست، مناسب اور مؤثر بنانا۔
☆ علم، مہارتوں، قابلیتوں، صلاحیتوں اور پیشہ وارانہ قابلیتوں کا بہتر طور پر تعین کرنا۔
☆ مختلف اقسام کے ٹیسٹوں اور داخلے کے امتحانات کا انعقاد۔
☆ بین الاقوامی اور ملکی سطح پر تعلیم اور پیشہ وارانہ شعبوں کے مابین ایک رابطے کا کام کرنا۔

اس وقت پی ایچ ڈی اور ایم فل کے داخلوں کے علاوہ ملک کی چند نمایاں یونیورسٹیوں، کالجز اور اداروں نے NTS کو داخلوں کے امتحانات منعقد کروانے کی ذمہ داری سونپی ہے اور وقت گزرنے کے ساتھ ساتھ ان کی تعداد میں مزید اضافہ ہونے کا امکان یقینی ہے۔

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پاکستان کے تعلیمی پس منظر میں تیار کی گئی ڈوگر پبلشرز کی چند شاہکار کتابیں

درج ذیل کتب کی ضرورت عرصہ دراز سے محسوس کی جارہی تھی کیونکہ یہ غیر ملکی کتب ہمارے لوکل ٹیسٹ سسٹم سے مطابقت نہیں رکھتی تھیں اور ہمارے ملک میں تیار کیے جانے والے ٹیسٹوں میں ان میں سے 30% سے زیادہ سوالات نہیں آتے تھے۔اب ہم نے ان کتب کو ایسے مرتب کیا ہے کہ انشاءاللہ 100% سوالات ہماری کتب میں سے پوچھے جائیں گے۔

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☆ An Extract of Variety of Books — ☆ 55 سے زائد غیر ملکی کتابوں کا نچوڑ

☆ Questions Collected from Previous Papers — ☆ سابقہ پرچہ جات کے سوالات کا مجموعہ

☆ Topic by Topic Explanation and Introduction — ☆ ہر مضمون اور عنوان کی علیحدہ وضاحت اور تعارف

☆ Special Stress Upon Mathematics — ☆ ریاضی پر خصوصی توجہ

☆ Guidance from Paper-Setting Authorities

مختلف اقسام کے پرچہ جات بنانے والے ماہرین کی خدمات حاصل کر کے ایسے سوالات کتابوں میں شامل کیے گئے ہیں جو کہ سو فیصد متوقع ہوں۔اپنی نوعیت' افادیت اور مواد کے اعتبار سے آپ ان کتب کو غیر ملکی کتب سے کہیں زیادہ منفرد' جامع اور آسان پائیں گے۔

(Be Pakistani and Buy Pakistani)

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Introduction

"NTS" is a new testing system introduced by the government of Pakistan for the search of national talent. For this purpose, NTS was established in July 2002. An experience of decades brought it to the forefront that a large number of scholarships were restricted to certain pockets of the country and many areas remained unrepresentative. In the light of this, the new scheme of testing system was launched. The fundamental aim of NTS is to evaluate the knowledge level of the students. It also aims at to provide a single standard for quality evaluation of the educated human resource. There is no single syllabus in our educational institutions right from basic to the advanced level. There is also a big variation in syllabi taught in various schools, colleges and universities. A huge gap is also found in the standard of education and institutions. It is a vital issue to evaluate the knowledge of students on the prevailing system of examinations. NTS was established to solve this issue by evaluating the skills and knowledge of the individuals on a standardized measure.

NTS is providing training and testing programs for individual students. Efficiency, transparency, reliability and consistency are few prominent features of the NTS which are the main causes of its popularity. Most of the educational institutions and organizations are attracting towards it. In the beginning, NTS was conducting test only for Ph.D and M.Phil scholarships, but not it is also conducting Entry Test for admission to different educational institutions. With the passage of time, it is gaining a prestigious position in the testing system. It has become a reliable name in the field of testing. It has gained this position in a very short span of time.

Objectives of NTS

NTS was established for the achievement of certain objectives and goals, so we can call it a national talent search scheme. It is a novel testing system introduced for the first time in Pakistan. Within the overall premise of contributing to the improvement of the quality of education in Pakistan, the important objectives of NTS are as under :

1. The main objective of NTS is to improve upon the shortcomings in our prevailing performance evaluation system of candidates through establishing a credible, technically appropriate and accurate system of performance evaluation for students of different educational institutions at various levels in the country.
2. Another important objective is to provide a standardized and uniform criterion for selecting candidates seeking admission to various universities in the country and abroad and also to provide a basis on which students, individuals and institutions can assess performance for preparation of various types of tests conducted by different international testing agencies.
3. To enable assessment and ranking not only in a particular subject on a general competency level but also help in identification of areas of strengths and weaknesses in its sub-contents on a disaggregated basis.
4. To provide a reliable and efficient system for evaluating the knowledge base and skills of candidates including professionals - employed and unemployed.
5. To establish a credible and reliable measure of assessing the students' knowledge in commonly taught programs for relative ranking.
6. To bridge up the gap between academic preparation and the practical market needs in a highly dynamic and competitive environment.

NTS Commitment

Promoting Overall Quality of Education

Based on a vision aimed at providing quality services in educational testing and assessment mechanisms for various stakeholders, NTS is committed to promoting the overall quality of education as a value-added adjunct.

Performance Evaluation

NTS undertakes programs that are aimed at establishing a credible, technically appropriate and accurate system of performance evaluation of students/candidates of various educational institutions at different levels.

Assessing Knowledge, Skills & Professional Abilities

It also extends services for the employed and unemployed professionals to get satisfactory jobs in their relevant area through an assessment of their knowledge base, skills, and professional abilities.

A Rapidly Growing & Credible Entity

NTS is a rapidly growing and credible entity of national repute. Its context and potential coincides with the expressed need of the Government for the establishment of a National Accreditation Council and Testing Service.

Conducting Various Types of Tests

While the beginnings were made by conducting General Tests through an IT-focused thrust, NTS has expanded its services to other disciplines of education testing. It now provides services for conducting Subject Tests by providing facilities both for individual candidates and institutions. These tests are prepared and assessed by experts and professionals from relevant fields.

Linkages & Networks

NTS now has growing linkages and associations with educational and professional organizations & institutions, including those engaged in talent search and career & educational promotion activities, at the national and international levels. (Courtesy to NTS)

****

NTS – SERVICES

NTS provides its services in vast areas:

- **National Aptitude/General Testing:**

NTS conducts General Tests so as to evaluate the critical thinking skills of analytical reasoning and also performance assessment of candidates with regard to quantitative & verbal ability. General Tests also called National Aptitude Tests, help establishing a reliable and credible measure of judging the knowledge base of students in commonly taught program.

- **Services – Measuring Knowledge Base and Skill Levels:**

The NTS Subject Tests are prepared and designed with a view to assess the qualifications, knowledge base, competency and skill level of the candidates in a specific area of study.

- **Services for Test Preparation:**

NTS provides complete guidelines with regard to Test Preparation. A guide for users is available on our website as well as in the market in printed form. Model Questions and Sample Tests are also available on the website for helping the candidates to practice and be well prepared for the test. Furthermore, Reference Books are also recommended by NTS to be an additional source of help for the test takers for Computer Science Subject Tests.

- **Administrative Services:**

NTS provides an administrative service for not only the candidates but also the institutions. All administrative arrangements regarding registration, test center selection, test execution, result preparation and then its reporting etc. are taken as a responsibility by NTS.

- **Admission Test Services for Institutions:**

NTS conducts Admission Tests for various institutes and organizations, all services like the arrangements of test registration, paper generation, test execution, paper marking, result declaration etc. are provided by NTS for these entities.

- **Services for Online Testing:**

NTS also conducts Online Tests for the candidates' efficient and credible evaluation of their knowledge and skills. Arrangements like the availability of labs, content development for the assessment, online paper generation, test execution and online result formation immediately after the test, are made by NTS.

- **Establishing Performance Ranking:**

NTS also assists in carrying out an estimated ranking of students as well as professionals. This ranking is not only in a particular subject on a general competency level but also helps in the identification of areas of strengths and weaknesses in its sub-contents on a disaggregated basis.

- **Services for Score Reporting:**

For General Tests, NTS declares its results electronically within seven days of the test being conducted, the results of each candidate are sent to their homes through a fast and reliable courier service. The result of Online Subject Tests is displayed online immediately after the candidate completes his/her test. The score of candidates is also reported in the form of a certificate at their respective addresses.

- **Services to Facilitate Employment:**

NTS extends its services for employed and unemployed candidates to get satisfactory jobs in their relevant areas through a very estimated and careful judgement of their knowledge base and skills. It facilitates access to job opportunities for various professionals and candidates both within the country and abroad on the basis of their scores in NTS Subject Tests.

- **Statistical Analysis and Reports:**

NTS by virtue of its flexible and user friendly solution can better help the institutions and organizations to have statistical analysis and report for administrative tasks and it will ultimately lead to true decision making and managerial tasks.

- **Survey:**

NTS has very robust and accurate system for processing forms and surveys. In addition to the educational and professional testing services, NTS has the survey facility for different organizations, government data collection and other methods to get feedback from people. (Courtesy to NTS)

Test Format

General Tests

Type & Design of Questions:

The General Test is generally divided into three sections, i.e., The Verbal Section, Quantitative Section and the section of Analytical Reasoning. All the questions are based on Multiple-Choice Question format.

The test of Verbal Section is based on completing sentences with the appropriate words, pointing out the errors in the sentences, comprehension exercises to assess the basic knowledge about the vocabulary and grammar of the language. Questions are basically about sentence completion, analogy and critical reading.

In the Quantitative Section, the NTS tries to measure your basic mathematical skills and check your understanding of the elementary level mathematics. The MCQs in this section are based on three areas, i.e., Arithmetic, Algebra and Geometry.

In the Analytical Section, various objective type questions are asked to analyze the capability of every student with regard to their problem solving aptitude in the daily life activities and also the academic interests. Questions asked are basically on the analytical reasoning and logical reasoning.

A few Model Questions in the website can help you in being familiarized with the nature of questions.

Duration of the Test:

The duration of NAT(S) is 120 minutes. The Verbal Section is given 20 minutes in which the candidates are asked to solve 20 questions. The Quantitative Section has been given 55 minutes for the solution of 35 questions. In the last section, i.e., the Analytical Section, 45 minutes are provided to the candidates to answer 35 questions.

The duration of NAT(G) is also 120 minutes. The time suggested for Verbal Section is 35 minutes. 28 and 57 minutes time is recommended for the Quantitative and Analytical Reasoning Section respectively.

The following time table is prepared which describes the number of questions, its time and marks distribution more clearly:

Science (NAT-S)

Test Sections	No. of Questions	Suggested Time	Marks
Verbal	20	20	20
Quantitative	35	55	35
Analytical	35	45	35

Science (NAT-S)

Test Sections	No. of Questions	Suggested Time	Marks
Verbal	35	35	35
Quantitative	15	28	15
Analytical	40	57	40

Subject Tests

Type & Design of Questions:

The test consists of 120 Multiple Choice Questions (MCQs) that range in difficulty level (easy, moderate and hard). The Questions have been selected from a standardized curriculum that has been designed by experts and professionals and is updated from time to time.

General Test (NAT)

The National Aptitude Test program (NAT) aims to assess the verbal, quantitative and analytical abilities that have been attained over a period of time and that are not necessarily related to any specific field of study.

The Verbal Section measures the ability to analyze and evaluate written material and synthesize information obtained from it; to analyze relationships among component parts of sentences and recognize relationships between words and concepts. There is a balance of different subject matters, i.e., humanities, social sciences, and natural sciences.

The Quantitative Section measures your basic mathematical skills, understanding of elementary mathematical concepts, and the ability to reason quantitatively and solve problems in a quantitative setting. There is a balance of questions requiring basic knowledge of arithmetic, algebra, geometry, and data analysis. These are essential content areas usually studied at the high school level.

The Analytical Reasoning tests your critical thinking and analytical writing skills. It assesses your ability to articulate and support complex ideas, analyze an argument, and sustain a focused and coherent discussion. It does not assess any specific content knowledge.

Present NAT programs have an IT & CS-focused thrust and have different levels that includes;

NAT-IS NAT-IIS NAT-IG NAT-IIG

NAT-IS & NAT-IIS programs are meant for candidates who hold a degree in Science discipline and wish to pursue further education in graduate or post graduate courses, respectively.

NAT-IG and NAT-IIG programs are designed for candidates who hold a degree in Humanities discipline and wish to pursue further education in graduate or post graduate courses, respectively.

Note: It may be noted that in case of the 2nd Masters in Development Studies Program (MDS-2) being offered in Fall-2005, candidates may indicate their preference to take the test in NAT-IIG or NAT-IIS, irrespective of their previous academic background being in Science or Humanities. The MDS Program is presently being offered by CIIT Abbottabad only by the Department of Development Studies. A specific Test Format, Test Scores and Test Fee are designed for the General Tests.

Subject Tests

NTS Subject Tests have their own special attributes:

- These tests help in comprehensive evaluation of a candidate emphasizing any one required area of study or specialization.
- The scores of NTS Subject Tests interpret the acquired knowledge base of candidates and predict their abilities regarding their specific knowledge, skills and abilities in a number of different ways. These also help in the overall ranking of students.

The NTS Subject Test focuses on evaluating the knowledge base and skills of the candidates in a specific area of study. These Subject Tests are extremely helpful in measuring the potentials, capabilities and knowledge of candidates in specific fields.

The NTS Subject Tests enable assessment and ranking not only in the particular subject on a general competency level but also help in identification of areas of strengths and weaknesses in sub-contents on a disaggregated basis in that particular subject.

NTS offers Subject Tests in various disciplines:

- Computer Science (CS)
- Physics
- Chemistry
- English
- Biology
- Mathematics
- Psychology

Graduate Assessment Test (GAT) - Subject

These tests are designed for those who have extensive background in that specific subject/discipline.

These tests help evaluate students' ability to analyze and solve problems, understand relationships, and interpret material in their particular field. These are prepared on national level, i.e., to assess students' capability and existing knowledge in their specific/major area of study.

Presently, these tests are being offered in **Computer Sciences** and **Management Sciences,** only. The tests will be held **biannually**. The scores of this test remain valid for **ONE WHOLE YEAR** from the test date.

A few of the other study areas are under process and are being considered for Subject Tests.

Test Format of GAT Subject

Total Questions (MCQs) = 100

Total Test Time = 120 Minutes (2 Hours)

Test Type: Paper Based

Test Contents

General Section = 30% of the Total Test

Subject Section = 70% of the Total Test

NTS New Testing System Guide

Areas of General Section 30%

English (Verbal) = 15%

Analytical Reasoning = 15%

Core Areas of Computer Sciences 70%

Sr. No.	*Core Areas of Computer Sciences*	*Area Percentage (100% of the 70%)*
1.	Discrete Structures	10%
2.	Algorithm & Complexity	12%
3.	Architecture & Organization	12%
4.	Operating System	12%
5.	Programming Fundamentals	09%
6.	Programming Languages	10%
7.	Information Management	12%
8.	Net-centric Computing	09%
9.	Software Engineering	10%
10.	Computational Science	04%

Core Areas of Management Sciences 70%

Sr. No.	Core Areas of Management Sciences	Area Percentage (100% of the 70%)
1.	Finance	33%
2.	Marketing	33%
3.	Management	20%
4.	Economic Analysis	14%

Test Schedule for GAT Subject 2007

The Subject Test dates for **Computer Sciences** and **Managements Sciences** in the year 2007 is as follows

1. First test **April, 2007**
2. Second test **October, 2007**

The schedule of these tests in detail is as under:

Sr #	Advertisement Date	Last Date for Application Submission	Test Date	Result Date	Result Card Issue Date
First Test	February, 2007 & will be repeated on February, 2007	March, 2007	April, 2007	April, 2007	Before April, 2007
Second Test	August, 2007 & will be repeated on September, 2007	September, 2007	October, 2007	October, 2007	October, 2007

Note: Lists of eligible candidates, Test venue and Candidate instructions will be displayed on NTS website at least 7 days prior to test date.
Any queries will be received through e-mail latest by 4 days prior to the test date.

Registration & Test Fee

Applicants will be charged Rs. 600/- as test fee, in shape of Bank Draft in favor of NTS Abbottabad (Bank Draft will be received along with Application Forms) as a matter of processing for registration.

Note: *Candidates with more than 40 years of age are not eligible to apply.*

Result Announcement and validity

1. Results will be declared within seven days from the test date.
2. The candidates will be issued the result card through authorized courier service/ Pakistan postal service (registered mail) within 14 days from test date.
3. The results of GAT Subject remains valid for One Year (1 Year) from the test date.
4. The candidates intending to improve there NTS test score will be eligible to reappear in the next test by applying afresh.
5. NTS will verify the test score of the candidate on written demand of the HEC within 48 hours.
6. NTS will conduct these tests **biannually** and will issue NTS GAT - Subject Score Card to the individual candidate.
7. Any candidate intending to improve his NTS GAT Subject Score can reappear in the next test.

NTS – TEST CENTERS

Test will be conducted in maximum 20 cities of Pakistan depending on the number of candidates applied for test. (Sr. No. 1-5 cities are compulsory test centers and Sr. No. 6-20 are optional test centers depending on number of candidates applied for the said center a Minimum of 100 candidates per center)

1. Islamabad	2. Lahore	3. Karachi	4. Peshawar
5. Quetta	6. Multan	7. Faisalabad	8. Gujrat
9. Sargodha	10. Bahawalpur	11. D.I. Khan	12. D.G. Khan
13. Sukkur	14. Larkana	15. Nawabshah	16. Hyderabad
17. Bannu	18. Abbottabad	19. Muzaffarabad	20. Gilgit

NTS Head Office is in Islamabad, NTS has also established its offices in big cities of Pakistan like Karachi, Lahore, Quetta, Peshawar etc. and NTS is in a position to conduct any kind of test that is paper based or online at a very short notice while providing all the logistics and support required for the conduction of exam, preparation of result and communicating the result to the candidates.

NTS – Partners

Recently, following universities, colleges and institutions are the partners of NTS, that are increasing with the passage of time:

1. Federal Urdu University of Arts, Science and Technology, Islamabad
2. Board of Intermediate and Secondary Education, Abbottabad
3. Liaquat University of Medical & Health Sciences, Jamshoro
4. Mehran University of Engineering & Technology, Jamshoro
5. Pakistan Telecommunication Corporation Limited (PTCL)
6. Dow University of Health Sciences
7. University of Engineering & Technology Peshawar
8. COMSATS Institute of Information Technology
9. Higher Education Commission (HEC)
10. Engineering University Khuzdar
11. Textile University Faisalabad
12. Ministry of IT & Education
13. Pakistan Computer Bureau
14. Bolan Medical College
15. Virtual University
16. And many more.

SCHEDULE AND INFORMATION

Information and schedules for the NTS Tests can be obtained from the following sources:

i. Advertisements through Printed Media (Newspapers, Magazines, etc.)
ii. Advertisements through Electronic Media (Radio, TV, Internet, etc.)
iii. NTS Allied Institutions
iv. NTS and its Allied Institutions' Website
v. NTS Website www.nts.org.pk

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New Testing System Guide

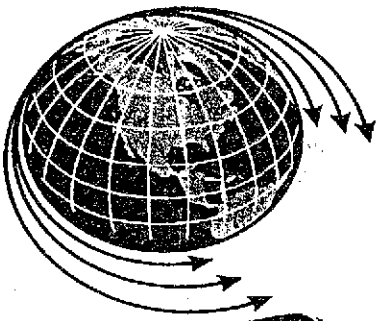
NTS کے حوالے سے اخبارات میں شائع ہونے والے ایک اشتہار کا عکس،
اس کے ذریعے آپ درخواست دینے کے طریقہ کار اور شرائط وغیرہ سے متعلق معلومات حاصل کر سکتے ہیں۔

National Testing Service (NTS)

Building Standards in Educational and Professional Testing

GRADUATE ASSESSMENT TESTS

GAT-SUBJECT FOR ADMISSIONS IN Ph.D



The Higher Education Commission (HEC) has assigned National Testing Service (NTS) the responsibility to conduct Graduate Assessment Test, GAT-SUBJECT for admissions in PhD in all Universities of Pakistan.

GAT-SUBJECT test will be held in the following subjects:

1. Agriculture Sciences
2. Economics
3. Electrical Engineering
4. Geography
5. Management Sciences
6. Computer Engineering
7. Electronics
8. Education
9. Islamic Studies
10. Veterinary/ Animal Sciences

- The test result will remain valid for **ONE YEAR** for Admissions & HEC Scholarship schemes.
- Candidates who want to improve their previous GAT Score can also apply.
- The candidates will have to qualify other specified criteria of the Universities/HEC schemes.

Candidate with a minimum 18 years degree is eligible to appear in the test

Candidates are advised to visit the website regularly for further details. The list of eligible candidates, test venue, test date and other information will be displayed on website:

Last date for Applications 25th September 2006

SCHEDULED NTS TESTS

GAT-GENERAL	GAT-SUBJECT
• M.Phil Admissions in all Universities of Pakistan • All HEC Scholarships. ► 1st October, 2006 ► 7th January, 2007 ► 8th April, 2007	Ph.D Admissions in all Universities of Pakistan. ► 8th October, 2006 ► 5th August, 2007

TEST DATE 8th October, 2006

Building Standards in Educational and Professional Testing

HOW TO APPLY:

Candidates are required to send their applications on prescribed Application Form alongwith an online Deposit slip (in original) at the address given below. Please deposit online Rs. 600/- (Non-Refundable) in any nearest branch of Allied Bank Ltd. Application form & online deposit slip are available on NTS website.

Application submitted by hand will not be entertained.

For further information, please contact:

National Testing Service (NTS) Pakistan

Office # 10-11, First Floor, Plaza 2000, I-8 Markaz, Islamabad
Email: support@nts.org.pk
Help Line: 051-9258478

www.nts.org.pk

New Testing System Guide

SECTION I

Quantitative Ability

The basic mathematical skills, understanding of elementary mathematical concepts, and the ability to reason quantitatively and solve problems in a quantitative setting are measured in the quantitative part of the test. The knowledge of **arithmetic, algebra, geometry** and **data analysis**, which are usually essential area of study of the high school level are measured in balanced questions. The questions about quantitative ability can also be asked from:

- Discrete Quantitative Questions
- Quantitative Comparison Questions
- Data Interpretation Questions etc.

This section is discussed and explained in detail in this book. Topic by topic explanation is given to facilitate the candidates. Explanatory answers are also given to avoid complications.

NTS Guide

Part 1 Arithmetic

Chapter-1: NUMBERS

Numbers:

In decimal number system, we use ten symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 called digits, to represent any number.

Note: A group of figures, denoting a number is called numeral.

Types of Numbers

Natural Numbers:

Numbers which we use for counting the objects are known as natural numbers. It is denoted by 'N'.

$$N = \{1, 2, 3, 4 \ldots\ldots\}$$

Whole Numbers:

All Natural Numbers together with zero form the set of all whole numbers. It is denoted by 'W'.

$$W = \{0, 1, 2, 3, \ldots\ldots\}$$

Integers:

The set of numbers which consists of whole numbers and negative numbers is known as integers. It is denoted by Z.

$$Z = \{\ldots\ldots -3, -2, -1, 0, 1, 2, 3, \ldots\ldots\}$$

Positive Integers:

The set $Z^+ = \{1, 2, 3, 4, \ldots\ldots\}$ is the set of all positive integers. It is clear that positive integers and natural numbers are synonyms.

Negative Integers:

The set $Z^- = \{-1, -2, -3, \ldots\ldots\}$ is the set of all negative integers.

Remember: "0" is neither positive nor negative.

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Non-Negative Integers:
The set {0, 1, 2, 3, ...} is a set of non-negative integers.
Non-Positive Integers:
The set {0, −1, −2, −3, ...} is the set of non-positive integers.
Even Numbers:
The numbers which are divisible by 2 are called Even Numbers.

$$E = \{2, 4, 6, ...\}$$

Odd Numbers:
The numbers which are not divisible by 2 are called Odd Numbers.

$$O = \{3, 9, 11, 17, 19, ...\}$$

Properties of zero:

1. 0 is neither positive nor negative.
2. 0 is an even integer.
3. 0 is smaller than every positive number.
4. 0 is greater than every negative number.
5. For any integer p; $p \times 0 = 0$.
6. For any integer p (including 0): $p \div 0 = 0$.
7. For any positive integer p; $0 \div p$; $\frac{0}{p}$ = undefined.
8. For every integer p; $p + 0$ and $p - 0 = p$.
9. If the product of two or more numbers is 0, then at least one of them is 0.

Properties of one:

1. For any number p: $p \times 1 = p$ and $\frac{p}{1} = p$.
2. 1 is the divisor of every integer.
3. 1 is an odd integer.
4. 1 is not a prime number, because prime numbers should be greater than 1.
5. 1 is the smallest positive integer.
6. For any number n : $1^n = 1$.

Factors and Multiples:
A number which divides a given number exactly is called a factor of the given number.
Example 1: Find the factors of (i) 64 and (ii) 75.
Solution: (i) $64 = 1 \times 64$
$= 2 \times 32$
$= 4 \times 16$
$= 8 \times 8$
The factors of 64 are 1, 2, 4, 8, 16, 32 and 64.
(ii) $75 = 1 \times 75$
$= 3 \times 25$
$= 5 \times 15$
The factors of 75 are 1, 3, 5, 15, 25 and 75.
Division Algorithm:
Let a and b be two given integers such that $b \neq 0$. On dividing a by b, let q be the quotient and r the remainder, then $a = bq + r$.
Clearly, $0 < r < b$
In general, we have

$$\text{Dividend} = (\text{Divisor} \times \text{Quotient}) + \text{Remainder}$$

Multiple of a Number:
A multiple of any natural number is a number obtained by multiplying that number by any natural number.
Example: Find the multiples of:
(i) 4 less than 30 (ii) 9 less than 60

Solution: (i) $4 \times 1 = 4$
$4 \times 2 = 8$
$4 \times 3 = 12$
$4 \times 4 = 16$
$4 \times 5 = 20$
$4 \times 6 = 24$
$4 \times 7 = 28$ etc.

$\therefore$ The multiples of 4 less than 30 are 4, 8, 12, 16, 20, 24 and 28.

(ii) $9 \times 1 = 9$
$9 \times 2 = 18$
$9 \times 3 = 27$
$9 \times 4 = 36$
$9 \times 5 = 45$
$9 \times 6 = 54$ etc.

$\therefore$ The multiples of 9 less than 60 are 9, 18, 27, 36, 45 and 54.

Divisible of a Number:

If a number divides a second number without leaving any remainder, then we say that the second number is divisible by the first number. For example, since the number 2 divides 14 without leaving any remainder, we say that 14 is divisible by 2.

Multiple Choice Questions (MCQs)

Q1. How many numbers between 200 and 500 are divisible by 13?
(A) 23 (B) 17
(C) 15 (D) 32

Q2. The first five multiples of 17 are:
(A) 0, 1, 17, 34, 51 (B) 17, 34, 51, 68, 85
(C) 38, 57, 76, 95, 114 (D) None of these

Q3. The number which is divisible by 7 but not by 14 is:
(A) 21 (B) 12
(C) 71 (D) None of these

Q4. The total number of even prime numbers is:
(A) 0 (B) 1
(C) 2 (D) None of these

Q5. The least prime number is:
(A) 0 (B) 1
(C) 2 (D) 3

Q6. The smallest member of set *W* is:
(A) 0 (B) 1
(C) 2 (D) 3

Q7. The smallest even number of three digits is:
(A) 98 (B) 102
(C) 998 (D) 100

Q8. The smallest 4-digit number using 7, 0, 8 and 9 is:
(A) 0879 (B) 0789
(C) 0978 (D) 7890

Q9. The cube of $\frac{1}{2}$ is:
(A) $\frac{1}{4}$ (B) $\frac{1}{8}$
(C) $\frac{1}{2}$ (D) $\frac{1}{16}$

Q10. $3 - 7 =$

(A) -7 (B) 7
(C) -4 (D) 4

Q11. **If 1 is added to the denominator of a fraction, it becomes $\left(\frac{1}{2}\right)$ and if 1 is added to the numerator, the fraction becomes 1. The fraction is:**

(A) $\frac{4}{7}$ (B) $\frac{10}{11}$
(C) $\frac{2}{3}$ (D) $\frac{5}{9}$

Q12. **How many two-digit numbers are there which are divisible by 6?**

(A) 17 (B) 18
(C) 16 (D) 15

Q13. **A number whose fifth part increased by 5 is equal to its fourth part diminished by 5, is:**

(A) 160 (B) 180
(C) 200 (D) 220

Q14. **If $(5^a)(5^b) = \frac{5^c}{5^d}$, what is d in terms of a, b and c?**

(A) $a + b - c$ (B) $a - b + c$
(C) $a + b + c$ (D) $c - a - b$

Q15. **Which of the following is equal to $(3^8 \times 3^9)^{10}$?**

(A) 3^{720} (B) 3^{170}
(C) 3^{27} (D) 3^{98}

Q16. **If $0 < p < 1$, which of the following lists the numbers are in increasing order?**

(A) $p, \sqrt{p}, p^2$ (B) $p, p^2, \sqrt{p}$
(C) $\sqrt{p}, p, p^2$ (D) $p^2, p, \sqrt{p}$

Q17. **The value of x satisfying $\sqrt{5 + \sqrt[3]{x}} = 3$ is:**

(A) 64 (B) 27
(C) 125 (D) 9

Q18. **If, $x^x\sqrt{x} = (x\sqrt{x})^x$, then $x =$**

(A) $\frac{1}{2}$ (B) $\frac{9}{4}$
(C) $\frac{3}{2}$ (D) $\frac{1}{4}$

Q19. **$(16)^{7/4}$ is equal to:**

(A) 28 (B) 128
(C) 27 (D) None of these

Q20. **$\frac{4}{5}$ of a number exceeds its $\frac{2}{3}$ by 8. The number is:**

(A) 30 (B) 60
(C) 75 (D) 90

Explanatory Answers

Q1.(A) Number of numbers up to 200 which are divisible by 13

$$= \frac{200}{13} = 15 + \frac{5}{13}, \text{ i.e., } 15$$

Number of numbers up to 500 which are divisible by 13

$$= \frac{500}{13} = 38 + \frac{6}{13} \text{ i.e., } 38$$

The required numbers = 38 – 15 = 23

Hence, the correct answer is choice A.

Q2.(B) The first five multiples of 17 are

$$17 \times 1 = 17$$
$$17 \times 2 = 34$$
$$17 \times 3 = 51$$
$$17 \times 4 = 68$$
$$17 \times 5 = 85$$

First five multiples of 17 are 17, 34, 51, 68 and 85.

Q3.(A) The number which is divisible by 7 but not by 14 is 21. Hence, the correct answer is choice A.

Q4.(B) There is only one even prime number, namely 2. Hence, the correct answer is choice C.

Q5.(C) The least prime number is 2. Hence, the correct answer is choice C.

Q6.(A) 0 is the smallest member of the set W. Hence, the correct choice is A.

Q7.(D) The smallest even number of three digits is 100. The correct choice is choice D.

Q8.(B) Using 0, 7, 8, 9, the smallest number is 0789. Hence, the correct answer is choice B.

Q9.(B) $\left(\frac{1}{2}\right)^3 = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

$$= \frac{1 \times 1 \times 1}{2 \times 2 \times 2} = \frac{1}{8}$$

Correct answer is choice B.

Q10.(C) $3 + (-7) = 3 - 7 = -4$

Correct answer is choice C.

Q11.(C) Let the fraction be $\frac{x}{y}$. Then $\frac{x}{y+1} = \frac{1}{2}$ and $\frac{x+1}{y} = 1$

First we solve $\frac{x}{y+1} = \frac{1}{2}$ $\Rightarrow y + 1 = 2x$

$\Rightarrow 2x - y = 1$ ...(i)

Similarly $\frac{x+1}{y} = 1$ $\Rightarrow x + 1 = y$

$\Rightarrow x - y = -1$ ...(ii)

Subtracting (ii) from (i), we have

$$\begin{array}{r} 2x - y = 1 \\ \underline{x - y = -1} \\ x = 2 \end{array}$$

$x = 2, \Rightarrow x - y = -1 \Rightarrow 2 - y = -1 \Rightarrow y = 3$

Hence, the required fraction is $\frac{2}{3}$.

The correct answer is choice C.

Q12.(D) Required numbers are 12, 18, 24, ... 96

Here, $a = 12$ and $d = 6$

$$\begin{aligned} Tn = 96 &\Rightarrow a + (n-1)d = 96 \\ &\Rightarrow 12 + (n-1)6 = 96 \\ &\Rightarrow 12 + 6n - 6 = 96 \\ &\Rightarrow 6(n+1) = 96 \\ &\Rightarrow n + 1 = \frac{96}{6} = 16 \\ &\Rightarrow \boxed{n = 15} \end{aligned}$$

Hence, the correct answer is choice D.

Q13.(C) $\frac{x}{5} + 5 = \frac{x}{4} - 5 \Rightarrow \frac{x}{4} - \frac{x}{5} = 10$

$$\Rightarrow 5x - 4x = 200$$
$$\Rightarrow x = 200$$

Hence, the correct answer is choice C.

Q14.(D) $(5^a)(5^b) = \frac{5^c}{5^d}$

$5^{a+b} = 5^{c-d}$ (By power rule)

$\Rightarrow a + b = c - d$

$\Rightarrow d = c - a - b$

Hence, the correct answer is choice D.

Q15.(B) Given that, $(3^8 \times 3^9)^{10}$

$= (3^{8+9})^{10}$ (By power rule)

$= (3^{17})^{10}$

$= 3^{17 \times 10}$

$= 3^{170}$

Hence, the correct answer is choice B.

Q16.(D) For any number p, between 0 and 1

$p^2 < p$ and $p < \sqrt{p}$

Hence, the correct answer is choice D.

Q17.(A) $\sqrt{5 + \sqrt[3]{x}} = 3$

$5 + \sqrt[3]{x} = 9$ (Squaring both sides)

$\sqrt[3]{x} = 9 - 5$

$\sqrt[3]{x} = 4$

$((x)^{1/3})^3 = (4)^3$

$x^{1/3 \times 3} = 4 \times 4 \times 4$

$\boxed{x = 64}$

Hence, the correct answer is choice A.

Q18.(B) $x^{x\sqrt{x}} = (x\sqrt{x})^x$

$x^{x\sqrt{x}} = (x \,.\, x^{1/2})^x$

$\Rightarrow x^{x\sqrt{x}} = (x^{3/2})^x$

$\Rightarrow x^{x\sqrt{x}} = (x^{3x/2})$

$\Rightarrow x\sqrt{x} = \frac{3x}{2}$

$\Rightarrow \sqrt{x} = \frac{3}{2}$

$\Rightarrow \boxed{x = \frac{9}{4}}$

Hence, the correct answer is choice B.

Q19.(B) $(16)^{7/4}$

$= (2^4)^{7/4}$

$= 2^{4 \times 7/4}$

$= 2^7$

$= 128$

Hence, the correct answer is choice B.

Q20.(B) $\frac{4}{5}x - \frac{2}{3}x = 8$

$\Rightarrow 12x - 10x = 120$

$\Rightarrow 2x = 120$

$\Rightarrow x = 60$

Hence, the correct answer is choice B.

****

Chapter 2

MULTIPLICATION AND DIVISION

MULTIPLICATION

Multiplication is a short method of adding the same number repeatedly.

PROPERTIES OF MULTIPLICATION

1. Multiplication is commutative for rational numbers.
   Example:
   $$\frac{a}{b}\times\frac{c}{d}=\frac{ac}{bd}=\frac{c}{d}\times\frac{a}{b}$$
   $$\frac{2}{3}\times\frac{5}{7}=\frac{10}{21}=\frac{5}{7}\times\frac{2}{3}$$
2. Multiplication is associative for rational numbers.
   Example:
   $$\frac{a}{b}\times\left(\frac{c}{d}\times\frac{e}{f}\right)=\frac{ace}{bdf}=\left(\frac{a}{b}\times\frac{c}{d}\right)\times\frac{e}{f}$$
   $$\frac{2}{3}\times\left(\frac{5}{7}\times\frac{11}{13}\right)=\frac{110}{273}=\left(\frac{2}{3}\times\frac{5}{7}\right)\times\frac{11}{13}$$
3. Multiplication is distributive over addition and subtraction for rational numbers.
   Example:
   $$\frac{a}{b}\times\left(\frac{c}{d}\pm\frac{e}{f}\right)=\frac{a}{b}\times\frac{c}{d}\pm\frac{a}{b}\times\frac{e}{f}$$
   $$\frac{2}{3}\times\left(\frac{5}{7}\pm\frac{11}{13}\right)=\frac{2}{3}\times\frac{5}{7}\pm\frac{2}{3}\times\frac{11}{13}$$
4. For any rational number $\frac{x}{y}, \frac{x}{y}\times 1=\frac{x}{y}=1\times\frac{x}{y}$, one is called multiplicative identity.
5. Two rational numbers $\frac{a}{b}$ and $\frac{b}{a}$ are the multiplicative inverses of each other.
   $$\frac{a}{b}\times\frac{b}{a}=1=\frac{b}{a}\times\frac{a}{b}$$

Note: The sign of the product is +ive, if there are an even number of negative factors or there are no negative factors. The sign of the product is –ive, if there are an odd number of negative factors

DIVISION

The process of subtraction of the same number form a given number for a few times is called division (÷), *i.e.*,

$$6 \div 2 = 3$$

(2 can be subtracted 3 times from 6)

IMPORTANT POINTS

1. Division is the inverse operation of multiplication. For example $6 \div 2 = 3$ means to find the number by which 2 should be multiplied so as to obtain 6.
   Because $3 \times 2 = 6$
   Therefore, $6 \div 2 = 3$
2. When a number is divided by another number, the first number *i.e.* the number which is being divided is called the ***dividend***, the second number which divides is called the ***divisor*** and the number obtained as a result of division is called the ***quotient***. In the above example, 6 is the dividend, 2 is the divisor and 3 is the quotient.
3. The operation of division starts from the left whereas the operations of addition, subtraction and multiplication start from the right.

Divisibility:

The following table gives the rules to test the divisibility from 2 to 19.

Divisibility by	If	Example
2	Any number in the unit's place which is either even or zero.	12, 10, 26, 32, 38, 567992, 11110234
3	The sum of digits is divisible by 3.	321 : 3 + 2 + 1 = 6 is divisible by 3.
4	The last two digits of a number is divisible by 4.	725324 : 24 is divisible by 4.
5	The number ends with 5 or zero.	4112370, 5321095, 3331210, etc.
6	A number is divisible by 2 and the sum of the digits of the number is multiple of 3.	342, 63924, 154, 261 etc.
8	The last three digits of a number is divisible by 8. or The last three digits of a number are zero.	2125000, 135923120, 7792320, 1256, etc.
9	The sum of all the digits of a number is divisible by 9.	33456735 : 3 + 3 + 4 + 5 + 6 + 7 + 3 + 5 = 36 divisible by 9.
10	Any number which ends with zero.	70, 789790, 7111130, 5773313570, 112300100 etc.
11	The sum of digits at odd and even places are respectively equal or differ by a number divisible by 11.	4235682 : Sum 1 = 4 + 3 + 6 + 2 = 15 Sum 2 = 2 + 5 + 8 = 15 Sum 1 = Sum 2, the number is divisible by 11. or 283712 : Sum 1 = 2 + 3 + 1 = 6 and Sum 2 = 8 + 7 + 2 = 17, their differ 17 – 6 = 11 is divisible by 1.
12	The number which is divisible by both 4 and 3.	135792 etc.
14	The number which is divisible by both 2 and 7.	98, 504 etc.
15	The number which is divisible by 3 and 5.	360, 733352215 etc.
16	The number whose last 4 digit number is divisible by 16.	253421020, 27954204 etc.
18	Any number which is divisible by 9 and has its last digit even (or zero).	2709360, 252630 etc.
25	The number formed by the last two digits of the number is divisible by 25.	257275, 25277750 etc.

Model Examples:

Q1. Multiply 63987 by 91763 is not more than 3 lines.

Solution:

```
        63986
   (×)  91763
      403 1181     Multiplication by    63
     4479 09××     Multiplication by   700
   5822817×××      Multiplication by 91000
   5871639081
```

Q2. Find the number, one-sixth of which exceeds its one-ninth by 654.

Solution: Let the number be x

$$\therefore \quad \frac{x}{6} - \frac{x}{9} = 654$$

$$\frac{x}{18} = 654$$

$$\Rightarrow \quad x = 654 \times 18 = 11772 \qquad \text{Ans.}$$

Q3. Find the quotient and remainder when $x^2 + bx - 5$ is divided by $x + 1$. For what value of 'b' will the remainder be zero?

Solution:

$$x+1\overline{)\,x^2+bx-5\,}\,(x+(b-1)$$
$$\begin{array}{l} x^2+x \\ \underline{-\quad -} \\ (b-1)x-5 \\ (b-1)x-1+b \\ \underline{-\qquad +\quad -} \\ \qquad -4-b \end{array}$$

So Quotient $= x+b-1$ Ans.

Remainder $= -(b+4)$

For remainder $= 0$

$-b-4=0$

$\Rightarrow \boxed{b=-4}$

Q4. The speed of mail train is 1,370 meters per minute. Express it in miles per hour correct to three significant figures, given that 1 metre = 39.37 inches.

Solution: Speed of mail train = 1,370 metres per minute

$= 1370 \times 60$ metres per hour

$= \dfrac{1370\times60\times39.37}{12\times3\times1760}$ miles per hour

$= 51.077$ miles per hour **Ans.**

Q5. A boy when asked to multiply a number by 7/8, divided this instead, by 7/8 and found the answer $1\frac{1}{14}$ too great. Find the number and the correct answer.

Solution: Let the number be 'x'

$$\therefore \quad (x\div\frac{7}{8})-\left(x\times\frac{7}{8}\right)=\frac{15}{14}$$

$$\frac{8x}{7}-\frac{7x}{8}=\frac{15}{14}$$

$$\frac{64x-49x}{56}=\frac{15}{14}$$

or

$$\frac{15x}{56}=\frac{15}{14}$$

$$\therefore \quad x=\frac{56\times15}{14\times15}=4 \quad \textbf{Ans.}$$

Correct answer $= 4\times\frac{7}{8}=3\frac{1}{2}$ **Ans.**

Q6. The sum of the squares of two consecutive integers is 1105. Find the integers and check your answer.

Solution: Let the two consecutive positive numbers be:

$x, x+1$

Then sum of the squares of these consecutive numbers = 1105

$$\begin{aligned} \therefore \quad x^2+(x+1)^2 &= 1105 \\ x^2+x^2+2x+1 &= 1105 \\ 2x^2+2x-1104 &= 0 \\ x^2+x-552 &= 0 \end{aligned}$$

or

$$\begin{aligned} x^2+24x-23x-552 &= 0 \\ x(x+24)-23(x+24) &= 0 \\ (x-23)(x+24) &= 0 \end{aligned}$$

$\Rightarrow \quad x=23 \quad$ or $x=-24$

As the two consecutive numbers are +ve integers, therefore, we neglect the –ve number.

Thus the two consecutive numbers are 23 and 24. **Ans.**

Chapter 3

HIGHEST COMMON FACTOR & LEAST COMMON MULTIPLE

The highest common factor of two or more numbers is the greatest number which divides each of them exactly.

Methods of finding H.C.F.

(i) ***By Prime Factors.***

Resolve the given number into their prime factors. The product of all prime common factors is known as H.C.F.

Model Example

Find the H.C.F. of 630, 1050 and 1260.

Solution: 630 = 2.3.3.5.7

1050 = 2.3.5.5.7

1260 = 2.2.3.3.5.7

∴ H.C.F. is 2.3.5.7 = 210. **Ans.**

(ii) ***By Division:***

Find the H.C.F. of 5133 and 3953

```
3953) 5133 (1
      3953
      1180) 3953(3
            3540
            413) 1180(2
                 826
                 354) 413(1
                      354
                       59) 354(6
                           354
                            ×
```

Various Steps:

Step I. Dividing the greatest number by the lesser, we get the remainder 1180.

Step II. Dividing the previous divisor 3953 by 1180, we get the remainder 413.

Step III. Dividing the previous divisor 1180 by 413 we get the remainder 354.

Step IV. Dividing the previous divisor 413 by 354 we get the remainder 59.

Step V. Dividing the previous divisor 354 by 59 we get no remainder.

∴ The last divisor 59 is the H.C.F.

H.C.F. is also known as Greatest Common Measure (G.C.M.)

LEAST COMMON MULTIPLE (L.C.M)

The Least Common Multiple of two or more given numbers is the least number which is exactly divisible by each of them.

Methods of Finding L.C.M.

(*i*) ***By Factors.*** Resolve the given numbers into prime factors, and find the product of the highest powers of all the factors that occur in the given number. The product will be the required L.C.M.

Model Example

Q1. Find the L.C.M. of 70, 80, 90.

Solution: $70 = 2 \times 5 \times 7$

$80 = 2^4 \times 5$

$90 = 2 \times 3^2 \times 5$

L.C.M. = $2^4.3^2.5.7 = 5040$ Ans.

(ii) ***With the help of H.C.F.*** The product of two numbers is equal to the product of their L.C.M. and H.C.F.

∴ L.C.M. of two numbers

$= \dfrac{\text{Product of numbers}}{\text{H.C.F.}}$

L.C.M. and H.C.F. of Factions.

L.C.M. of two or more fractions

$$= \frac{\text{L.C.M. of numerators}}{\text{H.C.F. of denominators}}$$

H.C.F. of two or more fractions

$$= \frac{\text{L.C.M. of numerators}}{\text{H.C.F. of denominators}}$$

Model Examples

Q1. The H.C.F. of two numbers is 34 and their L.C.M. is 4284. If one of the numbers is 204, find the other.

Solution: As product of 2 numbers
= their H.C.F. × L.C.M.

The other number is $= \frac{34 \times 4284}{204}$

$= 714$ **Ans.**

Q2. What is the highest number of four digits which will leave a remainder of 1 when divided by any of numbers 6, 9, 12, 15, or 18?

Solution: L.C.M. of 6, 9, 12, 15, 18 = 180

Greatest no. of 4 digits = 9999

Greatest no. of 4 digits divisible by

180 = 9999 – 99 = 9900

```
        55
180 ) 9999
      900
      999
      900
       99
```

∴ Reqd. No. = 9900 + 1 = 9901 **Ans.**

Q3. Three men A, B and C go walking round a circle one mile in circumference at the rates of 160, 120 and 105 yards per minute, respectively. If they all start together and walk in the same direction, when will they first be together again?

Solution: Circumference of the circle
= 1 mile or 1760 yds.

A will complete the circle in

$= \frac{1760}{160} = 11$ min.

B will complete the circle in

$= \frac{1760}{120} = \frac{44}{3}$ min.

C will complete the circle in

$= \frac{1760}{105} = \frac{352}{21}$ min.

L.C.M. of 11, $\frac{44}{3}, \frac{352}{21} = 352$ minutes.

i.e., they will be together again first after 352 min. or 5 hrs. 52 min. **Ans.**

Multiple Choice Questions (MCQs)

Q1. A neon sign flashes every 3 seconds, another sign flashes every 5 seconds, and a third flashes every 7 seconds. If they all flash together, how many seconds will pass before they all flash simultaneously again?

(A) 15 seconds (B) 35 seconds
(C) 105 seconds (D) 21 seconds

Q2. The greatest number which exactly divides 1155 and 735 is:

(A) 25 (B) 5

(C) 15 (D) 105

Q3. The least number which when divided by 35, leaves remainder of 25; when divided by 45 leaves a remainder of 35 and when divided by 55 leaves 45 as remainder, is:

(A) 3455 (B) 3465
(C) 3475 (D) 10

Q4. The L.C.M of 12,20,24,32 is:

(A) 240 (B) 360
(C) 480 (D) 600

Q5. How many whole bricks 6 × 12 × 24 cm will be sufficient to construct a solid cube of minimum size?

(A) 4 (B) 6
(C) 8 (D) 12

Q6. If the L.C.M and H.C.F of two numbers are 150 and 30 respectively, and one of the numbers is 18, find the other number?

(A) 250 (B) 180
(C) 150 (D) 170

Q7. The product of two numbers is 2500. If their L.C.M is 125, then their H.C.F is:

(A) 20 (B) 250
(C) 125 (D) None of these

Q8. It takes Riaz 30 minutes to mark a paper. Razi only need 25 minutes to mark a paper. If they both start marking papers at 11 : 00 AM, what is the first time they will finish marking a paper at the same time?

(A) 12 : 30 (B) 12 : 45
(C) 1 : 30 (D) 12 : 25

Q9. Sonia buys two off-cuts of ribbon in a sale. One is 153 cm long. The other is 204 cm long. She cuts them so that she ends up with a number of pieces all the same length. What is the greatest length each piece can be?

(A) 39 (B) 6
(C) 17 (D) 51

Q10. A farmer wants to fence a triangular field. He plans to put a fencing post in each corner and place other posts at equal distance along its sides. He wants the posts to be as far apart as possible. The sides of the field are 477 feet 2412 feet and 639 feet long. How far apart will the posts be?

(A) 18 feet (B) 9 feet
(C) 27 feet (D) 159 feet

Q11. Find the greatest number of 4 digits which when divided by 18, 24, 30 and 36 leaves a remainder 17 in each case.

(A) 360 (B) 9360
(C) 3600 (D) 9377

Q12. The least number which when divided by 12, 15 and 18 leaves 5 as remainder in each case is:

(A) 180 (B) 175
(C) 185 (D) 125

Q13. The greatest number which divides 2400 and 3600 leaving 48 and 60 respectively, as remainder is:

(A) 9 (B) 7
(C) 17 (D) 10

Q14. Ahmed has a rectangular garden measuring 4.32m by 3.36m. He wants to divide it into square plots of equal size. What is largest sized square he can use?

(A) 0.24 (B) $\sqrt{3}$
(C) 0.48m (D) 0.16

Q15. The chairs in the school hall can be set out in 35 equal rows or in 45 equal rows or in 105 equal rows are:

(A) 600 (B) 400

(C) 40 (D) 80

Q16. Three bells toll after intervals of 6, 9 and 15 minutes, respectively. If they toll together at 5 p.m., when will they toll together next?

(A) 6 : 30 (B) 5 : 30
(C) 6 : 45 (D) 5 : 45

Explanatory Answers

Q1. (C) The L.C.M of 3,5 and 7 will give the answer

3	3 – 5 – 7
5	1 – 5 – 7
7	1 – 1 – 7
	1 – 1 – 1

$= 3 \times 5 \times 7 = 105$

Q2. (D) The required number is the H.C.F of 1155 and 735

```
        1
735 ) 1155
      735    1
      420 ) 735
            420    1
            315 ) 420
                  315    3
                  105 ) 315
                        315
                          X
```

The greatest number required is 105.

Q3. (A) The least number which is completely divided by 35, 45 and 55, is their L.C.M. which is 3465. We want to find the least number which on dividing by 35, 45 and 55 leave remainders 25, 35 and 45 respectively *i.e.,* 10 less than the quotient in each case. Hence such a number is 3465 – 10 = 3455

Q4. (C)

2	12–20 – 24–32
2	6 – 10 – 12 – 16
2	3 – 5 – 6 – 8
2	3 – 5– 3 – 4
2	3 – 5– 3 – 2
3	3 – 5– 3 – 1
5	1 – 5 – 1 – 1
	1 – 1 – 1 – 1

The L.C.M. of 12, 20, 24 and 32 is

$2^5 \times 3 \times 5 = 32 \times 3 \times 5 = 480$

Q5. (C) One edge of the minimum cube must be 24 cms, the least common multiple of 6, 12 and 24. Thus, it will have a volume of 24 × 24 × 24 cubic centimeters which is equal to 8 bricks

i.e., $\frac{24 \times 24 \times 24}{6 \times 12 \times 24} = 8$

Q6. (A) Product of two numbers = L.C.M × H.C.F

18 × 2nd number = 150 × 30

2nd number $= \frac{150 \times 30}{18}$

= 250

Q7. (A) Product of two numbers = L.C.M × H.C.F

2500 = 125 × H.C.F

$\Rightarrow$ H.C.F $= \frac{2500}{125}$

$= 20$

Q8. (C) The question asks for the first time they will finish at the same time. So, we must find least common multiple

5	25 – 30
5	5 – 6
6	1 – 6
	1 – 1

$6 \times 5 \times 5 = 150$ minutes

$= 2 : 30$ hours

So they will finish marking at 1 : 30 PM.

Q9. (D) The HCF of 153 and 204 gives the wanted length

3	153
3	51
17	17
	1

2	204
2	102
3	51
17	17
	1

$153 = 3 \times ③ \times \boxed{17}$

$204 = 2 \times 2 \times ③ \times \boxed{17}$

HCF $= 17 \times 3 = 51$

Greatest length $= 51$cm

Q10. (B) The HCF of 477, 2412 and 639 gives the wanted length.

3	477
3	159
	53

2	2412
2	1206
3	603
3	201
	67

3	639
3	213
	71

$477 = ③ \times \boxed{3} \times 53$

$2412 = 2 \times 2 \times ③ \times \boxed{3} \times 67$

$639 = ③ \times \boxed{3} \times 71$

H.C.F $= 3 \times 3 = 9$

Q11. (D) The number which is divisible by 18, 24, 30 and 36 is divisible by their L.C.M

2	18 – 24 – 30 – 36
2	9 – 12 – 15 – 18
3	9 – 6 – 15 – 9
3	3 – 2 – 5 – 3
2	1 – 2 – 5 – 1
5	1 – 1 – 5 – 1
	1 – 1 – 1 – 1

$\therefore$ L.C.M $= 2 \times 2 \times 2 \times 3 \times 3 \times 5 = 360$

The greatest number of 4–digits is 9999. Now we find the greatest multiple of 360 less than 9999.

```
      26
360) 9999
     720
     2799
     2160
      639
```

Thus 9999 – 639 = 9360 is exactly divisible by 360. But the required number leaves a remainder of 17 in each case. Hence, the number is

$9360 + 17 = 9377$

Q12. (C) Required number = L.C.M of 12, 15, 18

2	12, 15, 18
3	6 – 15 – 9
2	2 – 5 – 3
3	1 – 5 – 3
5	1 – 5 – 1
	1 – 1 – 1

∴ L.C.M $= 2 \times 2 \times 3 \times 3 \times 5 = 180$

The required least number $= 180 + 5 = 185$

Q13. **(D)** As 48 and 60 remainders when 2400 and 3600 are divided by the numbers 2400 – 48 = 2352 and

3600 – 60 = 3540 must be exactly divisible by the number.

The H.C.F of 2352 and 3540 is the required number.

```
          1
2352) 3540
      2350   1
      1190) 2350
            1190   1
            1160) 1190
                  1160   38
                    30) 1160
                        90
                        260
                        240      1
                         20) 30
                             20   2
                             10) 20
                                 20
                                 x
```

a. The H.C.F of 2350 and 3540 is 10.

b. The required greatest number is 10.

Q14. **(C)**

```
          1
3.36) 4.32
      3.36   3.5
      .96) 3.36
           3.36
           x
```

$0.96 \div 2 = 0.48$m

Q15. **(A)**

5	25 – 40 – 120
5	5 – 8 – 24
8	1 – 8 – 24
3	1 – 1 – 3
	1 – 1 – 1

$= 5 \times 5 \times 8 \times 3 = 600$ chairs

Q16. **(A)**

3	6 – 9 – 15
2	2 – 3 – 5
3	1 – 3 – 5
5	1 – 1 – 5
	1 – 1 – 1

L.C.M of 6, 9 and 15 $= 3 \times 2 \times 3 \times 5$

$= 90$

∴ The bells will toll after 90 minutes, it mean at 6 : 30.

Chapter 4

SQUARE ROOT

Methods of Finding Square Root:

(*i*) ***By Factors.*** Resolve the number into its prime factors. The square root is the product of the prime factors taken half as many times as they occur in the number.

(*ii*) ***By Division.***

Model Example

Find the square root of 2480625.

Solution:

1	2480625	(1575
	1	
25	148	
	125	
307	2306	
	2149	
3145	15725	
	15725	
	×	

∴ **Ans.** 1575

Q2. Find the square root of 43.45 to four decimal places.

Solution:

6	43.45	(6.5916
	36	
125	7.45	
	6.25	
1309	12000	
	11781	
13181	21900	
	13181	
131826	871900	
	790956	
	80944	

As remainder is more than half 6.5917 **Ans.**

Q3. Find the value of $\sqrt{\dfrac{2+\sqrt{3}}{2-\sqrt{3}}}$ correct to three decimal places.

Solution: $\sqrt{\dfrac{2+\sqrt{3}}{2-\sqrt{3}}} = \sqrt{\dfrac{(2+\sqrt{3})(2+\sqrt{3})}{(2-\sqrt{3})(2-\sqrt{3})}}$ (Rationalization)

$$= \sqrt{\frac{(2+\sqrt{3})^2}{(2)^2-(\sqrt{3})^2}}$$

$$= \sqrt{\frac{(2+\sqrt{3})^2}{4-3}} = 2+\sqrt{3} = 2+1.732$$

$= 3.732$ **Ans.**

Unitary Method and Chain Rule

IMPORTANCE:

The unitary method and chain rule have quite an importance in our daily life. It is explained by the following model examples.

Model Examples:

Q1. In a kilometer race A can beat B by 40 metres and B can beat C by 50 metres. How many metres can A beat C in a 500 metres race?

Solution: When A covers 1000 m.

B covers $1000 - 40 = 960$ m.

and When B covers 1000 m.

C covers $1000 - 50 = 950$ m.

$\therefore$ When B covers 960 m.

C covers $\frac{950}{1000} \times 960\,\text{m} = 912$ m.

i.e., when A covers 1000 m. C covers $\frac{912}{2}$

$= 456$ m.

or when A covers 500 m. race, A will beat C by

$= 500 - 456 = 44$ m.

Q2. Divide Rs. 510 between A, B and C so that A gets 2/3 of what B gets and B gets ¼ of what C gets. Find the share of each.

Solution: Let C's share be Rs. x

$\therefore$ B's // // $= \frac{x}{4}$

A's // // $= \frac{2}{3} \times \frac{x}{4} =$ Rs. $\frac{x}{6}$

Total amount = Rs. 510

$$\therefore \quad x + \frac{x}{4} + \frac{x}{6} = 510$$

$$\frac{12x + 3x + 2x}{12} = 510$$

$$\therefore \quad x = \frac{12 \times 510}{17} = 360$$

$$\therefore \quad \left.\begin{array}{ll} \text{A's share} & = \frac{360}{6} = \text{Rs. } 60 \\ \text{B's } \quad // & = \frac{360}{4} = \text{Rs. } 90 \\ \text{C's } \quad // & = \text{Rs. } 360 \end{array}\right\} \textbf{Ans.}$$

Q3. Divide Rs. 600 among A, B, and C so that Rs. 40 more than 2/5 of A's share, Rs. 20 more than 2/7 of B's share, Rs. 10 more than $\frac{9}{17}$ of C's share may be equal.

Solution: Let $\frac{2}{5}$ of A's share + Rs. 40 = $\frac{2}{7}$ of B's share + Rs. 20 = $\frac{9}{17}$ of C's share + Rs. 10 be = x.

$\therefore$ $\frac{2}{5}$ of A's share $= x - 40$

or A's share $= \frac{5}{2}(x - 40)$

Similarly B's share $= \frac{7}{2}(x - 20)$

$$\text{C's share} = \frac{17}{9}(x-10)$$

As total amount = Rs. 600

$$\therefore \quad \frac{5x-200}{2}+\frac{7x-140}{2}+\frac{17x-170}{9}=600$$

$$\frac{45x-1800+63x-1260+34x+340}{18}=600$$

or $142x - 3400 = 600 \times 18 = 10800$

$142x = 10800 + 3400 = 14200$

$$x = \frac{14200}{142} = 100$$

$$\therefore \quad \left.\begin{aligned} \text{A's share} &= \frac{5}{2}(100-40) = \text{Rs. } 150 \\ \text{B's share} &= \frac{7}{2}(100-20) = \text{Rs. } 280 \\ \text{C's share} &= \frac{17}{9}(100-10) = \text{Rs. } 170 \end{aligned}\right\} \textbf{Ans.}$$

Q4. A garrison has enough provision for 52 days. After 20 days, a reinforcement of 400 men arrives and the food would then last for 24 days only. How many men were there in the garrison originally?

Solution: Let there be x men in the garrison originally. After 20 days no. of men = $x + 400$.

If these men had not joined, the provision would have lasted for $50 - 20 = 32$ days more.

$\therefore$ For x men the provision can last for 32 days

// 1 // // // $32x$ //

For $(x + 400)$ men of provision can last for $\frac{32x}{x+400}$ days

But provision lasted for 24 days

$$\therefore \quad \frac{32x}{x+400} = 24$$

or $32x = 24x + 9600$

$8x = 9600$

or $x = 1200$ men. **Ans.**

Multiple Choice Questions (MCQs)

Q1. What is the least positive integer which is to be added to 57592910 so that the sum may be a perfect square?

(A) 7588 (B) 7

(C) 11 (D) 15166

Q2. A rectangular field which is twice as long as it is broad, has an area of 14450 m^2, what is its perimeter?

(A) 85 m (B) 510 m

(C) 165 m (D) 170 m

Q3. The cost of the planting sugarcane at the rate of 6 paisa per square meter is Rs. 5840.64. What is the length of side of this square field:

(A) 312 m (B) 622 m

(C) 97344 m (D) 459 m

Q4. What is the smallest number which when subtracted from 1.00060219 gives a perfect square number?

(A) 0.00210 (B) 210

(C) 0.00000210 (D) 0.210

Q5. The product of 313 with itself is:

(A) 97969 (B) 17.69
(C) 5536.97 (D) 195938

Q6. The size of the square which can be made using 256 square shapes with a side length of 6 cm is:

(A) 16 cm (B) 36 cm
(C) 2.67 (approx) cm (D) 96 cm

Q7. An instructor having 9224 students under him, arranges them into a square and finds 8 students to be excess. What is the number of students in the front row?

(A) 97 (B) 88
(C) 104 (D) 96

Q8. The difference between the first two perfect squares that end with 9 is:

(A) 11 (B) 40
(C) 30 (D) 120

Q9. A rectangular field which is 10 times as long as its breadth has an area of 75690 *sq m*. What is its perimeter?

(A) 275 m (B) 2750 m
(C) 1914 m (D) 191 m

Q10. A square lawn having area 0.25 *sq km* has to be enclosed with iron railings at the rate of Rs. 101-00 per metre. What will be its cost?

(A) Rs. 20200 (B) Rs. 2000
(C) Rs. 202000 (D) Rs. 100100

Q11. How many 1cm square pieces of paper can Rachael cut from a square sheet of paper with a side length of 2.1m?

(A) 22100 cm^2 (B) 44100 cm^2
(C) 27100 cm^2 (D) 36100 cm^2

Q12. If length of a rectangular field is twice that of its width. What is the perimeter of the field if its area is 1152 m^2?

(A) 256 m (B) 144 m
(C) 24 m (D) 96 m

Q13. $\sqrt{x^2+y^2}$ is equal to:

(A) $(x+y)(x-y)$ (B) $\sqrt{x^2}+\sqrt{y^2}$
(C) $x+y$ (D) None of these

Q14. $\sqrt{\frac{a^2}{16}+\frac{a^2}{25}}=$

(A) $\frac{a}{10}$ (B) $\frac{2a}{9}$
(C) $\frac{a\sqrt{41}}{20}$ (D) $\frac{41a}{20}$

Q15. The product of $\sqrt{24a}$ and $\sqrt{6a}$ is:

(A) 12a (B) $6a^2$
(C) $12a^2$ (D) $3a\sqrt{8}$

Explanatory Answers

Q1. **(C)** 57592910 is greater than the square of 7588 (using calculator). The next squared is the square of 7589. $(7589)^2 = 57592921$.

$\therefore$ Now $57592921 - 57592910 = 11$, which is the required integer to be added.

Q2. **(B)** As the length is twice as long as width and so its rectangle can be divided into 2 square regions

The area of each square region $= \frac{14450}{2} = 7225\text{m}^2$

Now length of each region = Width of the field × 2

$= \sqrt{7225}$

$= 85\text{m}$

$= 85 \times 2 = 170\text{m}$

Perimeter $= 2(170 + 85)$

$= 2(255)$

$= 510\text{m}$

Q3. **(A)** Cost = Rs. 5840.64

= 584064 paisas

Area $= \frac{584064}{6} = 97344$

Side $= \sqrt{97344}$

$= 312$ m

Q4. **(C)**

```
        1 | 1.00 06 02 19 (1.0003
          | 1
2.0003    | 00 06 02 19
          |      6 00 09
          |          210
```

There are eight places after the decimal in the given number. so after subtracting .0000021 from the given number the remainder would be zero. So .0000021 is the required number.

Q5. **(A)** $313 \times 313 = 97969$

Q6. **(D)** Side length = 6cm

Sides of 256 square shapes $= \sqrt{256} = 16$

Size of the square is $= 16 \times 6 = 96$

Q7. **(D)**

Number of students in front row $= \sqrt{9224 - 8}$

$= \sqrt{9216}$

$= 96$

Q8. **(B)** The first two integers those square end with 9 are 3 and 7.

The difference between there is

$49 - 9 = 40$

Q9. **(C)** Suppose breadth $= x$

Length $= 10x$

Area $= (10x) \times (x) = 75690$

$\Rightarrow \quad 10x^2 = 75690$

$\Rightarrow \quad x^2 = 7569$

$\Rightarrow \quad x = 87 =$ breadth

$\Rightarrow \quad 10x = 870 =$ length

Perimeter $= 2(\text{Length} + \text{breadth})$

$= 2(87 + 870)$

$= 1914$ m

Q10. (C) Side of each square $= \sqrt{0.25} = 0.5$km

Length of the railings $= (.5 + .5)2 = 2$km

$= 2 \times 1000 = 2000$ m

Cost of railing $= 2000 \times 101 =$ Rs. 202000

Q11. (B) Size of the square sheet $= 2.1\text{m} = 2.1 \times 100 = 210\text{cm}$

Area of square root $= 210 \times 210$

$= 44100\ \text{cm}^2$

So, No. of 1cm square pieces will be $= 1 \times 44100 = 44100$

Q12. (C) Let the width $= b$, then length $= 2b$

Area $= b \times 2b = 2b^2$

Now $2b^2 = 1152$ (given) $\Rightarrow b^2 = 576$

$\Rightarrow \quad b = \sqrt{576} = 24$

length $= 2b \Rightarrow$ length $= 2 \times 24 = 48$

Perimeter of the field $= 2(24 + 48)$

$= 2(72) = 144$m

Q13. (D) There is no way to simplify $\sqrt{x^2 + y^2}$.

Q14. (C) $\sqrt{\frac{a^2}{16} + \frac{a^2}{25}} = \sqrt{\frac{25a^2 + 16a^2}{400}} = \sqrt{\frac{41a^2}{400}} = \frac{a\sqrt{41}}{20}$

Q15. (A) $\sqrt{24a} \times \sqrt{6a}$

$= \sqrt{144a^2}$

$= 12a$

****

Chapter 5

FRACTIONS AND DECIMALS

FRACTIONS:

If any unit be divided into any number of equal parts, one or more of these parts is called a fraction of the unit.

Example: The fractions one-fourth, two-third and three-fourth are respectively written as $\frac{1}{4}, \frac{2}{3}$ and $\frac{3}{4}$

NUMERATOR AND DENOMINATOR:

The upper number, which shows the number of parts taken to form the fraction, is called numerator.

The lower number, which indicates the number of equal parts in which the unit is divided, is called denominator.

Terms of The Fraction:

The numerator and the denominator of a fractions are called its terms.

Note: A fraction is also called a rational number.

Lowest Terms of a Fraction:

When the numerator and the denominators of a fraction have no common factor, the fraction is said to be is its lowest terms:

Example: $= \frac{6}{10} = \frac{3\times2}{5\times2}$

In the above example denominator and the numberator have a common factor, thus $\frac{6}{10}$ is not is its lowest terms. If we cancel out 2 by dividing numerator and denominator by 2 we find $\frac{3}{5}$, which has no common factor. hence $\frac{3}{5}$ is in its lowest terms.

Proper Fraction:

A proper fraction is one whose numerator is less than the denominator.

Example: $\frac{2}{3}, \frac{5}{7}, \frac{23}{46}$ are proper fractions.

Note: The value of proper fractions is always less than 1

IMPROPER FRACTION:

A fraction whose numerator is equal to or greater than the denominator is called improper fraction.

Example: $\frac{15}{13}, \frac{13}{5}$, and $\frac{21}{14}$ are improper fractions.

Note: The value of an improper fractions is always more than as equal to 1.

Mixed Fraction:

When an improper fraction is changed to consist of a whole number and a fraction, it is called a mixed fraction.

Example: The improper fraction $\frac{15}{13}$ can be written as $\frac{2}{13}$, which is a mixed fraction.

$$\begin{array}{r} 1 \\ 13\overline{)\,15} \\ \underline{13} \\ 2 \end{array}$$

Compound Fraction:

A fractions of a fraction is called a compound fraction.

Example: $\frac{1}{3}$ of $\frac{3}{5}$ is a compound fraction.

Thus $\frac{1}{3}$ of $\frac{3}{5} = \frac{1}{\cancel{3}} \times \frac{\cancel{3}}{5} = \frac{1}{5}$

Complex Fractions:

A complex fraction is one in which the numerator or denominator or both are fractions.

Example: $\frac{3/2}{5}$, $\frac{3}{2/5}$, $\frac{2/5}{3/7}$ and $\frac{1/3+1/2}{2/3-1/5}$ are complex fractions.

Example 1: One third of one-seventh of a plot is sold Rs. 45000. What is the value of six-twnety fifth of the plot.

Solution: One third of one seventh $=\frac{1}{3}\times\frac{1}{7}=\frac{1}{21}$

Now, $\frac{1}{21}$ of a plot costs = Rs. 45000.

$\frac{6}{25}$ of the plot will cost $=\frac{45000}{1/2}\times\frac{6}{25}$

$$=\frac{45000\times 21\times 6}{25}$$

Example 2: A sum of money increased by its sixth part amount to Rs. 56. Find the sum.

Solution: Let x be the amount of money, thus

$$x+\frac{x}{6}=56 \Rightarrow \frac{6x+x}{6}=56$$
$$\Rightarrow \frac{7x}{6}=56$$
$$\Rightarrow 7x=56\times 6$$
$$\Rightarrow x=\frac{56\times 6}{7}=48$$

VULGAR FRACTIONS

In questions of fractions signs +, –, ×, ÷, "*of*" ('of' signifies *multiplication*) and brackets are often involved. In simplifying these questions the following order must be followed:

IMPORTANT POINTS

(i) Remove the brackets.
(ii) Then quantities which are connected by *'of'* should be simplified.
(iii) Then division and multiplication are carried out.
(iv) Operation of addition and subtraction are performed at last.

Note: The above rules can be easily remembered by the word 'BODISA' of which 'B' stands for brackets, O for 'of', D for division, I for into, S for subtraction and A for addition.

Model Examples:

Example 3: $7\frac{1}{2}-\frac{1}{9}\left[3\frac{3}{4}\div\left\{\frac{5}{6}of\,\frac{2}{3}\left(\frac{1}{3}-\overline{\frac{1}{4}-\frac{1}{6}}\right)\right\}\right]$

Solution:

$$=7\frac{1}{2}-\frac{1}{9}\left[\frac{15}{4}\div\left\{\frac{5}{6}of\,\frac{2}{3}\left(\frac{1}{3}-\frac{3-2}{12}\right)\right\}\right]$$

$$=7\frac{1}{2}-\frac{1}{9}\left[\frac{15}{4}\div\left\{\frac{5}{6}of\,\frac{2}{3}\left(\frac{1}{3}-\frac{1}{12}\right)\right\}\right]$$

$$=7\frac{1}{2}-\frac{1}{9}\left[\frac{15}{4}\div\left\{\frac{5}{6}of\,\frac{2}{3}\left(\frac{3}{12}\right)\right\}\right]$$

$$=7\frac{1}{2}-\frac{1}{9}\left[\frac{15}{4}\div\left\{\frac{5}{6}of\,\frac{1}{6}\right\}\right]$$

$$=7\frac{1}{2}-\frac{1}{9}\left[\frac{15}{4}\div\frac{5}{36}\right]$$

$$= 7\frac{1}{2} - \frac{1}{9}\left[\frac{15}{4} \times \frac{36}{5}\right]$$

$$= 7\frac{1}{2} - \frac{1}{9} \times 3 \times 9 = 7\frac{1}{2} - 3 = 4\frac{1}{2} \text{ Ans.}$$

Example 4: Simplify

$$\frac{\frac{1}{6} + \frac{5}{12} \times \left(\frac{4}{5} - \frac{5}{7}\right)}{\frac{3}{4} \text{ if } 1\frac{2}{3} - \frac{3}{5} \text{ of } 1\frac{4}{7}} \div \frac{\frac{1}{3} + \frac{1}{7} - \frac{2}{5}}{\frac{1}{5} + \frac{1}{9} - \frac{2}{7}}$$

Solution:

$$= \frac{\frac{1}{6} + \frac{5}{12} \times \left(\frac{4}{5} - \frac{5}{7}\right)}{\frac{3}{4} \times \frac{5}{3} - \frac{3}{5} \times \frac{11}{7}} \div \frac{\frac{1}{3} + \frac{1}{7} - \frac{2}{5}}{\frac{1}{5} + \frac{1}{9} - \frac{2}{7}}$$

$$= \frac{\frac{1}{6} + \frac{5}{12} \times \left(\frac{28-25}{35}\right)}{\frac{5}{4} - \frac{33}{35}} \div \frac{\frac{35+15-42}{105}}{\frac{63+35-90}{315}}$$

$$= \frac{\frac{1}{6} + \frac{1}{28}}{\frac{5}{4} - \frac{33}{35}} \div \frac{8}{105} \times \frac{315}{8}$$

$$= \frac{\frac{14+3}{84}}{\frac{175-132}{140}} \div 3 = \frac{17}{84} \times \frac{140}{43} \times \frac{1}{3}$$

$$= \frac{85}{387} \text{ Ans.}$$

Continued Fraction:

The fractions of the form $a + \cfrac{b}{c + \cfrac{d}{e + \cfrac{f}{g}}}$

etc. are known as continued fractions where *a, b, c,* ..... etc., are any numbers.

Note: In order to simplify such fractions, we begin with the lowest part and proceed step by step, upwards.

Model Examples:

Example 5: Simplify:

$$\left\{1 + \cfrac{1}{2 + \cfrac{2}{3 + \cfrac{3}{4}}}\right\} \div \left\{\cfrac{4}{4 + \cfrac{4}{3 + \cfrac{3}{2}}}\right\}$$

Solution:

$$\left\{1+\frac{1}{2+\frac{2}{\frac{15}{4}}}\right\}\div\left\{\frac{4}{4+\frac{4}{\frac{9}{2}}}\right\}$$

$$=\left\{1+\frac{1}{2+\frac{8}{15}}\right\}\div\left\{\frac{4}{4+\frac{8}{9}}\right\}$$

$$=1+\left\{\frac{1}{\frac{38}{15}}\right\}\div\left\{\frac{4}{\frac{44}{9}}\right\}$$

$$=\left\{1+\frac{15}{38}\right\}\div\left\{\frac{36}{44}\right\}$$

$$=\frac{53}{38}\times\frac{44}{36}=\frac{583}{342}$$ **Ans.**

DECIMAL FRACTION

A fraction involving decimal point is called decimal fraction.

Conversion of a decimal fraction into vulgar fraction:

Rule. Write down the given number in the numerator omitting the decimal point and for the denominator write 1 followed by as many zeroes as there are figures on the right of the decimal point.

As $46.76=\frac{4676}{100}$

and $199.0083=\frac{1990083}{10000}$

Model Example

Q1. Simplify

$$\frac{0.1\times0.1\times0.1+0.01+0.01\times0.01}{0.2\times0.2\times0.2+0.08+0.04\times0.02}$$

Solution:

$$=\frac{\frac{1}{10}\times\frac{1}{10}\times\frac{1}{10}+\frac{1}{100}+\frac{1}{100}\times\frac{1}{100}}{\frac{2}{10}\times\frac{2}{10}\times\frac{2}{10}+\frac{8}{100}+\frac{4}{100}\times\frac{2}{100}}$$

$$=\frac{\frac{1}{1000}+\frac{1}{100}+\frac{1}{10000}}{\frac{8}{1000}+\frac{8}{100}+\frac{8}{10000}}$$

$$= \frac{\left(\frac{1}{1000}+\frac{1}{100}+\frac{1}{10000}\right)}{8\left(\frac{1}{1000}+\frac{1}{100}+\frac{1}{10000}\right)}$$

$$= \frac{1}{8}$$ **Ans.**

Multiple Choice Questions (MCQs)

Q1. If $\frac{5}{x}, \frac{8}{x}$, and $\frac{13}{x}$ are all in lowest terms. Then how many integers, x, between 30 and 40?

(A) 5 (B) 1
(C) 2 (D) 3
(D) None of these

Q2. $\frac{6}{6} \times \frac{6}{12} \times \frac{6}{18} \times \frac{6}{24} \times \frac{6}{30}$ equals:

(A) $\frac{1}{120}$ (B) $\frac{1}{2}$
(C) $\frac{1}{30}$ (D) 1
(D) None of these

Q3. If $\frac{4}{13}$ of a number is 39, what is $\frac{8}{13}$ of that number?

(A) $\frac{39}{4}$ (B) 78
(C) 16 (D) $\frac{39}{8}$

Q4. $\frac{3}{4}$ of 28 is equal to $\frac{30}{7}$ of what number?

(A) 90 (B) 45
(C) 30 (D) 56
(D) None of these

Q5. Which of the following is less than $\frac{5}{11}$?

(A) $\frac{3}{2}$ (B) $\frac{2}{3}$
(C) $\frac{1}{2}$ (D) $\frac{2}{5}$
(D) None of these

Q6. There are 20 boys in a class. Five of them are left-handed. What fraction of the class is left handed?

(A) $\frac{1}{5}$ (B) $\frac{1}{2}$
(C) $\frac{1}{4}$ (D) $\frac{2}{11}$

Q7. **A chemical solution contains 8% of acid. If there is 15ml of acid, what is the volume of the solution?**

(A) 125.5 mL (B) 187.5 mL
(C) 225.5 mL (D) 171.5 mL

Q8. **What fractional part of a week is 98 hours?**

(A) $\frac{7}{98}$ (B) $\frac{7}{12}$
(C) $\frac{1}{20}$ (D) $\frac{1}{7}$

Q9. **A village has 5860 voters, of whom 7% usually forget to vote. In order to win an election, a candidate must gain at least 50% of the remaining votes. How many votes does he need in order to win?**

(A) 2725 (B) 410
(C) 5450 (D) None of these

Q10. **What fraction is exactly midway between $\frac{1}{3}$ and $\frac{1}{4}$?**

(A) $\frac{7}{12}$ (B) $\frac{7}{24}$
(C) $\frac{29}{11}$ (D) $\frac{1}{2}$

Q11. **$\frac{4}{9}$ of a number is 12. What is the number?**

(A) 27 (B) 36
(C) 18 (D) 16

Q12. **Ali purchased some goldfish. During the first week, $\frac{1}{5}$ of them died, and during the second week, $\frac{3}{8}$ of those still alive at the end of the first week died. What is the fraction of the original goldfish still alive after two weeks?**

(A) $\frac{1}{2}$ (B) $\frac{3}{2}$
(C) $\frac{5}{2}$ (D) $\frac{4}{3}$

Q13. **$\frac{3}{8}$ of a number is 10. What is the number?**

(A) 91 (B) 81
(C) 23 (D) 27

Q14. **$\frac{5}{8}$ of 24 is equal to $\frac{15}{7}$ of what number?**

(A) 15 (B) 105
(C) 35 (D) 7

Q15. **A German class has 12 boys and 18 girls. What is the fraction of the class boys?**

(A) $\frac{1}{6}$ (B) $\frac{3}{5}$
(C) $\frac{2}{3}$ (D) $\frac{4}{15}$

Explanatory Answers

Q1. (D) If x is even, then $\frac{8}{x}$ will not be in lowest term. This is because, both x and 8 are divisible by 2.

Now we take the odd number between 30 and 40, these are; 31, 33, 35, 37, 39. In these numbers, we see that 35 and 39 are divisible by 5 and 13, respectively. Thus only 31, 33 and 37 are required numbers.

Q2. (A) Simplifying $\frac{6}{6} \times \frac{6}{12} \times \frac{6}{18} \times \frac{6}{24} \times \frac{6}{30}$

$\frac{1}{1} \times \frac{1}{2} \times \frac{1}{3} \times \frac{1}{4} \times \frac{1}{5} = \frac{1}{20}$

Q3. (B) As $\frac{4}{13}$ of a number is 39. Therefore the $\frac{8}{13}$ of that number will be 78

Because $\frac{8}{13} = \frac{4}{13} \times 2$, and $\frac{4}{13}$ of a number is 39, therefore double of $\frac{4}{13}\left(\frac{4}{13} \times 2 = \frac{8}{13}\right)$ should be equal to $39 \times 2 = 78$.

Q4. (A) Let x be the required number, then by given condition

$$28 \div \frac{4}{3} = x \div \frac{30}{7}$$

$$28 \times \frac{3}{4} = x \times \frac{7}{30}$$

$$21 = x \times \frac{7}{30}$$

$$\frac{21 \times 30}{7} = x$$

$$\Rightarrow \boxed{x = 90}$$

Q6. (C) Left handed $= 5$

Total $= 20$

So, fraction $= \frac{5}{20} = \frac{1}{4}$

Q7. (B) 8mL acid in solution $= 100$mL

1mL acid in solution $= \frac{100}{8} = 12.5$ mL

15mL acid in solution $= 12.5 \times 15$

$= 187.5$ mL

Q8. (B) There are 7 days in a week, and each day has 24 hours. Therefore, Hours in a week $= 24 \times 7 = 168$

The required fraction is: $\frac{98}{168} = \frac{7}{12}$

Q9. (A) People does not give vote $= \frac{7}{100} \times 5860$

$= 7 \times 58.6$

$= 410.2$

People does not give vote $\cong 410$ people

Remaining people $= 5860 - 410$
$= 5450$ people

Candidate must gain vote $= 5450 \times \frac{50}{100}$
$= 2725$ vote

Q10. **(B)** The midway fraction of the fractions $\frac{1}{3}$ and $\frac{1}{4} = \frac{1}{2}\left(\frac{1}{3}+\frac{1}{4}\right) = \frac{1}{2}\left(\frac{7}{12}\right) = \frac{7}{24}$

Q11. **(A)** Let the required number be "x", then according to given condition $\frac{4}{9} \times x = 12 \Rightarrow x = \frac{12}{\frac{4}{9}}$

$$= 12 \times \frac{9}{4} = 27$$

Q12. **(A)** Let the number of fish purchased $= x$

During first week (died fish) $= \frac{1}{5} \times x = \frac{x}{5}$

Still alive $= x - \frac{1}{5}x = \frac{4}{5}x$

During second week (died fish) $= \frac{4}{5}x \times \frac{3}{8} = \frac{3}{10}x$

Fish at the end of two weeks $= \frac{4x}{5} - \frac{3x}{10} = \frac{8x-3x}{10} = \frac{5x}{10} = \frac{1}{2}x$

So fraction $= \frac{\frac{1}{2}x}{x} = \frac{1}{2}$

Q13. **(D)** Let the number $= x$

Then $\frac{3}{8} \times x = 10$

$\Rightarrow \quad x = \frac{80}{3}$

$\Rightarrow \quad x = 26.67 = \boxed{27}$

Q14. **(D)** Let the number $= x$

Then $\frac{15}{7} \times x = \frac{5}{8} \times 24$

$\Rightarrow \quad \frac{15 \times x}{7} = 15$

$\Rightarrow \quad x = \frac{7 \times 15}{15} = \boxed{7}$

Q15. **(D)** No. of boys $= 12$
No. of girls $= 18$
Total $= 12 + 18 = 30$

Required fraction $= \frac{12}{30} = \boxed{\frac{4}{15}}$

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Chapter 6

PERCENTAGE

Percentage:

The term 'percent' is a short form of the Latin word 'per centum' meaning 'out of hundred'. It can best be defined as:

> "A fraction whose denominator is 100 is called a percentage and the numerator of the fraction is called the rate percent."

A rate percent is reduced to an equivalent fraction dividing it by 100.

Change of percentage into Fraction or Decimal:

To convert a percentage to a fraction, mixed number or decimal, divide it by 100, and reduce, if possible. If necessary, the relating fraction may then be changed to a decimal.

Example:

(i) Express $2\frac{1}{7}\%$ to a fraction

(ii) Change $\frac{3}{4}\%$ to a decimal.

Solution:

(i) $2\frac{1}{7}\% = \frac{15}{7}\%$

$$= \frac{15}{7} \times \frac{1}{100}\left(\text{Replace}\% \text{ by } \frac{1}{100}\right)$$

$$= \frac{3}{140}$$

$$= \frac{3}{140}$$

(ii) $\frac{3}{4}\% = \frac{3}{4} \times \frac{1}{100}\left(\text{Replace}\% \text{ by } \frac{1}{100}\right)$

$$= \frac{3}{400} = .0075$$

Change of Fraction into Percentage:

To change a fraction or a mixed numbers to a percent.

a. Multiply the fraction or mixed number by 100%.

b. Reduce, if possible

c. Affix a % sign.

Example 2:

(i) Change $\frac{1}{80}$ to a percent.

(ii) Change 0.05 to a percent

Solution:

(i) $\frac{1}{80} = \frac{1}{80} \times 100\%$

$= 1.25\%$

(ii) $0.05 = 0.05 \times 100\%$

$$= \frac{5}{100} \times 100\%$$
$$= 5\%$$

Expressing One Quantity as a Percentage of Another:

To express one quantity "p" as a percentage of another quantity "q".

a. Write p as a fraction of q.

b. Multiply the fraction $\frac{p}{q}$ by 100% to convert it to a percentage.

Example 3:

There are 56 boys in a class of 140 students. What is the percentage of the boys?

Solution:

Total students $= q = 140$

Boys $= p = 56$

Fraction $= \frac{p}{q}$

$= \frac{56}{140}$

Percentage $= \frac{56}{140} \times 100\%$

$= 40\%$

Important Tip:

If a salary of a man is first increased by x% and then it has decreased x%, the change in its initial salary is less by x% of x or $\frac{x^2}{100}$.

Note:

If two values are respectively a% and b% more than a third value, then the first is $\frac{100+a}{100+b} \times 100$'s of the second.

Example 4:

Two numbers are respectively 20% and 50% more than a third, what percentage is the first to the second?

Solution:

Following the above, we have the value

$$= \frac{100+20}{100+50} \times 100\%$$
$$= \frac{120}{150} \times 100\%$$
$$= 80\%$$

Important Tip:

If the first value is r% more than the second value, then the second is $\left[\frac{r}{100+r} \times 100\right]$% less than the first value.

Example 5:

If Hamza's salary is 35% more than that of Osama, then how much percent is Osama's salary less than that of Hamza?

Solution:

Following the above theorem, we have the value

$$= \left[\frac{35}{100+35} \times 100\right]\%$$

$$= \left[\frac{35}{135} \times 100\right]$$

Important Tip:

If the first value is r% less than the second value, then the second is $\left[\frac{r}{100-r} \times 100\right]\%$ more than the first value.

Example 6:

If Maryam's salary is 25% less than that of Fatima, then how much percent is Fatima's salary more than that of Maria?

Solution:

Following the above theorem, we have

$$\left[\frac{25}{100-25} \times 100\right]\%$$

$$= 33\frac{1}{3}\%$$

Important Tip:

a% of a quantity is taken by the first, *b*% of the remaining is taken by the second and *c*% of the remaining is taken by the third person. Now if X is left then there was

$$\frac{X \times 100 \times 100 \times 100}{(100-x)(100-y)(100-z)}$$

in the beginning.

Example 7:

After deduction 20% from a certain sum, and then 30% from the remainder, there is 3500 left. Find the original sum.

Solution:

Following the above theorem, we have

$$= \frac{3500 \times 100 \times 100}{(100-20)(100-30)}$$

$$= \frac{3500 \times 100 \times 100}{80 \times 70}$$

$$= 6250$$

Model Examples

Example 8:

In an examination paper of 5 questions, 5 percent of the candidates answered all of them and 5 percent none of the rest, 25 percent answered only one question, and 20 percent answered only 4. If 24½ percent of the entire candidates answered only 2 questions and 200 candidates answered only 3, how many candidates appeared at the examination?

Solution: Let the total no. of candidates be x

$\frac{5x}{100}$ answered all the questions and $\frac{5x}{100}$ answered none.

The remaining candidates $= x - \left(\frac{5x}{100} + \frac{5x}{100}\right)$

$= \frac{9x}{10}$

No. of candidates answering only one question $= \frac{25}{100} \times \frac{9x}{10} = \frac{9x}{40}$

No. of candidates answering four questions $= \frac{20}{100} \times \frac{9x}{10} = \frac{9x}{50}$

No. of candidates answering two questions $= \frac{49}{200} \times x$

No. of candidates who answered three questions

$$= x - \left(\frac{5x}{100} + \frac{5x}{100} + \frac{9x}{40} + \frac{9x}{50} + \frac{49x}{200}\right) = 200$$

$$\Rightarrow x - \left(\frac{10x + 10x + 45x + 36x + 49x}{200}\right) = 200$$

$$\Rightarrow \frac{(200 - 50)x}{200} = 200$$

$$\Rightarrow \quad 50x = 40000$$

$$x = 800 \text{ Ans.}$$

Example 9:

The following table gives the number of the candidates (boys and girls) who appeared an examination. To complete the missing figure, find the number of candidates and their passing percentage.

Candidates	Appeared	Passed	Passing percentage *(correct to one decimal place)*
Boys	6720	3528	---
Girls	4750	---	62.4
Total	11470	---	---

Solution: Passing percentage of boys $= \frac{3528}{6720} \times 100$

$= 52.5\%$ **Ans.**

No. of girls passed $= \frac{62.4}{100} \times 4750$

$= 2964$ **Ans.**

Total no. of candidates passed $= 3528 + 2964$

$= 6492$ **Ans.**

Total pass percentage $= \frac{6492}{11470} \times 100$

$= 56.6\%$ **Ans.**

Example 10:

In 1990, the population of a town is given below:

Men	7640
Women	6675
Boys	5628

Girls	4872
Total	24815

In 1992, men increased by 5 percent, women by 8 percent and total population by 20 percent, and for every 7 boys there are 6 girls. Find the number of boy in 1992.

Solution: Increase in the no. of men in 1992. $= \frac{5}{100} \times 7640 = 382$

Total no. of men in 1992 $= 7640 + 382 = 8022$

Increase in the no. of women in 1992 $= \frac{8}{100} \times 6675 = 534$

Total no. of women in 1992 $= 6675 + 534 = 7209$

Increase in the total population in 1992 $= \frac{20}{100} \times 24815 = 4963$

Total population in 1992 $= 24815 + 4963 = 29778$

Total no. of boys and girls in 1992 $= 29778 - (8022 + 7209)$

$= 29778 - 15231$

$= 14547$

No. of boys $= \frac{7}{13} \times 14547$

$= 7833$ **Ans.**

Example 11:

Complete the following table which gives the enrolment in professional courses of one country in (1990-91) and (1995-96) and calculate the percentage increase in the first three totals of 1995-96 over those of 1990-91.

Solution:

Categories	1990-91		
	Boy	Girls	Total
Higher Secondary stage	57,676	9,511	67,187
Undergraduate stage	169,259	15,715	184,974
Postgraduate & Research	12,052	898	12,950
Total of all stages	238,987	26,124	265,111

Categories	1995-96		
	Boy	Girls	Total
Higher Secondary stage	93,760	14,585	108,345
Undergraduate stage	256,040	33,415	289,455
Postgraduate & Research	20,200	2,000	22,200
Total of all stages	370,000	50,000	420,000

Solution: Increase in no. of boys

$= 370,000 - 238987 = 131013$

% increase $= \frac{131013}{238987} \times 100$

$= 54.8\ \%$ **Ans.**

Increase in no. of Girls

$= 50,000 - 26124 = 23876$

% increase $= \frac{23876}{26124} \times 100$

$= 91.39\ \%$ **Ans.**

Total increase $= 420{,}000 - 265{,}111$

$= 154889$

% increase $= \dfrac{154{,}889}{265{,}111} \times 100$

$= 58.42\ \%$ **Ans.**

Example 12:

The civilian industrial products of a country are given below in million dollars. Complete the columns of 'increase over the preceding year' and 'percentage increase'.

Solution:

		Increase over the preceding year	*Percentage increase*
1951	27,003		
1952	29,314	2,311	8.60
1953	32,439	3,125	10.68
1954	36,335	3,896	12.00
1955	40,033	3,698	10.18

Example 13:

In an examination, 75% of candidates passed in English and 65% in mathematics while 15% failed both in English as well as mathematics. If 495 candidates passed in both the subjects, find the total number of candidates who took the examination.

Solution: Let the total number of students be 100

25 failed in English

35 failed in Mathematics

15 failed in both

No. of failed students $= 25 + 35 - 15 = 45$

No. of passed in both subjects $= 100 - 45 = 55$

If no. of passed students is 55, then total no. of students = 100

If no. of passed students is 495, total no. of students $= \dfrac{100}{55} \times 495$

$= 900$ **Ans.**

Example 14:

A candidate who gets 30 percent marks in an examination fails by 30 marks, but another candidate who gets 42 percent marks gets 42 marks more than that necessary for passing. Find the maximum number of marks and the percentage necessary for passing.

Solution: Let the passing marks $= x$

Now the candidate gets 30% marks and by this he gets 30 marks less than passing marks.

It means, he gets $(x - 30)$ marks

If he gets 30 marks, it means, maximum marks $= 100$

If he gets $(x - 30)$ marks, maximum marks $= \dfrac{100}{30}(x - 30)$ ...(*i*)

In the second case the candidate gets 42% marks such that he gets 42 marks above the passing marks.

It means, he gets $(x + 42)$ marks.

If he gets 42 marks, then maximum marks $= 100$

If he gets $(x + 42)$ marks, maximum marks $= \dfrac{100}{42}(x + 42)$ ...(*ii*)

As (*i*) and (*ii*) are equal because maximum marks are the same in both cases.

$$\frac{100}{30}(x-30) = \frac{100}{42}(x+42)$$

or $$\frac{x-30}{5} = \frac{x+42}{7}$$

or $$7x - 210 = 5x + 210$$

$$\Rightarrow \quad x = 210$$

Putting the value of x in (*i*) we get

Maximum marks $= \frac{100}{30}(210-30) \quad = 600$ **Ans.**

If maximum marks are 600 then passing marks = 210

If maximum marks are 100 then passing marks $= \frac{210}{600} \times 100 = 35\%$

$\therefore$ Passing percentage = 35 % **Ans.**

Multiple Choice Questions (MCQs)

Q1. If the base of a rectangle is increased by 40% and its altitude is decreased by 20%, then its area is:

(A) decreased by 20% (B) increased by 12%
(C) decreased by 12% (D) increased by 16%

Q2. If x% of y is 20, then y =

(A) $2000\,x$ (B) $\frac{100}{x}$
(C) $\frac{2000}{x}$ (D) $\frac{x}{200}$

Q3. 12 is $\frac{1}{3}$% of what number?

(A) 4 (B) 400
(C) 36 (D) 3600

Q4. If p is a positive number, 400% of p is what percent of 400 p?

(A) 4 (B) 25
(C) 40 (D) 1

Q5. What is 10% of 30% of 40%?

(A) 0.12% (B) 0.012%
(C) 12% (D) 1.2%

Q6. What percent of 75 is x?

(A) $\frac{3}{4} - x$ (B) $\frac{4}{3}x$
(C) $4x$ (D) $3x$

Q7. If 35 students took an exam and 13 of them failed, what percent of them passed?

(A) 20% approx (B) 63% approx
(C) 25% approx (D) 22% approx

Q8. There are twice as many boys as girls in an economics class. If 20% of the boys and 35% of the girls have already handed over their result cards, what percent of the students have not yet handed over their cards?

(A) 75 (B) 65
(C) 55 (D) 15

Q9. A dealer bought an ornamental jar for Rs. 7,000 and after some days sold it for Rs. 21,000. By what percent did the value of jar increase?

(A) 300 (B) 200
(C) 150 (D) 20

Q10. On a test consisting of 60 problems, Sonia solved 75% of first 40 problems correctly. What percent of the other 20 questions does she need to solve correctly for her grade on the entire exam to be 90%?

(A) 95% (B) 65%
(C) 85% (D) cannot achieve 90%

Q11. If 60% of A is 30% of B, then B is what percent of A?

(A) 300% (B) 30%
(C) 200% (D) 3%

Q12. What percent of p is q?

(A) $\frac{q}{p}$ (B) $\frac{q}{p}$
(C) $\frac{100q}{p}$ (D) $\frac{100p}{q}$

Q13. What percent of $\frac{1}{2}$ is $\frac{5}{4}$?

(A) 2.5 (B) 1.5
(C) 250 (D) 150

Q14. In a school of 820 students, 55% are boys. The number of girls and the number of boys are:

(A) 369 boys, 451 girls (B) 281 boys, 539 girls
(C) 539 boys, 281 girls (D) 451 boys, 369 girls

Q15. Jafer drew a square. He then erased it and drew a second square whose sides were 3 times the sides of the first square. By what percent was the area of the square increased?

(A) 300% (B) 800%
(C) 400% (D) 200%

Q16. A team has won 60 percent of the 20 games for all this season. If the team plays a total 50 games all season and wins 80 percent of the remaining games, how many games will the team win for the entire season?

(A) 36 (B) 25
(C) 42 (D) 39

Q17. Local telephone calls increased in price from 25Pa to 30Pa. What percentage increase was this?

(A) 15% (B) 25%
(C) 5% (D) 20%

Q18. A worker pays Rs. 350 tax per month, which is 15% of his income. What is his income?

(A) 3500 (B) 5250
(C) 2333.30 (D) 2523.30

Q19. If "x" is a positive number, 400% of x is what percent of $400x$?

(A) 1 (B) 0.1
(C) 0.01 (D) 100

Q20. Babar gave 15% of his baseball cards to Laeeq and 20% to Sarfraz. If he still had 520 cards, how many did he have originally?

(A) 800 (B) 720
(C) 820 (D) 600

Q21. A certain country has an infant mortality rate of 6.8% of 20000 babies born in a certain year, how many survived?

(A) 1360 (B) 18640
(C) 18000 (D) 17640

Q22. 20% of 50% of 80 is:

(A) 40 (B) 16
(C) 8 (D) 60

Q23. The price of a can of acid was increased by 20%. How many cans can be purchased for the amount of money that used to buy 300 cans?

(A) 250 (B) 320
(C) 150 (D) 240

Q24. In a basket containing 180 pears, 9 Pears are spoiled. What percent of the pears in the basket are not spoiled?

(A) 85% (B) 5%
(C) 95% (D) 9%

Q25. A silo (container for storing grain) is filled to capacity with p kilograms of wheat. Rats eats q kilograms a day. After 21 days, what percentage of the silo's capacity have the rats eaten?

(A) $\frac{21q}{30p} \times 100$ (B) $\frac{q}{p} \times 100$
(C) $\frac{21q - p}{p} \times 100$ (D) $2100\left(\frac{q}{p}\right)$

Q26. A factory normally employs 100 people. During a slow spell, it fired 20% of its employees. By what percentage must it now increase its staff to return to full capacity?

(A) 25% (B) 20%
(C) 80% (D) 40%

Q27. Six students in a class failed in geometry. This represents $16\frac{2}{3}$% of the class. How many students passed the course?

(A) 36 (B) 30
(C) 42 (D) 24

Q28. If 30% of all women are voters and 42% of the population are women, what percent of the population are women voters?

(A) 17.4% (B) 25.20%
(C) 12.60% (D) None of these

Q29. If the length of the rectangle is increased by 16% and the width is decreased by 25%, then the area:

(A) increases by 9% (B) decreases by 41%
(C) decreases by 13% (D) increases by 59%

Q30. If the base of a rectangle is increased by 40% and the altitude is decreased by 30%, the area is

(A) increased by 10% (B) increased by 12%
(C) decreased by 10% (D) decreased by 2%

Explanatory Answers

Q1. (B) If the value firstly increased by x% and then decreased by y% then there is $\left[x - y - \frac{xy}{100}\right]$% increase or decrease according as the sign +ve or –ve, respectively. In this problem, $x = 40$ and $y = 20$. Therefore

$$\left[40 - 20 - \frac{(40)(20)}{100}\right]\%$$

$$\left[20 - \frac{800}{100}\right]\%$$

$[20 - 8]\% = 12\%$

Because sign is +ve therefore its area is increased by 12%.

Q2. (C) $y \times \frac{x}{100} = 20$

$\Rightarrow \quad xy = 20 \times 100 \quad \Rightarrow xy = 2000$

$\Rightarrow \quad y = \frac{2000}{x}$

Q3. (D) Using, $\frac{\text{Part}}{\text{Whole}} = Y$ percent, here $P = 12$, W = ? and Y percent $= \frac{1}{300}$

$\frac{P}{W} = \frac{Y}{100} \quad \Rightarrow \quad \frac{P}{W} = Y \times \frac{1}{100}$

$\frac{12}{W} = \frac{1}{3} \times \frac{1}{100} \quad \Rightarrow \quad W = 3 \times 1200 = 3600$

Q4. (D) 400% of $p = \frac{400}{100} \times p = 4p$, which is 1% of 400 p.

Q5. (D) 30% of 40% $= \frac{30}{100} \times \frac{40}{100} = \frac{12}{100} = 0.12$

Now 10% of 30% of 40% $= \frac{10}{100} \times 0.12 = 0.012 = 1.2$

Q6. (B) $\frac{P}{W} = \frac{y}{100} \quad \Rightarrow \quad x = \frac{y}{100} \times 75$

$\Rightarrow \quad x = \frac{3y}{4} \quad \Rightarrow \quad y = \frac{4x}{3}$

Q7. (B) If 13 students failed, then the number of passed students = 35 – 13 = 22

Thus, $\frac{22}{35} \times 100 = 63\%$ approx

Q8. (A) Let the number of girls = 100, then

Number of boys = 200

Then 35 girls (35% of 100) and 40 boys (20% of 200), have handed in their cards. Hence 75 of the 300 (100 + 200) students have handed them in. It means that 300 – 75 = 225 have not handed them in. Thus

$\frac{225}{300} \times 100 = 75\%$

Q9. (B) The increment in the value of the jar = Rs. 21000 – Rs. 7000 = Rs. 14000

The %age increase in the value of the jar

$$= \frac{\text{Increment}}{\text{Actual}} \times 100$$

$$= \frac{14000}{7000} \times 100 = 200\%$$

Q10.(D) To achieve 90% grade on the entire examination, Sonia needs 54 (as calculated below) problems:

$$\boxed{\frac{P}{W} = y\% \Rightarrow \frac{P}{60} = \frac{90}{100} \Rightarrow P = \frac{90}{100} \times 60 \quad \Rightarrow P = 54}$$

to solve correctly. So far she has solved 30 $\left(\frac{P}{40} = \frac{75}{100} \Rightarrow P = \frac{75}{100} \times 40 = 30\right)$ problems correctly. Therefore, on the last 20 problems she needed 54 – 30 = 24 correct answers, which is impossible to get from 20 problems.

Q11.(C) 60% of A is 30% of B, *i.e.*, $\frac{60}{100}A = \frac{30}{100}B$.

$\Rightarrow .60A = .30B, \Rightarrow B = \frac{.60}{.30}A \Rightarrow B = 2A$

Now we find B is what percent of A. *i.e.*,

$B = \frac{x}{100}A$ or $B = (x\%)(A)$

$\Rightarrow \quad B = (200\%)A$

Q12.(A) Using the relation $\frac{\text{Part}}{\text{Whole}} = y\%$

$$\frac{q}{p} = y\%$$

Second Method: What % p is q

x % p = q

$\Rightarrow x\% = \frac{q}{p}$

Q13.(C) Using $\frac{\text{Part}}{\text{Whole}} = y\%$

$$\frac{5}{4} \div \frac{1}{2} = y\%$$

$$\frac{5}{4} \times 2 = y\% \quad \Rightarrow y\% = \frac{5}{2} = 2.5$$

$$\Rightarrow y\% = 250\%$$

Q14. (D) Total no of students = 820

No. of boys $= 820 \times \frac{55}{100} = 451$ boys

No. of girls $= 820 - 451 = 369$ girls

Q15. (B)

Let the length of first square = 1 inch

Then Area of first square = 1 square inch

Then sides of the second square = 3 inch

Area of the second square = 9 square inch

∴ Increase in the area of the 2nd square = 8 square inches

%age increase in the second square = $\boxed{800\%}$

Q16. **(A)** Total No. of games that

the team has won so far $= \frac{60}{100} \times 20 = 12$ games

The total number of games left $= 50 - 20 = 30$

80% of 30 games will the team win

$\therefore \quad \frac{80}{100} \times 30 = 24$ games

The total number of wins $= 12 + 24 = \boxed{36}$

Q17. **(D)** Increase in local call $= 30 - 25 = 5$Pa

% increase $= \frac{5}{25} \times 100 = 20\%$

Q18. **(C)** Let "x" be his income then

15% of $x = 350$

$x = 350 \div 15\% = 350 \times \frac{100}{15}$

$x = 2333.33$

Q19. **(A)** 400% of $x = 4x$. Which is 1% of $400x$.

Q20. **(A)** Actually, Babar had 100% of the cards. After distributing 35% (20% + 15%) of them, he had 100% – 35% = 65% of them left. So

$$520 = \frac{65}{100}x \Rightarrow x = \frac{520 \times 100}{65} = \boxed{800}$$

Q21. **(B)** Infant mortality $= 20000 \times \frac{6.8}{100} = 1360$

survived bodies $= 20000 - 1360 = \boxed{18640}$

Q22. **(C)** 50% of 80 $= 80 \times \frac{50}{100} = 40$

20% of 40 $= 40 \times \frac{20}{100} = \boxed{8}$

Q23. **(A)** Let the can of acid used to cost Rs. 1

After increasing 20% cost it became $= 1 + \frac{1}{20} = 1.20$

Then 300 cans of acid used to cost = Rs. 300

Each can be bought for Rs 300

$\therefore 300 \div 1.20 = \boxed{250}$

Q24. **(C)** The pears that are not spoiled $= 180 - 9 = 171$

Percentage $= \frac{171}{180} \times 100$

$= 19 \times 5$

$= 95\%$

Q25. **(D)** After 21 days the rats have eaten wheat = 21q kilograms.

So, the required fraction in percentage $= \frac{21q}{p} \times 100$

$$= 2100\left(\frac{q}{p}\right)$$

Q26. (A) 20% of 100 = 20 employee

employees left = 100 – 20 = 80 employees

If it again increases by 20, the percentage of increase

$$= \frac{20}{80} \times 100 = 25\%$$

Q27. (B) Let x be the number of students, then

$$16\frac{2}{3}\% = \frac{1}{6} \Rightarrow \frac{1}{6}x = 6$$

$$\Rightarrow x = 36$$

36 students in class, 6 failed, 30 passed

Q28. (C) 30% of the 42% of the population who are women are voters so

(.30)(.42) = 0.126 = 12.60% of the population are women voters.

Q29. (C) Let L be the original length and W be the original width.

The new length = 100% + 16% = 116% of L $\Rightarrow$ $\boxed{1.16L}$

Since the width decreases by 25% so the new width is 75% of W $\Rightarrow$ .75W

Area = LW

$\Rightarrow$ New Area = (1.16)(.75)LW

= 0.87 LW = 87% of Area

Since the area is 87% of the original area. Thus the area has decreased by (100 – 87) = 13%

Q30. (D) Let "b" be the base and "a" altitude. Then the new base will be (b + 0.4b). The new altitude after decreasing 30% is (a – 0.3a).

So the area is

((a – .3a)(b + 0.4b) = (0.7)(1.4)ab

= 0.98ab

The new area is 42% of the old. So the new area (98% – 100%)

= –2% is decreased by 2%

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Chapter 7

RATIO AND PROPORTION

RATIO:

The number of times one quantity contains another quantity of the same kind is called the ratio of the two quantities.

Note: The ratio of two quantities is equivalent to the fraction that one quantity is to the other.

Example: There can be ratio between Rs. 30 and Rs. 40, but there can be no ratio between Rs. 30 and 40 apples.

Remember: The ratio 3:5 is written as 3:5 or $\frac{3}{5}$, 3 and 5 are called the terms of the ratio. 3 is the first and 5 is the second term.

Note: The first term of a ratio is called the antecedent and the second the consequent.

If a set of objects is divided into two groups in the ratio $a : b$, then the first group contains $\frac{a}{a+b}$ of the total objects. The second group contain $\frac{b}{a+b}$ of the total number of objects.

Important Example:

If a bag containing twelve mirrors is dropped, which of the following cannot be the ratio of the broken mirrors to unbroken mirrors?

(i) 2: 1 ii) 3:1 iii) 3:2 iv) 1:1 v) 7:5

Solution:

Since there are 12 mirrors is the bag. So 12 must be divisible by the sum of terms in the ratio exactly. We see that 2+1=3 divides 12 exactly 3+1=4 also divides exactly. Only the ratio 3+2=5 doesn't divide 12 exactly. Thus the correct answer is (iii)

PROPORTION:

The equality of ratios is called proportios.

Example:

Consider the two ratios

Ist ratio	2nd ratio
5:15	7:21

Since 5 is one-third of 15, and 7 is one-third of 21, the two ratios are equal.

Note: The first and fourth terms are called extremes, and the second and third terms, are called the means. In above example 5 and 21 are extremes, while 15 and 7 are means.

Important Points:

1. If four quantities be in proportion, the product of the extremes is equal to the product of the means.
2. Three quantities of the same kind are said to be in continued proportion when the ratio of the first to the second is equal to the ratio of the second to the third.

Aid to Memory:

The mean proportional between two numbers is equal to the square root of their product.

Example: Find

i). Fourth proportions to 5, 10, 5

ii) Third proportion of 5 and 10.

iii) Mean proportions between .04 and 0.09.

Solution:

(i) Let $5 : 10 :: 5 : x$

Then $5x = 10 \times 5 \Rightarrow 5x = 50 \Rightarrow \boxed{x = 10}$

(ii) Let $5 : 10 :: 10 : x$

Then $5x = 10 \times 10 \Rightarrow 5x = 100 \Rightarrow \boxed{x = 20}$

(iii) Mean proportion between .04 and .09

$$= \sqrt{.04 \times .09} \quad = \sqrt{.0036}$$

$$= \sqrt{\frac{36}{1000}} \quad = \frac{6}{100} = 0.06$$

Direct Proportion:

If the given two quantities are so related to each other that if one of them is multiplied (or divided) by any number, the other is also multiplied (or divided) by the same number.

Inverse Proportion:

If two quantities are so related that if one of them is multiplied by any number, the other is divided by the same number.

Example:

1. If 5 balls cost Rs. 7, what do 15 balls cost?

Solution: This example is an illustration of direct proportion. Therefore, setting a proportion.

$$\therefore \quad 5 : 7 :: 15 : x$$

$$\Rightarrow 5x = 15 \times 7 \Rightarrow x = \frac{15 \times 7}{5} = 21$$

2. If 5 men can build a house in 28 days, in how many days will 10 men build it?

Solution:

This example is an illustration of inverse proportion. Here, if we increase number of men.

2, 3, 4 …… times, the number of days will be decreased.

2, 3, 4…….. times, Thus the inverse ratio of the number of men is equal to the ratio of the corresponding number of days.

$$\therefore \quad \frac{1}{15} : \frac{1}{10} :: 28 : x \text{ days}$$

$$\Rightarrow \quad x = \frac{1}{5} = \frac{1}{10} \times 28$$

$$\Rightarrow \quad x = \frac{28 \times 15}{10} = 42 \text{ days.}$$

DOUBLE RULE OF THREE:

Example: If 8 men can reap 80 hectares in 24 days, how many hectares can 36 men reap in 30 days.

Solution: We resolve this problem in two parts.

Ist Part: If 8 men can reap 80 hectors, how many hectares can 36 men reap.

Setting a proportion

8 men : 36 men : : 80 hectares : x hecter

$$x = \frac{36 \times 80}{8} = 360 \text{ hectres}$$

2nd Part: If 360 hectares can be reaped in 24 days, how many hectares can be reaped in 30 days?

24 days : 30 days = 360 hectares : x hector

$$x = \frac{360 \times 30}{24} = 450$$

SINGLE STEP:

8 men : 36 men 24 days : 30 days	: : 80 hectare : x hector

Required No. of hectares : $= \dfrac{\text{Multiplication of means}}{\text{Multiplication of Ist terms}}$

$= \dfrac{80\times36\times30}{8\times24} = 450$

Model Examples:

Example 1: Three liquids contain petrol and spirit mixed in the ratio 2 : 3, 3 : 4 and 4 : 5, respectively. A motor owner mixes 20 litres of the first, 21 litres of the second and a few litres of third. If the ratio of petrol to spirit in the mixture is 29 : 39, find the number of litres of the third liquid taken for the mixture.

Solution: 20 litres of the first liquid has 2/5 × 20

= 8 litres of petrol and 12 litres of spirit

21 litres of the second liquid have 3/7 × 21

= 9 litres of petrol and 12 litres of spirit.

Suppose x litres of the third liquid are taken, it will have $\frac{4x}{9}$ litres of petrol and $\frac{5x}{9}$ litres of spirit.

$\therefore$ Total petrol in the mixture $= 8 + 9 + \frac{4x}{9}$ litres

and total spirit $= 12 + 12 + \frac{5x}{9}$ litres.

$$\text{Ratio of these} = \frac{17+\frac{4x}{9}}{24+\frac{5x}{9}} = \frac{29}{39}$$

$\therefore$ $663 + \frac{156}{9}x = 696 + \frac{145}{9}x$

or $\left(\frac{156}{9} - \frac{145}{9}\right)x = 696 - 663$

or $\frac{11}{9}x = 33$

or $x = \frac{9}{11}\times 33 = 27$ litres.

Example 2: In a regiment the number of officers to men was 3:31 before the battle. In the battle, 6 officers and 22 men were killed and the ratio become 1:13. Find the number of officers and men in the regiments.

Solution: In the beginning *i.e.*, before the battle

Let no. of officers $= x$

// // men $= y$

then $x : y :: 3 : 31 \Rightarrow \frac{x}{y} = \frac{3}{31}$ .....(*i*)

After the battle

No. of officers $= x - 6$

// // men $= y - 22$

Then $\frac{x-6}{y-22} = \frac{1}{13}$ .....(*ii*)

From (*i*) we get $x = \frac{3}{31}y$

Substituting this value in (*ii*), we get

$$\frac{\frac{3}{31}y-6}{y-22}=\frac{1}{13} \quad .....(iii)$$

Solving for y in (*iii*), we get

$$y = 217$$

$$x = \frac{3}{31}\times 217 = 21$$

$\therefore$ No. of officers = 21
No. of men = 217 } **Ans.**

Example 3: Of two kinds of alloy, silver and copper are contained in one in the ratio of 5 : 1 and in the other in the ratio of 7 : 2. What weights of the two alloys should be melted and mixed together so as to make up a 5 lb mass with 80% of silver?

Solution: Let the alloys taken be in the ratio of $6 : 9x$

So in 1st alloy wt. of silver = 5
// // // copper = 1
in 2nd alloy wt. of silver = $7x$
// // // copper = $2x$
$\therefore$ wt. of silver = $5 + 7x$
and Total wt. = $9x + 6$

But $$(5+7x)\frac{100}{80} = 9x+6$$

$$25 + 35x = 36x + 24$$

$\therefore$ $x = 1$

So alloys are taken in 6 : 9 ratio.
Total wt. of 2nd alloys = 5 lb

// // 1st alloy $= \frac{6}{15}\times 5 = 2$ lb
// // 2nd // $= \frac{9}{15}\times 5 = 3$ lb } **Ans.**

Example 4: An alloy contains copper and zinc in the ratio of 5 : 3 and another alloy contains copper and tin in the ratio 8 : 5. If equal weights of both the alloys are melted together find the weight of tin in the resulting alloy per kg.

Solution: Let weight of both alloys be taken to be 13 kg.

$\therefore$ wt. of resulting alloy = 26 kg.
In 2nd alloy in 13 kg. wt. of tin = 5 kg.
$\therefore$ in resulting alloy of
26 kg. wt. of tin = 5 kg.
$\therefore$ wt. of tin per kg. of resulting alloy $= \frac{5}{26}$ kg.

Multiple Choice Questions (MCQs)

Q1. In a city 90% of the population own a car, 15% own a motorcycle, and everybody owns one or the other or both. What is the percentage of motorcycle owners to who own cars?

(A) 15% (B) 5%
(C) 75% (D) $33\frac{1}{3}\%$

Q2. Concrete consists of cement, sand and screenings in the ratio of 1 : 5 : 4, what is the percentage of the sand mixed?

(A) 10% (B) 40%

(C) 50% (D) 60%

Q3. **Three business partners shares have profit of Rs. 24000 in the ratio 5 : 4 : 3. What is the amount of the least share?**

(A) 6000 (B) 8000
(C) 10,000 (D) 1200

Q4. **A machine produces 1280 parts in 16 hours. How many parts would it make in a working week of 44 hours?**

(A) 2530 (B) 3520
(C) 2122 (D) 3960

Q5. **If the ratio of x and y is $\frac{11}{3}$, what is the value of $2x$ to y?**

(A) $\frac{11}{6}$ (B) $\frac{22}{6}$
(C) $\frac{22}{3}$ (D) $\frac{11}{5}$

Q6. **If 80% application to a program were rejected, what is the ratio of the number accepted to the number rejected?**

(A) 1 : 4 (B) 4 : 1
(C) 1 : 8 (D) 3 : 8

Q7. **What is the ratio of the circumference of a circle to its radius?**

(A) π (B) $\frac{\pi}{2}$
(C) $2\pi r$ (D) 2π

Q8. **Win/Loss ratio for two teams are A, 5 : 2 and B, 7 : 3 which team has the better record?**

(A) A (B) B
(C) both A and B (D) wrong question

Q9. **If 15 workers can paint a certain number of houses in 24 days, how many days will 40 workers take, working at the same rate, to do the same job?**

(A) 12 days (B) 18 days
(C) 15 days (D) 9 days

Q10. **If a jet travels 1280 km in 2 hours, how far will it travel in $5\frac{1}{2}$ hours, at the same speed?**

(A) 2100 (B) 3300
(C) 2700 (D) 3520

Q11. **If the ratio of $a : b$ is 9 : 7 then $a + b$ is:**

(A) 14 (B) 16
(C) 63 (D) not possible

Q12. **If you can buy A apples for n nickels (five cent coin), how many apples can you buy for d dimes and q quarters?**

(A) $\frac{A(d+q)}{n}$ (B) $\frac{A}{n}(10d+25q)$
(C) $\frac{A}{n}(2d+5q)$ (D) $\frac{d+q}{An}$

Q13. **If the ratio of boys and girls in a class is 3 : 5 and the class contains 24 students, how many additional boys would have to enroll to make the ratio of boys to girls 1:1?**

(A) 9 (B) 15
(C) 6 (D) 12

Q14. **A recipe requires 13 gram of sugar and 18 gram of flour. If only 100gram of sugar is used, how much flour, to the nearest gram, should be used?**

(A) 16 (B) 13
(C) 14 (D) 17

Q15. **Green paint is obtained from blue and yellow paint in the ratio 3 : 5. How much of each colour is**

needed to make 40 litres of this green paint?

(A) Blue paint 15 litres, yellow paint: 25 litres
(B) Blue paint: 25 litres, yellow paint: 15 litres
(C) Blue paint: 10 litres, yellow paint: 30 litres
(D) Blue paint: 13 litres, yellow paint: 27 litres

Explanatory Answers

Q1. (D) Let x stand for the percentage who own both a car and a motorcycle. Then
(The %age who own a motorcycle) + (The %age who own a car) – (The %age who own one or the other or both) = 100% own one or other or both.

$\therefore \quad 15\% + 90\% - A = 100\%$

$\Rightarrow \quad 105\% - A = 100\% \Rightarrow \boxed{A = 5\%}$

The %age of motorcycle owners to who own car is

$$= \frac{5\%}{15\%} = \frac{1}{3} = \boxed{33\frac{1}{3}\%}$$

Q2. (C)

Ratio $= 1 : 5 : 4$

Sum of ratio $= 1 + 5 + 4 = 10$

Sand $= \frac{5}{10} \times 100 = 50\%$

Q3. (A)

Ratio $= 5 : 4 : 3$

Sum of ratio $= 5 + 4 + 3 = 12$

least share $= \frac{3}{12} \times 24000$

$= \text{Rs. } 6000$

Q4. (B) Let "x" be the number of parts in 44 hours

Then $16 : 1280 :: 44 : x$

$$\Rightarrow \quad \frac{16}{1280} = \frac{44}{x} \Rightarrow x = \frac{44 \times 1280}{16}$$

$$x = \boxed{3520}$$

Q5. (C) The ratio of x to y can be written as $\frac{x}{y}$. The ratio of x to y is $\frac{11}{3}$, which can be written as

$$\frac{x}{y} = \frac{11}{3}$$

If $\frac{x}{y} = \frac{11}{3}$, then $2\left(\frac{x}{y}\right) = 2\left(\frac{11}{3}\right)$

$$\frac{2x}{y} = \boxed{\frac{22}{3}}$$

Q6. (A) Since 80% of the application were rejected. Therefore, 20% = (100% – 80%) were accepted, the ratio of accepted to rejected is

$20\% : 80\% = 1 : 4$

Q7. (D) The ratio of the circumference to the diameter of the circle is π. Therefore

$$\pi = \frac{C}{d} \Rightarrow \frac{C}{2r} \Rightarrow 2\pi = \frac{C}{r}$$

Q8. (A)

A	B
$5 : 2$	$7 : 3$
$= \frac{5}{2} : 1$	$= \frac{7}{3} : 1$
$2.5 : 1$	$= 2.3 : 1$

Team A has the better record.

Q9. **(D)** Clearly, the more workers are there, the less time will be required, therefore, $15 : 40 :: \frac{1}{24} : \frac{1}{x}$

$$\Rightarrow \frac{15}{40} = \frac{x}{24} \quad \Rightarrow x = \frac{15 \times 24}{40} = 9 \text{ days}$$

Q10. **(D)** It's a direct variation question

$$1280 : 2 :: x : \frac{11}{2}$$

$$\frac{1280}{2} = \frac{x}{11/2} \quad \Rightarrow 2x = \frac{1280 \times 11}{2}$$

$$\Rightarrow x = 3520 \text{ km}$$

Q11. **(D)** In this question if a is 18 and b is 14, then the ratio a : b is 9 : 7 but a + b = 32. The point in this question that a and b can take on many possible values. It is not possible here to establish one definite value for the sum of a and b.

Q12. **(C)** $\frac{\text{A apples}}{\text{n nickels}} = \frac{\text{A apples}}{\text{5n cents}} = \frac{\text{x apples}}{(10d + 25q)\text{cents}}$

$$\Rightarrow \frac{A}{5n} = \frac{x}{10d + 25q} \quad \Rightarrow 5nx = A(10d + 25q)$$

$$\Rightarrow \quad x = \frac{A5(2d + 5q)}{5n}$$

$$\Rightarrow \quad x = \frac{A}{n}(2d + 5q)$$

Q13. **(C)** Given ratio 3 : 5 of boys and girls. Total number of students in the class is 24.

$$\text{Number of boys} = \frac{3}{8} \times 24 = 9 \text{ boys}$$

$$\text{Number of girls} = \frac{5}{8} \times 24 = 15 \text{ girls}$$

In order of have same number of boys and girls, 6 additional boys would have to enroll.

Q14. **(B)** This is a direct proportion, because the more sugar, the more flour

$$\frac{13}{18} = \frac{10}{x}$$

$$13x = 180$$

$$\Rightarrow x = 13\frac{11}{13}$$

Q15. **(A)**

The ratio 3 : 5 gives (3 + 5) = 8 parts

$$\text{Blue paints} = \frac{3}{8} \times 40 = 15 \text{ litres}$$

$$\text{Yellow paints} = \frac{5}{8} \times 40 = 25 \text{ litres}$$

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Chapter 8

AVERAGE

In Mathematics, average is a representative of a number of given quantities. Average is of several kinds.

METHOD OF FINDING AVERAGE

To find average of any number of quantities of the same kind is to add all the items together and then divide the sum by the number of items.

$$\therefore \quad \text{Average} = \frac{\text{Sum of all the items}}{\text{No. of items}}$$

Model Examples.

Example 1: The average daily temperature from 9th January to 16th January (both inclusive) was 38.6° and that from the 10th to 17th January (inclusive) was 39.2°. What was the temperature on 17th January?

Solution: Total temp. from 9th Jan. to 16th Jan.

$= 38.6 \times 8°C$

$= 308.8°C$

Since the temp. on 9th $= 34.6°C$

$\therefore$ Total temp. from 10th Jan. to 16 Jan.

$= 308.8 - 34.6$

$= 274.2°C$

Total temp. from 10 to 17th Jan.

$= 39.2 \times 8°C$

$= 313.6°C$

$\therefore$ Temp on 17th Jan. $= 313.6 - 274.2$

$= 39.4°C$

Example 2: A goods train in five successive minutes from its start runs 68 metres, 127 metres, 208 metres, 312 metres and 535 metres and for next five minutes in maintains average speed of 33 km/hr. Find the whole distance covered and the average speed of train in km/hour.

Solution: Distance covered in first five minutes.

$$= \frac{68+127+208+312+535}{1000} \text{ kms.}$$

$$= \frac{5}{4} \text{ kms.}$$

Now average speed for next five minutes

$= 33$ km/hr.

$\therefore$ Distance covered in next five minutes $= \frac{33 \times 5}{60} = \frac{11}{4}$ km

Total distance covered in 10 minutes $= \frac{5}{4} + \frac{11}{4} = \frac{16}{4}$

$= 4$ kms. **Ans.**

Average speed $= \frac{4}{10}$ km/min.

$$= \frac{4}{10} \times 60$$

$= 24$ km/hr. **Ans.**

Example 3: The average salary per head of all the workers of an institution is Rs. 60. The average salary per head of 12 officers is Rs. 400. The average salary per head of the rest is Rs. 56. Find the total no. of workers in the institute.

Solution: Let the total No. of workers $= x$

$\therefore$ Total salary drawn $= 60x$ .....(*i*)

Salary of 12 officers $= 12 \times 400 = 4800$ Rs.

// // // // the rest $= (x - 12) \times 56$

Hence total salary of the workers

$= (x - 12)56 + 4800$ .....(*ii*)

Equating (*i*) and (*ii*)

$60x = 4800 + 56x - 672 \quad \Rightarrow \quad 4x = 4128$

$\Rightarrow \quad x = 1032$ **Ans.**

Example 4: On a journey across Karachi the overage speed of a taxi 20 m.p.h. for 70% of the distance, 25 m.p.h. for 10% of it and 8 m.p.h. for the remainder. Find the average speed for the whole journey.

Solution: Let the distance be 100 miles.

Time taken for 70% journey at 20 m.p.h. $= \frac{70}{20} = 3.5$ hrs.

Time taken for 10% journey at 25 m.p.h. $= \frac{10}{25} = 2/5$ hrs.

Total taken for 20% journey at 8 m.p.h. $= \frac{20}{8} = 2.5$ hrs.

Total time taken $= 3.5 + \frac{2}{5} + 2.5 = 6\frac{2}{5}$ hrs.

$\therefore$ Average speed $= \frac{100}{\frac{32}{5}} = \frac{125}{8}$

$= 15.625$ m.p.h. **Ans.**

Example 5: A batsman has a certain average of runs for 16 innings. In the 17th innings, he makes a score of 85 runs there by increasing his average by 3. What is the average of the 17th inning.

Solution: To increase the average by 3 runs he has to make $17 \times 3 = 51$ runs more than the average of previous innings.

$\therefore$ Average of 16 innings $= 85 - 51 = 34$

// // 17 // $= 34 + 3 = 37$ **Ans.**

Example 6: A motorist set out at 10 a.m. to travel from Lahore to Gujrat, suppose a distance of 80 miles. He estimated that he could maintain an average speed of 25 m.p.h. For the first 44 miles from Lahore to Gujranwala his speed, averaged 30 m.p.h. but afterwards he was delayed by traffic and reached Gujrat 24 minutes later than the estimated time. Calculate:

(i) His time of arrival in Gujrat

(ii) His average speed from Gujranwala to Gujrat

Solution: Total distance from Lahore to Gujrat = 80 miles.

Average speed estimate from Lahore to Gujrat = 25 m.p.h.

Estimated time taken from Lahore to Gujrat $= \frac{80}{25} = \frac{16}{5}$ hours

But he was late by $\frac{24}{60}$ hours.

$\therefore$ Total actual time taken $= \frac{16}{5} + \frac{24}{60}$

$= \frac{18}{5}$ hours

$=$ 3 hours 36 minutes.

He starts at 10:00 A.M. and will reach there at Gujrat at 1:36 P.M.

Now distance from Lahore to Gujranwala $=$ 44 miles

Average speed from Lahore to Gujranwala $=$ 30 m.p.h.

Time taken from Lahore to Gujranwala $= \frac{44}{30}$ hours

$= \frac{22}{15}$ hours

Now distance from Gujranwala to Gujrat $= 80 - 44$

$=$ 36 miles.

Time taken from Gujranwala to Gujrat $= \frac{18}{5} - \frac{22}{15}$

$= \frac{32}{15}$ hours

$\therefore$ Average speed from Gujranwala to Gujrat $= \frac{36}{\frac{32}{15}}$ miles/hr.

$=$16.9 miles per hour (app.) **Ans.**

Multiple Choice Questions (MCQs)

Q1. The average of even integers from 2 to 100 inclusive is:

(A) 49 (B) 52

(C) 51 (D) 50

Q2. What is the average of first hundred natural numbers?

(A) 50 (B) 50.5

(C) 49.5 (D) 100

Q3. What is the average of x, y and z? If $x + y = 5$, $y + z = 8$ and $x + z = 11$.

(A) $\frac{11}{3}$ (B) $\frac{1}{2}$

(C) $\frac{13}{5}$ (D) 4

Q4. The average of five numbers is 54. If three of the numbers are 26, 28 and 30, what is the average of the other two?

(A) 91 (B) 93

(C) 54 (D) 186

Q5. Which of the following is the average of $x^2 - 16$, $39 - x^2$ and $3x + 10$?

(A) $x + 3$ (B) $2x + 13$

(C) $x + 11$ (D) $\frac{x + 11}{3}$

Q6. 8 students in a class obtained 60%, 3 obtained 75%, 2 obtained 80% and 7 obtained 45% in a class test. What is the average marks?

(A) 49% (B) 59%

(C) 29% (D) 51%

Q7. The average number of goals a team has scored in 7 matches is 8. They averaged 10 goals for the first 3 matches and they scored 5 goals in each of the next two matches. What is the average score of the last two matches?

(A) 5 goals (B) 4 goals

(C) 6 goals (D) 8 goals

Q8. If the mean (average) of 6 numbers is 4.5. What is the sum of the numbers?

(A) 0.75 (B) 10.5

(C) 12 (D) 27

Q9. A worker is paid R rupees per hour for the first 8 hours daily. For every hour after the first 8 hours, she is paid S rupees per hour. If she works 12 hours in one day, what is her average hourly for the day?

(A) 8R + S (B) $\frac{8R + 4S}{4}$

(C) $\frac{12R - 8S}{4}$ (D) $\frac{2R + S}{3}$

Q10. Asim had an average of 60 on his first four math tests. After taking the next test, his average dropped to 58. Find his recent test grade.

(A) 40 (B) 50

(C) 48 (D) 32

Q11. If a + b = 8, b + c = 9, and c + a = 11, what is the average of a, b and c?

(A) $\frac{14}{3}$ (B) $\frac{28}{3}$

(C) $\frac{14}{6}$ (D) $\frac{7}{3}$

Q12. If the average of 3,5, 10 and S is 6, what is the value of S?

(A) 4 (B) 6

(C) 12 (D) 0

Q13. What is the average of 3^{10}, 3^{20} and 3^{30}?

(A) 3^{59} (B) $3^9 + 3^{19} + 3^{29}$

(C) 3^{57} (D) $3^{11} + 3^{21} + 3^{31}$

Q14. If $20x + 20y = 70$, what is the average of x and y?

(A) $\frac{7}{2}$ (B) 7

(C) $\frac{7}{4}$ (D) $\frac{4}{7}$

Q15. Which of the following is the average of $x^4 - 20$, $40 - x^4$, and $3x + 4$?

(A) $x^4 - 24$ (B) $x + 8$

(C) $x^4 + 3x + 24$ (D) $x + 24$

Explanatory Answers

Q1. (C) As sum of the first n even numbers $= n(n+1)$
Now, the sum of even numbers from 2 to 100 is
$2 + 4 + 6 + 8 + \ldots\ldots\ldots + 100$ (or 50 even number)

$$= 50(50+1) = 2550$$

$$\text{Average} = \frac{\text{Sum of numbers}}{\text{Number of terms}}$$

$$= \frac{2550}{50} = 51$$

Q2. (B) The first 100 natural numbers are $\{1,2,3,\ldots\ldots,100\}$

Now, sum of all the first n numbers $= \frac{n(n+1)}{2}$

$$\text{Sum of first 100 natural numbers} = \frac{100(100+1)}{2}$$

$$= 5050$$

$$\text{Now, average} = \frac{\text{Sum of numbers}}{\text{Number of terms}}$$

$$= \frac{5050}{100} = 50.5$$

Shortcut: The average of first "n" natural number is $\frac{n+1}{2}$

Thus, average $= \frac{100+1}{2} = \frac{101}{2} = 50.5$

Q3. (D) Adding the given three equations:

$$(x+y) + (y+z) + (z+x) = 5 + 8 + 11$$
$$2x + 2y + 2z = 24$$
$$2(x+y+z) = 24$$

Dividing both sides by 2

$$x + y + z = 12$$

Now average of x, y and z is

$$\frac{x+y+z}{3} = \frac{12}{3} = 4$$

Q4. (B) Let the missing numbers be a and b, then by given condition,

$$\frac{a+b+26+28+30}{5} = 54$$

$$a + b + 84 = 270 \text{ (Multiplying both sides by 5)}$$
$$a + b = 186$$

Hence average of a and b is

$$\frac{a+b}{2} = \frac{186}{2} = 93$$

Q5. (C) $\text{Average} = \frac{\text{Sum of the terms}}{\text{No. of terms}}$

$$= \frac{x^2 - 16 + 39 - x^2 + 3x + 10}{3}$$

$$= x + 11$$

Q6. (B)
8 students with 60%, total = 480 marks
3 students with 75%, total = 225 marks
2 students with 80%, total = 160 marks
7 students with 45%, total = 315 marks
$\therefore$ 20 students obtain a total = 1180 marks

$\therefore$ Average $= \frac{1180}{20} = 59\%$

Q7. (D) Total goals for 7 matches $= 7 \times 8 = 56$
Total goals for 3 matches with average score of $10 = 30$
Total goals for 2 matches with average score of $5 = 10$
$\therefore$ Total goals for remaining 2 matches $= 56 - 30 - 10$
$= 16$

Average goals in last two matches $= \frac{16}{2}$
$= 8$ goals

Q8. (D) Average of 6 numbers $= \frac{\text{Sum of numbers}}{6}$

$\Rightarrow$ Sum of the numbers $=$ (Average of 6 numbers) $\times 6$
$= 4.5 \times 6 = 27$

Q9. (D)
For first 8 hours, she is paid $= 8R$
Next 4 hours, she is paid $= (12 - 8) = 4S$
Total pay $= 8R + 4S$

Average $= \frac{8R + 4S}{12} = \boxed{\frac{2R + S}{3}}$

Q10. (B) Let "x" be the required grade, then

$$\frac{4(60) + x}{5} = 58$$

$\Rightarrow \quad 240 + x = 290 \quad \Rightarrow x = 290 - 240 = 50$

Q11. (A)

$$\frac{(a+b)+(b+c)+(c+a)}{3} = \frac{8+9+11}{3}$$

$$\Rightarrow \quad \frac{2(a+b+c)}{3} = \frac{28}{3}$$

$\Rightarrow \quad a + b + c = 14 \quad$ .....(i)
Put $a + b = 8 \Rightarrow 8 + c = 14 \Rightarrow \boxed{c = 6}$
Now put $b + c = 9 \Rightarrow a + 9 = 14 \Rightarrow \boxed{a = 5}$
again put $c + a = 11 \Rightarrow 11 + b = 14 \Rightarrow \boxed{b = 3}$

Average of a, b and c $= \frac{6+5+3}{3} = \boxed{\frac{14}{3}}$

Q12. (B) $\frac{3+5+10+S}{4} = 6 \Rightarrow 18 + S = 24 \Rightarrow \boxed{S = 6}$

Q13. (B) $\frac{3^{10} + 3^{20} + 3^{30}}{3} = (3^{10} + 3^{20} + 3^{30})3^{-1}$
$= 3^{10-1} + 3^{20-1} + 3^{30-1}$
$= 3^9 + 3^{19} + 3^{29}$

Q14. (C) $20x + 20y = 70 \Rightarrow 20(x + y) = 70 \Rightarrow x + y = \frac{7}{2}$

$\Rightarrow$ Average of x and y $= \frac{x+y}{2} = \frac{7}{2 \times 2} = \boxed{\frac{7}{4}}$

Q15. (B) $\frac{(x^4 - 20) + (40 - x^4) + (3x + 4)}{3} = \frac{3x + 24}{3} = \frac{3(x+8)}{3}$
$= x + 8$

Part 2 Algebra

Chapter 1

POLYNOMIALS

Polynomial:

A sum of finite number of monomials is called a polynomial. Each monomial is called a term of the polynomial.

Monomial:

A monomial is a variable, or a constant, or a product of constant and one or more variables, with the variables having only non-negative integer in exponents.

Example:

$3x^2y$, $-5xy$, and $-7xy^3$ are monomials.

The algebraic expression

$$4y^{-3} \text{ and } \frac{3}{y}$$

are not monomials, because these expressions have not non-negative integer in exponent, and cannot be written as a product of a constant and a variable with a non-negative integer exponent.

Degree of Monomial:

In any monomial the sum of the exponents of the variables is called the degree of monomial.

Example:

What are the degrees of the monomials

$$-3x^2y,\ 7x^3y,\ -18xy^2$$

Solution:

In algebraic expression $-3x^2y$, the degree of the monomial is 3, because the exponents of x and y are 2 and 1 respectively therefore their sum is $(2 + 1 = 3)$. Similarly the degree of the expressions $7x^3y$ and $-18xy^2$ are 4 and 3 respectively.

Note:

In monomial, the constant is called the numerical coefficient or simply the coefficient of the monomial.
$-3x^2y$, $7x^3y$ and $-18xy^2$ are monomials of coefficient -3, 7, and -18 respectively.

Multiplication of Monomials:

The process of multiplication is illustrated in the following example:

Example:

What is the value of $-5xy^2$, when $x = -2$ and $y = -3$

Solution:

First of all write the coefficient of the monomial, then substitute the value of x and y in monomial. Then evaluate:

$$-5(-2)(-3)^2 = -5(-2)(9) = 90$$

Polynomial:

A sum of a finite number of monomials is called a polynomial. Each monomial in a polynomial is called a term of the polynomial.

Degree of a Polynomial:

The degree of a polynomial is the largest degree of the terms in the polynomial.

What are like terms in a Polynomial?

Terms of polynomial that have exactly the same variables raised to the same powers are called like terms.

Example:

Each of the following is a polynomial:

$3x^2 + 5$, $3x^2$, $2x^2 + 9x - 12$, $-4x^2$, $7x^2y$, $9x^2 - 8$

Explanation:

In above lists of polynomials; $3x^2 + 5$, and $9x^2 - 8$ are called binomial because each polynomial has two terms; the polynomials $3x^2$, $-4x^2$ and $7x^2y$ are monomials; the polynomial $2x^2 + 9x - 12$ is called trinomial because it has three terms. In above list of polynomials $3x^2$ and $-4x^2$ are like terms, similarly $3x^2 + 5$ and $9x^2 - 8$ are like terms, because they have exactly the same variables raised to the same power.

General Form of a Polynomial:

In a single variable x, the general form of a polynomial of degree n is

$a_nx^n + a_{n-1}x^{n-1} + \ldots\ldots\ldots + a_2x^2 + a_1x + a_0$ where n is a non negative integer, and $a_n \neq 0$

Combination of Like Terms in a Polynomial:

The polynomial $4x^2 + 3x + 6x + x^2 + x$ is equivalent to the binomial $5x^2 + 10x$, because like terms are combined in a polynomial. The process of combination is illustrated as:

$4x^2 + x^2 = 5x^2$ and $3x + 6x + x = 10x$

$\therefore\ 4x^2 + 3x + 6x + x^2 + x = 5x^2 + 10x$

Remember:

Only like terms in a polynomial can be combined.

Arithmetic Operations on Polynomials:

We use usual law of arithmetic, to add subtract, multiply and divide polynomials.

Addition and Subtraction:

Polynomials are added or subtracted by combining like terms.

Example:

$(2x^3 + 3x^2 + 7x + 6) + (4x^2 + 3x - 2) - (5x^2 + 4x)$

$= 2x^3 + (3x^2 + 4x^2 - 5x^2) + (7x + 3x - 4x) + (6 - 2)$

$= 2x^3 + 2x^2 + 6x + 4$

The rules for adding like terms are:

Rule 1:

If all the terms are positive in a polynomial, then add their coefficients.

Example:

Find the value of $8x^2 + 2x^2 + 7x^2$

Solution:

Here we have to increase 8 like things by 2 and 7 like things of the same kind, and aggregate is 17 of each thing.

Rule 2:

If all the terms in a polynomial are negative add the coefficient numerically and prefix the minus sign to the sum.

Example:

What is the sum of $-4x$, $-x$, $-3x$ and $-7x$

Solution:

In this example the word sum indicates the aggregate of 4 subtractive quantities of like terms. In this case we have to take away successively 4, 1, 3 and 7 like things, therefore the result is the same as taking away 15(4 +

1 + 3 + 7) such things in the aggregate.

∴ The sum of $-4x, -x, -3x, -7x$ is $-15x$.

Rule 3:

If all the terms have not same sign, add together separately the coefficient of all the negative terms and the coefficient of all the positive terms: Then find the difference of those two results, preceded by the sign of the greater, will give the coefficient of the sum required.

Example:

Find the sum of $12x^2 - 3x^2 + 15x^2 - 17x^2$

Solution:

The sum of the coefficient of positive terms is 12 + 15 = 27

The sum of the coefficient of negative terms is 3 + 17 = 20

The difference of these is 7, and the sign of the greater is positive: hence the required sum is $7x^2$.

Multiplication of Monomials:

To multiply two simple monomials together, first multiply their coefficients together and prefix their product to the product of the different letters, giving to each letter on index equal to the sum of the indices that letter has in the separate factors.

Example:

What is the product of $5x^2y^3$ and $-3xy^2$

Solution:

$$(5x^2y^3)(-3xy^2) = (5)(-3)(x^2 \times x)(y^3 \times y^2)$$
$$= -15x^3y^5$$

Note:

The product of a monomial by any polynomial is the algebraic sum of the partial products of each term of the polynomial by that monomial.

Example:

Find the product of $2xy^2$ and $(4x^2 + 3y + 7xy)$

Solution:

$$2xy^2(4x + 3y + 7xy) = 8x^3y^2 + 6xy^3 + 14x^2y^3$$

Multiplication of two Binomials:

The procedure of multiplication of two binomials is illustrated as:

1. Multiply each term of the first binomial by each term of the second.
2. When the terms multiplied together have like signs, prefix to the product the sign +, when unlike prefix –.
3. The algebraical sum of the partial products so formed gives the complete product.

Example:

Multiply $(x + 3)$ by $(x - 5)$

Solution: $(x + 3)(x - 5) = x(x - 5) + 3(x - 5)$

$$= x^2 - 5x + 3x - 15$$
$$= x^2 - 2x - 15$$

Example:

Find the value of $(x + 2)(x - 3) - (x + 4)(x - 5)$

Solution:

First of all, multiply both pairs of binomials separately, then subtract the second result from the first.

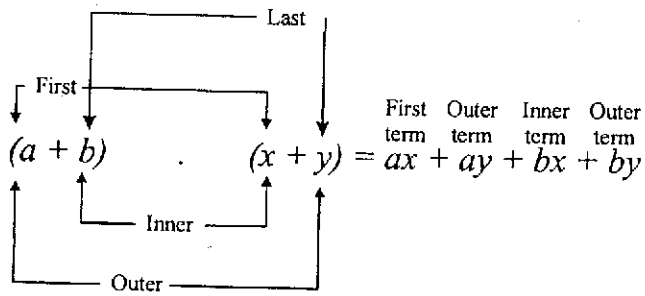
$$(x+2)(x-3) = x(x-3)+2(x-3) = x^2-3x+2x-6$$
$$= x^2-x-6$$
$$(x+4)(x-5) = x(x-5)+4(x-5) = x^2-5x+4x-20$$
$$= x^2-x-20$$

Suotracting: $(x^2-x-6)-(x^2-x-20) = x^2-x-6-x^2+x+20$

$$= 14$$

FOIL Method:

The product of the two binomials can be computed by the FOIL method. This method is illustrated in the following example



Example:

Find the product of $(2x-3)$ and $(4x+2)$ using FOIL Method

First Outer Inner Last

$$(2x-3)(4x+2) = (2x)(4x)+(2x)(2)+(-3)(4x)+(-3)(2)$$
$$= 8x^2+4x-12x-6$$
$$= 8x^2-8x-6$$

Important Binomial Formulas:

Following are most important binomial products, those occur frequently in algebra.

1. $(x+y)(x-y) = x^2-y^2$
2. $(x+y)^2 = x^2+2xy+y^2$
3. $(x-y)^2 = x^2-2xy+y^2$

Example:

Find each of the following products:

a) $(2a+3)(2a-3)$

b) $(a-5b)^2$

Solution:

a) Using formula $(x+y)(x-y) = x^2-y^2$

Here $(2a+3)(2a-3) = (2a)^2-(3)^2 = 4a^2-9$

b) Using formula $(x-y)^2 = x^2-2xy+y^2$

$(a-5b)^2 = (a)^2-2(a)(5b)+(5b)^2 = a^2-10ab+25b^2$

Example:

Given $x+y=5$, and $x^2-y^2=10$, what is the value of $x-y$?

Solution:

Using the fact $(x + y)(x - y) = x^2 - y^2$

$$(5)(x - y) = 10$$

$$x - y = \frac{10}{5}$$

$$\Rightarrow \quad x - y = 2$$

Example:

Find the value of xy, when $(x + y)^2 = 25$ and $x^2 + y^2 = 3$

Solution:

We know $(x + y)^2 = x^2 + 2xy + y^2$

We can write $(x + y)^2 = (x^2 + y^2) + 2xy$

Substituting the value of $(x + y)^2$, and $x^2 + y^2$ in above

$$25 = (3) + 2xy$$

$$22 = 2xy$$

$$xy = \frac{22}{2}$$

$$xy = 11$$

Division of Polynomial by Monomial:

Division is the inverse of multiplication. The object of division is to find out the quantity called quotient.

Thus $\frac{\text{divided}}{\text{divisor}} = \text{quotient}$

To divide a monomial by a monomial, use distributive law, the index of each letter in the quotient is obtained by subtracting the index of that letter in the divisor from that in the divided. To the result so obtained prefix its proper sign the quotient of the divided by that of divisor.

To divide a polynomial by a monomial, divide each term separately by that monomial, and take the algebraic sum of the partial quotient so obtained.

Example 1:

What is the quotient when $-4x^2y$ is divided by $2x$.

Solution:

The quotient $= \frac{-4x^2y}{2x} = -2xy$

Example 2:

Divide $12x^3 - 6x^2 - 9x$ by $3x$

Solution:

$$\frac{12x^3}{3x} - \frac{6x^2}{3x} - \frac{9x}{3x} = 4x^2 - 2x - 3$$

Evaluating a Polynomial:

To evaluate a polynomial, substitute the given value(s) for the variable(s) and then perform the given operation.

Example:

If $x = 3$, $y = -7$ and $z = -2$, find the value of $x^2 - 26y + 17z$

Solution:

$x^2 - 26y + 17z = (3)^2 - (26)(-7) + 17(-2)$

$= 9 + 182 - 34 = 157$

Factorising Polynomials:

Writing a polynomial as a product of polynomials of lower degree is called factoring.

When each of the terms which compose a polynomial is divisible by a common factor, the polynomial may be simplified by dividing each term separately by this factor, and enclosing the quotient within brackets; the common factor being placed outside as a coefficient.

Example 1:

Resolve into factors $4x^2 - 20x$

Solution:

The terms of the polynomials $4x^2 - 20x$ have a common factor $4x$;

$\therefore \quad 4x^2 - 20x = 4x(x - 5)$

Example 2:

Resolve into factors $x^2 - sx + tx - st$

Solution:

We see that the first two terms contain a common factor x, and the last two terms a common factor t, we enclose the first two terms in one bracket, and the last in another. Thus,

$x^2 - sx + tx - st \quad = (x^2 - sx) + (tx - st)$

$= x(x - s) + t(x - s)$; take $(x - s)$ common

$= (x - s)(x + t)$

Factorising Quadratic Trinomials:

Some trinomials of the form $x^2 + bx + c$ can be factorized by trial and error procedure. This method is the reverse of the FOIL method. This is illustrated in the following example.

Example:

Consider the following binomial expansion:

$(x + 5)(x + 6) \quad = \quad x(x + 6) + 5(x + 6)$

$= \quad x^2 + 6x + 5x + 30$

$\therefore \quad x^2 + 11x + 30 \quad = \quad (x + 5)(x + 6)$

Notice at $11 = 5 + 6$ and $30 = 5 \times 6$

This result can be used to factorize trinomials? For example, to factorize the trinomial $x^2 + 7x + 12$ we need to find two numbers so that:

Product $= 12$ and sum $= 7$

The two numbers are 4 and 3

$4 \times 3 \quad = 12$ and $4 + 3 = 7$

$\therefore x^2 + 7x + 12 \quad = (x + 4)(x + 3)$

Example:

Factorize *i)* $x^2 + 7x - 18$

ii) $m^2 - 9m + 14$

Solution:

i) $x^2 + 7x - 18$

Product $= 9 \times (-2) = -18$

Sum $= 9 + (-2) = 7$

$\therefore \quad x^2 + 7x - 18 \quad = (x+9)(x-2)$

ii) $m^2 - 9m + 14$

Product $= (-7)(-2) = 14$

Sum $= (-7) + (-2) = -9$

$\therefore \quad m^2 - 9m + 14 \quad = (x-7)(x-2)$

Example:

Find the value of $(10001)^2$

Solution:

$$\begin{aligned}(10001)^2 &= (10000+1)^2\\ &= (10000)^2 + 2(10000)(1) + (1)^2\\ &= 100000000 + 20000 + 1\\ &= 100020001\end{aligned}$$

Example:

What is the value of $(9999)^2$

Solution:

$$\begin{aligned}(9999)^2 &= (10000-1)^2\\ &= (10000)^2 - 2(10000)(1) + (1)^2\\ &= 100000000 - 20000 + 1\\ &= 99980001\end{aligned}$$

Algebraic Fraction:

An expression which has a variable in the denominator, is called an algebraic expression. Algebraic fractions are added and subtracted using the same method as for arithmetic fractions. The denominator must be the same before these operations can be carried out.

Example:

Simply *i)* $\frac{3x}{4} + \frac{x}{6}$

ii) $\frac{x+3}{8} - \frac{x-4}{4}$

Solution:

i) $\frac{3x}{4} + \frac{x}{6}$

$= \frac{9x + 2x}{12}$ (lowest common denominator is 12)

$= \frac{11x}{12}$

ii) $\frac{x+3}{8} - \frac{x-4}{4}$

$= \frac{(x+3) - 2(x-4)}{8} = \frac{(x+3) - (2x-8)}{8} = \frac{x+3-2x+8}{8}$

$= \frac{11-2x}{8}$

Multiplication and Division of Algebraic Fractions:

Algebraic fractions are multiplied and divided using the same method as for arithmetic fractions.

Example:

Simply *i)* $\frac{x}{15} \times \frac{9}{y}$

ii) $\frac{a^2}{2} \div \frac{a^3}{4}$

Solution:

i) $\frac{x}{15} \times \frac{9}{y} = \frac{3x}{5y}$

ii) $\frac{a^2}{2} \div \frac{a^3}{4}$

$= \frac{a^2}{2} \times \frac{4}{a^3}$

$= \frac{2}{a}$

Example:

Simplify $\frac{9x^3 - x}{(3x-1)(9x-3)}$, also find the value when $x = 39$

Solution:

$$\frac{9x^3 - x}{(3x-1)(9x-3)} = \frac{x(9x^2-1)}{(3x-1)3(3x+1)} = \frac{x(3x-1)(3x+1)}{3(3x-1)(3x+1)}$$

using $a^2 - b^2 = (a + b)(a - b)$

$= \frac{x}{3}$

Now when $x = 39$

$= \frac{39}{3} = 13$

Example:

Find the value of $\frac{a^2 - b^2}{(a-b)}$, when $a = 2.9$ and $b = 9.1$

Solution:

$\frac{a^2 - b^2}{a - b}$

$= \frac{(a-b)(a+b)}{(a-b)} = a + b$

$= (2.9 + 9.1) \quad = 12$

Example:

Simply $\frac{e}{4c} \div \frac{eb}{ac}$

Solution:

$\frac{e}{4c} \times \frac{ac}{eb} = \frac{a}{4b}$

Example:

What is the value of a and b, If $a^2 - b^2 = 36$ and $a + b = 6$?

Solution:

$a^2 - b^2 = 36 \Rightarrow (a - b)(a + b) = 36$

$\Rightarrow (a - b)(6) = 36$ as $a + b = 6$

$\therefore a - b = 6$ .....(1)

adding $a + b = 6$ .....(2)

$2a = 12$

$\Rightarrow a = 6$

Substituting $a = 6$ in (2) we have

$6 + b = 6, \quad b = 0$

$\therefore a = 6, \quad b = 0$

Example:

Find the value of xy, when $x^2 + y^2 = 58$ and $x^2 - y^2 = 42$

Solution:

adding

$(x^2 + y^2) + (x^2 - y^2) = 2x^2$

$(58) + (42) = 2x^2 \Rightarrow 2x^2 = 50 \Rightarrow x = \pm 5$

Substituting $x = \pm 5$ in $x^2 + y^2 = 29$

$(5)^2 + y^2 = 29 \Rightarrow y^2 = 4 \Rightarrow y = \pm 2$

$\Rightarrow \boxed{xy = (5)(2) = 10}$

Multiple Choice Questions (MCQs)

Q1. If $x = 235$ and $y = 117$, then $\frac{x^2 - y^2}{x - y} = ?$

(A) 118 (B) 100

(C) 115 (D) 352

Q2. If $x^2 - y^2 = 16$ and $x^2 + y^2 = 34$, which of the following could be the value of xy?

I 15 II −15 III 45

(A) only I (B) only II

(C) I and II only (D) III only

Q3. The average of the polynomials, $2x^2 + 5x - 6$, $5x^2 - 5x - 6$ and $30 - 7x^2$ is:

(A) 14 (B) 18

(C) 6 (D) $5x$

Q4. What is the value of $x^2 + 14x + 24$, when $x = 854$?

(A) 1000 (B) 100,000

(C) 741,296 (D) 742,398

Q5. If $x^2 + y^2 = 9$ and $(x - y)^2 = 3$, what is the value of xy?

(A) 16 (B) 9

(C) 6 (D) 3

Q6. The value of $(5x + 6)(x + 12) - (5x - 6)(x + 3)$ is:

(A) $2(5x^2 + 9x)$ (B) 14

(C) 4 (D) 22

Q7. If $\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$ and $xy = z$, what is the average of x and y?

(A) $\frac{1}{2}$ (B) 1

(C) $\frac{x+y+z}{3}$ (D) $\frac{x+y+z}{2}$

Q8. If $p^2 - q^2 = 48$ and $p - q = 12$, what is the average of p and q?

(A) 4 (B) 6

(C) 2 (D) 12

Q9. $\left(\frac{1}{x}+x\right)^2 - \left(\frac{1}{x}-x\right)^2 = ?$

(A) 4 (B) 2

(C) $2\left(\frac{1}{x^2}+x^2\right)$ (D) $2\left(\frac{1+x^2}{x^2}\right)$

Q10. If $\left(x+\frac{1}{x}\right) = 81$, then $x^2 + \frac{1}{x^2} = ?$

(A) 6563 (B) 6561

(C) 6559 (D) 79

Q11. If $x < 0$, then $-3x^2$ is:

(A) less than $(-3x)^2$ (B) greater than $(-3x)^2$

(C) equal to $(-3x)^2$

(D) greater than or equal to $(-3x)^2$

Q12. If $x > y$, then $(x - y)(x + y)$ is:

(A) equal to $(x - y)(x - y)$ (B) less than $(x - y)(x - y)$

(C) greater than $(x - y)(x - y)$ (D) options A and C

Q13. If $a = -5$ and $b = 3$ then $-a^2b^3$ is:

(A) less than 0 (B) equal to 0

(C) greater than 0 (D) options B and C

Q14. $(a + b)(a - b) =$

(A) $a(a - b) - b(a - b)$ (B) $a(b - a) + b(a - b)$

(C) $a(a + b) - b(b + a)$ (D) $a(a - b) + b(b - a)$

Q15. $\frac{3x^2 - 27}{x - 3}$ and $(x > 0)$, is:

(A) less than $2x + 9$ (B) equal to $2x + 9$

(C) greater than $2x + 9$ (D) cannot find

Q16. The sum of the polynomials, $6x^2 + 9x - 8$ and $2x^2 - 5x + 3$ is:

(A) $4x^2 - 14x - 5$ (B) $8x^2 + 14x + 11$

(C) $8x^2 - 4x + 5$ (D) $8x^2 + 4x - 5$

Q17. $(6x^2 + 9x - 8) - (4x^2 - 5x + 3) = ?$

(A) $2x^2 - 14x - 11$ (B) $2x^2 + 14x - 11$

(C) $10x^2 + 4x - 11$ (D) $2x^2 + 14x + 11$

Q18. The product of $-3x^2y$ and $2x^2y^2z$ is:

(A) $-6x^4y^3z^2$ (B) $-6x^4y^3z$

(C) $-6x^3y^3z$ (D) $6x^2y^2z^2$

Q19. What is the product of $2x$ and $6x^2 - 3xy^2 + 4$?

(A) $3x^2 - xy^2 + 2$ (B) $3x - \frac{3}{2}y^2 + \frac{2}{x}$

(C) $12x^3 - 6x^2y^2 + 8x$ (D) $6x^3 - 6x^2y^2 - 8x$

Q20. What is the product of $(2x + y)$ and $(4x^2 - 6xy^2)$?

(A) $8x^3 + 12x^2y^2 + 4x^2y - 6xy^3$ (B) $8x^3 - 12x^2y^2 + 4x^2y + 6xy^2$

(C) $8x^3 + 12x^2y^2 - 4x^2y + 6xy^3$ (D) $8x^3 - 12x^2y^2 + 4x^2y - 6xy^3$

Q21. What is quotient if $36x^2y + 21xy^3z$ is divided by $9xy$?

(A) $4x - \frac{7}{3}yz^2$ (B) $4y - \frac{3}{7}y^2z$

(C) $4x + \frac{3}{7}y^2z^2$ (D) $4x + \frac{7}{3}y^2z$

Q22. If $p = 3q - s$, then what is the value of q in terms of p and s?

(A) $\frac{p+s}{3}$ (B) $\frac{p-s}{3}$

(C) $\frac{s-p}{3}$ (D) $\frac{3}{p-s}$

Q23. If $x - 3 = 11$, what is the value of $x - 6$?

(A) 14 (B) 8

(C) 22 (D) 19

Q24. $\frac{x^2y^2 - 1}{xy - 1} = ?$

(A) $xy - 1$ (B) $(xy + 1)$

(C) $(xy + 1)^2$ (D) $(1 - xy)$

Q25. If $y = \frac{1}{\frac{1}{a} + \frac{1}{b}}$, when $a = 1$ and $b = \frac{1}{3}$, then $y =$

(A) $\frac{1}{3}$ (B) 1

(C) 3 (D) $\frac{1}{4}$

Q26. If $\frac{1}{1 + \frac{x}{1+x}} = 1$, then $x =$

(A) 0 (B) 1

(C) 2 (D) $\frac{1}{1+x}$

Q27. If $a^2 + b^2 = 16$ and $(a - b)^2 = 4$, then ab is equal to:

(A) -6 (B) 20

(C) -20 (D) 6

Q28. If $a^2 - b^2 = 27$ and $a^2 + b^2 = 13$ then the value of ab is equal to:

(A) $2\sqrt{5}$ (B) $2i\sqrt{35}$

(C) 14 (D) 20

Q29. What is the value of $\left[\frac{1}{x}+x\right]^2-\left[\frac{1}{x}-x\right]^2$?

(A) 4 (B) 0

(C) $2x^2$ (D) $\frac{2}{x^2}$

Q30. What is the arithmetic mean of x and y if $\frac{1}{x}=\frac{1}{z}-\frac{1}{y}$ and $xy = z$?

(A) 1 (B) $\frac{1}{2}$

(C) $\frac{x+y+z}{3}$ (D) $\frac{x+y}{2}$

Q31. If $x^2 - y^2 = 25$ and $x - y = 5$, then the average of x and y is:

(A) 2.5 (B) 5

(C) 15 (D) 7.5

Q32. What is the value of $\frac{1}{x^2}+x^2$, when $\left(x-\frac{1}{x}\right)^2 = 36$?

(A) 6 (B) 8

(C) 38 (D) 34

Q33. What is the value of $\frac{m^2-m-6}{m^2-6m+9}$, when m = 6666?

(A) 6666 (B) −1

(C) 0 (D) 1

Q34. If 7 + 4p = q − kp, what is value of p?

(A) $\frac{4+k}{q-7}$ (B) $\frac{4-k}{q+7}$

(C) $\frac{q-7}{4+k}$ (D) $\frac{k-4}{7-q}$

Q35. If $F = C + \frac{bv^2}{K}$, then v is terms of F, C, K and b is:

(A) $\pm\sqrt{\frac{K}{C}(F-b)}$ (B) $\pm\sqrt{\frac{C}{b}(FK-C)}$

(C) $\pm\sqrt{\frac{K}{b}(F-C)}$ (D) $\pm\sqrt{\frac{KF+C}{K}}$

Q36. If $x = 7$, what is the value of $x^{5/2} \div x^{1/2}$?

(A) $\sqrt{2}$ (B) $\sqrt{7}$

(C) 49 (D) 4

Q37. What is the value of $m^2 + 7m - 18$ when m = 91?

(A) 8882 (B) 8900

(C) 1260 (D) 8918

Q38. What is the value of ab, when $a^2 + b^2 = 9$ and $(a - b)^2 = 7$?

(A) 16 (B) 2

(C) 1 (D) 8

Q39. What is the difference of the reciprocals of x^2 and y^2?

(A) $\frac{y^2 - x^2}{x^2y^2}$ (B) $\frac{-y^2 - x^2}{x^2y^2}$

(C) $\frac{x^2y^2}{y^2 - x^2}$ (D) $x^2 + y^2$

Q40. What is the value of $a^2 - b^2$, when $a + b = 2.95$ and $a - b = 1000$?

(A) .000295 (B) .00000295

(C) 295 (D) 2950

Q41. What is the value of $(x - 7)(x + 8) - (x - 9)(x + 10)$?

(A) 34 (B) 146

(C) −146 (D) $-14x + 34$

Explanatory Answers

Q1. (D) $\frac{x^2 - y^2}{x - y} = \frac{(x - y)(x + y)}{x - y} = x + y = 235 + 117 = 352$

Q2. (C) Adding both equations *i.e.*,

$$x^2 + y^2 = 34$$
$$\underline{x^2 - y^2 = 16}$$
$$2x^2 = 50 \quad \Rightarrow x^2 = 25$$
$$\Rightarrow x = \pm 25$$

Now, $x^2 + y^2 = 34 \Rightarrow 25 + y^2 = 34 \Rightarrow y^2 = 9$

$\Rightarrow y = \pm 3$

Hence, $xy = (-5)(-3) = 15 = (5)(3)$

and $xy = (-5)(3) = -15 = (5)(-3)$

So correct answer is *C*.

Q3. (C) First of all we find the sum of the three polynomials, then divide the answer by 3.

Sum of the three polynomials

$$2x^2 + 5x - 6$$
$$5x^2 - 5x - 6$$
$$\underline{-7x^2 \qquad +30}$$
$$18$$

Now, Average $= \frac{\text{Sum of the three polynomials}}{3}$

$= \frac{18}{3} = 6$

Q4. (C) To avoid time consuming calculation, factorize the given polynomial

$x^2 + 14x + 24 = x^2 + 12x + 2x + 24 = x(x + 12) + 2(x + 12)$

$= (x + 2)(x + 12)$

Substituting the value of x in above

$= (854 + 2)(854 + 12) = (856)(866)$

$= 741{,}296$

Q5. (D) Solving $(x - y)^2 = 3 \Rightarrow x^2 + y^2 - 2xy = 3$

Substituting the value of $x^2 + y^2$ in above

$(x^2 + y^2) - 2xy = 3 \Rightarrow 9 - 2xy = 3$

$\Rightarrow 9 - 3 = 2xy \Rightarrow 6 = 2xy$

$\Rightarrow xy = 3$

Q6. (D) $(5x + 4)(x + 1) - (5x - 6)(x + 3) =$

$(5x^2 + 5x + 4x + 4) - (5x^2 + 15x - 6x - 18)$
$(5x^2 + 9x + 4) - (5x^2 + 9x - 18) = 4 + 18 = 22$

Q7. (A) $\frac{1}{z} = \frac{1}{x} + \frac{1}{y} \Rightarrow \frac{1}{z} = \frac{x+y}{xy} \Rightarrow \frac{1}{z} = \frac{x+y}{z} (\because z = xy)$

$$\Rightarrow 1 = x + y \Rightarrow \frac{1}{2} = \frac{x+y}{2}$$

Hence $\frac{x+y}{2} = \frac{1}{2}$

Q8. (C) $p^2 - q^2 = 38 \Rightarrow (p+q)(p-q) = 48$ ...(i)
Now, given that $p - q = 12$ ...(ii)
Dividing equation (i) by (ii), we have

$$\frac{(p+q)(p-q)}{(p-q)} = \frac{48}{12}$$

$p + q = 4$ ...(iii)

Dividing both sides of equation (iii) by 2, we get

$$\frac{p+q}{2} = \frac{4}{2} = 2$$

Q9. (A) Expanding each square of the polynomial, we get

$$\left(\frac{1}{x^2} + x^2 + 2\right) - \left(\frac{1}{x^2} + x^2 - 2\right)$$

$$\Rightarrow \left(\frac{1}{x^2} + x^2 + 2 - \frac{1}{x^2} - x^2 + 2\right) = 4$$

Q10.(D) Given $\left(x + \frac{1}{x}\right)^2 = 81 \Rightarrow x^2 + \frac{1}{x^2} + 2 = 81$

$$\Rightarrow x^2 + \frac{1}{x^2} = 81 - 2 \Rightarrow x^2 + \frac{1}{x^2} = 79$$

Q11.(A) Since x is negative ($\because x < 0$), therefore x^2 is positive, implies that $-(+3)(+x^2) = -3x^2$ is negative
Now, we take $(-3x)^2$, because x is negative.
$\therefore [-3(-x)]^2 = (-3)^2(-x)^2 = 9x^2$,
which is positive, hence $-3x^2$ is less than $(-3x)^2$.

Q12.(C) Since $x > y$, therefore, $x - y$ is positive. Thus dividing $(x-y)(x+y)$ and $(x-y)(x-y)$ by $(x-y)$, we have, $\frac{(x-y)(x+y)}{x-y}$ $\frac{(x-y)(x-y)}{x-y}$

$x + y$ $x - y$

Because both quantities are positive, but L.H.S is greater than R.H.S.

Q13.(A) As $a = -5$ and $b = 3$ $\Rightarrow -a^2b^3 = -(-5)^2(3)^3$
$\Rightarrow -a^2b^3 = -(25)(27)$
$= -675$
which is clearly less than 0.

Q14.(C) If we multiply $(a + b)$ by $(a - b)$, we proceed as
$a(a + b) - b(a + b)$ so option C is the correct answer.

Q15.(C) $\frac{3x^2 - 27}{x - 3} = \frac{3(x^2 - 9)}{x - 3} = \frac{3(x-3)(x+3)}{(x-3)} = 3(x + 3) = 3x + 9$
which is clearly greater than $2x + 9$.

Q16.(D) $(6x^2 + 9x - 8) + (2x^2 - 5x + 3)$
$= (6x^2 + 2x^2) + (9x - 5x) + (-8 + 3)$
$= 8x^2 + 4x - 5$

Q17.(B) $(6x^2 + 9x - 8) - (4x^2 - 5x + 3)$
$= (6x^2 - 4x^2) + (9x - (-5x)) + (-8 - 3)$
$= 2x^2 + (9x + 5x) + (-11)$

$= 2x^2 + 14x - 11$

Q18.(B)
$$\begin{array}{r} 2x^2y^2z \\ \times -3x^2y \\ \hline -6x^4y^3z \end{array}$$

Q19.(C)
$$\begin{array}{r} 6x^2 - 3xy^2 + 4 \\ \times 2x \\ \hline 12x^3 - 6x^2y^2 + 8x \end{array}$$

Q20.(D) $(2x + y) \times (4x^2 - 6xy^2) = 2x(4x^2 - 6xy^2) + y(4x^2 - 6xy^2)$
$= 8x^3 - 12x^2y^2 + 4x^2y - 6xy^3$

Q21.(D) $(36x^2y + 21xy^3z) \div 9xy$

$$= \frac{36x^2y + 21xy^3z}{9xy} = \frac{36x^2y}{9xy} + \frac{21xy^3z}{9xy}$$

$$= 4x + \frac{7}{3}y^2z$$

Q22.(A) $p = 3q - s$

$\Rightarrow p + s = 3q \Rightarrow q = \frac{p+s}{3}$

Q23.(B) Given, $x - 3 = 11$

Subtracting 3 both sides of the equation

$x - 3 - 3 = 11 - 3$

$\Rightarrow \boxed{x - 6 = 8}$

Q24.(B) Factorizing the numerator

$$\frac{x^2y^2 - 1}{xy - 1} = \frac{(xy + 1)(xy - 1)}{(xy - 1)} = xy + 1$$

Q25.(D) $y = \frac{1}{\frac{1}{a} + \frac{1}{b}}$, putting $a = 1$ and $b = \frac{1}{3}$, we get

$$y = \frac{1}{\frac{1}{1} + \frac{1}{1/3}} \Rightarrow y = \frac{1}{1+3} \Rightarrow y = \frac{1}{4}$$

Q26. A $\frac{1}{1 + \frac{x}{1+x}} = 1$, Solving for x

$\frac{1}{\frac{1+x+x}{1+x}} = 1$ (Taking "$1 + x$" L.C.M in denominator)

$\Rightarrow \frac{1 \times (1+x)}{\frac{1+2x}{(1+x)} \times (1+x)} = 1$ (Multiplying denominator and numerator by $(1 + x)$)

$\Rightarrow \frac{1+x}{1+2x} = 1 \quad \Rightarrow 1 + x = 1 + 2x \Rightarrow 1 - 1 = 2x - x$

$\Rightarrow x = 0$

Q27. d) $a^2 + b^2 = 16$ and $(a - b)^2 = 4$

$(a - b)^2 = 4 \Rightarrow a^2 + b^2 - 2ab = 4$

$\Rightarrow a^2 + b^2 - 4 = 2ab$

but $a^2 + b^2 = 16$ substituting above equation

$12 = 16 - 4 = 2ab$

$\Rightarrow \boxed{ab = 6}$

Q28. b) Given $a^2 - b^2 = 27$ and $a^2 + b^2 = 13$

Adding $a^2 + b^2 + a^2 - b^2 = 27 + 13$

$2a^2 = 40 \Rightarrow a^2 = 20$

Which gives $20 + b^2 = 13 \Rightarrow b^2 = -7$

$a^2b^2 = -140 = 140i^2$

$ab = 2\sqrt{35}\,i \Rightarrow 2i\sqrt{35}$

Q29. a) $\left[\frac{1}{x}+x\right]^2 - \left[\frac{1}{x}-x\right]^2$

Expand each square

$$\left[\frac{1}{x}+x\right]^2 = \frac{1}{x^2}+x^2+2$$

$$\left[\frac{1}{x}-x\right]^2 = \frac{1}{x^2}+x^2-2$$

$$\left[\frac{1}{x^2}+x^2+2\right]-\left[\frac{1}{x^2}+x^2-2\right] = \frac{1}{x^2}+x^2+2-\frac{1}{x^2}-x^2+2$$

$$= 4$$

Q30. d) $\frac{1}{x} = \frac{1}{z} - \frac{1}{y} \Rightarrow \frac{1}{z} = \frac{1}{x} + \frac{1}{y} \Rightarrow \frac{1}{z} = \frac{x+y}{xy}$

$$\Rightarrow \frac{xy}{z} = x + y$$

but $xy = z$ which gives

$\frac{z}{z} = x + y$

$\Rightarrow 1 = x + y$

$\Rightarrow \boxed{\frac{x+y}{2} = \frac{1}{2}}$

Q31. a) Given $x^2 - y^2 = 25$ and $x - y = 5$

$x^2 - y^2 = 25 \Rightarrow (x-y)(x+y) = 25$

but $x - y = 5$ (given)

$\Rightarrow 5(x+y) = 25$

$\Rightarrow (x+y) = 5$

$\Rightarrow \boxed{\frac{x+y}{2} = \frac{5}{2} = 2.5}$

Q32. c) $\left(x - \frac{1}{x}\right)^2 = 36$, Expanding the square

$x^2 + \frac{1}{x^2} - 2 = 36$

$\Rightarrow x^2 + \frac{1}{x^2} = 38$

Q33. d) $\frac{m^2 - m - 6}{m^2 - 6m + 9}$

$$= \frac{m^2 - 3m + 2m - 6}{(m-3)^2} = \frac{m(m-3) + 2(m-3)}{(m-3)^2}$$

$= \frac{(m+2)(m-3)}{(m-3)^2} = \frac{m+2}{m-3} = \frac{6666+2}{6666-3}$

$= \frac{6668}{6663} = 1$ (approx)

Q34. c) $7 + 4p = q - kp$

$4p + kp = q - 7$

$p(4 + k) = q - 7$

$p = \frac{q-7}{4+k}$

Q35. (C) $F = C + \frac{bv^2}{K}$

$FK - CK = bv^2 \Rightarrow v^2 = \frac{K(F-C)}{b}$

$v = \pm\sqrt{\frac{K}{b}(F-C)}$

Q36. (C) $x^{5/2} \div x^{1/2}$ and $x = 7$

$7^{5/2} \div 7^{1/2} \Rightarrow \frac{7^{5/2}}{7^{1/2}} = 7^{5/2} \times 7^{-1/2}$

$= 7^{5/2 - 1/2} = 7^2 = 49$

Q37. (B) Given $m^2 + 7m - 18$ and $m = 91$

$m^2 + 9m - 2m - 18$

$\Rightarrow m(m+9) - 2(m+9) \Rightarrow (m-2)(m+9)$

$\Rightarrow (91-2)(91+9) = (89)(100)$

$= 8900$

Q38. (C) Given $(a-b)^2 = 7$ and $a^2 + b^2 = 9$

$a^2 + b^2 - 2ab = 7 \Rightarrow 9 - 2ab = 7$

$\Rightarrow -2ab = -2$

$\Rightarrow \boxed{ab = 1}$

Q39. (A) Given x^2 and y^2

reciprocals of x^2 and y^2 are $\frac{1}{x^2}$ and $\frac{1}{y^2}$ and their difference is

$\frac{1}{x^2} - \frac{1}{y^2} \Rightarrow \frac{y^2 - x^2}{x^2y^2}$

Q40. (D) Given $a + b = 2.95$ $a - b = 1000$

$(a+b)(a-b) = (2.95)(1000)$

$a^2 - b^2 = 2950$

Q41. (A) Given $(x-7)(x+8) - (x-9)(x+10)$

$(x^2 + x - 56) - (x^2 + x - 90)$

$x^2 + x - 56 - x^2 - x + 90 = 34$

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Chapter 2

EQUATION

An equation is a statement that has an equal sign. The parts of an equation to the right and left of the sign of equality are called sides of the equation and are distinguished as the right side and left side.

Highest power of the variable determines the degree of the equation. The letters used for variables in an equation are called unknown quantity. The process of finding the values of variables is called solving the equation. The value so found is called the root or solution of the equation.

Linear Equation:

The equation in which the highest power of the variable is one, is called a simple or linear equation of the first degree.

Example:

$3x = 9,\ 2x + 5 = 7,\ x - 7 = 9 \quad \frac{x}{2} - \frac{2}{3} = 5$ are linear equations

Axioms of Solving Linear Equation:

The process of solving linear equation depends only upon the following axioms:

1. If we add equals in an equation on both sides, the sums are equal.
2. If from equals we take equals the remainders are equal.
3. If equals are multiplied to both sides of an equation the products are equal.
4. If equals are divided by equals then the quotients are equal.

Rules of Solving Linear Equation:

We use following rules to solving a linear equation.

Rule 1:

In a linear equation, any term may be transposed from one side of the equation to the other by changing sign.

Example 1:

Consider a equation

$$-7x + 14 = -3x - 18$$

Transposing $3x + 14 = 7x - 18$

or $18 + 14 = 7x - 3x$

which is the original equation with the sign of some terms are changed.

Example 2:

Solve $3x - 8 = 16$

Solution:

The variable x is multiplied by 3 and then 8 has been subtracted

$$\boxed{x} \xrightarrow{\times 3} \boxed{3x} \xrightarrow{-8} \boxed{3x - 8}$$

Transposing the operations of "×", "–" in other words "undo" or backtrack these two operations, first add 8, and then divide by 3.

$$\boxed{x} \xleftarrow[\div]{} \boxed{3x} \xleftarrow[+8]{} \boxed{3x - 8}$$

To keep this equation balance, the same operation must be carried out on both sides of the equation. The process of solving above equation is illustrated simply in two steps as fellows:

$$3x - 8 = 16$$
$$3x - 8 + 8 = 16 + 8$$
$$3x \div 3 = 24 \div 3$$

$$\boxed{x = 8}$$

Steps for Solving Linear Equations:

1. If the equation involves a fraction, first, if necessary, clear the fractions.
2. Transpose all the terms containing the unknown quantity to one side of the equation, and the known quantity to the other side of the equation.
3. Collect the terms on each side.
4. Divide both sides of the coefficient of the unknown variable.
5. Compute for the result.

Example:

Solve (i) $7x - 12 = 3x$ (ii) $\frac{4}{x} = \frac{-1}{3}$

(iii) $\frac{3}{1+x} = \frac{1}{2}$ (iv) $\frac{4}{3+a} + 1 = \frac{1}{3}$

Solution:

(i) $7x - 12 = 3x$

$7x - 3x = 12$

$4x = 12$

$x = 3$

Check

Substituting $x = 3$ in equation $7x - 12 = 3x$

$7(3) - 12 = 3(3)$

$21 - 12 = 9$

$9 = 9$

$\boxed{\text{Solution is correct}}$

(ii) $\frac{4}{x} = \frac{-1}{3}$

Multiplying both sides by $3x$

$\frac{4}{x} \times 3x = \frac{-1}{3} \times 3x$

$12 = -x$

$(-1)(x) \div (-1) = 12 \div (-1)$

$x = -12$

Note: $3x$ is LCD

Check

Substituting $x = -12$ in

$\frac{4}{x} = \frac{-1}{3}$

$\frac{4}{-12} = \frac{-1}{3}$

$\frac{-1}{3} = \frac{-1}{3}$

$\boxed{\text{Solution is correct}}$

(iii) $\frac{3}{1+x} = \frac{1}{2}$

Multiplying both sides by $2(1+x)$

$\frac{3}{1+x} \times 2(1+x) = \frac{1}{2} \times 2(1+x)$

$6 = 1 + x$

$6 - 1 = 1 + x - 1$

$x = 5$

Check

Substituting $x = 5$ in

$\frac{3}{1+5} = \frac{1}{2}$

$\frac{3}{6} = \frac{1}{2}$

$\frac{1}{2} = \frac{1}{2}$

$\boxed{\text{Solution is correct}}$

Check

(iv) $\frac{4}{3+a}+1=\frac{1}{3}$

Multiplying both sides by 3(3 + a)

$$\left(\frac{4}{3+a}\right)\times 3(3+a)+1\times 3(3+a)=\frac{1}{3}\times 3(3+a)$$

$$(4\times 3)+3(3+a)=3+a$$

Removing Brackets

$$12+9+3a=3+a$$
$$21+3a=3+a$$
$$21+3a-21-a=3+a-21-a$$
$$2a=-18$$
$$a=-9$$

Check

Substituting a = –9 in given equation

$$\frac{4}{3+(-9)}+1=\frac{1}{3}$$

$$\frac{4}{-6}+1=\frac{4-6}{-6}$$

$$=\frac{-2}{-6}=\frac{1}{3}$$

$$\frac{1}{3}=\frac{1}{3}$$

Solution is correct

Example:

If $\frac{1}{x}=\frac{1}{y}+\frac{1}{z}$, what is the value of x?

Solution:

$$\frac{1}{x}=\frac{1}{y}+\frac{1}{z}$$

$$\frac{1}{x}=\frac{z+y}{yz}$$

Multiplying both sides by (xyz)

$$\frac{1}{x}\times xyz=\frac{(z+y)}{yz}\times xyz$$

$$\frac{yz}{y+z}=\frac{x(y+z)}{(y+z)}$$

$$x=\frac{yz}{y+z}$$

Example:

If $x = y(a + b)$, find a in terms of x, y and b.

Solution:

$$x=y(a+b)$$

$$\frac{x}{y}=\frac{y(a+b)}{y}$$

$$\frac{x}{y}-b=a+b-b$$

$$\boxed{a=\frac{x}{y}-b}$$

Solving Second-Degree Equation:

A second-degree equation involving the variable x has the generalized form

$$ax^2+bx+c=0$$

where a, b, and c are constants with $a \neq 0$. Second-degree equations are usually called quadratic equations. A quadratic equation in which the term containing x is missing is called a pure quadratic equation. Examples of second-degree equations are

$$2x^2 - 5x + 12 = 0$$
$$4x^2 = 16$$
$$7x^2 - 12 = 3x + 5$$

Example:

If $z^2 = x^2 + y^2$ and $x > 0$, what is y in terms of x and z.

Solution:

$$z^2 = x^2 + y^2$$
$$z^2 - x^2 = x^2 + y^2 - x^2$$
$$\Rightarrow \quad y^2 = z^2 - x^2$$

Taking square root

$$\sqrt{y^2} = \sqrt{z^2 - x^2}$$

The value of $\sqrt{\ }$ is $\frac{1}{2}$

$$\therefore \quad (y^2)^{1/2} = \sqrt{z^2 - x^2}$$
$$\boxed{y = \sqrt{z^2 - x^2}}$$

Example:

If x is positive number and $x^2 - 25 = 56$, what is the value of x.

Solution:

$$x^2 - 25 = 56$$
$$x^2 - 25 + 25 = 56 + 25$$
$$x^2 = 81$$

Taking square root

$$\sqrt{x^2} = \sqrt{81}$$
$$x = \pm 9$$

But x is +ive (given)

$$\therefore \quad x = 9$$

Example:

What is the value of 2^{x+3}, when $3^{x+2} = 81$?

$$3^{x+2} = 81 \quad \Rightarrow \quad 3^{x+2} = 3^4 \Rightarrow x + 2 = 4$$
$$\Rightarrow \quad x = 2$$
$$\text{Now} \quad 2^{x+3} = 2^{2+3} = 2^5 = 32$$
$$\therefore \quad \boxed{2^{x+3} = 32}$$

The Index Laws:

For multiplying and dividing powers, we use some rules. These rules are called index laws. These rules are summarized below:

Multiplying powers	$x^a x^b = x^{a+b}$
Dividing powers	$\frac{x^a}{x^b} = x^{a-b}$

Power of a power	$(x^a)^b = x^{ab}$
Power of a quotient	$\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}$
Power of a product	$(xy)^a = x^a y^a$

Special Index:

Zero Index	$x^o = 1$
Index in fraction	$x^{1/a} = \sqrt[a]{x}$
Index in negative form	$x^{-a} = \frac{1}{x^a}$

Example:

Find the value of x when $27^{-2x+1} = 729^{-2x+3}$

Solution:

$$27^{-2x+1} = 729^{-x+3}$$

Take L.H.S. $27^{-2x+1} = (3^3)^{-2x+1} = 3^{3(-2x+1)} = 3^{-6x+3}$.....(1)

by Power of a power in Index law

Now take R.H.S. $729^{-x+3} = (3^6)^{-2x+3} = 3^{6(-2x+3)} = 3^{-12x+18}$.....(2)

comparing (1) and (2)

$$3^{-6x+3} = 3^{-12x+18}$$

$$\Rightarrow \quad -6x+3 = -12x+18$$

$$\Rightarrow \quad -6x+12x = 18-3$$

$$6x = 15 \Rightarrow x = \frac{15}{6}$$

$$\Rightarrow \quad \boxed{x = \frac{5}{2}}$$

Check:

Substitute $x = \frac{5}{2}$ in given equation

$$27^{-2\times 5/2+1} = 729^{-2\times 5/2+3}$$

$$27^{-4} = 729^{-2}$$

$$(3^3)^{-4} = (3^6)^{-2}$$

$$3^{-12} = 3^{-12}$$

Hence the solution is correct.

Systems of Linear Equations:

A system of equation is two or more equations considered together. If the equations in a system are linear, then it is called linear system of equations. The following system of the equations is a linear system of equations in two variables

$$\begin{cases} x+y=7 \\ x-y=3 \end{cases}$$

Simultaneous Equations:

A pair of equation which has two unknown, and are solved together, are called simultaneous equation. In simultaneous equations the values of unknown quantities satisfied both the given equations.

Example:

$$3x + 5y = 9$$
$$3x + 7y = -19$$

Solution of a System of Equations:

The solution of a system of equation is an order pair that is a solution of both equations.

This system of equations can be solved by following two method

(1) Substitution Method

(2) Elimination Method

(1) Substitution Method:

This method is illustrated in the following example:

Example:

Solving the following system of equations using substituting method

$$3x - 4y = 2$$
$$4x + 3y = 14$$

Solution:

$$3x - 4y = 2 \quad \text{.....(1)}$$
$$4x + 3y = 14 \quad \text{.....(2)}$$

Solving equation (1) for x in terms of y

$$3x - 4y = 2 \Rightarrow 3x = 4y + 2 \Rightarrow x = \frac{4y+2}{3}$$

Substituting the value of $x = \frac{4y+2}{3}$ in (2)

$$4\left(\frac{4y+2}{3}\right) + 3y = 14 \quad \text{.....(3)}$$

To get rid of friction multiply both sides of the equation (3) by 3

$$4(4y + 2) + 9y = 42$$
$$\Rightarrow 16y + 8 + 9y = 42$$
$$\Rightarrow 25y = 34$$
$$\Rightarrow \boxed{y = \frac{34}{25}}$$

To find the value of "x" substitute $y = \frac{34}{25}$, in equation (1) or (2). Here we substitute it in equation (1)

$$3x - 4\left(\frac{34}{25}\right) = 2$$
$$75x - 136 = 50$$
$$75x = 186$$
$$x = \frac{186}{75}$$
$$\boxed{x = \frac{62}{25}}$$

The solution of the equation in the form of order pair is $\left(\frac{62}{25}, \frac{34}{25}\right)$.

Elimination Method:

The process by which we get rid of either of the unknown quantities is called elimination. In this method one of the unknown is eliminated by adding or subtracting one equation from the other.

Note: Since multiplying each side of an equation by the same non-zero constant does not change the solution of the equation. Therefore, if the coefficient of the unknown are not the same size, one or both equations are first multiplied by an appropriate number.

Example 1:

Solve

$$x + 2y = 22 \quad \text{.....(1)}$$
$$x - 2y = 2 \quad \text{.....(2)}$$

Solution:

Since y terms have equal but opposite coefficient, eliminate by adding

$$x + \cancel{2y} = 22 \quad \text{.....(1)}$$
$$x - \cancel{2y} = 2 \quad \text{.....(2)}$$
$$\text{(by adding) } 2x = 24$$
$$\boxed{x = 12}$$

Substitute $x = 12$ in (1)

$$12 + 2y = 22 \Rightarrow 2y = 10$$
$$\boxed{y = 5}$$

Solution set is (12, 5)

Example 2:

Solve

$$3x + 6y = 11 \quad \text{.....(1)}$$
$$2x + 4y = 9 \quad \text{.....(2)}$$

Solution:

In above system of equations, to eliminate the x variable. Multiplying equation (1) by 2 and equation (2) by −3. Then add the resultant equation and solve for y

$$6x + 18y = 11 \quad \text{.....(1)}$$
$$-6x - 12y = -27 \quad \text{.....(2)}$$
$$6y = -5$$
$$\boxed{y = -\frac{5}{6}}$$

Note: The multipliers we chosen so that the coefficient of the variables we want to eliminate are additive inverses.

Substitute $y = -\frac{5}{6}$ in (1)

$$3x + 6\left(\frac{-5}{6}\right) = 11 \Rightarrow 3x = 16 \Rightarrow \boxed{x = \frac{16}{3}}$$

Solution is $\left(\frac{16}{3}, -\frac{5}{6}\right)$

Example 3:

What is the arithmetic mean (average) of x and y, when $3x + 4y = 21$, and $4x + 3y = 35$

Solution:

$$3x + 4y = 21 \quad \text{.....(1)}$$
$$4x + 3y = 35 \quad \text{.....(2)}$$
$$(1) + (2) \qquad 7x + 7y = 56$$
$$7x + 7y = 56 \Rightarrow 7(x + y) = 56 \Rightarrow x + y = 8$$

Arithmetic mean of x and y is $\frac{x+y}{2} = \frac{8}{2} = \boxed{4}$

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Multiple Choice Questions (MCQs)

Q1. If $3x + 9 = 18$, what is the value of $x + 3$?

(A) 3 (B) 6
(C) –3 (D) 36

Q2. If $5x + 12 = 44$, what is the value of $5x - 12$?

(A) 24 (B) 32
(C) 20 (D) 22

Q3. If $3x + 17 = 9 - x$, what is the value of x?

(A) 2 (B) 3
(C) –2 (D) –3

Q4. If $x - 5 = 9$, what is the value of $x^2 - 5$?

(A) 196 (B) 191
(C) 16 (D) 11

Q5. If $at - b = c - dt$, what is the value of t in terms of a, b, c and d?

(A) $\frac{b-c}{a-d}$ (B) $\frac{a}{b}$
(C) $\frac{c}{d}$ (D) $\frac{b+c}{a+d}$

Q6. If $\frac{1}{2}x + \frac{1}{4}x + \frac{1}{8}x = 22$, what is the value of x?

(A) 88 (B) 44
(C) 1 (D) 24

Q7. If $2x - 3 = 15$, what is the value of $(2x - 3)^2$?

(A) 81 (B) 227
(C) ± 225 (D) 225

Q8. If $81^{10} = 3^{x-7}$, what is the value of x?

(A) 47 (B) 27
(C) 51 (D) 14

Q9. If $\frac{1}{x-y} = 7$, then $x =$

(A) $x + \frac{1}{7}$ (B) $x - \frac{1}{7}$
(C) $\frac{1}{7} - x$ (D) $\frac{x}{7} - 1$

Q10. If $x = 2t + 5$, and $y = 4t^2$, what is y in terms of x?

(A) $\left(\frac{x-5}{2}\right)^2$ (B) $\frac{x+5}{2}$
(C) $(x-5)^2$ (D) $\frac{x-5}{4}$

Q11. If x is a positive number and $x^2 + 36 = 100$, what is the value of x?

(A) 6 (B) 8
(C) 14 (D) 64

Q12. If $4^{x+5} = 8^{x-1}$, what is the value of x?

(A) $\frac{3}{5}$ (B) $-\frac{5}{3}$

(C) $-\frac{3}{5}$ (D) 4

Q13. If $\sqrt{x} = 9$, then $x^2 - \sqrt{x}$ equals:

(A) 6561 (B) 2530

(C) 6552 (D) $\sqrt{6} - 9$

Q14. If $\frac{a+3}{6} = \frac{12}{a+4}$, then positive value of x equals:

(A) 5 (B) 12

(C) 15 (D) 18

Q15. If x and y are positive integers and $x^2 + 2y^2 = 41$, $2x^2 + y^2 = 34$, then $x^2 =$

(A) 6 (B) 8

(C) 75 (D) 9

Q16. For any positive integer p, $\$p = \frac{p^2}{3}$ and $£p = \frac{9}{p}$, which of the following is an expression for the product of $\$p$ and $£p$?

(A) $\frac{2}{p}$ (B) $\frac{1}{p}$

(C) p (D) $3p$

Q17. If a, b, and c are different positive odd integers and $a + b + c = 11$, what is the greatest positive value of c?

(A) 9 (B) 7

(C) 3 (D) 1

Q18. If $n + 5 = n \times 5$, then $n =$

(A) 1.25 (B) 1.5

(C) 0.5 (D) 5

Q19. If $\frac{a}{b} = .75$, then $4a - 3b =$

(A) 1 (B) 2

(C) 0 (D) 3

Q20. Let $ab = c$, where a, b and c are non zero numbers. If a is multiplied by 3 and c is divided by 3, this is equivalent to multiply b by:

(A) $\frac{1}{3}$ (B) 3

(C) $\frac{1}{9}$ (D) 9

Q21. If $5a = 3$, then $(5a + 3)^2 = ?$

(A) 9 (B) 36

(C) 4 (D) 25

Q22. If $a = \frac{1}{3}$, then $a^3 = ?$

(A) $\frac{1}{9}$ (B) $\frac{1}{3}$

(C) $\frac{3}{27}$ (D) $\frac{1}{27}$

Q23. If 3a – 5 = 7, what is the value of *a*?

(A) $\frac{2}{3}$ (B) 4

(C) 12 (D) 2

Q24. If $4 + \frac{5W}{2} = 19$, what is the value of W?

(A) 6 (B) 30

(C) 15 (D) 75

Q25. What is the value of (11 – y) when 121 – 11y = 77?

(A) 11 (B) 44

(C) 7 (D) 4

Q26. What is the value of $x^2 - 4$, when $x^6 - 4x^4 = 64$ and $x^2 = 4$?

(A) 16 (B) 20

(C) 12 (D) 8

Q27. One factor of $8x^3 - 27y^3$ is $(2x - 3y)$, what is the other factor?

(A) $(2x + 3y)$ (B) $(4x^2 + 9y^2)$

(C) $(4x^2 + 6xy + 9y^2)$ (D) $(4x^2 + 12xy + 9y^2)$

Q28. If $32^{x+y} = 16^{x+2y}$, then $x =$

(A) y (B) 5y

(C) $\frac{y}{3}$ (D) 3y

Q29. If $px - q = r - sx$, what is the value of x?

(A) $\frac{p+s}{r+q}$ (B) $\frac{r+q}{p+s}$

(C) $\frac{r-q}{p+s}$ (D) $\frac{r-q}{p-s}$

Q30. If one factor of $a^2 - b^2 + am + bm$ is a + b, then the other factor is

(A) (a + b – m) (B) (a – m)

(C) (b – m) (D) (a – b + m)

Explanatory Answers

Q1. (B) $3x + 9 = 18 \Rightarrow 3(x + 3) = 18$(Taking 3 common from L.H.S)

$\Rightarrow \frac{3(x+3)}{3} = \frac{18}{3} \Rightarrow x + 3 = 6$ (Dividing both sides by 3)

Q2. (C) Given that $5x + 12 = 44$, subtracting –24 on both sides of the given equation, we have $5x + 12 - 24 = 44 - 24$

$\Rightarrow 5x - 12 = 20$

Q3. (C) $3x + 17 = 9 - x \Rightarrow 3x + x = 9 - 17$

$\Rightarrow 4x = -8$

$\Rightarrow x = \frac{-8}{4} = -2$

Q4. (B) $x - 5 = 9 \Rightarrow x - 5 + 5 = 9 + 5$

(adding 5 both sides of the equation)

$\Rightarrow x = 14 \Rightarrow x^2 = (14)^2 = 196$

Now $x^2 - 5 = 196 - 5 \Rightarrow x^2 - 5 = 191$

Q5. (D) $at - b = c - dt \quad \Rightarrow \quad at + dt = b + c$

$\Rightarrow t(a + d) = b + c$

$\Rightarrow \quad t = \dfrac{b + c}{a + d}$

Q6. (D) $\frac{1}{2}x + \frac{1}{4}x + \frac{1}{8}x = 21$, (taking L.C.M, 8)

$\dfrac{4x + 2x + x}{8} = 21$, Multiplying both sides by 8, we have

$\dfrac{7x}{8} \times 8 = 21 \times 8 \quad \Rightarrow 7x = 21 \times 8$

$\Rightarrow \quad x = \dfrac{21 \times 8}{7}$

$\Rightarrow \quad x = 3 \times 8$

$\Rightarrow \quad x = 24$

Q7. (D) Given that $2x - 3 = 15$

Taking square both sides of the equation, we get

$(2x + 3)^2 = (15)^2$

$\Rightarrow \quad (2x + 3)^2 = 225$

Q8. (A)

$81^{10} = 3^{x-7}$

$(3 \times 3 \times 3 \times 3)^{10} = 3^{x-7}$

$(3^4)^{10} = 3^{x-7}$

$3^{40} = 3^{x-7}$

$\Rightarrow \quad 40 = x - 7$

$\Rightarrow \quad 40 + 7 = x - 7 + 7$

$\Rightarrow \quad \boxed{x = 47}$

Q9. (B) $\dfrac{1}{x - y} = 7$, Multiplying both sides of the equation by $(x - y)$, we have

$\dfrac{1}{x - y} \times (x - y) = 7 \times (x - y)$

$1 = 7x - 7y$

$\Rightarrow \quad 7y = 7x - 1 \quad \Rightarrow y = \dfrac{7x - 1}{7}$

$\Rightarrow y = x - \dfrac{1}{7}$

Q10.(C) Let $x = 2t + 5$...(i) and

$y = 4t^2$ ...(ii)

Solving (i), for t

$x = 2t + 5 \Rightarrow x - 5 = 2t + 5 - 5$

$\Rightarrow x - 5 = 2t \quad \Rightarrow \dfrac{x - 5}{2} = \dfrac{2t}{2}$

$$\Rightarrow \frac{x-5}{2} = t$$

Putting the value of t in (ii), we have

$$y = 4\left(\frac{x-5}{2}\right)^2$$

$$y = \frac{4}{4}(x-5)^2 \Rightarrow y = (x-5)^2$$

Q11.(D) $x^2 + 36 = 100 \quad \Rightarrow x^2 + 36 - 36 = 100 - 36$

$\Rightarrow \quad x^2 = 64$

$\Rightarrow \quad \sqrt{x^2} = \sqrt{64}$

$\Rightarrow \quad x = \pm 8$

Since x is positive $\Rightarrow \quad x = 8$

Q12.(B) $4^{x+5} = 8^{x-1}$

$\Rightarrow \quad (2 \times 2)^{x+5} = (2 \times 2 \times 2)^{x-1}$

$\Rightarrow \quad (2^2)^{x+5} = (2^3)^{x+5}$

$\Rightarrow \quad 2^{2 \times (x+5)} = 2^{3(x+5)}$

$\Rightarrow \quad 2^{2x+10} = 2^{3x+15}$

$\Rightarrow \quad 2x + 10 = 3x + 15$

$\Rightarrow \quad 2x - 3x = 15 - 10$

$\Rightarrow \quad -3x = 5$

$\Rightarrow \quad \frac{-3x}{-3} = \frac{5}{-3}$

$\Rightarrow \quad x = -\frac{5}{3}$

Q13.(C) $\sqrt{x} = 9 \Rightarrow x = 81 \Rightarrow x^2 = 6561$

Now $x^2 - \sqrt{x} = 6561 - 9$

$\Rightarrow x^2 - \sqrt{x} = 6552$

Q14.(A) $\frac{a+3}{6} = \frac{12}{a+4} \Rightarrow \quad (a+3)(a+4) = 12 \times 6$

$\Rightarrow \quad a^2 + 7a + 12 = 72$

$\Rightarrow \quad a^2 + 7a + 12 - 72 = 0$

$\Rightarrow \quad a^2 + 7a - 60 = 0$

$\Rightarrow \quad a^2 + 12a - 5a - 60 = 0$

$\Rightarrow a(a+12) - 5(a+12) = 0$

$\Rightarrow \quad (a-5)(a+12) = 0$

$\Rightarrow \quad \boxed{a = 5, -12}$

Q15.(D) Given $2x^2 + y^2 = 34$

$\Rightarrow \quad y^2 = 34 - 2x^2$

Substituting the value of y^2 in the first equation

$x^2 + 2y^2 = 41$, gives

$x^2 + 2(34 - 2x^2) = 41$

$\Rightarrow x^2 + 68 - 4x^2 = 41$

$\Rightarrow -3x^2 = 41 - 68 \Rightarrow -3x^2 = -27$

$\Rightarrow 3x^2 = 27 \Rightarrow x^2 = \frac{27}{3} \Rightarrow \boxed{x^2 = 9}$

Q16.(D) $\$p = \frac{p^2}{3}$ and $£p = \frac{9}{2p}$

$\therefore \quad \$p \,.\, £p = \frac{p^2}{3} \times \frac{9}{p} \Rightarrow \$p \,.\, £p = 3p$

Q17.(B) The set of positive odd integers is {1,3,5,7,9.....}

The sum of the three positive integers should be 11. If we take greatest possible value of c, then there exist least positive integers a and b, the value of least positive integers a and b is 1 and 3, so their sum = 1 + 3 = 4. Thus the greatest positive integer is $1 + 3 + c = 11 \Rightarrow 4 + c = 11$

$\Rightarrow \boxed{c = 7}$

Q18.(A) Given that $n + 5 = n \times 5$

Subtracting n both sides of the equation

$n + 5 - n = 5n - n$

$5 = n(5 - 1)$ (Taking n common)

$5 = 4n$

$\Rightarrow \quad \frac{5}{4} = \frac{4n}{4}$ (Dividing both sides by 4)

$\Rightarrow \quad n = 1.25$

Q19.(C) $\frac{a}{b} = .75 \quad \Rightarrow \frac{a}{b} = \frac{75}{100} \Rightarrow \frac{a}{b} = \frac{3}{4}$

$\Rightarrow 4a = 3b \Rightarrow 4a - 3b = 0$

Q20.(C) $ab = c \qquad \ldots(1)$

If a is multiplied by 3, and c is divided by 3, the above equation becomes

$3ab = \frac{c}{3} \qquad \ldots(2)$

The above equation (2), is equivalent to (1), if b is multiplied by $\frac{1}{9}$

$\therefore \quad 3a \times \frac{1}{9}b = \frac{c}{3}$

$\frac{1}{3}ab = \frac{c}{3} \qquad$ As $ab = c$

$\frac{1}{3}c = \frac{c}{3} \Rightarrow c = c$

Q21.(B) $5a = 3 \Rightarrow 5a - 3 = 0 \qquad \ldots(1)$

Adding 6 both sides of equation (1), we get

$5a - 3 + 6 = 0 + 6 \Rightarrow 5a + 3 = 6 \quad \ldots(2)$

Squaring both sides of equation (2), we get

$(5a + 3)^2 = (6)^2 = (5a + 3)^2 = 36$

Q22.(D) $a = \frac{1}{3} \quad \Rightarrow (a)^3 = \left(\frac{1}{3}\right)^3$

$\Rightarrow \quad a^3 = \frac{1 \times 1 \times 1}{3 \times 3 \times 3}$

$\Rightarrow a^3 = \frac{1}{27}$

Q23. (B) $3a - 5 = 7$

$\Rightarrow 3a - 5 + 5 = 7 + 5$ (adding 5 both sides)

$\Rightarrow 3a = 12$

$\Rightarrow \boxed{a = 4}$

Q24. (A) $4 + \frac{5W}{2} = 19$

$\Rightarrow 4 + \frac{5W}{2} - 4 = 19 - 4$ (To get rid 4 from L.H.S. subtract 4 both sides)

$\Rightarrow \frac{5W}{2} = 15$ (To get rid 2 from L.H.S. multiply 2 both sides)

$\frac{5W}{2} \times 2 = 15 \times 2 \Rightarrow 5W = 30$

last to get rid 5 from L.H.S. divide both sides by 5

$\frac{5W}{5} = \frac{30}{5} \Rightarrow \boxed{W = 6}$

Q25. c) Given $121 - 11y = 77$ taking 11 common from L.H.S.

$11(11 - y) = 77$ Dividing 11 both sides

$\frac{11(11 - y)}{11} = \frac{77}{11}$

$\Rightarrow 11 - y = 7$

Q26. a) Given $x^6 - 4x^4 = 64$ and $x^4 = 4$

$x^4(x^2 - 4) = 64$ (Taking x^4 common)

$4(x^2 - 4) = 64$ (Substituting the value of x^4)

$\Rightarrow x^2 - 4 = 16$ (Dividing both sides by 4)

Q27. (C) $8x^3 - 27y^3$

$\Rightarrow (2x)^3 - (3y)^3 = (2x - 3y)(4x^2 + 6xy + 9y^2)$ (factorizing)

Q28. (D) $32^{x+y} = 16^{x+2y}$

$(2)^{5(x+y)} = (2)^{4(x+2y)} \Rightarrow 2^{5x+5y} = 2^{4x+8y}$

$\Rightarrow 5x + 5y = 4x + 8y$

$\Rightarrow 5x - 4x = 8y - 5y$

$\boxed{x = 3y}$

Q29. (B) $px - q = r - sx$

$px + sx = r + q$

$\Rightarrow x(p + s) = r + q$

$x = \frac{r + q}{p + s}$

Q30. (D) $a^2 - b^2 + am + bm$

$(a - b)(a + b) + m(a + b)$ as $a^2 - b^2 = (a + b)(a - b)$

$(a + b)(a - b + m)$

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Chapter 3

INEQUALITY

An inequality, or inequation is a statement which involves one of the sign below:

$<$	Less than
$\leq$	Less than or equal to
$>$	Greater than
$\geq$	Greater than or equal to

Examples:

$$6x > 52$$
$$11y \geq -101$$
$$-3x \leq 8$$
$$-52w \leq 9$$

The set of all solutions of an inequality is called the solution set of the inequality. For example the solution of $x + 3 > 5$ is the set of all real numbers greater than 2.

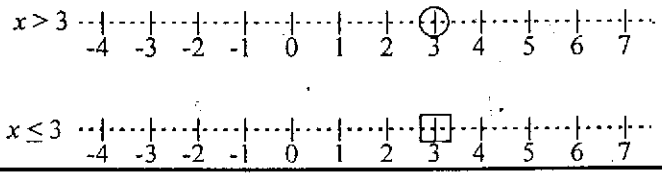
Note:

Equivalent Inequalities have the same solution set.

Representation of Inequality on number line:

Inequalities such as $x > 3$ or $x \leq 3$ can be represented on number line.

In following number line a circle "□" shows that x is included and a rectangle "O" shows that it is not



Note:

1. Any quantity x is said to be greater than another quantity y when $(x - y)$ is positive.

Example:

If $x = 2$ and $y = -3$, thus $x > y$ because $2 - (-3) = 5$ or positive.

2. y is said to be less than x when $y - x$ is negative.

Example:

If $x = 2$ and $y = -3$, than $y < x$ because $-3 - 2 = -5$ or negative.

Properties of Inequalities:

We apply the following properties to solve in-equalities.

1. An inequality will still hold after each side has been increased, decreased, multiplied or divided by the same positive quantity

If $x > y$

For example: $x + z > y + z$;

$x - z > y - z$;

$xz > yz$;

$\frac{x}{z} > \frac{y}{z}$

2. In an un-equality any term may be transposed from one side to the other if its sign be changed

If $x - y > z$

For example: $x > z + y$

3. If the sides of an inequality is transposed, then the sign of inequality is reversed

Example: If $x > y$, then evidently

$y < x$

4. If both sides of the inequality are multiplied or divided by a negative number, then direction of the inequalities sign is reversed

Example: If $x > y$, then $-x < y$ and

$\therefore \quad -xz < yz$

5. The square of real quantity is positive, therefore it is greater than zero.

Therefore $(x - y)^2$ is always positive

$\therefore (x - y)^2 > 0$

$\therefore x^2 + y^2 > 2xy$

6. If x and y are two positive quantities, then their arithmetic mean $\left(\frac{x+y}{2}\right)$ is greater than their geometric mean $\left(\sqrt{xy}\right)$.

$$\therefore \frac{x+y}{2} > \sqrt{xy}$$

Example:

Solve the following inequalities

(i) $3x - 11 < 13$ *(ii)* $\frac{-x}{2} \leq 2$

Solution:

(i)

$$3x - 11 < 13$$

$\therefore \quad 3x - 11 + 11 < 13 + 11$ (using property 1)

$\therefore \quad 3x < 24$

$\therefore \quad \frac{3x}{3} < \frac{24}{3}$ (using property 1)

$\therefore \quad x < 8$

(ii)

$$\frac{-x}{2} \leq 2$$

$\therefore \quad \frac{-x}{2} \times 2 < 2 \times 2$ (by property 1)

$\therefore \quad -x \leq 4$

$\therefore \quad -x \times -1 \geq 4 \times -1$ (using property 4)

$\therefore \quad x \geq -4$

Example: Find the greatest possible value of x, when the arithmetic mean of 5, 7 and x is less than 24.

Solution:

The arithmetic mean of three numbers 5, 7 and x is

$$\frac{5+7+x}{3}$$

By given condition $\frac{5+7+x}{3} < 24$

Now $\frac{(5+7+x)}{3} \times 3 < 24 \times 3$ (using property 1)

$\therefore \quad 12 + x < 72$

$\therefore \quad 12 + x - 12 < 72 - 12$ (using property 1)

$\therefore \quad x < 60$

Thus the greatest possible value of x is 59.

Example:

Solve $\frac{x}{4} - 4 > \frac{x}{5}$

Solution:

$$\frac{x}{4} - 4 > \frac{x}{5}$$

$$\frac{x}{4}-\frac{x}{5}-4+4 > \frac{x}{5}-\frac{x}{5}+4 \quad \text{(using property 1)}$$

$$\therefore \quad \frac{x}{4}-\frac{x}{5} > 4$$

$$\therefore \quad \frac{5x-4x}{20} > 4$$

$$\therefore \quad \frac{x}{20} > 4$$

$$\therefore \quad \frac{x}{20}\times 20 > 4\times 20 \quad \text{(using property 1)}$$

$$\therefore \quad x > 80$$

Multiple Choice Questions (MCQs)

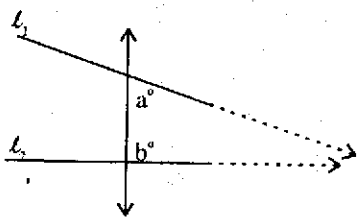
Q1. If $xy > 0$ and $x < 0$, which of the following negative?

(A) $-x$ (B) $-y$
(C) y (D) xy

Q2. If $a > 0$, $b > 0$ and $a - b < 0$, then?

(A) $a < b$ (B) $a + b < 0$
(C) $a > b$ (D) $b - a < 0$

Q3. If lines ℓ_1 and ℓ_2 meet when extended to the right, which inequality best expresses the relationship between a and b?



(A) $a = b$ (B) $a + b < 180$
(C) $a - b > 0$ (D) $b - a > 0$

Q4. If $a + b > 7$ and $a - b > 5$, then which of the following gives all possible values of a and only possible value of a?

(A) $a > 6$ (B) $a < 5$
(C) $a > 4$ (D) $a < 7$

Q5. If $A > B$ an $C < 0$, then which of the following is not true?

(A) $AC < BC$ (B) $A + C > B + C$
(C) $A - C < B - C$ (D) All of the above

Q6. If $a = 1$ and $1 > b > 0$, then which of the following statement is true?

(A) $a = b$ (B) $b > a$
(C) $\frac{1}{b} > a$ (D) $\frac{1}{a} < b$

Q7. If $a < c$ and $a < b$, assume $a \geq 0$ then which of the following statements are always true?

(i) $b < c$ (ii) $a < bc$ (iii) $2a < b + c$

(A) only (i) (B) only (ii)
(C) only (iii) (D) (i) and (ii)

Q8. If $6 - a > 7$, then

(A) $a > 1$ (B) $a > -1$
(C) $a < -1$ (D) $a < 1$

Q9. a has to be a whole number such that $0 \leq a \leq 10$. The solution for $a < 4$ and $a \geq 6$ is:

(A) 5 (B) 7
(C) 3 (D) no solution

Q10. If $5x > 2$ and $\frac{1}{2}x \leq 4$, list all the possible integral values of x?

(A) 2, 3, 4, 5, 6 (B) 1, 2, 3, 4, 5, 6, 7, 8
(C) 2, 3 (D) 1

Q11. The solution of the inequality $-1 < 5x - 6 \leq 4$ in whole number is

(A) 1 (B) 2
(C) 4 (D) 5

Q12. In inequality $y > 3x - 2$ if $a > b$, then which of the following statement is true?

(A) $x = 1$ (B) $x > 1$
(C) $x < 1$ (D) $x \geq 1$

Q13. If $\frac{a}{2} - 2 > \frac{a}{3}$, then which of the following statement is true?

(A) $a < 12$ (B) $a > 12$
(C) $a = 12$ (D) $a \geq 12$

Q14. Which of the following inequalities is the solution of the inequality $7a - 5 < 2a + 18$?

(A) $a < 23$ (B) $a > 13$
(C) $a \leq 23$ (D) $a \geq 13$

Q15. For which values of p is $p^2 - 5p + 6$ negative?

(A) $p < 0$ (B) $2 < p < 3$
(C) $x > 3$ (D) $x < 2$

Explanatory Answers

Q1. (C) The product of two numbers > 0 is only possible when either both numbers are positive or both are negative. Since $x < 0$, y must also be negative.

Q2. (A) In this case a and b are both positive ($a > 0$, $b > 0$), but $a - b$ is negative, which is only possible when $a < b$.

Q3. (B) When the lines will be extended to the right. They will make a triangle, and the sum of the angles of the triangle is 180°. Therefore, the sum of the two angles in a triangle is less than 180°.

Q4. (A) Since both inequalities have the same direction, therefore the corresponding sides can be added. Thus,

$$\begin{array}{r} a + b > 7 \\ a - b > 5 \\ \hline 2a > 12 \\ \boxed{a > 6} \end{array}$$

Q5. (C) If $A > B$ and $C < 0$, then multiplication of both sides by C reverses the inequality. Which implies

$AC < BC$. Also adding and subtracting in inequality, gives

$$A + C > B + C \quad \text{and}$$
$$A - C > B - C$$

But $A - C < B - C$ is not possible.

Q6. (C) Since b is a +ve fraction less than 1, therefore $\frac{1}{b}$ is a positive fraction greater than 1. Hence

$$\frac{1}{b} > a$$

Q7. (C) Statements (i) and (ii) are not always true.

Q8. (C) Given $6 - a > 7$

$\Rightarrow$ $-a > 1$

Dividing both sides by −1. This will reverse the inequality sign

$a < -1$

Q9. (D) Given set is {0,1,2,3,4,5,6,7,8,9,10}, the number $a < 4$ are {0,1,2,3} and the numbers $a \geq 6$ are {6,7,8,9,10}. Since there are no common elements between the last two sets. Therefore, there

is no solution of the inequality.

Q10. (B) $5x > 2$ $\quad \frac{1}{2}x \leq 4$

$x > \frac{2}{5}$ $\quad \frac{1}{2}x \times 2 \leq 4 \times 2$

$x \leq 8$

from above the integers greater than $\frac{2}{5}$ and less than and equal to 8 are 1,2,3,4,5,6,7,8

Q11. (B) Given $-1 < 5x - 6 \leq 4$, first of all get rid -6 then 5 in the middle term

To get rid -6, add 6 to each part

$-1 + 6 < 5x - 6 + 6 \leq 4 + 6$

$\Rightarrow \quad 5 < 5x \leq 10$

To get rid of 5, divide each part by 5.

$\frac{5}{5} < \frac{5x}{5} < \frac{10}{5}$

$1 < x \leq 2$

$\therefore$ only 2 is a whole number solution

Q12. (C) Since $x > y$ and $y > 3x - 2$, this implies that

$x > 3x - 2 \quad \Rightarrow \quad -2x > -2$

Dividing both sides by -2 will reverse the inequality symbol

$\frac{-2x}{-2} < \frac{-2}{-2}$

$\Rightarrow \quad \boxed{x < 1}$

Q13. b) Given $\frac{a}{2} - 2 > \frac{a}{3}$

adding $\frac{-a}{2}$ both sides of the inequality

$\frac{a}{2} - 2 - \frac{a}{2} > \frac{a}{3} - \frac{a}{2}$

$-2 > \frac{-a}{6} \Rightarrow \frac{-a}{6} < -2$

$-a < -12$

dividing both sides by -1 will reverse the inequality sign, therefore

$a > 12$

Q14. (A) $7a - 5 < 2a + 18$

$7a - 2a < 18 + 5$

$5a < 23$

$\Rightarrow \quad a < 23$

Q15. (B) Given $p^2 - 5p + 6$. The given expression factors into $(p - 3)(p - 2)$. If the expression is negative then the factors must have opposite signs. If $(p - 2)$ is negative and $(p - 3)$ is positive there are no such number. It is only possible when $(p - 3)$ is negative and $(p - 2)$ is positive, then $p > 2$ and $p < 3$. So, $2 < p < 3$ is the correct choice.

****

Chapter 4

WORD PROBLEMS

Multiple Choice Questions (MCQs)

Q1. **If 5 is subtracted from a certain number, the result is 7 less than twice the number. What is the number?**

(A) 2 (B) $\frac{1}{2}$

(C) 5 (D) 6

Q2. **Three times the first of three consecutive odd integers is 3 more than twice the third. What is the third integer?**

(A) 11 (B) 12

(C) 15 (D) 13

Q3. **Two-fifth of a certain number is 30. What is the number?**

(A) 75 (B) 25

(C) 90 (D) 150

Q4. **Saira weights 25 pounds more than Umbar. If together they weigh 205 pounds, what is the weight of Saira?**

(A) 90 (B) 105

(C) 115 (D) 135

Q5. **If the sum of two numbers is 36, and the larger is three times as larger as the smaller, what is the larger number?**

(A) 27 (B) 30

(C) 15 (D) 18

Q6. **The sum of integers p and q is 352. The units digits of p is 0. If p is divided by 10, the result is equal to q, what is the value of p?**

(A) 30 (B) 230

(C) 320 (D) 32

Q7. **A soap factory has 30 packers. Each packer can load $\frac{1}{8}$ of a box in 9 minutes. How many boxes can be loaded in $1\frac{1}{2}$ hours by all 20 packers?**

(A) 28 (B) $37\frac{1}{2}$

(C) 35 (D) $35\frac{1}{2}$

Q8. **Uzma is 15 years old. Asma is one-third older then Uzma. How many years ago when Asma was twice as old as Uzma is?**

(A) 5 (B) 12

(C) 15 (D) 10

Q9. **Mohin is now three times Mohsin's age. Four years from now Mohin will be y years old. In terms of y, how old will Mohsin be?**

(A) $\frac{x-4}{3}$ (B) $\frac{x+4}{3}$

(C) $x+4$ (D) $x-4$

Q10. **If the sum of one third of a number and twice the same number is 28, the number is:**

(A) 10 (B) 12

(C) 28 (D) 14

Q11. **A man's present age is x years. If his age in 8 years will be $\frac{4}{5}$ of what it will be in 20 years, then**

his present age is:

(A) 45 (B) 25
(C) 30 (D) 40

Q12. **When 42 is added to twice a number, the result is 346, the number is:**

(A) 304 (B) 242
(C) 152 (D) 265

Q13. **A man was 26 years old when his daughter was born. Now, he is three times as old as his daughter. How many years old is the daughter now?**

(A) 13 years (B) 22 years
(C) 15 years (D) 12 years

Q14. **13 years ago Shabbir's mother was 7 times as old as he was. She is now 48 years old. How many years old is Shabbir now?**

(A) 28 (B) 18
(C) 38 (D) 20

Q15. **If 5 years are added to a man's present age and that age is tripled, he will be 84. What is his present age?**

(A) 18 (B) 23
(C) 32 (D) 54

Explanatory Answers

Q1. (A) Let the required number be x. Then $x - 5 = 2x - 7$
$\Rightarrow$ $\boxed{x = 2}$. Thus the correct answer is 2.

Q2. (C) Let

$$\begin{aligned} x &= \text{first integer} \\ x+2 &= \text{second integer} \\ x+4 &= \text{third integer} \\ 3(x) &= 3 + 2(x+4) \\ 3x &= 3 + 2x + 8 \\ x &= 11 \\ \text{Third integer is } (x+4) &= \boxed{15} \end{aligned}$$

Q3. (A) Let the number $= x$, then

$$\frac{2}{5}x = 30$$

$$\Rightarrow \quad x = \frac{30 \times 5}{2}$$

$$\Rightarrow \quad x = 75$$

Q4. (C) Let the weight of Saira $= x$
and Umber's weight $= y$

$$x - 25 = y$$

and

$$\begin{aligned} x + y &= 205 \\ \Rightarrow \quad x - y &= 25 \\ x + y &= 205 \\ \hline 2x &= 230 \\ x &= \frac{230}{2} = 115 \text{ pound} \end{aligned}$$

Q5. (A) Let the smaller number $= x$
Then the larger number $= 3x$
Now $3x + x = 36$

$$\begin{aligned} 4x &= 36 \\ x &= 9 \end{aligned}$$

The larger number is $36 - 9 = 27$

Q6. (C) $p + q = 352$ and $\frac{p}{10} = q \Rightarrow p = 10q$

$$10q + q = 352 \Rightarrow 11q = 352 \Rightarrow q = 32$$
$$\text{Now } p + 32 = 352 \Rightarrow \boxed{p = 320}$$

Q7. **(B)** 30 packers will load $30 \times \frac{1}{8}$ or $\frac{30}{8}$ boxes in 9 minutes. There are 90 minutes in $1\frac{1}{2}$ hours. So the 30 packers will load $10 \times \frac{30}{8}$ or $37\frac{1}{2}$ boxes in $1\frac{1}{2}$ hours.

Q8. **(D)** Asma is one-third older or $\frac{1}{3} \times 15 = 5$ years older. Let x be the age of Uzma and $x + 5$ be Asma's age. When Asma was twice the age of Uzma, $2x = x + 5$ or $x = 5$. Uzma was 5 years old and Asma was $x = 5$ or 10 years old, twice Uzma's age. Since Uzma is 15 years old now, Uzma was 5 years old 10 years ago.

Q9. **(A)** Assume x for Mohin and y for Mohsin

$$x \text{ is three times y} \Rightarrow x = 3y$$
$$x \text{ in four years} \Rightarrow x = x + 4$$
$$\Rightarrow x = 3y + 4$$
$$\Rightarrow x - 4 = 3y$$
$$\frac{x-4}{3} = y$$

Q10. **(B)** Let x be the required number, then

$$\frac{1}{3}x + 2x = 28$$
$$\Rightarrow x + 6x = 84$$
$$\Rightarrow 7x = 84$$
$$\Rightarrow \boxed{x = 12}$$

Q11. **(D)** Present age $= x$

$$x + 8 = \frac{4}{5}(x + 20)$$
$$5x + 40 = 4x + 80$$
$$5x - 4x = 80 - 40$$
$$\boxed{x = 40}$$

Q12. **(C)** Let x be the required number, then

$$2x + 42 = 346$$
$$\Rightarrow 2x = 304$$
$$\Rightarrow \boxed{x = 152}$$

Q13. **(A)** Let x be the age of man and y be the age of his daughter

$$x - 26 = y \quad \text{.....(1)}$$
$$x = 3y \quad \text{.....(2)}$$

Substituting the value of x in (1)

$$3y - 26 = y$$
$$2y = 26 \Rightarrow \boxed{y = 13}$$

Q14. **(B)** Let x be the age of Shabbir

$$7(x - 13) = 48 - 13$$
$$7(x - 13) = 35$$
$$x - 13 = 5$$
$$\boxed{x = 18}$$

Q15. **(B)** Let x be the man's present age, then

$$3(x + 5) = 84$$
$$\Rightarrow x + 5 = 28$$
$$\Rightarrow \boxed{x = 23}$$

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Part 3 Geometry

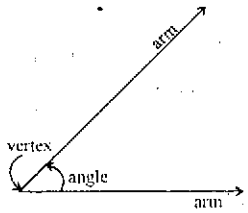
Chapter 1

LINES AND ANGLES

Angle:

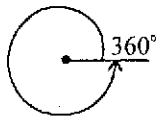
An angle is formed by the intersection of two line segments, which may be rays or lines.

In the diagram, an angle is shown by two lines (the arms) meeting at a point. The meeting point or point of intersection is called the vertex.

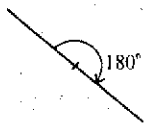


The unit of measure is the degree.

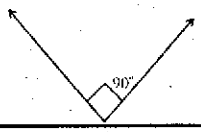
One full turn is 360 degrees (360°)



A half $\left(\frac{1}{2}\right)$ turn is 180 degrees (180°)



A $\frac{1}{4}$ turn is 90 degrees (90°).



Note:

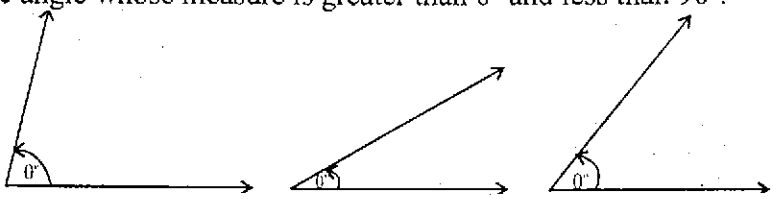
1. A half turn (180°) is also called straight angle.
2. A $\frac{1}{4}$ turn (90°) is also called a right angle.

Classification of Angles:

Angles are classified according to their degree measures.

Acute Angle:

An acute angle is the angle whose measure is greater than 0° and less than 90°.



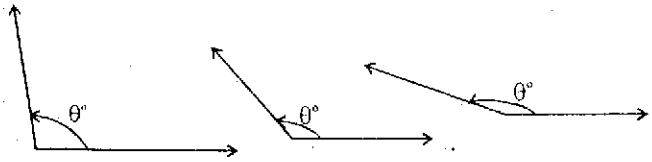
$0 < \theta < 90°$

In all above figures θ lies between 0 and 90°.

Obtuse Angle:

An angle whose measure is greater than 90° and less than 180° is called obtuse angle.

Examples:



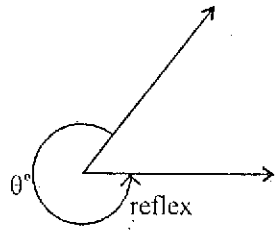
$90° < \theta < 180°$

In all above figures θ lies between 90° and 180°.

Reflex Angle:

A reflex angle is between 180° and 360°.

Example:

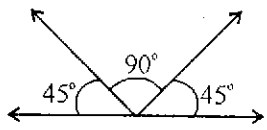


$180^\circ < \theta < 360^\circ$

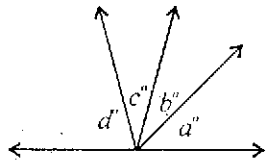
A reflex angle lies between 180° and 360° degrees.

Calculating Angles:

Angles on a straight line add up to 180°.



$45^\circ + 90^\circ + 45^\circ = 180^\circ$

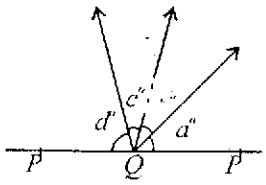


$a^\circ + b^\circ + c^\circ + d^\circ = 180^\circ$

A straight angle is 180°. So angles on a straight line add up to 180°.

Example 1:

What is the average of *a, b, c* and *d* in the following figure



Solution:

In the given figure since $\angle PQR$ is a straight angle. Because the angles on a straight line add up to 180°, therefore

$$a + b + c + d = 180^\circ$$

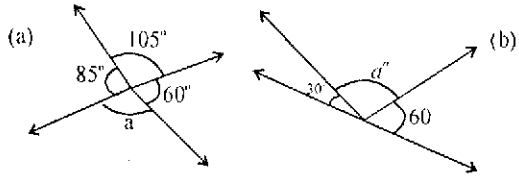
Average is

$$\frac{180^\circ}{4} = 45^\circ$$

Angles in a full turn add upto 360°.

Example 2:

Find the angle ***a*** in these diagrams.



Solution:

a) $a = 360^\circ - (60^\circ + 105^\circ + 85^\circ)$

$= 360^\circ - 250^\circ$

$= 110°$

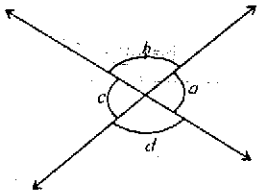
b) $a = 180° - (60° + 30°)$

$= 180° - 90°$

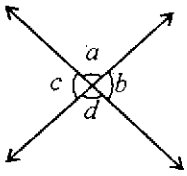
$= 90°$

Vertical Angles:

When two straight lines intersect, they make four angles. The two opposite angles are called vertical angles. In this diagram angles *a, c* and *b, d* are vertical angles.



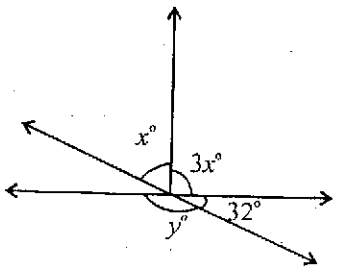
Vertically opposite angles are equal



$\angle a = \angle d$; $\angle c = \angle b$

Example 3:

Find the value of pronumerals in the following diagram, giving reasons:



Solution:

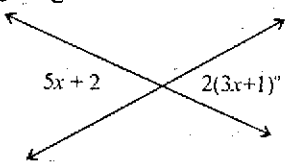
Because angles on a straight line add up to 180°, and vertically opposite angles are equal:

$\therefore \quad 3x + x + 32 = 180° \Rightarrow 4x = 148 \Rightarrow x = 37°$

Again $\quad y° = 3x° + x° \Rightarrow y = 4(37°) \Rightarrow y = 148°$

Example 4:

What is the value of x in the following diagram?



Solution:

Since the vertically opposite angles are equal:

$5x + 11 = 2(3x + 1)$

$5x + 11 = 6x + 2$

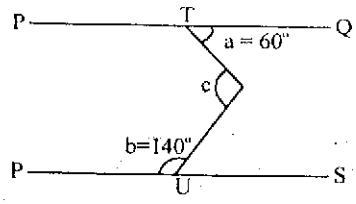
$\Rightarrow \quad x = 9$

Parallel Lines:

Parallel lines are always the same distance apart. They never meet, even if you make them longer. Parallel lines form no angles.

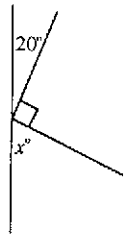
(A) 65 (B) 55
(C) 35 (D) Insufficient information

Q13. In the figure, line *PQ* is parallel to line *RS*, angle a = 60° and angle *b* = 140°. How many degrees are there in angle *c*?



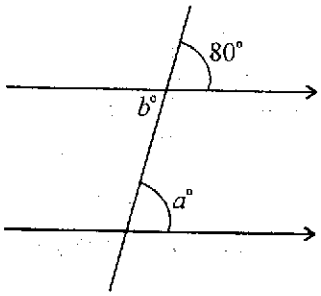
(A) 80 (B) 110
(C) 100 (D) 95

Q14. In the fig below, what is the value of *x*?



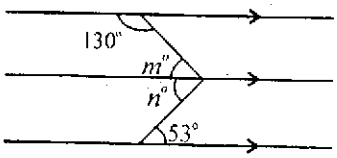
(A) 20° (B) 70°
(C) 100° (D) 110°

Q15. In the fig below, what is the value of *a*?



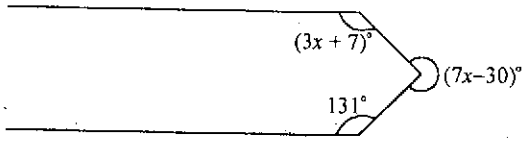
(A) 100° (B) 60°
(C) 80° (D) 40°

Q16. In the figure below, what is the value of *m* + *n*?



(A) 103° (B) 77°
(C) 130° (D) 85°

Q17. In the figure below, what is the value of *x*?



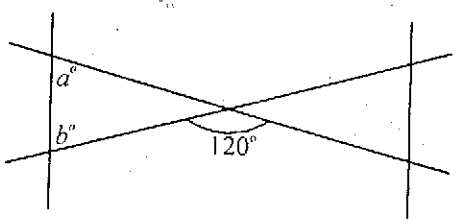
(A) 61 (B) 42

(C) 41 (D) 72

Q18. If P, Q and R are points on a line, with B between P and R. Let A and B be the mid points of PQ and QR, respectively. If $PQ : QR = 3 : 1$, what is $PQ : AB$?

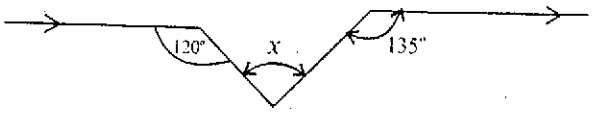
(A) 3 : 2 (B) 1 : 2
(C) 1.5 : .5 (D) 3 : 1

Q19. In the figure below, $a + b =$



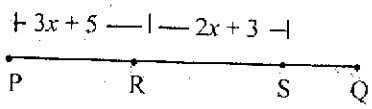
(A) 80 (B) 60
(C) 135 (D) 120

Q20. In the figure below, what is the value of x?



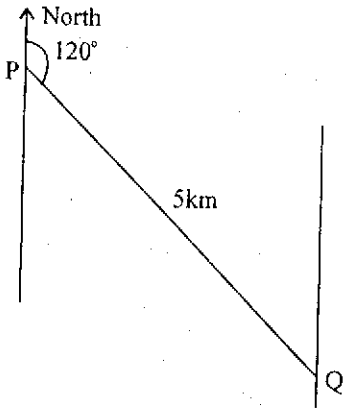
(A) 45° (B) 72°
(C) 57° (D) 65°

Q21. In the figure below, if the length of PQ is $5x + 9$, what is the length of SQ?



(A) 1 (B) 3
(C) 5 (D) 7

Q22. A ship leaves a port P and sails for 5km on a bearing of 120° to a port Q. What is the bearing of P from Q?



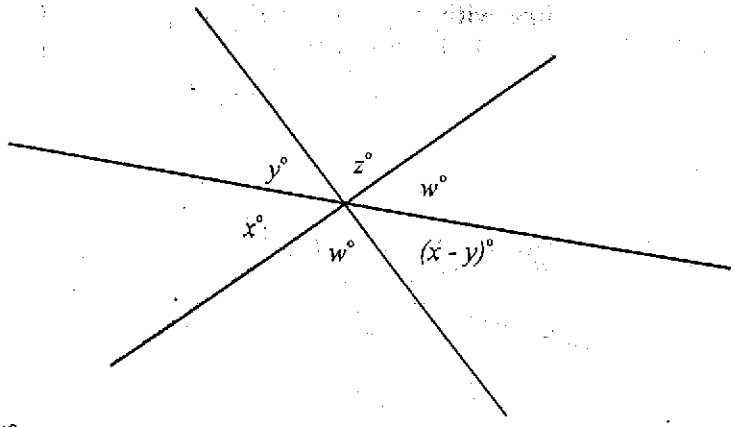
(A) 60° (B) 120°
(C) 40° (D) 300°

Q23. A hill-walker set off from P on a bearing of 225° to arrive at Q. What bearing must they take to retrace their steps?

(A) 55° (B) 135°
(C) 45° (D) 95°

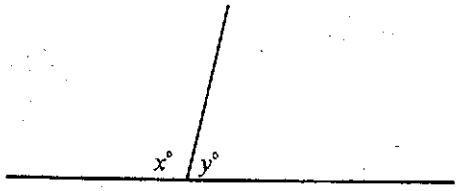
Q24. In the figure below, the straight line ABC is parallel to DE and BD is parallel to CF. $AD = BD$, $DBC = 110°$ and $FED = 45°$. What is the value of x?

Q4. In the following figure, what is the value of y?



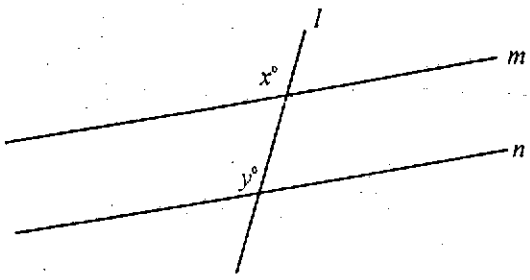
(A) 45°
(B) 36°
(C) 46°
(D) 35°

Q5. In the figure below, if x is 130 more than y, what is the value of y?



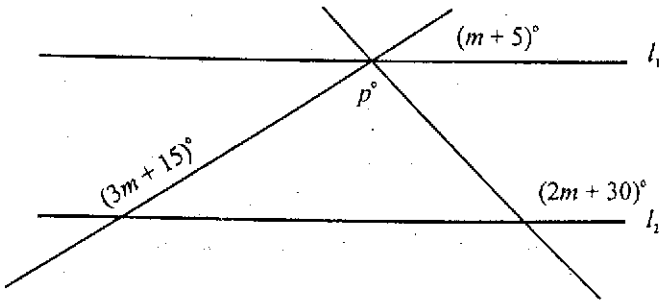
(A) 7
(B) 15
(C) 25
(D) 35

Q6. In the following figure, $m \parallel n$ and l is a transversal, then which of the following statement is (are) true?



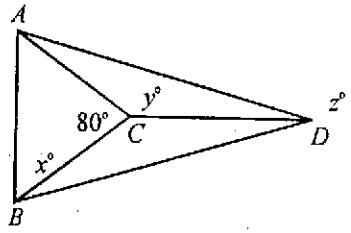
(A) $x + y = 180$
(B) $x - y = 180$
(C) $180 < x + y \leq 270$
(D) Insufficient information

Q7. In the following figure, if $l_1 \parallel l_2$, then what is the value of p?



(A) 70
(B) 45
(C) 40
(D) 65

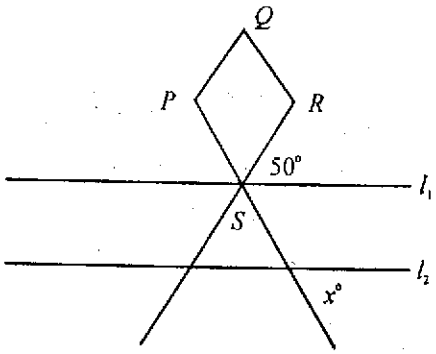
Q8.



In the above figure, AC = BC = CD and $m\angle ACB = 80°$. This information is sufficient to determine the value of which of the following?

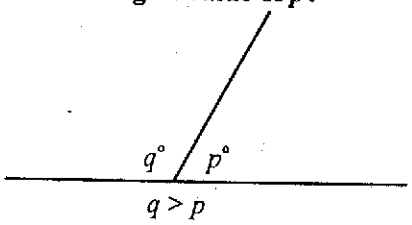
(A) x only (B) y only
(C) x and y only (D) y and z only

Q9. In the following figure, lines l_1 and l_2 are parallel, and line l_1 passes through S, one of the corners of square $PQRS$. What is the value of x?



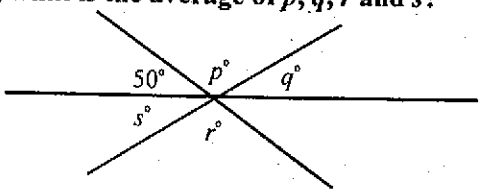
(A) 50 (B) 30
(C) 45 (D) 40

Q10. In the following figure, what is the largest value of p?



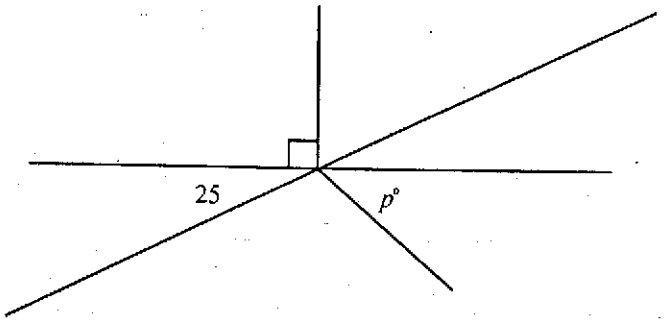
(A) 89 (B) 90.9
(C) 89.9 (D) 105

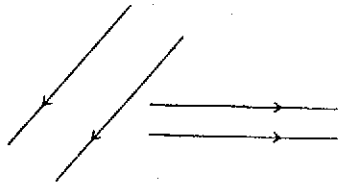
Q11. In the following figure, what is the average of p, q, r and s?



(A) 70° (B) 60°
(C) 75° (D) 65°

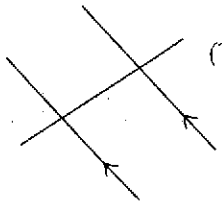
Q12. In the following diagram, what is the value of p?





Transversal:

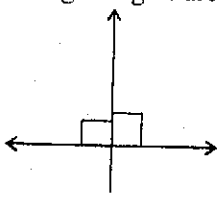
A straight line which cuts parallel lines is called a **transversal**.



(Transversal)

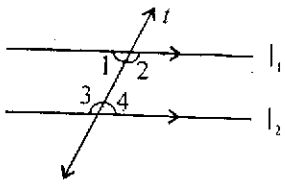
Perpendicular:

If two lines interest in such a way that they form right angles are called perpendicular

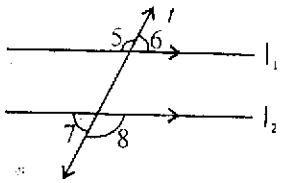


Interior Angles:

In the figure below, transversal t intersects lines ℓ_1 and ℓ_2 to form interior and exterior angle.



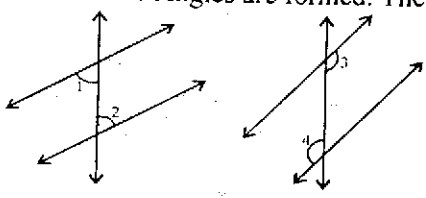
Interior Angles



Exterior Angles

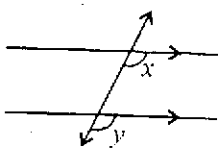
Alternate Angles:

When a transversal cuts parallel lines Alternative Angles are formed. These alternative angles are equal.

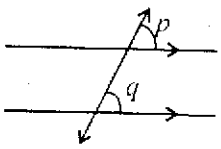


Corresponding Angles:

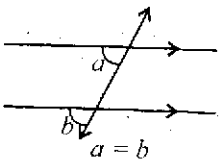
Corresponding angles are two angles in corresponding positions relative to the two lines and the transversal. These corresponding angles are also equal. A pair of equal corresponding angles is shown below.



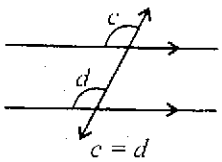
$x = y$



$p = q$

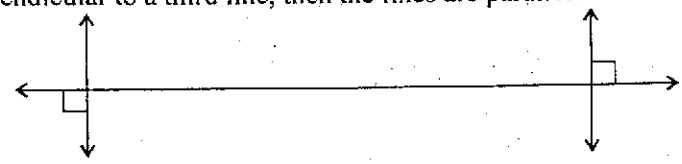


$a = b$



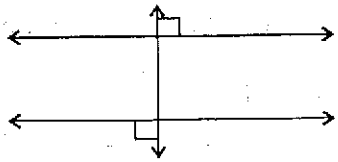
$c = d$

If two lines are both perpendicular to a third line, then the lines are parallel.



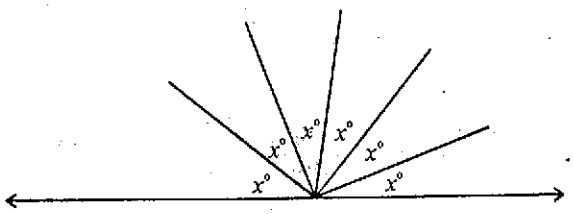
Alternatively

If a line is perpendicular to each of a pair of lines, then that pair of lines are parallel.



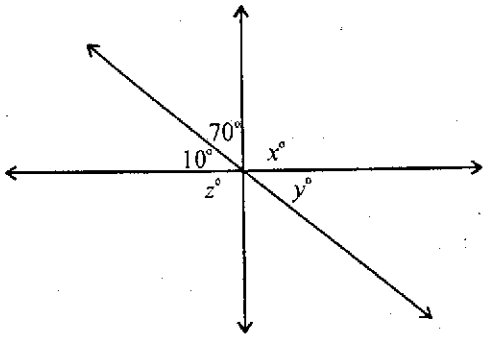
Multiple Choice Questions (MCQs)

Q1. In the following figure, what is the value of x?



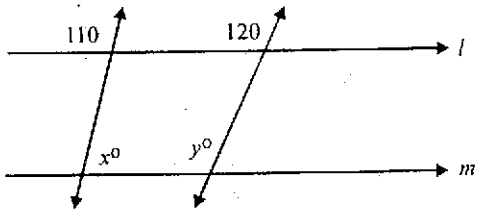
(A) 30 **(B)** 45
(C) 40 **(D)** 35

Q2. In the figure below, what is the value of $x + y + z$?



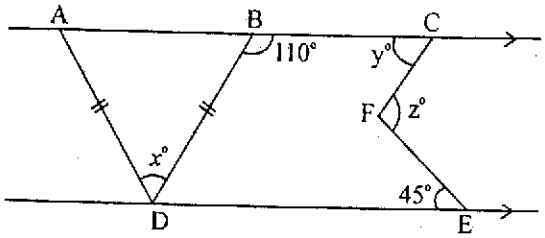
(A) 200 **(B)** 220
(C) 210 **(D)** 190

Q3. In the following figure, if $l \parallel m$



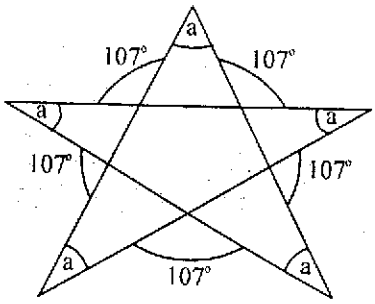
Then $x^\circ + y^\circ$ _____ 190°

(A) = **(B)** <
(C) > **(D)** ≑



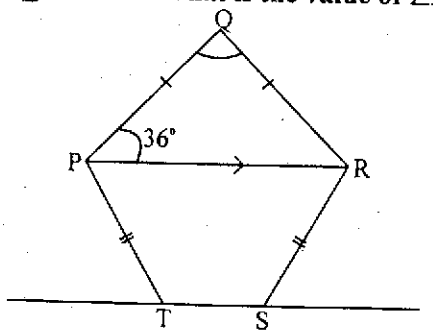
(A) 40 (B) 45
(C) 70 (D) 20

Q25. **In the copy of decoration given below, what is the value of angle *a*?**



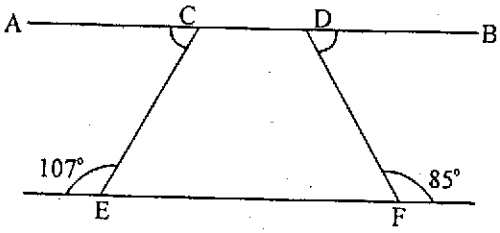
(A) 73° (B) 34°
(C) 107° (D) 45°

Q26. ***PQRST* represents a swimming pool with all its sides of equal length. A rope joins *PR* and is parallel to *TS*. Given that $\angle QPR = 36°$. What is the value of $\angle PQR$?**



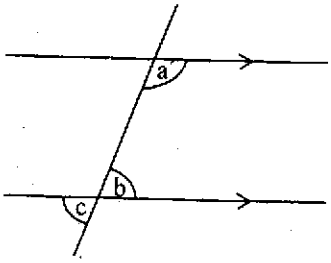
(A) 72 (B) 36
(C) 108 (D) 90°

Q27. **What is the sum of $\angle ACE + \angle BDF$?**



(A) 192 (B) 180
(C) 85 (D) 168

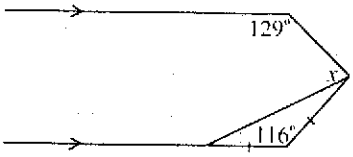
Q28. **In the figure below $a + b =$**



(A) $a + c$ (B) $a - b$

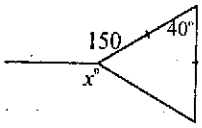
(C) $b + c$ (D) $c - a$

Q29. In the figure below, the value of x is:



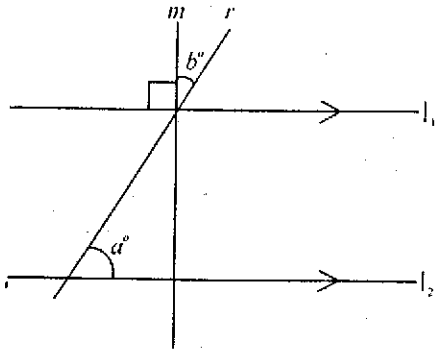
(A) 115° (B) 83°
(C) 69° (D) 43°

Q30. In the figure below, what is the value of x?



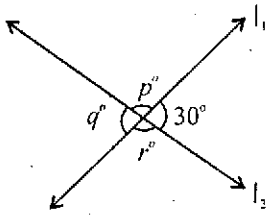
(A) 120 (B) 140
(C) 110 (D) 190

Q31. In the figure below, line ℓ_1 is parallel to line ℓ_2 and is perpendicular to line m. If $a = b$, what is the value of a?



(A) 30° (B) 90°
(C) 60° (D) 45°

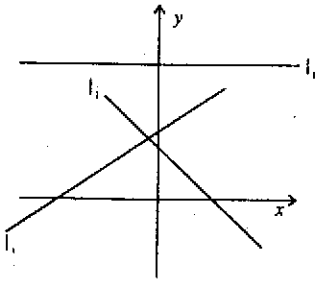
Q32. For the intersecting two lines ℓ_1 and ℓ_2 below, which of the following must be true?



I. $p > r$
II. $p = 5q$
III. $p + 30° = q + r$

(A) I only (B) I and II only
(C) III only (D) II and III only

Q33. In the figure below, lines ℓ_1, ℓ_2 and ℓ_3 have slopes a, b and c, respectively. Which of the following is a correct statement?

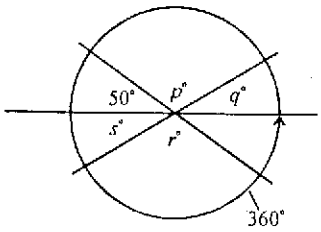


Q11. (D) $\because$ vertical angles are equal

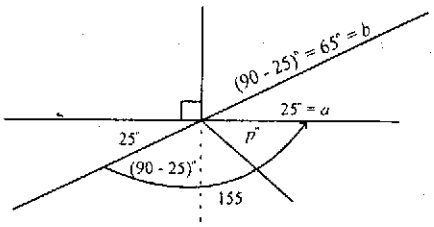
$\therefore p + q + r + s + 50 + 50 = 360$

$\Rightarrow p + q + r + s = 260$

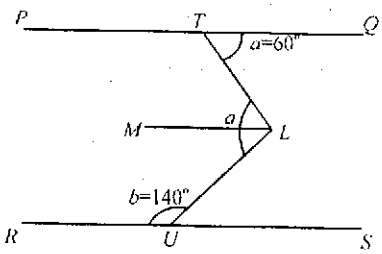
Average $\dfrac{p + q + r + s}{4} = \dfrac{260}{4} = 65$



Q12. (E) Since vertical angles are equal, here, $a = 25$, and so $p + q = 155$. We see that $b = 65$, and there is no other vertical angle. So it is impossible to determine p and q from the given information.



Q13. (C)



Through point L, draw ML parallel to RS and PQ.

$\angle c = \angle MLU + \angle MLT$

$\angle MLU = \angle LUS = 180° - 140° = 40°$

$\angle MLT = \angle LTQ = 60°$

Then $\angle c = 60° + 40° = 100°$

Q14. (B) Since $x + 90 + 20 = 180$, Therefore

$x = 70°$

Q15. (C) Since, vertical opposite angles are equal, therefore $\angle b = 80°$. Again because alternative angles are equal. Therefore, $\angle a = \angle b$

$\Rightarrow \angle a = 80°$

Q16. (D) $180 - 130 = 50°$

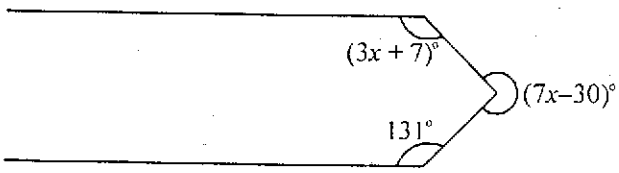
Because vertical opposite angles are equal

$\therefore \quad m = 50$

and $\quad n = 35°$

$m + n = 35° + 50° = 85°$

Q17. (B)



As $131° + (3x + 7)° = 7x - 30$

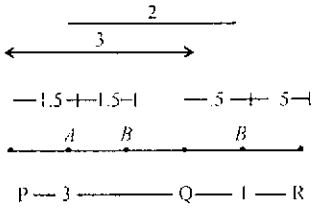
$131 + 3x + 7 = 7x - 30$

$\Rightarrow \quad 7x - 3x = 138 + 30$

$4x = 168°$

$x = 42°$

Q18. (A)

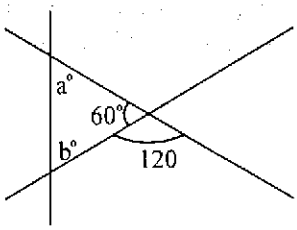


From above we see that

$$PQ : AB = 3 : 2$$

$$= \frac{3}{2} = 1.5$$

Q19. (D)



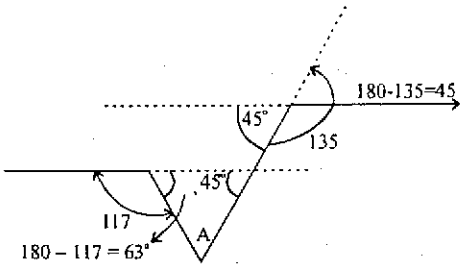
The 120 degrees angle and the angle next to it, is 60° because 180° – 120° = 60° (straight angle)

We know sum of all angles of any triangle is 180°

$$\therefore \quad a + b + 60^\circ = 180^\circ$$

$$a + b = 120^\circ$$

Q20. (B) The process of finding the value of x is illustrated by the following figure



Because the sum of the angles of a triangle always equal to 180°

$$\therefore \quad x + 45^\circ + 63^\circ = 180^\circ$$

$$\Rightarrow \quad x = 72^\circ$$

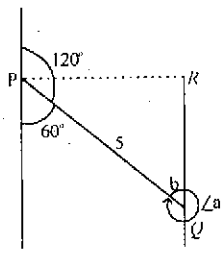
Q21. (A) Using figure we can see

$$SQ = PQ - (PR + RS)$$

$$SQ = 5x + 9 - (3x + 5 + 2x + 3)$$

$$SQ = 5x + 9 - 5x - 8$$

$$SQ = 1$$



Q22. (D) $\angle b = 180 - 120^\circ = 60^\circ$

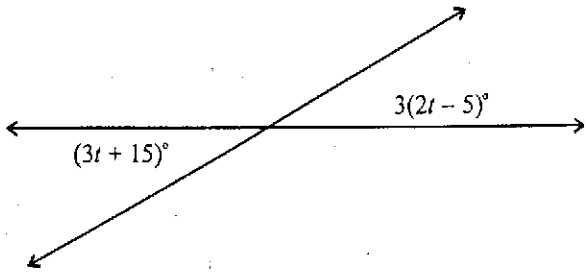
bearing of P from Q $= \angle a = 360 - \angle a$

$$= 360 - 60$$

$$= 300^\circ$$

(A) 30° (B) 40°
(C) 45° (D) 60°

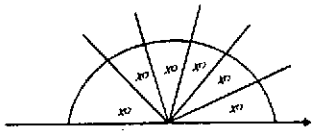
Q43. In the following figure, what is the value of t?



(A) 10 (B) 15
(C) 30 (D) 45

Explanatory Answers

Q1. (A) The sum of the given six angles make a straight angle, and the straight angle equal 180°. Thus

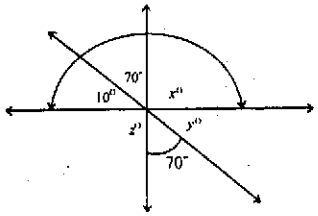


$x + x + x + x + x + x = 180^\circ$

$\Rightarrow \quad 6x = 180^\circ$

$\Rightarrow \quad x = 30^\circ$

Q2. (C) In the given figure, the arc shows a straight angle, hence



$10 + 70 + x = 180^\circ$

$x = 100$

Because opposite angles are equal, thus

$x = z^\circ = 100 \Rightarrow z = 100$

Similarly, $z + 70 + y = 180^\circ$

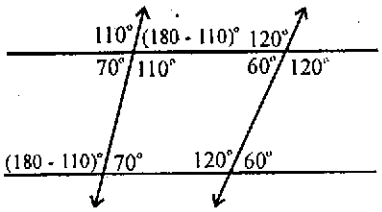
$100 + 70 + y = 180^\circ \quad (\because z = x = 100)$

$\Rightarrow \quad y = 10$

Thus,

Sum of the angles $x + y + z = 100 + 10 + 100$

$\Rightarrow \quad \boxed{x + y + z = 210}$



Q3. (A) Because when two straight lines intersect each other, the corresponding angles are equal. This fact is shown in the adjacent figure

Hence $x^\circ = 70$ and $y = 120$

$x^\circ + y^\circ = 120 + 70 \Rightarrow x^\circ + y^\circ = 190$

Q4. (B) Because vertical angles are equal, therefore

$x - y = y \Rightarrow x = 2y$

and $x = w$ also $z = w$

If we add y, z and w, then the sum of these angles is a straight angle which is equal to 180°.

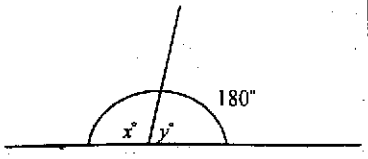
Thus, $y + z + w = 180$

$y + 2y + 2y = 180 \quad (\because z = w = x = 2y)$

$5y = 180$

$\Rightarrow \quad \boxed{y = 36}$

Q5. (C) In the given figure, the sum of the given two angles x and y is a straight angle, and straight angle equals to 180, therefore



$x + y = 180 \quad \ldots\ldots(i)$

By given condition

$$x = y + 130$$

$$\Rightarrow \quad x - y = 130 \qquad \text{.....(ii)}$$

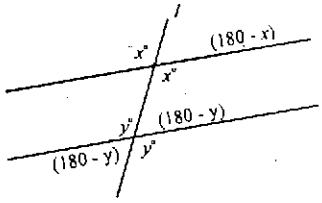
Adding (i) and (ii), we have

$$x + y = 180$$
$$x - y = 130$$
$$2x = 310$$
$$\Rightarrow \quad \boxed{x = 155}$$

Substituting the value of x, in (i), we have

$$155 + y = 180$$
$$\Rightarrow \quad y = 180 - 155$$
$$\Rightarrow \quad \boxed{y = 25}$$

Q6. (D) Since, there is not enough information, and the figure is not drawn in right scale, thus it is not possible to get the exact value of $x^o + y^o$.



Q7. (D) Since, when two straight lines intersect, the vertical angles are equal, therefore

$$x^o = m + 5$$

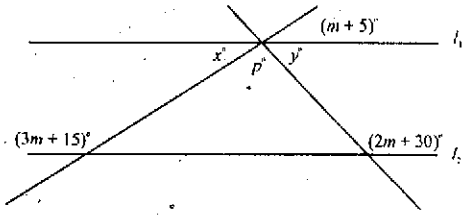
and $\quad x + (3m + 15) = 180$, but $x = m + 5$

Hence,

$$(m + 5) + (3m + 15) = 180 \Rightarrow 4m + 20 = 180 \Rightarrow 4m = 160$$

$$\Rightarrow \quad \boxed{m = 40}$$

As, $\quad x = m + 5 \Rightarrow x = 40 + 5 \Rightarrow \boxed{x = 45}$

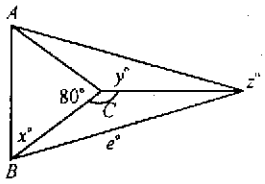


From figure, $x + p = 2m + 30$ ($\because$ Alternative angles are equal)

$45 + p = 2(40) + 30 \quad \Rightarrow 45 + p = 80 + 30 \Rightarrow 45 + p = 110$

$\Rightarrow p = 110 - 45 \quad \Rightarrow \boxed{p = 65}$

Q8. (A) Since $AC = BC$, we know that when two sides of a triangle are equal then their opposite angles are also equal. Hence $\angle A = \angle B$, and $\angle A = \angle B = x^o$. In any triangle, the sum of the three angles is equal to 180.



$\angle A + \angle B + \angle C = 180^o \Rightarrow x^o + x^o + 80^o = 180^o \Rightarrow 2x = 100$

$x = 50$

$\because$ y and e are not necessarily equal, therefore, we cannot determine y and z. The answer is x only.

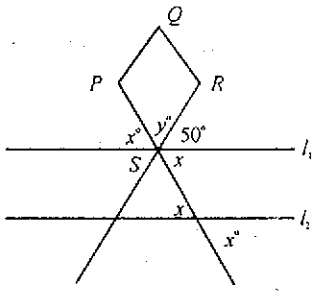
Q9. (D) $\because$ $PQRS$ is a square,

$\therefore \quad y = 90$. Then

$$x + y + 50 = 180$$
$$x + 90 + 50 = 180$$

($\because$ Alternative angles are equal)

$$x + 140 = 180 \quad \Rightarrow x = 40$$

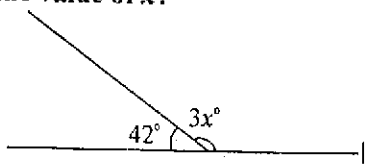


Q10. (C) $\because q > p$, then q must be greater than 90 and p less than 90. Therefore, the largest number less than 90 that can fit in the grid is 89.9.

(A) $b < a < c$ (B) $a < b < c$

(C) $c < b < a$ (D) $c < a < b$

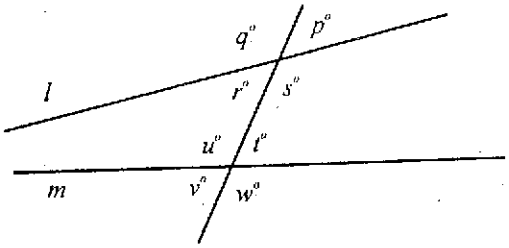
Q34. In the figure below, what is the value of x?



(A) 74° (B) 16°

(C) 46° (D) 14°

Q35. In the following diagram $l \parallel m$,

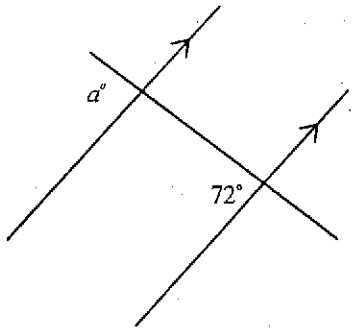


If A represents the average measure of all the eight angles, then A =

(A) 45° (B) 180°

(C) 90° (D) 360°

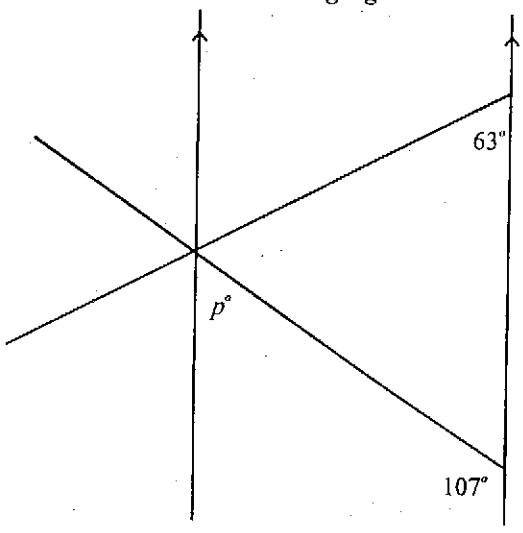
Q36. In the following figure, what is the value of $a°$?



(A) 108° (B) 72°

(C) 36° (D) 16°

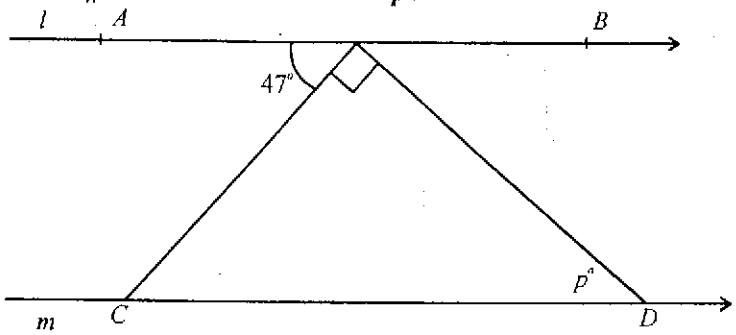
Q37. What is the value of pronumerals, in the following figure?



(A) 107° (B) 73°

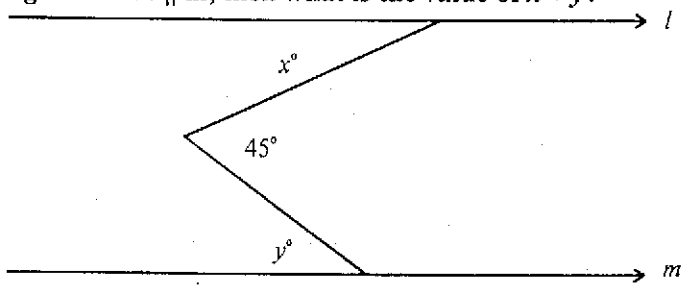
(C) 170° (D) 44°

Q38. In the figure below $l \parallel m$. What is the value of p?



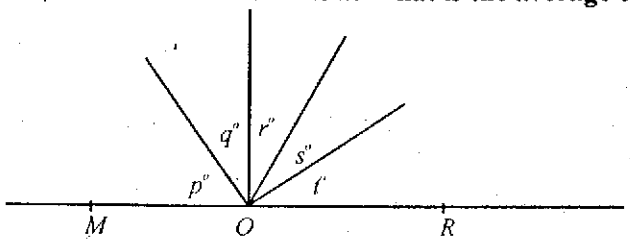
(A) 53° (B) 43°
(C) 47° (D) 30°

Q39. In the following figure lines $l \parallel$ m, then what is the value of $x + y$?



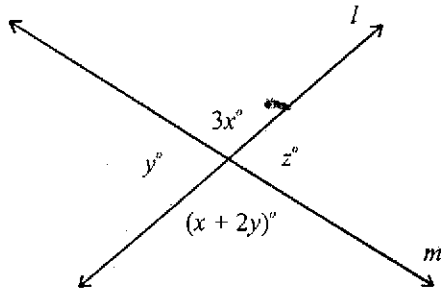
(A) 45° (B) 35°
(C) 135° (D) 180°

Q40. In the figure below M, O and N are all on line n. What is the average of p, q, r, s and t?



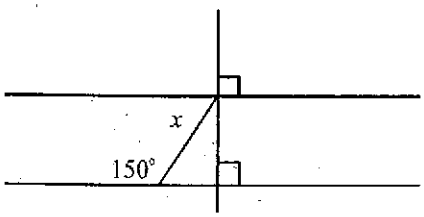
(A) 90 (B) 30
(C) 36 (D) 21

Q41. In the following figure, what is the value of x?



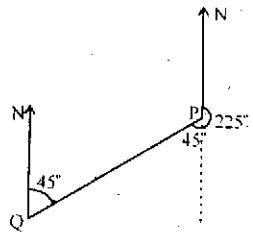
(A) 90° (B) 45°
(C) 85° (D) 180°

Q42. In the following figure, what is the value of x?



Q23. **(C)** The north lines are parallel and so $\angle NQP = 45^\circ$

So to retrace their steps, the hill-walker must take a bearing of 45° from Q



Q24. **(A)** $\angle BAD = \angle ABD$

$= 180^\circ - 110^\circ = 70^\circ$

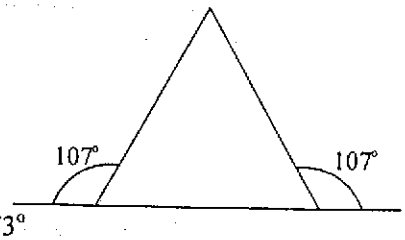
$\angle ADB = 180^\circ - 2\angle BAD$

$= 180^\circ - 140^\circ$

$= 40^\circ$

$\therefore \quad x = 40^\circ$

Q25. **(B)** Take one side of the copy of this decoration



from above $\angle b = 180 - 107 = 73^\circ$

In above triangle two angles are 73°, the third angle is

$\angle a = 180 - 73 - 73$

$= 34^\circ$

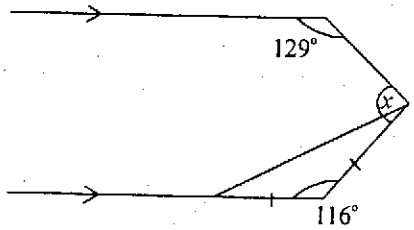
Q26. **(C)** Because In ΔPQR, the value of $\angle P$ and $\angle R$ is 36. Therefore the third angle $\angle PQR$ must equal to 108° (180 – 36 – 36)

Q27. **(D)** In the given figure $\angle CDF = 85^\circ$ and $\angle DCE = 107^\circ$ because the vertically opposite angles are equal. Now $\angle BDF = 95$ (180 – 85) and $\angle DCE = 73$, (180 – 107)

Sum of the angles $\angle BDF + \angle ACE = 168^\circ$.

Q28. **(A)** b and c are vertical angle and therefore equal angles. a + b = a + c

Q29. **(B)**



$116^\circ + y + y = 180^\circ$

$\Rightarrow \quad 2y = 180^\circ - 116 = 64$

$y = 32^\circ$

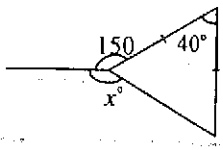
As $\quad x + y + 116 + 129 = 360^\circ$

$x + 32 + 116 + 129 = 360^\circ$

$x + 277 = 360^\circ$

$\Rightarrow \quad x = 360^\circ - 277 = 83^\circ$

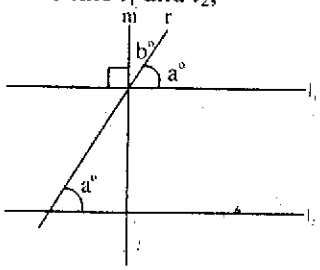
Q30. **(B)**



In triangle

$$40^\circ + y + y = 180^\circ$$
$$2y + 40 = 180^\circ$$
$$\Rightarrow \quad 2y = 180 - 40 = 140$$
$$y = 70^\circ$$
$$70^\circ + 150 + x = 360^\circ$$
$$x + 220 = 360^\circ$$
$$\Rightarrow \quad x = 360^\circ - 220 = 140^\circ$$

Q31. **(D)** Since line m is perpendicular to line l_1 and l_2,



$$a^\circ + b^\circ = 90^\circ$$
$$a^\circ + a^\circ = 90^\circ$$
$$2a^\circ = 90 \Rightarrow \boxed{a^\circ = 45}$$

Q32. **(D)** Since vertically opposite angles are equal. Therefore, $\angle q = \angle 30^\circ$ and sum of the straight angles is 180°. Thus

$$a^\circ + 30 = 180 \Rightarrow a = 150$$
$$\Rightarrow a = 5(q) \text{ and } p + 30^\circ = q + r$$

Q33. **(A)**

Q34. **(A)** $42 + 3x = 180^\circ \Rightarrow 3x = 138$

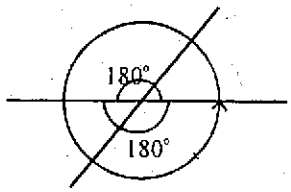
$$\Rightarrow \quad x = 46^\circ$$

Q35. **(C)** Sum of the angles $p + q + r + s = 360$

Similarly $t + u + v + w = 360$

Sum of the measure of above 8 angles $= 360 + 360 = 720$

Average $= \dfrac{720}{8} = 90^\circ$

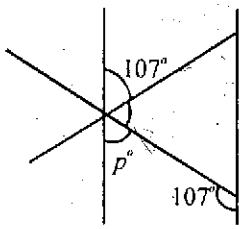


Q36. **(B)** Since corresponding angles of parallel lines are equal, therefore,

$$a = 72$$

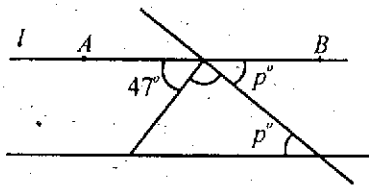
Q37. **(B)** $p^\circ = 73$

$$107^\circ + p^\circ = 180^\circ$$
$$p^\circ = 180^\circ - 107^\circ = 73$$



Q38. **(B)** The angles in the given figure, is decomposed as shown in the adjacent figure

Thus $47 + 90 + p^\circ = 180^\circ$



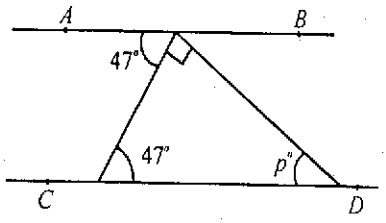
$137 + p^\circ = 180^\circ$

$p^\circ = 180 - 137$

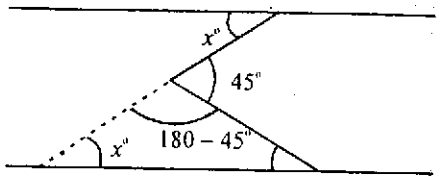
$\boxed{p^\circ = 43}$

Second Method: Because in any triangle, the sum of the three angles is equal to 180°, thus $47^\circ + 90^\circ + p^\circ = 180$

$\Rightarrow \boxed{p^\circ = 43}$

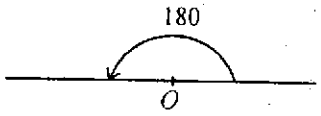


Q39. **(A)** Extend the line which makes angle x° with the upper line towards the down-ward line. Sine l and m are parallel, the measure in the bottom line in the triangle equals. In any triangle



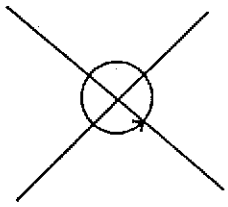
$$x + y + (180 - 45) = 180^\circ$$
$$x + y + 135 = 180^\circ$$
$$x + y = 180 - 135$$
$$x + y = 45$$

Q40. **(C)** In the figure, the angle MOR is straight angle. Thus, sum of the angles p, q, r, s and t is 180 and their average is



$$\frac{180}{5} = 36.$$

Q41. **(B)** Because, when the straight lines intersect each their then vertical angles are equal. Thus, in the given figure



$y^\circ = z^\circ$

Similarly $3x^\circ = (x + 2y)^\circ$

$\Rightarrow$ $3x - x = 2y \Rightarrow 2x = 2y \Rightarrow \boxed{x = y}$

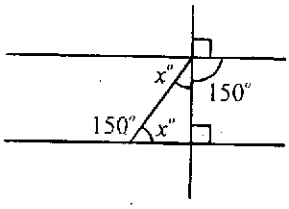
$\because$ $x = y = z \Rightarrow x = z$

Hence the four angles, are equal.

As the sum of angles $= 180^\circ$

Value of each angle $= \frac{180}{4} = \boxed{45^\circ}$

Q42. **(A)** Since a line is perpendicular to each pair of lines, thus the pair of lines are parallel, and when a line intersect pair of lines corresponding angles are equal.



First Method: Calculation of upper line

$x + 150 = 180$

$\Rightarrow$ $\boxed{x = 30}$

Second Method: Calculation of lower line

$150 + x^\circ = 180^\circ$

$\Rightarrow$ $\boxed{x = 30}$

Third Method: Calculation of triangle

$\because$ Sum of the three angles in a triangle equals 180°

$\therefore$ $x^\circ + 90 + (150 - 90) = 180^\circ$

$x^\circ + 90 + 60 = 180^\circ$

$x^\circ + 150 = 180^\circ$

$\Rightarrow$ $\boxed{x = 30^\circ}$

Q43. **(A)** When two lines intersect each other corresponding angles are equal, thus

$$3t + 15 = 3(2t - 5)$$
$$3t + 15 = 6t - 15$$
$$30 = 3t$$
$$t = 10$$

****

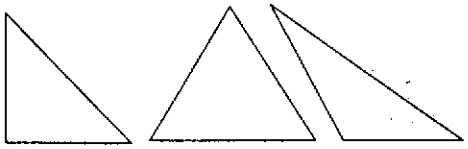
Chapter 2

TRIANGLES

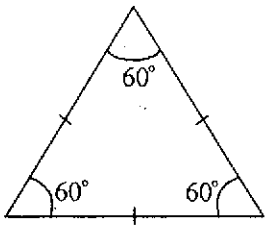
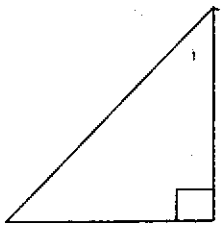
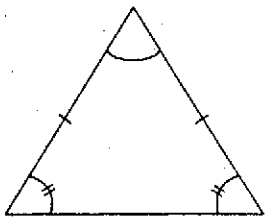
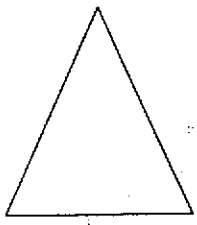
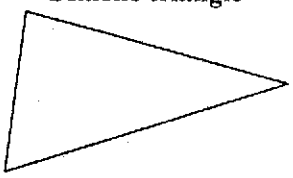
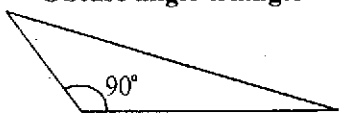
Triangle:

A three-sided polygon is called a triangle.

Examples:

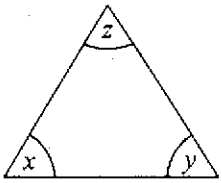


Types of Triangle:

Due to side	Due to Angle
Equilateral triangle  An equilateral triangle has 3 equal sides.	**Right angle triangle**  A right angle triangle has one angle has 90°.
Isosceles triangle  An isosceles triangle has 2 equal sides.	**Acute angle triangle**  An acute angle triangle all 3 angles measurement are less than 90°.
Scalene triangle  A scalene triangle has all 3 sides of different lengths.	**Obtuse angle triangle**  An obtuse angle triangle has one angle greater than 90°.

Angle's Sum of Triangle:

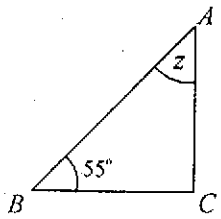
In any triangle, the sum of the measures of the three angles is 180°.



$$x + y + z = 180^\circ$$

Example 1:

In the figure below, what is the value of z?



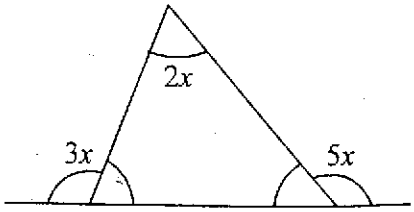
Solution:

Because, the angle of a triangle add up to 180°. Therefore

$$55^\circ + 90^\circ + Z = 180^\circ$$
$$\Rightarrow \quad Z = 180^\circ - 145^\circ$$
$$\Rightarrow \quad Z = 35^\circ$$

Example 2:

Calculate the value of x in the following figure:



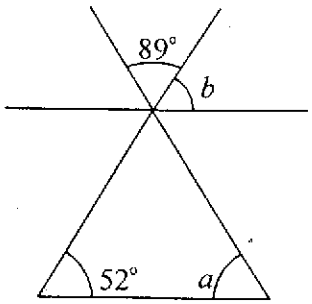
Solution:

Because the sum of the straight angles is 180°, therefore the missing angles of the triangle are $(180^\circ - 3x)$, $(180^\circ - 5x)$ and 2x.

$$(180^\circ - 5x) + (180^\circ - 3x) + 2x = 180^\circ$$
$$360^\circ - 6x = 180^\circ$$
$$-6x = 180^\circ - 360^\circ$$
$$-6x = -180^\circ$$
$$\boxed{x = 30^\circ}$$

Example 3:

Calculate the value of



Solution:

We know when two lines intersect each other then opposite angles are equal, therefore, the third angle of the triangle will be 89°. Hence

$$\angle 52^\circ + \angle a + \angle 89^\circ = 180^\circ$$
$$\angle a = 180^\circ - (52^\circ + 89^\circ)$$
$$\boxed{\angle a = 39^\circ}$$

Now, because corresponding angles are equal, here ∠52 and ∠b are pair of corresponding angies. Therefore

$$\angle b = \angle 52$$
$$\boxed{\angle b = 52^\circ}$$

Properties of Isosceles Triangle:

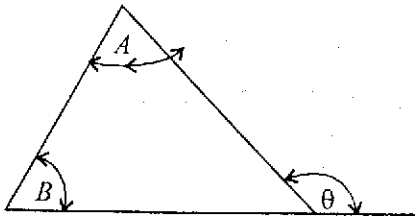
1. If two sides of a triangle are congruent, then the angles opposite to these sides are congruent.
2. If the three angles of a triangle are congruent, then the three sides are also congruent.
3. If two angles of a triangle are congruent, then the sides opposite these angles are also congruent.
4. If three sides of a triangle are congruent, then the three angles are also congruent.

Angle Properties of Triangle:

1. In every triangle the greatest angle is opposite to the longest side.
2. In every triangle the sum of the lengths of any two sides is always greater than the length of the third side.
3. In every triangle the shortest side is opposite to the smallest angle.
4. When the side of a triangle is produced the exterior angle so formed which is equal to the sum of the opposite interior angles.

Example:

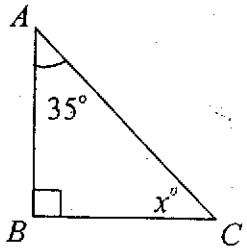
In the figure below



$$\angle\theta = \angle A + \angle B$$

5. In any right triangle, the sum of the measures of the two acute angles is 90°.

Example:



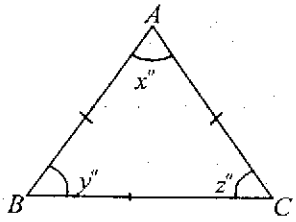
Find the value of x.

Solution:

Since, the sum of the measures of the two acute angles is 90°, therefore

$$x + 35^\circ = 90^\circ$$
$$\boxed{x = 90^\circ - 35^\circ = 55^\circ}$$

6. An equilateral triangle has three equal sides, and three equal angles of 60°.



Example:

The above triangle is an equilateral triangle. Therefore,

$$x^\circ = y = z = 60^\circ$$

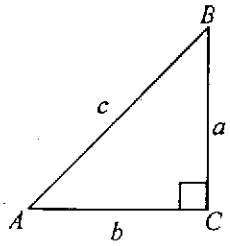
Right Triangle:

1. Pythagoras' Theorem:

It states that, in any right-angled triangle, the square on the hypotenuse is equal to the sum of the squares of the other two sides.

Using the letters in the diagram, the theorem is written as

$$c^2 = a^2 + b^2$$



This relation may be written as

$$a^2 = c^2 - b^2 \quad \text{or} \quad b^2 = c^2 - a^2$$

2. Pythagorean Triples:

Pythagorean triples are sets of numbers that satisfy Pythagorean theorem.

Let x be any positive number, then there is a right triangle whose sides are $3x$, $4x$ and $5x$.

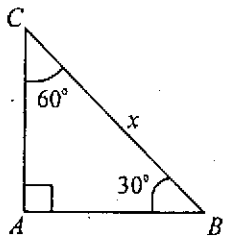
It mean, any multiples of this set such as $6x$, $8x$, $10x$ or $9x$, $12x$, $15x$ form a Pythagorean triple. The most common Pythagorean triples are:

3, 4, 5
5, 12, 13
7, 24, 25

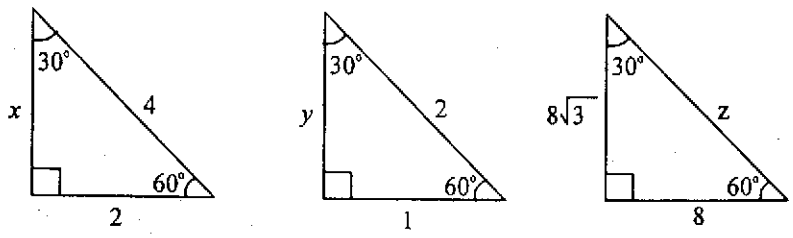
3. The 30°–60°–90° Triangle:

Let x be the hypotenuse of triangle ABC. Then

1) The leg opposite the 30° angle is $\frac{1}{2}(x)$.
2) The leg opposite the 60° angle is $\frac{1}{2}(x)(\sqrt{3})$.



Examples:



$x = 2\sqrt{3}$ $\quad$ $y = \sqrt{3}$ $\quad$ $z = 16$

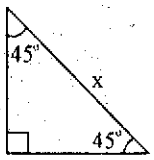
Note:

In an equilateral triangle, an altitude forms a 30°–60°–90° triangle and is equal to

$$\frac{1}{2}(\text{hyp}).\sqrt{3}$$

4. The 45°–45°–90° Triangle:

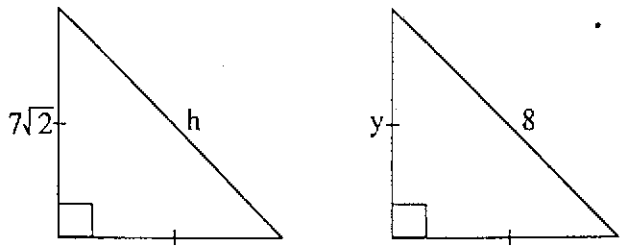
Let x be the hypotenuse of an isosceles right triangle, then



1) Each leg is $\frac{1}{2}(x).\sqrt{2}$

2) Hypotenuse = leg.$\sqrt{2}$

Examples:



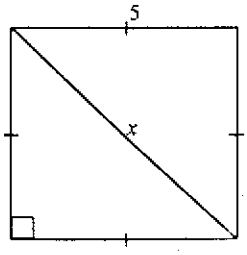
$h = (7\sqrt{2}) . (\sqrt{2}) = 14 \quad y = \frac{1}{2} . 8 . \sqrt{2} = 4\sqrt{2}$

Note:

In a square, the diagonal forms a 45°–45°–90° triangle. Thus, in a square

Diagonal = side.$\sqrt{2}$

Example:



In a square, diagonal = (Side).$\sqrt{2}$

$x = 5.\sqrt{2}$

$x = 5\sqrt{2}$

Example:

What is the area of the square whose diagonal is 12?

Solution:

Let S be the side of the square, then

$$\text{Diagonal} = (\text{Side}).\sqrt{2}$$

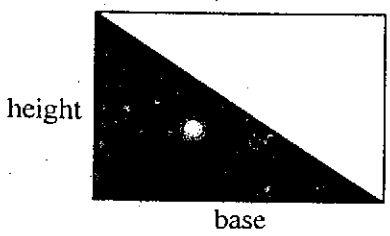
$$12 = S.\sqrt{2}$$

$$\Rightarrow \quad S = \frac{12}{\sqrt{2}}$$

$$\text{Area of square} = S^2 = \left(\frac{12}{\sqrt{2}}\right)^2 = \frac{144}{2} = 72$$

Area of Triangle:

To calculate the area of a triangle, first look at the following rectangle

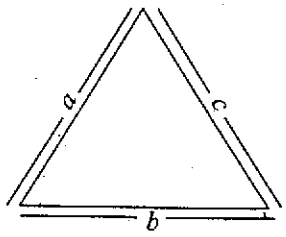


Area of rectangle = base × height

Area of triangle = $\frac{1}{2}$ × base × height

Perimeter of a triangle:

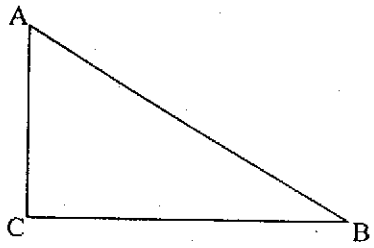
Perimeter of a triangle = sum of lengths of sides



Perimeter of a triangle = $a + b + c$

Triangle Inequality:

In ΔABC, given below



$AB > BC > AC$ and

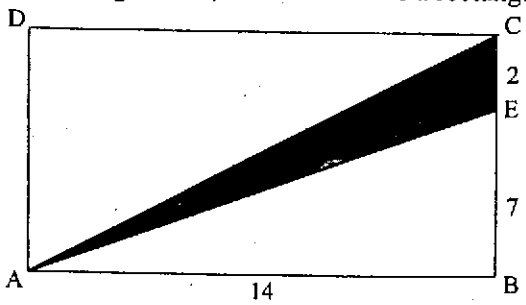
$\angle C > \angle A > \angle B$

These inequalities suggest the following theorems.

1. The perpendicular segment from a point to a line is the shortest distance from the point to the line.
2. **Triangle Inequality Theorem:**
   The sum of the lengths of two sides of a triangle is greater than the length of the third side.

Example:

What is the area and perimeter of the triangle AEC, where ABCD is a rectangular?



Solution:

The area of Rectangle ABCD is

$= 9 \times 14 = 126$

Now area of triangle ABE

$= \frac{1}{2}(14)(7) = 49$

and area of triangle ADC

$= \frac{1}{2}(9)(14) = 63$

Total area of the triangles ABE and ADC

$= 49 + 63 = 112$

Area of ΔAEC = (Area of the rectangle) – (Sun of the area of the triangle)

$= 126 - 112$

$= 14$

Perimeter of ΔAEC

In triangle ABE

$(AE)^2 = (14)^2 + (7)^2$

$= 196 + 49 = 245 \Rightarrow AE = 7\sqrt{5} = 16$

In triangle ADC

$(AC)^2 = (AD)^2 + (BC)^2$

$= (9)^2 + (14)^2 = 81 + 196 = 277 \Rightarrow AC = 17$

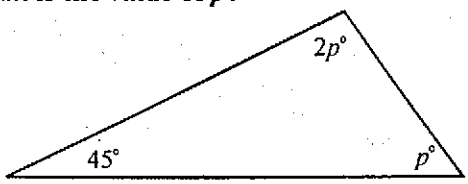
Perimeter of ΔAEC

$= AE + EC + CA$

$= 16 + 17 + 2 = 35$

Multiple Choice Questions (MCQs)

Q1. In the following triangle, what is the value of p?



(A) 35 (B) 45

(C) 55 (D) 40

Q2. The area of an equilateral triangle whose altitude is 10, is:

(A) $8\sqrt{3}$ (B) $2\sqrt{3}$

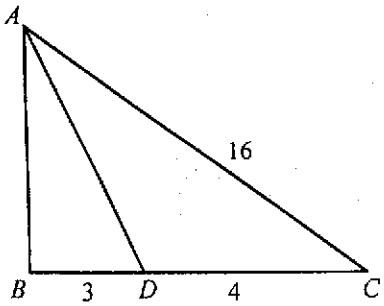
(C) $96\sqrt{3}$ (D) $4\sqrt{3}$

Q3. The two sides of a right triangle are 3 and 5. Then the length of the third side is:

(A) $\sqrt{34}$ (B) $\sqrt{22}$

(C) $2\sqrt{3}$ (D) $3\sqrt{2}$

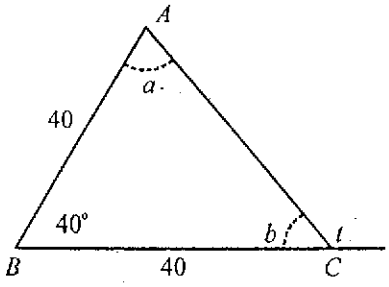
Q4. In the following triangle, AD =



(A) $3\sqrt{2}$ (B) $6\sqrt{6}$

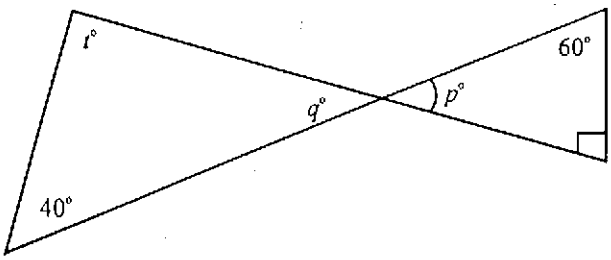
(C) $6\sqrt{3}$ (D) $3\sqrt{7}$

Q5. In the following figure, $t =$



(A) 110 (B) 115
(C) 70 (D) 140

Q6. What is the value of t, in the following diagram?



(A) 100° (B) 60°
(C) 30° (D) 110°

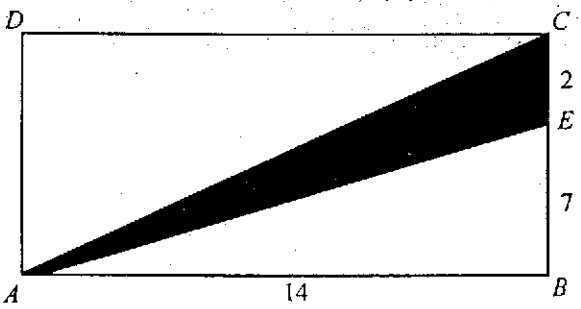


Fig. 1

Q7. What is the area of the triangle AEC, in the above figure 1?

(A) 12 (B) 49
(C) 14 (D) 21

Q8. What is the perimeter of ΔCEA, in the figure 1?

(A) 16 (B) 25
(C) 17 (D) $2 + 7\sqrt{5} + \sqrt{277}$

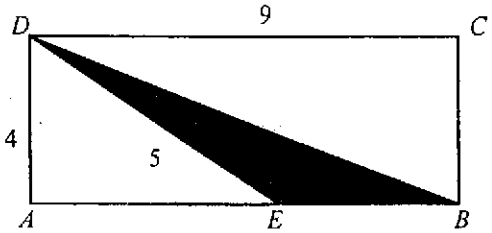


Fig. 2

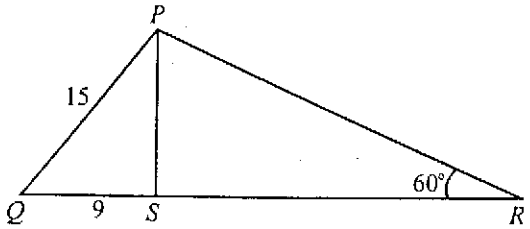
Q9. In figure 2, what is the area of ΔBED?

(A) 16 (B) 14
(C) 12 (D) 6

Q10. In figure 2, what is the perimeter of ΔBED?

(A) $3+\sqrt{93}$ (B) 11
(C) $11+\sqrt{97}$ (D) 81

Questions 11-12 refer to the following figure:



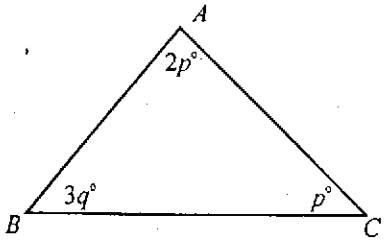
Q11. What is the area of ΔPQR?

(A) $3+4\sqrt{3}$ (B) $18(3+4\sqrt{3})$
(C) 54 (D) $84\sqrt{3}$

Q12. What is the perimeter of ΔPQR?

(A) 54 (B) 72
(C) $4+\sqrt{3}$ (D) $12(4+\sqrt{3})$

Q13. In the following figure, which of the following expresses a true relationship between p and q?



(A) $p = 180 + q$ (B) $q = 30 + p$
(C) $p = 90 + q$ (D) $p = 60 - q$

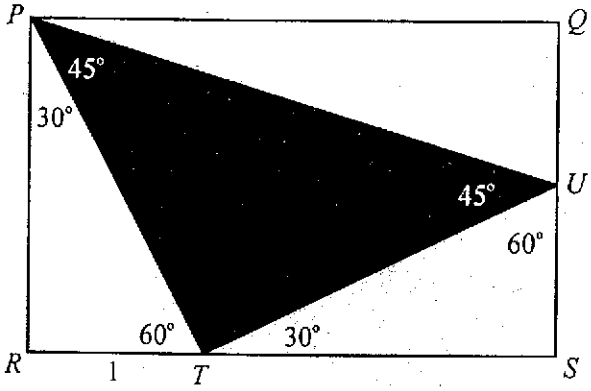


Fig. 3

Questions 14-15 refer to the above figure 3:

Q14. What is the perimeter of shaded triangle PTU?

(A) $2(2+\sqrt{2})$ (B) $4+\sqrt{2}$
(C) $2+\sqrt{2}$ (D) 4

Q15. What is the area of the shaded triangle?

(A) 2 (B) 4
(C) $2\sqrt{2}$ (D) $4\sqrt{2}$

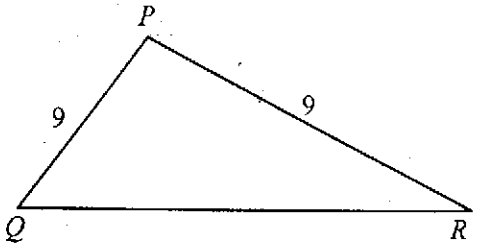
Q16. If the length of the two sides of a triangle are 4 and 6, then the length of the third side is:

(A) less than 11 (B) greater than 11
(C) less than or equal to 11 (D) None of these

Q17. What is the ratio of the diagonal to a side of a square?

(A) 1 : 1 (B) $\sqrt{2} : \sqrt{3}$
(C) $\sqrt{2} : 1$ (D) $\sqrt{2} : \sqrt{2}$

Q18. In the following figure, the perimeter of ΔPQR is:



(A) less than 18 (B) greater than 18
(C) equal to 18 (D) None of these

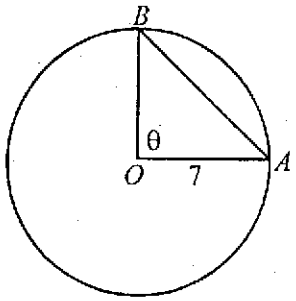


Fig. 4

Q19. In figure 4, if $\theta > 90$, then the length of AB is:

(A) less than 14 (B) greater than 14
(C) equal to 14 (D) not possible

Q20. In figure 4, if $\theta = 90$, then the perimeter of ΔAOB is:

(A) $7 + \sqrt{7}$ (B) $14 + \sqrt{2}$
(C) $14 + \sqrt{7}$ (D) $7(2 + \sqrt{2})$

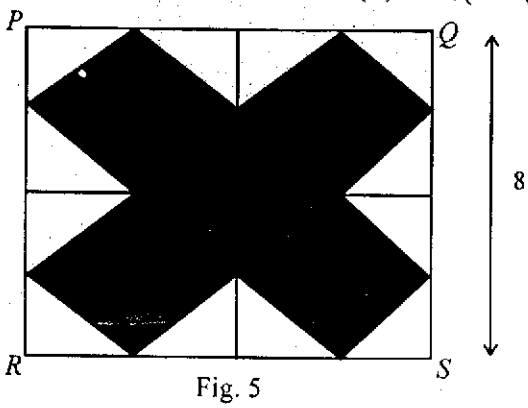


Fig. 5

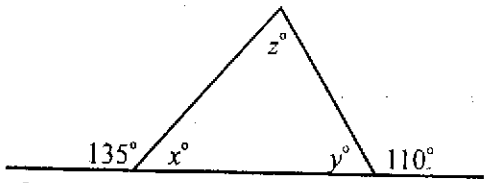
Q21. **In figure 5, the perimeter of shaded region is:**

(A) 17 (B) 24
(C) 34 (D) 21

Q22. **In figure 5, what is the area of the shaded region?**

(A) 40 (B) 24
(C) 20 (D) 12

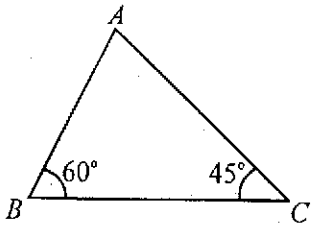
Q23.



Refer to the above figure, which of the following statement is true?

(A) $x + y > z$ (B) $x + y < z$
(C) $x + y = z$ (D) $x + y = x - z$

Q24. **In the following figure:**



which of the following statement is true?

(A) $AB > BC$ (B) $AB < BC$
(C) $AB = BC$ (D) $AC > BC$

Q25. **In a right triangle, if the difference between the measure of the two smaller angles is 30°, then what is the measure (in degrees) of the smallest angle?**

(A) 35 (B) 45
(C) 60 (D) 30

Q26. **In an isosceles triangle that is not equilateral, if all of its sides are integers and no side is longer than 25, then what is the largest perimeter?**

(A) 74 (B) 75
(C) 47 (D) 72

Q27. **What is the smallest integer, s, for which, s, $s + 3$, and $2s - 15$ can be the lengths of the sides of a triangle?**

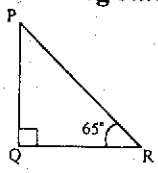
(A) 9 (B) 11
(C) 10 (D) 8

Q28. **If the perimeter of the triangle is $45 + 15\sqrt{3}$, and if the measure of the angles of the triangle are in the ratio of 1 : 2 : 3, then what is the length of the smallest side?**

(A) 10 (B) 15
(C) $3 + \sqrt{3}$ (D) $2 + \sqrt{2}$

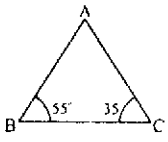
Q29. **Consider the accompanying diagram. Which of the following statements is true?**

(A) $PQ < QR$
(B) $PR < PQ$
(C) $PQ > QR$
(D) $PQ + QR < PR$



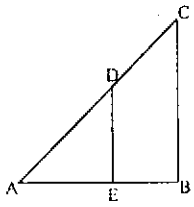
Q30. **Regarding the adjacent triangle, which of the following statements is true?**

(A) AB > AC (B) AB > BC
(C) AC > AB (D) AC > BC



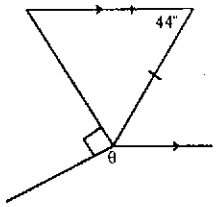
Q31. **In diagram, DE is parallel to CB, AE = BE, DE = 4, and EB = 3. What is CB?**

(A) 4
(B) 8
(C) 6
(D) 12

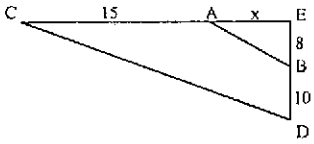


Q32. The value of θ is:

(A) 134°
(B) 148
(C) 112
(D) 206

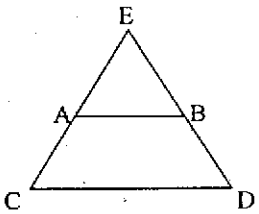


Q33. In ΔECD, $\overline{AB} \parallel \overline{CD}$. What is the value of x?



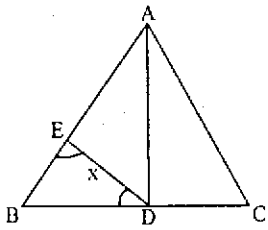
(A) 15 (B) 8
(C) 12 (D) 7

Q34. In ΔCDE, $\overline{AB} \parallel \overline{CD}$, then $\frac{EA}{AC}$ =



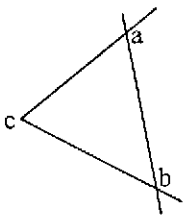
(A) $\frac{BD}{AC}$ (B) $\frac{EC}{AC}$
(C) $\frac{EB}{BD}$ (D) $\frac{AB}{CD}$

Q35. In figure given below, If AB = AC, AE = AD and ∠DAC = 20°. What is the value of x?



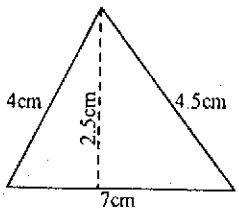
(A) 45° (B) 10°
(C) 40° (D) 35°

Q36. In figure below. What is the size of a + b – c



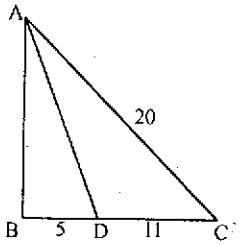
(A) 120° (B) 175°
(C) 110° (D) 180°

Q37. What is the area of the triangle?



(A) $8.75cm^2$ (B) $15.5cm^2$
(C) $17.5cm^2$ (D) $3.5cm^2$

Q38. What is the value of AD in the following triangle?



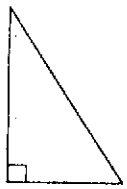
(A) 11 (B) 12
(C) $5\sqrt{2}$ (D) 13

Q39. If a triangle of base 4 has the same area as a circle of radius 4, what is the altitude of the triangle?

(A) 4π (B) 8π
(C) 2π (D) 10π

Q40. The area of the right triangle given below is $18cm^2$. The ratio of its legs is 2 : 3. What is the length of the hypotenuse?

(A) $\sqrt{39}cm^2$
(B) $3\sqrt{13}cm^2$
(C) $2\sqrt{18}cm^2$
(D) $6\sqrt{13}cm^2$



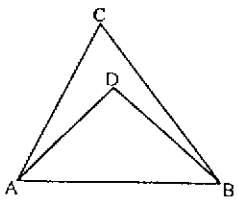
Q41. In a triangle, the ratio of the legs is 1 : 2. If the area of the triangle is $32cm^2$, what is the length of the hypotenuse?

(A) $4\sqrt{5}$ (B) $2\sqrt{3}$
(C) $3\sqrt{2}$ (D) $5\sqrt{6}$

Q42. The angles of a triangle are in the ratio 1 : 2 : 3. The largest angle in the triangle is:

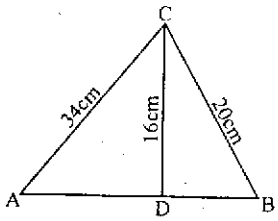
(A) 160° (B) 75°
(C) 40° (D) 90°

Q43. In triangle ABC below, angle BAC is greater than angle CBA. The bisector of angle A and angle B meet at point D. which of the following statement is (are) true?



(A) BD = AD (B) BD > AD
(C) BD ≤ AD (D) BD < AD

Q44. What is the length of AB in the figure below?

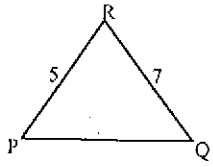


(A) 30cm (B) 12cm
(C) 24cm (D) 42cm

Q45. Two angles of a triangle are $(2a - 40)^\circ$ and $(3a + 10)^\circ$. The third angle is:

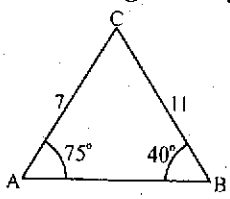
(A) $(230 + a)^\circ$ (B) $(180 + a)^\circ$
(C) $(210 - 5a)^\circ$ (D) $(220 - 5a)^\circ$

Q46. If the perimeter of ΔPQR below is 3 times the length of QR, then PQ =



(A) 3 (B) 9
(C) 7 (D) 5

Q47. Which of the following statements concerning the length of side AB is true?



(A) $AC < 7$ (B) $AB < 7$
(C) $7 < AB < 11$ (D) $AB > 11$

Q48. The three angles of a triangle are $(2a + 20)^\circ$, $(3a + 20)^\circ$ and $(a + 20)^\circ$. The value of a is

(A) 10 (B) 80
(C) 20 (D) 30

Q49. What is the area of an equilateral triangle PQR whose altitude is 6?

(A) $2\sqrt{3}$ (B) $\frac{1}{2}$
(C) 4 (D) $12\sqrt{3}$

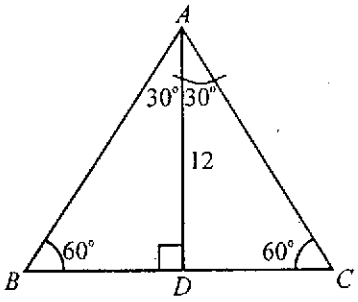
Explanatory Answers

Q1. (B) In any triangle, the sum of the angles = 180°

$\therefore \quad 45 + p + 2p = 180^\circ \Rightarrow 3p = 180 - 45$

$\Rightarrow \quad p = \frac{135}{3} = 45$

Q2. (C) To find the area, first of all we draw a equilateral triangle *ABC*, in which *AD* is altitude.



By, 30 – 60 Right Triangle Theorem,

$$BD = \frac{12}{\sqrt{3}} = \frac{4 \times \sqrt{3} \times \sqrt{3}}{\sqrt{3}} = 4\sqrt{3}$$

Now, Base= $4\sqrt{3} + 4\sqrt{3} = 8\sqrt{3}$ and altitude = 12

Thus, Area= Base × Altitude

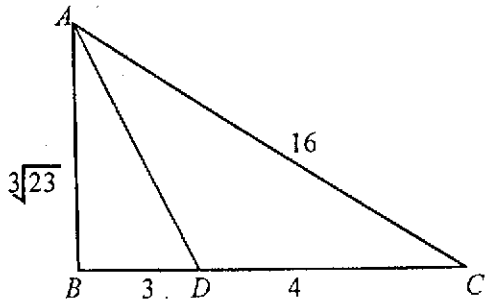
$= 8\sqrt{3} \times 12 = 96\sqrt{3}$

Q3. If the triangle is not right, then any number greater than 1 and less than 25 could be the length of the third side. Now, if the triangle is right, then there are only two possibilities:

(i) If 5 is the hypotenuse, then the legs are 4 and 3.

(ii) If 3 and 5 are two legs then hypotenuse is $\sqrt{34}$.

Q4. (B)



In ΔABC, $AC = 16$, $BC = 3 + 4 = 7$, using

Pythagorean theorem, $AC^2 = (AB)^2 + BC^2 \Rightarrow 16^2 = AB^2 + 7^2$

$\Rightarrow 256 = AB^2 + 49 \Rightarrow AB^2 = 256 - 49 = 207 \Rightarrow AB = \sqrt{207}$

$\Rightarrow AB = 3\sqrt{23}$.

Now, in ΔABD, $AD^2 = AB^2 + BD^2 \Rightarrow AD^2 = (3\sqrt{23})^2 + (3)^2$

$\Rightarrow AD^2 = 9(23) + 9 \Rightarrow AD^2 = 207 + 9 \Rightarrow AD^2 = 216$

$\Rightarrow AD = 6\sqrt{6}$

Q5. (A) Here, $\angle A + \angle B + \angle C = 180 \Rightarrow a + b + 40 = 180 \Rightarrow a + b = 140$

Because the given triangle is an isosceles, *i.e.*, $a = b$

Therefore, a and b are each 70, and

$x = 180 - 70 = 110°$

Q6. E Here $90° + 60° + p° = 180 \quad \Rightarrow p° = 30 \Rightarrow q° = 30°$

Now $q + t + 40 = 180 \quad \Rightarrow q + t = 140 \Rightarrow t = 140 - q$

$\Rightarrow t = 140 - 30 \quad \Rightarrow t = 110°$

Q7. (C) Area of the rectangle $ABCD = 14 \times 9 = 126$

To find the area of the shaded region, we subtract the two white areas of the right angled triangles ABE and ADC

Area of $\Delta ABE = \frac{1}{2}(14)(7) = 49$

Area of $\Delta ADC = \frac{1}{2}(9)(14) = 63$

Sum of the white areas = 49 + 63 = 112

Area of the shaded region = 126 – 112 = 14 square unit

Q8. (D) To calculate the value of AE in ΔABE, we use Pythagorean theorem,

$$(AE)^2 = (AB)^2 + BE^2$$

$$\Rightarrow \quad AE^2 = (14)^2 + (7)^2 \Rightarrow AE^2 = 196 + 49$$

$$\Rightarrow \quad AE^2 = 245 \quad \Rightarrow AE = \sqrt{245} = 7\sqrt{5}$$

Now, in ΔADC, we calculate AC

$$AC^2 = AD^2 + DC^2 \quad \Rightarrow AC^2 = 9^2 + 14^2$$

$$\Rightarrow \quad AC^2 = 81 + 196 \quad \Rightarrow AC^2 = 277 \Rightarrow AC = \sqrt{277}$$

Perimeter of $\Delta AEC = 2 + 7\sqrt{5} + \sqrt{277}$

Q9. (C) Area of the rectangle $ABCD = 4 \times 9 = 36$

Now, in ΔDAE,

$DE^2 = (4)^2 + AE^2 \Rightarrow (5)^2 = 4^2 + (AE)^2 \Rightarrow AE^2 = 25 - 16$

$\Rightarrow AE^2 = 9 \Rightarrow AE = 3$

Thus, $EB = AB - AE \Rightarrow ED = 9 - 3 = 6$ ($\because AB = DC = 9$)

Now Area of $\Delta DAE = \frac{1}{2}(3)(4) = 6$

and Area of the $\Delta BCD = \frac{1}{2}(9)(4) = 18$

Now Area of the shaded region

= Area of the rectangle $ABCD$ – (Area of ΔAED + Area of ΔBCD)

= 36 – (18 + 6) = 36 – 24 = 12 Square units

Q10. (C) In ΔDEB, $DE = 5$, $EB = 6$(from above Q), $DB = ?$

Now we find the value of DB

In ΔBDC,

$$(BD)^2 = (BC)^2 + (DC)^2 \Rightarrow BD^2 = 16 + 81$$

$$\Rightarrow \quad BD^2 = 97 \Rightarrow BD = \sqrt{97}$$

Now the perimeter of the ΔDEB

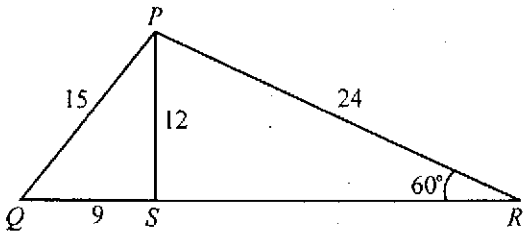
$$= DE + EB + BD = 5 + 6 + \sqrt{97} = 11 + \sqrt{97}$$

Q11. (B) ΔPQS is a right triangle, whose hypotenuse is 15 and its one leg is 9, using Pythagorean theorem

$$PQ^2 = QS^2 + PS^2$$

$$\Rightarrow \quad (15)^2 = (9)^2 + PS^2 \Rightarrow = 225 - 81 = PS^2$$

$$\Rightarrow \quad PS^2 = 144 \Rightarrow PS = 12$$



Now ΔPRS is a 30 – 60 – 90 right triangle, its shorter leg is 12. Then according to 30 – 60 Right Triangle Theorem hypotenuse PR will be 24 and leg RS will be $12\sqrt{3}$. So the area of the triangle PQR is

Area= $\frac{1}{2}$(Base) Altitude

$= \frac{1}{2}(9 + 12\sqrt{3})(12)$

$= 18(3 + 4\sqrt{3})$

Q12. (D) The perimeter of the triangle PQR is the sum of its sides

So perimeter of $\Delta PQR = PQ + QR + PR$

$= 15 + (9 + 12\sqrt{3}) + 24$

$= 48 + 12\sqrt{3}$

$= 12(4 + \sqrt{3})$

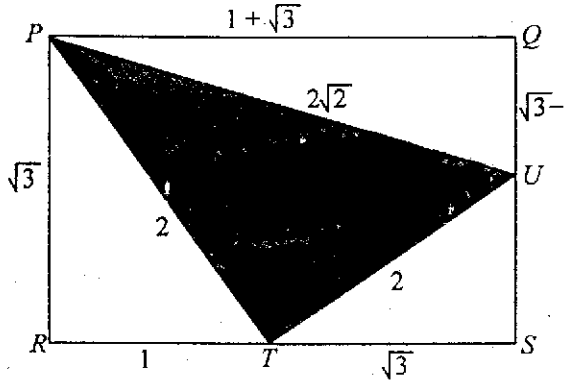
Q13. (D) In any triangle, sum of its interior angles is 180°. So

$p + 2p + 3q = 180°$

$\Rightarrow \quad 3p + 3q = 180 \quad \Rightarrow 3(p + q) = 180$

$\Rightarrow \quad p + q = 60 \quad \Rightarrow p = 60 - q$

Q14. (A) Triangles PRT and PQU both are 30 – 60 – 90 triangles. Thus both triangles will have sides 1, $\sqrt{3}$ and 2, and the inner triangle PTU is a 45 – 45 – 90 triangle and has sides 2, 2 and $\sqrt{2}$, as shown in the following figure



Also, $PQ = RS = 1 + \sqrt{3}$ and

$PR - US = QU = \sqrt{3} - 1$

Thus the perimeter of the shaded triangle is

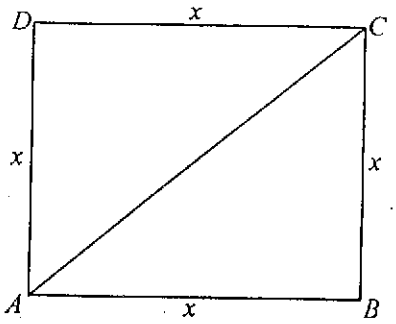
$2 + 2 + 2\sqrt{2} = 4 + 2\sqrt{2} = 2(2 + \sqrt{2})$

Q15. (A) The area of $\Delta PTU = \frac{1}{2}$(Base)(Altitude)

$= \frac{1}{2}(2)(2)$

$= 2$

Q16. (A) The sum of the two sides of a triangle always greater than the third side. Hence the third side is less than
$4 + 6 = 10 < 11$.



Q17. (C) First we draw the diagram, the diagonal of a square is the hypotenuse of each of the 45 – 90 – 45 triangle. Using

$(AC)^2 = (AB)^2 + (BC)^2$

$AC^2 = \sqrt{x^2 + x^2} = \sqrt{2x^2} = x\sqrt{2}$

Now $x\sqrt{2} : x \quad = \sqrt{2} : 1$

Q18. (B) In the given triangle, $QR < PQ + PR \quad \Rightarrow QR < 9 + 9$

$\Rightarrow QR < 18$

Therefore, the perimeter can be any number greater than 18.

Q19. (A) Since in the given figure OA and OB are radii, each is equal to 7. Thus AB could be any positive number less than 14.

Q20. (D) $(AB)^2 \quad = (OA)^2 + (OB)^2$

$= (7)^2 + (7)^2$

$(AB)^2 \quad = 49 + 49$

$AB \quad = \sqrt{98} \Rightarrow AB = 7\sqrt{2}$

Now perimeter of $AOB = AO + OB + AB$

$= 7 + 7 + 7\sqrt{2}$

$= 14 + 7\sqrt{2}$

$= 7(2 + \sqrt{2})$

Q21. (B) In the given figure, the perimeter of the shaded region consists of 12 line segments, these line segments are the hypotenuse of a 45 – 45 – 90 white triangle whose legs are 2. Then each line segment is, $\sqrt{(2)^2 + (2)^2} = \sqrt{4 + 4} = \sqrt{8} = 2\sqrt{2}$, and the perimeter of the shaded region is $24\sqrt{2} = 24$ approx.

Q22. (A) The white region consists of 12 right triangles, each of which has area of $\frac{1}{2}$ of the small square. Now the area of the small square is $2 \times 2 = 4$. The area of $\frac{1}{2}$ small square $= \frac{1}{2} \times 4 = 2$, the total area of the white half small squares is $= 12 \times 2 = 24$. Since the area of the large square is $8 \times 8 = 64$. Thus the area of shaded region is $64 - 24 = 40$.

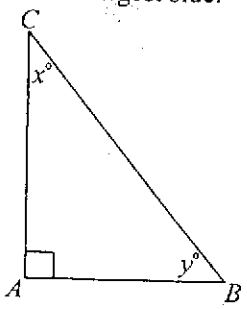
Q23. (A) Here, $x = 180 - 135 = 45$, and $y = 180 - 110 = 70$, then $x + y = 45 + 70 = 115$

Now, $x + y + z = 180 \Rightarrow 45 + 70 + z = 180 \Rightarrow z = 65$

Thus $115 > 65 \Rightarrow x + y > z$

Q24. (B) Since $60 + 45 = 105 \Rightarrow m\angle A = 75$, this shows that $\angle A$ is the largest angle and BC is the side opposite to the largest angle. Thus BC is the largest side.

Q25. (D) First of all we draw the diagram



By the given condition

$x - y = 30 \quad \ldots(i)$

Because, $\angle A + \angle B + \angle C \quad = 180$

$90 + y + x \quad = 180$

$x + y \quad = 90 \quad \ldots(ii)$

Adding (i) and (ii), we have

$x - y = 30$

$\underline{x + y = 90}$

$2x = 120 \quad \Rightarrow \boxed{x = 60^\circ}$

Put $x = 60$ in (ii), we get

$60 + y = 90 \quad \Rightarrow \boxed{y = 30^\circ}$

Q26. Since no side of the triangle can be longer than 25, thus, we suppose that both of the equal sides are 25. Then the largest possible value of the third side is 24. Its perimeter is

$$25 + 25 + 24 = 74$$

Q27. (C) In a triangle the sum of the sides of any sides must be greater than the third side. $S + (S + 3)$ to be greater than

$2S - 15$, thus $2S + 3$ must be greater than $2S - 15$; but that is always true. For $S + (2S - 15)$ to be greater than

$S + 3 \Rightarrow 3S - 15$ must be greater than $S + 3$;

but $3S - 15 > S + 3$ is true only if

$3S - S > 3 + 15$

$2S > 18 \Rightarrow S > 9$

Thus answer is 10.

Q28. (B) Since, angles are in ratio 1 : 2 : 3, thus

$\theta + 2\theta + 3\theta = 180 \Rightarrow 6\theta = 180 \Rightarrow \theta = 30$

Here, $\theta = 30$, $2\theta = 60$ and $3\theta = 90$, so the given triangle is a 30 – 60 – 90 triangle, thus its sides are x, $2x$ and $x\sqrt{3}$. Thus its perimeter is $3x + x\sqrt{3}$

but given that perimeter is $45 + 15\sqrt{3}$,

$\Rightarrow \quad 3x + x\sqrt{3} = 45 + 15\sqrt{3}$

$x(3 + \sqrt{3}) = 15(3 + \sqrt{3})$

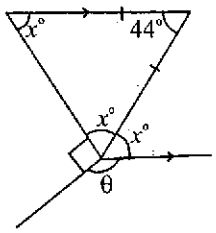
$\Rightarrow \quad \boxed{x = 15}$

Q29. **(C)** Since the measure of angle R is 65°, the measure of angle P is 25°. Since the larger side is opposite the larger angle, therefore, PQ > QR

Q30. **(C)** Since the larger side is always opposite to the larger angle. In the fig, angle A is 90° the larger side of the triangle is BC, followed by AC and then at last AB.

Q31. **(B)** Since DE is parallel CD, the triangle ADE and ACB are similar. Therefore, corresponding sides are proportional. So DE is to AB as AB to CB. Since AE = EB, $\frac{AB}{AE}$ is $\frac{1}{2}$. Therefore CB is twice or 8.

Q32. **(A)**



Since the triangle is isosceles triangle than

$x^\circ + x^\circ + 44^\circ = 180^\circ$

$\Rightarrow \quad 2x^\circ = 180^\circ - 44^\circ = 136^\circ$

and

$2x + \theta + 90^\circ = 360^\circ$

$136 + \theta + 90^\circ = 360^\circ$

$\theta + 226 = 360^\circ$

$\Rightarrow \quad \theta = 360^\circ - 226 = 134^\circ$

therefore $\theta = 134^\circ$

Q33. **(C)** Since, If a line interesting the interior of a triangle is parallel to one side, then the line divides the other two sides proportionally.

$$\frac{x}{15} = \frac{8}{10}$$

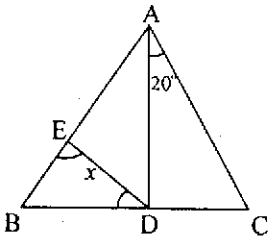
$$10x = 120$$

$\boxed{x = 12}$

Q34. **(C)** Since the line AB is parallel to line CD, therefore the line AD divides CE and ED proportionally.

$$\therefore \frac{EA}{AC} = \frac{EB}{BD}$$

Q35. **(B)**



As $AB = AC$

So, $m\angle ACB = m\angle ABC = y$

In ΔABC

$m\angle ACB + m\angle BCA + m\angle CAB = 180^\circ$

$y + y + 40 = 180^\circ$

$\Rightarrow 2y = 180^\circ - 40^\circ = 140$

$y = 70^\circ$

In ΔADE

$AD = AE$

So, $m\angle AED + m\angle ADE = z^\circ$

$m\angle AED + m\angle ADE + m\angle DAE = 180^\circ$

$z + z + 20^\circ = 180^\circ$

$\Rightarrow 2z = 180^\circ - 20^\circ = 160^\circ$

$\Rightarrow z = 80^\circ = m\angle AED$

$m\angle AED + m\angle DEB = 180^\circ$

$80^\circ + m\angle DEB = 180^\circ$

$\Rightarrow m\angle DEB = 180^\circ - 80^\circ = 100^\circ$

In ΔBED

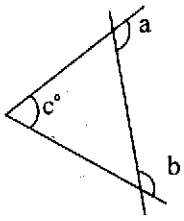
$m\angle BED + m\angle EDB + m\angle DBE = 180^\circ$

$100 + x + 70^\circ = 180^\circ$

$\Rightarrow x + 170^\circ = 180^\circ$

$x = 180^\circ + 170^\circ = 10^\circ$

Q36. **(D)**



$(180^\circ - a) + (180^\circ - b) + c = 180^\circ$

$180^\circ - a + 180^\circ - b + c = 180^\circ$

$360^\circ - (a + b - c) = 180^\circ$

$a + b - c = 360^\circ - 180^\circ$

$\Rightarrow a + b - c = 180^\circ$

Q37. **(A)** Area of the triangle $= \frac{1}{2}$ Base $\times$ Altitude

$$= \frac{1}{2} \times 7 \times 2.5$$
$$= 8.75 \text{ cm}^2$$

Q38. **(D)** Using the Pythagorean theorem in ΔABC

$$(per)^2 = (hyp)^2 - (base)^2$$
$$AB^2 = (20)^2 - (11 + 5)^2$$
$$AB^2 = 400 - 256 = 144$$
$$AB = 12$$

Again using the Pythagorean theorem in ΔABD

$$(hyp)^2 = (base)^2 + (per)^2$$
$$AD^2 = (12)^2 + (5)^2$$
$$= 144 + 25 = 169 \Rightarrow \boxed{AD = 13}$$

Q39. **(B)** Area of the circle $= \pi r^2$

$$= \pi(4)^2 = 16\pi$$

Area of the triangle $= \frac{1}{2} \times 4 \times$ Altitude

$$= 2 \times \text{Altitude}$$

Since area of the given triangle is equal to the area of the circle of radius 4, therefore

$$16\pi = 2 \times \text{Altitude}$$
$$\Rightarrow \quad \text{Altitude} = 8\pi$$

Q40. **(D)** Let legs be $2x$ and $3x$, by Pythagorean theorem

$$(hyp)^2 = (2x)^2 + (3x)^2$$

But $\frac{1}{2}.2x.3x = 18$

$$3x = 18 \Rightarrow \boxed{x = 6}$$
$$(hyp)^2 = (12)^2 + (18)^2$$
$$= 144 + 324 = 468$$
$$hyp = \sqrt{468} = 6\sqrt{13}$$

Q41. Let legs of the triangle be x and $2x$, then

$$hyp^2 = (x)^2 + (2x)^2$$

But $32 = \frac{1}{2}.x.2x$

$$\Rightarrow \quad x^2 = 16 \quad \Rightarrow \quad x = 4$$
$$hyp^2 = (4)^2 + (8)^2 = 16 + 64 = 80$$
$$\Rightarrow \quad hyp = \sqrt{80} = \boxed{4\sqrt{5}}$$

Q42. **(D)** The sum of the angles in a triangle $= 180^\circ$

Given ratio 1 : 2 : 3

Sum of the ratios $= 1 + 2 + 3 = 6$

Largest angle $= \frac{3}{6} \times 180 = 90^\circ$

Q43. If $\angle CAB > \angle ABC$, then $\angle A > \frac{1}{2}\angle B$. Then $\angle DAB$ greater than $\angle DBA$. Therefore

$DB > DA$ (opposite sides of the angle)

Q44. **(D)** Solving ΔCAD

$$(AD)^2 = (34)^2 - (16)^2$$
$$= 1156 - 256 = 900$$

$\boxed{AD = 30}$

Now solving ΔCBD

$(BD)^2 = (20)^2 - (16)^2$

$= 400 - 256$

$(BD)^2 = 144$

$\boxed{BD = 12}$

$AB = AD + BD$

$= 30 + 12 = \boxed{42}$

Q45. **(C)** Let the angles of triangle be

$(2a - 40)^\circ, (3a + 10)^\circ, x$ then

$(2a - 40)^\circ + (3a + 10^\circ) + x = 180^\circ$

$\Rightarrow \quad x = 180^\circ - [(2a - 40) + (3a + 10)]$

$= 180^\circ - [5a - 30^\circ]$

$= 180^\circ - 5a + 30^\circ$

$x = (210 - 5a)$

Q46. **(B)** The perimeter of the triangle is the sum of the lengths of the 3 sides. Since the perimeter is equal to 3 times the length of QR ($3 \times 7 = 21$)

$5 + 7 + PQ = 21$

$PQ = 21 - 12$

$\boxed{PQ = 9}$

Q47. **(C)** $\angle C + 75^\circ + 40^\circ = 180 \Rightarrow \angle C = 65$. Here $\angle A$ is the largest angle, B is the smallest and C is in between. Therefore

$CA < AB < BC$

$7 < AB < 10$

Q48. **(C)** $(2a + 20) + (3a + 20) + (a + 20) = 180$

$6a = 180 - 60$

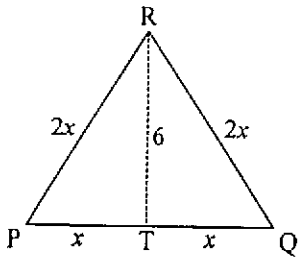
$6a = 120$

$\Rightarrow \quad \boxed{a = 20}$

Q49. **(D)**

As PQR is equilateral triangle

$\therefore \overline{PQ} = \overline{QR} = RP = 2x$



In ΔPTR

$(hyp)^2 = (base)^2 + (prep)^2$

$(2x)^2 = (x)^2 + (6)^2$

$4x^2 = x^2 + 36$

$4x^2 - x^2 = 36$

$3x^2 = 36$

$x^2 = 12$

$x = 2\sqrt{3}$

So,

$PQ = 2x = 2(2\sqrt{3})$

$= 4\sqrt{3}$

Area of PQR $= \frac{1}{2} \times$ base $\times$ Altitude

$= \frac{1}{2} \times 4\sqrt{3} \times 6$

Area of PQR $= 12\sqrt{3}$

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Chapter 3

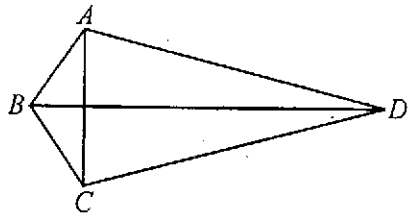
QUADRILATERALS AND POLYGONS

Quadrilateral:

A quadrilateral is a plane figure with four straight sides. The elements of a quadrilateral are its four sides and four angles.

Diagonal of Quadrilateral:

A diagonal of a quadrilateral is a line segment joining two non-consecutive vertices. In the following figure, the diagonals of the quadrilateral *ABCD* are *AC* and *BD*.



Family or Types of Quadrilateral:

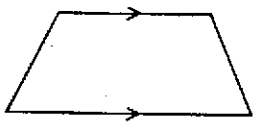
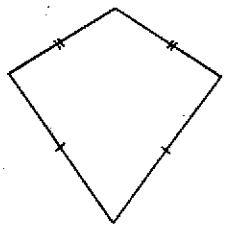
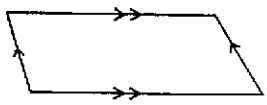
The properties of a quadrilateral are the features that are characteristic of that shape. They can include any of the following:

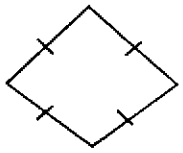
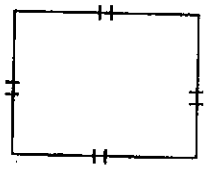
Sides: Are the side lengths equal? Are the sides parallel?

Angles: Are any angles equal? Are any angles right angles?

Diagonals: Are the diagonals equal? Do the diagonals bisect each other? Do the diagonals bisects the angles through which they pass? Do the diagonals cut at right angles?

The combination of properties is different for each quadrilateral.

Quadrilateral		Properties
Trapezium		❖ A quadrilateral with one pair of opposite sides parallel.
Kite		❖ A quadrilateral with two pairs of equal adjacent sides.
Parallelogram		❖ A quadrilateral with opposite sides parallel.
Rectangle		❖ All properties of parallelogram plus. (i) All angles are right angles. Diagonals are equal. (ii) Two axes of symmetry (perpendicular to sides)

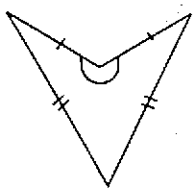
Rhombus		❖ All properties of parallelogram plus. (i) All sides are equal. (ii) Diagonals bisect at right angles. (iii) Diagonals bisect the angles through which they pass. (iv) Two axes of symmetry.
Square		❖ All properties of rectangle plus. ❖ All sides are equal ❖ Four axes of symmetry.

Convex Quadrilateral:

A quadrilateral is called convex if each of its interior angles is less than two right angles.

Re-entrant Quadrilateral:

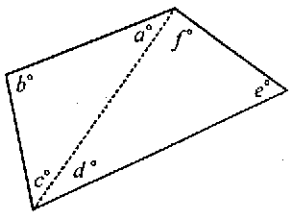
A quadrilateral is called re-entrant if one of its interior angles is reflex. A kite with one reflex interior angle is an example of a re-entrant quadrilateral.



Angle Sum of a Quadrilateral:

If we split a quadrilateral into two triangles as shown, then we can calculate the angle sum.

Angle sum $= a + b + c + d + e + f$



But $a + b + c = 180$

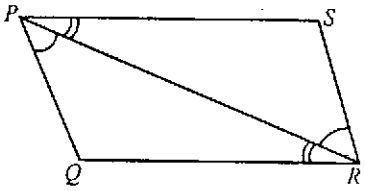
and $d + e + f = 180$

So $a + b + c + d + e + f = 180 + 180$

$= 360°$

Additional Properties of Parallelograms:

We have learnt that, in a parallelogram, it has two pairs of parallel sides. Opposite sides are equal. Opposites angles are equal. Diagonals bisect each other. Now, we shall derive some of the properties of parallelogram using the properties of parallel lines and angles. Consider a parallelogram $PQRS$ with diagonal $\overline{PR}$.



Then $\angle SPR = \angle PRQ$ ($\because$ Alternate angles)

$\angle QPR = \angle PRQ$ ($\because$ Alternate angles, $PS \parallel QR$)

$\therefore \angle SPR + \angle QPR = \angle PRQ + \angle PRS$

$\therefore \quad \angle QPS = \angle QRS$

Now, in ΔPQR

$\angle PQR + \angle QPR + \angle PRQ = 180°$

$\therefore \quad \angle PQR = 180 - (\angle QPR + \angle PRQ)$

$= 180 - (\angle PRS + \angle SPR) = \angle PSR$

$\therefore \quad \angle QPR = QRS$ and $PQR = PSR$ .....(i)

Hence, from (i), we can say that in a parallelogram "opposite angles are congruent."

Now, also in parallelogram $PQRS$,

$\angle QPS + \angle PQR + \angle QRS + \angle RSP = 360°$

But $\angle QPS = \angle QRS$ and $\angle PQR = \angle RSP$

$\therefore \quad 2(\angle QPS + \angle PQR) = 360^\circ$

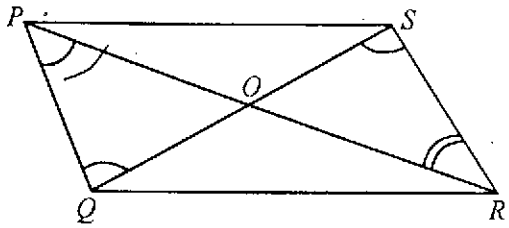
$\therefore \quad \angle QPS + \angle PQR = 180^\circ$

Similarly, $\angle QRS + \angle RSP = 180^\circ$ .....(ii)

Thus, from (ii), we can say, that, in a parallelogram,

"Pairs of adjacent angles are supplementary."

Now in triangle PQR and RSP,



$\angle PRQ = \angle SPR$

$\angle QPR = \angle PRS$

$\Rightarrow \quad PR = PR$ (common)

$\therefore \quad \Delta PQR = \Delta RSP$ ($\because$ Two angles and a side are congruence)

$\Rightarrow \quad PQ = RS$ and $PS = QR$ .....(iii)

Hence, in a parallelogram,

"Opposite sides are congruent."

In triangles PQO and RSO

$PQ = RS$

$\angle PQO = \angle RSO$

$\angle QPO = \angle SRO$

$\therefore \quad \Delta PQO \cong \Delta RSO$ (by AAS congruence)

$\Rightarrow \quad PO = RO$ and $QO = SO$ .....(iv)

Hence, in a parallelogram

"Two diagonals bisect each other."

Now in triangles PQR and RSP

$QR = PS$

$PQ = RS$

$PR = PR$ (common)

$\therefore \quad \Delta PQR \cong \Delta RSP$

Similarly $\Delta PQS \cong \Delta RSQ$

Hence, in a parallelogram

"A diagonal divides into two congruent triangles."

Test for Quadrilaterals:

We have seen, that each of the quadrilaterals has several properties, but it is not necessary to check all the properties when trying to identify a shape.

Tests for a Parallelogram:

To identify the shape as a parallelogram, satisfying any of these conditions is sufficient.

1. Both pairs of opposite sides are parallel or equal.

2. Both pairs of opposite angles are equal.
3. One pair of opposite sides is equal and parallel.
4. Diagonals bisect each other.

Tests for a Rhombus:

To identify the shape as a rhombus, satisfying any of these conditions is sufficient.

1. All sides are equal.
2. Diagonals bisect at right angles.

Common Properties in all Quadrilaterals:

The common properties in all quadrilaterals are:

1. Diagonals bisect each other.
2. Opposite angles are equal.

Polygons:

A simple closed figure formed by three or more line segments is known as a polygons. Polygons are named by the number of sides they have. The following is a list of the names given to polygons according to the number of their sides.

Number of sides	Name of the polygon
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nanagon
10	Decagon

Vertex of a Polygon:

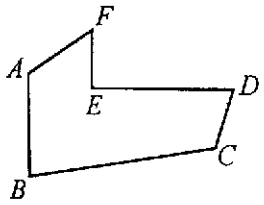
The angular points of a polygon are called its vertices, and the number of sides of a polygon is equal to the number of its vertices.

Other Names of Polygons:

Other words used to describe polygons are:

Concave:

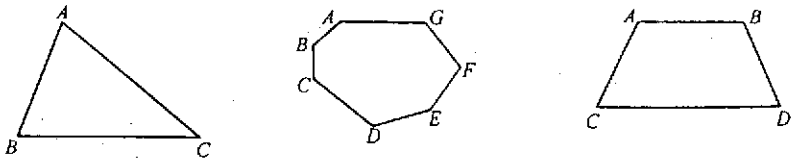
A polygon is said to be concave when one or more of its interior angles is greater than two right angles.



The above figure shows a concave polygon: the measure of interior angle *DEF* is greater than 180°.

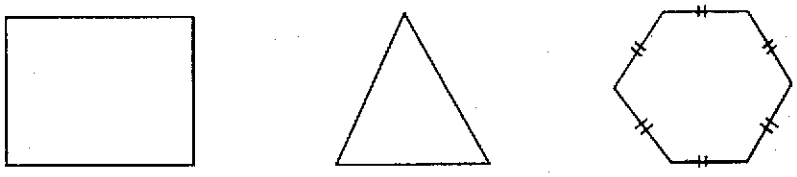
Convex Polygon:

A polygon is said to be convex when all its interior angles are less than two right angles. The following figures are examples of convex polygon.



Regular Polygon:

A polygon is said to be regular if all its sides as well as its angles are equal. The following figures are some examples of regular polygons:



Angle Sum of a Polygon:

The angle sum of a polygon with n sides is equal to

$$(n-2)\times 180^\circ$$

The size of each interior angle of a regular polygon with n sides is

$$\frac{(n-2)\times 180^\circ}{n}$$

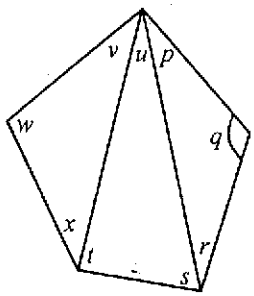
or $S = (2n-4)$ right angles.

In words

The sum of the angles in a polygon is equal to 'the number of sides less 2' multiplied by 180°.

Example 1:

What is the sum of the angles in a pentagon?



Solution:

$S = p + q + r + s + t + u + v + w + x$

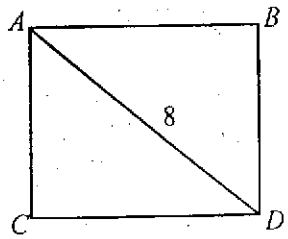
$S = (n-2)\,180^\circ \qquad n = 5$

$S = (5-2)\,180^\circ$

$S = 540^\circ$

Example 2:

What is the length of each side of a square if its diagonals is 8?



Solution:

In the adjacent diagram, diagonal AD is the hypotenuse of a 45 – 45 – 90 right triangle.

$$\text{Then} = \frac{AC}{\sqrt{2}} = \frac{8}{\sqrt{2}} = \frac{8\sqrt{2}}{2} = 4\sqrt{2}$$

Example 3: A decagon is drawn in which each angle has the same measure. What is the measure of each angle?

Solution: The sum of the measures of n sided is

$$(n-2)\,180^\circ$$

Here $n = 10$

$\therefore$ Sum of the angles of a decagon $= (10-2)\,180$

$= 8(180)$

$= 1440$

$\because$ Angles are equal measure, so each angle is

$= 1440 \div 10 = 144$

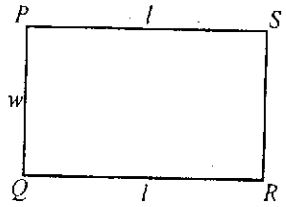
Perimeter:

The distance all around a shape is called its perimeter. Or

The perimeter of a figure is the measure of its bounding line-segments or curves.

Perimeter of a Quadrilateral:

For a quadrilateral $PQRS$



Perimeter = Sum of all the sides

$= PQ + QR + RS + SP$

$= 2(PQ + QR)$

$= 2(l + w)$

Perimeter of a Rectangle:

Similarly, the perimeter of rectangle $= 2(l + w)$, where l and w are the length and the width, respectively.

Perimeter of a Square and Rhombus:

Since, in a square and rhombus all sides are equal, so the perimeter of a rhombus or a square $= 4l$

Where l is the length of each side.

Area:

The area of a shape is the amount of flat space taken up by the shape.

Area of the Rectangle and Square:

Area of a rectangle = length × width

$= l \times w$

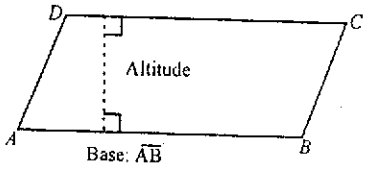
For a square

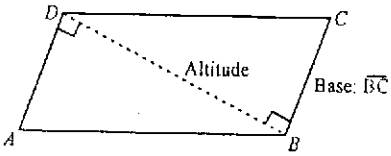
length = width *i.e.*, $w = l$

$\therefore$ Area of square $= l \times l = l^2$

Area of Parallelograms and Triangles:

Any side of a parallelogram may be called the base of the parallelogram. For each base, there is a corresponding altitude.





An altitude of a parallelogram is a perpendicular segment whose end points lie on opposite sides of a parallelogram.

In a parallelogram $ABCD$, let

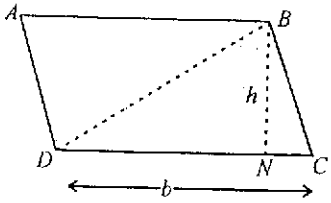
$DC = b$(base) and

distance between the parallel lines

AB and DC be h. Then

area of parallelogram = base × altitude

$= b \times h = bh$



Area of Triangle:

Since a diagonal of a parallelogram divides it into two congruent triangles, also the area of the triangles are

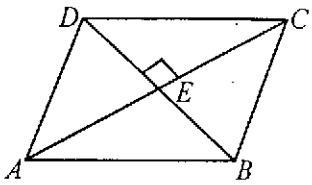
equal, therefore

Area of BDC = Area of ABD

Area of $BDC = \frac{1}{2}$ [Area of parallelogram $ABCD$]

$= \frac{1}{2} b \times c$

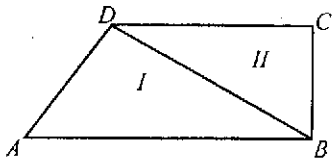
Area of Trapezium and Rhombuses:



In a rhombus, all sides are congruent and the diagonals are perpendicular to each other.

The area A of a rhombus is one-half the product of the lengths of its diagonals, d_1 and d_2, that is

$$A = \frac{1}{2} d_1 d_2$$



The formula for the area of a trapezium can be found by finding the areas of the triangles formed by drawing a diagonal.

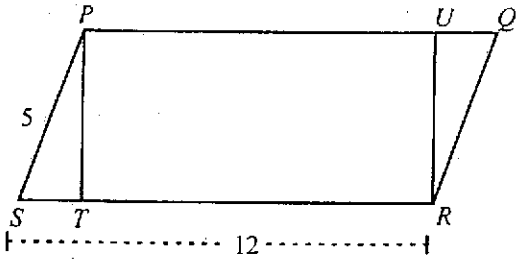
Area of $ABCD$ = Area of Triangle I + Area of Triangle II

Thus, the area A of a trapezium is one-half the product of its altitude and the sum of its bases, b and b'. That is

Area of Trapezium ABCD $= \frac{1}{2}bh + \frac{1}{2}b'h = \frac{1}{2}h(b + b')$

Multiple Choice Questions (MCQs)

Q1. In the following figure, the area of parallelogram *PQRS* is 36. What is the area of rectangle *PURT*?



(A) 60 (B) 40

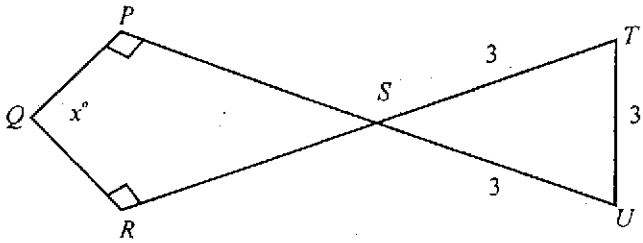
(C) 24 (D) 36

Q2. The length of a rectangle is twice its width. If the perimeter of a rectangle is the same as the perimeter of a square of size 9, what is the length of a diagonal of the rectangle?

(A) 180 (B) $3\sqrt{5}$

(C) 36 (D) $6\sqrt{5}$

Q3. In the figure below, what is the value of x?



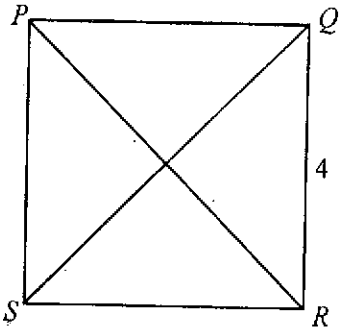
(A) 120 (B) 60

(C) 90 (D) 30

Q4. **A triangle has sides, 5 inches, 12 inches and 13 inches, respectively. A rectangle equal in area to that of the triangle has a width of 4 inches. The perimeter of the rectangle, expressed in inches, is:**

(A) 23 (B) 28
(C) 60 (D) 32

Q5. **In the following figure, square *PQRS* has divided into four triangles by drawing diagonals *PR* and *QS*. What is the sum of the perimeters of triangles?**



(A) $2+\sqrt{2}$ (B) $4+\sqrt{2}$
(C) $4(1+\sqrt{2})$ (D) $16(1+\sqrt{2})$

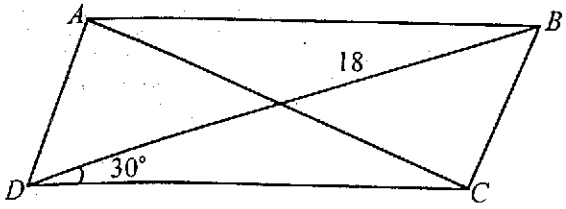
Q6. **If the length of a rectangle is 4 times its width, and if its area is 196, what is its perimeter?**

(A) 60 (B) 28
(C) 35 (D) 70

Q7. **If the angles of a hexagon are in the ratio 2 : 4 : 4 : 4 : 5 : 5, what is the degree measure of the smallest angle?**

(A) 30 (B) 60
(C) 40 (D) 70

Questions 4-5 refer to the following figure



Q8. **What is the area of the rectangle *ABCD*?**

(A) $3\sqrt{3}$ (B) $36\sqrt{3}$
(C) 81 (D) $81\sqrt{3}$

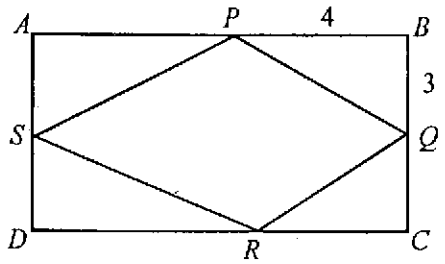
Q9. **What is the perimeter of the rectangle *ABCD*?**

(A) $81+81\sqrt{3}$ (B) $18+18\sqrt{3}$
(C) $12(1+2\sqrt{3})$ (D) $12+12\sqrt{3}$

Q10. **The length of a rectangle is 3 more than the side of a square, and the width of the rectangle is 3 less than the side of the square. If the area of the square is 58, what is the area of the rectangle?**

(A) 40 (B) 20
(C) 39 (D) 49

Question 11-12 refer to the following figure, in which *P*, *Q*, *R* and *S* are midpoints of the sides of rectangle *ABCD*.



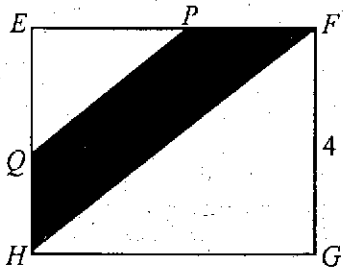
Q11. What is the perimeter of quadrilateral *PQRS*?

(A) 20 (B) 40
(C) 60 (D) 30

Q12. What is the area of the quadrilateral *PQRS*?

(A) 20 (B) 24
(C) 40 (D) 96

Question 13-14 refer to the following figure, in which P and Q are midpoints of two of the sides of square $EFGH$.



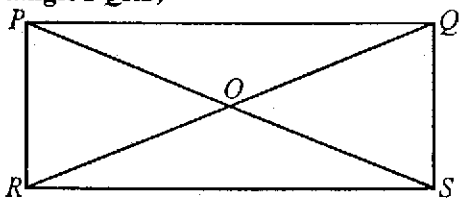
Q13. What is the perimeter of the shaded region?

(A) $2+3\sqrt{2}$ (B) 8
(C) $4+6\sqrt{2}$ (D) $3+2\sqrt{2}$

Q14. What is the area of the shaded region?

(A) 4 (B) 4.5
(C) 6 (D) 1.5

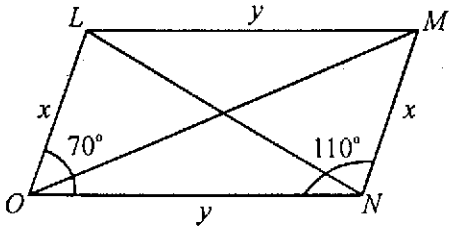
Q15. Refer to the following rectangle *PQRS*,



which of the following statements is true?

(A) Area of ΔPOR > Area of ΔORS
(B) Area of ΔPOR = Area of ΔORS
(C) Area of ΔORS > Area of ΔPOR
(D) $\Delta POR \cong \Delta ORS$

Q16. Refer to the following figure,



which of the following statements is true?

(A) Diagonal LN < Diagonal MO
(B) Diagonal LN < Diagonal MO

(C) Diagonal MO = Diagonal LN

(D) $LA \parallel MO$

Q17. What is the perimeter of a 30-60 right triangle whose longer leg is $2s$?

(A) $4S(2 + 2\sqrt{3})$ (B) $S(4 + 2\sqrt{3})$

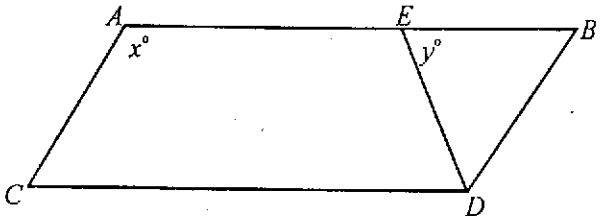
(C) $S(2 + 2\sqrt{3})$ (D) $4S + 2S\sqrt{2}$

Q18. If the area of a rectangle is 40, then its perimeter will:

(A) equal to 24 (B) less than 24

(C) greater than 24 (D) less than or equal to 22

Q19. In the following figure,

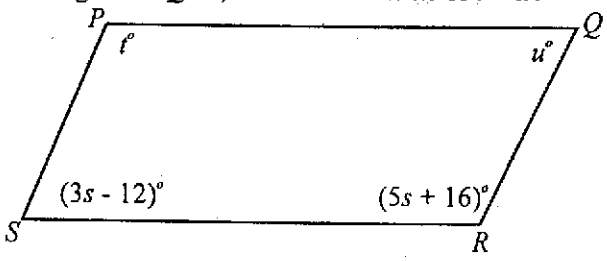


$x =$

(A) Y (B) $\frac{1}{y}$

(C) $4y$ (D) $2y$

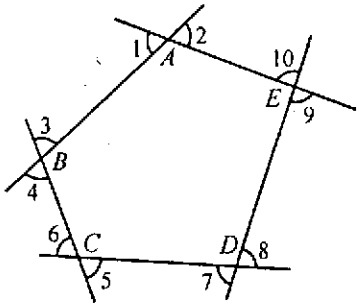
Q20. In the following parallelogram $PQRS$, what is the value of $t - u$?



(A) 72 (B) 50

(C) 91 (D) 22

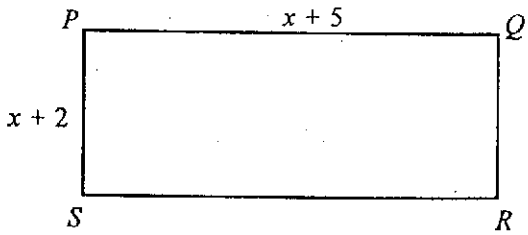
Q21. In the following pentagon $ABCDE$, what is the sum of all marked exterior angles?

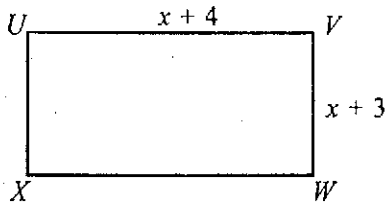


(A) 720 (B) 540

(C) 360 (D) 300

Q22. Following, two rectangles are given, the area of the rectangle $PQRS$ is 90, what is the area of the rectangle $UVWX$?

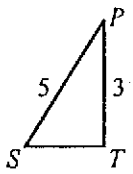




(A) 90 (B) 60
(C) 68 (D) 92

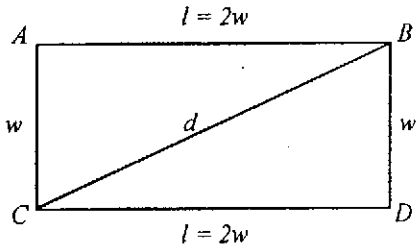
Explanatory Answers

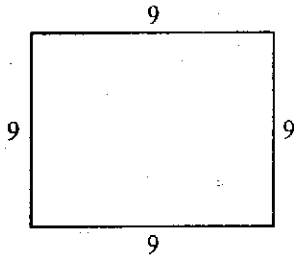
Q1. (C) Area of the parallelogram: $A = bh$, in the given figure,
$A = 36$ and $b = 12$, therefore
$36 = 12h \Rightarrow \boxed{h = 3}$
Now, in ΔPTS, $(SP)^2 = (PT)^2 + (ST)^2$
$\Rightarrow 25 = 9 + ST^2$
$\Rightarrow 16 = ST^2 \Rightarrow ST = 4$



Now $TR = SR - ST \Rightarrow TR = 12 - 4 = 8$
Now, Area of rectangle $= A = lw$
Hence, Area of rectangle $PURT = 8 \times 3 = 24$

Q2. (D) Now, Perimeter of square = Perimeter of rectangle
Since, Perimeter of square = 4(9)
= 36
Therefore, Perimeter of rectangle $= 2(l + w)$
$\Rightarrow 2(l + w) = 36 \Rightarrow l + w = 18$ ...(i)
But $l = 2w$ (Given)
Putting the value of l in (i), we have
$2w + w = 18 \Rightarrow 3w = 18$
$\Rightarrow w = 6$
Also, since $l = 2w$, therefore
$l = 2(6) = 12$
Using Pythagorean theorem, to find the length of diagonal BC
$(BC)^2 = (BD)^2 + (CD)^2 \Rightarrow d^2 = (6)^2 + (12)^2$
$\Rightarrow d^2 = 36 + 144$
$\Rightarrow d = \sqrt{180} = 6\sqrt{5}$





Q3. Since, ΔSTU is equilateral, therefore, all of its angles measure 60°. Now, at S the two angles are vertical, and since vertical angles are equal, therefore, the measure of $\angle S$ in quadrilateral PQRS is 60°.
Now, sum of the angles of PQRS = 360°
$90° + x° + 90° + 60° = 360°$
$\Rightarrow x° + 240° = 360°$
$\Rightarrow x = 360 - 240$
$\boxed{x = 120}$

Q4. The sides of the triangle are 5, 12 and 13 therefore, the given triangle is a right triangle.
Let $h = 5$ and $b = 12$, then its area

$A = \frac{1}{2}bh \Rightarrow \quad A = \frac{1}{2}(12)(5)$

$\Rightarrow \quad A = 30$

Now area of the rectangle is

Area $= w \times h$

Area $= 4 \times h$

Since the area of the rectangle equals to area of the triangle, therefore

$30 = 4 \times h \Rightarrow h = \frac{30}{4} \Rightarrow h = 7.5$ cm

The perimeter of the rectangle $= 2(w + h)$

$= 2(4 + 7.5)$

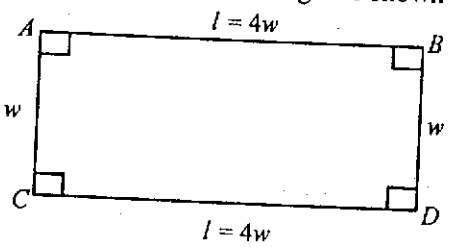
$= 23$ cm

Q5. (D) Each of the four triangle is a right triangle having hypotenuse 4. Therefore, each leg $= \frac{4}{\sqrt{2}} = \frac{2 \times \sqrt{2} \times \sqrt{2}}{\sqrt{2}} = 2\sqrt{2}$

The perimeter of each small triangle is $4 + 4\sqrt{2} = 4(1 + \sqrt{2})$

and the sum of the perimeter $= 4(4\ (1 + \sqrt{2}) = 16(1 + \sqrt{2})$

Q6. (D) According to the given condition, we draw a rectangle as shown in the figure



NowArea $= A = lw = 4w \times w = 4w^2$

and $196 = 4w^2 \Rightarrow w^2 = 49 \Rightarrow \boxed{w = 7}$

Now$w = 7 \Rightarrow l = 7 \times 4 = \boxed{28}$, so its perimeter

Perimeter $= 2(7 + 28) = 2(35) = \boxed{70}$

Q7. (B) The sum of the degree measures of the angles of a hexagon (six-sided polygon) is

$(6 - 2) \times 180° = 4 \times 180 = 720$

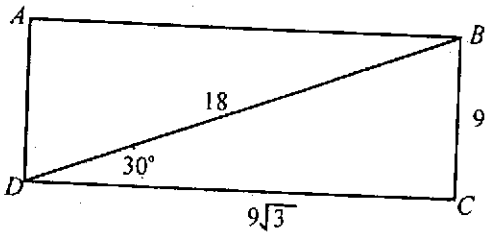
Now, sum of the ratio $= 2 + 4 + 4 + 4 + 5 + 5 = 24$

Now, degree measure of the smallest angle is

$\frac{2}{24} \times 720 = 2 \times 30 = 60$

Q8. (D) In the diagram, first we solve ΔBCD

$\frac{BC}{DB} = \sin 30° \Rightarrow \frac{BC}{18} = \frac{1}{2} \Rightarrow BC = 9$



and $\frac{DC}{BD} = \cos 30° \Rightarrow DC = BD \cos 30° \Rightarrow DC = 18 \times \frac{\sqrt{3}}{2}$

$\Rightarrow DC = 9\sqrt{3}$

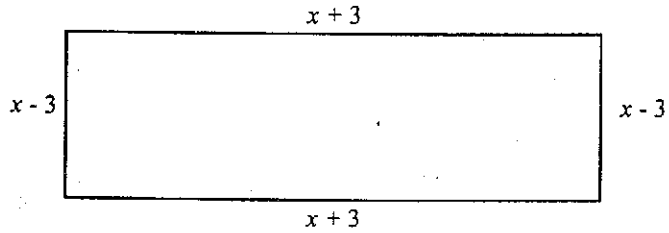
Area $= l \times w = 9 \times 9\sqrt{3} = 81\sqrt{3}$

Q9. (B) The perimeter of the rectangle $= 2(l + w)$

$= 2(9 + 9\sqrt{3})$

$= 18 + 18\sqrt{3}$

Q10.(D) Let x be the length of the square, then according to the given condition, the length and width of the rectangle is $x + 3$ and $x - 3$, respectively, as shown in the following figure



Then, area of the rectangle $A = (x + 3)(x - 3)$

$A = x^2 - 9$

But, area of the square $x^2 = 58$

Now, area of rectangle $A = 58 - 9$

$A = 49$

Q11.(A) In triangle PBQ, $PQ = \sqrt{(PB)^2 + (BQ)^2}$

$= \sqrt{(4)^2 + (3)^2} = \sqrt{25} = 5$

Hence perimeter of $PQRS = 4 \times 5 = 20$

Q12.(B) The area of the triangle $= \frac{1}{2}bh$

$= \frac{1}{2}(3)(4) = 6$

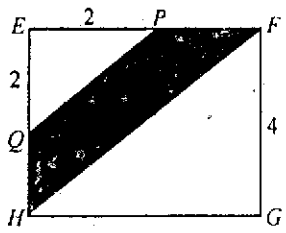
The total area of 4 triangles $= 6 \times 4 = 24$

Now, area of the triangle $ABCD = 8 \times 6 = 48$

Thus, area of the rectangle $PQRS$ = Area of rectangle $ABCD$ – Total area of the triangle

$= 48 - 24 = 24$

Q13.(C) Since P and Q are the midpoints of the sides of length 4. Therefore, EP, EQ, PF and QH are all equal to 2.



Also $PQ = \sqrt{(2)^2 + (2)^2} = \sqrt{4 + 4} = 2\sqrt{2}$

Area of $\Delta PEQ = \frac{1}{2}bh = \frac{1}{2}.2.2 = 2$

In ΔFEH, $FH = \sqrt{(EF)^2 + (EH)^2}$

$= \sqrt{16 + 16}$

$= \sqrt{32} = 4\sqrt{2}$

Perimeter of shaded region $= PQ + PF + FH + HQ$

$= 2\sqrt{2} + 2 + 4\sqrt{2} + 2$

$= 4 + 6\sqrt{2}$

Q14. Area of $\Delta EFH = \frac{1}{2}.4.4 = 8$

Area of $\Delta PEQ = \frac{1}{2} . 2 . 2 = 2$

Now area of shaded region = Area of ΔFEH – Area of ΔPEQ

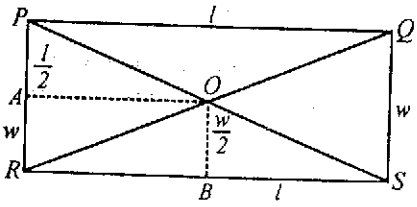
$= 8 - 2 = 6$

Q15. (B) The area of ΔPOR

$= \frac{1}{2}$ Base × Altitude

$= \frac{1}{2}(PR)(OA)$

$= \frac{1}{2}(w)(\frac{l}{2}) = \frac{wl}{4}$



Also, the area of $\Delta ORS = \frac{1}{2}$ Base × Altitude

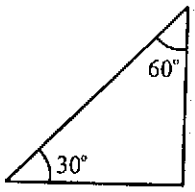
$= \frac{1}{2}(RS)(OB) = \frac{1}{2}(l)(\frac{w}{2})$

$= \frac{lw}{4}$

Q16. (B) Since angle O is acute ($\because m\angle O \angle 90$) and angle N is obtuse ($\because 90 < m\angle O \angle 180$), thus $(LN)^2 \angle x^2 + y^2$, where $(MO)^2 > x^2 + y^2$

$\Rightarrow (MO)^2 > (LN)^2 \Rightarrow MO > LN$

Q17. (C) 30 – 60 Right Triangle theorem states, that, in any right triangle with acute angle measures of 30 and 60 and with hypotenuse of length x, the length of the leg opposite the angle with measure 30(shorter leg) is $\frac{x}{2}$ and the length of the leg opposite the angle with measure 60 (longer leg) is $\frac{x}{2}\sqrt{3}$. Now the longer leg is given which is $2S$, therefore the hypotenuse will be $\frac{4S}{\sqrt{3}}$ ($\because \frac{4S}{\sqrt{3}} \times \frac{\sqrt{3}}{2} = 2S$) and the perpendicular will be $\frac{2S}{\sqrt{3}}$ ($\because \frac{4S}{\sqrt{3}} \times \frac{1}{2} = \frac{2S}{\sqrt{3}}$). Thus the perimeter of the triangle is



$$2S + \frac{2S}{\sqrt{3}} + \frac{4S}{\sqrt{3}} = \frac{2S\sqrt{3} + 2S + 4S}{\sqrt{3}} = \frac{6S + 2\sqrt{3}S}{\sqrt{3}}$$

$$= \frac{2 . \sqrt{3} . \sqrt{3}S}{\sqrt{3}} + \frac{2\sqrt{3}}{\sqrt{3}} . S$$

$$= 2S + 2S\sqrt{3}$$

$$= S(2 + 2\sqrt{3})$$

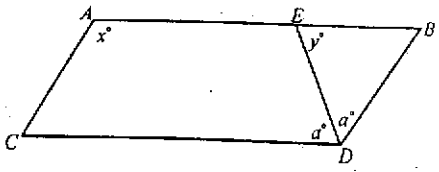
Q18. (C) The perimeter of a rectangle of area 40 is smallest when the rectangle is a square (because all sides are equal). In that case, each side is $\sqrt{40}$, which is greater than 6, and so the perimeter is greater than $24 (P = 6 + 6 + 6 + 6 = 24)$

Q19. (D) Given that ED is a transversal, which is cutting AB and CD, where $AB \parallel CD$, therefore, $y = a \Rightarrow 2y = 2a$.

We know that in a parallelogram opposite angles are equal, thus

$x^\circ = a + a \quad \Rightarrow x = 2a$ but $a = y$

$\Rightarrow x = 2y$



Q20. (A) As, in a parallelogram, opposite angles are equal, therefore

$\therefore \quad t = 5s + 16$

and $\quad u = 3s - 12$

Now $\quad t - u = (5s + 16) - (3s - 12)$

$= 5s + 16 - 3s + 12$

$t - u = 2s + 28$ .....(i)

Because, the sum of the measure of two consecutive angles of a parallelogram is 180, therefore

$(3s - 12) + (5s + 16) = 180 \quad \Rightarrow 8s + 4 = 180$

$\Rightarrow \quad 8s = 176$

$\Rightarrow \quad \boxed{s = 22}$

Now substituting the value of s in (i), we have

$t - u = 2(22) + 28$

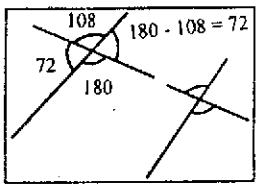
$t - u = 44 + 28 = \boxed{72}$

Q21. (A) The sum of the angles of n-gives $= (n - 2)180$

$\because \quad n = 5$, thus sum of the angles in a pentagon

$= (5 - 2)180 = 3 \times 180 = 540$

Average of each angle in a pentagon $= \frac{540}{5} = 108$



Since each interior angle has three exterior angle, so the sum of two exterior angles is $72 + 72 = 144$

Thus the sum of 5 pairs of exterior angles $= 5 \times 144$

$= 720$

Q22. E The area of the rectangle $= lw$

The area of the rectangle $PQRS = 90 = (x + 5)(x + 2)$

$\Rightarrow x^2 + 7x + 10 = 90$

and the area of the rectangle, $UVWX$

Area $= (x + 4)(x + 3) = x^2 + 7x + 12$

Which is exactly 2 more than the area of $PQRS$

hence the Area of $UVWX = 90 + 2$

$= 92$

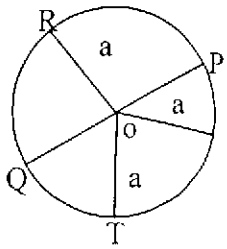
****

Chapter 4

CIRCLES

Circle:

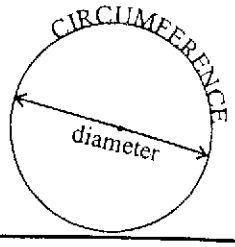
A circle is a set of all points in a plane at a given distance from a fixed point of the plane. The fixed point is the centre of the circle and the given distance is the radius. The adjacent figure is a circle of radius *a* unit whose centre is at the point *O*. The point *P, Q, R, S* and *T* lies on the circle, each *a* unit from *O*. Therefore, the following statement follows from the definition of a circle.



All radii (plural of radius) of the same circle are congruent.

Circumference:

The perimeter of a circle is called its circumference.



Note:

If "*c*" stands for the circumference of the circle and "*d*" is the diameter of the circle, then $\frac{c}{d}$ *(circumference ÷ diameter)* is the same for all circles. Its value cannot stated exactly.

The Greek letter π*(pi)* is used to stand for it.

$\therefore \quad \frac{c}{d} = \pi$

$c = \pi d$

or $\quad c = \pi \times 2 \times r$

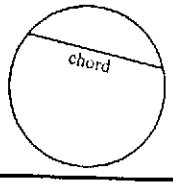
$c = 2\pi r$

π is a special number and is equal to 3.14159..... or $\frac{22}{7}$

Angles and Circles:

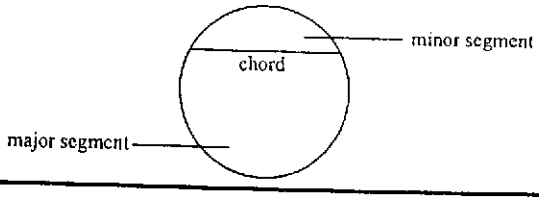
Chord:

A line joining two points on the circumference of a circle is called a chord.



Note:

A chord divides a circle into two segments.

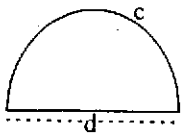


Diameter:

A chord which passes through the centre of the circle is a diameter.

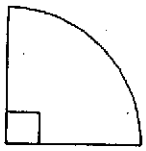
Arc Length of a Circle:

Arc length of a sector of half a circle is



$$c = \frac{\pi d}{2}$$

Arc length of a sector of quarter of a circle is



$$c = \frac{\pi d}{4}$$

Semicircles:

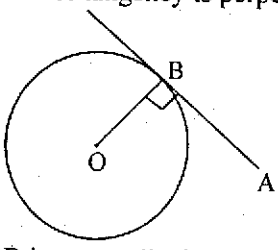
A diameter divides a circle into two congruent halves which are called semicircles.

Tangent of a Circle:

A line that intersects the circle in exactly one point is a tangent to the circle. The point of intersection is called the point of tangency.

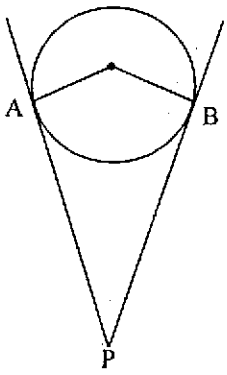
Note:

1. The radius from the centre to the point of tangency is perpendicular to the tangent.



AB is tangent to the circle with centre O. OB is perpendicular to BA.

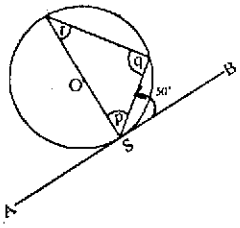
2. Tangents from the same point are equal



PA = PB

Example 1:

In the given figure, if AB is tangent to the circle. Calculate the sizes of angles, p, q, r.



Solution:

OS is the radius and AB is the tangent. By tangent – radius theorem $\angle OSB = 90°$

$\angle p = 90° - 50° = 40°$

Now $\angle q = 90°$ because angle in a semicircle is right. Now angle p, q and r the angles of triangle.

$\therefore \quad \angle p + \angle q + \angle r = 180°$

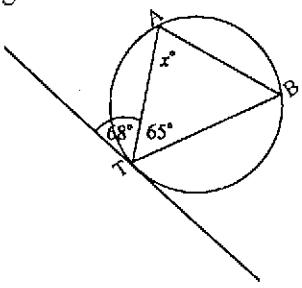
$\therefore \quad 40^\circ + 90^\circ + r = 180^\circ$

$\therefore \quad r = 180^\circ - 130^\circ$

$r = 50^\circ$

Example 2:

What is the value of x in the following diagram?

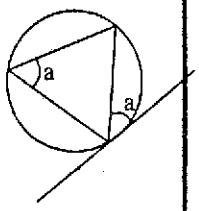


Solution:

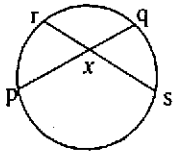
In above diagram $\angle TBA = 68^\circ$ because angles in alternate segment are equal

$\therefore \quad x^\circ + 65^\circ + 68^\circ = 180^\circ \Rightarrow x = 47^\circ$

Theorems: Tangents and Secants



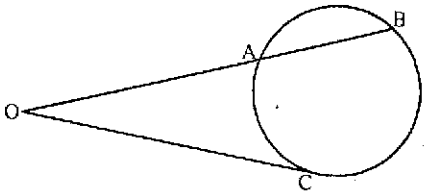
1. The angle between a tangent and a chord drawn to the point of contact is equal to the angles in the alternate segment.



2. The products of the intercepts of two intersecting chords of a circle are equal.

   That is: $px \,.\, qx = rx \,.\, sx$

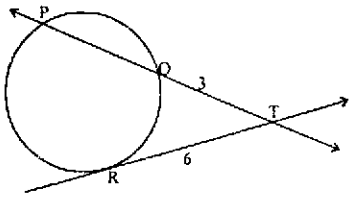
3. If a tangent and a secant intersect in the exterior of a circle, the square of the tangent segment equals the product of the secant segment and the external secant segment.



$OC^2 = OA \,.\, OB$

Example:

In the following figure. What is the value of TP?



Solution:

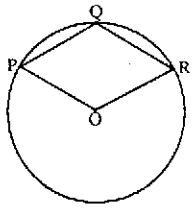
$(TR)^2 = TQ \,.\, TP$

$(6)^2 = 3 \,.\, TP$

$36 = 3 \,.\, TP$

$\Rightarrow \quad TP = 12$

Cyclic Quadrilateral:



A quadrilateral which has all its vertices laying on the circumference of a circle is called a cyclic quadrilateral.

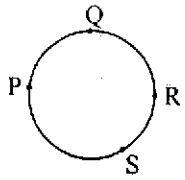
Note:

Opposite angles of a cyclic quadrilateral add up to 180°.

Common Arc Theorem:

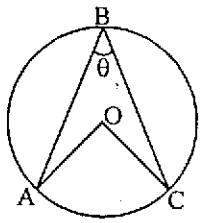
If four points on a circle are, in order and

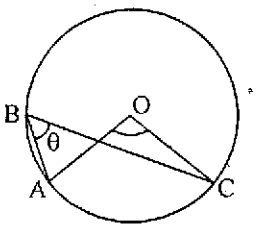
$\widehat{PQ} \cong \widehat{RS}$ Then $\widehat{PR} \cong \widehat{QS}$



Central Angle Theorems:

1. The angle subtended at the centre of a circle by an arc is twice the angle subtended at the circumference by the same arc.





In above, angle AOC = 2 × angle ABC.

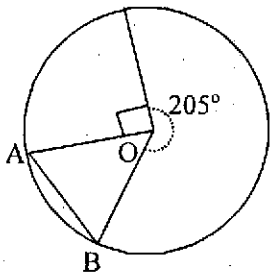
2. The angle measured in degree to complete one revolution in a circle is 360°.

Example:

What size angle is subtended at the centre (O) by chord AB?

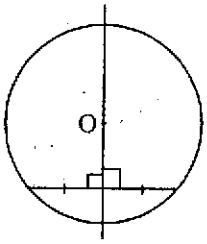
Solution:

$$\angle AOB + 90° + 205° = 360°$$
$$\angle AOB + 295° = 360°$$
$$\angle AOB = 360° - 295°$$
$$= 65°$$



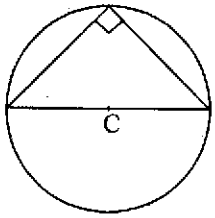
∴ Chord AB subtends an angle of 65° at the centre.

Theorem: A line from the centre of a circle through the mid-point of a chord meets the chord at right angles.



Theorem: Angle In a Semicircle:

An angle in a semicircle is a right angle.



Converse of Theorem:

If a circle passes through the ventricles of a right-angled triangle, then the hypotenuse of the triangle is a

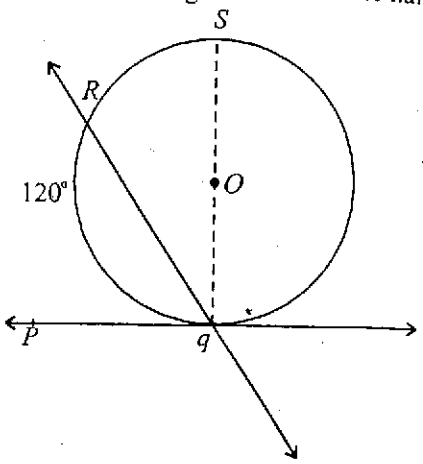
diameter of the circle.

More Angles and Arcs:

Theorem 1:

If a tangent and a secant (or chord) intersect in a point on a circle, the measure of the angle formed is one half the measure of the intercepted arc.

Explanation:



In circle O as shown in the figure, secant QR and tangent PQ intersect at point Q on the circle, forming angle PQR. The above theorem focuses upon the relationship between the measure of this angle and the degree measure of the intercepted arc, $\widehat{QR}$.

According to this theorem, $\angle PQR = \frac{1}{2}(120)$

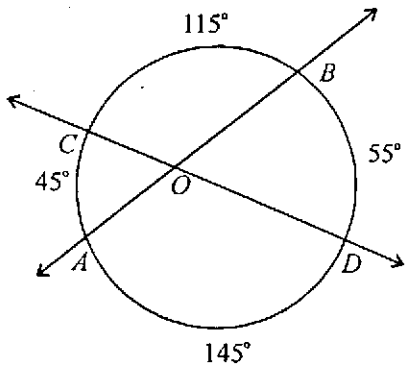
$= \frac{1}{2}(\text{Arc } QR)$

$= 60°$

Theorem 2:

If two secants (or chords) intersect in the interior of a circle, the measure of an angle formed is one half the sum of the measures of he arcs intercepted by the angle and its vertical angle.

Explanation:



When two secants AB and CD intersect in the interior of a circle, as circle O shows to the right, two pair of vertical angles are formed. According to the given theorem

$\angle AOC = \angle DOB = \frac{1}{2}(55° + 45°)$

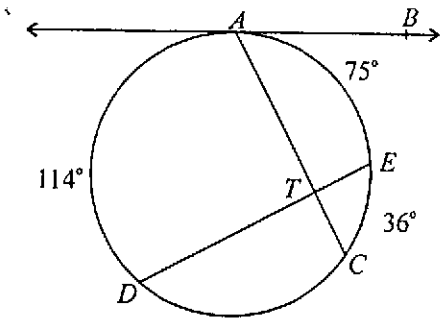
$= \frac{1}{2}(100) = 50°$

and $\angle AOD = \angle COB = \frac{1}{2}(145° + 115°)$

$= \frac{1}{2}(260)$

$= 130°$

Example 1:



Chord AC and DE intersect at T, $\overleftrightarrow{AB}$ is tangent to the circle at A.

$m\widehat{AD} = 114$, $m\widehat{EC} = 36°$ and $m\widehat{AE} = 75$

a. Find $m\angle CAB$

b. Find $m\angle ATD$

Solution: a. By theorem 1

$m\angle CAB = \frac{1}{2}m\widehat{AC}$

$= \frac{1}{2}(m\widehat{SR} + m\widehat{RT})$

$= \frac{1}{2}(75° + 36°)$

$= \frac{1}{2}(111)$

$= 55.5$

b. By theorem 2

$$m\angle ATD = \frac{1}{2}(m\widehat{EC} + m\widehat{AD})$$

$$= \frac{1}{2}(36^\circ + 114^\circ)$$

$$= \frac{1}{2}(150)$$

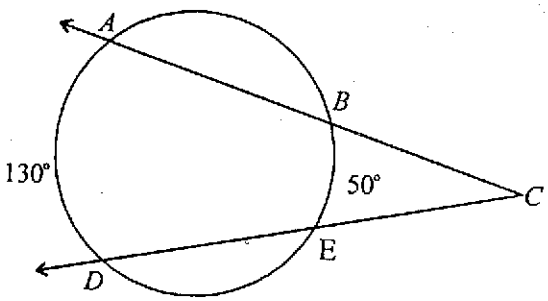
$$= 75$$

Theorem 3:

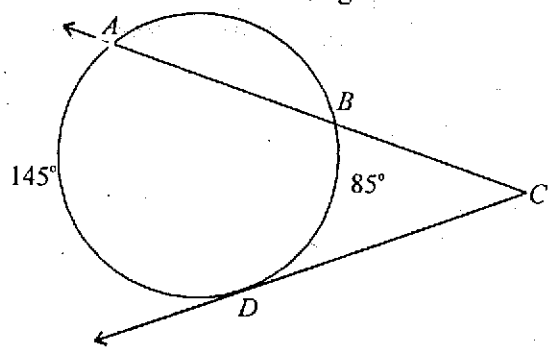
If two secants, a tangent and a secant, or two tangents intersects in the exterior of a circle, the measure of the angle formed is one-half the difference of the measures of the intercepted arcs.

Example 2: In each case of the following figure, find m$\angle C$.

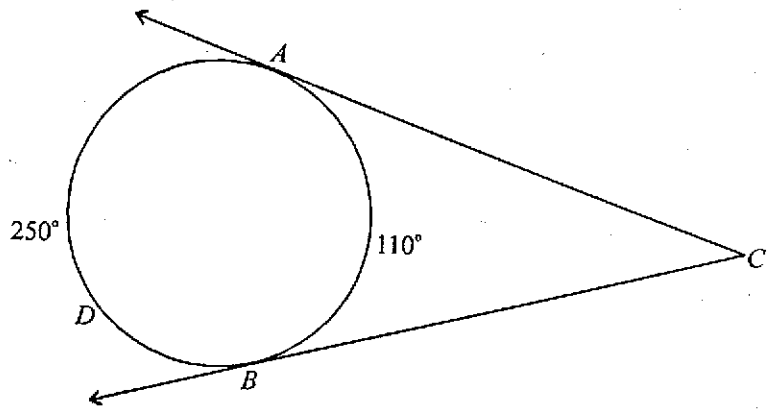
a. Two secants



b. One secant one tangent



c. Two tangents



Solution:

a. Applying theorem 3

$$m\angle C = \frac{1}{2}(m\widehat{AD} - m\widehat{BE})$$

$$= \frac{1}{2}(130 - 50)$$

$$= \frac{1}{2}(80) = 40$$

b. Applying theorem 3

$$m\angle C = \frac{1}{2}(m\widehat{AD} - m\widehat{BD})$$

$= \frac{1}{2}(250 - 110)$

$= \frac{1}{2}(140)$

$= 70$

c. Applying theorem 3

$m\angle C = \frac{1}{2}(m\widehat{ADB} - m\widehat{AB})$

$= \frac{1}{2}(250 - 110)$

$= \frac{1}{2}(150)$

$= 75$

Multiple Choice Questions (MCQs)

Q1. If the area of a circle is 81π, then its circumference is:

(A) 61π (B) 20π

(C) 18π (D) 16π

Q2. If circumference of a circle is 3π, then its area is:

(A) $\frac{7\pi}{2}$ (B) $9\pi^2$

(C) $4\pi^2$ (D) $\frac{9\pi}{4}$

Q3. If a circle is inscribed in a square of area 4, then the area of the circle is:

(A) π (B) $\frac{\pi}{2}$

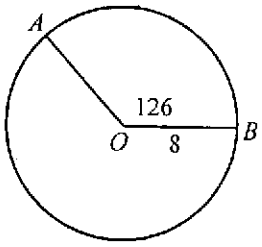
(C) $\frac{\pi}{4}$ (D) $\frac{3\pi}{4}$

Q4. If a square of area 3 is inscribed in a circle, then the area of the circle is:

(A) $\frac{9}{4}\pi$ (B) $9\pi^2$

(C) 3π (D) $\sqrt{3}\pi$

Questions 5 – 6 refer to the following figure



Q5. What is the length of arc AB?

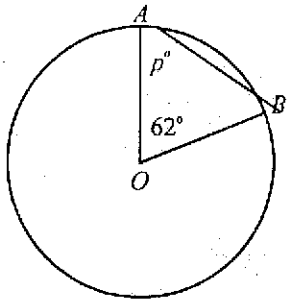
(A) 2.6π (B) 5.6π

(C) 7.6π (D) $\frac{1}{2}\pi$

Q6. What is the area of the shaded sector?

(A) 22.9π (B) 22.4π
(C) 60π (D) 62.3π

Q7. In the following figure, what is the value of p?



(A) 49 (B) 39
(C) 59 (D) 69

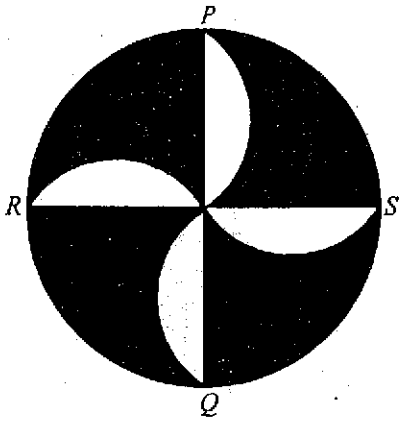
Q8. If P represents the area and W represents the circumference of the circle, then P in terms of W is:

(A) $\frac{2\pi}{W}$ (B) $\frac{4\pi^2}{W}$
(C) $\frac{2\pi^2}{W^2}$ (D) $\frac{W^2}{4\pi}$

Q9. What is the area of a circle whose radius is the diagonal of a square whose area is 9?

(A) $\sqrt{3}\pi$ (B) 12π
(C) 4π (D) 13π

Q10. In the following figure, PQ and RS are perpendicular, and each of the unshaded regions is a semicircle. What is the ratio of the white area to the shaded area?



(A) $\frac{4}{\pi}$ (B) $\frac{1}{1}$
(C) $\frac{2}{3}$ (D) $\frac{1}{2}$

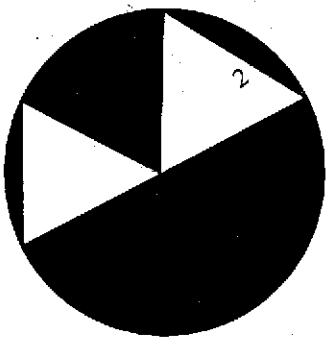
Q11. If C is the circumference of a circle of radius r, then which of the following statement is true?

(A) $\frac{C}{r} < 6$ (B) $\frac{C}{r} = 6$
(C) $\frac{C}{r} > 6$ (D) $\frac{C}{r} = \pi$

Q12. If C is the circumference of a circular disk in centimeters, and A is the area of the same circular disk in square centimeter. Then $\frac{C}{A} = \frac{A}{C}$, iff $r =$

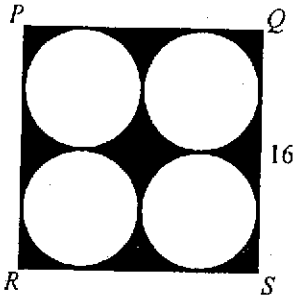
(A) 1 (B) 2
(C) 3 (D) 4

Q13. In the following figure, what is the area of the shaded region, if each of the triangle is equilateral?



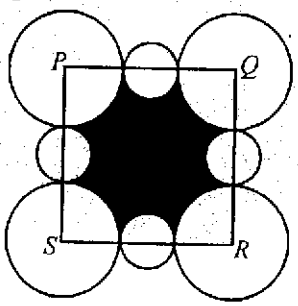
(A) 8π (B) $\frac{8}{3}\pi$
(C) 3π (D) 6π

Q14. In the following figure, *PQRS* is a square, and all the circles are tangent to one another and to the sides of the squares. What is the area of the shaded region?



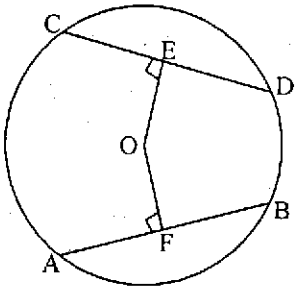
(A) 256 (B) 64π
(C) 256π (D) $64(4-\pi)$

Q15. In the following figure, the large circles have radius 4, and the small circles have diameter 4. What is the area of the square *PQRS*?



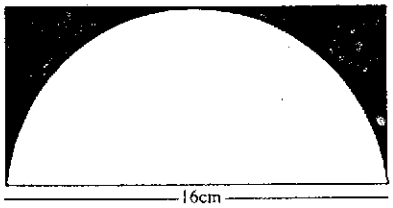
(A) 144 (B) 169
(C) 100 (D) 64

Q16. In the following figure, if AB = CD and OE = 2.5, what is the value of OF?



(A) 6.25 (B) 5
(C) 1.58 (D) 2.5

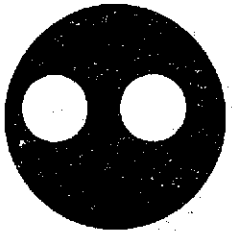
Q17. A semicircle is drawn inside a rectangle as shown.



The shaded area is closest to:

(A) 50 (B) 40
(C) 30 (D) 45

Q18. In the following figure, if the radius of the outer circle is p and the radius of each of the circles inside the larger circle is $\frac{p}{3}$, then what is the area of the shaded region?

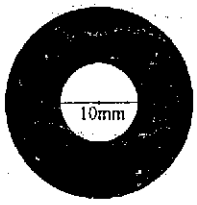


(A) $\frac{2}{9}\pi p^2$ (B) $\frac{11}{9}\pi p^2$
(C) $\frac{22}{9}\pi p^2$ (D) $\frac{7}{9}\pi p^2$

Q19. A circle is inscribed in a square of area $\sqrt{6}$. What is the area of the circle?

(A) $\frac{3}{2}\pi$ (B) 6π
(C) π (D) 9π

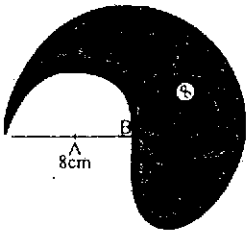
Q20. A circle of radius 5 mm is removed from the centre of a circular piece of metal of radius 7 mm to make a washer as shown below:



What is the area of the shaded region?

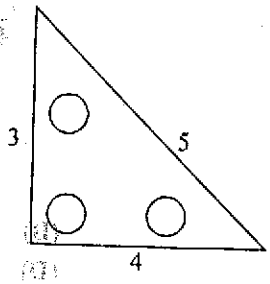
(A) 25π (B) 49π
(C) $35\pi^2$ (D) 24π

Q21. In the following metal cam, A, B and C are the centres of the semicircles shown. What is the area of the cam?



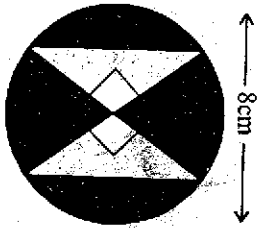
(A) 32.07 cm^2 (B) 157.08 cm^2
(C) 101.53 cm^2 (D) 201.06 cm^2

Q22. The sketch below shows a triangular copper plate with sides of 3cm, 4cm and 5cm. It has three small circular holes cut out of it. The radius of each circle is 3mm. What is the area of the copper triangle?



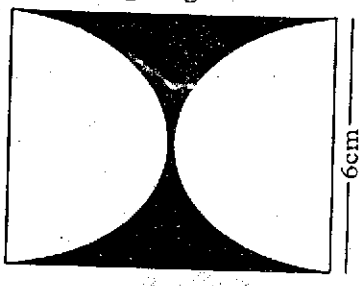
(A) 6 cm^2 (B) 5.16 cm^2
(C) 5.9973 cm^2 (D) 5.71 cm^2

Q23. What is the shaded area in the following diagram?



(A) 50.29 cm^2 (B) 16 cm^2
(C) 16.29 cm^2 (D) 34.29 cm^2

Q24. What is the shaded area in the following diagram?



(A) 36 cm^2 (B) 7.72 cm^2
(C) 28.3 cm^2 (D) 14.14 cm^2

Explanatory Answers

Q1. (C) Area of a circle: $A = \pi r^2 = 81\pi \quad \Rightarrow r^2 = 81 \Rightarrow r = 9$

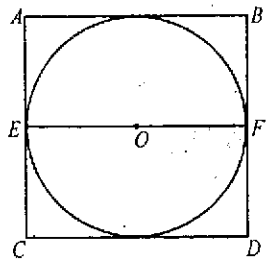
Circumference of a circle: $C = 2\pi r \Rightarrow C = 2\pi(9) = 18\pi$

Q2. (D) Circumference of a circle: $C = 2\pi r \Rightarrow 2\pi r = 3\pi$

$$\Rightarrow r = \frac{3}{2}$$

Area of a circle: $A = \pi r^2 \Rightarrow A = \pi \cdot \left(\frac{3}{2}\right)^2 \Rightarrow A = \frac{9\pi}{4}$

Q3. (A) First of all, we draw the diagram



Since the area of the square is 4 (given), therefore $AC = 2$, as in a square all sides are equal, therefore $AC = AB = BD = CD = 2$.

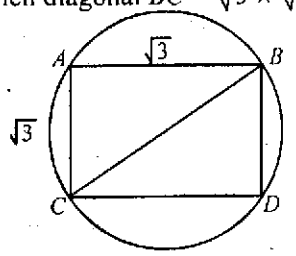
$\because$ Diameter of the circle, $EF = CD = AB \Rightarrow EF = 2$

and radius of the circle $= r = OF = OE = 1$ (half the diameter)

Hence the area of the circle with radius 1 is

$$\pi(1)^2 = \pi$$

Q4. (A) First we draw a diagram, because area of the square is 3, thus $AC = \sqrt{3}$, then diagonal $BC = \sqrt{3} \times \sqrt{3} = 3$, but BC is also the diameter of the circle, hence the diameter is 3 and radius is 1.5



Now, the area of the circle $A = \pi r^2 \Rightarrow A = \pi(1.5)^2 \Rightarrow A = 2.25\pi = \frac{9}{4}\pi$

Q5. (B) Setting a proportion

$$AB : OB :: 126 : 360$$

$$\frac{\overline{AB}}{2\pi r} = \frac{126}{360}$$

$$\overline{AB} = \left(\frac{126}{360}\right) \times 2\pi r$$

$$\overline{AB} = \left(\frac{126}{360}\right) \times 2\pi \times 8$$

$$\overline{AB} = 5.6\pi$$

Q6. (B) The area of shaded sector is $\frac{126}{360}\pi(8)^2$

$$= (0.35)\pi(64)$$

$$= 22.4\pi$$

Q7. (C) Because, the triangle is isosceles therefore, the angle B is also p, thus

$$p^\circ + p^\circ + 62^\circ = 180^\circ$$

$$\Rightarrow \quad 2p^\circ = 118 \Rightarrow p = 59$$

Q8. D Since, P is the area, so $P = \pi r^2$, and

W is the perimeter, thus $W = 2\pi r \Rightarrow r = \frac{W}{2\pi}$

$$\Rightarrow \quad P = \pi\left(\frac{W}{2\pi}\right)^2$$

$$P = \pi\frac{W^2}{4\pi^2}$$

$$\Rightarrow \quad P = \frac{W^2}{4\pi}$$

Q9. (B) Since the area of the square is 9, so its each side is 3, and the length of the diagonal will be $\sqrt{3} + \sqrt{3} = 2\sqrt{3}$

Thus area of the circle of radius $2\sqrt{3}$ is

$$A = \pi r^2 \Rightarrow A = \pi(2\sqrt{3})^2 \Rightarrow A = 12\pi$$

Q10. (B) Let the radius of the big circle be r, then its area will be πr^2, also the radius of the semicircle becomes $\frac{r}{2}$, so area of the small circle will be $\pi\frac{r^2}{2}$ and the area of the each semicircle is

$$\left(\pi\frac{r^2}{4}\right)\times\frac{1}{2} = \frac{\pi r^2}{8}$$

Then the area of the four small semicircles is

$$= 4\left(\frac{\pi r^2}{8}\right) = \frac{\pi r^2}{2}$$

So, shaded area = Total area – White area

$$= \pi r^2 - \frac{\pi r^2}{2} = \frac{\pi r^2}{2}$$

Therefore the ratio of shaded area is

$$\frac{\frac{\pi r^2}{2}}{\frac{\pi r^2}{2}} = \frac{1}{1}$$

Q11. (C) Since $C = 2\pi r \Rightarrow C = \pi(2r)$, but $2r = d$

Hence $C = \pi d \Rightarrow \pi = \frac{C}{d} \Rightarrow \frac{C}{r} = 2\pi$

$$\Rightarrow \frac{C}{2} = 2\left(\frac{22}{7}\right) > 6$$

Q12. (B) As $C = 2\pi r$ and $A = \pi r^2$, so

$$\frac{C}{A} = \frac{2\pi r}{\pi r^2} = \frac{2}{r}$$

and $\frac{A}{C} = \frac{\pi r^2}{2\pi r} = \frac{r}{2}$

Thus $\frac{C}{A} = \frac{A}{C}$, only possible, when $r = 2$

Q13. (B) Because the triangles are equilateral, then the white central angles each measure 60°, so their sum = 60 + 60 = 120. Then, the unshaded area is $\frac{120}{360} = \frac{1}{3}$ of the circle, so the shaded area of $\frac{2}{3}$ of the circle.

As the area of the circle = $\pi r^2 = \pi(2)^2 = 4\pi$

and the area of the shaded region $= \frac{2}{3}\times 4\pi = \frac{8}{3}\pi$

Q14. (D) Since $QS = 16$, thus the diameter of each circle is 8, and radius of each circle is 4.

$\therefore$ The area of each circle $= \pi r^2 = \pi(4)^2 = 16\pi$

Thus, the area of four circles $= 4(16\pi) = 64\pi$

Now, the area of the square $= 16 \times 16 = 256$

Area of the shaded region = Area of the square – Area of the circle

$= 256 - 64\pi$

$= 64(4 - \pi)$

Q15. (A) Since the radius of the large circle is 4, and diameter of the small circle is 4, so each side of the square is 4 + 4 + 4 = 12, so area of the rectangle = 12 × 12 = 144

Q16. **(D)** $\boxed{OF = 2.5}$

Because equal chords are equidistant from centre.

Q17. **(C)** Area of the rectangle $= 16 \times 8 = 128$

Area of the circle of radius 8cm $= \pi(8)^2$

$= 64\pi$

Area of the semicircle $= \frac{64}{2}\pi = 32\pi$

$= 32(3.14) = 100.48$

$= 100.48$

Area of the shaded region = Area of rectangle – Area of the semicircle

$= 128 - 100.48$

$= 27.52$

which is closest to 30

Q18. **(D)** Area of the outer circle $= \pi(p)^2 = \pi p^2$

Area of the inner circle $= \pi\left(\frac{p}{3}\right)^2 = \frac{\pi p^2}{9}$

Total area of the inner circles

$= \frac{\pi p^2}{9} + \frac{\pi p^2}{9}$

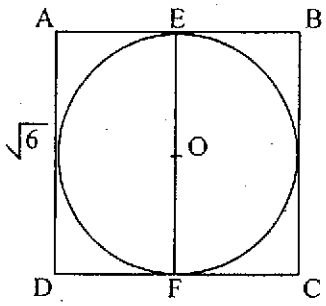
$= \frac{2}{9}\pi p^2$

Area of the shaded region $= \pi p^2 - \frac{2\pi p^2}{9}$

$= \frac{9\pi p^2 - 2\pi p^2}{9}$

$\boxed{= \frac{7}{9}\pi p^2}$

Q19. **(A)** The inscribed circle in a square of area 6 is



The side AD $= \sqrt{6}$ and also the diameter $= \sqrt{6}$. The radius of the circle O is OF $= \frac{\sqrt{6}}{2} \Rightarrow$ OF $= \sqrt{\frac{3}{2}}$

The area of the circle of radius $\sqrt{\frac{3}{2}}$ is

$$\text{Area} = \pi\left(\sqrt{\frac{3}{2}}\right)^2 \Rightarrow \boxed{\text{Area} = \frac{3}{2}\pi}$$

Q20. (D)

Area of the washer removed $= \pi r^2 = \pi(5)^2 = 25\pi$

Area of the metal $= \pi r^2 = \pi(7)^2 = 49\pi$

Area of the shaded region $= 49\pi - 25\pi = 24\pi$

Q21. (C) When we shift the semicircle of diameter 8cm in the space the shape becomes semicircle of diameter 16cm

$\therefore$ Area $= \pi r^2$

$= \pi \times (8)^2 = 64\pi$

Area of semicircle $= \frac{64\pi}{2} = 32\pi = \boxed{101.53\text{cm}^2}$

Q22. (B) Area of the triangle $= \frac{1}{2} \times \text{base} \times \text{Altitude}$

$= \frac{1}{2} \times 4 \times 3 = 6\text{cm}^2$

Now 3mm $= 0.3\text{cm}$

Area of the circle $= \pi r^2 = 0.28\text{cm}^2$

Area of 3 circles $= 3 \times 0.28 = 0.84\text{cm}^2$

Area of copper in plate $= 6 - 0.84 = \boxed{5.16\text{cm}^2}$

Q23. (D) Diameter of the circle $= 8\text{cm}$

Radius $= 4\text{cm}$

Area of the circle $= \pi r^2 = \frac{22}{7} \times 16 = 50.29\text{cm}^2$

Area of triangle $= \frac{1}{2}\text{bases} \times \text{altitude}$

$= \frac{1}{2} \times 4 \times 4$

$= 8\text{cm}^2$

Area of 2 triangles $= 2 \times 8 = 16\text{cm}^2$

Area of the shaded region $= 50.29 - 16 = \boxed{34.29\text{cm}^2}$

Q24. (B) Area of the semicircle $= \frac{1}{2}\pi d$

$= \frac{1}{2} . \pi . r^2 = \frac{1}{2}\pi(3)^2$

$= 14.14\ \text{cm}^2$

Area of the 2 same semicircles $= 2 \times 14.14 = 28.28\text{cm}^2$

Area of the square $= 6 \times 6$

$= 36\text{cm}^2$

Area of the shaded region $= 36 - 28.28 = \boxed{7.72\text{cm}^2}$

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Chapter 5

AREA

Some Important Formulae

(1) Area of a rectangle or square
$=$length$\times$breadth

(2) Perimeter of a rectangle or square
$= 2$ (length + breadth)

(3) Area of a triangle $= \frac{1}{2} \times$ base $\times$ height

(4) Area of a triangle of sides a, b, and c
$= \sqrt{s(s-a)(s-b)(s-c)}$

where $s = \frac{(a+b+c)}{2}$

(5) Area of a circle $= \pi r^2$

(6) Circumference of a circle
$= 2\pi r$

(7) Area of four walls of a room
$= 2$(length + breadth) $\times$ height

(8) Area of a parallelogram
$=$base $\times$ height

(9) Area of a trapezium
$= \frac{1}{2} \times$ sum of two parallel sides $\times$ width

(10) Area of a regular hexagon of side a
$= 6a^2\sqrt{\frac{3}{4}}$.

Model Examples

Q.1. The difference between the circumference of a circle and its diameter is 135 ft. Find the area of the circle.

$$\left(Take \pi = \frac{22}{7}\right)$$

Sol. Let r be the radius of the circle, then circumference of circle

$= 2\pi r$

Diameter of circle $= 2r$

By the given condition

$= 2\pi r - 2r = 135$

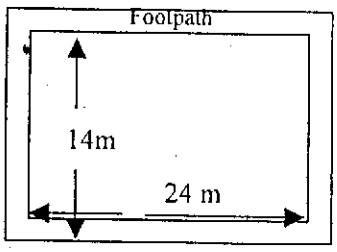
$r = \frac{63}{2}$ ft.

Area of circle $= \pi r^2$

$= \frac{22}{7} \times \frac{63}{2} \times \frac{63}{2}$ sq. ft.

$= 3118.5$ sq. ft. **Ans.**

Q.2. How many tiles 20 cm. square will be required to have a foot path 1 metre wide carried round the outside of grass plot 24 meter long by 14 metres broad?



Sol. Area of the grass plot $= 24 \times 14$
$= 336$ sq. m.

Length and breadth of the plot including the path is = 26 m and 16 m

Area of plot including path $= 26 \times 16$
$= 416$ sq. m.

$\therefore$ Area of path $= 416 - 336 = 80$ sq. m.

Area of one til $= \frac{20}{100} \times \frac{20}{100} = \frac{1}{25}$ sq. m.

$\therefore$ No. of tiles required $= \frac{80}{\frac{1}{25}} = 80 \times 25$

$= 2000$ **Ans.**

Q.3. If the length of a rectangular piece of land were 5 metres less and the breadth 2 metres more, the area would be 10 sq. m. less; but if the length were 10 metres more and breadth 5 metres more, the area would have been 275 sq. m. more. Find its length and breadth.

Sol. Let the length be 'l' breadth 'b'

$\therefore$ area $= l \times b$

Then $(l - 5)(b + 2) = lb - 10$.....(i)

And $(l + 10)(b + 5) = lb + 275$.....(ii)

From (i)

$lb + 2l - 5b - 10 = lb - 10$

$2l = 5b \quad \Rightarrow \quad l = \frac{5}{2}b$

From (ii)

$lb + 5l + 10b + 50 = lb + 275 \quad \Rightarrow \quad 5l + 10b = 225$

Putting $l = \frac{5}{2}b$

$\frac{25}{2}b + 10b = 225 \quad \Rightarrow \quad \frac{45}{2}b = 225$

$b = \frac{2 \times 225}{45} = 10$ and $l = 25$ m

Q.4. The perimeter of one square exceeds the perimeter of another square by 120 metres and the area of the larger square exceeds twice the area of the smaller square by 900 square metres. Find the length of the sides of the squares.

Sol. Let the length of one side of larger square $= x$ sq. m

Let the length of one side of smaller square $= y$ mts.

Now by the given condition the primeter ($4x$) of larger exceeds the perimeter of smaller ($4y$) by 120 sq. m

i.e., $4x - 4y = 120$

or $x = 30 + y$ ..... (i)

Again by the second condition

$$x^2 - 2y^2 = 900 \qquad \ldots\ldots (ii)$$

Putting the value of x from (i) in (ii), we get

$$(30 + y)^2 - 2y^2 = 900 \Rightarrow -y^2 + 60y = 0$$

or $\qquad y = 60$

Putting the value in (i) we get $\quad x = 90$

$\therefore$ Larger square side length is = 90 sq. m

Smaller square side length is = 60 sq. m **Ans.**

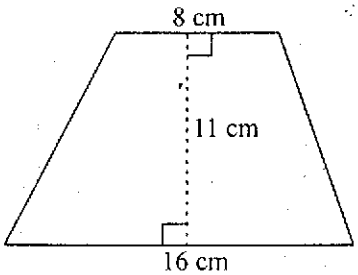
Multiple Choice Questions (MCQs)

Q1. The surface area of sphere of radius $3\frac{1}{2}$ cm is:

(A) 130 sq.cm (B) 69 sq.cm

(C) 154 sq.cm (D) 98 sq.cm

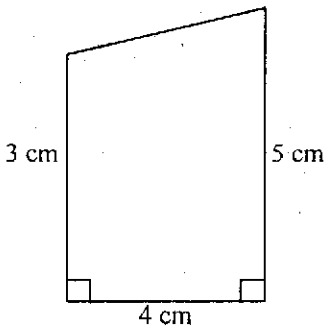
Q2. The area of the following trapezium is:



(A) 125 sq.cm (B) 132 sq.cm

(C) 139 sq.cm (D) 97 sq.cm

Q3. The area of the following figure is:



(A) 16 sq.cm (B) 15 sq.cm

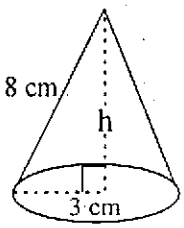
(C) 60 sq.cm (D) None of these

Q4. The height of a triangle of base 3 cm and area 9 cm is:

(A) 6 cm (B) 9 cm

(C) 18 cm (D) 22 cm

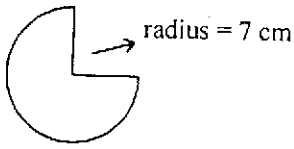
Q5. What is the surface of the following figure?



(A) 33π (B) 24π

(C) 25π (D) 11π

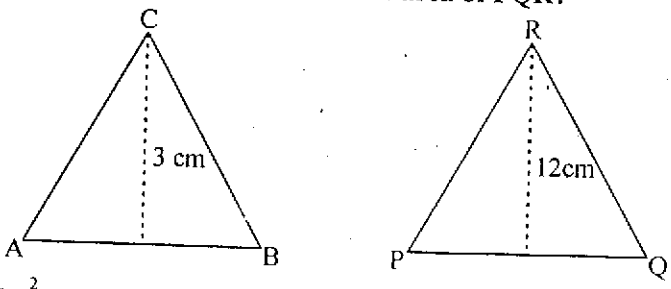
Q6. What is the area of the following figure?



(A) 125 sq.cm (B) 150.72 sq.cm

(C) 64 sq.cm (D) 56 sq.cm

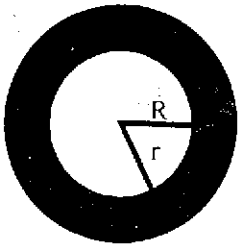
Q7. The following two triangles are similar. Find the area of PQR?



(A) $54\ cm^2$ (B) $24\ cm^2$

(C) $96\ cm^2$ (D) $59\ cm^2$

Q8. The area of the shaded region of the following figure is?



(A) $\pi^2(r^2 - R^2)$ (B) $\pi r^2 + \pi R^2$

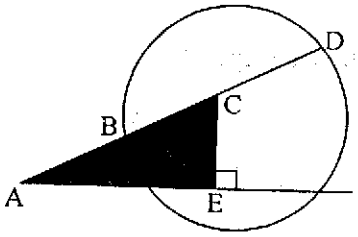
(C) $\pi^2(R + r)(R - r)$ (D) $\pi(R + r)(R - r)$

Q9. The area of the sector which contains an angle of 60° of circle of radius 7 cm, is:

(A) $25\frac{2}{3}\ cm^2$ (B) $27\frac{2}{3}\ cm^2$

(C) $41\ cm^2$ (D) $\sqrt{3}\,\frac{5}{28}\ cm^2$

Q10. In the following figure, what is the area of the right triangle? If $\overline{AE}$ = 16 cm and $\overline{BD}$ = 12 cm

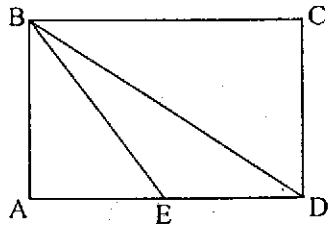


(A) 6 (B) 49

(C) 8 (D) Not possible

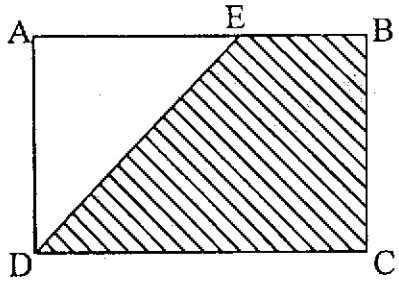
Q11. **In the figure below, ABCD is a rectangle and E is the mid point of one side, what is the area of triangle BCD?**

If $\overline{BE}$ = 5 cm and $\overline{CD}$ = 3cm



(A) 12 cm^2 (B) 25 cm^2
(C) 9 cm^2 (D) 17.23 cm^2

Q12. **In the figure below, given that AD = 6, CD = 8, AE = x. What is the area of the shaded region?**



(A) $28 - 2x$ (B) $4(14 - 4x)$
(C) $48 - 3x$ (D) $7(6 - 3x)$

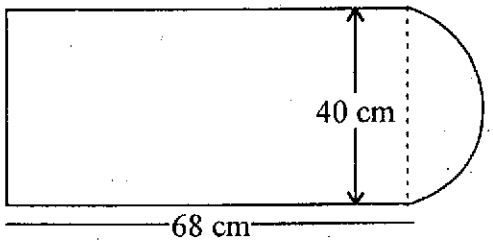
Q13. **If the radius of the circle is decreased by 20%, what happens the area?**

(A) 10% in crease (B) 20% decrease
(C) 80% increase (D) 36% decrease

Q14. **A square, with perimeter 16, is inscribed in a circle, what is the area of the circle?**

(A) 3π (B) $2\sqrt{2}\pi$
(C) 32π (D) 8π

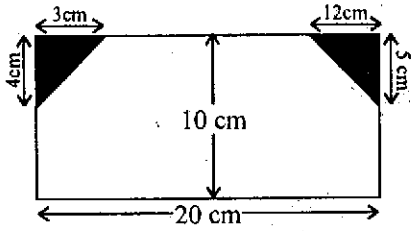
Q15. **Calculate the area of the following figure which is consists of:**



The rectangle is of 40 cm and 68 cm and half a circle of diameter 40 cm

(A) 2535 cm^2 (B) 2720 cm^2
(C) 3348 cm^2 (D) 628 cm^2

Q16. **What is the area of the following shake if the shaded areas are cut away?**



(A) 100 cm^2 (B) 125 cm^2

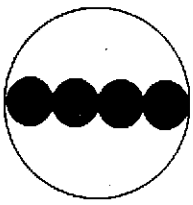
(C) $120\ cm^2$ (D) $164\ cm^2$

Q17. The length of rectangle is decreased by 15% and its width is increased by 40%. The area is:

(A) decreases by 25% (B) no effect

(C) increases by 36% (D) increases by 19%

Q18. In following figure equal circles lie along the diameter of the large circle. If the circumference of the circle is 64π. What is the area of the shaded region?



(A) 64π (B) 256π

(C) 16π (D) None of these

Explanatory Answers

Q1. (C) $A = 4\pi r^2$

$= \frac{4}{1} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$ sq.cm

$= 154$ sq.cm

Q2. (B) A = Average width × Height

$= \frac{(8+16)}{2} \times 11$ sq.cm

$= 132$ sq.cm

Q3. (A) $A = \frac{(3+5)}{2} \times 4$

$= 16$ sq.cm

Q4. (A) $9 = \frac{1}{2} \times 3 \times h$

$\therefore h = \frac{2 \times 9}{3}$

$= 6$ cm

Q5. (A) $S = \pi rs + \pi r^2$

$= (\pi \times 3 \times 8) + \pi(3 \times 3)$

$= 24\pi + 9\pi = 33\pi$

Q6. (B) The figure is $\frac{3}{4}$ of the circle, Now

Area of the whole circle $= \pi r^2$

$\frac{3}{4}$ of the circle $= \frac{3}{4}(3.14)(18)^2$

$= \frac{3}{4} \times 200.96$

$= 150.72\ cm^2$

Q7. (A) $\frac{AB}{DE} = \frac{\text{height of the triangle ABC}}{\text{height of the triangle DCP}}$

$$\text{height} = \frac{36\text{ cm}}{4} = 9\text{cm}$$

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 12 \times 9$$

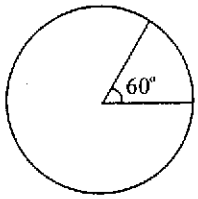
$$= 54\text{ cm}^2$$

Q8. **(D)** Area= Whole – hole

$$= \pi R^2 - \pi r^2$$

$$= \pi(R^2 - r^2) = \pi(R - r)(R + r)$$

Q9. **(A)** Sector $= \frac{\theta^o}{360^o} \times \pi r^2$



$$= \frac{60}{360} \times \frac{22}{7} \times \frac{7}{1} \times \frac{7}{1}\text{ cm}^2$$

$$= 25\frac{2}{3}\text{ cm}^2$$

Q10. **(D)** The area of the height triangle equals half the product of two legs. The point E is not on the circle, The length CE is given. We cannot find the exact value of CE. So lacking the length of CE, we cannot calculate the area.

Q11. **(A)** In the given rectangle, $\overline{AB} = \overline{CD} = 3\text{cm}$. The length of the side AE of triangle BAE can be found with the Pythagorean theorem as:

$(BF)^2 = (AB)^2 + (AE)^2 \Rightarrow 25 = 9 + (AE)^2$

$\Rightarrow (AE)^2 = 16 \Rightarrow AE = 4$

As point E is the mid point of $\overline{AD}$, the length of the rectangle is twice $\overline{AE}$ or 8 cm. The area of the right triangle BCD is half the product of the sides adjacent to the right angle

$$A = \frac{1}{2}bh = \frac{1}{2}(8)(3) = 12\text{ cm}^2$$

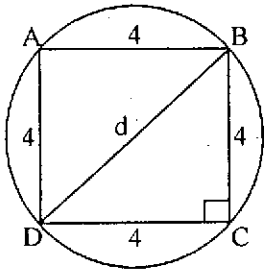
Q12. **(C)** The difference between the entire rectangle and the white triangle is the shaded area. Thus

$$(6 \times 8 = 48) - \frac{6 \times x}{2} = 48 - 3x$$

Q13. **(D)** If the radius of a circle is 1, after reducing 20% it becomes 0.8. Since Area = 0.8 × 0.8 = 0.64 = 1 – 0.64 = 0.36 = %

Q14. **(D)** Since the area of the square is 16.

∴ each side of the square is 4. In ΔBCD



$$(4)^2 + (4)^2 = (BD)^2 \Rightarrow BD = \sqrt{32} = 4\sqrt{2}$$

$$\text{The radius of the circle} = \frac{d}{2} = \frac{4\sqrt{2}}{2} = 2\sqrt{2}$$

$$\text{Area of the circle} = \pi r^2 = \pi(2\sqrt{2})^2 = \pi(4)(2) = 8\pi$$

Q15. **(C)** Area of the work surface = Area of the rectangle + area inside half a circle

Area of the rectangle $= 68 \text{ cm} \times 40 \text{ cm} = 2720 \text{ cm}^2$

Area of the circle $= \pi r^2 = 3.14 \times \left(\frac{40}{2}\right)^2 = 3.14 \times (20)^2$

$= 628 \text{ cm}^2$

Area of the shape $= 2720 \text{ cm}^2 + 628 \text{ cm}^2$

$= 3348 \text{ cm}^2$

Q16. (D) Area of the unshaded region = Whole Area of the rectangle – Area of first triangle – Area of second triangle

Area of rectangle $= 20 \times 10 = 200 \text{ cm}^2$

Area of first triangle $= \frac{1}{2}(3)(4) = 6 \text{ cm}^2$

Area of second triangle $= \frac{1}{2}(12)(5) = 30 \text{ cm}^2$

Area of unshaded region $= 200 - 6 - 30 = 164 \text{ cm}^2$

Q17. (D) The 85%L shows a 15% decrease in length 140% W represents a 40% increase in width. The news rectangle will be

Area = (new length)(new width)

$= (80\% \text{ L})(40\% \text{ W}) = \frac{85}{100}\text{L} \times \frac{40}{100}\text{W}$

$= \frac{119}{100}\text{LW}$

$= 119\% \text{ LW}$

$\therefore$ The area of new rectangle will increase $(119\% - 100\%) = 19\%$

Q18. (B) The circumference of big circle $= 64\pi$

We know circumference $= 2\pi r$

$\Rightarrow 64\pi = 2\pi r \Rightarrow r = 32$

From given radius of the small circle $= 8$

Area of the small circle $= \pi r^2 = \pi(8)^2$

$= 64\pi$

Area of 4 small circles = 4(area of 1 circle)

$= 4(64\pi)$

$= 256\pi$

****

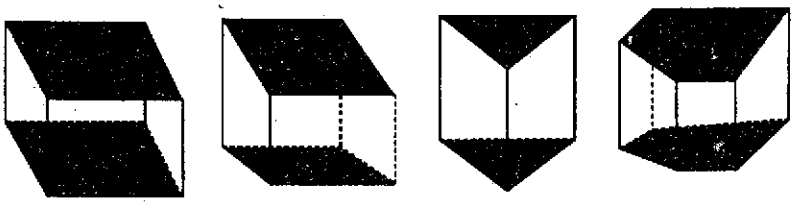
Chapter 6

SOLID GEOMETRY

Prism:

A prism is a solid figure bounded by plane faces, two of which (called the ends) are called congruent figures in parallel planes, and the others (called the sides) are parallelogram.

The solids shown below are prisms. The parts of intersecting planes that determine each prism are its faces. Two faces, F and F', are bases of each prism. The other faces are lateral (side) faces. The intersections of the faces are called edges.



1 2 3 4

Prism	Name	Description
1	Rectangular solid	All six faces are rectangles.
2	Cube	A rectangular solid in which all edges are congruent.
3	Triangular prism	The bases are triangles.
4	Pentagonal prism	The bases are pentagons.

Right Prism:

The above four prisms in the figure are right prisms because a lateral edge is perpendicular to the plane of a base.

Oblique Prism:

A prism that does not have the right prisms property is oblique prism.

Altitude of a Prism:

An altitude of a prism is a segment that is perpendicular to the planes of the bases and that has an end point in each plane. In a right prism, any lateral edge is an altitude.

Surface Area of Prism:

The total surface area of prism is the sum of its lateral surface areas and its two bases.

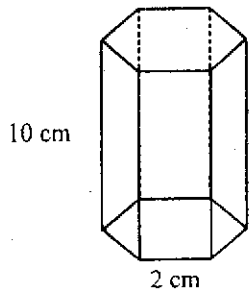
Literal Surface Area:

The literal surface area of a prism is the sum of the areas of its lateral faces.

The lateral surface area L of a right prism is the product of the perimeter P of its base and its lateral edge, e. That is

$$L = Pe$$

Example: Find the total area of the following regular hexagonal prism



Solution:

Step 1: First, find the lateral area

$L = Pe$

Here, $P = 6 \times 4 = 24$ and $e = 20$

Hence $L = 24 \times 20 = 480$

Step 2: Area of one base

Since, the area A of a regular polygon equals one half the product of its perimeter P and the apothem a

$$A = \frac{1}{2}ap$$

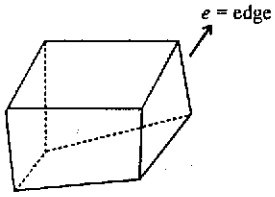
$$= \frac{1}{2} \cdot 2\sqrt{3} \cdot 24$$

$$= 24\sqrt{3}$$

Step 3: Total area

Total area $(480 + 24\sqrt{3})$ cm^2

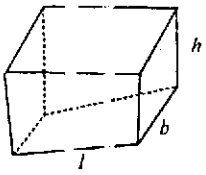
Surface Area of Rectangular Solid:



The surface area of rectangular solid

= 2(length × breadth + breadth × height + length × height)

$= 2(lb + bh + lh)$



In the case of cube, $l = b = h = e$

$\therefore$ Surface Area of a cube $= 2(e \times e + e \times e + e \times e)$

$= 2(e^2 + e^2 + e^2)$

$= 2(3e^2)$

$= 6e^2$

Volume of Rectangular Solid:

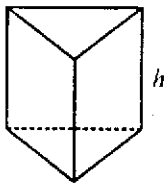
The volume of a rectangular prism is the product of its altitude h, the length of the base l, and the width of the base l and the width of the base w. That is

$$V = lwh$$

Volume of Prism:

The volume of a prism is the product of its altitude h and the area of the base, B.

$$V = Bh$$



Volume of Cube:

The volume V of a cube with edge e is the cube of e. That is

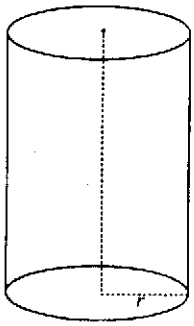
$$V = e^3$$

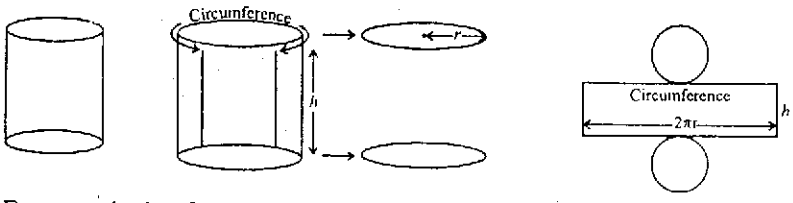
Cylinder:

A right circular cylinder is formed by the revolution of a rectangle around one of its sides. The ends of a right circular cylinder are congruent circles and the line joining the centre of the circular is perpendicular to the plane of the ends.

Surface of a Cylinder:

The surface of a cylinder consists of two congruent circular ends and a curved surface between them. If the circles are removed from the ends, and the curved surface is cut, a rectangle is produced.





From analysis of cylinder, we conclude, that

The height of the rectangle is the same as the height of the cylinder.

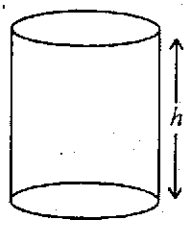
The length of the rectangle is the same as the circumference of the circle at the end.

The area of the two circles is $2\pi r^2$.

The area of the curved surface is $2\pi rh$ ($l \times b$ of the rectangle).

Surface Area of a Cylinder:

The surface area of a cylinder is double the area of one end plus the area of the curved surface.



Surface Area $= 2\pi r^2 + 2\pi rh$

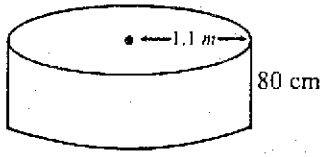
here $2\pi r^2$ is the area of the two circles

and $2\pi rh$ is the area of the curved surface.

Thus Surface area $= 2\pi r(r + h)$

Example:

Calculate the total surface area of this cylinder.



Solution:

Surface area $= 2\pi r^2 + 2\pi rh$

$= 2 \times \pi \times (1.1)^2 + 2 \times \pi \times 1.1 \times 0.8$

$= 13.1318$

$= 13.13\ m^2$

Volume of a Cylinder:

The formula for the volume of a solid cylinder is the same as the formula for the volume of a solid prism, that is

$$V = Bh$$

where B is the area of the base and h is the height. Since, for a circular cylinder, the area B of the base is πr^2, this formula reduces in this case to

$$V = \pi r^2 h$$

Multiple Choice Questions (MCQs)

Q1. The surface area of a cube is 180, its volume is:

(A) 125 (B) 30

(C) 164 (D) $30\sqrt{30}$

Q2. The volume of a cube is 216, its surface area is:

(A) 64 (B) 216

(C) 25 (D) 96

Q3. A solid metal cube of side 5 inches is placed in a rectangular tank whose length, breadth and height are 5, 6 and 7 inches, respectively. What is the volume in cubic units, of water that the tank can now held?

(A) 210 cubic units (B) 85 cubic units

(C) 125 cubic units (D) 216 cubic units

Q4. A cylinder and the cube have the same volume, if the height, h, of that cylinder is equal to the edge e of the cube, the radius of the cylinder is:

(A) $\frac{\sqrt{\pi}}{h}$ (B) $\frac{\pi}{h}$

(C) $\frac{h}{\sqrt{\pi}}$ (D) $\frac{2\pi}{h}$

Q5. If the height of a cylinder is double to its circumference, what is the volume of the cylinder in terms of its circumference, C?

(A) $\frac{C}{\pi}$ (B) $\frac{C^3}{\pi}$

(C) $\frac{C^3}{4\pi^2}$ (D) $\frac{C^3}{2\pi}$

Q6. Fatima and Maryium each roll a sheet of 9 × 18 paper to form a cylinder. Fatima tapes the two 9-inch edge together and Maryium tapes the two 18-inch edge together. Refer to this question and tell which of the following statement is true?

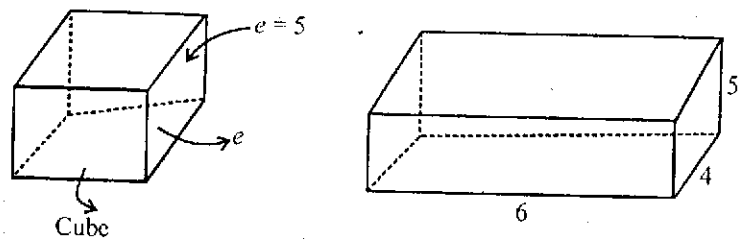
(A) The volume of Fatima's cylinder is greater than Maryium's cylinder

(B) The volume of Fatima's cylinder is less than Maryium's cylinder

(C) The volume of Fatima's cylinder is equal to the Maryium's cylinder

(D) Both cylinders have the same circumference

Q7.



Refer to the above figure, which of the following statement is true?

(A) Volume of the cube is less than the volume of the box

(B) Volume of the cube is greater than the volume of box

(C) Volume of the cube is equal to the volume of box

(D) The surface area of both cube and box are equal

Explanatory Answers

Q1. (D) ∵ Surface Area : $A = 180$

and Surface Area of the cube is $6e^2 \Rightarrow 6e^2 = 180$

$\Rightarrow e^2 = 180 \div 6 \Rightarrow e^2 = 30 \Rightarrow e = \sqrt{30}$

Therefore, edge = $e = \sqrt{30}$, hence its volume = $(\sqrt{30})^3$

$e^3 = 30\sqrt{30}$

Q2. (B) The volume of the cube: $e^3 = 216 \Rightarrow (e^3)^{1/3} = (216)^{1/3}$

$\Rightarrow e = (6^3)^{1/3} \Rightarrow e = 6$

Now Surface Area of a cube: $A = 6e^2 \Rightarrow A = 6(6)^2$

$\Rightarrow A = 216$

Q3. (B) Volume of the tank = $h \times b \times l = 5 \times 6 \times 7 = 210$ cubic units, but the volume of the solid cube, $e^3 = 5^3$.

$\Rightarrow e^3 = 125$ cubic units, therefore the tank can held 210 – 125 = 85 cubic unit of water.

Q4. (C) Volume of the cube $= e^3$ and the

Volume of the cylinder = $\pi r^2 h$

Since both are equal, therefore

$\pi r^2 h = e^3$ also $h = e$

$\therefore \quad \pi r^2 h = h^3 \Rightarrow \pi r^2 = h^2$

$$\Rightarrow \quad r^2 = \frac{h^2}{\pi} \Rightarrow r = \boxed{\frac{h}{\sqrt{\pi}}}$$

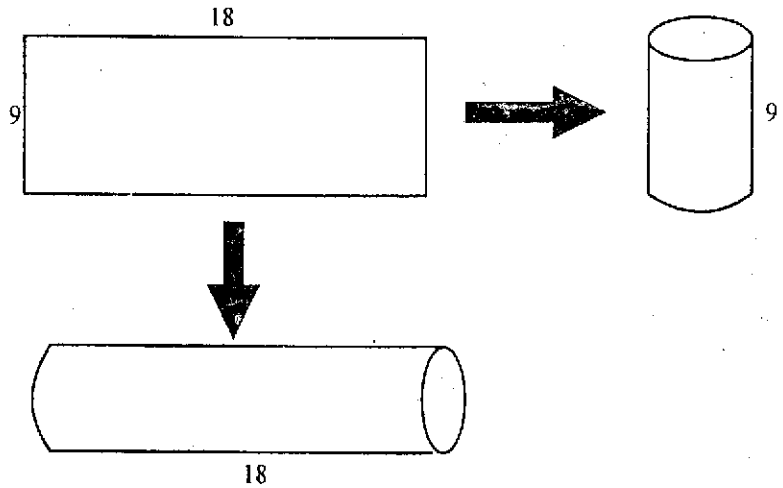
Q5. (D) Since, volume of the cylinder = $\pi r^2 h$, according to the given condition $h = 2C$.

Now $C = 2\pi r \Rightarrow r = \frac{C}{2\pi}$, therefore

$$V = \pi\left(\frac{C}{2\pi}\right)^2 \times 2C \Rightarrow V = \pi\left(\frac{C^2}{4\pi^2}\right) \times 2C$$

$$\Rightarrow V = \frac{C^3}{2\pi}$$

Q6. For sophistication, drawing the figure,



Now, the volume of the cylinder = $\pi r^2 h$

Here, we know only height only, to calculate the radius of the cylinder, we proceed as

The circumference of the cylinder made by Maryium is 9.

$$\therefore 2\pi r = 9 \Rightarrow r = \frac{9}{2\pi}$$

The circumference of the cylinder made by Fatima is 18.

$$\therefore 2\pi r = 18 \Rightarrow r = \frac{18}{2\pi} \Rightarrow r = \frac{9}{\pi}$$

Thus the volume of the cylinder made by Maryium is

$$V = \pi r^2 h \Rightarrow V = \pi\left(\frac{9}{2\pi}\right)^2 18 \Rightarrow V = \pi \times \frac{81}{4\pi^2} \times 18$$

$$\Rightarrow V = \frac{729}{2\pi}$$

and, volume of the cylinder made by Fatima is

$$V = \pi r^2 h \Rightarrow V = \pi\left(\frac{9}{\pi}\right)^2 \times 9 \Rightarrow V = \frac{729}{\pi}$$

Hence the volume of the cylinder made by Fatima is greater than the cylinder made by Maryium

Q7. (B) Volume of the cube : $V_c = 5^3 = 125$

Volume of the cube : $V_b = 4 \times 5 \times 6 = 120$

Thus the volume of the cube is greater than the volume of the box.

Chapter 7

COORDINATE GEOMETRY

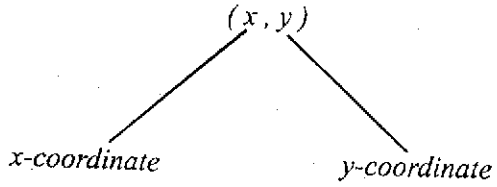
A French mathematician named Descartes invented a method of representing a pair of numbers as a point in a plane. The numbers are measured on a pair of axes which are at right angles to each other and which intersect at the origin O(0, 0).

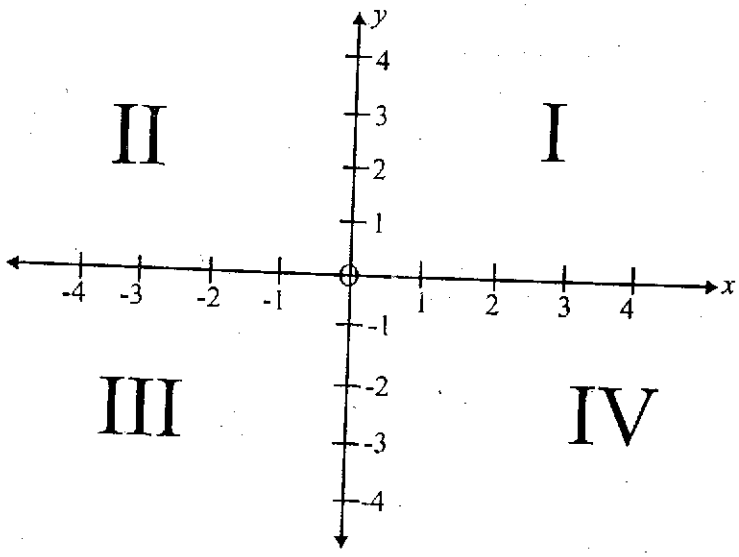
The plane is called the co-ordinate plane or the Cartesian plane.

Coordinates:

Coordinates are an ordered pairs of numbers. These ordered pairs give the position of a point using axes and an origin.

The coordinates of any point P are (x, y), where



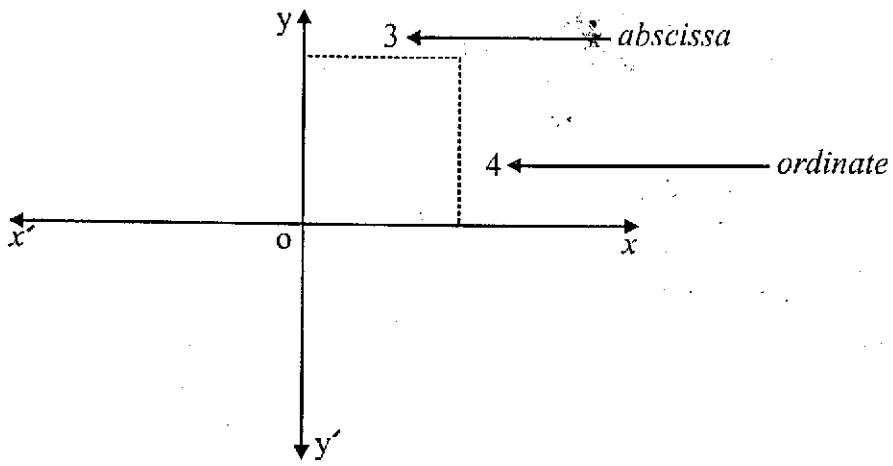


The figure above shows the (rectangular) coordinate plane. The horizontal line is called the x-axis and the perpendicular vertical line is called the y-axis. The point at which these two axis intersect, designated 0, is called the origin. The x-axis and y-axis divide the plane into four parts known as quadrants, I, II, III and IV, as shown.

Since, the coordinates of any point P are (x, y). The first number is always 'across'. The across axis is the x-axis. So the first number is called the x-coordinate.

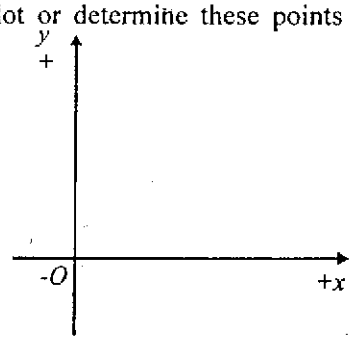
The second number is always 'up or down'. The 'up and down' axis is the y-axis. So the second number is called the y-coordinate.

Thus, in a Cartesian plane, the two perpendicular distances of the point from these axes are called the coordinates of the point. The distance of the point from the y-axis is known as the x-coordinate or the abscissa and that from the x-axis is known as the y-coordinate or ordinate of the point. Thus an order pair (3, 4) represents a point where abscissa is 3 and ordinate is 4.



Negative Coordinates:

Coordinates can be positive or negative numbers. To plot or determine these points you must have both positive and negative numbers on the axes.



On the *x*-axis:

(i) Values to the right of O are positive (+),

(ii) Values to the left of O are negative (–).

On the *y*-axis:

(iii) Values above O are positive (+),

(iv) Values below O are negative (–).

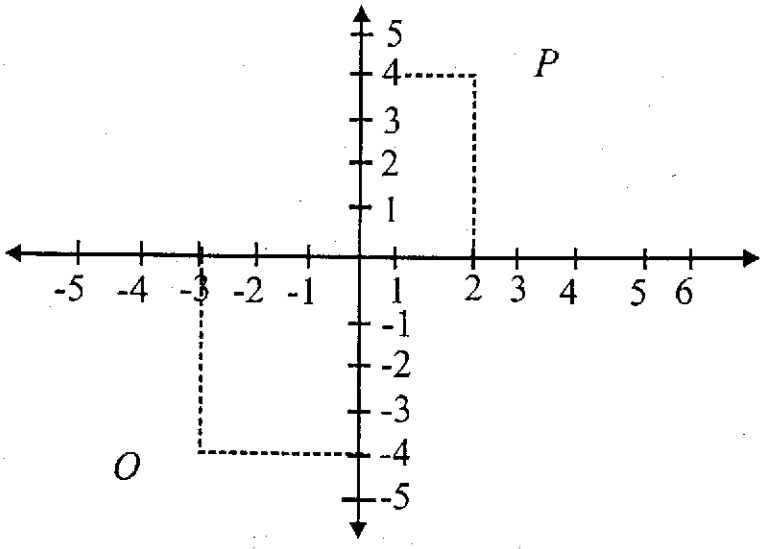
Remember

(i) The coordinates of the origin are (0, 0).

(ii) Any point on the *x*-axis has its ordinate zero.

(iii) Any point on the *y*-axis has its abscissa zero.

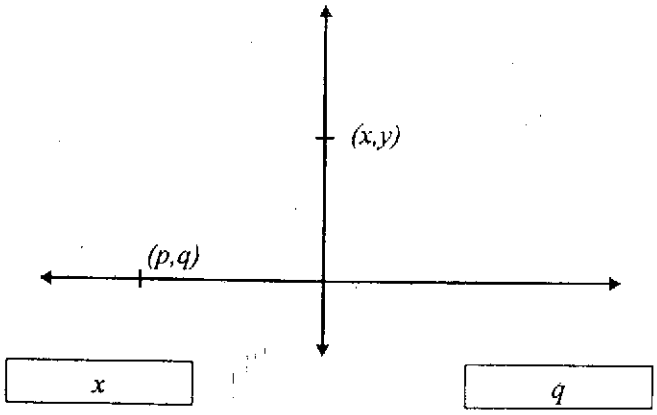
Example 1:



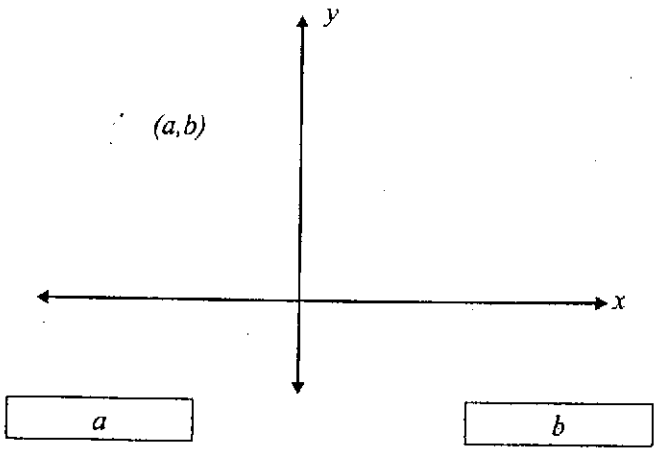
In the above graph, the (x, y) coordinates of point P are (2, 4), it is because P is 2 units to the right of the y-axis (*i.e.*, $x = 2$) and 4 units above the x-axis (*i.e.*, $y = 4$). In the same way, the (x, y) coordinates of point Q are (–3, –4), and the origin O has the coordinates (0, 0).

Column A **Column B**

Example 2:



Example 3:



Solution 2:

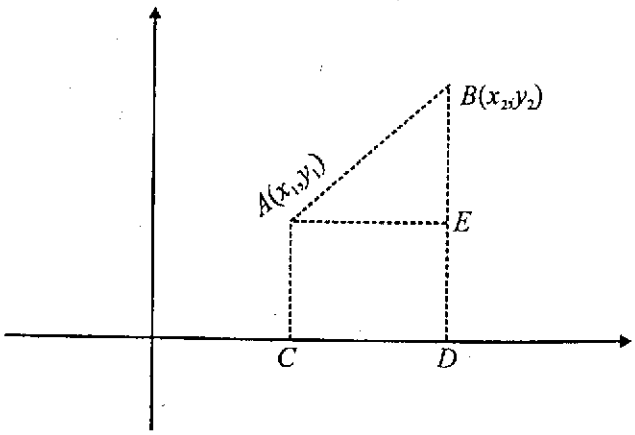
In the Cartesian plane, (p, q) lies on the x-axis, we know, on x-axis the y-coordinate of a point is zero, hence, $q = 0$. Similarly, on the y-axis, the x-coordinate of a point is zero. Now, because, point (x, y) lies on y-axis, hence $x = 0$. Thus, the correct answer is "=".

Solution 3:

In the second example, point (a, b) lies in the second quadrant and in second quadrant a is negative and b is positive, so $a < b$, is the correct answer. The answer is <.

Distance between Any Two Points in A Cartesian Plane:

Consider two points A and B with coordinates (x_1, y_1) and (x_2, y_2). Join A and B. Draw perpendicular



$\overline{AC}$ and $\overline{BD}$ on the x-axis from A and B respectively. Also from A, draw $\overline{AE}$ perpendicular to $\overline{BD}$. Then from figure

$$AE = CD$$

$= OD - OC$

$= x_2 - x_1$

$BE = BD - ED$

$= y_2 - y_1$

$\therefore$ In right-angled triangle ABE

$AB^2 = AE^2 + BE^2$

$= (x_2 - x_1)^2 + (y_2 - y_1)^2$

$\therefore \quad AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Thus

The distance d between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Note:

The distance of a point $P(x, y)$ from the origin is

$$OP = \sqrt{x^2 + y^2}$$

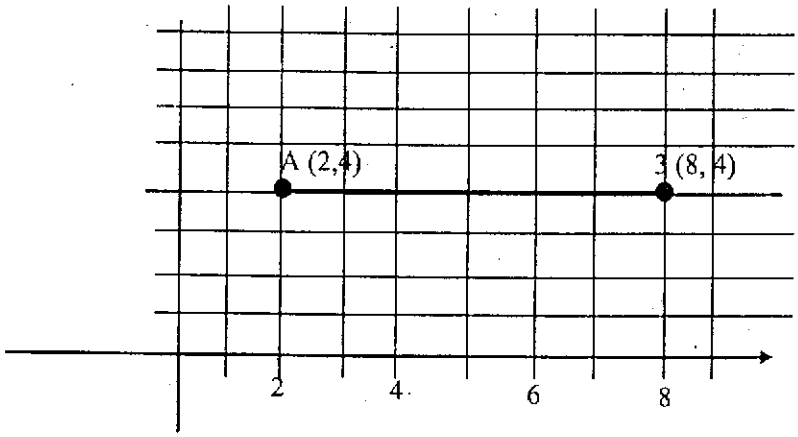
The Midpoint Formula

The coordinates (x_m, y_m) of the midpoint M of the segment whose endpoints are $P(x_1, y_1)$ and $P_2(x_2, y_2)$ are

$$x_m = \frac{x_1 + x_2}{2} \text{ and } y_m = \frac{y_1 + y_2}{2}$$

Example 4:

Find the coordinates of the midpoint $M(x_m, y_m)$ of $\overline{AB}$.



Solution:

Since $\overline{AB}$ is horizontal, the y-coordinate of any point on $\overline{AB}$ is 4. Thus y coordinate of M is 4.

Now $\quad x_m = \dfrac{x_1 + x_2}{2}$

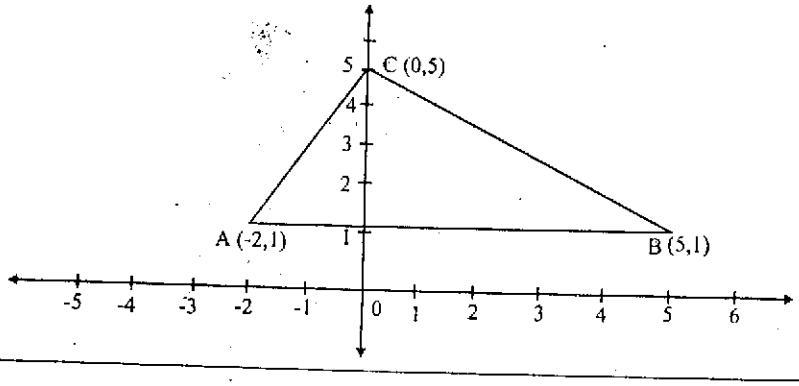
$= \dfrac{2 + 8}{2}$

$= \dfrac{10}{2}$

$= 5$

$\therefore \quad M(x_m, y_m) = (5, 4)$

Example 5-6 refer to a triangle in the following figure



Example 5:

What is the area of ΔABC?

A. 9 B. 15

C. 14 D. 12

Solution:

The base of the triangle ABC is AB, and A(–2, 1), B(5, 1) lies on a same horizontal line so length of AB is,

$$AB = \sqrt{(5+2)^2 + (1-1)^2}$$

$$AB = \sqrt{7^2} = 7$$

Now, the altitude of the triangle is the distance between point C to line AB, Hence

Altitude $= \sqrt{(0-0)^2 + (1-5)^2} = \sqrt{-4^2} = \sqrt{16}$

$= 4$

Thus, the Area $= \frac{1}{2}$(Base)(Altitude)

$= \frac{1}{2}(7)(4)$

$= (7)(2)$

$= 14$ (C)

Example 6:

What is the perimeter of ΔABC?

A. $14 + \sqrt{41}$ B. $\sqrt{41} + 6\sqrt{5}$

C. $\sqrt{41} + 14\sqrt{5}$ D. $7 + \sqrt{41} + 2\sqrt{5}$

Solution:

The perimeter of the triangle ABC is $AB + BC + CA$.

From example, 5 $AB = 7$, Now we find BC and AC

$\therefore$ $BC = \sqrt{(0-5)^2 + (5-1)^2} = \sqrt{25+16} = \sqrt{41}$

$AC = \sqrt{(0+2)^2 + (5-1)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$

Hence .Perimeter $= AB + BC + CA$

$= 7 + \sqrt{41} + 2\sqrt{5}$ (D)

Slope of a Line:

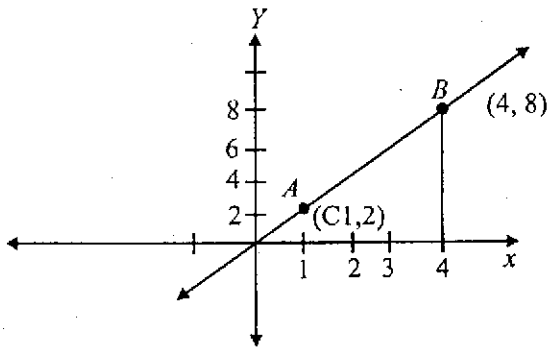
The gradient is the slope of a line. A line that slopes upwards ↗ from left to right has a positive gradient, while a line that slopes downwards ↘ from left to right has a negative gradient. In mathematics, the gradient or slope is represented by the letter m. Thus

Gradient (m) of a straight line $= m = \frac{\text{Rise}}{\text{Run}} = \frac{\text{Difference in } y \text{ values}}{\text{Difference in } x \text{ values}}$

$$= \frac{y_2 - y_1}{x_2 - x_1}$$

Example 7:

Find the gradient (slope) of line AB.



Solution:

$$m = \frac{\text{Difference between } y\text{-coordinate}}{\text{Difference between } x\text{-coordinate}}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{8-2}{4-1} = \frac{6}{3} = 2$$

Note:

1. The slope of any horizontal line is 0.
2. The slope of any vertical line is undefined.
3. Two non-vertical lines are parallel (||) if and only if they have the same slope.
4. Two non-vertical lines are perpendicular if and only if their slopes are negative reciprocals.

Example 8:

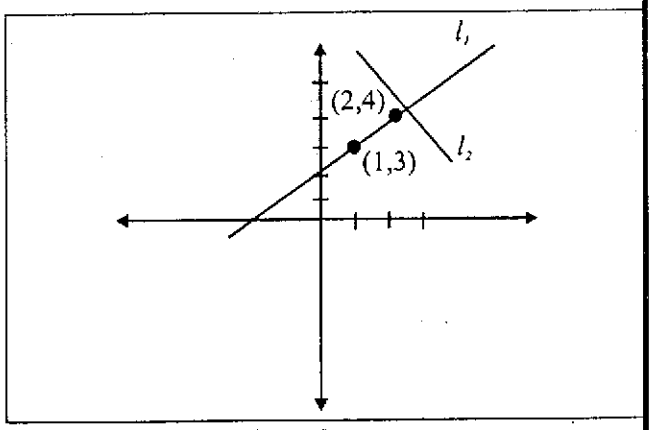
Column A	Column B
Line l_1 passes through (1, 3) and (2, 4) Line l_2 is perpendicular to l_1	
The slope of l_1	The slope of l_2

Solution:

First we sketch the line l_1, and then draw line l_2 perpendicular to l_2. The line l_1 slopes upward so it has positive slope and the line l_2 is sloped downward having negative slope, so

$$m_1 > m_2$$

where m_1 is the slope of line l_1 and m_2 is the slope of line l_2.



Multiple Choice Questions (MCQs)

Q1. If $A(-2, 3)$ and $B(5, -1)$ are the endpoints of one side of a square $ABCD$, what is the area of the square?

(A) 20 (B) 65

(C) 30 (D) 35

Q2. **If $A(3, 2)$ and $B(7, 2)$ are two vertices of a rectangle, which of the following could not be the another vertices of that rectangle?**

(A) (3, 7) (B) (7, 3)
(C) (3, −7) (D) (−3, 7)

Q3. **A circle whose center is at (3, 4) passes through the origin. Which of the following points is not on the circle?**

(A) (−1, 3) (B) (−1, 1)
(C) (0, 0) (D) (7, 7)

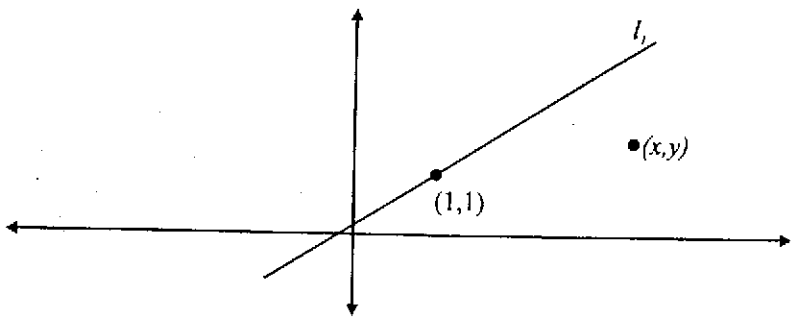
Q4. **If a line passes through the points (x, y) and $\left(\frac{1}{x}, y\right)$, then its slope is**

(A) $\frac{1}{x}$ (B) 0
(C) $\frac{1-x^2}{x}$ (D) 1

Q5. **The slope of the line passing through $(-b, b)$ and $(3b, a)$ is 1 and $b \neq 0$, which of the following is an expression for a in terms of b?**

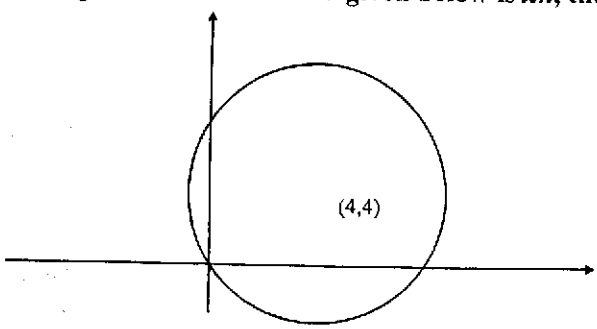
(A) $\frac{1}{4b}$ (B) $3b$
(C) $5b$ (D) $2b$
(E) $4b$

Q6. **In the figure given below, $x - y$ is**



(A) positive (B) negative
(C) less than zero
(D) less than or equal to zero
(E) cannot find from the given information

Q7. **If the area of the following circle with center O given below is $x\pi$, then x = ?**



(A) 28π (B) 32π
(C) 9π (D) 7π

Direction, **Questions 8 – 9** are referred to parallelogram $ABCD$, whose coordinates are $A(-4, 2)$, $B(-3, 6)$, $C(4, 6)$, $D(3, 2)$.

Q8. What is the area of the parallelogram $ABCD$?

(A) 24 (B) 21
(C) 29 (D) 28

Q9. What is the perimeter of parallelogram $ABCD$?

(A) $14+\sqrt{17}$ (B) $16+\sqrt{7}$
(C) $2+\sqrt{17}$ (D) $2(7+\sqrt{17})$

Explanatory Answers

Q1. (B) $S = AB = \sqrt{(5+2)^2+(-1-3)^2}$

$= \sqrt{(7)^2+(-4)^2}$

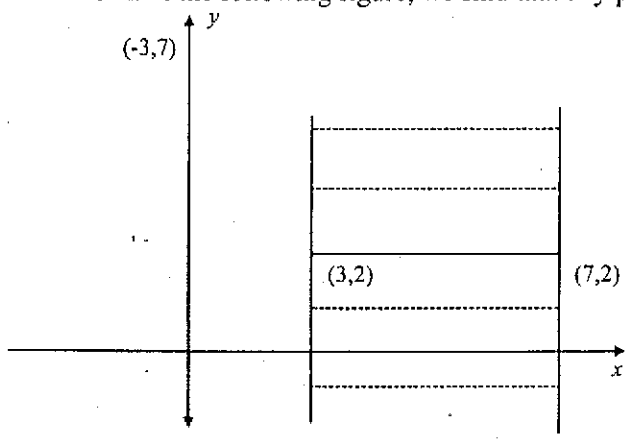
$= \sqrt{49+16}$

$S = \sqrt{65}$

$S^2 = (\sqrt{65})(\sqrt{65})$

$S^2 = 65$

Q2. (D) By drawing diagram as shown in the following figure, we find that any point whose x-coordinate is

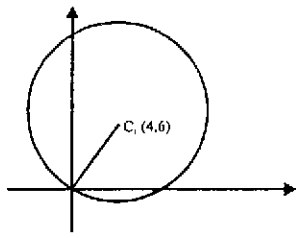


3 or 7 could be another vertex, so $(-3, 7)$ is not possible.

Q3. (A) First of all, we draw a diagram. Thus its radius is the distance from the origin to its center. By using distance formula

$r = \sqrt{(3-0)^2+(4-0)^2}$

$= \sqrt{9+16} = \sqrt{25} = 5$



Since the distance of any point of a circle from the center of that circle is same, so any point whose distance is greater than 5 would not be on the circle. Now check these points given in the options:

A: $(-1, 3) \Rightarrow r = \sqrt{(3+1)^2+(4-3)^2} = \sqrt{16+1} = \sqrt{17}$ which is not equal to 5, hence $(-1, 3)$ will not lie on the circle.

B: $(-1, 1)$ $r = \sqrt{(3+1)^2+(4-1)^2} = \sqrt{16+9} = \sqrt{25} = 5$

Since its distance from the center of the given circle is equal to the radius of the given circle, so given point will lie on the circle. Similarly, we can prove the options C, D & E.

Q4. (B) Slope of the given line $= \dfrac{y_2 - y_1}{x_2 - x_1}$

Here $y_2 = y$, $y_1 = y$ and $x_2 = \dfrac{1}{x}$, $x_1 = x$, thus

$$m = \frac{y - y}{\frac{1}{x} - x} = \frac{0}{\frac{1}{x} - x} = 0$$

Q5. (C) The slope of the line passing through two points (x_1, y_1) and (x_2, y_2) is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Here $y_2 = a, y_1 = b$ and $x_2 = 3b, x_1 = -b, m = 1$

Hence, $1 = \frac{a - b}{3b + b}$

$4b = a - b \Rightarrow a = 4b + b$

$\Rightarrow a = 5b$

Q6. Since the line l passes through origin (0, 0) and (1, 1), if any point (p, q), other than (1, 1) on the line l, then any point (p, r), below the point (p, q) will be less than (p, q). Thus $a > b \Rightarrow a - b > 0$

Q7. (B) Since the line segment joining (4, 4) and (0, 0) is the radius r of the circle

$\therefore \quad r = \sqrt{(4-0)^2 + (4-0)^2} \Rightarrow r = \sqrt{16 + 16}$

$\Rightarrow \quad r = \sqrt{32} \Rightarrow r = 4\sqrt{2}$

Since Area, $A = \pi r^2 \quad \Rightarrow A = \pi(4\sqrt{2})^2$

$\Rightarrow A = \pi(16(2))$

$\Rightarrow A = 32\pi$

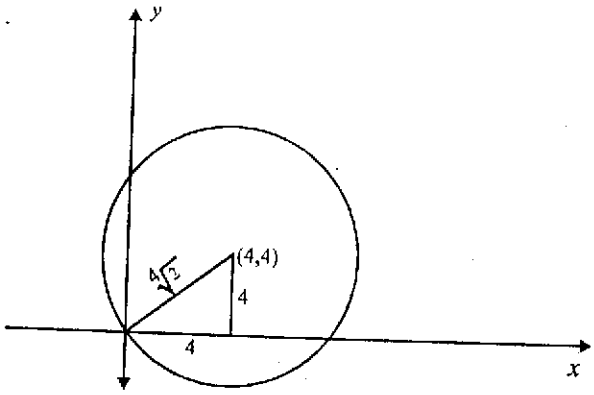
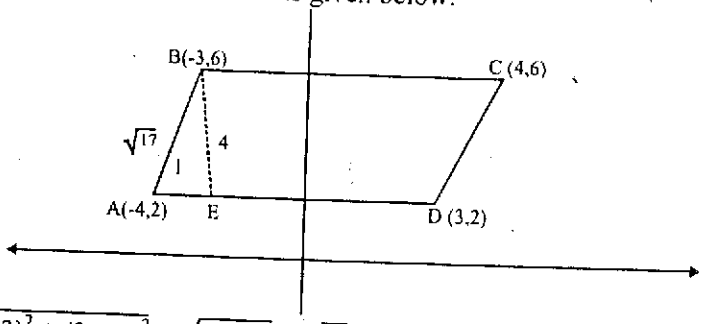


Diagram for solution question 8 and 9 is given below:



$AB = \sqrt{(-4+3)^2 + (2-6)^2} = \sqrt{1 + 16} = \sqrt{17}$

$AE^2 = AB^2 - BE^2 \Rightarrow AE^2 = (\sqrt{17})^2 - (4)^2$

$\Rightarrow AE^2 = 1 \Rightarrow AE = 1$

$AD = \sqrt{(-4-3)^2 + (2-2)^2} \Rightarrow AD = \sqrt{49 + 0} \Rightarrow AD = 7$

Q8. (D) Since the base is 7 and the height is 4, thus

Area = 7 × 4 = 28 square unit.

Q9. (D) Since AD and BC are each 7 and AB and CD both are equal to $\sqrt{17}$, thus the perimeter is

$7 + \sqrt{17} + 7 + \sqrt{17} = 14 + 2\sqrt{17} = 2(7 + \sqrt{17})$

****

Chapter 8

COUNTING AND PROBABILITY

In mathematics, some questions begin with "How much, how many,....." In these type of questions you are being asked to count something: how many bananas Sonia bought, how many Rupees did Rizwan spend, how many pages did Fatima read, how many numbers are required to satisfy a certain formula, or how many ways are there to complete a given task. Usually these problems can be solved by simple arithmetic. Sometimes it helps to use basic probability rules, and Vann diagrams. In this chapter, we shall deal with such rules and counting principles that helps to solve such problems.

Using Arithmetic to Count:

This method is illustrated by the following three examples:

Example 1:

Osama bought some chocolates. If he entered the shop with Rs. 215 and left with Rs. 195, how much did the chocolates cost?

Example 2:

Usman was selling tickets for the magic show. One day he sold tickets numbered 98 through 111. How many tickets did he sell that day?

Example 3:

In a line outside the utility store, Hamza is the 29th person and Osama is the 38th person. How many people are there between Hamza and Osama?

Solution:

These questions require a simple subtraction. In example 1, Osama did spend (215 – 195 = 20) 20 on chocolates; in example 2, however, Usman sold 14 tickets; and in example 3, only 8 persons are on line between Hamza and Osama.

In example 1, we simply subtract the amount after purchasing and before purchasing. In example 2, you need to subtract and then add 1: 111 – 98 = 13 + 1 = 14. And in example 3, you need to subtract and then subtract 1 more: 38 – 29 = 9 – 1 = 8.

In these examples, the issue is that, whether or not the first and last numbers are included or excluded.

From above examples, we find the following rules.

To find how many numbers there are between two numbers, apply following rules:

1. If exactly one of the endpoints is included, then subtract these values.
2. If both endpoints values is included, then subtract these values and add 1 in the answer.
3. If endpoints values are not included, then
   (i) Subtract these values, and
   (ii) Subtract 1 more from the answer.

The Counting Principle:

The Sum Rule

If a first task can be done in m ways and a second task in n ways, and if these tasks cannot be done at the same time, then there are $m + n$ ways to do either task.

Example 1:

A student can choose a computer project from one of the three lists. The three lists contain 21, 13 and 17 possible projects, respectively. How many possible projects are there to choose from?

Solution:

The student can choose a project from the first list in 21 ways, from the second list in 13 ways, and from the third list in 17 ways. Hence there are 21 + 13 + 17 = 51 projects to choose from.

The Product Rule:

Suppose that a procedure can be broken down into two tasks. If there are m ways to do the first task and n ways to do the second task after the first task has been done, then there are $m \times n$ ways to do the procedure.

The following examples illustrate how the product rule is used:

Example 1:

There are 28 computers in a computer center. Each computer has 22 parts. How many different parts to a computer in the center are there?

Solution:

The procedure of choosing a part consists of two tasks, first picking a computer and then picking a part on this computer. Since there are 28 ways to choose a computer and 22 ways to choose the part no matter which computer has been selected, the product rule shows that there are 616 (28×22) parts.

Example 2:

Sana has 6 different baskets in the basement. She is going to bring up 2 of them and placed 1 in her den and 1 in her bedroom. In how many ways can she choose which baskets go in each room?

Solution:

The first job was to pick 1 of the 6 baskets and place it in the bedroom. That could be done in 6 ways. The second job is to pick a second basket and place in the den. That could be done by choosing any of the remaining 5 baskets. So there are $6 \times 5 = 30$ ways to place 2 of the baskets.

The Inclusion-Exclusion Principle:

When two tasks can be done at the same time, we cannot use the sum rule to count the number of ways to do one of the two tasks. Adding the number of ways to do each task leads to an overcount, since the ways to do both tasks are counted twice. To correctly count the number of ways to do one of the two tasks, we add the number of ways to do each of the two tasks and then subtract the number of ways to do both tasks. This technique is called the principle of inclusion-exclusion.

Permutations:

Each one of the total arrangements that can be made by taking some or all of a number of different objects is called permutation. This actually presents any arrangement of a set of objects in a definite order. Permutations of the set of letters a, b, c, d taken all at a time are $abcd$, $acbd$, $bacd$ etc. An ordered arrangement of r elements of a set is called an r-permutation. For example, permutations of $n = 4$ letters a, b, c, d taken $r = 2$ at time are ab, ac, ad, bc, bd, ba, etc. Thus there are twelve permutation of four letters, taken two at a time.

The number of r-permutations of a set with n elements is denoted by ${}^{n}p_r$ or $P(n, r)$.

Note:

We can find ${}^{n}p_r$ using the product rule.

Theorem:

The number of r-permutations of a set with n distinct elements is

$$ {}^{n}p_r = n(n-1)(n-2) \ldots\ldots (n-r+1) $$

Example 1:

How many different arrangements are there of the letters A, B, C, D and E?

A. 24 B. 20 C. 25 D. 120

Solution:

In this problem, the first job is to choose one of the five letters to write in the first position; there are 5 ways to complete that job. The second job is to choose one of the remaining 4 letters to write in the second position; there are 4 ways to complete that job. The third job is to choose one of the 3 remaining letters to write in the third position; there are 3 ways to complete that job. The fourth job is to choose one of the two remaining letters to write in the fourth position; there are 2 ways to complete that job. At last, the fifth job is to choose the only remaining letter and to write it.

$$ 5 \times 4 \times 3 \times 2 \times 1 = 120 $$

Using Permutation:

The 5 letters can be written in

$^5P_4 = 5(5-1)(5-2)(5-3)(5-4)$

$= 5 \times 4 \times 3 \times 2 \times 1$

$= 120$ ways

Example 2:

In how many different ways can 6 persons be seated on a bench?

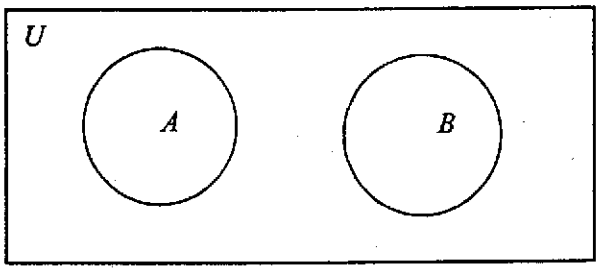
Solution:

Six persons can be seated in different position on a bench is

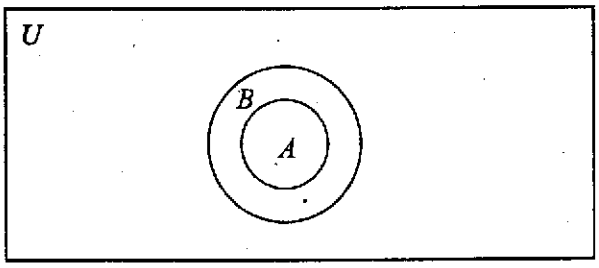
$$= 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720 \text{ ways}$$

Venn Diagrams:

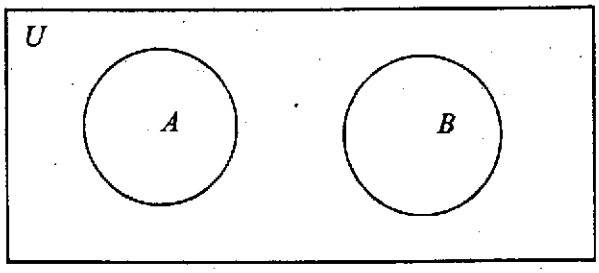
In a Venn diagram, a rectangular region represents a universal set and regions bounded by simple closed curves represent other sets, which are subsets of the universal set. In the following figure, the circular regions represents set A and set B and the remaining position of rectangle representing the universal Set U.



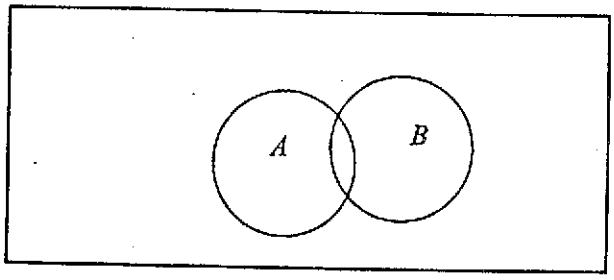
If $A \subset B$, then the circle representing A will be entirely within the circle representing B as shown in the following figure



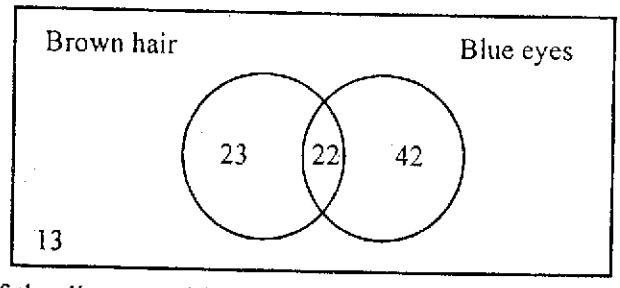
If A and B are disjoint, *i.e.*, have no elements common, then the circle representing A will be separated from the circle representing B as shown in the following figure



However, if A and B are two arbitrary sets, then it is possible that some elements are in A but not in B, some are in B but not in A, some are in both A and B, and some are in neither A nor B; hence in general we represent A and B; as in following figure



Many verbal statements can be translated into equivalent statements about sets which can be determined by Vann diagrams. To illustrate this assume that department of Pharmacy of Punjab University has 100 students. The following diagram, which divides the rectangle into four regions, shows the distribution of these students in the brown hair and blue eyes.

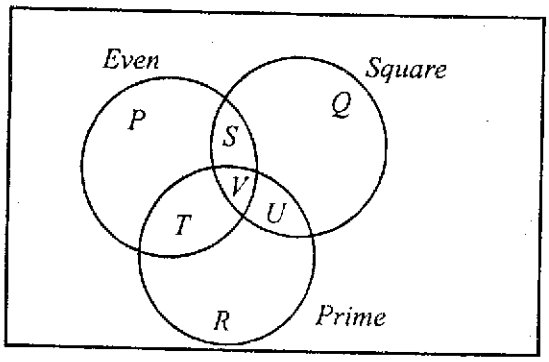


The 22 written in the part of the diagram where the two circles overlap represents the 22 students who have both brown hair and blue eyes. The 23 written in the circle on the right represents the 23 students who have brown hairs but not blue eyes, while the 42 written in the left circle represents the 42 students who have blue eyes but not brown hairs. At last, the 13 written in the rectangle outside the circles represents the 13 students who have neither brown hair nor blue eyes. The numbers in all four regions must add up to the total number of the students.

22 + 23 + 42 + 13 = 100. In this diagram, we see that, there are 64 students having blue eyes – 22 who are also having brown hair and 42 not having brown hair. Similarly, there are 23 + 22 = 45 students having brown hair. Note that, if we add both brown hair and blue hair, then we find 64 + 45 = 109 students—more than the number of students in the department. That's because 22 names are on both lists and so have been counted twice. The number of students having brown hair and blue eyes are 109 – 22 = 87. Those 87 together with the 13 students who have neither brown hair nor blue eyes make up the total of 100.

Example:

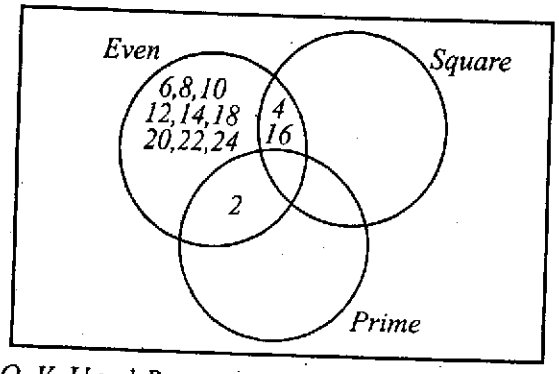
If the integers from 1 through 25 are each placed in the following Venn diagram. Then identify the empty region.



A. Q only B. V only
C. U and V only D. R only
E. Q, V, U and R only

Solution:

Put each of the number from 1 through 25 in the appropriate region



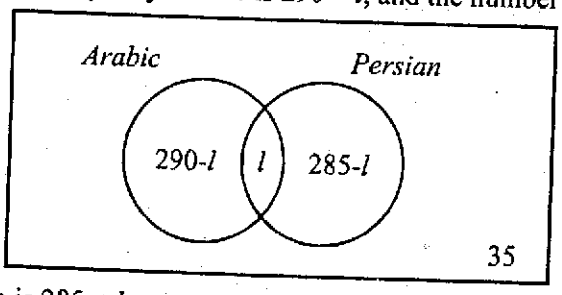
From above figure, we see that Q, V, U and R are empty regions. Hence the true choice is E.

Example:

There are 560 students of 10th class at Muslim High School, 290 of them study Arabic and 285 study Persian. If 35 students study neither language, how many study both?

Solution:

First of all, draw a Venn diagram. Let l represents the number of students who study both Persian and Arabic. Then the number of students who study only Arabic is $290 - l$, and the number of



students who study only Persian is $285 - l$.

The number of students who study at least one of the languages is $560 - 35 = 525$, thus

$$525 = (290 - l) + l + (285 - l)$$

$\Rightarrow \quad 525 = 290 - l + l + 285 - l$

$\Rightarrow \quad 525 = 575 - l \Rightarrow l = 575 - 525 \Rightarrow l = 50$

Thus 50 students study both languages.

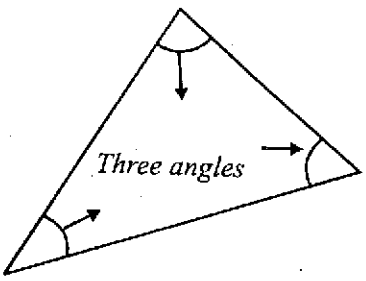
Probability:

Definition:

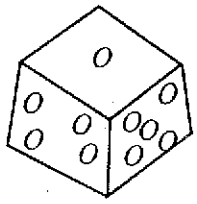
Probability is the numerical evaluation of a chance that a particular event would occur.

Probability is about how likely something is to happen. We often use the word chance for probability.

Something always happen, we say they are certain to happen. For example, if you draw a triangle it has three sides and three angles. That's certainty!



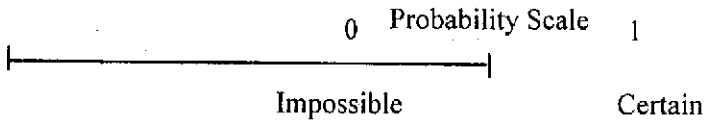
Something will never happen. We say they are impossible. For example, if you roll an ordinary dice you will never get a 7 or 8. That's impossible.



Something are not certain, but not impossible either. They may or may not happen. For example, if you toss a coin it may land heads, or it may not.

Scale of Probability:

In mathematics when we talk about the probability of 'something' happening, we call the something an event. Probability is a measure of how likely an event is to happen. We use a number, not words, to describe its size. We can give a probability a number from 0 to 1.



On this Probability Scale:

An event that is impossible is given a probability of 0. An event that is certain is given a probability of 1.

For example:

The probability that you get a 7 on an ordinary dice is 0, the probability that a triangle you draw will have three angles is 1.

All other probabilities on this scale are between 0 and 1. They must be less than 1. We write them as proper fractions or decimal fractions.

Remember:

In a proper fraction, the top number is smaller than the bottom.

To compare probabilities, we compare the sizes of the fractions. The less likely an event is to happen, the smaller the fraction. The more likely an event is to happen, the larger the friction.

Some Basic Definitions:

Equally Likely Events:

A set of events is said to be equally likely if none of them is expected to occur in preference to the other.

For example, when a fair coin is tossed, then occurrence of head or tail are equally likely events and there is no reason to expect a 'head' or a 'tail' in preference to the other.

Exhaustive Events:

A set of events is said to be exhaustive when a random experiment always results in the occurrence of at least one of them.

For example, if a die is thrown, then the events

$$E_1 = \{1, 2\}, E_2 = \{2, 3, 4\}$$

are not exhaustive as we cannot get 5 as outcome of the experiment which is not the member of any of the events E_1 and E_2. While, if $E_3 = \{1, 2, 3\}$ and $E_2 = \{2, 4, 5, 6\}$, then the set of events E_3 and E_4 is exhaustive.

Independent Events:

Two events are said to be independent, if the occurrence of one does not depend on the occurrence of the other.

For example, when a coin is tossed twice, the event of occurrence of head in the first throw and the event of occurrence of head in the second throw are independent events.

Mutually Exclusive Events:

A set of events is said to be mutually exclusive if occurrence of one of them precludes the occurrence of any of the remaining events.

In other words, events $E_1, E_2, \ldots\ldots E_n$ are mutually exclusive if and if only

$$E_i \cap E_j = \phi \qquad (i \neq j)$$

Complement of An Event:

The complement of an event E, denoted by $\overline{E}$ or E' or E^c, is the set of all sample points of the space other than the sample points in E.

For example, when a die is thrown, we get the sample space

$$S = \{1, 2, 3, 4, 5, 6\}$$

If $E = \{1, 4, 5, 6\}$, then $\overline{E} = \{2, 3\}$

It is noted that $E \cup \overline{E} = S$

Calculating Probabilities:

When all the outcomes of an activity are equally likely, you can calculate the probability of an event happening.

$$\text{Probability of an event} = \frac{\text{Number of favourable outcomes for that event}}{\text{Total number of possible outcomes}}$$

This is sometimes called the probability fraction.

To calculate the probability of an event, you may need to list all the outcomes and the favourable outcomes first.

Note:

Outcomes which give the 'event' you are interested in are called favourable outcomes for that event, and, outcomes which have an equal chance of happening are called equally likely outcomes.

Example:

Sadaf throw an ordinary dice. Write down the probability that she gets

a) a four b) more than two c) a seven

Solution:

Sample space: Possible outcomes = $\{1, 2, 3, 4, 5, 6\}$

Total number of possible outcomes $= 6$

a) Event: get a four

Favourable outcome: 4

Number of favourable outcomes = 1

Probability of getting a 4 $= \frac{1}{6}$

b) Event: get more than two

Favourable outcomes: 3, 4, 5, 6

Number of favourable outcomes = 4

Probability of getting more than 2 $= \frac{4}{6} = \frac{2}{3}$

c) Event: 'get a seven'

No. of favourable outcomes = 0

Probability of getting a seven $= \frac{0}{6} = 0$

Example:

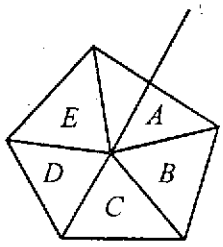
This spinner is made from a regular pentagon. When Fatima spins it once what is the probability that she get

a) the letter B

b) the vowel

c) a consonant

d) the letter F



Solution:

Sample space : Possible outcomes $= \{A, B, C, D, E\}$

Total number of possible outcomes = 5

a) Event: 'get a letter B'
Favourable outcomes = B
No. of favourable outcomes = 1
Probability of favourable outcomes $= \frac{1}{5}$

b) Event: 'get a vowel'
Favourable outcomes = A, E
No. of favourable outcomes = 2
Probability of favourable outcomes $= \frac{2}{5}$

c) Event: 'get a consonant'
Favourable outcomes : B, C, D
No. of favourable outcomes : 3
Probability of favourable outcomes $= \frac{3}{5}$

d) Event: 'get a letter F'
Favourable outcomes: 0
Probability of favourable outcomes $= \frac{0}{5} = 0$

Some Important Results on Probability:

1. $0 \le P(E) \le 1$, *i.e.*, the probability of occurrence of an event is a number laying between 0 and 1.
2. $P(\phi) = 0$, *i.e.*, probability of occurrence of an impossible event is zero.
3. $P(S) = 1$, *i.e.*, probability of occurrence of a sure event is 1.
4. $P(\overline{E}) = 1 - P(E)$, where $\overline{E}$ is the event that will not occur.
5. $P(\overline{E}) + P(E) = 1$
6. If A and B are mutually exclusive events, then

$$P(A \cup B) = P(A) + P(B)$$

Product Rule:

When more than one event occurs (*e.g.*, tossing 2 coins, planting 5 seeds, choosing 3 people, throwing 2 die), multiply the probabilities together.

$$P(AB) = P(A) \,.\, P(B)$$

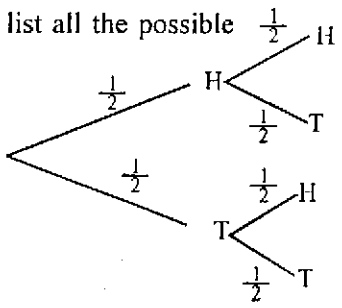
Tree Diagrams:

When using the product rule there may be more than one possible result. For example, the result when tossing two coins could be HT or TH (Head / tail or Tail / head).

When there is more than one possible result we add them together

$$P(A \text{ or } B) = P(A) + P(B)$$

This is called the additional rule of probability. A tree diagram helps to list all the possible outcomes. Tree diagrams combine the addition and product rules.



Example 1:

If 2 coins are tossed, find the probability of tossing a head and a tail.

Solution:

$P(\text{head or tail}) = P(HT) + P(TH)$

$$= \left(\frac{1}{2} \times \frac{1}{2}\right) + \left(\frac{1}{2} \times \frac{1}{2}\right)$$

$$= \frac{1}{4} + \frac{1}{4}$$

$$= \frac{1}{2}$$

Example 2:

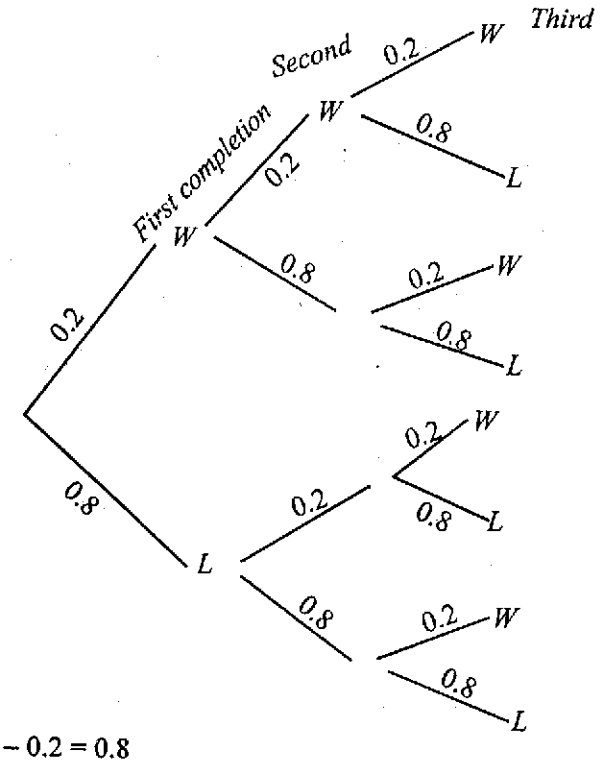
Fahad has probability of 0.2 of winning a prize in a competition. If he enters 3 competitions, find the probability of his winning:

a) 2 competition

b) at least 1 competition

Solution:

First, we construct a tree diagram of the example



a) Probability of losing = 1 – 0.2 = 0.8

$P(2W) = P(WWL) + P(WLW) + P(LWW)$

$= (0.2 \times 0.2 \times 0.8) + (0.2 \times 0.8 \times 0.2) + (0.8 \times 0.2 \times 0.2)$

$= 0.032 + 0.032 + 0.032$

$= 0.096$

b) $P(\text{at least one } W) = 1 - P(LLL)$

$= 1 - (0.8 \times 0.8 \times 0.8)$

$= 1 - 0.512$

$= 0.488$

Multiple Choice Questions (MCQs)

Q1. Ayesha completed questions 4 – 18 of a mathematics exercise in 30 minutes. At this rate, how long, in minutes, will it take her to complete questions 27 – 55?

(A) 59 (B) 29

(C) 30 (D) 58

Q2. Munir was born on August 14, 1934 and died on February 28, 1999. What was his age, in years, at the time of his death?

(A) 64 (B) 65

(C) 66 (D) 68

Q3. How many three-digit number have only even digits?

(A) 48 (B) 58

(C) 500 (D) 300

Q4. There are 28 players in a college cricket team. What is the probability that at least 3 of them

have their birthday in the same month?

(A) 0 (B) $\frac{1}{5}$

(C) 1 (D) $\frac{1}{2}$

Q5. A bag has 7 marbles, one of each of colours, green, blue, brown, yellow, red, white and pink. If 6 marbles are removed from the bag, what is the probability that the red one was removed?

(A) $\frac{1}{7}$ (B) $\frac{4}{3}$

(C) $\frac{2}{7}$ (D) $\frac{6}{7}$

Q6. A bag contains 20 marbles: 6 green, 10 brown, and 4 white. If one marble is removed randomly, what is the minimum number that must be removed to be certain that you have at least 2 marbles of each colour

(A) 16 (B) 18

(C) 10 (D) 15

Q7. In a squash tournament that has 75 entrants, a player is eliminated whenever he loses a match. How many matches will be played in the entire tournament?

(A) 74 (B) 18

(C) 34 (D) 36

Explanatory Answers

Q1. (D) Ayesha completed $18 - 4 + 1 = 15$ mathematics exercises in 30 minutes. It means that she complete one exercise every 2 minutes. Thus, to complete $55 - 27 + 1 = 29$ questions would take $29 \times 2 = 58$ minutes.

Q2. (A) Munir's last birthday was August 1998, when he turned $1998 - 1934 = 64$.

Q3. (A) We use counting principle to solve this problem. The first digit can be chosen in any 3 ways (2, 4, 6) whereas the second, third can be chosen in any 4 ways (0, 2, 4, 6). Therefore the total number of 4-digit numbers all of whose digits are even is $3 \times 4 \times 4 = 48$.

Q4. (C) Suppose, there were no month in which at least 3 players had a birthday, then each month would have the birthdays of at most 2 players. But it is not possible. Now, if there were two birthdays in January, 2 in February, ... and 2 in December, that would be 24 players only. Now, it is sure that with more than 24 players, (28 given) at least one month will have 3 or more birthdays. This is the sure event. The probability of the sure event is 1. Hence *C* is the correct choice.

Q5. (D) It is an equally likely event, any one of the 7 marbles will be the one that is not removed, so the probability that red one is left is $\frac{1}{7}$ and the probability that it is removed is $\left(1 - \frac{1}{7}\right) = \frac{6}{7}$.

Q6. (B) You might have a chance to remove 10 brown ones in a row, followed by all 6 green ones. At that point you have removed 16 marbles, and you still wouldn't have even 1 white one. Now, the next two marbles must both be white. Hence the answer is $16 + 2 = 18$.

Q7. (A) Since the winner never loses and the other 74 players each lose once. Since each match has exactly one loser, there must be 74 matches.

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Chapter 9

INTERPRETATION OF DATA

The Mathematical Reasoning section also include few questions about tables, charts and graphs. In these type of questions, you should know how to:

1. Read and understand information that is given.
2. Calculate, analyze and apply the information given.
3. Manipulating and predicting some future trends.

Two types of questions are asked based on the same set of data.

- The first question is quite simple and easy. It requires only that you read the facts or information in the table or graph.
- The second question is usually somewhat difficult, in which you are asked questions about Interpret, manipulate or predict data.

Let's start by looking at a pictogram.

Pictograms:

A pictograms uses simple symbols or pictures to show data. Pictograms gives you a quick impression of the information.

It is quite easy to compare data in a pictograms. In pictogram, you just compare how many pictures each item has.

Remember

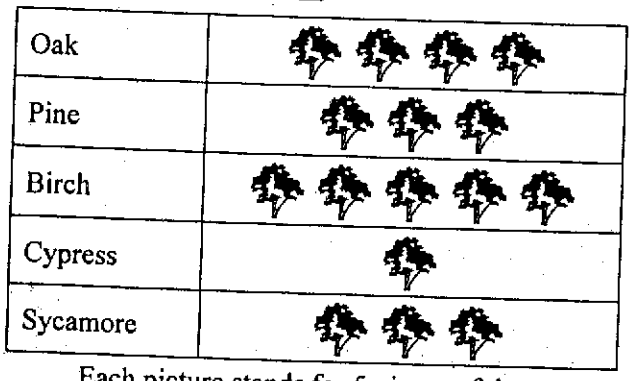
When you look at a pictogram.

(i) Read the title carefully, it tells you what the pictogram is about.

(ii) Read and understand the key, the key shows you what each little picture stands for.

In the following pictogram

Title → Trees in National Park = 5 tree ← Key



Each picture stands for 5 pieces of data.

Example 1:

What is the total number of trees in National Park.

Solution:

Just read the number of trees and multiply them by 5 *i.e.*, $16 \times 5 = 80$

Example 2:

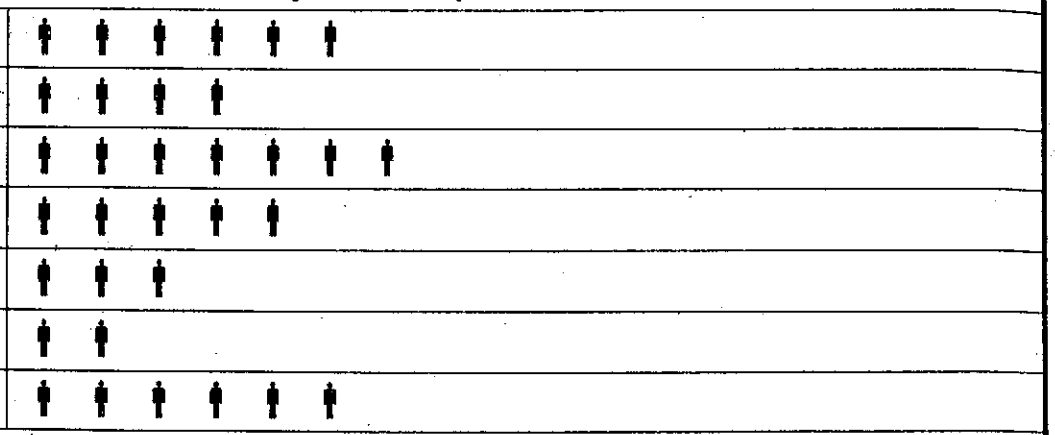
What percent of oak trees with respect to other trees in the park?

Solution:

There are, $4 \times 5 = 20$, trees of oak in the park and total number of trees is $16 \times 5 = 80$

$\therefore$ The %age of each oak trees in the park $= \frac{20}{80} \times 100 = 25\%$

Number of Students Enrolled in M.A. (Languages)
In Punjab University in 2005



Examples 3–5 refer to the above graph. Each 🧍 represents 30 students.

Example 3:

What is the total number of students enrolled language classes in Punjab University?

Solution:

Just read the pictogram and multiply the figure by 30,

$$33 \times 30 = 990$$

Example 4:

What is the average (Arithmetic Mean) number of students studying each language, if "other" category includes three languages.

Solution:

Total number of students $= 33 \times 30 = 990$

No. of languages $= 6 + 3 = 9$

Average $= 990 \div 9$

$= 110$

Example 5:

If the number of students studying English next year is the same as the number taking Arabic this year, by what percent will the number of students taking English increase?

Solution:

The number of students taking English next year = number of students taking Arabic this year = 150

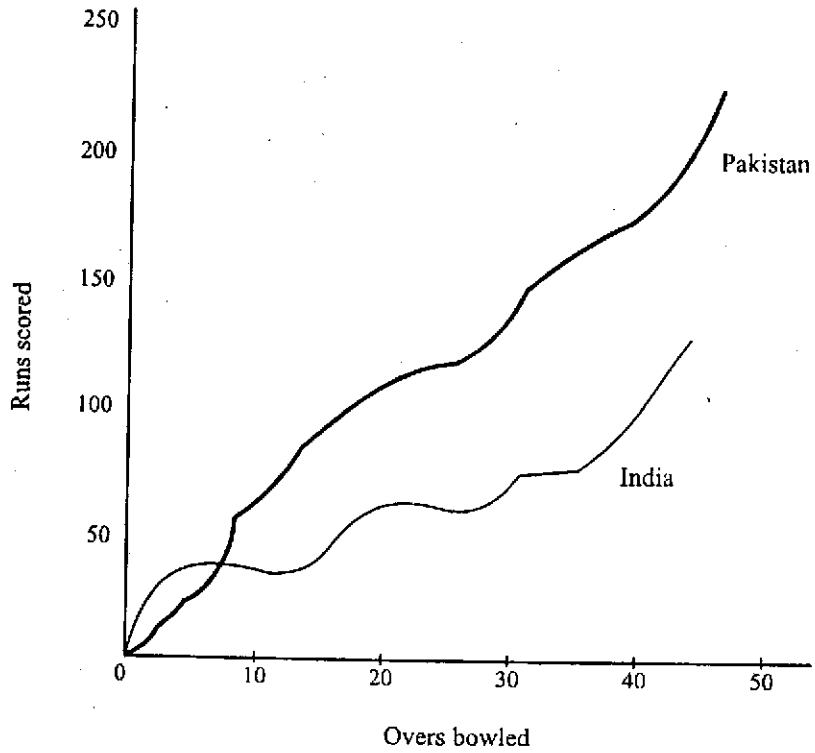
Increment $= 150 - 120 = 30$

%age increase $= \frac{30}{120} \times 100 = 25\%$

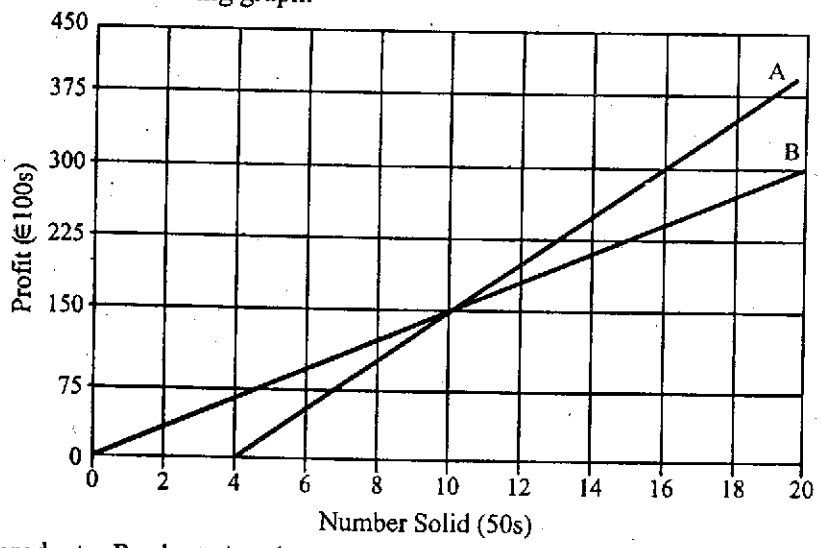
Line Graphs:

A line graph specifies how one or more quantities change over time. These types of graphs appear on the television screen during one-day cricket matches. The graph shows the accumulation of runs by each team as the match progresses. In this graph, Pakistan have already finished batting. India is still batting.

From graph, many questions could be asked. Here are some



Example 6:
How many scores did Pakistan score in his innings?
Solution:
The solid line in the graph represents Pakistan inning. You can see that at the end of 50 overs the grey line is at 250.
Hence, Answer is 250
Example 7:
How many runs have India scored so far?
Solution:
India have scored 125 in 40 overs.
Example 8:
How many runs had Pakistan scored at the same stage in their innings?
Solution:
175
Examples 9 – 11 refer to the following graph.



A firm produces 2 products. Product *A* makes a profit only when more than 200 items are sold; after that a

steady profit is made. Product B makes a steady profit of € 10 on each item. Use the above graph to solve the following examples.

Example 9:

What is the profit for selling 1000 items of A?

Solution:

Since the product A starts its profit when more than 200 items have been sold, thus its profit is

$$300 \times 100 = 30{,}000 \text{ €}$$

Example 10:

What does the profitability of A reach that of B?

Solution:

When 500 items of A are sold.

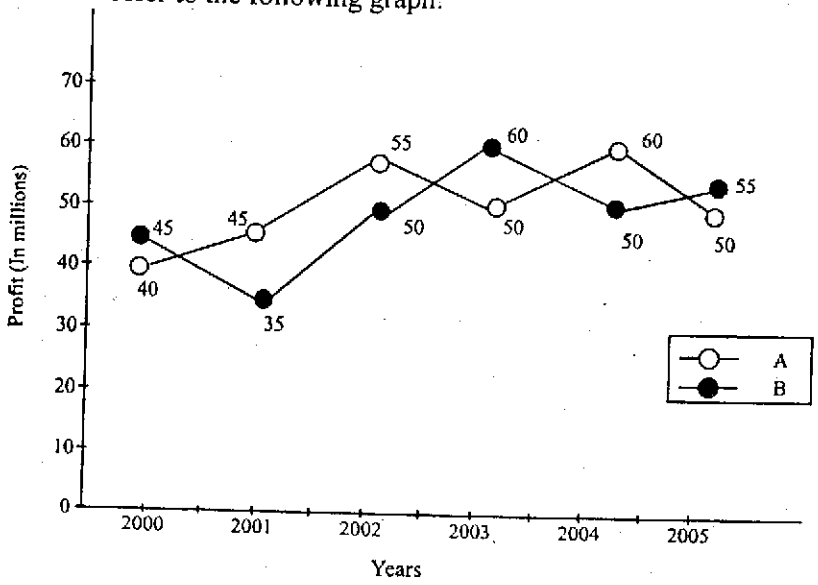
Example 11:

What is the profit from selling 800 items of B?

Solution:

€ 230,00

Directions: **Examples 12-14** refer to the following graph:



Example 11:

Firm A spent 3,50,000 in the year 2002. What is the income of Firm A in that year?

A. 15,42,500 **B.** 20,00,000

C. 15,25,000 **D.** 90,00,000

Solution: D

Profit of firm A in 2002 is 60,00,000

Income = Profit + Expenditure

= 3,50,000 + 55,00,000

= 900000

Example 12:

If the expenditure of both the firms A and B in the year 2004 was equivalent, then what was the ratio between the income of firm A to firm B?

A. 15 : 6 **B.** 6 : 15

C. 6 : 5 **D.** 16 : 5

Solution: C

Let X be the expenditure of the firm A, since expenditure of the firms A and B are equivalent, then set the ratio

Firm A		Firm B
$X + 60{,}00{,}000$	:	$X + 50{,}00{,}000$

$$\Rightarrow x + 60{,}00{,}000 : x + 50{,}00{,}000 \Rightarrow \text{ratio is ; } \boxed{6:5}$$

Example 13:

In which of the following years was the maximum percentage of growth/decline with respect to the previous

years in case of company B?

A. 2000 – 2001 B. 2001 – 2002

C. 2002 – 2003 D. 2003 – 2004

Solution: Profit in 2000 = 450000

Profit in 2001 = 350000

Decline = 450000 – 350000 = 100000

$= \frac{100000}{350000} \times 100 = 29\%$ (approx.)

Bar Charts:

A bar chart uses bars, side by side, to display data. The bars can go up or across the page. The length of a bar stands for the size of the data it shows. This makes the data easy to compare. Just compare the lengths of the bar.

Reading Bar Charts:

To read information from bar charts, here are some points to remember:

■ **Title**

Make sure that you know what the bar chart is about.

■ **Axes**

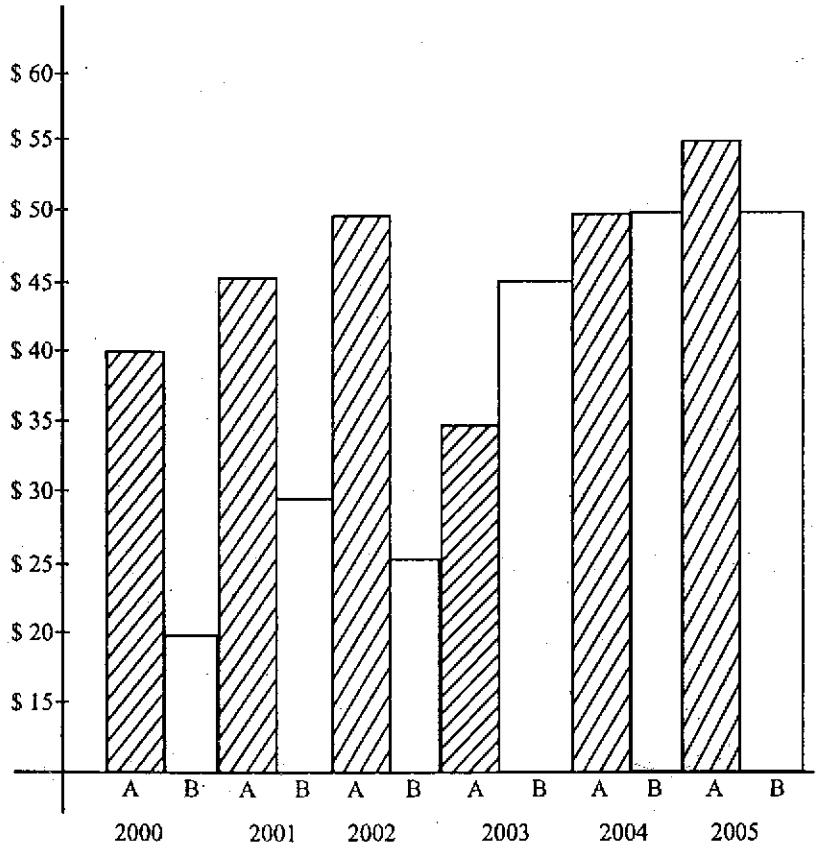
Carefully, check the labels on the axes.

■ **Scale**

Carefully, look at the scale on the number line.

Directions: **Examples 14 – 18** refer to the following graph.

Price per share of Stocks *A* and *B* on June 5 of 6 years



Example 14:

What is the difference, in dollars, of a share of stock A, between the highest and lowest value?

A. \$10 B. \$15

C. \$20 D. \$25

Solution:

According to the bar graph, the highest value of the share of stock *A* was \$55 in 2005 and the lowest value the share of stock A was \$35 in 2003. Thus

Difference = 55 – 35 = \$20. (C)

Example 15:

In which year, there was the greatest difference between the values of the share of the stock *A* and a share of stock *B*?

A. 2003 B. 2004

C. 2002 D. 2005

Solution:

According to the bar chart, clearly in 2002. There is greatest difference between the share of stock *A* and *B*. (C)

Example 16:

In which year, the ratio of the value of a share of stock *A* to the value of a share of stock *B* the greatest?

A. 2 : 1 B. 3 : 1

C. 5 : 1 D. 1 : 1.5

Solution: According to the bar chart, period from 2003 to 2005, the values of the shares of two stocks are very close, so we neglect these years. Now, there is a greatest difference between the shares of stock *A* and *B* in 2002, which is 50 : 25 ⇒ 2 : 1.

Answer is option A.

Example 17:

n which year was the percent increase in the value of a share of stock B the greatest?

A. 2001 B. 2002

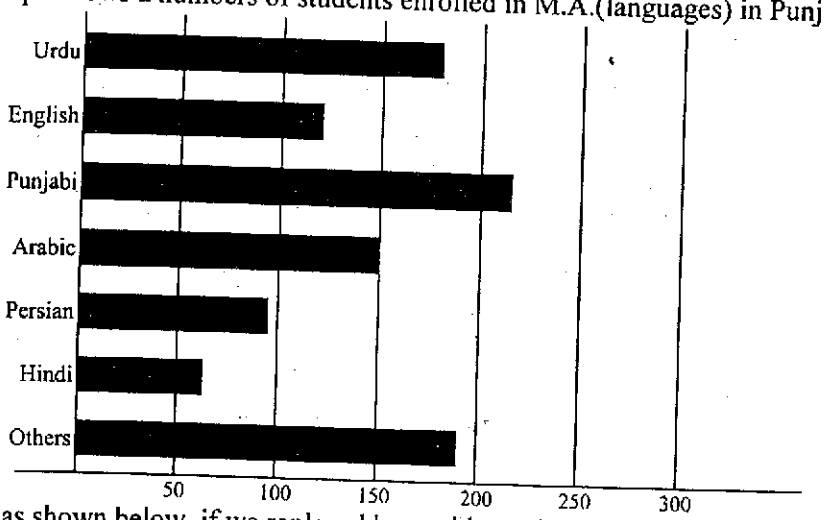
C. 2003 D. 2004

olution: According to the bar chart, the gradient of the line emerging in case of stock *B* in 2002 is the leepest.

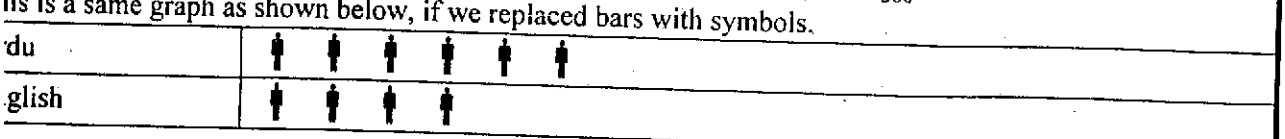
lote:

n a bar graph, the taller the bar, the greatest is the value of the quantity.

he following bar graph shows a numbers of students enrolled in M.A.(languages) in Punjab University.



is is a same graph as shown below, if we replaced bars with symbols.

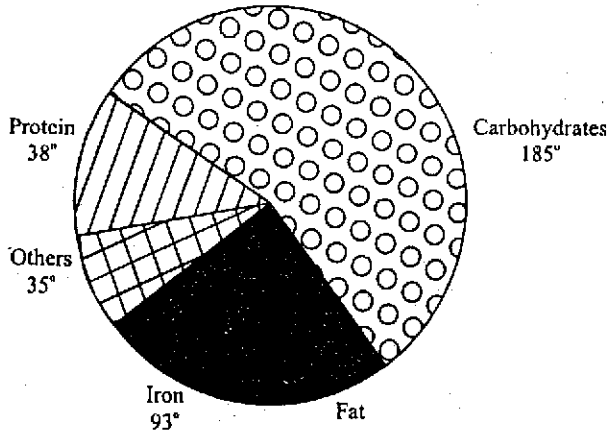


Punjabi	
Arabic	
Persian	
Hindi	
Others	

Pie Chart:

A pie chart is a circular diagram used to display data. It shows how the data are divided into group, so it looks like a pie cut into slices.

Example 17-18 refer to the following graph. The following chart shows what is in cereal.



Example 17:

Calculate the angle for fat on the diagram.

A. 23° B. 32°
C. 10° D. 20°

Solution: $360° - 185° - 37° - 35° - 93° = 10°$

Example 18:

An average 'serving' of this cereal weights 45g. How much protein is in this?

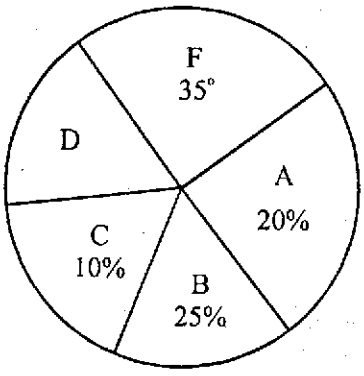
A. 1.26 g B. 2.35 g
C. 7 g D. 4.625 g

Solution: $\frac{37}{360}$ of 45 g $= 45 \times \frac{37}{360} = 4.625$ g

Multiple Choice Questions (MCQs)

Direction: Question 1-2 refer to the following graph.

Grades achieved on the Final Exam in Physics.



Q1. If 250 students took the exam, how many earned grades of D?

(A) 35 (B) 25
(C) 10 (D) 29

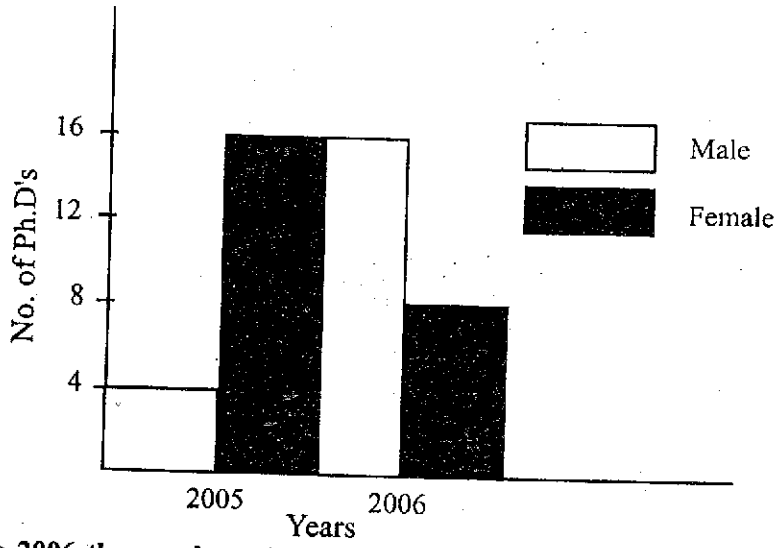
Q2. If 500 students took the exam, how many earned the grades of C?

(A) 50 (B) 20
(C) 35 (D) 10

Q3. What percent of the students who failed the exam would have had to pass it, in order for percent of students passing the exam to be at least 77% out of 500?

(A) 15% (B) 21%
(C) 23% (D) 27%

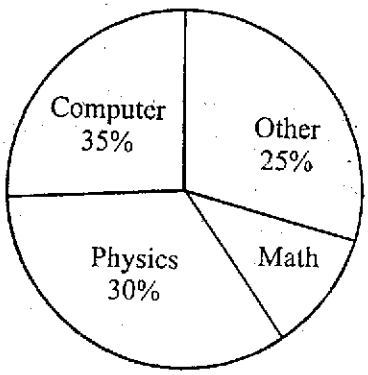
Direction: The following bar chart shows the number of male and female who earned Ph.D's in physics at Punjab University in 2005 and 2006:



Q4. From 2005 to 2006 the number of male earning Ph.D.'s increased by s%, and the number of female earning Ph.D's decreased by t%. What is the value of $s-t$?

(A) 225 (B) 200
(C) 275 (D) 250

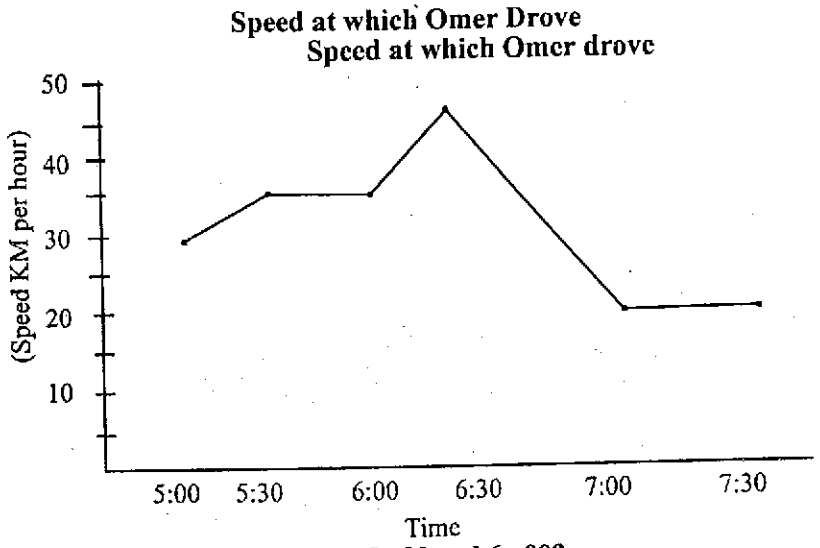
Q5. Refer to the following pie chart



every student at Crescent High School is taking exactly one science subject. This distribution has been illustration in above circular diagram. If the "other" category in order of number of students taking each subject, consists of biology, geology and astronomy. Then which of the following statement is true?

(A) Number of students taking math is equal to the number of students taking astronomy.
(B) Number of students taking math is less than the number of students taking astronomy.
(C) Number of students taking math is greater than the number of students taking astronomy.
(D) Cannot find from the given information.

Direction: **Questions 6 – 8** refer to the following graph.



Q6. **How far, in KM, did Omer drive between 5 : 30 and 6 : 00?**

(A) 20 (B) $17\frac{1}{2}$

(C) 35 (D) $12\frac{1}{2}$

Q7. **What was Omer's average speed, in KM per hour, between 5 : 30 to 6 : 30?**

(A) 25.25 (B) 36.5

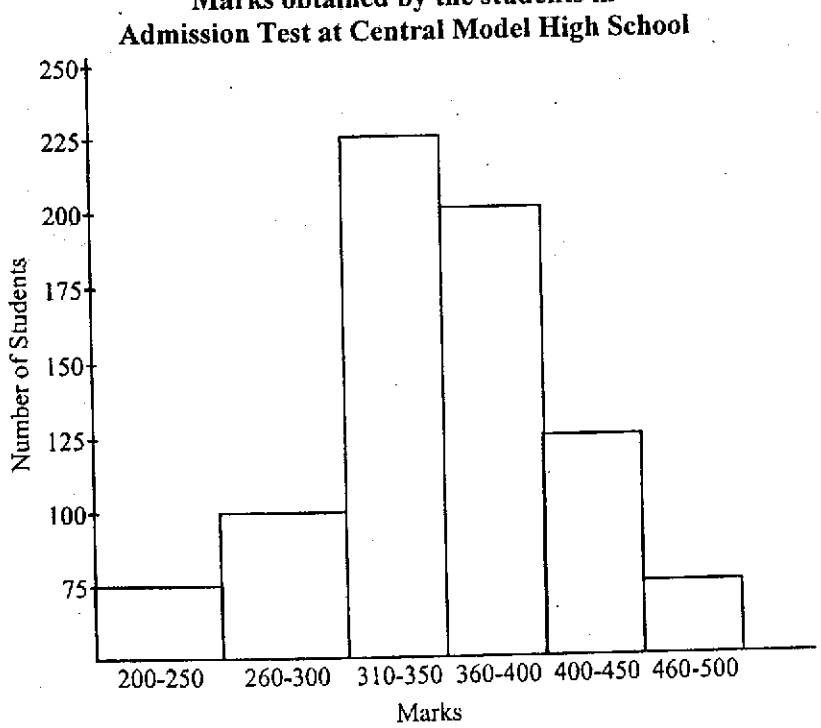
(C) 42.5 (D) 35

Q8. **For what percent of time was Omer driving at 35 km per hour or faster?**

(A) 35 (B) 25

(C) 27 (D) 50

Question 9-11 refer to the following graph.



Q9. **How many students took the admission test?**

(A) 400 (B) 375

(C) 600 (D) 800

Q10. What percent of the students had scored of less than 400?

(A) 50 (B) 75

(C) 25 (D) 65

Q11. How many candidates had scored between 250 to 350?

(A) 325 (B) 225

(C) 125

(D) cannot be exactly determined from the given information

Explanatory Answers

Q1. (B) In the given pie diagram

$$A\% + B\% + C\% + D\% + F\% = 100\%$$

$$20\% + 25\% + 10\% + D\% + 35\% = 100\%$$

$\Rightarrow$ $90\% + D\% = 100\%$

$\Rightarrow$ $D\% = 10\%$

Now, 10% of 250 is $= 250 \times \frac{10}{100} = 25.$

Q2. (A) 10% of 500 $= 500 \times \frac{10}{100} = 50.$

Q3. No. of passed students $= 500 \times \frac{77}{100} = 385.$

Thus for the passing rate to have been at least 77%, no more than 115 students, which is 23% of 500.

Q4. (D) No. of male students in 2005 = 4

No. of male students in 2006 = 16

Increase $= 16 - 4 = 12$

% Increase $= \frac{12}{4} \times 100\% = 300\%$

$\Rightarrow$ $s = 300$

During this period, the number of female students fell from 16 to 8, a decrease of 8.

$\therefore$ %age decrease $= \frac{8}{16} \times 100\% = 50\%$

$\Rightarrow$ $t = 50$

Hence $s - t = 300 - 50$

$= 250$

Q5. (B) Let m = number of students taking math

Then:

$25\% + 35\% + 30\% + m\% = 100\%$

$\Rightarrow$ $90\% + m\% = 100\%$

$\Rightarrow$ $m\% = 10\% \Rightarrow m = 10$

which is less than the number of students taking astronomy.

Q6. (B) Since Omer is driving at constant rate 35 km/hour during. 5 : 30 to 6 : 00. Thus in half hour he drove

$$\frac{1}{2} \times 35 = 17\frac{1}{2} \text{ km}$$

Q7. (D) From the graph, we see that, clearly from 5 : 30 to 6 : 00 Omer's average speed was clearly 35 km/hour, and from 6:00 to 6:30 Omer's speed steadily increased from 35 to 40 km/hour, so during 6:00 to 6:30 his average speed was $\frac{35+40}{2} = \frac{75}{2} = 37.5$ km/hour.

Thus, in the given hour, his average speed was

$$\frac{37.5+35}{2} = 36.25 \text{ km/h}$$

Q8. (D) From the graph, we see that from 5 : 30 to 6 : 45 $\left(1\frac{1}{2} \text{ hour}\right)$ the car is driven 35 or more than 35 km/hour which is half time of the total time (5:00 to 7:30) $2\frac{1}{2}$ hours.

Hence, the answer is 50%.

Q9. (D) Just add the number of candidates by reading the graph carefully

$$75 + 100 + 225 + 200 + 125 + 75 = 800.$$

Q10.(B) Number of students who scored less than 400

$$= 75 + 100 + 225 + 200 = 600.$$

Thus, 600 students had scored below or equal to 400 out of 800 candidates. Hence

$$\frac{600}{800} \times 100 = 75\%$$

Q11.(D) We cannot find exact number of students who scored between 250 to 350, because in every interval (260 – 300, 200, 250, etc.) every lower term is included in the previous term.

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SECTION II

NTS Guide

Analytical Reasoning

Each analytical reasoning question is designed to test the analytical skills. Usually, each logical reasoning question is a logical puzzle, based on given conditions. These questions have only one correct answer, which is asked to be selected.

Analytical reasoning questions consist of groups of four or five questions. At begging, a short passage followed by a set of conditions is given. Sometimes, graphs and tables are given instead of passage. To understand this type of questions, we define some logical terms and their applications with examples.

NTS Guide

Preposition:

A declarative statement which may be true or false but not both is called a preposition. For example, the statement x = y can be either true or false and there could not be any other possibility.

Symbols Used in Logic:

Capital letters are used to represent specific statements.

For Example:

A: A triangle has four angles.

B: $\sqrt{17}$ is an irrational number.

C: $20 + 18 = 36$

Solution: B is true; A and C are false.

Lower case letters, such as p, q are used to refer the preposition that are not specific. The following table is a brief list of the symbols which can be used:

Symbol	Meaning	Expression	How to read
~	not	$\sim p$	not p; negation of p
^	and	$p \wedge q$	p and q
v	or	$p \vee q$	p or q
→	If...then, implies	$p \rightarrow q$	If p then q p implies q
↔	If and only if is equivalent to	$p \leftrightarrow q$	p if and only if q p is equivalent to q

Explanation:

1. Negation:

The negation of a statement has the opposite truth value of the statement. The symbol for negation is ~, thus, $\sim p$ is read: "not p", "the negation of p" or "it is false that p". The adjoining table called the truth table, gives the possible truth values of p and $\sim p$. Thus

p	$\sim p$
T	F
F	T

The negation of true statement is a false statement.
The negation of false statement is a true statement.

2. Conjunction:

Conjunction of two statements p and q is denoted by $p \wedge q$ (p and q) and it is considered to be true only if both of its components are true.

3. Disjunction:

Disjunction of p and q is p or q. It is symbolically written $p \vee q$. The disjunction $p \vee q$ is considered to be true when at least one of the components p and q is true, and false if both components are false.

4. Implication or Conditional:

A compound statement of the form if p then q, also written p implies q, is called an implication or conditional. In the conditional statement below, p is called the antecedent or hypothesis, and q is called the consequent.

$$p \rightarrow q$$

Examples:

(i) If he works hard, then he will got through.

(ii) If he wastes time, then he will fail.

(iii) If $a = 3$, then $a^2 = 9$.

(iv) If $a = -3$, then $a^2 = 9$.

5. Biconditional:

If $p \rightarrow q$ and $q \rightarrow p$, then the preposition p and q are said to be biconditional and shortly written as "p if and only if q". Symbolically, it is written as:

$(p \rightarrow q) \wedge (q \rightarrow p) = p \rightarrow q$

Examples:

(i) If an triangle is an isosceles triangle then, its two angles are congruent.

(ii) If $x = y$, then $ax = ay$, where a, x and y are real numbers and $a \neq 0$.

Note: $p \leftrightarrow q$ is true only when both p and q are true or both p and q are false.

Model Example:

At All-Pakistan Weightlifting Championship, seven college athletes...M, N, O, P, Q, R and S... are being weighted. In order to make categories, the coach has given the following information:

(i) Each athlete has not exactly the same weight as another athlete.

(ii) R is heavier than S, but lighter than O.

(iii) P is heavier than S.

(iv) Both M and N are heavier than O.

1. ***Which of the following could not be true?***

(A) M is the heaviest. **(B)** N is the heaviest.

(C) P is the heaviest.

(D) More than three athletes are heavier than R.

(E) More than three athletes are lighter than R.

2. ***Which of the following, if true, would be sufficient to determine the lightest athlete?***

(A) P is the heaviest. **(B)** P is lighter than R.

(C) R is heavier than Q. **(D)** Q is heavier than R.

(E) Five candidates are lighter than M.

3. ***If Q is heavier than M, how many different ranking by weight, of the athletes are possible?***

(A) 1 (B) 2
(C) 3 (D) 5 (E) 7

4. ***If O is heavier than P, which of the following cannot be true?***

(A) P's weight is equal to the average of R's weight and S's weight.
(B) Q's weight is equal to the average of R's weight and S's weight.
(C) P's weight is equal to the average of M's weight and N's weight.
(D) Q is the second lightest.

Solution: First of all, we decompose the given information, symbolically:

(i) $M \neq N \neq O \neq P \neq Q \neq R \neq S$
(ii) $R > S, R < O$
(iii) $P > S$
(iv) $M > O \wedge N > O$

1. **(E)** From above, we find the exact answer.

Take false option, "M is the heaviest"

From the given information, $M > O$ and $N > O$, also $R < O$. Thus option A must be true.

Now, take second option, "N is the heaviest"

From above conclusion, it may be possible that N is the heaviest.

Third option is, "P is the heaviest". We solve it symbolically:

$\because$ $R > S \wedge R < O \Rightarrow S < R < O$

also $P > S \Rightarrow P \wedge R > S$

But $M \wedge N > O \Rightarrow R < M \wedge N$

From above, we cannot deduct that option "P is the heaviest" is wrong.

In third option, more than three athletes are heavier than R. Since, from the given information, $R > S$ But $R < O$ also $M > O$ and $N > O \Rightarrow M, N > O$.

$\Rightarrow M, N, O > R$. Because the information about Q not given, therefore, Q may be greater than R. This option may be true. Now we take the option E, according to this option, more than three athletes are lighter than R, Here we analyze it. Information (i), (ii) and (iv) can be written symbolically, as

$$\boxed{M, N > O} > R > S$$

Since, from above the three circled athletes are heavier than athlete R, and athlete R is heavier than only S. Suppose if the remaining two athletes P and Q are lighter than R. In this condition only three athletes are lighter than athlete R. Thus it is impossible that more than three athletes are lighter than R.

Thus the answer is "Choice E".

2. **(D)** The first option is "P is the heaviest". Suppose P is the heaviest, then $P > S$, also $R > S$

$\Rightarrow P > R$, But $R < O \Rightarrow P > O$ (given)

$\Rightarrow P > M, N$, from this evaluation we cannot determine the lightest.

Take a look at the second option, "P is lighter than R", Suppose P is lighter than R, the $R > P \wedge R > S$ $\Rightarrow R > P \wedge S$.

This is also unsufficient to find the lightest weight.

In option C, R is heavier than Q, if R is heavier than Q, then, symbolically

$$R > Q \wedge R > S. \because O > R \Rightarrow O > R, Q, S$$

Hence it is not possible to find the lightest weight. In option D, Q is heavier than R. If this is true, then

$Q > R \wedge R > S \Rightarrow Q > R > S$

But $R < O$, therefore, $Q > O > R > S$, also

$M, N > O \quad \Rightarrow Q > M, N > O > R > S$

$\Rightarrow Q > M, N > O > R, P > S$

Thus, we can find the lightest weight after accepting this option.

Hence, the correct choice is choice D.

3. (C) If Q is heavier than M, then

$Q > M \Rightarrow Q > O$

$O > R \Rightarrow Q > O > R$

$R > S \Rightarrow Q > O > R > S$

Thus, the three categories are possible, which are

(i) $Q > O$ (ii) $O > R$ (iii) $R > S$

4. If O is heavier than P, then according to the first option which says that, P's weight is equal to the average of R's weight and S's weight.

$P > S \wedge O > P \Rightarrow O > S$, *i.e.*, $O > P > S$ also $R < O$. But $R > S \Rightarrow O > R > S$

Multiple Choice Questions (MCQs)

Questions 1-3:

A chemist is preparing a nutriment using eight different vitamins and minerals...A, B, C, D, E, H, F (Ferric), and Z (Zinc). According to the recipes, the following requirements apply to the use of ingredients:

(i) If B is used, both C and Z must also be used.
(ii) E and H must always be used together.
(iii) If C is used, at least two of A, B and F must also be used.
(iv) C and H cannot be used together.
(v) E, F and Z cannot all be used in a same nutrient.
(vi) A, D and Z cannot all be used in the same nutriment.

Question 1:

1. ***Which of the following is a suitable combination of vitamins and minerals for a nutriment?***

(A) A, B, C, F (B) D, E, H, Z
(C) A, D, E, Z (D) C, D, E, F (E) E, H, F, Z

2. ***Which of the following cannot be included in a nutriment that contains E?***

(A) B (B) D
(C) H (D) F (E) Z

3. ***By the addition of exactly one more mineral, which of the following could make an acceptable combination of vitamins and minerals?***

(A) A, D, Z (B) B, H, E
(C) C, D, H (D) C, E, Z (E) E, H, F

Questions 4-6: A railway track from Lahore to Islamabad consists of six main stations, P, Q, R, X, Y and Z. Trains run only according to the following condition:

(i) From P to Q
(ii) From Q to P and from Q to R
(iii) From R to X
(iv) From X to Q and from X to Y
(v) From Z to P, from Z to Y and from Z to R.
(vi) From Y to X.
(vii) It is possible to transfer a station for another train.

4. ***The complete and accurate listing of the stations from which it is possible to reach R with exactly one transfer, is:***

(A) *P* and *Q* (B) *P* and *X*
(C) *X* and *Y* (D) *X* and Z (E) *X*, *P* and Z

5. ***The greatest number of stations that can be visited without visiting any station more than once, is:***

(A) 4 (B) 5
(C) 6 (D) 3 (E) 2

6. ***The trip which requires greatest number of transfers, is:***

(A) *P* to *R* (B) *Q* to *Y*
(C) *Z* to *Q* (D) *Z* to *R* (E) *Z* to *Y*

7. ***A group of dog lovers declare that the principal virtue of the dog is its general friendliness towards all people. But, another group of cat lovers declare that the principal virtue of the cat is its peculiar friendliness towards its provider.***

Which of the following is true of the claims of both dogs and cats lovers?

(A) Animals have not a sense to understand human behaviour.
(B) Groups of animal lovers are friendly.
(C) Friendliness is a virtue.
(D) They apply the same standard.
(E) Uncommon virtue of friendliness.

8. ***Rizwan was born in 1956, and so in 1965 he was nine years old. If we per use this example, it is clear that the last two digits of a person's birth will be the same as the last two digits of the year of that person's ninth birthday, except that the position of the digits will be reversed.***

Identify, which is the best criticism of the assertions made above.

(A) The generalization is valid only for those, in which last digit of their birth years is greater than four.
(B) The generalization is applicable only for those birth years that do not end in two zeroes.
(C) This example is not best illustration of the fact.
(D) The generalization is valid only for those birth years in which the difference of the last two digits is one.
(E) The generalization is valid only for those birth years that ends with 6.

Questions 9-10: At IOWA University, Students of Economics must complete a total of twelve courses selected from three different parts of the syllabus...comparative economics, environmental economics, and regional economics...in order to graduate, the students must meet the following course distribution requirements:

(i) At least six of the required twelve courses must be from the environmental economics.
(ii) At least five of the required twelve courses must be from comparative economics and regional economics, with at least one, but no more than three, selected from comparative economics.

9. ***The minimum number of regional economics courses required to fulfil the course distribution requirements is***

(A) 1 (B) 3
(C) 5 (D) 2 (E) 4

10. ***If the student has completed six environmental economics courses and one regional economics course, the possible groups of courses to fulfil the course distribution requirements must include at least:***

(A) One Environmental Economics Course
(B) Three Regional Economics Courses
(C) One Regional Economics Course

(D) Two Comparative Economics Courses

(E) Three Comparative Economics Courses

Questions 11-15: Six candidates.....Ali, Amin, Omer, Hamza, Saleem and Osama are being interviewed for a job. The interview will take place over four consecutive days, starting on Thursday. Each candidate will have only one interview. The day on which the different candidates will interview must confirm to the following conditions:

(i) At least one interview will take place each day.

(ii) No more than two interviews will take place on any day.

(iii) No more than three interviews will take place on any two consecutive days.

(iv) Ali's interview must take place on Saturday.

(v) Amin's interview must take place on the same day with another interview.

(vi) Saleem's interview must take place on the day before Osama's interview.

(vii) Omer's interview must take place on a day after Hamza's interview.

11. ***If only one interview takes place on Thursday which candidate could have that interview?***

(A) Ali (B) Amin

(C) Omer (D) Saleem

(E) Osama

12. ***If the director decides to take two interviews on Thursday and two on Sunday, how many candidates would be eligible to interview on Friday?***

(A) 1 (B) 2

(C) 3 (D) 4

(E) 5

13. ***If Hamza and Osama have their interviews on the same day which of the following must be true?***

(A) Hamza's interview will take place on Thursday.

(B) Saleem's interview will take place on Friday.

(C) Amin's interview will take place on Saturday.

(D) Osama's interview will take place on Saturday.

(E) Amin's interview will take place on Sunday.

Explanatory Answers

Q1. (B) Take first option, which is *A, B, C, B*. By the first condition, $B \rightarrow (C \wedge Z)$. But *D* has not given, so this combination is not suitable. Now take the second combination, which is *D, E, H*, Z. This option is a correct choice, since it satisfies all the given conditions.

The third option is rejected due to the sixth condition. According to this condition, *A*, *D* and *Z* cannot be used altogether in a nutriment. Since *H* is not present with *E*, thus we reject option *D* due to second condition. The fourth condition is rejected due to fifth condition. So the correct answer is choice *B*.

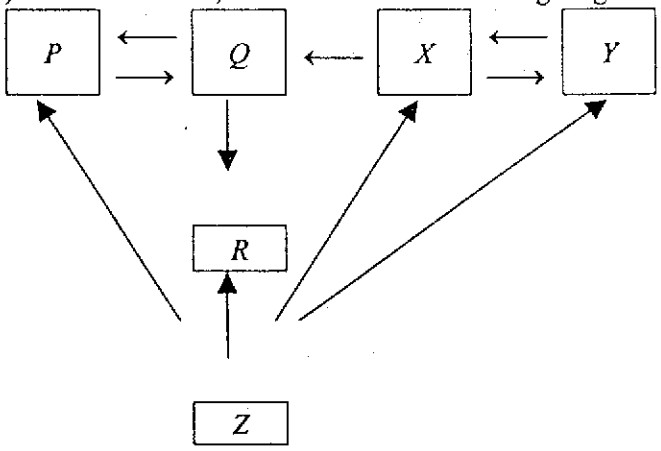
Q2. (A) If nutriment contains *E*, then according to second condition, $E \leftrightarrow H$. But according to fourth condition $H \rightarrow \sim C$.

If we include *B* in the nutriment, then according to first condition $B \rightarrow (C \wedge Z)$ *i.e.*, *C* and *Z* must also be used. But if we use *C* then *H* must be absent in the nutriment. But *E* and *H* must always be used together. Thus we cannot include *B* in the presence of *E*. The correct choice is option *A*.

Q3. (E) Since A, *D* and *Z* cannot all be used in a nutriment, so we reject option A. The option *B* is, *B, H, E*, Since, C and *Z* must also be used with *B* (first condition) so we reject option B. Since C and *H* cannot

be used together, so we reject option *C*. Since *E* and *H* must always be used together, so we reject option D. So the correct choice is option E. Which satisfies all the given conditions.

Q4. (B) For convenience, here we draw the following diagram:



It is clear from the diagram that from *P* to *R*, there is one transfer, but from *Q* to *R*, there is not any transfer, the track is direct. Thus, the option A is not possible. Now, from *P* to *R* these is only one transfer and same is the case from *X* to *R*. Thus option B is the correct choice.

Q5. (C) Here, the following list shows the track of trains that can be visited without visiting any station more than once:

$R \rightarrow X$

$Q \rightarrow R$

$Z \rightarrow R$

$Z \rightarrow P$

$Z \rightarrow Y$

$Q \rightarrow X$

The stations used in above tracks on

R, X, Q, Z, Y, P

Thus option (C) is the correct choice.

Q6. (B) From P→*R*, there is only one transfer, *i.e.*, $P \rightarrow Q$. From $Q \rightarrow Y$, there are two transfers, *i.e.*, $Q \rightarrow R$ and $R \rightarrow X$. From *Z* to *Q*, there is only one transfer *i.e.*, $Z \rightarrow P$. From *Z* to *R*, there is not any transfer. At last, *Z* to *Y* there is not any transfer. Thus the correct choice is option B, *i.e.*, 2.

Q7. (C) This question asks you to identify the main point of both dog and cat lovers. According to first option, animals have not a sense to understand human behaviour. Since animals have a great sense to understand human behaviour, like, love, anger, etc., so we reject this option. It is not sufficient that only the pet lovers should be friendly, the animals (cats, dogs etc.) should also be friendly. Thus, option *B* is not a the best choice. The best choice is option C, since it focusses upon the characteristic of animals and their lovers, which they require.

Q8. (D) The given generalization is true only if the difference of the last two digits of birth year is 1. Suppose a man was born in 1959 and so in 1995, he was 36 years old. This is a same example as given, the only difference is the, difference of the last two digits of birth year which is not 1. Now, suppose that a man was born in 1956 and so in 1965 he was 9 years old. This is so because the difference of the last two digits of the birth year in 1 *i.e.*, (6 – 5 = 1). Thus the correct answer is the option D.

Q9. (D) Since at least five courses must be from comparative and regional economics, with at least one, but no more than three selected from comparative economics. Thus if we select three courses from comparative economics, we must select two courses from regional economics group. Thus the correct choice is option D.

Q10(C)

C	E	R
	6	1
③	①	1
3 +	7 +	2 = 12

From above, we took 1 regional economics which is least. Thus if we take 3 from comparative and one from environmental economic, it will fulfil the requirement.

Q(11).D Here, first, we express the given conditions symbolically. Name of candidates Ali, Amin, Omer, Hamza, Saleem and Osama.

Days: Thu, Fri, Sat, Sun

Each days condition: 1 or 2 interviews

2 consecutive days: 2 or 3 interviews

Ali's interview = Saturday

According to condition (iv), Ali's interview will take place on Saturday. According to condition (v), Amin's interview must take place on the same day, an another interview. Thus choices C and E are impossible. Since Omer's interview must take place on a day after Hamza's interview and Osama's interview must take place on a day after Saleem's interview, cannot take place on Thursday. Thus the correct choice is choice D.

Q(12). According to third condition, no more than three interviews will on any two consecutive days. Thus only one interview can take place on Friday. Therefore according to fifth condition, it cannot be Amin. Since Ali's interview is on Saturday. Thus, it cannot be Ali. Any of the other four candidates could be interviewed on Friday as indicated in the following points:

(i)	Thu Hamza/Saleem	Fri Omer	Sat Ali	Sun Amin/Osama
(ii)	Thu Amin/Saleem	Fri Hamza	Sat Ali	Sun Omer/Osama
(iii)	Thu Amin/Hamza	Fri Saleem	Sat Ali	Sun Omer/Osama
(iv)	Thu Hamza/Saleem	Fri Osama	Sat Ali	Sun Amin/Omer

Thus the correct choice is D.

Q13. (E)The possible schedule to fulfil this condition is:

Thu	Fri	Sat	Sun
Saleem	Hamza/Osama	Ali	Amin/Omer

Thus the correct choice is choice (E).

****

SECTION III

NTS Guide

Verbal Ability

Verbal means 'pertaining to words' and ability means 'power of mind to do things', so in verbal test questions are stated in the form of words (language). The candidates are supplied with a question paper which contains variegated exercises designed to test their knowledge and intelligence. The purpose of the 'Verbal Test' is to evaluate and analyze candidate's English comprehension and understanding towards the language. These tests can be of various kinds but the questions about sentence completion and analogy testing will be asked randomly. There will be also a question about critical reading (comprehension) that will be asked separately. The brief explanation about these questions will be given on the next pages. This section is consisted of following types of questions:

1. Sentence Completion 2. Analogy Test 3. Critical Reading

NTS Guide

Part 1 Sentence Completion

In such type of questions, one or two blanks are given in a sentence, each blank indicates that something has been omitted. Four or five lettered words or sets of words are given below the sentence. The candidate is asked to choose the word or set of words, when inserted in the sentence, best fits the meaning of the sentence as a whole. Various choices i.e., (A) (B) (C) (D) are provided in these kinds of questions. The candidate is asked to complete the sentence by filling in the blanks with the most suitable choice. These questions are designed to determine the candidate's ability to recognize the following areas:

- Correct Sentence Structure
- Correct Choice of Vocabulary
- Applied Grammar (Rules)

The questions about sentence completion can be related to any of the other areas of study i.e., science, geography, general knowledge, history, literature etc., but the subject matter would not hinder the candidate's language ability. The knowledge of correct grammar and vocabulary is required to complete the sentence.

In this test, words and their correct use is judged. This test gives a good idea of the memory and the power to apply it at an appropriate time.

In sentence completion questions, you are given a sentence containing one or more blanks. A number of words or pair of words are suggested to fill the blank spaces. You must select the word or pair of words that will best complete the meaning of the sentence as a whole.

In a typical sentence completion question, if any of the answer choices is inserted into the blank spaces, the resulting sentence will be technically correct, but it may not make sense. Usually, more than one choice makes sense, but only one completely carries out the full meaning of the sentence. There is one best

answer.

HOW TO ANSWER SENTENCE COMPLETION QUESTIONS

1. Read the sentence carefully. Try to understand what it means.
2. Consider the blank or blanks with relation to the meaning of the sentence. Is a negative connotation called for or a positive one? If there are two blanks, should the pair be comparative, contrasting, or complementary? Are you looking for a term that best defines a phrase in the sentence?
3. Eliminate those answer choices that do not meet the criteria you established in step two.
4. Read the sentence to yourself, trying out each of the remaining choices, one by one. Which choice is the most exact, appropriate, or likely considering the information given in the sentence? Which of the choices does the best job of completing the sentence?
5. First answer the questions you find easy. If you have trouble with a question, leave it and go back to it later. If a fresh look does not help you to come up with a sure answer make an educated guess.

EXAMPLES WITH EXPLANATORY ANSWERS

Direction: Select appropriate word from the choices to fill blanks.

Example 1. My father _____ me that I should have informed him.

(A) said **(B)** told **(C)** asked **(D)** tell

The correct answer is (B). The sentence is the indirect narration so 'said' cannot be used. Asked cannot be used in the indirect narration 'if' or what, where etc. must used.

Example 2. He was _____ of all the valuable possessions.

(A) robbed **(B)** stolen **(C)** pinched **(D)** established

The correct answer is (A). Stolen cannot be used because a man cannot be stolen, only goods can be stolen; Similarly 'pinched' has no sense of 'deprive'.

Example 3. Those who feel that war is stupid and unnecessary think that to die on the battlefield is ____.

(A) courageous **(B)** pretentious **(C)** useless **(D)** illegal

The correct answer is (C). The key to this answer is the attitude expressed -- that war is stupid and unnecessary. Those who are antagonistic toward war would consider a battlefield death to be useless. While it is true that giving one's life in the field of battle is courageous (A), that is not the answer in the context of this sentence. Choice (B), pretentious, meaning "affectedly grand or ostentatious," does not go along with the idea that war is stupid. Choice (D) does not make sense in relation to a battlefield death.

Example 4. If you hear the ____ of a gun, don't worry; it's only my car backfiring.

(A) burst **(B)** report **(C)** retort **(D)** flash

The correct answer is (B) report. The sound of an explosion, whether from a gun or a car, is called a report.

Example 5. He demanded _____ obedience from us, and was always telling us we must be ____ subjects.

(A) total, foolish **(B)** partial, cringing **(C)** formal, rigorous **(D)** complete, compliant

The correct answer is (D). You may assume that no one demands partial or marginal obedience. Compliant is the best adjective for subjects.

Example 6. We are ____ going to have to face the reality that the resources of Earth are ____.

(A) finally, worthless **(B)** gradually, limitless **(C)** eventually, finite **(D)**quickly, unavailable

The correct answer is (C). As the Earth's resources are not limitless, worthless, or unavailable, only (C) logically completes this sentence.

Example 7. One reunion was completely ____ who'd have guessed we would have booked the same flight?

(A) illogical **(B)** fortuitous **(C)** expected **(D)** abandoned

The correct answer is (B) fortuitous. The sentence implies that the reunion occurred by chance so it was fortuitous.

Example 8. The presence of armed guards _____ us from doing anything disruptive.

(A) defeated **(B)** excited **(C)** irritated **(D)** prevented

The correct answer is (D) prevented. Armed guards are intended to prevent any kind of disruption. Answer (D) is the only logical and grammatical choice.

Example 9. Held up only by a _____ steel cable, the chairlift was _____ to carry only two people.

(A) slender, instructed **(B)** single, intended **(C)** sturdy, obliged **(D)** massive, designed

The correct answer is (B). This sentence is concerned with the design of the lift. As it says "held up only by", you may assume that the cable is not large, which eliminates (C) and (D). Of the three remaining

options, only intended (B) completes the sentence logically.

Example 10. ____ the factories had not closed, and those who needed work most were given a chance to survive during the economic disaster.

(A) Unintentionally (B) Mercifully (C) Blithely (D) Importunately

The correct answer is (B). According to the sense of this sentence, it was merciful, not unintentional, blithe, importunate, or tragic, that the factories remained open.

KINDS OF SENTENCE COMPLETION

TYPE I: SENTENCE COMPLETION USING VOCABULARY
TYPE II: SENTENCE COMPLETION USING GRAMMAR
TYPE III: SENTENCE COMPLETION USING APPROPRIATE FILLER

TYPE-I
SENTENCE COMPLETION USING VOCABULARY

Part A: Selecting the appropriate word from five alternative provided.

Learn by Example

Some people _____ themselves into believing that they are indispensable to the organisation they work for.

(A) Keep (B) Fool (C) Force (D) Denigrate (E) Delude

Answer: The correct word must be 'delude'. So answer is (E).

Multiple Choice Questions (MCQs)

Directions: Pick out the most effective word from the given words to fill in the blanks to make the sentence meaningfully complete.

1. ***Kamal's friends had nothing to offer him other than _____ in his grief.***
   (A) Solution (B) Consolation
   (C) Friendship (D) Kindness
   (E) Happiness
2. ***There is no doubt that one has to keep _____ with the changing times.***
   (A) Pace (B) Himself
   (C) Aside (D) Oneself
   (E) Tuning
3. ***Belying has mother's worries, Amir's behaviour throughout the function was _____.***
   (A) Imaginable (B) Imperial
   (C) Immodest (D) Impeccable
   (E) Impervious
4. ***After reaching New York, Azhar will have to _____ himself to the new surroundings.***
   (A) Submit (B) Adapt
   (C) Mix (D) Develop
   (E) Acquaint
5. ***Dowry is no longer permitted by law even in _____ marriage.***
   (A) Natural (B) Bigamous
   (C) Love (D) Conventional
   (E) Polygamous
6. ***Research has also _____ the illusion that childhood dreams are pure innocence.***
   (A) Dispelled (B) Discovered
   (C) Accepted (D) Observed
   (E) Established
7. ***Everyone should _____ himself against illness since medical care has now become expensive.***
   (A) Brace (B) Ensure
   (C) Vaccinate (D) Insure
   (E) Inoculate
8. ***How much did it _____ to reach Bombay by car?***
   (A) Cost (B) Estimate

(C) Charge (D) Price
(E) Pay

9. *In the departmental inquiry, it was denied that the police had committed any _____ on people under their custody.*
(A) Blunder (B) Beatings
(C) Injuries (D) Crime
(E) Excesses

10. *The petition before the Court prayed for _____ the appointment orders issued by the management.*
(A) Posting (B) Quashing
(C) Granting (D) Removing
(E) Dismissing

11. *Man power is the _____ means of converting other resources to mankind's use and benefit.*
(A) Indivisible (B) Indispensable
(C) Insuperable (D) Inimitable
(E) Inequitable

12. *I am given to _____ that you are going abroad.*
(A) Predict (B) Understand
(C) Learn (D) Think
(E) Apprehend

13. *_____ by people's perception it seems that democracy has succeeded in Pakistan.*
(A) Making (B) Planned
(C) Following (D) Going
(E) Liked

14. *The passengers and crew members of the aeroplane had a _____ escape when it was taking off from the runway.*
(A) Little (B) Brief
(C) Narrow (D) Large
(E) Better

15. *He very successfully _____ all the allegations leveled against him.*
(A) Retaliated (B) Rebutted
(C) Extricated (D) Eradicated
(E) Protected

16. *A glue produced by bees to _____ their hives appears to contain antibiotic substances.*
(A) Build (B) Decorate
(C) Collect (D) Design
(E) Structure

17. *He applied for and was _____ legal aid by the Labour Ministry.*
(A) Allowed (B) Awarded
(C) Offered (D) Granted
(E) Implemented

18. *The defending champion justified his top _____ by clinching the title.*
(A) Technique (B) Supremacy
(C) Skill (D) Form
(E) Billing

19. *He has _____ people visiting him at his house because he fears it will cause discomfort to neighbours.*
(A) Stopped (B) Warned
(C) Curtailed (D) Requested
(E) Forbidden

20. *There are _____ views on the issue of giving bonus to the employees.*
(A) Modest (B) Adverse
(C) Independent (D) Divergent
(E) Valuable

21. *Freedom is not a _____ but our birth right.*

(A) Farce (B) Illusion
(C) Sin (D) Gift
(E) Presentation

22. *My father keeps all his _____ papers in a lock and key.*
(A) Useful (B) Confidential
(C) Required (D) Necessary
(E) Enclosed

23. *I am _____ forward to our picnic scheduled in the next month.*
(A) Looking (B) Planning
(C) Seeing (D) Going
(E) Organizing

24. *Akram is too _____ as far as his food habits are concerned.*
(A) Curious (B) Involved
(C) Enjoyable (D) Fastidious
(E) Interesting

25. *Most of the issues discussed in the meeting were trivial and only a few were _____.*
(A) Interesting (B) Irrelevant
(C) Practical (D) Complex
(E) Significant

ANSWERS

1.	(B)	2.	(A)	3.	(D)	4.	(B)	5.	(C)
6.	(A)	7.	(D)	8.	(A)	9.	(E)	10.	(B)
11.	(B)	12.	(B)	13.	(D)	14.	(C)	15.	(B)
16.	(A)	17.	(D)	18.	(E)	19.	(A)	20.	(D)
21.	(D)	22.	(B)	23.	(A)	24.	(D)	25.	(E)

Part B: Selecting the appropriate word from four alternatives provided.

Learn by Example

Many women in developing countries experience a cycle of poor health that _____ before they are born and persists through adulthood passing from generation to generation.

(A) Derives (B) Establishes
(C) Begins (D) Originates

Answer: The correct word must be 'Begins'. So answer is (C).

Multiple Choice Questions (MCQs)

Directions: In each of the following questions, a sentence is given with a blank to be filled in with an appropriate word. Four alternatives are suggested for each question. Choose the correct alternative.

1. *He lives in the world of _____.*
(A) Conclusions (B) Delusions
(C) Allusions (D) Illusions

2. *He did not register his _____ to the proposal.*
(A) Disfavour (B) Dissent
(C) Deviation (D) Divergence

3. *He _____ that he could speak five languages.*
(A) Submitted (B) Suggested
(C) Challenged (D) Boasted

4. *Though Bonsai, a well-known art form, originated in China, it was _____ by the Japanese.*
(A) Cultivated (B) Perfected
(C) Finished (D) Borrowed

5. *Will you, like the _____ gentleman and soldier you are, leave at once before he finds you here?*
(A) Chivalrous (B) Luminous
(C) Barbarous (D) Ostentatious

6. *A crescendo of metallic thuds arose from the market, where the iron-smiths were _____ the pieces of metals.*
   (A) Thrashing (B) Striking
   (C) Hammering (D) Flattening
7. *The child's earliest words deal with concrete objects, only later he is able to grapple with _____.*
   (A) Opinions (B) Decisions
   (C) Abstractions (D) Maxima
8. *He lost the match easily because he had played a _____ five set match in the earlier round.*
   (A) Wonderful (B) Controversial
   (C) Sensational (D) Grueling
9. *In these days of inflation, the cost of consumer goods is _____.*
   (A) Climbing (B) Raising
   (C) Ascending (D) Soaring
10. *The most important task of the Air Force is to _____ the country against an air attack by an enemy.*
   (A) Defend (B) Secure
   (C) Protect (D) Save
11. *The Committee's appeal to the people for money _____ little response.*
   (A) Gained (B) Provided
   (C) Evoked (D) Provoked
12. *The manager tried hard to _____ his men to return to work before declaring a lockout.*
   (A) Encourage (B) Permit
   (C) Motivate (D) Persuade
13. *Our flight was _____ from Lahore to Islamabad airport.*
   (A) Diverted (B) Reverted
   (C) Deflected (D) Shifted
14. *Once I forgot the piece of paper on which the name of the hotel was written, I was as _____ as lost.*
   (A) Good (B) Bad
   (C) Much (D) Sure
15. *He is very _____ on meeting foreigners and befriending them.*
   (A) Insistent (B) Keen
   (C) Anxious (D) Fond
16. *Colgate has also set an ambitious aim of _____ an eight percent value share of the tooth paste market by the end of the first year.*
   (A) Cornering (B) Soliciting
   (C) Keeping (D) Distributing
17. *A person who constantly calls attention to his trails and sufferings is in danger of developing a martyr complex and impressing others that he is _____ seeking sympathy.*
   (A) Consciously (B) Willingly
   (C) Purposefully (D) Emphatically
18. *He is working under such _____ conditions that it is difficult to maintain his self-respect.*
   (A) Inimical (B) Humiliating
   (C) Low (D) Difficult
19. *The Hubble Space Telescope will search for planets around other stars, a key to the _____ for extra terrestrial life, and examine interstellar dust and gases out of which stars are born.*
   (A) Quest (B) Enquiry
   (C) Discovery (D) Perception
20. *The manner in which bombs exploded in five trains within a short span of time suggests that it is a part of a _____.*
   (A) Conspiracy (B) Sabotage
   (C) Game (D) Villainy
21. *I _____ a car to be absolutely necessary these days.*
   (A) Consider (B) Agree

(C) Think (D) Regard

22. *He didn't have the _____ idea of the villager's problems.*
(A) Faintest (B) Feeblest
(C) Smallest (D) Finest

23. *This is a _____ translation of the speech.*
(A) Literal (B) Literary
(C) Verbatim (D) Verbal

24. *The news of the secret deal soon _____ despite official silence.*
(A) Leaked out (B) Divulged
(C) Discovered (D) Disclosed

25. *No man had a more _____ love for literature, or a higher respect for it, than Dr. Shabbir.*
(A) Animated (B) Adroit
(C) Arduous (D) Ardent

26. *I have often _____ why he went to live abroad.*
(A) Thought (B) Surprised
(C) Puzzled (D) Wondered

27. *He lives near a lonely _____ of countryside.*
(A) Stretch (B) Section
(C) Piece (D) Length

28. *When their examinations were over, the children gleefully _____ the books they had been reading.*
(A) Shelve (B) Overthrew
(C) Despised (D) Neglected

29. *Not only did she condemn vice, she almost equally despised the _____ acceptance of an advantage.*
(A) Unholy (B) Furtive
(C) Commendable (D) Tacit

ANSWERS

1.	(D)	2.	(B)	3.	(D)	4.	(B)	5.	(A)
6.	(D)	7.	(C)	8.	(D)	9.	(D)	10.	(A)
11.	(C)	12.	(D)	13.	(A)	14.	(A)	15.	(B)
16.	(A)	17.	(A)	18.	(B)	19.	(A)	20.	(A)
21.	(A)	22.	(A)	23.	(A)	24.	(A)	25.	(D)
26.	(D)	27.	(A)	28.	(D)	29.	(D)		

Part C: Selecting the appropriate pair of words to fill in two blanks in a sentence

Learn by Example

The Deputy Manager _____ to resign because all his proposals were _____ down by his superiors.
(A) Offered, thrown (B) Gave, held
(C) Began, kept (D) Willing, knocked
(E) Threatened, turned

Answer: The pair of words are 'threatened', 'turned'. So answer is (E).

Multiple Choice Questions (MCQs)

Directions: In each sentence below, there are two blank spaces. Below each sentence some pairs of words are given. Pick out the most appropriate pair to fill in the blanks in the same order, to make the sentence meaningfully complete.

1. *He granted the request because he was _____ to _____ his friend.*
(A) Sure, displease (B) Unwilling, please
(C) Bound, hurt (D) Destined, agonise
(E) Reluctant, disappoint

2. *The charges made in the system were so _____ that they didn't require any _____.*
(A) Marginal, expenses (B) Certain, expertise
(C) Big, time (D) Genuine, intelligence
(E) Obvious, modifications

3. *In _____ of International matters, there is always an element of risk in _____ one might do.*
   (A) Defence, wrong (B) Case, whatever
   (C) View, whichever (D) Many, doing
   (E) Spite, whatever

4. *_____ is a criminal _____ in England and covers cases where offensive descriptions of Christianity are published.*
   (A) Blasphemy, offence (B) Obscenity, deviation
   (C) Heathenism, act (D) Sacrilege, violence
   (E) Impiety, transgression

5. *We are _____ to have him _____ here to make this function a great success.*
   (A) Sure, come (B) Pleased, over
   (C) Prood, leave (D) Happy, arrive
   (E) Wonderful, again

6. *He was so convinced that people were driven by _____ motives that he believed there was no such thing as a purely _____ act.*
   (A) Altruistic, praiseworthy (B) Ulterior, selfless
   (C) Personal, anti-social (D) Personal, eternal
   (E) Sentimental, divine

7. *We _____ him with many promises, but nothing would _____ him.*
   (A) Attracted, fascinate (B) Gave, deprive
   (C) Tempted, influence (D) Provoked, dessicate
   (E) Negotiated, please

8. *The counter clerk was very busy and _____ not pay _____ to Sameer's request.*
   (A) Had, cash (B) Did, attention
   (C) Can, help (D) Could, respect
   (E) Certainly, acceptance

9. *The construction of the hall has been _____ because of the _____ of cement in the market.*
   (A) Hampered, shortage (B) Prevented, supply
   (C) Held, non-availability (D) Denied, restrictions
   (E) Completed, disappearance

10. *The bandit _____ the traveller of his purse of gold and _____ him grievously.*
   (A) Demanded, beat (B) Robbed, wounded
   (C) Snatched, hurt (D) Stole, injured

11. *She was _____ because all her plans had gone _____.*
   (A) Distraught, awry (B) Frustrated, magnificently
   (C) Elated, wild (D) Dejected, splendidly

12. *The candidates' _____ at the polls was _____ as he won with a striking margin.*
   (A) Image, real (B) Strategy, unsuccessful
   (C) Claim, unrealistic (D) Victory, overwhelming
   (E) Candidature, inappropriate

13. *He is so _____ that everyone is always _____ to help him in his work.*
   (A) Adamant, enthusiastic (B) Miserly, ignorant
   (C) Helpful, reluctant (D) Aloof, cooperative
   (E) Magnanimous, eager

14. *The activities of the association have _____ from the _____ objectives set for it in the initial years.*
   (A) Details, grand (B) Emerged, total
   (C) Grown, simple (D) Deviated, original
   (E) Increased, perverse

15. *The leaders were _____ needed by those to _____ they were addressed.*
   (A) Scarcely, whom (B) Rarely, where
   (C) Angrily, who (D) Readily, which
   (E) Joyfully, when

16. *Instead of _____, prove your worth by _____ something.*

(A) Worrying, paying (B) Writing, reading
(C) Begging, demanding (D) Talking, doing
(E) Donating, demanding

17. *Due to _____ rainfall this year, there will be _____ cut in water supply.*
(A) Sufficient, no (B) Surplus, abundant
(C) Enough, substantial (D) Meager, least
(E) Abundant, considerable

18. *We must explore _____ sources of energy as our supply of fossil fuel has been _____.*
(A) Natural, exhausted (B) Sufficient, increased
(C) Alternative, depleted (D) Guaranteed, over
(E) Innovative, augmented

19. *He had managed to _____ several times, but was finally _____ by the police.*
(A) Escape, arrested (B) Cheat, robbed
(C) Deceive, cheated (D) Defend, acquitted
(E) Abscond, kidnapped

20. *Children are more _____ than adults, it is _____ their quickness in learning a new language.*
(A) Conservative, seen in (B) Susceptible, demonstrated in
(C) Intelligent, disproved by (D) Adaptable, reflected in
(E) Resourceful, proportionate to

21. *The Education Minister emphasized the need to discover and _____ each student's _____ talents.*
(A) Suppress, potential (B) Flourish, hidden
(C) Enlarge, dormant (D) Belittle, concealed
(E) Develop, intrinsic

22. *He is usually _____, but today he appears rather _____.*
(A) Quiet, calm (B) Happy, humorous
(C) Strict, unwell (D) Tense, restless
(E) Calm, disturbed

23. *The war _____ immediately after the cease-fire proposal was _____ bilaterally.*
(A) Began, thwarted (B) Extended, mitigated
(C) Receded, exchanged (D) Started, prepared
(E) Ended, accepted

24. *Yousaf _____ another feather _____ his cap by his wonderful performance in the one day match.*
(A) Took, in (B) Created, for
(C) Kept, by (D) Captured, from
(E) Added, to

25. *Any system is likely to _____ for _____ of support from the public.*
(A) Fail, want (B) Finish, failure
(C) Survive, lack (D) Succeed, reason
(E) Launch, paucity

ANSWERS

1.	(E)	2.	(E)	3.	(B)	4.	(A)	5.	(B)
6.	(B)	7.	(C)	8.	(B)	9.	(A)	10.	(B)
11.	(A)	12.	(D)	13.	(E)	14.	(D)	15.	(A)
16.	(D)	17.	(A)	18.	(A)	19.	(A)	20.	(D)
21.	(E)	22.	(E)	23.	(E)	24.	(E)	25.	(A)

TYPE-II
SENTENCE COMPLETION USING GRAMMAR

Learn by Example

Genius does what it must, and talent does what it _____.
(A) Can (B) Would
(C) May (D) Should

Answer: The correct word is 'can'. Hence the answer is (A).

Multiple Choice Questions (MCQs)

1. *The passengers were very happy _____ the friendly and warm treatment given to them.*
   (A) From (B) To
   (C) By (D) About
2. *The higher you go, the more difficult it _____ to breathe.*
   (A) Is becoming (B) Became
   (C) Has become (D) Becomes
3. *The children were disappointed because they had hoped _____ with us.*
   (A) Would have gone (B) To had gone
   (C) To have gone (D) To go
4. *She stood _____ Ahsan, but could not utter a single word for quite some time.*
   (A) About (B) Before
   (C) For (D) To
   (E) Towards
5. *He is the friend _____ I trust most.*
   (A) Which (B) Who
   (C) Him (D) Whom
6. *You must dispense _____ his service.*
   (A) With (B) Of
   (C) In (D) At
7. *The telephone _____ several times before I answered it.*
   (A) Has rung (B) Was ringing
   (C) Would ring (D) Had rung
8. *I shall not desert him _____ all the world.*
   (A) By (B) For
   (C) With (D) From
9. *I bought a new car last year, but I _____ my old car yet, so at present I have two cars.*
   (A) Have sold (B) Did not sell
   (C) Could not sell (D) Have not sold
10. *No sooner did he go in _____ he came out.*
    (A) Than (B) And
    (C) Then (D) When
11. *The judge acquitted the prisoner _____ the charge of murder.*
    (A) About (B) From
    (C) Of (D) With
12. *An argument _____ between the two friends.*
    (A) Broke out (B) Broke in
    (C) Sprang up (D) Rose up
13. *When I was a child, I _____ to school everyday instead of going by cycle.*
    (A) Had walked (B) Have walked
    (C) Walked (D) Have been walking
14. *Nobody has come to see us _____ we bought these fierce dogs.*
    (A) Since (B) For
    (C) When (D) Till
15. *He : an _____.*
    (A) Quickly (B) So quickly
    (C) Quick (D) So quick
16. *In Bush, Saddam was up _____ more than his match.*
    (A) Into (B) For
    (C) To (D) Against
17. *The doctor advised him to go _____ several medical tests.*
    (A) Through (B) Into

(C) Under (D) About

18. *If you persist in telling lies to me I shall sue you _____ slander.*
(A) For (B) On
(C) With (D) To

19. *The waiter hasn't bought the coffee _____ I've been here an hour already.*
(A) Up (B) Till
(C) Still (D) Yet

20. *The modern club is simply a more refined substitute _____ the old fashioned tavern.*
(A) For (B) With
(C) Of (D) To

21. *After the advice of his father, he was reconciled _____ his wife.*
(A) With (B) To
(C) Into (D) By

22. *The doctor tried both penicillin and sulphanilamide; the penicillin proved to be the _____ effective drug.*
(A) Most (B) Bad
(C) Very (D) More

23. *However honest he _____, I do not trust him.*
(A) Might be (B) Could be
(C) Is (D) May be

24. *He became the Governor of a Province _____.*
(A) In course of time (B) At times
(C) Little by little (D) By and large

25. *_____ the rain stopped, the play had to be suspended.*
(A) When (B) Since
(C) While (D) Until

ANSWERS

1.	(C)	2.	(D)	3.	(D)	4.	(B)	5.	(D)
6.	(A)	7.	(D)	8.	(B)	9.	(D)	10.	(A)
11.	(C)	12.	(C)	13.	(C)	14.	(A)	15.	(A)
16.	(D)	17.	(A)	18.	(A)	19.	(D)	20.	(A)
21.	(B)	22.	(D)	23.	(D)	24.	(A)	25.	(D)

TYPE-III
SENTENCE COMPLETION USING APPROPRIATE FILLER

Learn by Example

The notice at the petrol pump should be _____.
(A) All engines need to be switched off
(B) All engines have to be switched off
(C) All engines must have to be switched off
(D) All engines must be switched off.

Answer: (D)

Multiple Choice Questions (MCQs)

Directions: In each of the following questions, an incomplete statement followed by some fillers is given. Pick out the best one which can complete the incomplete statement correctly and meaningfully.

1. *If they share burden alternately, they _____.*
(A) Won't get tired (B) Will get fatigued
(C) Can't feel tiring (D) Will get tired soon
(E) Don't get tired

2. *When I saw him through the window _____.*
(A) I ran out to open the door (B) I have run out to open the door
(C) I should run out to open the door (D) I am running out to open the door

3. *Every person must learn _____.*
   (A) That his time needs a wise use
   (B) Wise ways in his time's use
   (C) To make wise use of his time
   (D) To using his time in a wisely manner
   (E) That how wisely his time can be used

4. *The income tax raid was too sudden _____.*
   (A) So that the man escaped
   (B) For the man escaping
   (C) Then the man escaped
   (D) For the man to escape

5. *Many people have law degrees _____.*
   (A) But not all of them practice law
   (B) However it isn't practised by all
   (C) And some of them do have practice also
   (D) But some of them do not practice it
   (E) Yet some are not undergoing practices

6. *He passed the examination in the first class because he _____.*
   (A) Worked hardly for it
   (B) Was hard working for it
   (C) Was working hard for it
   (D) Had worked hard for it

7. *'Where are my spectacles?' _____*
   (A) There are they, on your nose!
   (B) There they are, on your nose!
   (C) Here are they, on your nose!
   (D) Here they are, on your nose!

8. *With great difficulty, _____.*
   (A) He could keep his cool
   (B) He could get annoyed
   (C) He could not tolerate his nonsense
   (D) He could lose his temper
   (E) He could perform his usual functions easily

9. *He always stammers in public meetings, but his today's speech _____.*
   (A) Was not liked by the audience
   (B) Was not received satisfactorily
   (C) Was surprisingly fluent
   (D) Was fairly audible to everyone present in the hall
   (E) Could not be understood properly

10. *The Chairman rejected the proposal of increasing employee's salary because:*
   (A) The company had already gained three thousand crore profit
   (B) The company did not have sufficient funds to afford the rise
   (C) The number of employees in the company was very small
   (D) The employees had been demanding it for a long time
   (E) It was not difficult for the company to bear additional burden

11. *To succeed in a difficult task, _____.*
   (A) You need a person of persistent
   (B) One needs to be persistent
   (C) One needs to be persistence
   (D) Persistent is needed
   (E) Persistent is what one needs

12. *I shall not be late for dinner _____.*
   (A) Unless the train will be late
   (B) Unless the train will not be late
   (C) If the train is late
   (D) Unless the train is late

13. *_____ I would not have helped such an ungrateful man.*
   (A) Had I been in your place
   (B) Even after knowing that he was ungrateful
   (C) Had I asked him for his help
   (D) Though he did not deserve any help at all
   (E) If he had shown due respect to me

14. *Unless you work very hard, _____.*
   (A) You are not being successful
   (B) You will not be successful
   (C) You ought to be successful
   (D) You be not successful

15. *The more we looked at the piece of modern art, _____.*
   (A) We liked it less
   (B) The less we liked it
   (C) It looked better
   (D) The more we like it
   (E) Better we liked it

16. *In order to raise the company profit, the employees:*
   (A) Decided to raise the cost of raw material
   (B) Requested the management to implement new welfare schemes
   (C) Demanded two additional increments
   (D) Decided to go on paid holidays
   (E) Offered to work over time without any compension

17. *He seized control of the country _____.*
   (A) By using diplomacy and force
   (B) By diplomacy and being forceful
   (C) By being a diplomat and forceful
   (D) Not only because of diplomacy but force

18. *The doctor warns him that unless he gives up smoking:*
   (A) His health will soon be recovered
   (B) He will not recover
   (C) Will he be able to recover
   (D) He will not suffer

19. *Whichever way you approach the problem, _____*
   (A) It will not solve
   (B) It will not be solved
   (C) No one will not solve it
   (D) It will not be solve

20. *He is so lazy that he:*
   (A) Can't delay the schedule of completing the work
   (B) Can't depend on others for getting his work done
   (C) Always extends help to others to complete their work
   (D) Can seldom complete his work on time
   (E) Dislikes to postpone the work that he undertakes to do

21. *How much a man earns is as important as _____.*
   (A) Where does he earn
   (B) Why does he earn of all
   (C) When does he do so
   (D) How does he do it
   (E) How well he spends it

22. *Shan, where are you? _____, up this tree.*
   (A) There I am
   (B) There am I
   (C) Here I am
   (D) Here am I

23. *My mother is so poor:*
   (A) To get medical help for my father
   (B) That she cannot buy food for us
   (C) To send me to school
   (D) Because she will not work

24. *He tames animals because he:*
   (A) Is fond of them
   (B) Hates them
   (C) Seldom loves them
   (D) Is afraid of them
   (E) Wants to set them free

25. *He has no money now:*
   (A) Because he was very rich once
   (B) As he has given up all his wealth
   (C) Because he always spends money with utmost care
   (D) Because he had received huge donations once
   (E) Although he was very poor once

ANSWERS

1.	(A)	2.	(A)	3.	(C)	4.	(D)	5.	(A)
6.	(D)	7.	(B)	8.	(A)	9.	(C)	10.	(B)
11.	(B)	12.	(D)	13.	(A)	14.	(B)	15.	(B)
16.	(E)	17.	(A)	18.	(B)	19.	(B)	20.	(D)
21.	(D)	22.	(C)	23.	(B)	24.	(A)	25.	(B)

Multiple Choice Questions (MCQs)

(Sentence Completion)

TEST NO. 1

◈ *Choose the word which best completes each sentence.*

1. *We lost confidence in Salim because he never _____ the grandiose promises he had made.*
   (A) Tired of (B) Delivered on
   (C) Retreated from (D) Forgot about
2. *The driver suddenly applied the brakes when he saw a _____ truck ahead of him.*
   (A) Stationary (B) Moving
   (C) Static (D) Immobile
3. *Knowledge is like a deep well fed by _____ springs, and your mind is the little bucket that you drop in it.*
   (A) External (B) Perennial
   (C) Immortal (D) Inehaustible
4. *Salma is much too _____to have anything to do with that obnoxious affair.*
   (A) Noble (B) Proud
   (C) Happy (D) Difficult
5. *There is no incentive for America to sign the treaty since there is every reason to _____ no other nation intends to honour its provisions.*
   (A) Regret (B) Inform
   (C) Believe (D) Occupy
6. *A legislation was passed to punish brokers who _____ their clients funds.*
   (A) Defalcate (B) Devastate
   (C) Devour (D) Embezzle
7. *Normally, an individual thunderstorm _____ about 45 minutes.*
   (A) Lasts (B) Ends
   (C) Remains (D) Continues
8. *The task seemed impossible but somehow Jalil _____very skilfully in the end.*
   (A) Pulled it up (B) Pulled it off
   (C) Pulled it away (D) Pulled it out
9. *The unruly behaviour of the children _____ their parents.*
   (A) Aggrieved (B) Impeached
   (C) Incensed (D) Tempered
10. *We were amazed that a man who had been heretofore the most _____ of public speakers could, in a single speech, electrify an audience and bring them cheering.*
    (A) Pedestrian (B) Accomplished
    (C) Masterful (D) Auspicious
11. *The chairperson is a scintillating speaker whose lectures completely _____ students.*
    (A) Entertain (B) Absorb
    (C) Enthrall (D) Alienate
12. *Ali _____ force himself to work on till late in the night.*
    (A) Would (B) Would be
    (C) Could (D) Used to
13. *The officers threatened to take reprisals if the lives of their men were _____ by the conquered natives.*
    (A) Destroyed (B) Endangered
    (C) Enhanced (D) Irritated
14. *His moral decadence was marked by his _____from the ways of integrity and honesty.*
    (A) Obsession (B) Declivity
    (C) Departure (D) Opprobrium

15. *Her reaction was not the only _____ one.*
(A) Workable (B) Possible
(C) Likely (D) Good

16. *After a period of protracted disuse, a muscle will atrophy, _____ both its strength and the ability to perform its function.*
(A) Insuring (B) Regaining
(C) Sustaining (D) Losing

17. *True health and true success go together for they are inseparably _____ in the thought realm.*
(A) Interwined (B) Tied up
(C) Bound up (D) Inter-related

18. *If you are trying to make a strong impression on your audience you cannot do so by being understated, tentative, for _____.*
(A) Passionate (B) Authoritative
(C) Restrained (D) Argumentative

19. *Although, I had pledged not to tell anyone of the previous evening's trauma, the compulsive urge to unburden myself became _____.*
(A) Preposterous (B) Overwhelming
(C) Impassive (D) Irresistible

20. *The _____ arguments put forth for not disclosing the facts did not impress anybody.*
(A) Specious (B) Intemperate
(C) Spurious (D) Convincing

ANSWERS

1.	(B)	2.	(A)	3.	(B)
4.	(A)	5.	(C)	6.	(D)
7.	(A)	8.	(D)	9.	(C)
10.	(A)	11.	(C)	12.	(A)
13.	(B)	14.	(C)	15.	(B)
16.	(D)	17.	(A)	18.	(C)
19.	(D)	20.	(A)		

TEST NO. 2

◈ *Choose the word which, when inserted in the sentence, best fits the meaning of sentence.*

1. *Some officers have _____ their previous statements denying any involvement on their part with the contra aid network.*
(A) Recanted (B) Protracted
(C) Justified (D) Repeated

2. *As the market becomes _____ competitive, some companies will make larger profits.*
(A) Well (B) More
(C) Less (D) Fully

3. *Although, the conditions in which Riaz chooses to live suggest that he is miserly, his contributions to worthwhile charities show that he is _____.*
(A) Intolerant (B) Stingy
(C) Generous (D) Thrifty

4. *He suggests that the meeting _____ postponed.*
(A) Be (B) Is
(C) Must (D) Would be

5. *Modern architecture has discarded the _____ trimming on buildings and emphasises simplicity of life.*
(A) Flamboyant (B) Flabbergasting
(C) Gaudy (D) Gaunt

6. *Can he see his wife again? No, he _____.*
(A) Could (B) Can
(C) Will not (D) Cannot

7. *A man is one of those blessed artists who combine profundity and _____.*
(A) Fun (B) Education
(C) Depth (D) Wisdom

8. *The accused was released on _____ pending hearing of his case.*
(A) Bond (B) Bale
(C) Bail (D) Deposit

9. *Sadiq's _____ in his family's position is great but he does not boast about it.*
(A) Status (B) Proud
(C) Pride (D) Presumption

10. *There are many dialects of English with radically different pronunciations of the same word, but the spelling of these words is _____.*
(A) Shortened (B) Inconstant
(C) Contemplated (D) Uniform

11. *New concerns about growing religious tension in Kashmir were _____ this week between Hindus and Muslims.*
(A) Dispersed (B) Fueled
(C) Invalidated (D) Restrained

12. *Wasim was so good at Mathematics that his friends considered him to be a _____.*
(A) Prodigy (B) Prodigal
(C) Primeval (D) Profligate

13. *The majority report issued by the committee was completely _____, extolling in great detail the plan's strengths but failing to mention at all its shortcomings.*
(A) Skewed (B) Unbiased
(C) One-sided (D) Comprehensive

14. *I decided to sell a piece of land when I was offered a more _____ price.*
(A) True (B) Realistic
(C) Exact (D) Correct

15. *The enemy paid a large sum as _____.*
(A) Punishment (B) Reward
(C) Restitution (D) Compensation

16. *Despite the mixture's _____ nature, we found that by lowering its temperature in the laboratory we could reduce its tendency to vaporize.*
(A) Homogeneous (B) Resilient
(C) Volatile (D) Acerbic

17. *Many boys were _____ at street corners for the coffee bar to open.*
(A) Hanging upon (B) Hanging about
(C) Hanging back (D) Hanging on

18. *His monotonous voice acted like _____ and his audience was soon asleep.*
(A) An emetic (B) An anaesthetic
(C) A sedative (D) A purgative

19. *My finger is still _____ where I caught it in the door yesterday.*
(A) Sore (B) Wounded
(C) Injured (D) Bruised

20. *The flood water pushed against the river wall and _____ from a sudden break made by it.*
(A) Ran out (B) Serged up
(C) Gushed out (D) Flowed

ANSWERS

1.	(A)	2.	(B)	3.	(C)
4.	(A)	5.	(C)	6.	(D)
7.	(A)	8.	(C)	9.	(A)
10.	(D)	11.	(B)	12.	(A)
13.	(C)	14.	(B)	15.	(D)
16.	(C)	17.	(B)	18.	(C)
19.	(D)	20.	(B)		

TEST NO. 3

◈ *Complete the sentences by given choices.*

1. *Measurement is, like any other human endeavour, a complex activity, subject to error, not always used _____, and frequently misinterpreted and misunderstood.*
   (A) Properly (B) Innovatively
   (C) Mistakenly (D) Systematically
2. *Non-violence is the law of saints as violence is the law of the _____.*
   (A) Coward (B) Foolish
   (C) Brute (D) Ignorant
3. *His injury was very painful but not incapacitating and he managed to _____ the game in spite of it.*
   (A) Interrupt (B) Concede
   (C) Abandon (D) Finish
4. *They have some difficulty _____ all the employees, especially the smaller ones to confirm _____ the adopted scale of wages.*
   (A) Getting, to (B) In getting, upon
   (C) To get, over (D) To getting, with
5. *This contract was _____; it was not valid now.*
   (A) Nullified (B) Annulled
   (C) Invalid (D) Canceled
6. *To the dismay of the student body, the class president was _____ berated by the principal.*
   (A) Privately (B) Magnanimously
   (C) Inconspicuously (D) Ignominously
7. *We never believed that he would resort to _____ in order to achieve his end, we always regarded him as an honest man.*
   (A) Logic (B) Subterfuge
   (C) Charm (D) Diplomacy
8. *It was the help he got from his parents which _____ him through the tragedy.*
   (A) Boosted (B) Supported
   (C) Helped (D) Parked
9. *The plot of the play was extremely complicated and included many minor characters _____ to the central events.*
   (A) Tangential (B) Contemporary
   (C) Essential (D) Momentous
10. *It is a marble wall, _____ no bills.*
   (A) Stick (B) Affix
   (C) Paste (D) Attach
11. *You will have to catch the morning flight, so you _____ better get ready.*
   (A) Would (B) May
   (C) Had (D) Should
12. *The controversy is likely to create _____ between the two rivals.*
   (A) Doubt (B) Amity
   (C) Bitterness (D) Revenge
13. *The authority of voice in Faraz writing strikes many readers today as _____ colonialism.*
   (A) Cognizant of (B) Detrimental to
   (C) Consonant with (D) Independent of
14. *Beauty is to ugliness as adversity is to _____.*
   (A) Prosperity (B) Cowardice
   (C) Miser (D) Happiness
15. *Whenever Imran refers to his favourites he is voluble, but when he talks of his adversaries he is _____.*
   (A) Rough (B) Reticent

(C) Miserly (D) Aggressive

16. *The chairman's intolerance of _____ among his aides was intensified by his insistence upon total loyalty from all.*
(A) Compliance (B) Flattery
(C) Dissent (D) Dishonesty

17. *Despite the millions of rupees spent on improvements, the telephone system in Pakistan remains _____.*
(A) Suspicious (B) Primitive
(C) Outdated (D) Impartial

18. *Do you think Irum is avoiding you? Yes, I _____.*
(A) Do (B) Agree
(C) Did (D) Have not

19. *Few plants can grow beneath the canopy of the sycamore tree, whose leaves produce a natural herbicide that leaches into the surrounding soil, _____ other plants that might compete for water and nutrients.*
(A) Refreshing (B) Nourishing
(C) Distinguishing (D) Inhibiting

20. *Even when Akram's reputation was in _____ almost everyone was willing to admit that he had genius.*
(A) Eclipse (B) Peregrination
(C) Rebuttal (D) Accumulation

ANSWERS

1.	(A)	2.	(C)	3.	(D)
4.	(A)	5.	(B)	6.	(D)
7.	(B)	8.	(C)	9.	(A)
10.	(B)	11.	(D)	12.	(C)
13.	(C)	14.	(A)	15.	(B)
16.	(C)	17.	(B)	18.	(C)
19.	(D)	20.	(A)		

TEST NO. 4

◈ *Select anyone the most suitable answer.*

1. *I do not think, Javed will gain anything by insulting and _____ the man Javed do not agree with.*
(A) Depicting (B) Revamping
(C) Defaming (D) Charging

2. *Although, a few years ago the fundamental facts about the silky way seemed fairly well _____, now even its mass and its radius have come into question.*
(A) Ignored (B) Established
(C) Determined (D) Problematic

3. *Naveed's _____ of the topic was so good that students had few doubts to raise at the end.*
(A) Clarity (B) Exposure
(C) Picturisation (D) Exposition

4. *A good lawyer will argue only what is central to an issue, eliminating _____ information which might jeopardize the client.*
(A) Extraneous (B) Prodigious
(C) Seminal (D) Erratic

5. *Ali got the company car for a _____ price as he was the senior most employee in the company.*
(A) Nominal (B) Fixed
(C) Discounted (D) Reduced

6. *His novel is both so eloquent in its passion and so searching in its candor that it is bound to _____ any reader.*
(A) Bore (B) Disappoint
(C) Unsettle (D) Embarrass

7. *We felt as if the ground was _____ beneath our feet.*

(A) Digging (B) Slipping
(C) Sinking (D) Bursting

8. *It is irritating to try to keep a commitment that you know you are not going to _____.*
(A) Honour (B) Decorate
(C) Glorify (D) Dignify

9. *Because Rehana had a reputation for _____ we were surprised and pleased when she greeted us so affably.*
(A) Graciousness (B) Insolence
(C) Arrogance (D) Querulousness

10. *Through a _____ circumstance, they unexpectedly found themselves on the same bus with Mohsin.*
(A) Referential (B) Fortuitous
(C) Lambent (D) Elusive

11. *Could you appease her curiosity? No, I _____.*
(A) Did not (B) Could not
(C) Have not (D) Could never

12. *Unlike the images in symbolist poetry which are often vague and obscure, the images of surrealist poetry are startlingly _____ and bold.*
(A) Trivial (B) Concrete
(C) Furtive (D) Virulent

13. *The child was so spoiled by his parents that he pouted and become _____ when he did not receive all of their attention.*
(A) Sullen (B) Discreet
(C) Suspicious (D) Elated

14. *Everyone in this universe is accountable to God _____ his actions.*
(A) Of (B) Against
(C) For (D) About

15. *I am not concerned _____ him in that business.*
(A) By (B) About
(C) For (D) With

16. *Just as disloyalty is the mark of the renegades _____ is the mark of the craven.*
(A) Cowardice (B) Avarice
(C) Vanity (D) Temerity

17. *The new owners of the paper changed the _____ completely.*
(A) Outlook (B) Outlet
(C) Layout (D) Outlay

18. *Contrary to popular opinion, bats are not generally aggressive and rabid, most are shy and _____.*
(A) Innocuous (B) Turgid
(C) Disfigured (D) Punctual

19. *Sadia _____ at me in doubt and disbelief.*
(A) Watched (B) Gazed
(C) Gaped (D) Looked

20. *If you are seeking _____ that will resolve all our ailments, you are undertaking an impossible task.*
(A) A direction (B) A contrivance
(C) A panacea (D) A precedent

ANSWERS

1.	(C)	2.	(B)	3.	(D)
4.	(A)	5.	(A)	6.	(C)
7.	(B)	8.	(C)	9.	(D)
10.	(B)	11.	(A)	12.	(B)
13.	(A)	14.	(C)	15.	(D)
16.	(A)	17.	(C)	18.	(A)
19.	(B)	20.	(C)		

TEST NO. 5

1. *Didn't you tell me that you would come to see me? No, I _____.*
   (A) Didn't (B) Had not
   (C) Have not (D) Could not
2. *We must _____ the tickets for the movie in advance.*
   (A) Remove (B) Take
   (C) Draw (D) Buy
3. *Only _____ were present at the seminar.*
   (A) A few people (B) A little people
   (C) A few people (D) The little people
4. *The stenographer is very efficient. He is _____ to his firm.*
   (A) An asset (B) A boon
   (C) A credit (D) A blessing
5. *The twins are so alike that I cannot _____ one from the other.*
   (A) Say (B) Notice
   (C) Discern (D) Tell
6. *Just _____ the files on my table.*
   (A) Let (B) Leaves
   (C) Stay (D) Leave
7. *I never miss a cricket match. I _____ fond of cricket since childhood.*
   (A) Am (B) Has been
   (C) Have been (D) Will be
8. *All of us should abide _____ the laws of our country.*
   (A) By (B) In
   (C) To (D) With
9. *The period of the fall of the Roman Empire was a dark period for _____ as well as for other arts.*
   (A) Aesthetics (B) Gastronomy
   (C) Astrology (D) Histrionics
10. *Multan _____ a very hot climate.*
    (A) Has (B) Have
    (C) Has been (D) With
11. *That professor enjoys teaching and _____.*
    (A) Writing (B) Written
    (C) To write (D) Write
12. *She came _____ with me to see the circus.*
    (A) After (B) Across
    (C) Along (D) Off
13. *The police has been looking for him _____ four weeks.*
    (A) During (B) For
    (C) Since (D) Till
14. *_____ discovery of insulin, it was not possible to treat diabetes.*
    (A) Prior (B) Before to the
    (C) Prior to the (D) To prior the
15. *Do no hanker _____ worldly pleasures.*
    (A) For (B) Towards
    (C) After (D) About
16. *In partnership with Pakistan, South Korea _____ on Motorway.*
    (A) Helped worked (B) Helping work
    (C) Helped working (D) To help working
17. *He is too dull _____ the problem.*
    (A) Solving (B) To solving
    (C) Solves (D) To solve
18. *Distribute the handouts _____ the candidates.*

(A) Between (B) Among
(C) To (D) In

19. *Thank you for _____ me your book.*
(A) Borrowing (B) Lending
(C) Borrowed (D) Had lent

20. *Although he is blind, he is very fast _____ calculations.*
(A) At (B) About
(C) In (D) With

ANSWERS

1.	(C)	2.	(D)	3.	(A)	4.	(A)	5.	(D)
6.	(D)	7.	(C)	8.	(A)	9.		10.	(A)
11.	(A)	12.	(C)	13.	(B)	14.	(C)	15.	(C)
16.	(C)	17.	(D)	18.	(B)	19.	(B)	20.	(A)

TEST NO. 6

◈ *Select the word that best completes each of the following sentences:*

1. *The reasoning in this editorial is so _____ that we cannot see how anyone can be deceived by it.*
(A) Dispassionate (B) Cogent
(C) Specious (D) Coherent

2. *Sometimes, it is necessary for an author to know what is going on in the minds of his characters. This is called _____.*
(A) Omniscience (B) Omnipotence
(C) Truclence (D) Omnipresence

3. *The press conference did not clarify many issues since the president responded with obfuscation and _____ rather than clarity and precision.*
(A) Lucidity (B) Vagueness
(C) Formality (D) Humor

4. *Sri Lanka, for the present, is deeply _____ in economic difficulties, but, the Government has taken a pledge to set everything right within 2 years.*
(A) Ruined (B) Swamped
(C) Saturated (D) Engrossed

5. *I don't know _____ to value your qualities.*
(A) Only how (B) How
(C) So how (D) That how

6. *The _____ of evidence was on the side of the plaintiff since all but one witness testified that Salim's story was correct.*
(A) Brunt (B) Accuracy
(C) Propensity (D) Preponderance

7. *Patriotism, like so many other objects of this imperfect world, is a _____ web of good and evil.*
(A) Tangled (B) Entrapped
(C) Entangled (D) Complicated

8. *It is difficult for a modern audience, accustomed to the minutiae of film and TV; to appreciate opera with its grand spectacle and _____ gestures.*
(A) Subtle (B) Inane
(C) Monotonous (D) Extravagant

9. *She should continue to remain cold towards her lover till the latter has taken _____ to move her heart to kindness.*
(A) Suffering (B) Pain
(C) Trouble (D) Pains

10. *Though Akram was theoretically a friend of labour, his voting record in party _____ that impression.*
(A) Belied (B) Confirmed
(C) Maintained (D) Implied

11. *To _____ a bus here is not so easy.*
(A) Obtain (B) Get
(C) Gain (D) Acquire

12. *Because of the moon's _____ gravity, it has little or no substance.*
(A) Weak (B) Dull
(C) Frail (D) Unsubstantial

13. *Behaviorism was a protest against the _____ psychological tradition which held that the proper data of psychology were mentalistic.*
(A) Moralistic (B) Orthodox
(C) Redoubtable (D) Rudimentary

14. *He had a terrible night caused by an _____ during his sleep.*
(A) Delusion (B) Hypochondria
(C) Debility (D) Obsession

15. *A metaphorical statement is an _____ comparison; it does not compare things explicitly, but suggests a likeness between them.*
(A) Implied (B) Ardent
(C) Unfair (D) Inherent

16. *Any leader who allows nepotism to flourish should be subject to _____.*
(A) Stringency (B) Punish
(C) Autopsy (D) Condemnation

17. *A lively joke is a _____ expression for the moment.*
(A) Apt (B) Befitting
(C) Proper (D) Correct

18. *A scathing review of the recent performance of dancers called the production grotesque and the conducting of the orchestra _____.*
(A) Munificent (B) Pedestrian
(C) Prejudicial (D) Heretical

19. *The appropriate word used for marriage between people of different races is _____.*
(A) Embolism (B) Scurrility
(C) Reverberation (D) Nonsequitur

20. *The ravages of time had left the caste _____; it towered above the village, looking much as it must have done in King Faisal's time.*
(A) Untouched (B) Lonely
(C) Destroyed (D) Alone

ANSWERS

1.	(C)	2.	(A)	3.	(B)
4.	(A)	5.	(B)	6.	(D)
7.	(A)	8.	(D)	9.	(C)
10.	(A)	11.	(B)	12.	(A)
13.	(B)	14.	(D)	15.	(A)
16.	(D)	17.	(C)	18.	(B)
19.	(D)	20.	(A)		

TEST NO. 7

◈ *Choose the word for blank spaces.*

1. *I have no _____ motive in offering this advice; I seek no personal advantage.*
(A) Ulterior (B) Nominal
(C) Disinterested (D) Incongrous

2. *We had a wonderful view of the day through the _____.*
(A) Proximity (B) Hole
(C) Window (D) Vicinity

3. *Because customers believe that there is a direct correlation between price and value, software manufacturers continue to _____ their prices at an astonishing rate.*
(A) Control (B) Raise

(C) Inflate (D) Determine

4. *To find the volume of an irregular solid you must _____ Archimedes Principle.*
(A) Practice (B) Exploit
(C) Apply (D) Employ

5. *Hamid and Javed were part of that inner _____ of the police whose distinguished legacy dominated national domestic policy.*
(A) Sanctum (B) Core
(C) Life (D) Circle

6. *Artists and poets are possessed of the same qualities of mind, governed by the same principles of taste, and are consistently is sympathy and never in _____ with one another.*
(A) Danger (B) Accord
(C) Collusion (D) Disagreement

7. *That charming girl was the _____ of all eyes.*
(A) Cynosure (B) Aim
(C) Target (D) Ambition

8. *This garden has been preserved in all its _____ wildness so that visitors in future years may see how people lived during the previous centuries.*
(A) Esoteric (B) Pristine
(C) Hedonistic (D) Prospective

9. *The gold ornament was _____ with precious stones.*
(A) Beautiful (B) Studded
(C) Adorned (D) Decorated

10. *The most difficult modern compositions for the piano _____ the audience.*
(A) Incited (B) Excited
(C) Thrilled (D) Disappointed

11. *The High Court, is striking down the state law, ruled the statute had been enacted in an atmosphere charged with religious convictions which had _____ the law-making process.*
(A) Repealed (B) Infected
(C) Written (D) Influenced

12. *Nighat is kind; her husband is not _____ a man.*
(A) Such (B) Like
(C) So (D) As well

13. *If you carry this truculent attitude to the conference, you will _____ any supporters you may have at this moment.*
(A) Alienate (B) Delight
(C) Attract (D) Defer

14. *I have no _____ to listen to his silly talk.*
(A) Trouble (B) Convenience
(C) Patience (D) Perseverence

15. *The latest research indicates that feelings of love occur in the nonverbal part of brain, which helps to explain why people are often able to experience such feelings but not _____ them.*
(A) Convey (B) Remove
(C) Explain (D) Believe

16. *The young man was quickly promoted when his boss saw how _____ he was.*
(A) Assiduous (B) Cursory
(C) Lethargic (D) Indigent

17. *The courtiers had to swear _____ to the new king.*
(A) Allegiance (B) Obedience
(C) Loyalty (D) Faithfulness

18. *He never _____ to her in the near past.*
(A) Wrote (B) Write
(C) To write (D) Written

19. *Progress in government, literature, art, religion, science and philosophy _____ great civilisations from mere groups of society.*

(A) Relinquish (B) Distinguish
(C) Describe (D) Extol

20. *The _____ pittance the widow receives from the society cannot keep her from poverty.*
(A) Indulgent (B) Munificent
(C) Niggardly (D) Magnanimous

ANSWERS

1.	(A)	2.	(C)	3.	(B)
4.	(C)	5.	(B)	6.	(D)
7.	(A)	8.	(B)	9.	(D)
10.	(C)	11.	(B)	12.	(D)
13.	(A)	14.	(B)	15.	(C)
16.	(A)	17.	(D)	18.	(A)
19.	(B)	20.	(C)		

TEST NO. 8

◈ *Complete the sentences by given choices:*

1. *For Arshad, art became a _____ ritual; paper and pencils were holy objects to him.*
(A) Futile (B) Fascinating
(C) Sacred (D) Superficial

2. *Since there was adequate grazing area for the herds, the land was _____ populated.*
(A) Sparsely (B) Disproportionately
(C) Rustically (D) Inadequately

3. *It is _____ that students do not repay their loans.*
(A) Laudable (B) Unfortunate
(C) Unforgivable (D) Regrettable

4. *The tapeworm is an example of _____ organism.*
(A) Parasitic (B) Protozoan
(C) Exemplary (D) Hospitable

5. *He said that there was no going back because his decision was _____.*
(A) Palatable (B) Peremptory
(C) Premeditated (D) Revolutionary

6. *Although his initial success was _____ by the fact that Faisal was the son of a famous actor, the critics later acclaimed him as a star in his own right.*
(A) Refuted (B) Superceded
(C) Enhanced (D) Trivialized

7. *That was an _____ addition of this book.*
(A) Summarized (B) Abbreviated
(C) Shortened (D) Abridged

8. *Shy and hypochondriacal Akbar was uncomfortable at public gatherings, his character made him a most _____ lawmaker and practicing politician.*
(A) Fervent (B) Effective
(C) Unlikely (D) Gregarious

9. *The event came _____ as he had predicted it.*
(A) Up (B) Off
(C) By (D) About

10. *Moeen is _____ opponent, you must respect and fear him at all times.*
(A) Craven (B) Redoubtable
(C) Insignificant (D) Disingenuous

11. *Now she feels the folly of _____ him.*
(A) Quarreling with (B) Quarrel on
(C) Quarreling to (D) Quarreling against

12. *His answer was such _____ I expected him to give.*
(A) As (B) Like
(C) Which (D) That

13. *His true feelings _____ themselves in his sarcastic asides; only then was his bitterness revealed.*
(A) Developed (B) Concealed
(C) Manifested (D) Grieved

14. *Contrary to popular belief, they were not peace-loving astronomers but _____ warriors who viewed their gods as cruel and vengeful.*
(A) Reluctant (B) Amicable
(C) Skilled (D) Formidable

15. *We must try to understand his momentary aberration for Ashraf has _____ more strain and anxiety than any among us.*
(A) Undergone (B) Forgotten
(C) Described (D) Understood

16. *I do not think, you will gain anything by insulting and _____ the man you do not agree with.*
(A) Depicting (B) Revamping
(C) Defaming (D) Charging

17. *Although, alcoholism has long been regarded as a personality disorder, there is evidence to suggest that alcoholics are often the children of alcoholics and that they are born with a _____ the disease.*
(A) Deterioration of (B) Respect for
(C) Liability for (D) Predisposition to

18. *Milk is a _____ food.*
(A) Wholesome (B) Nutritious
(C) Health giving (D) Pure

19. *So many servants attended _____ him during his illness.*
(A) On (B) With
(C) At (D) Upon

20. *Because it arrives so early in the season, before many other birds, the robin has been called the _____ of spring.*
(A) Compass (B) Harbinger
(C) Autocrat (D) Hostage

ANSWERS

1.	(C)	2.	(A)	3.	(D)
4.	(A)	5.	(B)	6.	(C)
7.	(B)	8.	(C)	9.	(D)
10.	(B)	11.	(C)	12.	(A)
13.	(C)	14.	(D)	15.	(A)
16.	(C)	17.	(D)	18.	(B)
19.	(A)	20.	(B)		

TEST NO. 9

◈ *Choose the word which when inserted in the sentence, best fits the meaning of sentence.*

1. *Overindulgence _____ character as well as physical stamina.*
(A) Maintains (B) Debilitates
(C) Stimulates (D) Strengthens

2. *_____ the event of Aslam's resigning his job, his family would starve.*
(A) In (B) On
(C) At (D) Within

3. *Despite the fact that Nadia was much _____, the scientist continued to present her controversial theory to the public.*
(A) Imitated (B) Chastened
(C) Maligned (D) Admired

4. *Traffic speed limits are set at a level that achieves some balance between the danger of _____ speed and the desire of most people to travel as quickly as possible.*
(A) Excessive (B) Prudent

(C) Marginal (D) Normal

5. *Wooden surface was glued _____ the steel surface.*
(A) Within (B) Against
(C) Onto (D) Into

6. *His _____ tactics may compel me to cancel the contract as the job must be finished on time.*
(A) Offensive (B) Dilatory
(C) Infamous (D) Confiscatory

7. *In the world of today, material values take precedence _____ spiritual values.*
(A) Over (B) At
(C) On (D) About

8. *The supposedly impregnable defenses of the country's southern border became _____ when the officer discovered that the defenses could be circumvented by an approach from the east.*
(A) Useless (B) Flexible
(C) Valuable (D) Worthless

9. *Work hard lest you _____ fail.*
(A) Will (B) Should
(C) Can (D) Should not

10. *Usman is not attracted by the peripatetic life of the _____ always wandering through the countryside.*
(A) Vagabond (B) Mendicant
(C) Almsgiver (D) Philosopher

11. *For Amna, each new school year was an_____ experience, but her brother awaited the coming of autumn with dread.*
(A) Exhausting (B) Illuminating
(C) Exciting (D) Exhilarating

12. *_____ is a person who dabbles in art and letters.*
(A) Philosopher (B) Philistine
(C) Chauvinist (D) Dilettante

13. *Faced with these massive changes, the government keeps its own counsil, although generally benevolent, it has always been _____ regime.*
(A) Reticent (B) Indifferent
(C) Altruistic (D) Unpredictable

14. *Train is _____ of different bogeys.*
(A) Made of (B) Make up of
(C) Made with (D) Made up

15. *The legal system of Russia can no longer regard itself as _____ and standing apart from those of other countries.*
(A) Damaging (B) Arbitrary
(C) Binding (D) Independent

16. *The members of the religious sect ostracized the _____ who had abandoned their faith.*
(A) Recreant (B) Coward
(C) Suppliant (D) Litigant

17. *Saima _____ my name from across the river.*
(A) Cried (B) Called
(C) Cried out (D) Shouted

18. *After having worked in the soup kitchen feeding the hungry, the volunteer began to see her own good fortune as a _____.*
(A) Threat (B) Fluke
(C) Omen (D) Reward

19. *To the ambassador, who believed that some measure of _____ should always characterize junior staffers, the new aid seemed very presumptuous.*
(A) Energy (B) Integrity
(C) Humility (D) Sincerity

20. *He worked _____, because he aspired to stand first in the examination.*

(A) Very hard (B) Harder
(C) Much hard (D) The harder

ANSWERS

1.	(B)	2.	(A)	3.	(C)
4.	(A)	5.	(C)	6.	(B)
7.	(A)	8.	(D)	9.	(B)
10.	(A)	11.	(C)	12.	(D)
13.	(A)	14.	(C)	15.	(D)
16.	(A)	17.	(D)	18.	(B)
19.	(C)	20.	(A)		

TEST NO. 10

◈ *Choose the word that is most nearly the same in meaning to the key word.*

1. *Those defenders of the tobacco industry who deny that there is a casual linkage between cigarette smoking and many diseases all but _____ the statistical evidence that very clearly demonstrates the connection.*
(A) Ignore (B) Signify
(C) Explain (D) Refute

2. *I have come to see the loss, I won't see any one _____.*
(A) Except (B) Else
(C) Or else (D) Other

3. *This state is a colony, however, in most matters, it is _____ and receives no order from the mother country.*
(A) Distant (B) Autonomous
(C) Submissive (D) Amorphous

4. *Take any apple _____ you like.*
(A) What (B) As
(C) Which (D) That

5. *By the middle of January, the river had become so choked with ice as to be _____ even for the smallest of boats.*
(A) Inactive (B) Unreliable
(C) Impassable (D) Unattractive

6. *They fired upon the enemy from behind the trees, walls and any other _____ point they could find.*
(A) Vantage (B) Indefensible
(C) Exposed (D) Definitive

7. *The dog is _____ faithful animal.*
(A) The (B) Not
(C) A (D) Very

8. *In their most recent report to the shareholders, the directors _____ financial information on international sales.*
(A) Distort (B) Omit
(C) Invent (D) Substitute

9. *Maria has been waiting for you _____ morning.*
(A) Till (B) From
(C) For (D) Since

10. *The evil of class and race hatred must be eliminated while it is still in an _____ state.*
(A) Independent (B) Amorphous
(C) Embryonic (D) Uncultivated

11. *The candidate later _____ the broad generalization concerning welfare recipients by noting that the vast majority are not able to find significant employment.*
(A) Verified (B) Qualified
(C) Withdrew (D) Clarified

12. *Haider is either a scholar _____ a professional teacher.*
(A) Else (B) Or else

(C) Or (D) And

13. *In many countries, the proponents of a centrally controlled economy, although in _____, continue to constitute a powerful political force.*
(A) Control (B) Restart
(C) Disarray (D) Error

14. *Raheela spoke _____ that it was applauded by all.*
(A) Well (B) As well
(C) Very well (D) So well

15. *Studded starfish are well protected from most predators and parasites by _____ surface whose studs are actually modified spines.*
(A) A fragile (B) An obtuse
(C) An armoured (D) A brittle

16. *The committee censured the member, noting that his behaviour was the very _____ of ethical conduct.*
(A) Essence (B) Embodiment
(C) Nature (D) Antithesis

17. *Stand here, _____ I speak.*
(A) While (B) As
(C) When (D) The time

18. *We need more men of culture and enlightenment, we have too many _____ among us.*
(A) Students (B) Philistines
(C) Philosophers (D) Visionaries

19. *The successful _____ of a novel or a poem requires fluency in two languages.*
(A) Publication (B) Reproduction
(C) Writing (D) Translation

20. *Let us wait _____.*
(A) So little (B) Little
(C) A little (D) The little

ANSWERS

1.	(A)	2.	(B)	3.	(B)
4.	(D)	5.	(C)	6.	(A)
7.	(C)	8.	(A)	9.	(D)
10.	(C)	11.	(B)	12.	(C)
13.	(B)	14.	(D)	15.	(C)
16.	(D)	17.	(A)	18.	(B)
19.	(D)	20.	(C)		

TEST NO. 11

Direction: The following sentences are given with blanks to be filled with prepositions. Four alternatives are suggested. Choose the correct option.

1. The party had to struggle _____ the wet grass for half an hour _____ it reached dry land.
(A) along; before (B) through; before
(C) across; till (D) along; until

2. _____ the weight of the elephant the old bridge collapsed, and the elephant had much difficulty _____ getting _____ the bank of the river.
(A) under; for; at (B) beneath; for; to
(C) under; in; to (D) with; in; on

3. I was dead _____ time; you are the one who was late _____ an hour.
(A) on; over (B) within; for
(C) in; for (D) on; by

4. _____ half an hour all those who were trapped _____ the building were rescued _____ the brave firemen.
(A) within; near; by (B) before; within; by
(C) after; outside; with (D) within; inside; by

5. The man was _____ measure rich, but he was _____ compassion _____ the poor.
(A) beyond; for; for (B) within; without; to
(C) without; for; to (D) beyond; without; for

6. _____ two weeks, he did not venture _____ the safety of his house, even once.
(A) within; for (B) for; beyond
(C) for; towards (D) over; for

7. Most of the members were enthusiastically _____ him, but there were a few who were very much _____ him.
(A) against; with (B) behind; for
(C) for; against (D) with; without

8. Half a mile from the house he turned _____ the main road, and walked _____ the railway line.
(A) off; along (B) from; beside
(C) towards; beside (D) off; on

9. He laboured _____ the hill; sat watching the city; then ran _____ the hill.
(A) along; towards (B) down; up
(C) up; down (D) towards; from

10. The dog ran _____ me; I ran _____ the dog; but he ran faster _____ me.
(A) before; behind; than (B) before; after; than
(C) behind; with; to (D) after; behind; than

11. We got _____ the train _____ Rawalpindi.
(A) on; in (B) into; at
(C) into; in (D) in; at

12. Put the milk _____ the table and cover it _____ a cloth.
(A) on; with (B) near; by
(C) on; by (D) in; with

13. The plane flew _____ our house and disappeared _____ the clouds.
(A) above; beyond (B) over; in
(C) across; in (D) over; into

14. Come and stand _____ me _____ my umbrella or you will get quite wet.
(A) with; under (B) by; beneath
(C) beside; under (D) near; below

15. The child ran _____ the street to the other side and climbed _____ the bridge.
(A) across; up (B) along; over
(C) into; under (D) over; into

ANSWERS

1.	(B)	2.	(C)	3.	(D)	4.	(D)	5.	(D)
6.	(B)	7.	(C)	8.	(A)	9.	(C)	10.	(B)
11.	(B)	12.	(A)	13.	(D)	14.	(C)	15.	(A)

TEST NO. 12

Direction: Each sentence has two blanks. Under each sentence are given four alternative sets of words, marked A, B, C and D. Select the set which most appropriately completes the sentence.

1. James Watt first realized the _____ of steam when he saw a _____ boiling on the fire.
(A) strength; pot (B) force; kettle
(C) force; tea-pot (D) power; vessel

2. You will _____ difficulty and danger and reach the height of power through _____.
(A) vanquish; self-knowledge (B) defeat; worship
(C) overcome; self-discipline (D) conquer; soul-power

3. On the top of Mount Everest, I did not feel anything _____; I rather felt a great _____ to God.
(A) sinful; love (B) superstitious; similarity
(C) superhuman; closeness (D) strange; nearness

4. Viruses are invisible through the _____ microscope; but we know that they are there because we can see the _____ they cause.
(A) cheap; damage (B) elementary; harm
(C) simple; danger (D) ordinary; havoc

5. Man punishes the _____, but God punishes the _____.

(A) deed; reason (B) action; intention
(C) wickedness; temptation (D) crime; sin

6. The _____ the church; the _____ from God.
(A) closer; further (B) nearer; farther
(C) holier; nearer (D) greater; closer

7. The _____ is not steel, but it _____.
(A) tongue; cuts (B) word; pricks
(C) mouth; hurts (D) mouth; eats

8. Today in cases of _____, deaths from blood poisoning are _____.
(A) injection; common (B) operations; rare
(C) surgery; seldom (D) infection; frequent

9. Scientists and _____ tracking down germs have shown _____ in their fight against disease.
(A) doctors; heroism (B) surgeons; boldness
(C) dentists; courage (D) artists; skill

10. Hydrogen balloons, which were much _____ than hot-air balloons became very _____.
(A) smaller; cheap (B) lighter; popular
(C) cheaper; fashionable (D) brighter; common

11. The sea was coming after me as high as a great hill and as _____ as a _____ enemy.
(A) furious; charging (B) dreadful; advancing
(C) terrible; charging (D) angry; attacking

12. But as I looked about me in fear, I felt _____ miserable and _____ cheerful.
(A) quite; very (B) more; less
(C) extremely; quite (D) less; more

13. Everywhere, the Iron Horse replaced the living _____ to pull trains carrying _____ and goods.
(A) animals; passengers (B) horses; passengers
(C) horses; labourers (D) animals; men

14. "Some are _____ and some are _____."
(A) wise; otherwise (B) clever; cheats
(C) foolish; stupid (D) good; saintly

15. The _____ to hell is _____ with good intentions.
(A) road; made (B) highway; lined
(C) road; paved (D) path; filled

ANSWERS

1.	(B)	2.	(C)	3.	(D)	4.	(D)	5.	(D)
6.	(B)	7.	(A)	8.	(B)	9.	(A)	10.	(B)
11.	(A)	12.	(B)	13.	(B)	14.	(A)	15.	(C)

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Part 2 Analogy Questions

The word analogy means "an agreement or correspondence in certain respect between things otherwise different ---- a resemblance of relations, as in the phrase, "knowledge is to the mind, what light is to the eye": relation in general: likeness: correspondence of a word or a phrase with the genius of a language, as learned from the manner in which its words and phrases are ordinarily formed: similarity of derivative or inflectional processed."

The Urdu meaning of the word analogy is

Example 1: interesting the missing word.

Days is to night as truth is to falsehood.

Answer: In the above sentence, the word falsehood has been inserted.

Example 2: which choice gives the answer?

1. Man is to run as bird is to _____.

Choices: (i) fly, (ii) run, (iii) weak.

Answer: (i)

Example 3: ring is to finger as watch is to ____

Choices: (i) arm, (ii) wrist, (iii) leg.

Answer: (ii)

TYPES OF ANALOGY TESTS

First Type: The first type is that in which two words which have some relationship with each other are presented. These are followed by another word and a number of choices. One word from the choices is to be picked up to establish the same relation with the third word as the first two have. For example----

Day is no Night as Cold is to?

(A) Ice (B) Wet
(C) Warm (D) Snow

The correct answers is (C).

Day and Night bear the relation of the opposites. As cold is opposite of warm.

Second Type: Part relationship----- In this type of relationship, the two words represent the parts of a bigger thing. For example----

Lyric is to Ode as

Head is to legs
Sky is to earth
Bomb is to science
Newspaper is to journalist

The correct answer is **(A)**.

In the above quoted example, Lyric and Ode are two types of poems. Similarly, head and legs are two parts of the human body.

Third Type: Another type of analogy is in which one of the four relationship element is not given. One out of the choices is selected. Example----

Ship is to Fish as

(A) Kite (B). Feather
(C) Tree (D) Chirp

is to bird

The correct answer is **(A)**.

Explanation ---- Both ship and fish are found in water. This is the relationship between the two words. For bird, we will have to pick up kite because both are seen in air.

HOW TO ATTEMPT THIS QUESTION

Step One --- Establish the relationship between the first two words.

Step Two ---- Find the same relationship among the choices which follows the pattern of the two words.

KINDS OF RELATIONSHIP

1. ***Purpose relationship __ Example __ Glove: Balls as***
   (A) Hook: Fish (B) Winter: Weather
   (C) Games: Sports (D) Stadium: Seats

The correct answer is (A)

The purpose of glove is to help in catching the ball and the purpose of hook is to catch fish. The correct answer is (A)

2. ***Cause and effect relationship_Example__ Race: Fatigue as:***
   (A) French: Athlete (B) Fast: Hunger
   (C) Art: Bug (D) Walking: Running

   The correct answer is (B) because fatigue is the effect of race; hunger is the effect of fast.

3. ***Part whole relationship __Example__ Snake: Reptile as***
   (A) Patch: Thread (B) Removal: Snow
   (C) Struggle: Wrestle (D) Hand: Clock

4. ***Action to object and object to action relationship, Examples__***
   A. Kick: Football:
   (A) Kill: Bomb (B) Break: Pieces
   (C) Question: Team (D) Smoke: Pipe
   B. Steak: Broil:
   (A) Bread: Bake (B) Food: Sell
   (C) Wine: Pour (D) Sugar: Spill

   Kick shows action and Football the object of action. This very relationship is represented in (D) i.e., smoking is action and pipe is the object. For B answer is (A).

5. ***Synonym relationship—***

Enormous: Huge as
   (A) Rough: Rock (B) Muddy: Unclear
   (C) Purse: Kitchen (D) Black: White

   The correct answer is (B) because "muddy" and unclear are synonyms.

6. ***Antonym relationship—Example——***
   Purity: Evil as
   (A) Suavity: Bluntness (B) North: Climate
   (C) Angle: Horns (D) Boldness: Victory

   The correct answer is (A) because the two words are antonyms.

7. ***Place relationship—Example—Faisal Mosque: Islamabad as***
   (A) Red Square: Moscow (B)Albany: New York
   (C) India: Madras (D) Pakistan: Nepal

   Faisal Mosque is situated in Islamabad so is Albany in New York.

8. ***Degree relationship—Example—Warm: Hot as***
   (A) Glue: Paste (B) Climate: Weather
   (C) Bright: Genius (D) Frown: Anger

   Warm is less hot and frown is less anger.

9. ***Sequence relationship—Spring: Summer at.***
   (A) Thursday: Wednesday
   (B) Wednesday: Monday
   (C) Monday: Wednesday
   (D) Wednesday: Thursday

   Summer comes after Spring so does Thursday after Wednesday.

10. ***Association relationship—Example—Devil: Wrong as***
   (A) Colour: Sidewalk (B) Slipper: State
   (C) Ink: Writing (D) Picture: Bed

   Devil is associated with wrongs so ink is associated with writing.

11. ***Grammatical relationship—Example—Restore: Climb as***
   (A) Segregation: See (B) Nymph: In

(C) Room: Although (D) Run: See

Restore and climb are verbs so are run and see.

EXAMPLES WITH EXPLANATORY ANSWERS

Example 1 BRIM: HAT::

(A) hand: glove
(B) spoke: umbrella
(C) skirt: hem
(D) snood: hood
(E) lace: shoe

Answer: The correct answer is (E)

What is the relationship between BRIM AND HAT? A brim is a part of a hat, so the relationship is that of part to whole. The next step is to examine the answer choices to find another pair of words which bear the same relation to each other. Consider each answer choice in turn.

(A) A hand is not a part of a glove, so eliminate **(A)**.

(B) A spoke is part of an umbrella, so (B) is a likely choice. But don't mark your answer yet. You must always look at all five choices before making your final decision.

(C) A hem is part of a skirt, but BEWARE. The relationship in (C) is whole (the skirt) to part (the hem), which is the reverse of the initial relationship. Your answer must maintain the same relationship in the same sequence as the original pair. Eliminate (C).

(D) If you know that a snood is a hair net, you can see that snood, hood, and hat are all headgear. However, a snood is not a part of a hood, so (D) is incorrect. If you do not know the meaning of one word among the choices, do not fall into the trap of choosing that answer just because it's unfamiliar. Consider all the choices carefully before you mark an unknown answer as correct.

(E) A lace is a part of a shoe, so (E) appears to be a perfectly good answer.

Having found two likely answers, (B) and (E), you must go back to the original pair and determine its other distinguishing characteristics. A brim is a part of a hat, but it is not a necessary part. Not all hats have brims. A lace is a part of a shoe, but it is not a necessary part. Some shoes have buckles and some are slip-ons. A spoke, however, is a necessary part of an umbrella. Furthermore, a brim is a part of a hat, which is wearing apparel. A lace is part of a shoe, which is also wearing apparel. But an umbrella is not something to wear. Thus there are two counts on which to eliminate (B) and to choose (E) as the best answer.

Usually, the problem with analogies is refining the relationship to find the best answer. Sometimes, however, the difficulty will be in finding even one correct answer. If this happens, you may have to redefine the relationship. Consider an analogy which begins LETTER: WORD. You first thought is probably that a letter is part of a word, and so you look for an answer choice that shows a part-to-whole relationship. However, suppose the question looks like this:

Example 2. LETTER: WORD::

(A) Procession: Parde **(B)** Dot: Dash
(C) Whisper: Orate **(D)** Song: Note
(E) Spell: Recite

Answer: The correct answer is (D).

Not one of these choices offers a part-to-whole relationship. Returning to the original pair, you must then consider other relationship between letter and word. If letter is not "letter of the alphabet," but, rather, "written communication," then a word is part of a letter and the relationship becomes that of the whole to its part. Now the answer is immediately clear. A song is the whole of which a note is a part.

Example 3. PILOT: STEER::

(A) Chef: Dine **(B)** Boss: Obey
(C) Lawyer: Retain **(D)** Guard: Protect

Answer: The correct answer is (D)

At first glance several of these answers may seem to work. "A pilot is someone who steers." "A soldier is someone who is commanded." The relationship looks promising, but it's not correct. Ask yourself who is doing what to whom? In the original pair, the pilot is doing something: the pilot is steering. The choices B and C: a boss is someone who is obeyed: a lawyer is someone who is retained (hired). Again, the original grammatical relationship is reversed.

By definition, a pilot is a person who steers. In the same way, by definition, a guard is a person who protects.

Example 4. INTERLOPE: INTRUSION::

(A) Witness: Interrogation (B) Actor: Intermission
(C) Recluse: Interference (D) Mediator: Intercession

Answer: The correct answer is (D).

Again, ask yourself who is doing what to whom. An interloper is a person who butts in or thrusts himself into the business of others. An interloper commits an intrusion; he or she intrudes. A witness, on the other hand, is not the person who conducts the interrogation. A witness is the person who is being interrogated.

You can eliminate choice (A) and any other answer choices in which the original relationship is reversed. The mediator or go-between is the person who acts, trying to reconcile quarrelling parties by means of intercession.

Example 5. CONSTELLATION: STARS::

(A) Prison: Bars (B) Assembly: Speaker
(C) Troupe: Actors (D) Mountain: Peak

Answer: The correct answer is (C)

A constellation is made up of stars. A troupe (not troop but troupe) is made up of actors. Choice C is correct.

Note, by the way, the characteristics of the analogy you have just analyzed, CONSTELLATION: STARS. It is a good analogy. The relationship between the words is built-in; if you look up constellation in a dictionary, you will see that a constellation is a group of stars. The words are related by definition.

Your correct answer choice has got to have the same characteristics as the original pair. The words must have a clear relationship. They must be related by definition. If you substitute them in your test sentence, they must fit it exactly.

Example 6. FISH: TROUT::

(A) Ocean: Wave (B) Mammal: Whale
(C) Bird: Aviary (D) Antenna: Insect

Answer: The correct answer is (B)

A trout is a kind of fish. A whale is a kind of mammal. (Class and Members)

Example 7. DIMMED: LIGHT::

(A) Beached: Texture (B) Muffled: Sound
(C) Measured: Weight (D) Tragrant: Smell

Answer: The correct answer is (B)

Light that is dimmed is lessened in brightness. Sound that is muffled is lessened in volume.

Example 8. DOCTOR: DISEASE::

(A) Moron: Imbecility
(B) Pediatrician: Senility
(C) Psychiatrist: Maladjustment
(D) Broker: Stocks

Answer: The correct answer is (C)

A doctor attempts to treat a disease. A psychiatrist attempts to treat a maladjustment.

Example 9. PATRON: SUPPORT::

(A) Spouse: Divorce
(B) Restaurant: Management
(C) Counselor: Advice
(D) Host: Hostility

answer: The correct answer is (C). A patron by definition provides patronage or support. A counselor by definition provides advice.

Example 10. CLOCK: TIME::

(A) Watch: Wrist (B) Odometer: Speed
(C) Hourglass: Sand (D)Yardstick: Distance

Answer: The correct answer is (D)

A clock measures time. A yardstick measures distance. (Function).

The candidate should know about the different types of analogies that are more frequently asked in the question paper. Some of the common analogy types are as follows:

i. Synonyms:

Some words are linked together in a pair which means the same or has a similar dictionary definition e.g., Large-Big.

ii. Describing Qualities:

Some pairs have some words in which one word describes the other word, e.g., Hot-Iron.

iii. Class and Member:

Some pairs have words which are based on class and member basis, e.g., Vehicle-Car

iv. Antonyms:

Some pairs consist of the words that are opposite to each other, e.g., Confess-Deny.

v. Describing Intensity:

Some pairs consist of the words in which one describes the intensity of the other, e.g., Anger-Rage (violent anger).

vi. Function:

In some pairs, a word describes the function of the other word, e.g., Football-Pay.

vii. Manners:

Some words in a speech describe the manners and behavior e.g., Weep-Bitterly.

viii. Worker-Workplace

Some pairs in a word describe the profession and its workplace, e.g., Teacher-Class.

Multiple Choice Questions (MCQs)

TEST-1

Directions: In the following questions, select the pair of words given against *(A)*, *(B)*, *(C)* or *(D)* which has the same relation as between the first two words.

1. ***VERITY: CASUISTRY::***
   - (A) Egalitarian: Equality
   - (B) Sweet: Mellifluous
   - (C) Constant: Capricious
   - (D) Milk: Cream
2. ***NAIVE: INGENUE::***
   - (A) Ordinary: Genius
   - (B) Venerable: Celebrity
   - (C) Urbane: Sophisticate
   - (D) Crafty: Artisan
3. ***HORSE: CORRAL::***
   - (A) Oyster: Reef
   - (B) Dog: Muzzle
   - (C) Sheep: Flock
   - (D) Pig: Sty
4. ***PROOF: ALCOHOL::***
   - (A) Cream: Milk
   - (B) Canteen : Water
   - (C) Tanker: Oil
   - (D) Octane: Gasoline
5. ***APOSTATE: RELIGION::***
   - (A) Potentate: Kingdom
   - (B) Traitor: Country
   - (C) Bureaucrat: Government
   - (D) Jailer: Law
6. ***ANIMAL : MONKEY::***
   - (A) Zebra : Giraffe
   - (B) Stationery : Pencil
   - (C) Book: Cap
   - (D) Tree: Wood
7. ***MARATHON: STAMINA::***
   - (A) Relay: Independence
   - (B) Hurdle: Perseverance
   - (C) Sprint: Celerity
   - (D) Jog: Weariness
8. ***DESCRY: DISTANT::***
   - (A) Mourn: Lost
   - (B) Whisper: Muted
   - (C) Discern: Subtle
   - (D) Destroy: Flagrant
9. ***FOX: CUNNING::***
   - (A) Dog: Playful
   - (B) Hyena: Amusing
   - (C) Beaver: Industrious
   - (D) Vixen: Cute
10. ***HOUSE : BIG::***
    - (A) Home : Live
    - (B) School : Daily

(C) Water : Cold (D) Clothes : Socks

11. *PRIDE : LIONS::*
(A) Gaggle : Geese (B) Honor: Thieves
(C) Snarl: Wolves (D) Arrogance: Kings

12. *BIBULOUS: DRINK::*
(A) Rapacious: Clothing (B) Gluttonous: Food
(C) Altruistic: Money (D) Vegetarian: Meat

13. *IMPROMPTU: REHEARSAL::*
(A) Practiced: Technique (B) Makeshift: Whim
(C) Offhand: Premeditation (D) Glib: Fluency

14. *HEAVY : LIGHT*
(A) Fat : Thin (B) Stupid : Idiot
(C) Rough: Surface (D) Beautiful: Diary

15. *HERMIT: GREGARIOUS::*
(A) Miser: Penurious (B) Ascetic: Hedonistic
(C) Coward: Pusillanimous (D) Scholar: Literate

16. *BELLWETHER: BAROMETER::*
(A) Proselyte: Spark plug (B) Panhandler: Kill
(C) Embezzler: Abduct (D) Cynosure: Magnet

17. *PERJURY: OATH::*
(A) Plagiarism: Authority (B) Embezzlement: Trust
(C) Disrespect: Age (D) Testimony: Court

18. *ELISION: SYLLABLES::*
(A) Contraction: Letters (B) Thesis: Ideas
(C) Diagnosis: Symptoms (D) Almanac: Facts

19. *OBLIVIOUS: AWARENESS::*
(A) Comatose: Consciousness (B) Serene: Composure
(C) Erudite : Knowledge (D) Adroit: Skill

20. *DIDACTIC: TEACH ::*
(A) Sophomoric: Learn (B) Satiric: Mock
(C) Reticent: Complain (D) Chaotic : Rule

21. *ACT : ACTION::*
(A) Therapy : Thermometer (B) Oblivion: Obvious
(C) Liturgy: Literature (D) Image: Imagine

22. *SONG: RECITAL::*
(A) Author: Bibliography (B) Episode: Series
(C) Coach: Team (D) Dancer: Agile

23. *MENDACITY: HONESTY ::*
(A) Courage: Cravenness (B) Truth: Beauty
(C) Courage: Fortitude (D) Unsophistication: Ingenuousness

24. *HEIGHT: MOUNTAIN::*
(A) Depth: Trench (B) Shade: Tree
(C) Weight: Age (D) Speed: Highway

25. *DETRITUS: GLACIER::*
(A) Thaw: Snowfall (B) Snow: Ice Cap
(C) Silt: River (D) Range: Mountain

TEST-2

Directions: In the following questions, select the pair of words given against *(A), (B), (C), (D), (E)* which has the same relation as between the first two words:

1. ***Oasis: Desert***
(A) Lunch: Dessert (B) Inveigh: Inveigle
(C) Forest: Jungle (D) Ocean: Bay
(E) Affluence: Poverty

2. *Sad: Morose*
   (A) Robust: Weak (B) Ingress: Egress
   (C) Grass: Food (D) Glad: Happy
3. *Work: Tired*
   (A) Player: Field (B) Swim: Pool
   (C) Race: Fatigue (D) Book: Knowledge
   (E) Cook: Eat
4. *Thin: Sparse*
   (A) Tract: Tome (B) Prologue: Epilogue
   (C) Preface: Book (D) Tree: Tall
   (E) Corpulent: Obese
5. *Sad: Melancholic*
   (A) Over: Under (B) Joy: Ecstasy
   (C) Weak: Robust (D) Book: Writer
   (E) Floor: Ceiling
6. *Flurry: Confusion*
   (A) Water: Thirst (B) Night: Sleep
   (C) Colour: Attraction (D) Job: Pay
   (E) Intimidate: Fear
7. *Turbid: Muddy*
   (A) River: Lake (B) Tree: Darkness
   (C) Bell: Ring (D) Huge: Colossal
   (E) Deep: Shallow
8. *Fresh: New*
   (A) Disease: Malaise (B) Supercilious: Meek
   (C) Epical: Humorous (D) Indigent: Affluent
   (E) Strident: Polite
9. *Wheel: Hub*
   (A) Sea: Island (B) Body: Heart
   (C) Ruling Party: Cabinet (D) Life: Happiness
   (E) Watch: Needle
10. *Speech: Peroration*
   (A) Dinner: Dessert (B) Country: Boundary
   (C) Argument: Conclusion (D) Style: Debate
   (E) Money: House
11. *Death: Lament*
   (A) Impose: Fine (B) Cast: Vote
   (C) Make: Furniture (D) Celebrate: Centenary
   (E) Learn: Lesson
12. *Virus: Disease*
   (A) Discussion: Fight (B) Desire: Success
   (C) Suggestion: Acceptance (D) Bombardment: Destruction
   (E) Clothes: Gentleman
13. *Foot: Toe*
   (A) Body: Legs (B) Belly: Intestines
   (C) Nail : Fingers (D) Hand : Fingers
   (E) Leaves: Tree
14. *Zenith: Nadir*
   (A) Serious: Sober (B) Food: Hungry
   (C) Fat : Protein (D) Majestic: Sublime
   (E) Peak: Foot
15. *Giggle: Chortle*
   (A) Smile: Grin (B) Melancholy: Antipathy
   (C) Emancipation: Bondage (D) Insipid: Charming

(E) Edifice: Facade

16. *Immaculate: Spotless*
(A) Fastidious: Punctilious (B) Careless: Carefree
(C) Mitigate: Pain (D) Allay: Aggravate
(E) Ally: Lesson

17. *Fever: Flush*
(A) Liquid: Gas (B) Malaria: Shiver
(C) Wings: Flap (D) Rehearsal: Drama
(E) Cough: Cough

18. *Pistol: Shoot*
(A) Stone: Throw (B) Catechism: Church
(C) Ball: Goal (D) Shaft: Fling
(E) Sword: Cut

19. *Prelude: Symphony*
(A) Drama: Epilogue (B) Epilogue: Prologue
(C) Meal: Dessert (D) Ticket: Show
(E) Preamble: Constitution

20. *Chains: Clang*
(A) Leaves: Twigs (B) Coins: Jingle
(C) Door: Lock (D) Wolf: Roar
(E) Wire: Snap

21. *Loud: Blatant*
(A) High: Low (B) Circle: Oval
(C) Penchant: Trenchant (D) Noisome: Fetid
(E) Good: Bad

22. *Earth: Sun*
(A) Venus: Mars (B) Pluto: Moon
(C) Sun: Galaxy (D) Sun: Star
(E) Universe: Pole-Star

23. *Room: Cell*
(A) Fort: Battlements (B) Arcade: Arbour
(C) Hall: Room (D) Building: Office
(E) Bedroom: Bed

24. *Coal: Mine*
(A) Rice: Husk (B) Heart: Body
(C) Seed: Pod (D) Marble: Quarry
(E) Paper: Pulp

25. *Lion: Pride*
(A) Tiger: Strength (B) Wolf: Cub
(C) Building: Height (D) Fish: School
(E) Book: Study

26. *Voice: Aphasia*
(A) Hearing: Ear (B) Nose: Smell
(C) Sleep: Somnambulism (D) Hunger: Starvation
(E) Sight: Blindness

27. *Swan: Cygnet*
(A) Horse: Filly (B) Elephant: Goad
(C) Pig: Sty (D) Tree: Forest
(E) Leaf: Flower

28. *Reaper: Scythe*
(A) Surgeon: Scalpel (B) Barber: Hair
(C) Butcher: Sheep (D) Batsman: Bat

(E) Writer: Ink

29. *Hassock: Kneeling*
(A) Kitchen: Cooking (B) Bedroom: Sleeping
(C) Field: Running (D) Chair: Sitting
(E) Hearth: Cooking

30. *Barometer: Pressure*
(A) Humidity: Hygrometer (B) Blood Circulation: Stethoscope
(C) Height: Pole (D) Sea: Wave
(E) Height: Depth

31. *Bugle: Sound*
(A) Horn: Head (B) Horn: Blow
(C) Flute: Hole (D) Flute: Blow
(E) Drum: Hit

32. *Say: Assert*
(A) Request: Implore (B) Tell: Narrate
(C) Paint: Describe (D) Tea: Coffee
(E) Solve: Sum

33. *Grain: Skin*
(A) Wheat: Barley (B) Wood: Table
(C) Wood: Grain (D) Book: Wrapper
(E) Book: Alphabet

34. *Staircase: Degrees*
(A) Ladder: Rung (B) House: Storey
(C) Tree: Trunk (D) Table: Legs
(E) Book: Leaves

35. *Mascot: Ominous*
(A) Auspices: Auspicious (B) Loss: Defeat
(C) Game: Player (D) Beginning: Introduction
(E) Victory: Defeat

36. *Book: Leaves*
(A) Table: Legs (B) Teacher: Books
(C) Rosary: Beads (D) Country: Government
(E) Classroom: Desks

37. *Peacock: Pride*
(A) Goat: Kid (B) Man: Manikin
(C) Lion: Bravery (D) Monkey: Man
(E) Loan: Debt

38. *Painter: Brush*
(A) Student: Paper (B) Sculptor: Chisel
(C) Farmer: Field (D) Writer: Ink
(E) Doctor: Medicine

39. *Wool: Shear*
(A) Bicycle: Pedal (B) Fan: Switch
(C) Tree: Planted (D) Feather: Pluck
(E) Bell: Ring

40. *Wool: Cashmilon*
(A) Sheep: Grass (B) Cloth: Dress
(C) Wood: Furniture (D) Paper: Bamboo
(E) Butter: Margarine

41. *Slip: Hurt*
(A) Neglect: Punishment (B) Plant: Tree
(C) Hard Work: Deprecate (D) Stand: Walk
(E) Cry: Pain

42. ***Society: Ostracise***

(A) Family: Member (B) Zoo: Animal
(C) Judge: Order (D) Government: Exile

43. ***Wolf: Cub***

(A) Old Man: Young Man (B) Forest: Tree
(C) Parliament: Cabinet (D) Swan: Cygnet
(E) Book: Booklet

44. ***Speak: Shout***

(A) Whimper: Bang (B) Silence: Whisper
(C) Gallop: Stop (D) Eat: Digest
(E) Word: Sentence

45. ***Friend: Love***

(A) Enemy: Hate (B) Woman: Marry
(C) Officer: Defied (D) Leader: Hear
(E) Neighbour: Talk

46. ***Cool: Liked***

(A) Cold: Disliked (B) Hot: Welcomed
(C) Truth: Hidden (D) Weather: Pleasant
(E) Question: Solved

47. ***Expand: Expensive***

(A) Expand: Expansive (B) Deceive: Deception
(C) Defend: Intrusive (D) Think: Pensive
(E) Sensitive: Obstrusive

48. ***Watch: Wrist***

(A) Clock: Time (B) Room: Sitting
(C) Success: Work (D) Ornaments: Decoration
(E) Grain: Field

49. ***Bulb: Socket***

(A) Pen: Inkpot (B) Necklace: Neck
(C) Foot: Knee (D) Eye: Socket
(E) Hair: Scalp

50. ***Aeroplane: Pilot***

(A) House: Mason (B) School: Principal
(C) Assembly: Speaker (D) Car: Chauffeur
(E) Machine: Engineer

ANSWERS

1.	(E)	2.	(D)	3.	(C)	4.	(E)	5.	(B)
6.	(E)	7.	(D)	8.	(A)	9.	(C)	10.	(C)
11.	(D)	12.	(D)	13.	(D)	14.	(E)	15.	(A)
16.	(A)	17.	(B)	18.	(D)	19.	(E)	20.	(B)
21.	(D)	22.	(C)	23.	(C)	24.	(D)	25.	(D)
26.	(E)	27.	(A)	28.	(A)	29.	(D)	30.	(A)
31.	(B)	32.	(A)	33.	(C)	34.	(A)	35.	(E)
36.	(C)	37.	(C)	38.	(B)	39.	(D)	40.	(E)
41.	(A)	42.	(D)	43.	(D)	44.	(A)	45.	(A)
46.	(A)	47.	(A)	48.	(B)	49.	(D)	50.	(D)

SYNONYM/ANTONYM TESTS

Directions. In the following questions, each word is followed by four responses *a, b, c* and *d*. You should choose the correct synonym of the given word out of the four choices.

1. ***Abandon :***
   (A) vacate (B) foil
   (C) lose (D) gain
2. ***Abdicate :***
   (A) give up (B) imperious
   (C) rude (D) dissent
3. ***Abhor :***
   (A) crave (B) reconcile
   (C) detest (D) rude
4. ***Abnegation :***
   (A) indulgence (B) rejection
   (C) complete (D) final
5. ***Axiom :***
   (A) absurdity (B) shirk
   (C) elude (D) maxim
6. ***Bloated :***
   (A) privileged (B) emaciated
   (C) swollen (D) rapture
7. ***Blemish :***
   (A) disgrace (B) eccentric
   (C) young (D) fair
8. ***Bizzare :***
   (A) normal (B) strange
   (C) logical (D) tense
9. ***Bawl :***
   (A) mulberry (B) mutter
   (C) vociferate (D) daub
10. ***Bequeath :***
    (A) alienate (B) stab
    (C) obstruct (D) dispose of
11. ***Cataclysm :***
    (A) reverse (B) upheavel
    (C) pungent (D) trash
12. ***Cupidity :***
    (A) extravagance (B) shrewd
    (C) complaisant (D) avarice
13. ***Cumbersome :***
    (A) awkward (B) analyse
    (C) decay (D) grow
14. ***Culmination :***
    (A) nadir (B) apex

(C) lenient (D) blast

15. ***Crestfallen :***
(A) humiliated (B) hard
(C) elite (D) frustrated

16. ***Deluge :***
(A) flood (B) object
(C) annihilate (D) restore

17. ***Dereliction :***
(A) attention (B) neglect
(C) divorce (D) restore

18. ***Derogate :***
(A) exaggerate (B) calm
(C) deflenerate (D) ordain

19. ***Dormant :***
(A) active (B) vigilant
(C) warbling (D) inert

20. ***Don :***
(A) doff (B) assume
(C) pine (D) blithe

21. ***Exemplify :***
(A) reprehensible (B) illustrate
(C) empty (D) finish

22. ***Entramel :***
(A) hamper (B) extricate
(C) apathy (D) amuse

23. ***Emendation :***
(A) irritant (B) stoicism
(C) barren (D) rectification

24. ***Elude :***
(A) evade (B) rule
(C) sway (D) fascinate

25. ***Exudation :***
(A) aridity (B) ecstasy
(C) percolation (D) horrid

26. ***Frivolous :***
(A) serious (B) trivial
(C) peculiar (D) candid

27. ***Furtive :***
(A) public (B) open
(C) secretive (D) combine

28. ***Fugitive :***
(A) escaping (B) enduring
(C) vain (D) weak

29. ***Fulminate :***
(A) clamour (B) misfire
(C) barren (D) prodigal

30. *Frugal :*
(A) prolific (B) efficacious
(C) clamour (D) thrifty

31. *Guile :*
(A) candour (B) wander
(C) deceit (D) sense

32. *Grotesque :*
(A) ordinary (B) archaic
(C) usual (D) crawl

33. *Gluttony :*
(A) frugality (B) torment
(C) voracity (D) thwart

34. *Grardiloquence :*
(A) simplicity (B) wanton
(C) shear (D) verbosity

35. *Grief :*
(A) delight (B) sorrow
(C) redundancy (D) beauty

Directions. In the following questions, each word is followed by four responses *a, b, c* and *d*. You should choose the correct antonym of the given word out of the four choices.

36. *Abbreviate :*
(A) abridge (B) expand
(C) achieve (D) accept

37. *Abjure :*
(A) acknowledge (B) disown
(C) deny (D) hate

38. *Absurd :*
(A) scarcity (B) adorn
(C) rational (D) flourish

39. *Abundant :*
(A) ample (B) enough
(C) great (D) scant

40. *Acerbity :*
(A) Sweat (B) gentleness
(C) tasty (D) account

41. *Blooming :*
(A) Fading (B) flowering
(C) quiet (D) mild

42. *Bliss :*
(A) merge (B) disseminate
(C) dull (D) suffering

43. *Bewitch :*
(A) disenchant (B) rapture
(C) profit (D) avail

44. *Bauble :*
(A) plaything (B) valuable
(C) modest (D) besiege

45. ***Busy :***
(A) bully (B) curb
(C) indolence (D) occupied

46. ***Celebrity :***
(A) celebration (B) solemnity
(C) obscurity (D) hazy

47. ***Cataclysm :***
(A) peace (B) deluge
(C) quibble (D) conjecture

48. ***Concord :***
(A) consolidate (B) bestow
(C) outline (D) discord

49. ***Compliance :***
(A) condone (B) clamour
(C) resistance (D) condense

50. ***Circumlocution :***
(A) amass (B) ambiguity
(C) effusion (D) simplicity

51. ***Despicable :***
(A) worthy (B) contemptible
(C) earn (D) purity

52. ***Ductile :***
(A) indistinct (B) vague
(C) inelastic (D) quiescent

53. ***Dormant :***
(A) Inert (B) doff
(C) indulgence (D) active

54. ***Dissipate :***
(A) accumulate (B) distil
(C) percolate (D) emanate

55. ***Disdain :***
(A) haughtiness (B) erroneous
(C) respect (D) contempt

56. ***Ephemeral :***
(A) transient (B) perpetual
(C) disencumber (D) demote

57. ***Erudition :***
(A) ignorance (B) evict
(C) scholarship (D) dubious

58. ***Exultation :***
(A) ecstasy (B) frugality
(C) mourning (D) severe

59. ***Expunge :***
(A) erase (B) dilate
(C) entity (D) imprint

60. ***Extenuate :***
(A) palliate (B) quality
(C) enhance (D) offhand

61. *Flux :*
(A) stillness (B) motion
(C) swerve (D) wince

62. *Fatigue :*
(A) vitality (B) weariness
(C) solemn (D) enchant

63. *Fume :*
(A) frown (B) chafe
(C) comply (D) dupe

64. *Fugitive :*
(A) evanescent (B) captive
(C) unkempt (D) smart

65. *Fulminate :*
(A) murmur (B) clamour
(C) efficacious (D) vain

66. *Gallant :*
(A) bold (B) fine
(C) frolic (D) coward

67. *Garrulity :*
(A) reticence (B) gaudy
(C) superb (D) abettor

68. *Generous :*
(A) magnanimous (B) visual
(C) vivid (D) stingy

69. *Genuine :*
(A) real (B) voracious
(C) spurious (D) insincere

70. *Germinate :*
(A) shoot (B) sprout
(C) alien (D) decay

ANSWERS

1	(A)	2	(A)	3	(C)	4	(B)	5	(D)
6	(C)	7	(A)	8	(B)	9	(C)	10	(D)
11	(B)	12	(D)	13	(A)	14	(B)	15	(D)
16	(A)	17	(B)	18	(C)	19	(D)	20	(B)
21	(B)	22	(A)	23	(D)	24	(A)	25	(C)
26	(B)	27	(C)	28	(A)	29	(A)	30	(D)
31	(C)	32	(B)	33	(C)	34	(D)	35	(B)
36	(B)	37	(A)	38	(C)	39	(D)	40	(B)
41	(A)	42	(D)	43	(A)	44	(B)	45	(C)
46	(C)	47	(A)	48	(D)	49	(C)	50	(D)
51	(A)	52	(C)	53	(D)	54	(A)	55	(C)
56	(B)	57	(A)	58	(C)	59	(D)	60	(C)
61	(A)	62	(A)	63	(C)	63	(B)	65	(A)
66	(D)	67	(A)	68	(D)	69	(C)	70	(D)

Part 3 Critical Reading

COMPREHENSION

Introduction:

Comprehension means the act of comprehending or the capacity of the mind to understand. In the examination papers, questions on comprehension test are included to judge the ability of the students to understand the given passage.

In the English language paper, questions on comprehension test are very important for the students appearing in the competitive examinations. Therefore, they should try to learn how to solve these questions. Practice of solving these questions will greatly help them in the examination.

LONG PASSAGE COMPREHENSION
PASSAGES WITH EXPLANATORY ANSWERS

PASSAGE – 1

The Romans – for centuries is the masters of war and politics across Europe, Northern Africa, and Asia Minor – have often been criticized for producing few original thinkers outside the realm of politics. This criticism, while in many ways true, is not without its problems. It was, after all, the conquest of Greece that provided Rome with its greatest influx of educated subjects. Two of the great disasters in intellectual history – the murder of Archimedes and the burning of Alexandria's library – both occurred under Rome's watch. Nevertheless, a city that was able to conquer so much of the known world could not have been devoid of the creativity that characterizes so many other ancient empries.

Engineering is one endeavour in which the Romans showed themselves capable. Their aqueducts carried water hundreds of miles along the tops of vast arcades. Roman roads built for the rapid deployment of troops, criss-cross Europe and still form the basis of numerous modern highways that provide quick access between many major European and African cities. Indeed, a large number of these cities owe their prominence to Rome's economic and political influence.

Many of those major cities lie for beyond Rome's original province, and Latin-derived languages are spoken in most Southern European nations. Again a result of military influence, the popularity of Latin and its off spring is difficult to overestimate. During the centuries of ignorance and violence that followed Rome's decline, the Latin language was the glue that held together the identity of an entire continent. While seldom spoken today, it is still studied widely, if only so that such master or rhetoric as Cicero can be read in the original.

It is Cicero and his like who are perhaps the most overlooked legacy of Rome. While far from being a democracy, Rome did leave behind useful political tool that serve the American republic today. "Republic" itself is Latin for "the people's business," a notion cherished in democracies worldwide. Senators owe their name to Rome's class of elders; Representatives owe theirs to the Tributes who seized popular prerogatives from the Senatorial class. The veto was a Roman notion adopted by the historically aware framers of the Constitution, who often assumed pen names from the lexion of Latin life. These accomplishments, as monumental as any highway or coliseum, remain prominent features of the Western landscape.

Multiple Choice Questions (MCQs)

1. The author describes "two of the great disasters in intellectual history" in order to
   - (A) establish a point directly related to the main argument
   - (B) show that certain historical claims are inaccurate
   - (C) demonstrate the importance of certain historical data
   - (D) disprove the claims made by other with a different view
   - (E) concede the partial accuracy of an opposing view
2. According to the passage, ancient Roman roads
   - (A) connected many major cities in ancient Europe
   - (B) are engineering marvels unequalled in modern times
   - (C) are similar in some respects to modern highways

(D) were products of democratic political institutions
(E) caused the development of modern European cities

3. According to the passage, which of the following accurately describes the Latin language?
I. It spread in part due to Rome's military power
II. It is reflected in modern political concepts
III. It is spoken today in some parts of Europe
(A) I only (B) II only
(C) I and II only (D) I and III only
(E) II and III only

4. It can be inferred from the passage that the framers of the Constitution
(A) were familiar with certain aspects of Roman government
(B) were similar to the Roman elders
(C) embraced the veto as the hallmark of Roman democracy
(D) overlooked Cicero's contributions to the theory of democracy
(E) formed a government based on world-wide democracy

5. The primary purpose of the passage is to
(A) reveal the indifferent attitude taken by the ancient Romans toward the fine arts
(B) discuss the lasting accomplishments achieved by ancient Romans
(C) analyse the use of the Latin language by the framers of the Constitution
(D) show that the construction of roads and aqueducts could not have been accomplished in ancient Greece
(E) Compare the destruction of the library at Alexandria to the murder of Archimedes

6. Which of the following is NOT described in the passage as a part of ancient Roman life that left a lasting legacy?
(A) The Latin languages
(B) Military accomplishments
(C) An extensive system of roads
(D) A democratic system of government
(E) Wide-ranging economic influence

Explanatory Answers

Ans. 1 (E). This specific question has a line number. Remember to read a bit above and below the cited lines. The two disasters citied are mentioned to give an example of Rome's failings, before countering with a number of Rome's successes. The best answer was choice (E).

Ans. 2 (A). The specific question also has a good lead phrase : "Roman roads." You'll find it in the second paragraph. Choice (C) is wrong because these roads are not just similar to modern highways – they form the basis of these highways. Choice (E) is a bit too extreme. The cities may owe their prominence to these roads, but they were not "caused" by the roads. The correct answer is choice (A).

Ans. 3 (C). This is a specific question with a good lead phrase: "Latin language." You'll find it in the third paragraph. The correct answer is choice (C).

Ans. 4 (A). The specific question also has a good lead phrase: "framers of the Constitution". You'll find it in the last paragraph. You may have been to choices (A) and (C). Choice (C) used extreme language and went a bit too far. Great as the veto is, did the founding fathers consider it the "hallmark" of democracy? The passage doesn't say so. The correct answer was choice (A).

Ans. 5 (B). The first paragraph puts forth the idea that, despite criticism levelled against it, ancient Rome had many lasting accomplishments. The following three paragraphs give examples of these accomplishments.

Ans. 6 (D). In the fourth paragraph, it is explicitly stated that despite its political innovations, Rome was "far from being a democracy." All of the other choices are touched upon somewhere in the passage.

PASSAGE 2

Anthropologists who study orangutans, distant cousins of the human race, find in the animals behaviour hints of how our earliest ancestors may have lived. It has long been accepted that primates originally dwelt in the treetops and only migrated to the ground as forests began to dwindle. While to a certain extent, all primates except humans spend at least some time dwelling in trees, the orangutans can grow as heavy as 330

pounds and live for decades, requiring copious amounts of fruit simply to stay alive. Thus, they become very jealous of the territory where they find their food. Compounding this territoriality are the breeding habits of orangutans, since females can only breed every few years and, like humans, give birth not to litters but single off-spring.

Consequently, orangutans are solitary, territorial animals who have difficulty foraging in any part of the forest where they were not raised. Orangutans take from poachers by customs agents undergo incredible hardship on their return to the wild. Incorrectly relocating a male orangutan is especially problematic, often ending in the animal's death at the hands of a rival who sees not only his territory but also the females of his loosely knit community under threat from an outsider. While humans, like chimpanzees, are more gregarious and resourceful than orangutans, the latter provide anthropologists with useful information about the behaviour of prehominid primates and how apelike behaviour influenced out ancestors' search for the food and family beneath the forest's canopy.

Multiple Choice Questions (MCQs)

1. The primary purpose of this passage is to
   - **(A)** Describe some behavioural and evolutionary characteristics of orangutans
   - **(B)** Analyse the reasons why early primates left their forest dwellings
   - **(C)** Illustrate the dangers posed to orangutans by poachers
   - **(D)** Show how orangutan behaviour differs from that of other primates
   - **(E)** Criticise anthropologists who misinterpret orangutan behaviour
2. The author of the passage discusses "orangutans taken from poachers" in order to
   - **(A)** Stress the importance of preserving orangutans as a species
   - **(B)** Indicate the widespread practice of animal poaching
   - **(C)** Refute the theory that orangutans can live in a variety of environments
   - **(D)** Contrast the behaviour of orangutans with that of other apes
   - **(E)** Emphasize the consequences of orangutan territoriality
3. The passage indicates that it is difficult to return orangutans to the wild for which of the following reasons?
   - A. The threat posed by new comers to other orangutans' territory
   - B. The conflict between males over available females
   - C. The scarcity of available food in the orangutan's environment
   - **(A)** A only **(B)** A and B only
   - **(C)** A and C only **(D)** B and C only
   - **(E)** A, B, and C.
4. Which of the following can be inferred about differences between the behaviour of orangutans and that of other ape species?
   - **(A)** While orangutans spend much of their time on the treetops, other apes live exclusively on the ground
   - **(B)** Orangutans and other types of apes are all sociable species, but orangutans are more likely to bond for life
   - **(C)** Apes such as chimpanzees rely less upon their size than the average orangutans do
   - **(D)** Orangutans spend less time in the company of their members of their species than do some other apes
   - **(E)** Because of their stringent territoriality, orangutans are less likely to elude capture by poachers than are other apes
5. According to the author, anthropologists study the behaviour of orangutans in order to
   - **(A)** Prevent orangutans from becoming the target of poaching
   - **(B)** Assist customs agents in the relocation of orangutans
   - **(C)** Analyse the causes and consequences of contemporary human behaviour
   - **(D)** Prevent larger orangutans from eliminating their weaker rivals
   - **(E)** Better understand the factors that influenced human evolution
6. Which of the following are factors that the author indicates contribute to the orangutan's territoriality?
   - **(A)** The lack of available food and the antisocial nature of orangutans

(B) The orangutan's need for large quantities of food and the infrequency with which it mates
(C) The threat posed by poachers and the orangutan's inability to protest itself from them
(D) The difficulties that orangutans face when compelled to socialize with other species such as chimpanzees
(E) The constant dangers that present themselves whenever one orangutan encounters another

7. It can be inferred from the passage that one development responsible for the evolution of distinct ape species was
(A) Early primates inability to survive in the forest
(B) The shrinking of the available primitive forest
(C) The growth of human and chimpanzee communities
(D) The orangutan's eventual dominance of the treetops
(E) The encroachment of other species into the primitive forest

Explanatory Answers

Ans. 1 (A). The answer to this general question came from understanding the main idea. The passage did not analyse the reasons primates left trees (B), or devote itself to a discussion of poachers (C), or do a point-by-point comparison of orangutans with other primates (D), or criticize anthropologists (E). The correct answer is choice (A).

Ans. 2 (E). The specific question has a line number. Remember to read a bit above and below the cited lines. The answer to this question actually came just *below* the quote. The poacher example is simply a further illustration of orangutans' territorial nature. The correct answer is choice (E).

Ans. 3 (B). Both statements I and II were mentioned, statement III was not. The correct answer is choice (B).

Ans. 4 (D). This is a specific question with no line number, and, really, no lead word. We're looking for differences between orangutans and other types of apes. The only other types of monkey mentioned is the chimpanzee who is said to be more gregarious. The correct answer is choice (D).

Ans. 5 (E). The specific question also has a good lead word: "anthropologists." It is found in two places, at the very beginning and the very end of the passage. You were probably down to choices (C) and (E). Why was (C) wrong? Because anthropologists only see parallels with *early* man.

Ans. 6 (B). The specific question also has a good lead word: "Territoriality," you'll find it in the second half of the first paragraph, which discusses two cases: the need for large amounts of food, and breeding habits. You were probably down to (A) or (B). Why was (A) wrong? It didn't discuss both food and breeding habits. Choice (B) was correct.

Ans. 7 (B). The second sentence of the first paragraph ends, ... "only migrated to the ground as forests began to dwindle." That gives us choice (B).

NOTE

This is a science passage. Paragraph one says the orangutan, studied by scientists for its resemblance to early humans, lives in trees and is very territorial. Paragraph two describes the orangutan's solitary territorial behaviour, which resembled that of early humans.

PASSAGE 3

Though the U.S. prides itself on behing a leader in the world community, a recent report shows that it lags far behind other industrialized countries in meeting the needs of its youngest and most *vulnerable* citizens. The U.S. has a higher infant mortality rate, a higher proportion of low birth weight babies, a smaller proportion of babies immunized against childhood diseases and a much higher rate of adolescent pregnancies. These findings, described as a "quiet crisis" requiring immediate and far-reaching action, appeared in a report prepared by a task force of educators, doctors, politicians and business people. According to the report, a fourth of the nation's 12 million infants and toddlers live in poverty. As many as half *confront* risk factors that could harm their ability to develop intellectually, physically and socially. Child immunizations are too low, more children are born into poverty, more are in *substandard* care while their parents work and more are being raised by single parents. When taken together, these and other risk factors can lead to educational and health problems that are much harder and more costly to reverse.

The crisis beings in the womb with unplanned parenthood. Women with unplanned pregnancies are less likely to seek pre-natal care. In the U.S., 80% of teenage pregnancies and 56% of all pregnancies are unplanned. The problems continue after birth where unplanned pregnancies and *unstable* partnerships often go

hand in hand. Since 1950, the number of single parent families has nearly tripled. More than 25 percent of all births today are to unmarried mothers. As the number of single parent families grows and more women enter the work force, infants and toddlers are increasingly in the care of people other than their parents.

Most disturbingly, recent statistics show that American parents are increasingly neglecting or *abusing* their children. In only four years from 1987-1991, the number of children in foster care increased by over 50 percent. Babies under the age of one are the fastest growing category of children entering foster care. The crisis affects children under the age of three most *severely*, the report says. Yet, it is this period-from infancy through preschool years-that sets the stage for a child's future.

Multiple Choice Questions (MCQs)

1. The main focus of the passage is on the plight of
   (A) Low birth weight babies (B) Unwed mothers
   (C) Orphaned children (D) Teenage mothers
   (E) None of these
2. Children falling in which age group are most severely affected by the 'quiet crisis'?
   (A) Between 2 & 3 years (B) Between 1 & 3 years
   (C) Below 1 year (D) Below 3 years
   (E) None of these
3. Which of the following does not constitute the 'quiet crisis' in the U.S. as per the task force report?
   (A) Lower rate of babies surviving childhood diseases
   (B) Larger proportion of babies who are deprived of immunization
   (C) Lower proportion of new born babies with normal weight
   (D) higher incidence of adolescent girls becoming mothers
   (E) Increasing cases of teenage couples getting divorced
4. Which of the following statements is not true in the context of the passage?
   (A) In the U.S., the number of infants living in poverty is about 3 million
   (B) Only 20 percent of all the pregnancies in the U.S. are planned
   (C) The number of single-parent families today is approximately three times that of four decades ago
   (D) The number of children in the U.S. entering foster care has decreased after 1991
   (E) About 6 million infants in the U.S. are likely to develop educational and health problems
5. The number of children born to married mothers in the U.S. is approximately how many times the number of children born to unwed mothers?
   (A) 3 times (B) 3.5 times
   (C) 1.5 times (D) 2 times
   (E) Not mentioned in the passage
6. Children born out of unplanned pregnancies are highly vulnerable because
   (A) They are mostly malnourished
   (B) They are less likely to receive parental care
   (C) They are raised by single parents
   (D) Their parents are mostly poor
   (E) Their parents are emotionally immature
7. Decide which of the following factors is/are responsible for the physical, intellectual and social under-development of infants in the U.S.?
   (A) Illiteracy of parents (B) Lack of parental care
   (C) Poverty
   (A) Only A (B) Only B
   (C) Only C (D) Both A & C
   (E) Both B & C
8. An increasing number of infants in the U.S. are in the foster care on account of
   (A) An increasing number of employed couples who are required to stay apart
   (B) An increasing number of women getting divorced and abandoning their babies
   (C) An increasing number of single parent families with the female member working

(D) An increasing number of women maintaining the status of unwed motherhood and becoming economically independent
(E) An increasing number of parents who lack awareness about baby-care

9. The task force report seems to be based on the data pertaining to the period
(A) 1987 onwards till date (B) 1950-91
(C) 1987-91 (D) 1950 onwards till date
(E) 1991 onwards till date

Directions (Q 10-12): Choose the word which is most nearly the same in meaning as the given word as used in the passage.

10. Confront
(A) Succumb (B) Eliminate
(C) Face (D) Tolerate
(E) Oppose

11. Vulnerable
(A) Risky (B) Promising
(C) Insecure (D) Indispensable
(E) Delicate

12. Abusing
(A) Ill treating (B) Accusing
(C) Cursing (D) Beating
(E) Oppressing

Directions (Q 13-15): Choose the word which is most opposite in meaning to the given word as used in the passage.

13. Severely
(A) Minutely (B) Normally
(C) Drastically (D) Intensely
(E) Slightly

14. Unstable
(A) Stagnant (B) Confined
(C) Changing (D) Steady
(E) Constant

15. Substandard
(A) Excellent (B) Valuable
(C) Impoverished (D) Compassionate
(E) Beneficial

ANSWERS

1.	(A)	2.	(D)	3.	(E)	4.	(D)
5.	(A)	6.	(B)	7.	(E)	8.	(C)
9.	(B)	10.	(C)	11.	(E)	12.	(A)
13.	(E)	14.	(D)	15.	(B)		

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SHORT PASSAGE COMPREHENSION

Learn by Example

Read the passage carefully once and detect its theme. Note what the questions are about. Answer the questions.

PASSAGE WITH EXPLANATORY ANSWERS

It is difficult to reconcile the ideas of different schools of thought on the question of education. Some people maintain that pupils at school should concentrate on a narrow range of subjects which will benefit them directly in their subsequent careers. Others contend that they should study a wide range of subjects so that they have not only the specialized knowledge necessary for their chosen careers but also sound general knowledge about the world they will have to work and live in. Supporters of the first theory state that the greatest contributions to civilization are made by those who are most expert in their trade or profession. Those on the

other side say that, unless they have a broad general education, the experts will be too narrow in their outlook to have sympathy with their follows or a proper sense of responsibility towards humanity as a whole.

Multiple Choice Questions (MCQs)

1. 'Schools of thought' can be explained as:
   - (A) Groups of people who study in a particular school thoughtfully
   - (B) Groups of people having the same ideas but with different perception on a particular subject
   - (C) Groups of people whose job is to think
   - (D) Groups of people who are schooled to think
2. Broad general knowledge is necessary because
   - (A) It teaches us about different things
   - (B) It broadens one's outlook
   - (C) Specialisation is incomplete without it
   - (D) Without it no one would get a job
3. The idea of the first school of thought in the passage is that
   - (A) Students should study all the subjects they want to
   - (B) Students should study a few subjects that will help them in their profession
   - (C) Students should concentrate on studies
   - (D) Students should not undertake any specialized work
4. Supporters of the first theory say that
   - (A) Experts have done nothing to help mankind
   - (B) People with general knowledge are more useful than experts
   - (C) Experts have contributed most to progress in the modern world
   - (D) People with general knowledge have contributed to civilization
5. According to the second school of thought, education will not be very effective if pupils
   - (A) Ignore the study of fine arts
   - (B) Have nothing but general knowledge
   - (C) Have inadequate knowledge of their own work
   - (D) Do not have a wide general education

Explanatory Answers

1. (B) 'Schools of thought' means two persons or groups having different ideas or opinions on the same subject or topic.
2. (B) The second school of thought supports the idea of having knowledge of a wide range of subjects for wider perception and outlook.
3. (B) The idea of the first school of thought is that people should focus on few subjects to benefit their career.
4. (C) The statement, 'Supporters of the first theory…expert in their trade or profession' gives the answer.
5. (D) The second school of thought opines that pupils should only concentrate on subjects of their interest to have an effective education and career.

SHORT PASSAGES WITH EXPLANATORY ANSWERS

Passage 1

Books are, by far, the most lasting product of human effort. Temples crumble into ruin. Pictures and statues decay, but books survive. Time does not destroy the great thoughts which are as fresh today as when they first passed through their author's mind. These thoughts speak to us through the printed page. The only effect of time has been to throw out of currency the bad products. Nothing in literature which is not good can live for long. Good books have always helped man in various spheres of life. No wonder that the world keeps its books with great care.

Multiple Choice Questions (MCQs)

1. Of the product of human effort, books are the most
   - **(A)** Permanent
   - **(B)** Important
   - **(C)** Enjoyable
   - **(D)** Useful
2. Time does not destroy books because they contain
   - **(A)** Useful material
   - **(B)** Subject-matter for education
   - **(C)** High ideals
   - **(D)** Great ideas
3. "To throw out of currency" means
   - **(A)** Destroy
   - **(B)** Put out of use
   - **(C)** Extinguish
   - **(D)** Forget
4. The world keeps its books with care because
   - **(A)** They bring great ideas to us
   - **(B)** They educate us
   - **(C)** They make us successful
   - **(D)** They help us in various spheres of life

Explanatory Answers

1. **(A)** The phrase 'books survive' indicates that books are permanent and cannot be destroyed easily.
2. **(D)** 'Time does not destroy the great thoughts', provides the correct answer.
3. **(B)** The author implies that bad products have always been discarded or 'thrown out of currency' with time while good things like books have always withstood the test of time.
4. **(D)** The author says that good books have always been handled with care by the world as they have helped man in different phases of life.

Passage 2

The low unit of gas is a real temptation to anyone choosing between gas and electrical processes. But gas-fired processes are often less efficient, require more floor space, take longer and produce more variable product quality. The drawbacks negate the savings many businesses believe they make.

By contrast, electricity harnesses a unique range of technologies unavailable with gas. And many electric processes are well over 90 percent efficient, so far less energy is wasted with benefits in terms of products quality and overall cleanliness, it can so often be the better and cheaper choice. Isn't that tempting?

Multiple Choice Questions (MCQs)

1. The passage can be described as
   - **(A)** An advertisement for electricity and its efficiency
   - **(B)** An extract from a science journal
   - **(C)** An account of the growth of technology
   - **(D)** An appeal not to use gas
2. What does the writer mean by 'variable quality'?
   - **(A)** The quality of the products cannot be assessed
   - **(B)** Products from gas-fired processes are inefficient

(C) The kind of products vary from time to time
(D) The quality of the products is not uniform

3. "Electricity harnesses a unique range of technologies" – What does the writer mean? Electricity
(A) Has developed new technologies
(B) Ensures power for electricity and its efficiency
(C) Depends on new kinds of technology
(D) Makes use of several technologies

Explanatory Answers

1. (B) The passage brings to attention that the 'low unit of gas' should not lead anyone to use gas processes as it has many negative factors. It serves to make aware the reader of the scientific reason for using electric processes and hence the passage looks to be an extract from a science journal.
2. (D) 'Variable quality' means quality which is not consistent or uniform.
3. (B) According to the passage, electricity provides a wide range of technologies and such processes are far more efficient and consumes less energy when compared to gas processes.

Passage 3

There was a marked difference of quality between the personages who haunted the near bridge of brick and the personages who haunted the far one of stone. Those of lowest character preferred the former, adjoining the town; they did not mind the glare of the public eye. They had been of no account during their successes; and though they might feel dispirited, they had no sense of shame in their ruin. Instead of sighing at their adversaries they spat, and instead of saying the iron had entered into their luck. The miserables who would pause on the remoter bridge were of a politer stamp – persons who did not know how to get rid of the weary time. The eyes of this species were mostly directed over the parapet upon the running water below. While one on the townward bridge did not mind who saw him so, and kept his back to the parapet to survey the passerby, one on this never faced the road, never turned his head at coming foot-steps, but, sensitive to his own condition, watched the current whenever a stranger approached, as if some strange fish interested him, though every finned thing had been poached out of the river years before.

Multiple Choice Questions (MCQs)

1. The two bridges were known
(A) For attaching dejected people to them
(B) For being equi-distant from town
(C) For being haunted places
(D) For their similar design

2. People belonging to the lower strata, in their moments of distress:
(A) Felt ashamed of their failures
(B) Dressed shabbily to earn sympathy
(C) Visited the brick-made bridge
(D) Remembered their days of glory

3 The bridge of stone was frequented by
(A) All the sections of society
(B) The sophisticated but luckless
(C) Those fond of fishing
(D) None of the above

4. The attitude of the lowly and genteel towards strangers was
(A) Virulently hostile (B) Completely indifferent
(C) Entirely different (D) Virtually the same

5. In this passage, the author is trying to
(A) Explain the variety of ways in which strangers can be treated

(B) Describe how people of different classes behaved when unhappy
(C) Explain the difference between the construction of two bridges
(D) Describe the way different sections of people like to dress

Multiple Choice Questions (MCQs)

1. (A) The passage talks of dispirited people standing on the bridges. Hence one can say that the two bridges were known for 'the miserables' or the dejected people be coming frequently to those bridges.
2. (D) The statement, 'they did not mind the glare of the public eye', gives the correct answer.
3. (D) The bridge of stone was frequented by mostly dispirited and miserable people. Hence the answer is (D).
4. (C) The altitude of the lonely and genteel towards strangers was entirely different. According to the passage, they pretended to inspect the river under the bridge for some strange fish, whenever a stranger came nearby.
5. (B) They author tried to explain the different kinds of behaviour of people of different classes when they were unhappy. Here he used the bridge to explain the behaviour of these different classes of people.

Passage 4

It is to progress in the human sciences that we must look to undo the evils which have resulted from a knowledge of the physical world hastily and superficially acquired by populations unconscious of the changes in themselves that the new knowledge has made imperative. The road to a happier world than any known in the past lies open before us if atavistic destructive passions can be kept in leash while the necessary adaptations are made. Fears are inevitable in our time, but hopes are equally rational and far more likely to bear good fruit. We must learn to think rather less of the dangers to be avoided than of the good that will lie within our grasp if we can believe in it and let it dominate our thoughts. Science, whatever unpleasant consequences it may have by the way, is in its very nature a liberator, a liberator of bondage to physical nature and in to come, a liberator from the weight of destructive passions. We are on the threshold of utter disaster or unprecedentedly glorious achievement. No previous age has been fraught with problems so momentous; and it is to science that we must look to for a happy future.

Multiple Choice Questions (MCQs)

1. What does science liberate us from? It liberates us from
   (A) Idealistic hopes of a glorious future
   (B) Slavery to physical nature and from passions
   (C) Bondage to physical nature
   (D) Fears and destructive passions
2. To carve out a bright future a man should
   (A) Cultivate a positive outlook
   (B) Analyse dangers that lie ahead
   (C) Try to avoid dangers
   (D) Overcome fears and dangers
3. If man's bestial yearning is controlled
   (A) The future will be brighter than the present
   (B) The future will be tolerant
   (C) The present will be brighter than the future
   (D) The present will become tolerant
4. Fears and hopes, according to the author
   (A) Are irrational
   (B) Are closely linked with the life of modern man
   (C) Can yield good results
   (D) Can bear fruit

5. Should human sciences be developed because they will
   - (A) Make us conscious of the changing world
   - (B) Provide more knowledge of the physical world
   - (C) Eliminate the destruction caused by a superficial knowledge of the physical world
   - (D) Make us conscious of the changes in ourselves

Explanatory Answers

1. (B) The phrase, 'liberator from the weight of destructive passions', provides the correct answer.
2. (A) According to the passage, despite problems and dangers, we must adopt an optimistic or a positive outlook and look at science as a means to secure a bright future.
3. (A) The sentence, 'The road to a happier world...adaptations are made,' provides the correct answer. [Bestial means 'beast-like' or brutal]
4. (B) Fears and hopes are bound to occur in one's life, according to the passage.
5. (C) The statement, 'Science, whatever unpleasant... passions,' provides the correct answer.

Passage 5

The Nobel Committee, in fact, a notoriously conservative body which among other things had a marked antipathy to pure science, especially to Mathematical Physics. Restrained by a clause in Alfred Nobel's will that the prize should go to the person whose 'discovery or invention' shall have conferred the greatest benefit to mankind, the committee initially ignored the great theoretical advances in Physics. Wrecked, no doubt, by guilt that he had become a merchant of death through his invention of dynamite and smokeless powder and plagued by sadistic fantasies of destruction, the Swedish chemist, engineer and aspiring poet, Alfred Nobel, who has been described as Europe's richest vagabond, left his colossal fortune to the cause of progress in human knowledge. Five prizes were installed, one each for Literature ('to the person who shall have produced in the field of Literature, the most outstanding work of an idealistic tendency'), Physics, Chemistry, Medicine and Peace ('to the person who shall have done the most or the best work for fraternity among nations for abolition or reduction of armies, and for holding or promotion of Peace').

Multiple Choice Questions (MCQs)

1. The Nobel Committee has been called a conservative body because
   - (A) Its members are old fashioned
   - (B) It awards prizes only to those people who adopt a conservative approach
   - (C) It is conservative in choice of subjects
   - (D) Its members believe in a conservative ideology
2. In the beginning, the Nobel Committee ignored the great advances in theoretical physics because
   - (A) The Committee felt that the discovery and invention in the field did not contribute to the benefit of mankind
   - (B) They proved to be merchants of death and hence dangerous to mankind
   - (C) They were different from other branches of Physics
   - (D) They were concerned with theory and did not suggest its application
3. Noble Prize would not be given to
   - (A) A composer who composed a symphony
   - (B) An author who wrote a novel
   - (C) A diplomat who negotiated a peace settlement
   - (D) A doctor who discovered a vaccine
4. Alfred Nobel left his colossal fortune to the cause of progress in human knowledge because he
   - (A) Felt guilty for having invented highly destructive things
   - (B) Felt guilty for having earned so much money
   - (C) Was 'Europe's richest vagabond'
   - (D) Was a chemist, engineer and an aspiring poet

Explanatory Answers

1. (C) The Nobel Committee is a conservative body because it does not favour subjects like 'Mathematical Physics'.
2. (A) The statement, 'will that the prize should go to the person ... in Physics,' provides the correct answer.
3. (A) The five prizes installed for conferring 'Nobel Prize...' do not include music.
4. (A) The phrase, 'wrecked, no doubt, by guilt...,' gives the correct answer.

Passage 6

Mountaineering is now looked upon as the king of sports. But men have lived amongst the mountains since prehistoric times and in some parts of the world, as in the Andes and Himalayas, difficult mountain journeys have inevitably been part of their everyday life. However, some of the peaks were easily accessible from most of the cities of Europe. It is quite interesting that while modern mountaineers prefer difficult routes for the greater enjoyment of sport, the early climbers looked for the easiest ones, for the summit was the prize they all set their eyes on. Popular interest in mountaineering increased considerably after the ascent of the Alpine peak of Matterhorn in 1865 and Edward Whymper's dramatic account of the climb and fatal accident which occurred during the descent.

In the risky sport of mountaineering, the element of competition between either individuals or teams is totally absent. Rather one can say that the competition is between the team and the peaks themselves. The individuals making up a party must climb together as a team, for they depend upon one another for their safety. Mountaineering can be dangerous unless reasonable precautions are taken. However, the majority of fatal accidents happen to parties which are inexperienced or not properly equipped. Since many accidents are caused by bad weather, the safe climber is the man who knows when it is time to turn back, however, tempting it may be to press on and try to reach the summit.

Multiple Choice Questions (MCQs)

1. Mountaineering is different from other sports because
   - (A) There is no competition between individuals
   - (B) It is most thrilling and exciting
   - (C) It can be fatal
   - (D) It is risky and dangerous
2. People living in the Andes and the Himalayas made mountain journeys because
   - (A) It was a kind of sport
   - (B) They had to undertake them in their day-to-day life
   - (C) They lived in pre-historic times
   - (D) Of the challenge offered by the difficult journey
3. Mountaineers climb as a team because
   - (A) The height is too much for one individual
   - (B) They have to rely on each other for safety
   - (C) There is no competition among them
   - (D) The competition is between the team and the peak
4. ".....the summit was the prize they all set their eyes on". In the context of the passage this means
   - (A) They chose a route from which they could see the summit clearly
   - (B) They cared for nothing but the prize of reaching the summit
   - (C) They kept their eyes steadily on reaching the peak
   - (D) Reaching the top was their exclusive concern
5. "to press on" in the last sentence means
   - (A) To continue in a determined manner
   - (B) To work fearlessly
   - (C) To force upon others
   - (D) To struggle in a forceful manner

Explanatory Answers

(D) The phrase 'In the risky sport of mountaineering...', suggests that this kind of sport is different from other sports in being more dangerous and fatal.

(B) The statement, '...difficult mountain journeys have been part of their everyday life,' provides the correct answer.

(B) The statement, '...climb together as a team, for they depend upon one another for their safety' provides the correct answer.

(D) The challenge of the sport lay in reaching the peak. Hence the answer is (D).

(A) 'To press on' means to be persistent in one's struggle to achieve the objective or in other words to continue in a determined manner.

Passage 7

After submitting his resignation, Albert came out and took the long narrow road leading to the railway station which was one of the busiest roads in the city. Sad and depressed and worried about looking for a new job, Albert looked around for a cigarette shop. He walked up to the end of the road but found no tobacconist. It was odd that such a busy thoroughfare with thousands of people passing through did not even have a single cigarette shop. He suddenly felt that it was no longer necessary for him to hunt for a job. He decided to open a tobacco shop himself. It was bound to be profitable, he felt.

Multiple Choice Questions (MCQs)

1. After submitting his resignation, Albert came out worried about
   - (A) A job
   - (B) The next available rain
   - (C) A shelter
   - (D) Cigarettes
2. Albert was sad and depressed because
   - (A) He was not able to buy cigarettes
   - (B) He was worried about finding a job
   - (C) He had no money for the train journey
   - (D) He had to walk on a long road
3. There was no cigarette shop on that road because
   - (A) It was a very narrow road
   - (B) Cigarette-shop owners do not make any profit
   - (C) Smoking is banned in that area
   - (D) Just by chance nobody had opened one on that road
4. Albert decided not to look for a new job because
   - (A) There was no hope of finding a job
   - (B) He saw the possibility of self-employment
   - (C) The thought of having to look for a job greatly distressed him
   - (D) He did not want to work at all
5. A cigarette shop on a busy road was bound to be profitable because
   - (A) Cigarettes are inexpensive items and people buy them willingly
   - (B) A cigarette shop on a busy road would attract a large number of customers
   - (C) Cigarette shops are known to make a great deal of profit
   - (D) Any shop on a busy street would attract a large number of customers

Explanatory Answers

1. (A) According to the author, Albert gave resignation from his job to look for a new job. Hence the answer is (A).
2. (D) Albert was worried and depressed about looking for a new job.
3. (D) According to the author, it was strange that there was not any single cigarette shop on such a busy road and hence assumed that it must be chance that nobody opened one.

4. (B) Albert thought of opening a cigarette shop on that busy road thereby indicating that he wanted to be self-employed.

5. (B) A cigarette shop on a busy road was bound to be profitable, especially when there was none there, and would be attracting many customers.

Passage 8

In this work of incessant and feverish activity, men have little time to think, much less to consider ideals and objectives. Yet how are we to act, even in the present, unless we know which way we are going and what our objectives are? It is only in the peaceful atmosphere of a University that these basic problems can be adequately considered. It is only when the young men and women, who are in the university today and on whom the burden of life's problems will fall tomorrow, learn to have clear objectives and standards of values that there is hope for the next generation. The past generation produced some great men but as a generation it led the world repeatedly to disaster. Two world wars are the price that has been paid for the lack of wisdom on man's part in this generation.

I think that there is always a close and intimate relationship between the end we aim at and the means adopted to attain it. Even if the end is right but the means are wrong, it will vitiate the end or divert us in a wrong direction. Means and ends are thus intimately and inextricably connected and cannot be separated. That, indeed, has been the lesson of old taught us by many great men in the past, but unfortunately it is seldom remembered.

Multiple Choice Questions (MCQs)

1. People have little time to consider ideals and objectives because
   - (A) They have no inclination for such things
   - (B) They are excessively engaged in their routine activities
   - (C) They consider these ideals meaningless
   - (D) They do not want to burden themselves with such ideas
2. 'The burden of life's problems' in the fourth sentence refers to
   - (A) The onerous duties of life
   - (B) The sorrows and sufferings
   - (C) The incessant and feverish activities
   - (D) The burden of family responsibilities
3. The two world wars are the price that man paid due to
   - (A) His ignoring the ideals and objectives of life
   - (B) His excessive involvement in feverish activities
   - (C) The absence of wisdom and sagacity
   - (D) His not caring to consider the life's problems
4. According to the writer the adoption of wrong means even for the right end would
   - (A) Impede our progress
   - (B) Deflect us from the right path
   - (C) Not let us attain our goal
   - (D) Bring us dishonour
5. The word 'vitiate' used in the second paragraph means
   - (A) Tarnish
   - (B) Destroy
   - (C) Negate
   - (D) Debase

Explanatory Answers

1. (B) The phrase, 'In this work of incessant and feverish activity' means that men are always busy with their daily work and activities.

2. (A) 'The burden of life's problems' means burdened with life's responsibilities or undertaking the onerous (burdensome) duties of life.

3. (C) The statement, 'Two world wars...has been paid for the lack of wisdom in man's part in this generation' gives the correct answer.

4. (B) The statement, 'Even if the end is right...divert us in the wrong direction', gives the correct answer. 'Deflect' means 'to divert' or to turn away from a certain direction.

5. (D) 'Vitiate' means to spoil or impair. Hence it means debase meaning 'to pollute' or to contaminate.

Passage 9

In the past thirty years, drugs have been discovered that prevent and cure physical disease and reverse the disturbances that occur in certain mental illness. Excitement over what drugs can do has led people to believe that any ailment, infective or psychic, can be relieved by taking a pill. At the first sign of nervousness, they try pep-up pills. Medical journals now advertise tranquillizers, and other mood-altering drugs; doctors prescribe them; and the public expect miracles from them. In such an atmosphere, it is not surprising that drug abuse has spread.

Multiple Choice Questions (MCQs)

1. According to the author, in recent years there has been
   - (A) Recognition of the ill-effects of medicine
   - (B) A misplaced trust in drugs
   - (C) A distrust of drugs
   - (D) None of the above
2. According to the passage, the medicines that have been discovered in recent times
   - (A) Can reduce mental illnesses
   - (B) Cannot cure mental illnesses
   - (C) Can cure mental illnesses
   - (D) Can help treat some symptoms of mental illnesses
3. People often believe that
   - (A) Medicines can cure all the diseases
   - (B) Doctors cannot cure all the diseases
   - (C) Medicines cannot cure all the diseases
   - (D) Doctors can cure all the diseases

Explanatory Answers

1. (B) The author talks of the immense trust of people in drugs followed by the statement '...drug abuse has spread'. This indicates a misplaced trust in drugs.

2. (D) The statement 'drugs...reverse the disturbances that occur in some mental illness' gives the answer.

3. (A) The sentence 'Excitement...pill' gives us the answer.

Passage 10

When we are suddenly confronted with any terrible danger, the change of nature we undergo is equally great. In some cases, fear paralyses us. Like animals, we stand still, powerless to move a step in fright or to lift a hand in defence of our lives, and sometimes we are seized with panic, and again, act more like the inferior animals than rational beings. On the other hand, frequently in cases of sudden extreme peril, which cannot be escaped by fright, and must be instantly faced, even the most timid men at once as if by miracle, become possessed of the necessary courage, sharp quick apprehension, and swift decision. This is a miracle very common in nature. Man and the inferior animals alike, when confronted with almost certain death 'gather resolution from despair' but there can really be no trace of so debilitating a feeling in the person fighting, or prepared to fight for dear life. At such times, the mind is clearer than it has ever been; the nerves are steel, there is nothing felt but a wonderful strength and daring. Looking back at certain perilous moments in my own

life, I remember them with a kind of joy, not that there was any joyful excitement then; but because they brought me a new experience, a new nature, as it were and lifted me for a time above myself.

Multiple Choice Questions (MCQs)

1. An appropriate title for the above passage would be
   - **(A)** The change of nature
   - **(B)** Courage and panic
   - **(C)** The will to Fight
   - **(D)** The miracle of confronting danger
2. The author names three different ways in which a man may react to sudden danger. What are they?
   - **(A)** He may be paralysed with fear, or seized with panic, or as if by miracle, become possessed of the necessary courage, and face the danger
   - **(B)** He may be paralysed with fear, run away or fight
   - **(C)** He may flee in panic, or fight back or stand still
   - **(D)** He may be paralysed with fear, seized with panic or act like an inferior animal
3. The distinction between 'inferior animals' and 'rational beings' is that
   - **(A)** The latter are stronger
   - **(B)** The latter are capable of reasoning things out whereas the former cannot do so
   - **(C)** The former are incapable of fighting
   - **(D)** The latter are clever
4. Explain the phrase 'gather resolution from danger'.
   - **(A)** Not to lose hope, but fight
   - **(B)** Find courage to face the danger
   - **(C)** Find hope and courage
   - **(D)** A state of utter hopelessness steels one to fight out the danger
5. The author feels happy in the recollection of danger faced and overcome because
   - **(A)** He survived his ordeal
   - **(B)** He was lucky to be alive
   - **(C)** They brought him a new experience
   - **(D)** They brought him a new experience, and lifted him above himself for a time

Explanatory Answers

1. **(D)** The passage talks of the different ways in which people react when faced with any kind of danger and hence the title should be 'The miracle of confronting danger'
2. **(A)** In the first few lines, the author talks of fear paralyzing a person or being seized with panic when faced with danger. Then he goes on to explain that some people become amazingly courageous in times of peril and danger.
3. **(B)** The author while stating the different reactions of men when confronted with danger makes a comparison with animals who tend to become powerless or behave irrationally in times of danger.
4. **(D)** The phrase 'gather resolution from danger' means to brace oneself or assemble the strength to face danger.
5. **(D)** The statement, 'Looking back…joyful excitement' gives the answer.

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NTS Guide

GRE-GAT GENERAL TESTS
With Explanatory Answers

NTS Guide

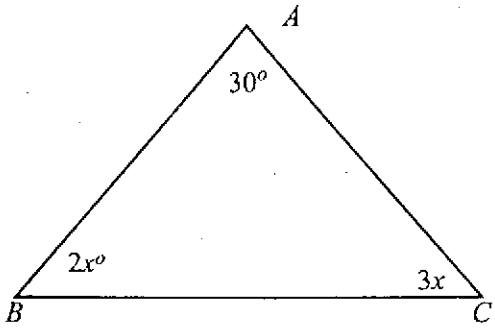
GRE - GAT TEST 1

Quantitative Section	No. of Questions = 20

Q1. If it is now March, what month will be after the 100 months from now?

(A) March (B) April
(C) July (D) August

Q2. What is the value of x in the following figure?



(A) 30° (B) 40°
(C) 50° (D) 60°

Q3. What is the value of x if $3^{x+1} = 243$?

(A) 3 (B) 5
(C) 7 (D) 4

Q4. If x is a multiple of 5 and $y = 5x$, then which of the following could be the value of $x + y$?

I. 60
II. 110
III. 50

(A) I only (B) II only
(C) I and II only (D) I and III only

Q5. **If Riaz can mow $\frac{3}{4}$ of a lawn each hour, how many lawns can he mowed in k hours?**

(A) $\frac{4k}{3}$ (B) $\frac{3k}{4}$

(C) $\frac{2k}{3}$ (D) $\frac{3k}{2}$

Q6. **If $2^a = x$ and $2^b = y$, then $xy =$**

(A) 9^{a+b} (B) 2^{ab}

(C) 4^{a+b} (D) 2^{a+b}

Q7. **If the average (arithmetic mean) of three consecutive integers is M, then which of the following must be true?**

I **Any one of the three numbers is M**

II **The average of two of the three numbers is M.**

III **M is also an integer**

(A) I only (B) II only

(C) III only (D) I, II and X

Q8. **If $a^2 = 17$, then $(a + 1)(a - 1) = ?$**

(A) 15 (B) 12

(C) $\sqrt{18}$ (D) 16

Q9. **Which of the following cannot be expressed as the sum of three consecutive integers?**

(A) 27 (B) 26

(C) 21 (D) 42

Q10. **Ali and Omer share an apartment. If each month Ali pays x dollars and Omer pays y dollars, what percent of the total cost does Ali pay?**

(A) $(x+y)100$ (B) $\frac{x}{y}\%$

(C) $\frac{100x}{y}\%$ (D) $\frac{100x}{x+y}\%$

Q11. **If $(a-b)^2 = a^2 - b^2$ and $a \neq b$, then which of the following is true?**

I $a = 0$

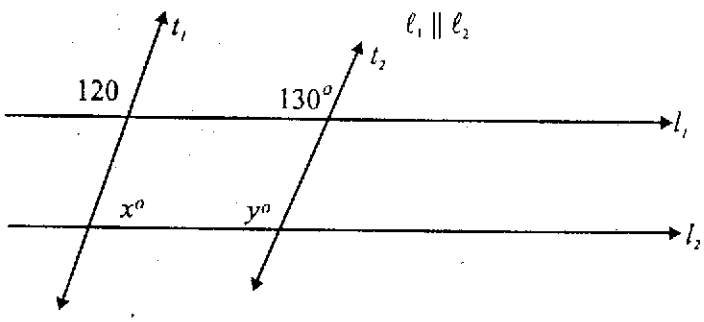
II $b = 0$

III $a = -b$

(A) I only (B) II only

(C) I and II only (D) I and III only

Q12. **In the following figure**

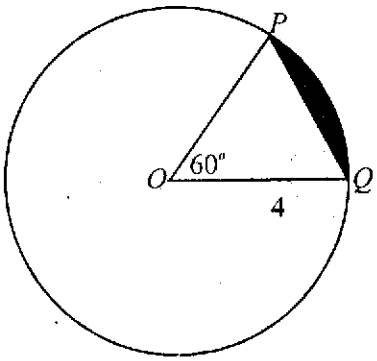


$x + y =$

(A) 230 (B) 210

(C) 110 (D) 190

Q13. **In the following figure the radius of the circle is 4, and m∠POQ = 60. What is the perimeter of the shaded region?**



(A) $4+\frac{2\pi}{3}$ (B) $4+\left(2+\frac{\pi}{3}\right)$

(C) $4+\frac{5\pi}{3}$ (D) $4+\frac{4\pi}{3}$

Q14. **If S_1 is the sum of integers from 1 to 60 and S_2 is the sum of the integers from 61 to 100, what is the value of $S_2 - S_1$?**

(A) 2500 (B) 2100

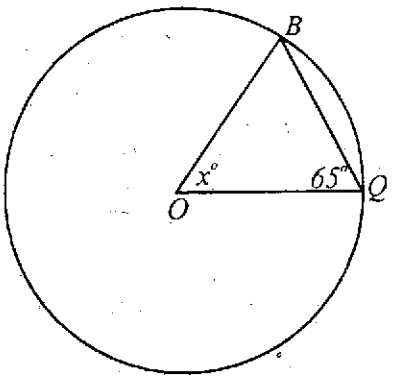
(C) 1800 (D) 1390

Q15. **If p, q and r are different prime numbers less than 15, what is the greatest possible value of $\frac{p+q}{r}$?**

(A) 9 (B) 2

(C) 13 (D) 12

Q16. **In the following figure O is the center of the circle. What is the value of x?**



(A) 65° (B) 50°

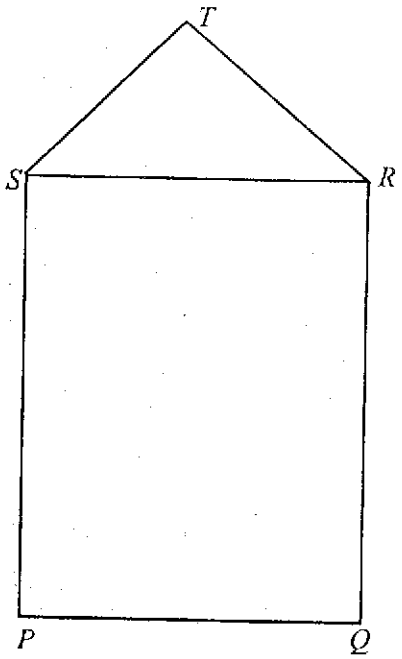
(C) 45° (D) 35°

Q17. **If x is increased by 10% and y is decreased by 10%, the resulting numbers will be equal. What is the ratio x to y?**

(A) $\frac{3}{4}$ (B) $\frac{9}{11}$

(C) $\frac{4}{3}$ (D) $\frac{5}{3}$

Q18. **In the following figure, the area of the isosceles triangle RST is 8 and the area of the square PQRS is 64. What is the distance from *P* to *T*.**



(A) 64 **(B)** 5

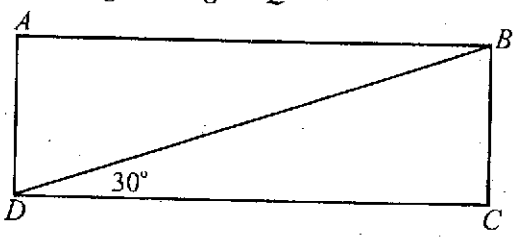
(C) 41 **(D)** $2\sqrt{29}$

Q19. **If $5x + 3 = 3x + 5$, then $x = ?$**

(A) 1 **(B)** 2

(C) $\frac{1}{2}$ **(D)** $\frac{1}{3}$

Q20. **What is the area of the following rectangle *PQRS*?**



(A) $25\sqrt{3}$ **(B)** $16\sqrt{3}$

(C) $12\sqrt{3}$ **(D)** 64

Explanatory Answers

Q1. (C) In a year there are 12 months, so

100 month $= (12 \times 8) + 4$

$= 96 + 4$

= (March) + April + May + June + July

Explanation: 8 years from now, it will again be March, and 4 months later it will be July.

Q2. (A) In any triangle,

The sum of three angles = 180°

$\therefore \quad 30^\circ + 2x + 3x = 180$

$\Rightarrow \quad 5x = 180 - 30$

$\Rightarrow \quad x = \frac{150}{5} \Rightarrow x = 30$

Q3. (D) $3^{x+1} = 243$

$\Rightarrow 3^{x+1} = 3^5$

$\Rightarrow x + 1 = 5$

$\Rightarrow x = 4$

Q4. (A) As x is a multiple of 5, then for any integer n, x can be written as

$$x = 5n$$

Also $y = 5x$

$\Rightarrow x + y = x + 5x \Rightarrow x + y = 6x$

$\Rightarrow x + y = 6(5n) \Rightarrow x + y = 30n$

It means, that $x + y$ is the multiple of 30.

Now, w check I, II and III

(i) Could $x + y = 60$?

Yes, because $x + y = 30(2) \Rightarrow x + y = 30n$

i.e., $(a = 10, b = 50)$

(ii) Could $x + y = 110$?

No, because 110 is not multiple of 30.

(iii) Could $x + y = 50$?

No, because 50 is not multiple of 50.

Therefore, only option A is true.

Q5. (B) Simply multiply $\frac{3}{4}$ by k.

$$\frac{3}{4}(k) = \frac{3k}{4}$$

Q6. (D) $x = 2^a$ and $y = 2^b$ (given)

$xy = 2^a \times 2^b$

$\Rightarrow xy = 2^{a+b}$

Q7. (D) Let the three consecutive numbers be 5, 6 and 7. Its average is

$$M = \frac{5+6+7}{3} = \frac{18}{3} = 6$$

Hence I is true.

Now, $\frac{5+7}{2} = \frac{12}{2} = 6$

Hence, II is true.

Also, III is true.

Q8. (D) $a^2 = 17 \Rightarrow a^2 - 1 = 17 - 1$

$\Rightarrow a^2 - 1 = 16$

$\Rightarrow (a-1)(a+1) = 16$

Q9. (B) Let the three consecutive numbers be x, $x + 1$ and $x + 2$, then their sum $(x) + (x + 1) + (x + 2) = 3x + 3 = 3(x + 1)$ which is multiple of 3. Only 26 is not multiple of 26.

Q10.(D) The total rent is $x + y$, so Ali's share is $\frac{x}{x+y}$. To convert into percentage we multiply $\frac{x}{x+y}$ by 100 and place %age sign.

Q11.(B) $(a-b)^2 = a^2 - b^2$

$a^2 + b^2 - 2ab = a^2 - b^2$

Case I: If $a = 0$, then

$(0)^2 + b^2 - 2(0)(b) \quad = (0)^2 - b^2$

$b^2 \quad = -b^2$

which is not true.

Case II: If $b = 0$, then

$a^2 + b^2 - 2ab = a^2 - 0^2$

$a^2 + (0)^2 - 2a(0) = a^2 - (0)^2$

$\Rightarrow a^2 = a^2$

which is true.

Case III: If $a = -b$, then

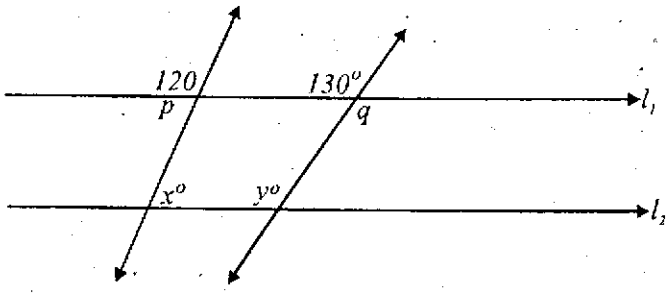
$(-b)^2 + b^2 - (2)(-b)(b) = (-b)^2 - b^2$

$b^2 + b^2 + 2b^2 = b^2 - b^2$

$\Rightarrow \quad 4b^2 = 0$

which is not true.

Q12.(D) Here $120 + p = 180 \Rightarrow p = 60$



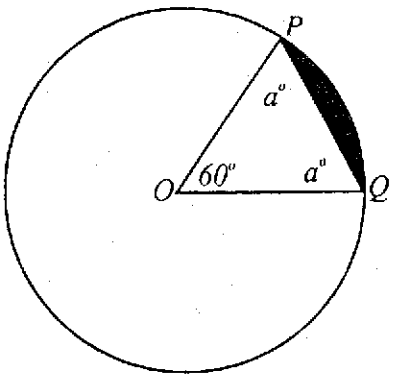
and $q = 130$ (corresponding angles)

Since l_1 and l_2 are parallel, so

$p = x^o = 60$ and $q = y^o = 130$

Hence $x + y = 60 + 130 = 190$.

Q13.(D) Since each radius of a circle is always equal, so



$OP = OQ = 4$

Because, two sides of a triangle. POQ are equal, so their opposite angles must be equal. Let one angle be a^o then the other will also be a^o.

Thus in ΔPOQ

$60^o + a^o + a^o = 180 \quad \Rightarrow 2a^o = 180 - 60$

$$\Rightarrow a^o = \frac{120}{2} = 60$$

Thus the length of $\overline{PQ}$ also 4, and the length of arc PQ is

$\frac{60}{360} = \frac{1}{6}$ of the circumference.

Now, circumference, $C = 2\pi r \Rightarrow C = 2\pi(4)$

$$\Rightarrow C = 8\pi \Rightarrow \frac{1}{6}C = 8\pi \times \frac{1}{6}$$

$$= \frac{4\pi}{3}$$

Hence the perimeter of the region is

$$4 + \frac{4\pi}{3}$$

Q14.(D) To find the sum of $1 + 2 + 3 + \ldots\ldots\ldots + 60$, use the following formula

$$S_1 = \frac{n}{2}\{2a + (n-1)d\}$$

Here, $n = 60, a = 1, d = 2 - 1 = 1$

$$S_1 = \frac{60}{2}\{2(1) + (60-1)1\}$$

$$S_1 = 30(2 + 59) \Rightarrow S_1 = 30(61)$$

$\Rightarrow \quad S_1 = 1830$

Now, we find the sum of $\{61 + 62 + 63 + \ldots\ldots + 100\}$

Here, $a = 61, n = 40 \quad d = 62 - 61 = 1$

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

$$S_2 = \frac{40}{2}\{2(61) + (40-1)1\}$$

$$S_2 = 20\{122 + 39\} \Rightarrow S_2 = 20(161)$$

$\Rightarrow \quad S_2 = 3220$

Now, $S_2 - S_1 = 3220 - 1830$

$= 1390$

Q15.(D) The prime numbers less than 17 are 2, 3, 5, 7, 11, 13

To make a larger fraction, make the numerator as large and denominator as small. So, Let $p = 13$ and $q = 11$

and $r = 2$ (smallest prime number)

$$\therefore \quad \frac{p+q}{2} = \frac{13+11}{2} = \frac{24}{2} = 12$$

Q16.(B) Since all the radii of a circle have the same magnitude, thus $OA = OB$. Therefore $m\angle A = m\angle B = 65^o$

Hence, $\quad x + m\angle A + m\angle B = 180^o$

$\Rightarrow \quad x + 65 + 65 = 180$

$\Rightarrow \quad x + 130 = 180$

$\Rightarrow \quad x = 50$

Q17.(B) $x + \frac{10}{100}(x) = x + 0.1x = 1.1x$

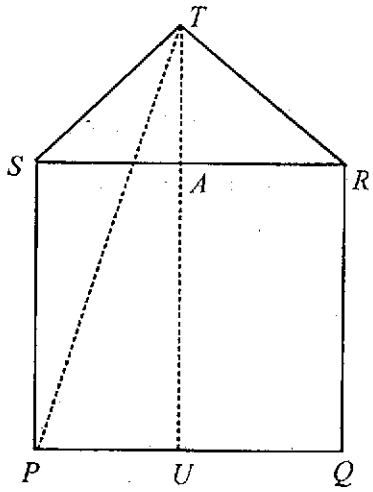
Now $y - \frac{10}{100}(y) = y - 0.1y = 0.9y$

Setting ratio

$1.1\,x = 0.9\,y \Rightarrow \frac{x}{y} = \frac{0.9}{1.1}$

$\Rightarrow \frac{x}{y} = \frac{9}{11}$.

Q18.(D) Join the point P to T by line segment. From T draw TU perpendicular on PQ, which cut the line RS at A.



Then $AU = 8$

In $\Delta\, RST$, base = 8 and Area = 8

As Area $= \frac{1}{2}$(base)(Altitude)

$\Rightarrow 8 = \frac{1}{2}(8)AT \Rightarrow AT = \frac{8 \times 2}{8} = 2$

$\Rightarrow TU = TA + AU \Rightarrow TU = 2 + 8 \Rightarrow \boxed{TU = 10}$

Now, In $\Delta\, PTU$

Altitude $PU = 4$ and Base, $TU = 10$

We know, $(PT)^2 = (PU)^2 + (TU)^2$

$\Rightarrow \quad (PT)^2 = (4)^2 + (10)^2 \Rightarrow (PT)^2 = 16 + 100$

$\Rightarrow \quad PT = \sqrt{116} \Rightarrow 2\sqrt{29}$

Q19.(A) $5x + 3 = 3x + 5$

$\Rightarrow \quad 5x - 3x = 5 - 3$

$\Rightarrow \quad 2x = 2$

$\Rightarrow \quad x = 1$

Q20.(B) PR is the hypotenuse of a 30 – 60 – 90 triangle. Thus, QR, the opposite leg of the 30° angle, is 4 (half of PR), and PQ is $4\sqrt{3}$. Then the area of the rectangle $PQRS$.

$$4 \times 4\sqrt{3} = 16\sqrt{3}$$

Analytical Section	No. of Questions = 20

For questions 1 to 4

Three women — X, Y, and Z are traveling in a van with five children — A, B, C, D and E. The van has a driver's seat and one passenger seat in the front, and two benches behind the front seats, one bench behind the first. Each bench has room for exactly three people. Everyone must sit in a seat or on a bench and seating is subject to the following restrictions: A women must sit on each bench. Either X or Y must sit in the driver's seat. C must sit immediately beside E.

Q1. Which of the following can sit in the front passenger seat?

(A) C (B) D
(C) X (D) Y
(E) Z

Q2. Which of the following groups of three can sit together on a bench?

(A) A, C and E (B) A, C and Z
(C) A, Y and Z (D) B, D and Y
(E) D, E and X

Q3. If A sits immediately beside Z, which of the following CANNOT be true?

(A) C sits immediately beside Y. (B) D sits immediately beside Z.
(C) B sits in the front passenger seat. (D) A sits on the same bench as B.
(E) B sits on the same bench as X.

Q4. If Y sits on a bench that is behind where C is sitting, which of the following must be true?

(A) B sits in a seat or on a bench that is in front of where E is sitting.
(B) D sits in a seat or on a bench that is in front of where A is sitting.
(C) A sits on the same bench as B.
(D) D sits on the same bench as Y.
(E) E sits on the same bench as Z.

For questions 5 to 7

Four computer operators (Ali, Babar, Cheema and Dar) each have to perform duties at the NADRA on four different days, Thursday through Sunday. The following is their duty schedule: Cheema has his duty day before Ali. Dar has his duty day later than Babar.

Q5. Which of the following is a possible order of duty days for the four operators?

(A) Cheema, Dar, Ali and Babar (B) Dar, Cheema, Ali and Babar
(C) Babar, Cheema, Dar and Ali (D) Ali, Cheema, Dar and Babar
(E) Ali, Babar, Dar and Cheema

Q6. If Cheema has his duty day on Saturday, who must have his duty day on Thursday?

(A) Either Ali or Dar (B) Dar
(C) Ali (D) Either Babar or Dar
(E) Babar

Q7. Each of the following possible EXCEPT:

(A) Cheema has his duty on Thursday. (B) Babar has his duty on Thursday.
(C) Dar has his duty on Saturday. (D) Babar has his duty on Sunday.
(E) Ali has his duty on Sunday.

Two statements, labeled *X* and *Y*, follow each of the following questions. The statements contain certain information. In the questions you do not actually have to compute an answer, rather you have to decide whether the information given in the statements *X* and *Y* is sufficient to find a correct answer by using basic mathematics and everyday facts.

Q8. How much time will computer need to solve 150 problems?

***X*. The computer needs 50 seconds to solve one problem.**

***Y*. Computer never takes more than 60 seconds to solve a problem.**

(A) Statement *X*. Alone is sufficient but *Y*. Alone is not sufficient to answer this question.

(B) Statement *Y*. Alone is sufficient but *X*. Alone is not sufficient to answer this question.

(C) Statements *X* and *Y*. TOGETHER are sufficient to answer the question but NEITHER of them is sufficient ALONE.

(D) Statements *X* and *Y*. COMBINED are NOT sufficient to answer the question and additional information is needed to find the correct answer.

Q9. A horse ran 80 miles without stopping. What was its average speed in miles per hour?

***X*. The journey started at 6 PM and ended at 2 AM the following day.**

***Y*. The horse ran 20 miles per hour for the first 40 miles.**

(A) Statement *X*. ALONE is sufficient but *Y*. ALONE is not sufficient to answer this question.

(B) Statement *Y*. ALONE is sufficient but *X*. ALONE is not sufficient to answer this question.

(C) Statements *X* and *Y*. TOGETHER are sufficient to answer the question but NEITHER of them is sufficient ALONE.

(D) Statements *X* and *Y*. COMBINED are NOT sufficient to answer the question and additional information is needed to find the correct answer.

Q10. In a B.Sc. class at G.C. University, 40 boys and 15 girls registered for Calculus and Analytical geometry. How many boys passed the course?

***X*. 7 students could not pass.**

***Y*. There were 3 girls who obtained A grade.**

(A) Statement *X*. ALONE is sufficient but *Y*. ALONE is not sufficient to answer this question.

(B) Statement *Y*. ALONE is sufficient but *X*. ALONE is not sufficient to answer this question.

(C) Statements *X* and *Y*. TOGETHER are sufficient to answer the question but NEITHER of them is sufficient ALONE.

(D) Statements *X* and *Y*. COMBINED are NOT sufficient to answer the question and additional information is needed to find the correct answer.

Q11. A runner has just completed 46 miles running. How long did it take him to finish the journey?

***X*. His record speed is 13.2 miles per hour.**

***Y*. His average speed through the journey was 9.2 miles per hour.**

(A) Statement *X*. ALONE is sufficient but *Y*. ALONE is not sufficient to answer this question.

(B) Statement *Y*. ALONE is sufficient but *X*. ALONE is not sufficient to answer this question.

(C) Statements *X* and *Y*. TOGETHER are sufficient to answer the question but NEITHER of them is sufficient ALONE.

(D) Statements *X* and *Y*. COMBINED are NOT sufficient to answer the question and additional information is needed to find the correct answer.

Q12. Captain of national hockey team should be the most popular member of the team. Who is the captain of Pakistan's national hockey team?

***X*. Saqlain is the best player on the team.**

***Y*. Junaid is the senior-most member.**

(A) Statement *X*. ALONE is sufficient but *Y*. ALONE is not sufficient to answer this question.

(B) Statement *Y*. ALONE is sufficient but *X*. ALONE is not sufficient to answer this question.

(C) Statements *X* and *Y*. TOGETHER are sufficient to answer the question, but NEITHER of them is sufficient ALONE.

(D) Statements *X* and *Y*. COMBINED are NOT sufficient to answer the question and additional information is needed to find the correct answer.

Q13. The principal of a college is forming a committee. There are to be five members: three teachers, chosen from Mr. *A*, Mr. *B*, Mr. *C*, Mr. *D* and Mr. E; and two students, chosen from *L*, *M*, *N*, and *O*. The composition of the committee must conform the following conditions:

Mr. A will serve only if *O* is also on the committee. Mr. *C* will not serve unless Mr. *B* and *L* also serve. Neither Mr. *D* nor Mr. *E* will serve without the other. If *M* serves, either *N* nor *O* can serve. Which of the following is an acceptable committee?

(A) *A, C, D, E, L* (B) *B, C, E, L, M*
(C) *B, D, E, L, O* (D) *C, D, E, L, M*
(E) *D, E, L, M, N*

Q14. How many different committees could include Mr. A and *N*?

(A) 1 (B) 2
(C) 3 (D) 4
(E) 5

Q15. If *N* and *O* are both on the committee, who else must be on the committee?

(A) *A* (B) *B*
(C) *C* (D) *D*
(E) *L*

Q16. In how many different ways can the principal select an acceptable committee?

(A) less than 3 (B) 3
(C) 7 (D) 5
(E) more than 7

Explanatory Answers

Solution 1-4

We summarize the given paragraph as:

i) Everyone must sit in a seat or on a bench but
   a) A women must sit on each bench.
   b) Either *X* or *Y* must sit in the driver's seat.
   c) C must sit immediately beside E.

ii) There are three women and 5 children in the van.

iii) There are two benches behind the front seats.

iv) Each bench has room for exactly three people.

v) The van has a driver's seat and one passenger seat in the front.

Q1. (B) There are five options given and asked which of the following sit on the front passenger seat. *C* cannot sit in the front passenger's seat, because by given restrictions *C* must sit immediately beside *E*, and there are only one passenger seat in front, so *C* cannot sit in the front passenger seat. *X*, *Y* and *Z* also could not sit in front seat, because by the given restriction, either *X* or *Y* must sit in the driver's seat and a woman must sit on each bench. So, if *Y* sits on driver's seat, then *X* and *Z* must be on benches, similarly, if *X* sits on driver's seat then *Y* and *Z* must be on the benches. Hence, *X*, *Y* and *Z* cannot sit on front passenger's seat. The possible children who can sit on front passenger's seat are *A*, *B* and D. But *D* is the only name from these three (*A*, *B* and D) names in the given choices. Hence the correct answer is choice (B).

Q2. (D) Take first choice *A*, *C* and E. Here we see that *C* and *E* must sit together, it is also restricted that a woman must sit on a bench, but A is not a women. Hence, choice A is not a correct answer. Take second choice, *A*, *C* and Z. Because *C* must sit beside *E*, so this choice is not acceptable. Take third choice, *A*, *Y* and Z. In this choice *Y* and *Z* are women, and according to given restriction, a woman must sit on each bench. Now if *Y* and *Z* sit on a same bench, then *X* will be on driving seat. In this case a bench will be without a woman. Hence, choice *C* is also not acceptable combination. Now take choice *D*, *i.e.*, *B*, *D* and *Y*. In this choice, there is only one woman *Y* since *C* and *E* not occurring separately. So this choice is acceptable. In last choice, since *E* is sitting without *C*, so this choice is also not acceptable. Hence the best answer is choice D.

Q3. (E) In this question, we should choose a wrong combination in case of, if A sits immediately beside Z. Take choice *A*, *C* sits immediately beside *Y*. This is correct choice, because there is only one woman *Y* and one child C. The third child may be E. According to second choice, "D sets immediately beside Z". It may also be possible that *D* may sit beside Z. Here *Z* is only one woman on the bench, the third child may be *A*, according to this question. According to third choice, *B* sits in the front passenger seat. This is also possible. Because, women *X*, *Y*, *Z* and children *A*, *C* and *E* cannot sit on front passenger's seat. So *B* may sit in front passenger's seat. The fourth choice, *D* may also be true, because if A sits immediately beside Z, then there is one seat of a child is empty on that bench. Since, *C* sits beside *E*, so *C* and *E* may not be sit on this bench. Only *B* or *D* can sit on that empty seat. Hence, choice *D* may be possible. According to choice *E*, *B* sits on the same bench as *X*." Since, *X* is a woman and she cannot sit on that bench with other woman Z. So, *B* cannot sit as *X*. This choice may not be possible. Hence the correct answer is choice E.

Q4. (E) In this question, it is understood that *C* and *Y* are not on the same bench. Thus *E* is also not on the same bench where *Y* is. Take choice *A*, "B sits in a seat or on a bench that is in front of where *E* is sitting. From above discussion we concluded that woman *Y* is on the last bench since *C* and *E* must sit together, therefore *B*, A and *D* can sit only in front passenger's seat. Now, take second choice, according to this, *D* sits in a seat or on a bench that is in front of where A is sitting." This is also incorrect choice, because *C* and *E* sit between front and a last bench, so *D* can sit with A or behind *C* and E. According to third choice, "A sits on the same bench as B." A can sit both *B* and *D* not only B. Choice *C*, may also be true, but choice *D* may not be possible, because *C* and *E* must sit together. Thus if *E* sits as *Z* then the third may be *X*.

Solution 5-7

Here, we decompose the given paragraph:

i) There are four computer operators, Ali, Babar, Cheema and Dar.

ii) Each have to perform their duties on four different days.

iii) Days of duty are: Thursday, Friday, Saturday and Sunday.

iv) Cheema has his duty day before Ali.

v) Dar has his duty day later than Babar.

Q5. (C) In choice *A*, Cheema will perform his duty a day before Ali, this is according to the given restriction, but by the given restriction, Dar's duty should be a day later than Babar. Here, the Dar's duty is before Babar. Thus, this combination is not acceptable. In second choice, there is not a day's gap between Cheema and Ali, so this is also not acceptable. Choice *C*, is a right combination because,

Cheema will perform his duty a day before Ali, and Dar will perform his duty a day later than Babar. Hence, the correct answer is choice C.

Q6. (E) If Cheema has his duty on Saturday, then the possible schedule is:

Thursday	Friday	Saturday	Sunday
Babar	Dar	Cheema	Ali

Hence, the correct answer is choice E.

Q7. (D) If Cheema has his duty on Thursday, the possible schedule is:

Thursday	Friday	Saturday	Sunday
Cheema	Babar	Ali	Dar

which is acceptable according to the given restrictions.

If Babar has his duty on Thursday, then the possible schedule is:

Thursday	Friday	Saturday	Sunday
Babar	Cheema	Dar	Ali

which is also acceptable according to the given restrictions.

If Dar has his duty on Saturday, then the above schedule is formed, which is acceptable. If Babar has his duty on Sunday, then it is not possible to follow the given restrictions. Because, Dar has his duty later than Babar. So, it is not possible to perform Dar duty if Babar perform his duty on Sunday. Because Sunday is the last day in the given schedule. Hence the correct answer is choice D.

Q8. (A) We set a proportion, to solve this problem

Time in seconds		Problems		Time in second		Problems
50	:	1	::	x	:	150

$$\therefore \frac{50}{1} = \frac{x}{150}$$

$$\Rightarrow x = 150 \times 50 \quad \Rightarrow X = 7500 \text{ seconds} = 2 \text{ hours and } 5 \text{ minutes}$$

So, statement X, ALONE is sufficient to solve this problem. Hence, the correct answer is choice A.

Q9. (A) Since, the journey started at 6 PM and ended at 2 AM, so this journey is 8 hours. The average speed of the horse is $= \frac{80}{8} = 10$ miles per hour. So, statement X, ALONE is sufficient to solve this problem. Hence, the correct answer is choice A.

Q10.(D) Statements X and Y are not sufficient to answer.

Q11.(B) Average speed of the runner $= 9.2$ m/hour

Distance traveled $= 46$ miles

$$\because \quad \text{Average speed} = \frac{\text{Distance travelled}}{\text{time}}$$

$$9.2 = \frac{46}{t}$$

$$\Rightarrow t = \frac{46}{9.2} = 5 \text{ hours}$$

So, statement Y, ALONE is sufficient but X alone is not sufficient to answer this question. Hence, the correct answer is choice B.

Q12.(D) COMBINED statements X and Y are not sufficient to answer the question and some additional information is needed to find the correct answer.

Solution 13-16

Here, we decompose the given paragraph:

i) The principal of college is forming a committee of five members.

ii) Three teachers chosen from five teachers, Mr. *A*, Mr. *B*, Mr. *C*, Mr. *D* and Mr. E.

iii) Two students chosen from four students, *L*, *M*, *N* and *O*.

iv) Mr. *A* will serve only if *O* is also in the committee.

v) Mr. *C* will not serve unless Mr. *B* and *L* also serve.

vi) Neither Mr. *D* nor Mr. *E* will serve without the other.

vii) If *M* serves, neither *N* nor *O* can serve.

Q13.(C) Take choice *A i.e., A, C, D, E* and *L*, this choice will not be acceptable because according to the given condition, Mr. A will serve only if *O* is also in the committee. Take choice *B i.e., B, C, E, L*, M. This choice is also not acceptable, because according to the given restriction, neither Mr. *D* not Mr. *E* will serve without the other. Here, *E* is without D. Take choice *C i.e., B, D, E, L, O*. Since, this choice satisfies all the given restrictions. So this combination is acceptable for committee.

Q14.(A) There is only possible committee can be formed including Mr. A and N. The combination of this committee is *A, D, E,* N, *O*. Hence the correct answer is choice A.

Q15.(C) If *N* and *O* are both on the committee, then the other three members should all be teachers. Since, Mr. *C* will not serve unless Mr. *B* and *L* also serve. So Mr. *C* would not be on the committee. If he is on the committee, the third student *L* must include the other two students *N* and *O*. Thus the correct answer is choice C.

Q16.(C) The principal can select acceptable committee in the following ways:

1. *B, D, E, L, O*
2. *A, D, E, N, O*
3. *A, C, B, L, O*
4. *D, E, M, B, L*
5. *C, B, L, N, A*
6. *A, B, O, L, C*
7. *D, E, B, L, N*

Hence. the correct answer is choice C.

III. Verbal Section	No. of Questions = 25
Select the correct answer for each question and blacken the corresponding circle in the answer sheet.	

Instructions (1-10): In this part of test, you have 10 MCQs about English. Each sentence below has one or two blanks, each blank shows that something has been omitted. Choose the correct answer from the four answer choices given with each question, numbered *(A)*, *(B)*, *(C)*, *(D)*.

1. ***She turned this place _____ looking for her keys.***

(A) Thoroughly (B) Inside out
(C) Up and down (D) In and out

2. ***It is widely believed that a nuclear war could _____ enough smoke and dust to black out the sun and freeze the earth.***

(A) Billow (B) Extinguish
(C) Generate (D) Duplicate

3. ***Many kinds of harmful viruses are unhindered when passing through different parts of the host organism; indeed, there are few organic substances which such viruses cannot _____.***

(A) Undermine (B) Disseminate
(C) Aerate (D) Exterminate

4. ***The light kept flashing _____ all night long.***

(A) One and of (B) On and off
(C) Up and down (D) In and out

5. *Some illnesses such as smallpox, which have been almost eliminated in the United States are still _____ in many places abroad.*
   (A) Discussed (B) Prevalent
   (C) Scarce (D) Unknown

6. *The Inuit natives of Alaska's North Slope worry that _____ oil exploration might _____ their sensitive natural environment.*
   (A) Additional; Assist (B) Current; Bolster
   (C) Curtailed; Shatter (D) Unregulated; Damage

7. *Everywhere the iron horse replaced the living _____ to pull trains carrying _____ and goods.*
   (A) Animals; Passengers (B) Horses; Passengers
   (C) Horses; Labourers (D) Animals; Men

8. *James Watt first realized the _____ of steam when he saw a _____ boiling on the fire.*
   (A) Strength; Pot (B) Force; Kettle
   (C) Force; Tea-Pot (D) Power; Vessel

9. *A _____ response is one that is made with _____.*
   (A) Stupid, Fear (B) Speedy, Alacrity
   (C) Sure, Slowness (D) Harmful, Grimaces

10. *A _____ glance pays _____ attention to details.*
   (A) Furtive; Meticulous (B) Cursory; Little
   (C) Cryptic; Close (D) Keen; Scanty

Instructions (11-20): Each question below consists of a related pair of words or phrases, followed by four lettered pairs of words or phrases numbered *(A), (B), (C), (D)*. Choose the lettered pair that best expresses a relationship similar to that expressed in the pair given in the question.

11. *INDUSTRIOUS: ASSIDUOUS*
   (A) Affluent: Impoverished (B) Fortuitous: Fortunate
   (C) Impecunious: Poor (D) Impartial: Biased

12. *DISBAND: ARMY*
   (A) Convene: Assembly (B) Muster: Platoon
   (C) Dissolve: Corporation (D) Abandon: Navy

13. *VIBRATION: SOUND*
   (A) Gravity: Pull (B) Watercolor: Paint
   (C) Accident: Death (D) Worm: Reptile

14. *FIRE: ASHES::*
   (A) Event: Memories (B) Accident: Delay
   (C) Wood: Splinters (D) Water: Waves

15. *LOYALTY: TRAITOR::*
   (A) Truthfulness: Liar (B) Longevity: Corne
   (C) Hope: Optimist (D) Understanding: Sage

16. *ENERVATE: STRENGTHEN*
   (A) Aver: Attribute (B) Divert: Turn
   (C) Apprise: Appraise (D) Stultify: Enliven

17. *LAUREL: VICTOR*
   (A) Chevrons: Army (B) Oscar: Movie Star
   (C) Power: Glory (D) Blue Ribbon: Cooking

18. *TEAMMATE: ADVERSARY::*
   (A) Felon: Criminal (B) Enemy: Associate
   (C) Pacifier: Agitator (D) Friend: Foe

19. *HACKNEYED: ORIGINAL::*

(A) Mature: Juvenile
(B) Withdrawn: Reserved
(C) Evasive: Elusive
(D) Derivative: Traditional

20. *SALVAGE: TREASURE*

(A) Settle: Argument
(B) Incorporate: Company
(C) Send: Correspondence
(D) Rescue: Victim

Read the following passage carefully and answer the question given at its end:

Something is radically wrong with the entire structure of human relationship that makes man delight in killing man, whether it be in the name of civilization or religion or anything else. Two wrongs do not make a right, hatred must beget hatred. It is this fundamental truth that women have got to bring home to the people in their respective countries. no peace treaties can avail that have revenge as their basis and self righteous arrogance and hypocrisy in the so called victors. But women are the natural preservers of life.

21. ***Which of the following expresses most accurately the idea contained in the opening sentence of the passage?***

(A) Man destroying another man is a painful practice.
(B) A social structure that permits people to kill each other for religion is inherently rotten.
(C) It is strange that one religion encourages its followers to kill the followers of another religion.
(D) It is wrong on man's part to derive pleasure out of killing others for any motive whatsoever.

22. ***The expression "Two wrongs do not make a right' means that:***

(A) A wrong action in retaliation does not mend matters.
(B) Hatred destroys the person who perpetrates it.
(C) A tit for tat policy aggravates hatred.
(D) Even repeated assertions of a wrong statement do not make it right.

23. ***Which of the following statements is not implied in the passage?***

(A) It is human tendency to kill others professing different religion.
(B) It is the duty of women to foster peace and harmony amongst their countrymen.
(C) Peace treaties among nations tend to establish peace in the world.
(D) If you hate someone, he is bound to respond with the same feeling.

24. ***Which word is opposite in meaning to 'preserver' as used in the passage?***

(A) Enemy
(B) Destroyer
(C) Rival
(D) Belligerent

25. ***Which of the following would sum up most suitably the central idea of the passage?***

(A) The role of women in the world of hatred and violence
(B) Man's instinct of destroying others
(C) Hatred leads to further hatred
(D) The significance of peace treaties

ANSWERS

1.	(B)	2.	(C)	3.	(D)	4.	(B)	5.	(B)
6.	(D)	7.	(B)	8.	(B)	9.	(B)	10.	(B)
11.	(C)	12.	(C)	13.	(A)	14.	(D)	15.	(A)
16.	(D)	17.	(B)	18.	(D)	19.	(A)	20.	(D)
21.	(D)	22.	(A)	23.	(C)	24.	(B)	25.	(A)

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GRE - GAT TEST 2

Quantitative Section	No. of Questions = 20

Q1. If p and q are two non-zero numbers and if $69(p + q) = (69 + p)q$, then which of the following must be true?

(A) $p = 69$ (B) $p + q = pq$
(C) $p < 1$ (D) $q = 69$

Q2. The Ravi Town is divided into p divisions. Each division has c cricket team, and each cricket team has x players. How many players are there in the entire town?

(A) pcx (B) $p + c + x$
(C) $\frac{pc}{x}$ (D) $\frac{px}{c}$

Q3. What is the value of x if $2^{10} \times 8^2 = 4^2 \times 2^x$?

(A) 2 (B) 8
(C) 10 (D) 12

Q4. A bag contains 35 cards, on each of which different integers from 1 to 35 are written. Hamza chooses a card randomly. He wins if the number on the card he chooses is a multiple of 3 or 7. What is the probability that Hamza to win?

(A) $\frac{2}{5}$ (B) $\frac{1}{2}$
(C) $\frac{1}{7}$ (D) $\frac{3}{7}$

Q5. Which of the following cannot be expressed as the sum of two or more consecutive positive integers?

(A) 24 (B) 26
(C) 32 (D) 19

Q6. An international conference has a total of x delegates from y countries. If each country is represented by the same number of delegates, how many delegates does each country has?

(A) $\frac{x}{y}$ (B) $\frac{y}{x}$
(C) xy^2 (D) yx^2

Q7. For how many positive numbers x is it true that

$$x \times x \times x = x + x + x$$

(A) 1 (B) 2
(C) 0 (D) 3

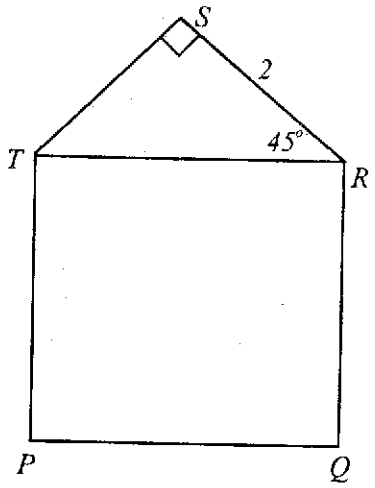
Q8. The lengths of the sides of a triangle are represented by $p + 3$, $2p - 3$ and $3p - 5$. If the perimeter of the triangle is 25, what is the length of the shortest side?

(A) 7 (B) 12
(C) 5 (D) 3

Q9. Fatima is now 5 times as old as Maryium, but after 6 years from now she will be 3 times as old as she will be then. How old is Maryium now?

(A) 25 (B) 18
(C) 12 (D) 30

Q10. What is the perimeter of pentagon $PQRST$, in the figure given below? Where $PQRS$ is



a square and RST is a right triangle?

(A) $2+2\sqrt{2}$ (B) $8+12\sqrt{2}$
(C) $4+6\sqrt{2}$ (D) $4+\sqrt{2}$

Q11. If $4-(4-m)=4$, then $m=$

(A) 4 (B) 8
(C) -4 (D) 12

Q12. Munir purchased some shares of stock at \$20 per share. Three months later the stock was worth \$40 per share. What was the percentage increase in the value of Munir's shares?

(A) 50% (B) 100%
(C) 200% (D) 400%

Q13. If $a^4=10$, then $a^6=?$

(A) 100 (B) 10000
(C) $10\sqrt{10}$ (D) $100\sqrt{10}$

Q14. If $5x=15$, then $3x=$

(A) 15 (B) 18
(C) 12 (D) 9

Q15. If $4x=144$, then $\frac{x}{4}=$

(A) 36 (B) 9
(C) 16 (D) 12

Q16. The following chart shows the value of an investment in January of each year from 2000 to 2005. In which year the percent increase in the value of the investment is the greatest?

Year	Values of Investment
2000	\$175
2001	\$275
2002	\$475
2003	\$775
2004	\$1225
2005	\$1825

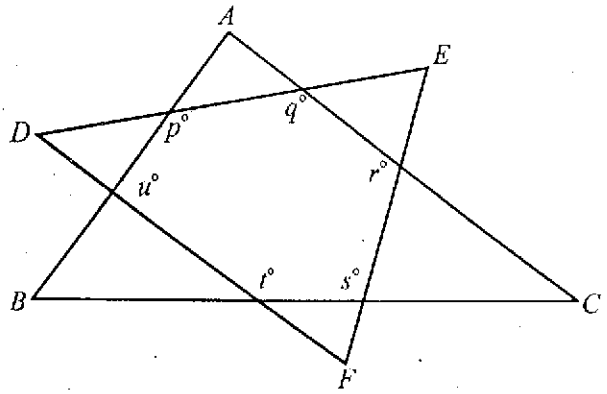
(A) 2001 (B) 2002
(C) 2003 (D) 2005

Q17. The average (Arithmetic Mean) of two numbers is m. If one of the number is 12, what is the other?

(A) $m - 6$ (B) $2m - 6$

(C) $m - 12$ (D) $2(m - 6)$

Q18.



In the figure above, what is the value of $p + q + r + s + t + u$?

(A) 380 (B) 180

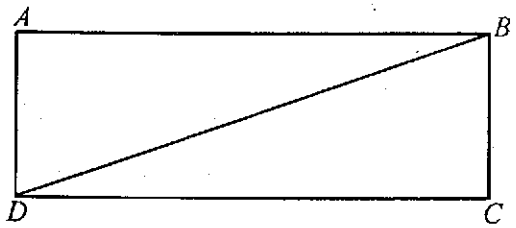
(C) 720 (D) 1260

Q19. If $x\left(\frac{3}{7}\right) = \left(\frac{3}{7}\right)y$, then $\frac{x}{y} =$

(A) 1 (B) $\frac{3}{7}$

(C) $\frac{7}{3}$ (D) $\frac{4}{3}$

Q20. If the perimeter of the rectangle $ABCD$ is 16, what is the perimeter of ΔBCD?



(A) 7 (B) 8

(C) $8\sqrt{2}$

(D) It is not possible to determine from the given information.

Explanatory Answers

Q1. (D) $69(p + q) = 69p + 69q$ (By distributive law)

$(69 + p)q = 69q + pq$ (//)

Then $69p + 69q = 69q + pq$

$\Rightarrow$ $69p = pq$

$\Rightarrow$ $q = 69$ (Dividing both sides by p)

Q2. (A) Since, Ravi Town is divided into p divisions and each division has C team. So, there are pc teams in Ravi Town. Now, because there are x players in each team, thus, there are $pc \times x = pcx$ players in Ravi Town.

Q3. $2^{10} \times 8^2 = 4^2 \times 2^x$

$\Rightarrow \quad 2^{10} \times (2^3)^2 \quad = (2^2)^2 \times 2^x$

$\Rightarrow \quad 2^{10} \times 2^6 \quad = 2^4 \times 2^x$

$\Rightarrow \quad 2^{10+6} \quad = 2^4 \times 2^x$

$\Rightarrow \quad \dfrac{2^{16}}{2^4} \quad = 2^x$

$\Rightarrow \quad 2^{16} \times 2^{-4} \quad = 2^x$

$\Rightarrow \quad 2^{16-4} \quad = 2^x$

$\Rightarrow \quad 2^{12} \quad = 2^x$

$\Rightarrow \quad x \quad = 12$

Q4. (D) Let E_1 be the event that the outcome is multiple of 3, then

$E_1 = \{3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33\}$

$\Rightarrow \quad n(E_1) = 11$

Let E_2 be the event that outcome is multiple of 7, then

$E_2 = \{7, 14, 21, 28, 35\}$

$n(E_2) = 5$

Now $E_1 \cup E_2 \quad = \{3,6,7,9,12,14,15,18,21,24,27,28,30,33,35\}$

$\Rightarrow \quad n(E_1 \cup E_2) \quad = 15$

$P(E_1 \cup E_2) \quad = \dfrac{15}{35}$

$= \dfrac{3}{7}$

Q5. (C) Any odd number can be expressed as the sum of two consecutive integers:

$19 = 9 + 10, 23 = 11 + 12, 24 = 7 + 8 + 9$

$26 = 5 + 6 + 7 + 8$

So eliminate *A, B, D* and *E*

Thus the answer is 32.

Q6. (A) Dividing the number of delegates by the number of countries, we have

$$\frac{x}{y}$$

Q7. (A) The given equation can be written as $x^3 = 3x$. Since x is positive, dividing each side of the equation by x, we have

$$x^2 = 3 \Rightarrow x = \pm\sqrt{3}$$

But x is positive, so there is only 1 positive integer that satisfies this equation.

Q8. Since the perimeter of the triangle is 19,

$p + 3 + 2p - 3 + 3p - 5 = 19$

$6p - 5 \quad = 19$

$\Rightarrow \quad 6p \quad = 19 + 5$

$\Rightarrow \quad 6p \quad = 24$

$\Rightarrow \quad \boxed{p \quad = 4}$

Now, substitute the value of p in the given sides, we get

$$4 + 3 = 7, 2(4) - 3 = 5, 3(4) - 5 = 7$$

Hence the shortest side is 5.

Q9. (D) Let x be the age of Maryium, then

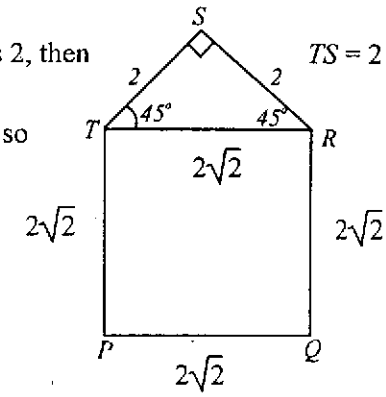
Now the age of Fatima is 5x.

Six years before "now" age of Maryium was $x + 6$ and age of Fatima was $5x + 6$. Then

$$5x + 6 = 3(x + 6)$$
$$\Rightarrow 5x + 6 = 3x + 18$$
$$\Rightarrow 5x - 3x = 18 - 6$$
$$\Rightarrow 2x = 12$$
$$\Rightarrow x = 6$$

Thus, age of Maryium is $5(6) = 30$ years.

Q10.(C) In the given figure RST is a 45 – 45 – 90 triangle, thus if one side is 2, then and $RT = 2\sqrt{2}$.



Since all sides of a square have equal length and $PQRT$ is a square, so its all sides are also equals to $2\sqrt{2}$.

Now perimeter of the pentagon $PQRST$ is given by

$$2\sqrt{2} + 2\sqrt{2} + 2 + 2 + 2\sqrt{2} = 4 + 6\sqrt{2}$$
$$= 2(2 + 3\sqrt{2})$$

Q11.(A) $4 - (4 - m) = 4$

$$\Rightarrow 4 - 4 + m = 4$$
$$\Rightarrow m = 4$$

Q12.(B) Increment in the share's $= 40 - 20 = 20\$$

Percentage increment $= \frac{20}{20} \times 100$

$= 100\%$

Q13.(C) $a^4 = 10$

$$\Rightarrow (a^2)^2 = 10 \Rightarrow \sqrt{(a^2)^2} = \sqrt{10}$$
$$\Rightarrow a^2 = \sqrt{10}$$

Now $a^6 = a^4 \times a^2 = 10 \times \sqrt{10}$

$$\Rightarrow a^6 = 10\sqrt{10}$$

Q14.(D) Given that $5x = 15$

$$\Rightarrow \frac{5x}{5} = \frac{15}{5} \Rightarrow x = 3$$
$$\Rightarrow 3x = 3 \times 3$$
$$\Rightarrow 3x = 9$$

Q15.(B) $4x = 144$

$$\Rightarrow x = \frac{144}{4} \Rightarrow x = 36$$

Now $\frac{x}{4} = \frac{36}{4} \Rightarrow \frac{x}{4} = 9$

Q16.(B) %age increase in a quantity

$$= \frac{\text{increment}}{\text{original}} \times 100$$

Now, we check the %age increment given in the table

In 2001, %age increase $= \frac{275 - 175}{175} \times 100$

$$= \frac{100}{175} \times 100 = 57.12$$

In 2002, %age increase $= \frac{475-275}{275} \times 100$

$$= \frac{200}{275} \times 100 = 72.73$$

In 2003, %age increase $= \frac{775-475}{475} \times 100$

$$= \frac{300}{475} \times 100 = 63.15$$

In 2004, %age increase $= \frac{1225-775}{775} \times 100$

$$= \frac{450}{775} \times 100 = 58.06$$

In 2005, %age increase $= \frac{1825-1225}{1225} \times 100$

$$= \frac{600}{1225} \times 100 = 48.98$$

Q17.(D) Let the other number be y, then by given condition

$m = \frac{12+y}{2} \quad \Rightarrow \quad 2m = 12 + y$

$\Rightarrow 2m - 12 = y$

$\Rightarrow \quad y = 2(m-6)$

Q18.(C) The interior side of the shape (star) is a six-sided figure (hexagon).
Now the sum of the angles of six-sided figure is $(n-2)180$
$\Rightarrow (6-2)180$
$\Rightarrow 4(180) = 720$

Q19.(A) $x\left(\frac{3}{7}\right) = \left(\frac{3}{7}\right)y$

$\Rightarrow x = y$ (Dividing both sides by $\frac{3}{7}$)

$\Rightarrow \frac{x}{y} = 1$

Q20. Since, we cannot find the value of BD from the given information. Therefore, we cannot find the area of $\Delta\, BCD$.

II. Analytical Section	No. of Questions = 20

For questions 1 to 4

A builder will build five houses in New Housing Scheme on a street that currently has no houses. The builder will select from seven different models of houses — *L, M, N, O, P, Q* and *R*. The Development Authority has placed the following restrictions on the builder: No model can be selected for more than one house. Either model *O* must be select or model *R* must be selected, but both cannot be selected. If model *Q* is selected, then model *N* cannot be selected. If model *M* is selected, then model *O* cannot be selected.

Q1. If model *M* is one of the models selected for the street, then which of the following models must also be selected?

(A) L (B) O
(C) P (D) Q
(E) R

Q2. If *L, M* and *P* are three of the models selected for the street, then which of the following must be the other two models selected?

(A) N and O (B) N and Q
(C) N and R (D) O and Q
(E) Q and R

Q3. Which of the following is an acceptable combination of models that can be selected for the street?

(A) *L, M, N, P, Q* (B) *L, M, P, Q, R*
(C) *L, N, P, Q, R* (D) *M, N, O, P, Q*
(E) *N, O, P, Q, R*

Q4. The model *R* is one model not selected for the street, then the other model NOT selected must be which of the following?

(A) *L* (B) *M*
(C) *N* (D) *O*
(E) *P*

For questions 5 to 7

An English speaking class in a college has a circular table with eleven seats around it. Five girls (Fatima, Maryam, Iram, Sana and Amna) and five boys (Bilal, Najam, Hamza, Osama, Javed) are seated around the table. None of the girls are seated in a seat adjacent to another girl. Fatima sits between Bilal and Najam, and next to each of them Javed does not sit next to Osama.

Q5. Which of the following is a possible seating order around the table?

(A) Empty seat, Bilal, Fatima, Najam, Maryam, Iram, Hamza, Osama, Amna, Javed and Sana
(B) Empty seat, Bilal, Fatima, Najam, Maryam, Javed, Amna, Sana, Osama, Iram, Hamza
(C) Empty seat, Bilal, Fatima, Najam, Sana, Javed, Amna, Osama, Iram, Hamza, Maryam
(D) Empty seat, Sana, Bilal, Fatima, Najam, Maryam, Javed, Osama, Amna, Hamza, Iram
(E) Empty seat, Iram, Bilal, Fatima, Najam, Maryam, Javed, Amna, Osama, Sana, Hamza

Q6. If Javed leaves his seat and occupies the empty seat, his new seating position would be between:

(A) Bilal and Fatima (B) Iram and Najam
(C) Fatima and Najam (D) Osama and Maryam
(E) Amna and Maryam

Q7. If Maryam, Hamza, Iram, Javed and Najam are seated in that order, which of the following is a correct completion of the seating order after Najam?

(A) Fatima, Bilal, Sana, Osama, Amna, empty seats
(B) Fatima, Bilal, Osama, Sana, empty seat, Amna
(C) Bilal, Amna, Fatima, Osama, Sana, empty seats
(D) Fatima, Bilal, Amna, Osama, empty seats, Sana
(E) Fatima, Bilal, Sana, empty seats, Amna, Osama

For questions 8 to 12

The accounts staff of the Mark corporation presently consists of three book-keepers (X, *Y* and Z) and five Data Entry Operators (*M, N, O, P* and Q). Management is planning to open a new office in another city sending three Data Entry Operators and two book-keepers from the present staff. To do so they plan to separate certain individuals who do not function well together. The following guidelines were established to set up the new office:

(i) Book-keepers *X* and *Z* are constantly finding faults with one another therefore should not be sent together to the new office.

(ii) Z and *N* function well alone but not as a team. They should be separated.

(iii) M and *P* have not been on speaking terms for many months. They should not go together.

(iv) Since *M* and *O* have been competing for a promotion, they should not be in one team. Based on the above information, find the correct answers to the following questions:

Q8. If *Y* insists on staying back then how many combinations are possible?

(A) 3 (B) 2
(C) 1 (D) None

Q9. If *X* is to be moved as one of the book-keepers, which of the following CANNOT be a possible working unit?

(A) XYMNQ (B) XYNOQ
(C) XYMPQ (D) XYNPQ

Q10. If *Z* is sent to the new office then which member of the staff CANNOT be sent?

(A) N (B) Y
(C) O (D) P

Q11. If *M* is sent to the new office then which of the following is a possible team?

(A) XYMNP (B) YZMOQ
(C) YZMNQ (D) XYMNQ

Q12. If both *Z* and *O* are moved to the new office, how many combinations are possible?

(A) 1 (B) 4
(C) 3 (D) 2

Direction: For questions 13 to 16

Each of the following problems has a question and two statements which are labeled 1 and 2. Use the data given in 1 and 2 together with other information given in the statement, and find a correct answer by using basic mathematics and everyday facts.

Q13. How many bulbs does Munir have?

1. He bought two boxes each containing 12 bulbs.

2. He lent three bulbs to Khalid.

(A) Statement 1. ALONE is sufficient but 2. ALONE is not sufficient to answer this question.

(B) Statement 2. ALONE is sufficient but 1. ALONE is not sufficient to answer this question.

(C) Statements 1 and 2. TOGETHER are sufficient to answer the question but NEITHER of them is sufficient ALONE.

(D) Statements 1 and 2. COMBINED are not sufficient to answer the question and additional information is needed to find the correct answer.

Q14. If $M > N$ and $O > P$, then, $M + O > N + P$. Is $S > T$?

1. $S + A > T + B$

2. $A > B$

(A) Statement 1. ALONE is sufficient but 2. ALONE is not sufficient to answer this question.

(B) Statement 2. ALONE is sufficient but 1. ALONE is not sufficient to answer this question.

(C) Statements 1 and 2. COMBINED are sufficient to answer the question but NEITHER of them is sufficient ALONE.

(D) Statements 1 and 2. COMBINED are not sufficient to answer the question and additional information is needed to find the correct answer.

Q15. In Lahore Zoo, there are 37 deer. How many small black deer are there?

1. 12 of deer are small.

2. There are 20 black deer in the Zoo.

(A) Statement 1. ALONE is sufficient but 2. ALONE is not sufficient to answer this question.

(B) Statement 2. ALONE is sufficient but 1. ALONE is not sufficient to answer this question.

(C) Statements 1 and 2. TOGETHER are sufficient to answer the question but NEITHER of them is sufficient ALONE.

(D) Statements 1 and 2. COMBINED are not sufficient to answer the question and additional information is needed to find the correct answer.

Q16. Can there be more than 150 pictures in a 30-page book?

1. There is at least two pictures in each page.

2. There are no more than 4 pictures in any page.

(A) Statement 1. ALONE is sufficient but 2. ALONE is not sufficient to answer this question.

(B) Statement 2. ALONE is sufficient but 1. ALONE is not sufficient to answer this question.

(C) Statements 1 and 2. TOGETHER are sufficient to answer the question but NEITHER of them is sufficient ALONE.

(D) Statements 1 and 2. COMBINED are not sufficient to answer the question and additional information is needed to find the correct answer.

Explanatory Answers

Solution 1 to 4

From the given paragraph, we write the following main points:

1. A builder will build five houses on a street that currently has no house on it.
2. There are 7 different models of houses *L, M, N, O, P, Q* and R.
3. The builder will select five different models from *L, M, N, O, P, Q* and R.
4. No model can be selected for more than one house.
5. Either model *O* must be selected or model *R* must be selected, but both cannot be selected.
6. If model *Q* is selected, then model *N* must also be selected.
7. If *M* is selected, then model *O* cannot be selected.

Q1. (E) If model *M* is selected, then builder should ignore *O*. We see from above point 5 that, either *O* must be selected or model *R* must be selected, but both cannot be selected. So, if builder cannot select *O*, he would select *R*. Hence the correct answer is choice E.

Q2. (C) If models *L*, M, and *P* have already been selected, then model *O* cannot be selected because *O* will not be selected if *M* has already selected. Since, *O* will not be selected, therefore *R* will be surely selected. Hence, the fourth model is *R*. Now, fifth model will be selected from *N* and Q. Here, we suggest *N* as a fifth model, because if the builder choose *Q* as a model, then model *N* must also be chosen, and there will be only five models must be selected. Hence the correct answer is choice C.

Q3. (C) Take choice "A", *L, M, N, P, Q*. This choice is not acceptable, because, either model *O* must be selected or model *R* must be selected. In this choice, any of them (O or R) is not present. Take choice

"B", *L*, M, *P*, *Q*, *R*. According to the given condition, if model *Q* is selected, then model *N* must also be selected. In this choice, *Q* is without N. Hence, this choice is also not acceptable. Take choice "C", *L*, N, *P*, *Q*, *R*. Since, this choice satisfies all the conditions.

Hence, the correct answer is choice C.

Q4. (B) If, the model *R* is one model not selected, then model *O* must be selected, because by the given condition, either model *O* must be selected or model *R* must be selected. Now in this case *O* must be selected, but *M* cannot be selected, because, according to the given restriction, if *M* is selected, then model *O* cannot be selected. Hence, the correct answer is choice B.

Solution 5 to 7

We simplify the given problem in the following points:

1. There are eleven seats around a circular table.
2. There are five girls (Fatima, Maryam, Iram, Sana and Amna) who will be seated.
   For convenience, we denote them by F, M, I, S and A.
3. There are five boys (Bilal, Najam, Hamza, Osama and Javed) who will be seated.
   For convenience, we denote them by *B*, N, H, *O* and J.
4. None of the girls are seated in a seat adjacent to another girl.
5. Fatima sits between Bilal and Najam, and next to each of them Javed does not sit next to Osama.

Q5. (E) Take choice, "A". The choice is not acceptable because in this choice, Maryam sits adjacent to another girl Iram. But by the given restriction none of the girls are seated in a seat adjacent to another girl. In choice "B", since Amna sits adjacent to Sana, so this choice is also not acceptable. Clearly, the choice "E" is the only choice that satisfies all the condition. Hence, the correct answer is choice E.

Q6. (E) The correct order of seating is

Empty, Iram, Bilal, Fatima, Najam, Maryam, Javed, Amna, Osama, Sana, Hamza.

If Javed leaves his seat empty then the above setting becomes Iram, Bilal, Fatima, Najam, Maryam, Empty, Amna, Osama, Sana, Hamza.

Clearly more setting is required between Maryam and Amna, because none of the girls are seated in a seat adjacent to another girl. Hence the correct answer is choice E.

Q7. (A) If Maryam, Hamza, Iram, Javed and Najam are seated in that order, the correct completion of the seating order is

Fatima, Bilal, Sana, Osama, Amna, empty seat.

Hence, the correct answer is choice A.

Solution 8 to 12

Brief points from the given paragraph are given below:

1. Accounts staff of Mark Corporation consists of three book-keepers (X, *Y* and Z) and five data entry operators (*M*, *N*, *O*, *P* and Q).
2. Management sending three Data entry operators and two book-keepers from the present staff to new office located to another city.
3. Book-keepers *X* and *Z* should not be sent together to the new office.
4. Z and *N* should be separated.
5. M and *P* should not go together.
6. M and *O* should not be in one team.

Q8. (D) Since, book-keepers *X* and *Z* are constantly finding faults with one another, therefore they should not be sent together to the new office. But the management has decided to sent two book-keepers, now, if *Y* insists on staying back, then there would not be another combination of two book-keepers except *X* and Z. Hence the correct answer is choice D.

Q9. (C) Since *M* and *P* have not been on speaking terms for many months, so they should not go together. Hence, choice "C" cannot be a possible working unit.

Q10.(A) If *Z* is sent to the new office, then *X* should not be sent and *Y* should be sent to the new office. Now, if *Z* is sent to new office, then *N* should not sent. Hence the correct answer is choice A.

Q11.(D) If *M* is sent to the office, then we analyze the given options as:

Choice "A", XYMNP, this choice is not acceptable because *M* and *P* cannot be sent together.

Choice "B", YZMOQ, this choice is also not acceptable, because *M* and *O* should not be in one team.

Choice "C", YZMNQ, because *Z* and *N* should be separated, therefore, this choice is also not acceptable.

Choice "D", XYMNQ, since this combination satisfies all the conditions, so this choice is acceptable. Hence, the correct answer is choice "D".

Q12.(A) If *Z* and *O* both are moved to the new office, then *M* should not be included in the team because *M* and *O* should not be in one team, also *N* should not be included in the team because *Z* and *N* should be separated. Therefore, only one combination YZOPQ, is possible, if *Z* and *O* both are moved to the new office. Hence, the correct answer is choice A.

Q13.(C) Take 1 statement, Munir bought two boxes each containing 12 bulbs, so

Munir initially has $12 \times 2 = 24$ bulbs

Now, take 2 statement, he lent three bulbs to Khalid, combining the two statements we find that Munir has 21 $(24 - 3 = 21)$ bulbs. So, statements 1 and 2 TOGETHER are sufficient to answer the question but neither of them alone is sufficient. Hence, the correct choice is choice C.

Q14.(C) If $M > N$ and $O > P$, then $M + O > N + P$

$S > T = ?$

Statement (1), $S + A > T + B$

Statement (2), $A > B$

We analyze the given problem, by supposing values of the variables involved in this problem

Let $S = 7$ and $T = 6$, then

and Let $A = 4$ and $B = 3$, then

clearly, $S > T$ and $A > B$, but

$S + A > T + B$

$7 + 4 > 6 + 3$

$11 > 9$

Hence, statements 1 and 2. TOGETHER are sufficient to answer the question but NEITHER of them alone is sufficient. Hence, the correct answer is choice C.

Q15.(D) Total deer in the Zoo = 37

Small deer = 12

Black deer = 20

Small black deer = ?

Since, a small deer may be black or not black, and a black deer may be small or not small, so we cannot find the exact answer from the given two statements, 1 and 2. Hence, the correct answer is choice D.

Q16.(B) Since, there are no more than 4 pictures in any page, so there are maximum $30 \times 4 = 120$ pictures in the 30 pages book. Thus, statement (2) ALONE is sufficient but (1) ALONE is not sufficient to answer the question.

III. Verbal Section	No. of Questions = 25
Select the correct answer for each question and blacken the corresponding circle in the answer sheet.	

Instructions (1-10): In this part of test, you have 10 MCQs about English. Each sentence below has one or two blanks, each blank shows that something has been omitted. Choose the correct answer from the four answer choices given with each question, numbered *(A)*, *(B)*, *(C)*, *(D)*.

1. ***He couldn't decide whether to read or study, he was _____.***
   - (A) Betwixt and between
   - (B) For and against
   - (C) Pulled down
   - (D) Over and out

2. ***Why are you still sleeping? You'll be late. _____ of bed now.***
   - (A) Out
   - (B) Get out
   - (C) Get up
   - (D) Up and out

3. ***You have to wait _____ Thursday.***
   - (A) Since
   - (B) From
   - (C) By
   - (D) Until

4. ***All the children went down _____ measles.***
   - (A) With
   - (B) From
   - (C) Due to
   - (D) Without

5. ***Ants live in colonies based on _____; each member contributes to the good of all by actively working with others in performing necessary tasks.***
   - (A) Heredity
   - (B) Individualism
   - (C) Cooperation
   - (D) Reasoning

6. ***A recent study indicates that the crime rate in the United States remains _____ and that one in three households _____ some form of major crime in any year.***
   - (A) Incredible, Witnesses
   - (B) Astronomical, Experiences
   - (C) Simultaneous, Perpetrates
   - (D) Defeated, Prosecutes

7. ***We got _____ the train _____ Rawalpindi.***
   - (A) On; In
   - (B) Into; At
   - (C) Into; In
   - (D) In; At

8. ***On the top of Mount Everest, I did not feel anything _____; I rather felt a great _____ to God.***
   - (A) Sinful; Love
   - (B) Superstitious; Similarity
   - (C) Superhuman; Closeness
   - (D) Strange; Nearness

9. ***A _____ is a _____.***
   - (A) Norm, Standard
   - (B) Criterion, Mistake
   - (C) Discipline, School
   - (D) Doctrine, Follower

10. ***You should _____ this paragraph in order to make your essay more _____.***
    - (A) Delete, Succinct
    - (B) Enlarge, Redundant
    - (C) Remove, Discursive
    - (D) Revise, Abstruse

Instructions (11-20): Each question below consists of a related pair of words or phrases, followed by four lettered pairs of words or phrases numbered *(A)*, *(B)*, *(C)*, *(D)*. Choose the lettered pair that best expresses a relationship similar to that expressed in the pair given in the question.

11. ***SILO: CORN***

(A) Mill: Grain (B) Reservoir: Water
(C) Acre: Wheat (D) Paddy: Rice

12. ***GAGGLE: GEESE***

(A) Coop: Chickens (B) Muzzle: Dogs
(C) Gill: Fish (D) Swarm: Bees

13. ***OBSTRUCTION: BUOY::***

(A) Construction: Building (B) Boy: Girl
(C) Danger: Red Light (D) Iceberg: Titanic

14. ***MARATHON: STAMINA::***

(A) Hurdle: Perseverance (B) Sprint: Celerity
(C) Relay: Independence (D) Ramble: Directness

15. ***INTEREST: FASCINATE***

(A) Vex: Enrage (B) Vindicate: Condemn
(C) Regret: Rue (D) Appall: Bother

16. ***HORNS: BULL::***

(A) Hoofs: Horse (B) Wings: Eagle
(C) Mane: Lion (D) Antlers: Stge

17. ***TOSS: HURL::***

(A) Speak: Shout (B) Sense: Flourish
(C) Prepare: Emit (D) Consider: Formulate

18. ***SHALE: GEOLOGIST::***

(A) Catacombs: Estomologist (B) Reef: Astrologer
(C) Obelisk: Fireman (D) Aster: Botanist

19. ***PERMEATE: REFUEL***

(A) Truculent: Merciful (B) Sadden: Pitiful
(C) Evaporate: Mournful (D) Penetrate: Sorrowful

20. ***MUMBLE: SPEAK::***

(A) Adorn: Denude (B) Convert: Preach
(C) Plagiarize: Write (D) Delimit: Expand

Read the following passages carefully and answer the questions given at its end:

Passage: Each nation has its own peculiar character which distinguishes it from others. But the people of the orld have more points in which they are all like each other than points in which they are different. One type of rson that is common in every country is the one who always tries to do as little as he possibly can and to get as uch in return as he can. His opposite, the man who is in the habit of doing more than is strictly necessary and is dy to accept what is offered in return, is rare everywhere. Both these types are usually unconscious of their aracter. The man who avoids effort is always talking about his rights, he appears to think that society owes him a asant easy life. The man who is always doing more than his sheer talks of duties feels that the individual is in

debt to society, and not society to the individual. As a result of their view, neither of these men thinks that he behaves at all strangely.

21. ***What type of person is common in every nation?***

(A) A person who want to do little and get more

(B) A person who want to do more and get little

(C) Each person is different

(D) There is no such type of person that is common in every country

22. ***A man who talks about his rights:***

(A) Avoids meeting other people

(B) Avoids hard work

(C) Knows his duties well

(D) Believes in hard work

23. ***Which one of the following thinks that the individual is in debt to society?***

(A) A person who talks of his rights only

(B) A person who is always doing more than his sheer talks of duties

(C) Every citizen of the country

(D) A person who talks of his duties only

Passage: A man who has no sense of history, Hitler declared is like a man who has no ears or eyes. He himself claimed to have had a passionate interest in history since his school days and he displayed considerable familiarity with the course of European history. His conversation was studied with historical reference and historical parallels. More than that Hitler's whole cast of thought was historical and his sense of mission was derived from his sense of history. Like his contemporary Spengler, Hitler was fascinated by the rise and fall of civilizations. He was himself born at a critical moment in European history when the liberal bourgeois world of nineteenth century was disintegrating. What would take its place? The future lay with the 'Jew-Bolshevik' ideology of the masses unless Europe could be saved by the Nazi racist ideology of the elite.

24. ***Who has no ears or eyes?***

(A) A man having sense of history

(B) A man who has no sense of history

(C) A man who has extra knowledge

(D) A man having passionate interest in history

25. ***Hitler displayed familiarity with:***

(A) Scientific facts (B) American history

(C) European history (D) None of these

ANSWERS

1.	(A)	2.	(D)	3.	(D)	4.	(A)	5.	(C)
6.	(D)	7.	(B)	8.	(D)	9.	(A)	10.	(A)
11.	(B)	12.	(D)	13.	(C)	14.	(B)	15.	(A)
16.	(D)	17.	(A)	18.	(D)	19.	(D)	20.	(C)
21.	(A)	22.	(B)	23.	(C)	24.	(B)	25.	(C)

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GRE - GAT TEST 3

I. Quantitative Section	No. of Questions = 20

Q1. **If n is a multiple of 5 and $m = 5n$, which of the following will be the value of $n + m$?**

I. 60 II. 100 III. 150

(A) I only (B) II only
(C) I and II only (D) I and III only

Q2. **If the sum of the edges of a cube is 48 inches, the volume of the cube in inches is:**

(A) 4096 cubic inches (B) 512 cubic inches
(C) 64 cubic inches (D) 1728 cubic inches

Q3. **The two numbers, whose sum is –3 and product is –40, are:**

(A) 15, –2 (B) 8, 5
(C) –8, 5 (D) –8, –5

Q4. **If $n + 3 = n \times 3$, then $n =$**

(A) 1 (B) $\frac{1}{2}$
(C) $1\frac{1}{2}$ (D) None of these

Q5. **The integral part of logarithm is called:**

(A) Characteristic (B) Mantissa
(C) Real (D) Imaginary

Q6. **Which of the following expression has the greatest value?**

(A) $3 \times 3 \div 3 + 3$ (B) $3 \div 3 + 3 + 3$
(C) $3 \times 3 - 3 \times 3$ (D) $3 \div 3 + 3 \times 3$

Q7. **On the x-axis, the y-coordinate is:**

(A) 1 (B) 0
(C) –1 (D) ∞

Q8. **If scales are bought at 35 rupees per dozen and sold at 3 scales for 10 rupees, the total profits on $5\frac{1}{2}$ dozen is:**

(A) 35 rupees (B) 23.5 rupees
(C) 26.5 rupees (D) 27.5 rupees

Q9. **If x books cost \$3 each and y books cost \$7 each, then the average (arithmetic mean) cost, in dollars per book, is equal to:**

(A) $\frac{3x + 7y}{10}$ (B) $\frac{x + y}{10}$
(C) $\frac{3x + 7y}{x + y}$ (D) $\frac{10}{x + y}$

Q10. **If $y < 2$ and $2x - 3y = 0$, which of the following must be true?**

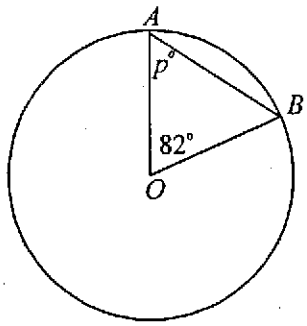
(A) $x > -3$ (B) $x < 2$

(C) $x = 3$ (D) $x < 3$

Q11. The angles of a pentagon are in the ratio 1:2:3:5:9. The smallest angle is:

(A) 72° (B) 45°
(C) 54° (D) 27°

Q12. In the following figure, what is the value of p?

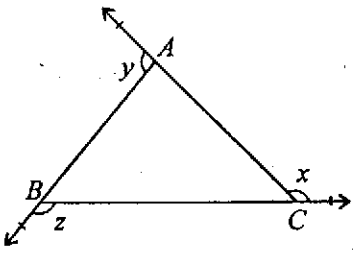


(A) 49 (B) 42
(C) 52 (D) 78

Q13. The number of cubic inches in one cubic foot is:

(A) 144 cubic inches (B) 1728 cubic inches
(C) 1000 cubic inches (D) 27 cubic inches

Q14. In the following figure, the sides of a triangle are produced. The sum of the exterior angles *i.e.*, $\angle x + \angle y + \angle z =$



(A) 180° (B) 360°
(C) 90° (D) 270°

Q15. In a right triangle, one of the angle is 60°. The side opposite to the angle is:

(A) $\frac{1}{2} \times$ hypotenuse (B) $\frac{1}{\sqrt{2}} \times$ hypotenuse
(C) $\frac{2}{3} \times$ hypotenuse (D) $\frac{\sqrt{3}}{2} \times$ hypotenuse

Q16. The number of degrees through which the hour hand of a clock moves in 2 hours and 12 minutes is:

(A) 72° (B) 66°
(C) 60° (D) 13°

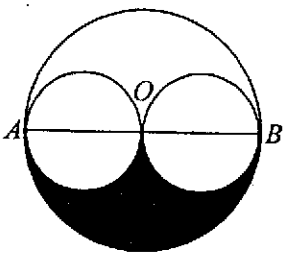
Q17. The height of a cylinder is 4 times its circumference, what is the volume of the cylinder in terms of its circumference, C?

(A) $\frac{C^3}{2\pi}$ (B) $\frac{2\pi}{C^3}$
(C) $\frac{2C^3}{\pi}$ (D) $\frac{\pi}{2C^2}$

Q18. What is the area of the circle which is inscribed in an equilateral triangle of side 24 cm^2?

(A) $24\pi\ cm^2$ (B) $36\pi\ cm^2$
(C) $48\pi\ cm^2$ (D) $18\pi\ cm^2$

Q19. In the following figure, the larger circle with radius 4 cms is touched internally by two smaller circles that also touch each other externally at the centre *O* of the larger circle. The area of the shaded region is:



(A) 4π (B) 7π
(C) 12π (D) 16π

Q20. The number of common tangents that can be drawn to two given circles at the most:

(A) one (B) two
(C) three (D) four

Explanatory Answers

Q1. (D) Since n is the multiple 5, *i.e.*,

$n = 5, 10, 15, 20, 25.....$

Now $m = 5n \Rightarrow m = 25, 50, 75, 100,.....$

Now $m + n = (5 + 25), (10 + 50), (15 + 75), (20 + 100), (25 + 125),.....$

$= 30, 60, 80, 120, 150$

Hence the correct answer is choice D.

Q2. (C) As we know, a cube is a rectangular solid in which length, width and height are equal. Note that, length, width and height are the edges of the cube. Let e be the edge of the cube, then according to given condition:

Since a cube has 12 edges, $12e = 48 \Rightarrow e = \frac{48}{12}$

$\Rightarrow e = 4$ inches

Now, the volume of a cube $= e^3 = (4)^3 = 64.$

Q3. (C) The product of two numbers could be negative only if one of them is negative and the other is positive

Now, $40 = 2 \times 2 \times 2 \times 5$

$= 8 \times 5$

If we take 8, –ive and 5 as positive then, $-8 \times 5 = 40$ and $-8 + 5 = -3$

Hence the required numbers are –8 and 5, and the correct answer is choice C.

Q4. (A) Given that $n + 3 = n \times 3$, we substitute the numbers in the options

When, $n = 1 \Rightarrow 1 + 3 \neq 1 \times 4, \Rightarrow 4 \neq 3$

When, $n = \frac{1}{2} \Rightarrow \frac{1}{2} + 3 \neq \frac{1}{2} \times 3, \Rightarrow 3.5 \neq 1.5$

When, $n = 1.5 \Rightarrow 1.5 + 3 = 1.5 \times 3 \Rightarrow 4.5 = 4.5$

Hence the correct answer is choice A.

Q5. (A) The common logarithms consists of two parts, the integral part is known as characteristic and the decimal part is known as Mantissa. Hence the correct answer is choice A.

Q6. (D) We solve expressions given in the options

$3 \times 3 \div 3 + 3 = 9 \div 6 = 1.5$

$3 \div 3 + 3 + 3 = 1 + 6 = 7$

$3 \times 3 - 3 \times 3 = 9 - 9 = 0$

$3 \div 3 + 3 \times 3 = 1 + 9 = 10$

Hence the correct answer is choice D.

Q7. (B) On the x-axis the y-coordinate are always zero. Hence the correct answer is choice B.

Q8. (D) We solve it by unitary method

12 scales costs = Rs. 35

1 scale cost $= \frac{35}{12}$

Now, 3 scales are sold = Rs. 10

1 scale will sold $= \frac{10}{3}$

Profit in one scale $= \frac{10}{3} - \frac{35}{12}$

$= \frac{5}{12}$

Profit of 66 scales ($5\frac{1}{2}$ dozen) $= \frac{5}{12} \times 66$

$= \frac{55}{2} = 27\frac{1}{2}$

Hence the correct answer is choice D.

Q9. (C) The total number of books is $x + y$, and their total cost is $3x + 7y$ dollars. Therefore, the average cost per book is

$$\frac{3x + 7y}{x + y} \text{ dollars.}$$

Thus, the best answer is choice C.

Q10.(D) It follows from $2x - 3y = 0$ that $y = \frac{2}{3}x$

So $y < 2 \quad \Rightarrow \frac{2}{3}x < 2 \Rightarrow 2x < 6$

$\Rightarrow x < 3$

Hence the best answer is choice D.

Note that choices A and B may be true.

But the exact and best choice is D.

Q11.(D) Sum of all angles of a pentagon = $[(2 \times 5 - 4) \times 90]$

$= (6 \times 90) = 540$

Let the angle be x, $2x$, $3x$, $5x$ and $9x$. Then

$x + 2x + 3x + 5x + 9x = 540$

$\Rightarrow \quad 20x = 540 \Rightarrow x = 27^\circ$

Q12.(A) In triangle AOB, since two of the sides are radii of the circles, and all the radius of a circle are equal, so the triangle is isosceles. Hence the unmarked angle is also p.

$180 = 82 + x + x \quad \Rightarrow 2x + 82 = 180$

$\Rightarrow 2x = 180 - 82 = 98$

$\Rightarrow x = \frac{98}{2} = 49$

Hence the correct answer is choice A.

Q13.(B) Volume of a cube $= e^3$

If $e = 1$ foot, then volume $= (1)^3$

As ($e = 1$ feet $= 12$ inches), volume $= (12 \text{ inches})^3$

$= 1728$ cubic inches

Hence the correct answer is choice B.

Q14.(B) Since every exterior angle is equal to the sum of opposite interior angles, so

$\angle x = \angle A + \angle B$, $\angle y = \angle B + \angle C$

and $\angle z = \angle A + \angle C$

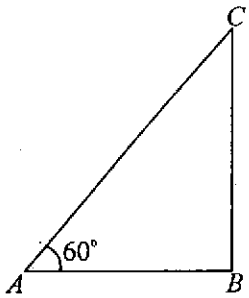
$\therefore \quad \angle x + \angle y + \angle z = 2(\angle A + \angle B + \angle C)$

$= 2(180)$

$= 360^\circ$

Hence the correct answer is choice B.

Q15.(D) Let ABC be a triangle in which $\angle A$ is 60°.



In ΔABC, $\frac{BC}{AC} = \sin 60^\circ$

$\Rightarrow \quad BC = (AC) \times \sin 60^\circ$

$\Rightarrow \quad BC = (\text{hypotenuse}) \times \frac{\sqrt{3}}{2}$

Hence the correct answer is choice D.

Q16.(B) The complete revolution of an hour hand subtends an angle of 360°. Since 360° is divided into 12 equal parts as an hour, thus each hour hand is $\frac{360}{12} = 30^\circ$ with minute hand. If we divide 30° into 5 parts we get 12 minute rotation hour hand, *i.e.*, $\frac{30}{5} = 6$. Hence the angle of the hour hand at 2 hour and 12 minute is

$30 \times 2 + 6 \quad = (60 + 6)^\circ$

$= 66^\circ$

Q17.(C) Volume of a cylinder $= V \quad = \pi r^2 h$

According to given condition, $h \quad = 4C$,

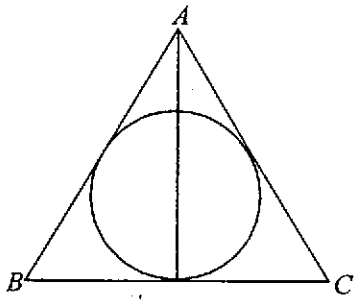
$\because C = 2\pi r \quad \Rightarrow \quad r \quad = \frac{C}{2\pi}$

$$\text{Now } V = \pi\left(\frac{C}{2\pi}\right)^2(4C)$$

$$\Rightarrow V = \frac{2C^3}{\pi}$$

Hence the correct answer is choice C.

Q18.(C) Here, we draw a figure



$$\frac{1}{2} \times 24 \times h = \frac{\sqrt{3}}{4} \times 24 \times 24$$

$$\Rightarrow \frac{h}{2} = \frac{\sqrt{3}}{4} \times 24$$

$$\Rightarrow \boxed{h = 12\sqrt{3}}$$

$\therefore$ $3r = 12\sqrt{3}$ or $\boxed{r = 4\sqrt{3}}$

$\therefore$ Area of the circle $= \pi r^2$

$= \pi(4\sqrt{3})^2 = \pi(16(3))$

$= 48\pi$

Hence the correct answer is choice C.

Q19.(B) Since the two smaller circles touch internally at the centre of the larger circle, they have equal radius; the diameter of each being 2 cm and radius of each is 1 cm.

Required Area = (Area of semi-circle with radius 4 cm) – (Area of semi-circle with radius 1 cm)

$= \frac{1}{2} \times \pi \times (4)^2 - 2 \times \frac{1}{2} \times \pi \times (1)^2$

$= 8\pi - \pi$

$= 7\pi$

Q20.(B) At the most two common tangents can be drawn to two circles.

II. Analytical Section	No. of Questions = 20

For questions 1 to 2

A city map representing roads *M, N, O, P, Q* and *R*. Link roads cannot have the same colour in the map. The roads link to each other are as under:

Each *M, N, P* and *Q* has link to *O*.

P has a link to Q.

Each of *M* and *N* has a link to R.

Q1. Which of the following roads can be the same colour as *O* on the map?

(A) N **(B)** P

(C) Q **(D)** R

Q2. Which of the following is a pair of roads that can be the same colour?

(A) M and N (B) N and O
(C) O and P (D) P and Q

Questions 3 to 8 depend on the following passage

A Government College sports president wishes to select four members of a sports-wing committee as special representatives to meet the requirements of college's sports activities.

The committee consists of eight members four of which (K, *L*, *M* and N) are sports teachers whereas the other four (*P*, *Q*, *R* and S) are students.

The president can select any four of the eight committee members as long as the following rules are observed:

The four representatives must consist of exactly two sports teachers and two students.

Either *K* or *L* must be one of the representatives but *K* and *L* both cannot be the representatives. If *P* is a representative then *M* must also be a representative.

If *R* is a representative then *L* cannot be a representative.

Q3. If *R* is a representative but *M* is not a representative then the whole group can be determined if it were also true that:

(A) K is a representative (B) N is a representative
(C) P is a representative (D) S is not a representative

Q4. If *P* is a representative then which of the following CANNOT be a representative?

(A) M (B) N
(C) Q (D) R

Q5. If *L* is a representative then which of the following can be the other three representatives?

(A) K, *Q* and S (B) M, *N* and P
(C) M, *P* and Q (D) N, *P* and S

Q6. If neither *Q* nor S is a representative then which of the following is a pair of teachers representatives?

(A) *K* and *L* (B) *K* and *M*
(C) *K* and *N* (D) *L* and *M*

Q7. If *L*, *N* and *Q* are representatives then which of the following must also be a representative?

(A) *M* (B) *P*
(C) *R* (D) *S*

Q8. If *K* and *N* are representatives then which of the following is not a representative?

(A) *Q* (B) *R*
(C) *P* (D) None

Two statements, labeled (1) and (2), follow each of the following given questions. The statements contain certain information. In the question you do not actually have to compute an answer, rather than you have to decide whether the information given in the statements (1) and (2) is sufficient to find a correct answer by using basic mathematics and everyday fact?

Q9. What day of the week is today?

1. **Today is December 25.**
2. **Amjad left Pakistan on Monday.**

(A) Statement 1. ALONE is sufficient but 2. ALONE is not sufficient to answer this question.
(B) Statement 2. ALONE is sufficient but 1. ALONE is not sufficient to answer this question.

(C) Statements 1 and 2. TOGETHER are sufficient to answer the question but NEITHER of them is sufficient ALONE.

(D) Statements 1 and 2. COMBINED are not sufficient to answer the question and additional information is needed to find the correct answer.

Q10. Can any of the four rivers be more than 200 meters wide?

1. The narrowest of the four rivers is 140 meters wide.

2. Average width of the four rivers is 200 meters.

(A) Statement 1. ALONE is sufficient but 2. ALONE is not sufficient to answer this question.

(B) Statement 2. ALONE is sufficient but 1. ALONE is not sufficient to answer this question.

(C) Statements 1 and 2. TOGETHER are sufficient to answer the question but NEITHER of them is sufficient ALONE.

(D) Statements 1 and 2. COMBINED are not sufficient to answer the question and additional information is needed to find the correct answer.

Q11. If it is raining then there must be clouds. Are there clouds?

1. Today is Saturday. It is not raining.

2. It rained Friday.

(A) Statement 1. ALONE is sufficient but 2. ALONE is not sufficient to answer this question.

(B) Statement 2. ALONE is sufficient but 1. ALONE is not sufficient to answer this question.

(C) Statements 1 and 2. TOGETHER are sufficient to answer the question but NEITHER of them is sufficient ALONE.

(D) Statements 1 and 2. COMBINED are not sufficient to answer the question and additional information is needed to find the correct answer.

For questions 12 to 13

There are seven cages next to each other in a zoo. The following is known about the cages. Each cage has only one animal, which is either a lion or a monkey. There is a lion in each of the first and last cages. The cage in the middle has a monkey. No two adjacent cages have monkeys in them. The monkey's cage in the middle has two lion cages on either side. Each of the other monkey cages are between and next to two lion cages.

Q12. How many cages have lions in them?

(A) 3 (B) 2

(C) 4 (D) 6

(E) 5

Q13. The monkey cage in the middle must have:

(A) No other monkey cage to its left. (B) No lion cage on its right.

(C) A lion cage to its left and to its right. (D) Other monkey cages next to it.

(E) No lion cage to its left.

For questions 14 to 16

Seven children — *M, N, O, P, Q, X* and *Y* are eligible to enter a drawing contest. From these seven, two teams must be formed, a blue team and a yellow team, each team consisting of exactly three of the children. No child can be selected for more than one team. Team selection is subject to the following restrictions: If *P* is on the blue team, *O* must be selected for the yellow team. If *M* is on the blue team, *Q*, if selected must be on the yellow team. *Q* cannot be on the same team as *X*. *N* cannot be on the same team as *O*.

Q14. Which of the following can be the three members of the blue team?

(A) M, *N* and O (B) M, *Q* and Y

(C) N, *O* and Y (D) O, *P* and Q

(E) *P*, *Q* and Y

Q15. If *P* and *M* are both on the blue team, the yellow team can consist of which of the following?

(A) N, *O* and Q
(B) N, *X* and Y
(C) O, *Q* and X
(D) O, *Q* and Y
(E) Q, *X* and Y.

Q16. If *P* is on the blue team, which of the following if selected, must also be on the blue team?

(A) M
(B) N
(C) Q
(D) X
(E) Y

Explanatory Answers

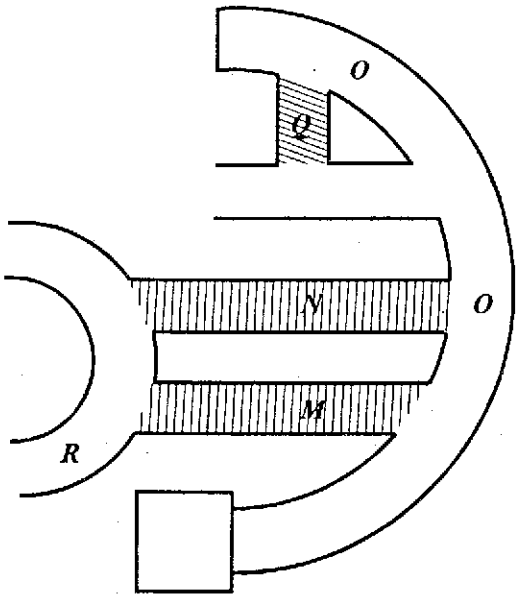
Solution 1 to 2

Here, first of all we decompose the given problem in the shape of important points.

1. A city map representing roads, *M, N, O, P, Q* and R.
2. Link roads cannot have the same colour on the map.
3. Each *M, N, P* and *Q* has link to *O*.
4. P has a link to Q.
5. Each of *M* and *N* has a link to R.

Q1. (D) Since, link roads cannot have the same colour on the map. Now, according to the given condition, each, *M, N, P* and *Q* has link to *O*. As *R* has no link to *O*, so its colour should be same as *O*. Hence, the correct answer is choice D.

Q2. (A) We draw the following estimated sketch of the roads on the map:



From above diagram, clearly roads *M* and *N* have no link each other. Hence, *M* and *N* should have the same colour. So, the correct answer is choice A.

Solution 3 to 8:

The important conclusions from the given problem are given below:

1. Sports committee consists of eight members.
2. There are four (K, *L, M* and N) sports teachers and four (*P, Q, R* and S) students in the committee.
3. President can select any four of the eight committee members.
4. The four representatives must consist of exactly two sports teachers and two students.
5. Either *K* or *L* must be one of the representatives but *K* and *L* both cannot be representatives.

6. If P is representative, then M must also be a representative.
7. If R is a representative then L cannot be a representative.

Q3. (D) Take choice "A", which says, K is a representative. We analyze the given statement, according to this statement, if R is representative, but M is not representative. If K is representative, then R combine K. Since M is not representative, so according to above point 6, P will also not representative. This situation is given in the following table "✓" indicates the representation and "×" represents "not representation".

✓	×	×	
K	L	M	N
×		✓	
P	Q	R	S

From above table, we find two groups, *i.e.*, KNRQ, KNRS. Thus we cannot find a single group. So, choice "A" is not correct choice. Now, if we accept choice "B", then the possible outcomes are given in the following table.

	×	×	✓
K	L	M	N
×		✓	
P	Q	R	S

It is clear from above table, that if we accept choice "B" then whole group cannot be determined. Now, take choice, "C", according to this choice, the following table formed.

	×	×	
K	L	M	N
✓		✓	
P	Q	R	S

It cannot be acceptable, because if P is accepted then M will also be accepted. But in the given statement M is not representative.

Lastly, we prepare the table according to the choice "D"

	×	×	
K	L	M	N
×		✓	×
P	Q	R	S

Thus a whole group KNQR can be determined. Hence, the correct answer is choice D.

Q4. (B) If P is representative then M must also be representative. Thus, choice "A" is not acceptable. The correct choice is choice "B".

Q5. (C) If "L" is a representative, then choice "A" is not acceptable, because K and L both cannot be representative. Choice B is also not acceptable, because there will three teachers (L, M, N) be joined in one group. The choice C is acceptable.

Q6. (B) If neither Q nor S is a representative, then choice "A" cannot be accepted, because K and L both cannot be representative. The choice "B" is acceptable. Hence the correct answer is choice B.

Q7. (D) If K, N and Q are representative, then choice "A" is not true choice, because if M join with K, N and Q, then the teacher representatives become three, but the four representative must consist of two teachers. Now, take choice, B, if we accept this choice then the group of representative is K, N, P, Q. But, according to the given condition, if P is representative then M must also be a representative. Here, P is without M, so this choice is also rejected. Now, if we accept choice "C" then the group of representative is K, N, Q, R, which is also not acceptable. If we accept choice "D", then the group of

representative becomes K, N, *Q*, S, which is a acceptable representative group. Hence, the correct answer is choice D.

Q8. (C) If *K* and *N* are representative, then, clearly, choice "A", which is "Q" is not acceptable. The second choice *R* is also not acceptable, because if *R* is a representative, then *L* cannot be a representative. Here *R* is with *K* not with L. So *R* may be form a representative group. Hence this choice is also not acceptable. The choice "C" is acceptable, because if *P* is representative then *M* must also be a representative. Then the group becomes KNMP. This group is not a representative because in this group there are three teachers (K, *N* and M) which is not according to the given condition. Hence, the correct answer is choice C.

Q9. (D) Since, there is not any link between two statements and statements (1) and (2) COMBINED are not sufficient to answer the question and additional information is needed to find the correct answer. The correct answer is choice D.

Q10.(C) Since, according to statement (1), the narrowest of the four rivers is 140 meters and according to the statement (2), average width of the four rivers is 200 meters. We are asked, can any of four rivers be more than 200 meters. Let we suppose the width of the rivers; $R_1 = 200$, $R_2 = 200$, $R_3 = 200$, $R_4 = 140$ meter.

$$\text{Average} = \frac{200 + 200 + 200 + 140}{4} = 185$$

But according to second statement, the average of four rivers is 200. It is only possible if any one or more rivers have their width more than 200. Hence, statements, (1) and (2) TOGETHER are sufficient to answer the question but NEITHER of them is sufficient ALONE.

Q11.(D) Since, Statements 1 and 2. COMBINED are not sufficient to answer the question and additional information is needed to find the correct answer.

Solution 12–13

We write the important points from the given problem briefly:

1. There are seven cages next to each other in the zoo.
2. Each cage has only one animal.
3. The animal is either a lion or a monkey.
4. There is a lion in the first and last cage.
5. No two adjacent cages have monkeys in them.
6. The monkey's cage in the middle has two lion cages on either side.
7. Each of the other monkey cages are between and next to two lion cages.

From above points we sketch the following diagram.

Lion	Monkey	Lion	Monkey	Lion	Monkey	Lion

1st cage Last cage

Q12.(C) From above diagram, clearly there are four lion cages.

Q13.(C) From above fig., we find two cages of lion to left and right of Monkey's cage.

Solution 14 to 16

Important points deduction from the given problem are given below:

1. Seven children — *M, N, O, P, Q, X* and *Y* are eligible to enter a drawing context.
2. From these seven children, two teams must be formed
   A blue team, a yellow team and each team consisting of exactly three of the children.
3. No child can be selected for more than one team.
4. If *P* is on the blue team, *O* must be selected for the yellow team.
5. If *M* is on the blue team, *Q*, if selected must be on the yellow team.

6. Q cannot be on the same team as *X*.
7. N cannot be on the same team as *O*.

Q14.(E) Take choice "A" that is M, *N* and *O*. This choice is not acceptable because *N* cannot be on the same team as *O*. Take choice "B", that is, *M*, *Q* and *Y*. This choice is also not acceptable, because *M* and *Q* could not be in the same team. Take choice "C", that is *N*, *O* and *Y*. Since *N* and *O* cannot be in the same team, so choice "C" also not acceptable. Take choice "D", that is, *O*, *P* and *Y*. Since *P* and *O* cannot be on the same team, so choice, *D*, also not acceptable. The correct choice is *E*, since it meets all the requirements given in the problem.

Q15.(D) Here, we draw the following table, according to the given condition.

Blue team	Yellow team
P	*O* ✓
M	Q
X	Y
N	

If P, M, *X* and *N* are in blue team, then *O* and *Q* must be in yellow team, *Y* can be either on blue and yellow team, so the yellow team may be *O*, *Q* and *Y*. Hence, the correct answer is choice D.

Q16.(B) By the given restrictions, if *P* is on the blue team, *O* must be selected for the yellow team. Similarly, if *M* is on the blue team, *Q* must be on the yellow team. The most important point to solve this problem is that, *Q* cannot be on the same team as *X* and *N* cannot be on the same team as *O*. Now, if *P* is on blue team *O* must be on yellow team, so *N* cannot be on the yellow team. Hence, the correct answer is choice B.

III. Verbal Section	No. of Questions = 25
Select the correct answer for each question and blacken the corresponding circle in the answer sheet.	

Instructions (1-10): In this part of test, you have 10 MCQs about English. Each sentence below has one or two blanks, each blank shows that something has been omitted. Choose the correct answer from the four answer choices given with each question, numbered *(A)*, *(B)*, *(C)*, *(D)*.

1. ***Normally an individual thunderstorm _____ about 45 minutes.***

(A) Lasts (B) Ends
(C) Remains (D) Continues

2. ***The _____ arguments put forth for not disclosing the facts did not impress anybody.***

(A) Specious (B) Intemperate
(C) Spurious (D) Convincing

3. ***Modern architecture has discarded the _____ trimming on buildings and emphasises simplicity of life.***

(A) Flamboyant (B) Flabbergasting
(C) Gaudy (D) Gaunt

4. ***I decided to sell a piece of land when I was offered a more _____ price.***

(A) True (B) Realistic
(C) Exact (D) Correct

5. ***Because of moon's _____, it has little or no substance.***

(A) Weak (B) Dull
(C) Frail (D) Unsubstantial

6. ***His habit of spending more than he earned left him in a state of perpetual _____ but he _____ hoping to see a more affluent day.***

(A) Indigence: Persevered In
(B) Confusion: Compromised By
(C) Enervation: Retaliated By

(D) Motion: Responded

7. *Come and stand _____ me _____ my umbrella or you will get quite wet.*
   (A) With: Under (B) By: Beneath
   (C) Beside: Under (D) Near: Below

8. *Hydrogen balloons, which were much _____ than hot-air balloons became very _____.*
   (A) Smaller; Cheap (B) Lighter; Popular
   (C) Cheaper; Fashionable (D) Brighter; Common

9. *Surprisingly enough, it is more difficult to write about the _____ than about the _____ and strange.*
   (A) Specific, Foreign (B) Abstract, Prosaic
   (C) Commonplace, Exotic (D) Simple, Routine

10. *Known for his commitment to numerous worthy causes, the philanthropist deserved _____ for his _____.*
   (A) Recognition: Folly (B) Blame: Hypocrisy
   (C) Reward: Modesty (D) Credit: Altruism

Instructions (11-20): Each question below consists of a related pair of words or phrases, followed by four lettered pairs of words or phrases numbered *(A), (B), (C), (D)*. Choose the lettered pair that best expresses a relationship similar to that expressed in the pair given in the question.

11. *VESSEL: FLEET::*
   (A) Wolf: Pack (B) Forest: Clearning
   (C) Vehicle: Truck (D) Carriage: Horse

12. *BUSTLE: MOVE::*
   (A) Hum: Sing (B) Lope: Run
   (C) Glide: Dance (D) Chatter: Talk

13. *PRECEDENT: JUSTIFICATION::*
   (A) Kindness: Obedience (B) Authority: Sanction
   (C) Usage: Submission (D) Tradition: Novelty

14. *NOTE: SCALE::*
   (A) Conductor: Orchestra (B) Singer: Music
   (C) Musician: Instrument (D) Letter: Alphabet

15. *STUDYING: LEARNING::*
   (A) Running: Jumping (B) Investigating: Discovery
   (C) Reading: Writing (D) Dancing: Swimming

16. *HELMET: HEAD::*
   (A) Pendant: Neck (B) Breastplate: Chest
   (C) Pedal: Foot (D) Knapsack: Back

17. *SOLDIER: REGIMENT*
   (A) Colonel: Martinet (B) Dancer: Balletomane
   (C) Singer: Chorus (D) Trooper: Rifle

18. *RETOUCH: PHOTOGRAPH::*
   (A) Finger: Fabric (B) Hang: Painting
   (C) Compose: Melody (D) Refine: Style

19. *YAWN: BORFDOM::*
   (A) Dream: Sleep (B) Anger: Madness
   (C) Smile: Amusement (D) Face: Expression

20. *BALLAD: SONG::*
   (A) Melody: Rhythm (B) Novel: Chapter
   (C) Portrait: Painting (D) Credit: Movie

Read the following passages carefully and answer the questions given at its end:

Suppression of people's feelings in any form not only provokes mental reaction but also amounts to asking for trouble. It is always hazardous to deny self-expression either to the born rebel or to the cool and satirical critic, for the former will go the whole hog in denouncing the policy of repression while the latter may reserve his right to support the government and may even turn ironical in his approach.

A tolerant government will see to it that it enlists the backing of all sections including the extremists and humorists. England has won a number of friends following the policy of tolerance by granting asylum to staunch opponents of their respective governments. Even from practical point of view, it is safe to allow people to vomit their venom against the government rather than deny their self-expression and incite them to go for pernicious activities.

Even if a government happens to be despotic, it will do well to provide and facilitate opportunities at self-expression. It will be well advised to avoid wasting money on security arrangements and go in for utilizing stipulated sum for facilitating the openings for expression. Hence no government, despotic or otherwise, should persecute people for holding political views contrary to their own.

21. ***It can be inferred that the author favours:***

(A) Suppression (B) Despotism
(C) Criticism (D) Self-expression

22. ***According to the passage:***

(A) Supporting government is good
(B) Denouncing government is good
(C) Even despotic governments should allow self-expression
(D) Denying self-expression is good

23. **The passage indicates that:**

(A) The government should allow people to go against it
(B) England has developed tolerance towards the opponents of the government
(C) Self-expression incites people towards destructive works
(D) All governments should restrict free expression

24. **Security arrangements to suppress self-expression are:**

(A) Preferable (B) Advisable
(C) Not advisable (D) None of the above

25. **Which of the following is incorrect ?**

(A) It is dangerous to deny self-expression
(B) A tolerant government wins over extremists and humorists
(C) It is safe to allow self-expression
(D) Government should persecute people for holding political view against it

ANSWERS

1.	(A)	2.	(A)	3.	(C)	4.	(B)	5.	(A)
6.	(A)	7.	(C)	8.	(B)	9.	(C)	10.	(D)
11.	(A)	12.	(D)	13.	(B)	14.	(D)	15.	(B)
16.	(B)	17.	(C)	18.	(D)	19.	(C)	20.	(C)
21.	(D)	22.	(C)	23.	(B)	24.	(C)	25.	(D)

****

GRE - GAT TEST 4

Quantitative Section	No. of Questions = 20

Q1. The sum of a number and its reciprocals is thrice the difference of the number and its reciprocal. The number is:

(A) $\pm\sqrt{2}$ (B) $\pm\frac{1}{\sqrt{2}}$

(C) $\pm\frac{1}{\sqrt{3}}$ (D) $\pm\sqrt{3}$

Q2. When the integer k is divided by 17, the quotient is p and the remainder is 5. When k is divided by 23, the quotient is q and the remainder is 14. Which of the following is true?

(A) $23p + 17q = 19$ (B) $14p + 5q = 6$.

(C) $17p - 23q = 9$ (D) $5p - 14q = 6$

Q3. What is the area of a circle whose radius is the diagonal of a square whose area is 9?

(A) 8π (B) 18π

(C) 3π (D) 9π

Q4. The circumference of a circle is $x\pi$ units, and the area of the circle is $y\pi$ square units. If $x = y$, then radius of the circle is:

(A) 1 (B) 2

(C) 3π (D) 2π

Q5. A cylindrical sillo (container for storing grain) has a diameter of 14 inches and a height of 6 inches. Since one gallon equals 231 cubic inches, the capacity of the sillo is approximately:

(A) 4 gallons (B) $1\frac{1}{7}$ gallons

(C) $2\frac{2}{7}$ gallons (D) $2\frac{1}{2}$ gallons

Q6. What is the volume of a cube whose surface area is 294?

(A) 125 (B) 216

(C) 294 (D) 343

Q7. The ratio of boys to girls in a certain classroom was 2 : 3. If boys represented five more than one-third of the class, how many people were in the classroom?

(A) 15 (B) 23

(C) 21 (D) 27

Q8. What is the average of 5^{30}, 5^{60}, 5^{17}, 5^{13} and 5^{90}?

(A) 5^{210} (B) 5^{209}

(C) $5^{29} + 5^{59} + 5^{16} + 5^{12} + 5^{89}$ (D) 5^{205}

Q9. If $p + 1 < 3p + 5$, then:

(A) $p < -2$ (B) $p > -2$

(C) $p = 0$ (D) $p > 2$

Q10. Which of the following numbers cannot be represented by a repeating decimal?

(A) $\frac{23}{7}$ (B) $\frac{13}{3}$

(C) 7 (D) $\sqrt{5}$

Q11. **If an equilateral triangle and a square have the same perimeter, what is the ratio of the lengths of the sides of the equilateral triangle to the lengths of the sides of the square?**

(A) 3 : 4 (B) 4 : 3
(C) 1 : 2 (D) 1 : 3

Q12. **If $\frac{1}{x}+\frac{1}{x}+\frac{1}{x}=12$, then $x=$**

(A) $\frac{3}{4}$ (B) $\frac{1}{4}$
(C) $\frac{1}{3}$ (D) $\frac{1}{12}$

Q13. **If $a+2b=x$ and $a-2b=y$, which of the following expression is equal to ab?**

(A) $\frac{x+y}{2}$ (B) $\frac{x-y}{2}$
(C) $\frac{x^2+y^2}{4}$ (D) $\frac{x^2-y^2}{8}$

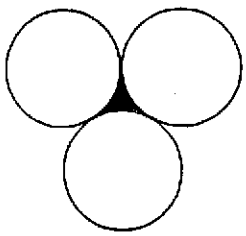
Q14. **If the sum of the two integers is 42 and their difference is 22. Then the greater of the two integers is:**

(A) 25 (B) 32
(C) 8 (D) 10

Q15. **Each of Nazir's buckets has a capacity of 11 gallons, while each of Osama's buckets can hold 8 gallons. How much more water in gallons can 7 of Nazir's buckets held compared to 7 of Osama's buckets?**

(A) 7 gallons (B) 9 gallons
(C) 21 gallons (D) 24 gallons

Q16.



In the above figure, three equal circles of radius 3 cm each touch each other, then the area of the shaded portion is:

(A) $\frac{3}{2}(\sqrt{3}-\pi)\text{ cm}^2$ (B) $\frac{9}{2}(2\sqrt{3}+\pi)\text{ cm}^2$
(C) $\frac{9}{2}(2\sqrt{3}-\pi)\text{ cm}^2$ (D) $\frac{\sqrt{3}}{2}(2-\pi)\text{ cm}^2$

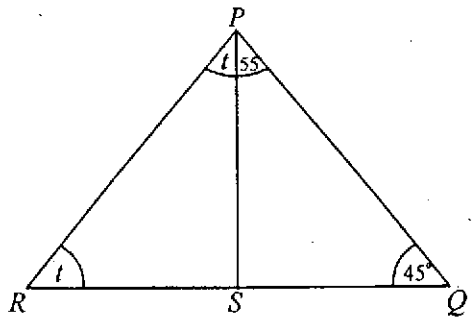
Q17. **If two circles touch each other externally at C and AB is a common tangent to the circles. Then $\angle ACB$ is:**

(A) greater than 120° (B) greater than 90°
(C) less than 90° (D) equal to 90°

Q18. **A train running between two towns arrives at its destination 10 minutes late when it goes 40 miles per hour and 16 minutes late when it goes 30 miles per hour. The distance between two towns is:**

(A) 12 (B) 720
(C) $8\frac{1}{2}$ (D) $12\frac{1}{7}$

Q19. What is the value of t in the following figure?



(A) 45 (B) 30
(C) 40 (D) 55

Q20. Two trains 132 meters and 108 meters in length are running towards each other on parallel lines, one at the rate of 32 km per hour and another at 40 km per hour. In what time will they be clear off each other from the moment they meet?

(A) 20 sec (B) 15 sec
(C) 12 sec (D) 29 sec

Explanatory Answers

Q1. (A) Let the number be x. Then, its reciprocal is $\frac{1}{x}$. According to the given condition,

$$x + \frac{1}{x} = 3\left(x - \frac{1}{x}\right)$$

$$\Rightarrow \frac{x^2+1}{x} = 3\left(\frac{x^2-1}{x}\right)$$

$$\Rightarrow x^2 + 1 = 3x^2 - 3 \quad \Rightarrow 3x^2 - x^2 = 1 + 3$$

$$\Rightarrow 2x^2 = 4$$

$$\Rightarrow x^2 = 2$$

$$\Rightarrow x = \pm\sqrt{2}$$

Hence the correct answer is choice A.

Q2. (C) According to the first condition, $k = 17p + 5$

According to the second condition $k = 23q + 14$

$\Rightarrow 17p + 5 = 23q + 14$

$\Rightarrow 17p - 23q = 9$

Hence the correct answer is choice C.

Q3. (B) Since the area of the square is 9, therefore, each side is 3. Thus the length of the diagonal is $3\sqrt{2}$. Now the area of the circle whose radius is $3\sqrt{2}$ is

Area $= \pi r^2 \quad \Rightarrow$ Area $= \pi(3\sqrt{2})^2$

$\Rightarrow$ Area $= \pi[9(2)]$

$\Rightarrow$ Area $= 18\pi$

Thus, the best answer is choice B.

Q4. (B) It is given:

Circumference, $C = x\pi$ and

Area, $A = y\pi$

$\because x = y \Rightarrow C = A \Rightarrow 2\pi r = \pi r^2$

$\Rightarrow \quad 2 = r$

Hence, the correct answer is choice B.

Q5. (A) Volume of the cylinder $= \pi r^2 h$

Here, $r = \frac{1}{2}$ diameter $= \frac{1}{2}(14) = 7$ inches

and Height, $h = 6$ inches

$$V = \pi(7)^2(6)$$
$$= \pi(49)(6)$$
$$= 294 \times \frac{22}{7} = 924 \text{ cubic inches}$$

$\because$ 231 cubic inches = 1 gallon

1 cubic inch $= \frac{1}{231}$ gallon

924 cubic inches $= \frac{1}{231} \times 924$ gallons

$= 4$ gallons

Hence the correct answer is choice A.

Q6. (D) Let e be the each face of the cube, then

$e^2 + e^2 + e^2 + e^2 + e^2 + e^2 = 6e^2 = 294$

$$\Rightarrow e^2 = \frac{294}{6} = 49$$

$$\Rightarrow e = 7$$

So each edges are all 7.

Hence the volume is $e^3 = 7^3 = 343$

The correct answer is choice D.

Q7. (A) Let x be the total number of boys in the class and b, be the total boys in the class, then by the given condition

$\frac{2}{5} \times x = b$ ...(i)

$\frac{1}{3}x = b - 5$

$\Rightarrow \frac{1}{3}x + 5 = b$ ...(ii)

Substituting the value of b from (ii) in (i), we have

$\frac{2}{3}x = \frac{1}{3}x + 5$

$\Rightarrow \frac{2}{3}x = \frac{x+15}{3} \Rightarrow 2x = x + 15$

$\Rightarrow \boxed{x = 15}$

The correct answer is choice A.

Q8. (C) $\frac{5^{30} + 5^{60} + 5^{17} + 5^{13} + 5^{90}}{5}$

$= \frac{5^{30}}{5} + \frac{5^{60}}{5} + \frac{5^{17}}{5} + \frac{5^{13}}{5} + \frac{5^{90}}{5}$

$= 5^{29} + 5^{59} + 5^{16} + 5^{12} + 5^{89}$

Hence the correct answer is choice C.

Q9. (B) $p+1 < 3p+5$

$\Rightarrow p+1-p < 3p+5-p$

$\Rightarrow 1 < 2p+5$

$\Rightarrow 1-5 < 2p+5-5$

$\Rightarrow -4 < 2p$

$\Rightarrow \frac{-4}{2} < \frac{2p}{2}$

$\Rightarrow -2 < p$

$\Rightarrow p > -2$

Hence, the correct answer is choice B.

Q10.(D) The square root of any prime number cannot be represented by a repeating decimal. Hence the correct answer is choice D.

Q11.(B) Let s be the side of equilateral triangle and t be the side of the square, then

Perimeter of triangle $= s+s+s = 3s$

Perimeter of square $= t+t+t+t = 4t$

Since the perimeter of the square and the triangle is equal, in other words $3s = 4t$

the two sides must be equal only if $s = 4$ and $t = 3$, hence the ratio between the sides of the equilateral triangle and a square is 4 : 3

Hence the correct answer is choice B.

Q12.(B) $\frac{1}{x}+\frac{1}{x}+\frac{1}{x} = 12$

$\frac{1+1+1}{x} = \frac{12}{1}$

$\Rightarrow \frac{3}{x} = \frac{12}{1} \Rightarrow 12x = 3$

$\Rightarrow x = \frac{3}{12}$

$\Rightarrow \boxed{x = \frac{1}{4}}$

Hence, the best answer is choice B.

Q13.(D) $a+2b = x$ ...(i)

$a-2b = y$ ...(ii)

$2a = x+y$ (Adding (i) and (ii))

$\Rightarrow a = \frac{x+y}{2}$ ...(iii)

Now, subtracting (ii), from (i), we get

$4b = x-y$

$\Rightarrow b = \frac{x-y}{4}$ ...(iv)

Now, multiplying (iii) and (iv), we get

$ab = \left(\frac{x+y}{2}\right)\left(\frac{x-y}{4}\right)$

$\Rightarrow \quad ab = \dfrac{x^2 - y^2}{8}$

Hence the correct answer is choice D.

Q14.(B) Let the two integers be x and y, then

$x + y = 42 \quad \ldots(i)$

$x - y = 22 \quad \ldots(ii)$

Adding (i) and (ii), we get

$2x = 64$

$\Rightarrow \boxed{x = 32}$

Subtracting (ii) from (i), we get

$2y = 20$

$\Rightarrow \boxed{y = 10}$

Clearly, the greater integer is x having value 32. Hence, the correct answer is choice B.

Q15.(C) Capacity of Nazir's bucket = 11 gallons

7 bucket of Nazir will contain water $= 11 \times 7 = 77$ gallons

Capacity of Osama = 8 gallons

7 buckets of Osama will contain water $= 8 \times 7 = 56$ gallons

The difference $= 77 - 56 = 21$ gallons

Hence the correct answer is choice C.

Q16.(C) Let A, B and C be the centres of the three circles, respectively. The ABC is an equilateral triangle with each side equal to 6 cm

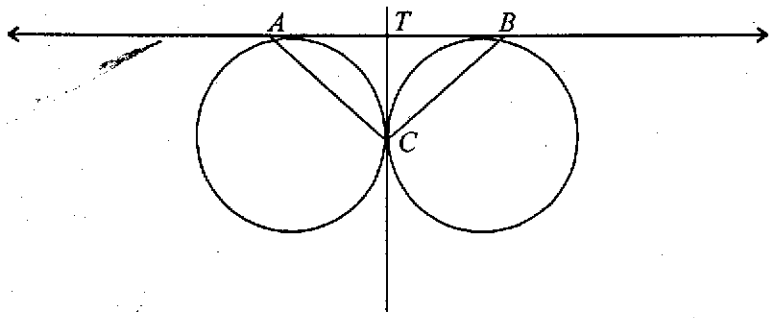
Required area = Area of $\Delta ABC - 3 \times$ area of sector of a circle with $r = 3$ and $\theta = 60^\circ$

$= \left[\dfrac{\sqrt{3}}{4} \times 36 - 3 \times \pi \times 3^2 \times \dfrac{60}{360}\right]$

$= \left[9\sqrt{3} - \dfrac{9\pi}{2}\right] = \dfrac{9}{2}(2\sqrt{3} - \pi)\text{cm}^2$

The correct answer is choice C.

Q17.(D) Let AB be a common tangent touching the circles at A and B



Let CT be the common tangent at C, meeting AB at T

Then, tangents to a circle from a point outside it being equal, we have

TA = TC & TB = TC

$\therefore \angle TAC = \angle TCA$ and $\angle TBC = \angle TCF$

So, $\angle ACB = \angle TCA + \angle TCB$

$= \angle TAC + \angle TBC$

$\Rightarrow 2\angle ACB = \angle TAC + \angle TBC + \angle ACB = 180^\circ$

$\Rightarrow \angle ACB = 90^\circ$

Hence the correct answer is choice D.

Q18.(A) Let the required distance be x km with two speeds, the difference of time taken is (16 – 10) 6 minutes

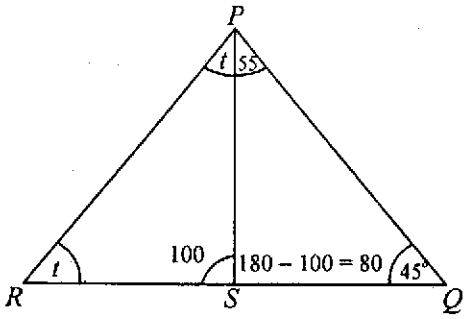
Using the formula

$$\text{Distance} = \frac{\text{Product of two speeds}}{\text{Difference of two speeds}} \times \text{Difference between time arrival}$$

$$= \frac{30 \times 40}{40 - 30} \times \frac{6}{60} = 12 \text{ km}$$

Hence the correct answer is choice A.

Q19.(C) We simplify the figure in the following manner:



$\because$ The sum of the angles of a triangle = 180

$\therefore t + t + 100 = 180 \quad \Rightarrow 2t + 100 = 180$

$\Rightarrow \quad 2t = 80 \quad \Rightarrow \quad \boxed{t = 40}$

Q20. Relative speed of the trains = (32 + 40)km/hour

= 72 km/h

$= \left(72 \times \frac{5}{18}\right)$ m/sec

= 20 m/sec

Time taken by the trains in passing each other

$$= \left[\frac{\text{Sum of lengths}}{\text{Relative speed}}\right]$$

$$= \left(\frac{240}{20}\right) \text{sec} = 12 \text{ sec}$$

Hence the correct answer is choice C.

II. Analytical Section	No. of Questions = 20

For questions 1 to 4

A carrier must deliver mail by making a stop at each of six buildings: S, T, U, V, W and *X*. Mail to be delivered are of two types, ordinary mail and priority mail. The delivery of both types of mail is subject to the following conditions: Regardless of the type of mail to be delivered, mail to W and mail to *X* must be delivered, mail to W and mail to *X* must be delivered before mail to U is delivered. Regardless of the type of mail to be delivered, mail to T and mail to S must be delivered before mail to *X* is delivered. Mail to buildings receiving some priority mail must be delivered, as far as the above conditions permit, before mail to buildings receiving only ordinary mail.

Q1. If S is the only building receiving priority mail, which of the following lists the buildings in an order, from first through sixth, in which they can receive their mail?

(A) *T, S, W, X, V, U* (B) *T, S, X, W, U, V*

(C) *S, T, W, U, X, V* (D) *S, W, T, X, V, U*
(E) *V, S, T, W, X, U*

Q2. If T, U and *X* are each receiving priority mail, which of the following lists the buildings in an order, from first through sixth, in which they can receive mail?

(A) *S, T, W, X, V, U* (B) *T, S, V, W, X, U*
(C) *T, S, X, W, U, V* (D) *U, T, X, W, S, V*
(E) *X, T, U, W, S, V*

Q3. If the sequence of buildings to which mail is delivered is V, W, T, S, X, U and if *X* is receiving priority mail, which of the following is a complete and accurate list of buildings that must also be receiving priority mail?

(A) *V, T* (B) *V, W*
(C) *W, T* (D) *W, U*
(E) *V, W, T, S*

Q4. If only one building is to receive priority mail, and as a result, V can be no earlier than fourth in the order of buildings, which of the following must be the building receiving priority mail that day?

(A) *S* (B) *T*
(C) *U* (D) *W*
(E) *X*

Questions 5–9

During 2006, from January through June, the Chairman of Physics Department will be on Sabbath. The Dean of College has asked each of the college six professors in the department — Akhter, Bilal, Chohan, Fraz, Hamid and Noman — to serve as acting chairman during one of these months. The physicists can decide the order in which they will serve, subject only to the following criteria established by the dean.

i. Chohan will serve as chairman in February.

ii. Akhter will serve as chairman before Hamid does.

iii. Bilal and Fraz will serve as chairman in consecutive months.

Q5. Which of the following professors could serve as chairman in January?

(A) Bilal (B) Chohan
(C) Fraz (D) Hamid
(E) Noman

Q6. In how many ways can the schedule be made up if Noman has to serve as Chairman in May?

(A) 1 (B) 3
(C) 6 (D) 4
(E) 2

Q7. If Noman serves in April, all of the following could be true except:

(A) Akhter serves in January (B) Hamid serves in March
(C) Bilal serves in May (D) Bilal serves in June
(E) Hamid serves in June

Q8. If Bilal serves in May, what is the latest month in which Akhter could serve?

(A) March (B) April
(C) January (D) February
(E) June

Q9. Which of the following CANNOT be true?

(A) Akhter and Noman serve in consecutive months
(B) Noman and Hamid serve in consecutive months
(C) Hamid and Fraz serve in consecutive months
(D) Akhter and Chohan serve in consecutive months
(E) Bilal and Chohan serve in consecutive months

Questions 10–14

During practice matches, before a major tournament, in a football ground, one team can practice at a time. There are seven teams — the Argentine, the Brazil, the Senegal, the Dubai, the England, the France and the Germany. The football ground is open seven evenings a week from Monday to Sunday (Sunday being considered the last day of the week), and the allocation of the practice times is governed by the following rules:

(i) On any evening, only one team can play.
(ii) The Argentine must practice on Monday.
(iii) The Dubai practice exactly one day before the France practice.
(iv) The France practice exactly one day before the Germany practice.
(v) The Senegal and the Brazil must practice earlier in the week than the England.

Q10. The latest day in the week that the Brazil can practice is:

(A) Tuesday
(B) Wednesday
(C) Thursday
(D) Friday
(E) Saturday

Q11. If a person went to the football ground on three consecutive evenings, her or she could see which of the following teams in the order listed?

(A) the France, the Germany, the Senegal
(B) the France, the Germany, the Dubai
(C) the Argentine, the Dubai, the Senegal
(D) the Brazil, the Senegal, the France
(E) the Dubai, the England, the France

Q12. On week, the Senegal practiced on Wednesday and the Dubai practiced the next day. That week, the Brazil must have practiced on:

(A) Monday
(B) Tuesday
(C) Friday
(D) Saturday
(E) Sunday

Q13. If the Germany practice on Thursday, the England and the Dubai must practice on which days, respectively?

(A) Sunday and Tuesday
(B) Saturday and Tuesday
(C) Friday and Wednesday
(D) Wednesday and Thursday
(E) Tuesday and Monday

Q14. If the France practice on Saturday, the England must practice on what day?

(A) Tuesday
(B) Wednesday
(C) Thursday
(D) Friday
(E) Sunday

Questions 15 to 17

At a meeting of the Ruling Party, the seven top party leaders, who are all cabinet ministers, are seated on a platform in order of rank the Prime Minister being in the center. The closer a person is to the Prime Minister; the higher is his/her rank. Moreover, a person sitting on the right of the Prime Minister outranks the one sitting equidistant on the left of the Prime Minister. The seven leaders are *L, M, N, O, P, Q* and R.

Q is four places left to the Minister of Agriculture, who is two places to the right of N.
M's neighbours are *L* and the Minister of Agriculture *R* is two places to the left of *O*.
The Minister of Education, Mining and Culture are seated together, in order, from left to right.
The remaining Ministers are these of Social Welfare and Defence.

Q15. The fifth ranking person in the party hierarchy is:

(A) R, the Minister of Mining (B) Q, the Minister of Culture
(C) O, the Prime Minister (D) P, the Minister of Defence

Q16. How many of the seven party leaders outrank the Minister of Education?

(A) 2 (B) 3
(C) 4 (D) 5
(E) 6

Q17. The lowest ranking Minister is:

(A) Minister of Education (B) Minister of Social Welfare
(C) Minister of Mining (D) Minister of Defense
(E) Minister of Culture

Explanatory Answers

Solution 1-4

Here, we illustrate the given problem into important points:

1. A courier must deliver mail by making a stop at each of the six buildings: S, T, U, V, W and *X*.
2. There are two types of mail: Ordinary mail and priority mail.
3. Mail to W and mail to *X* must be delivered before mail to U be delivered.
4. Mail to T and mail to S must be delivered before mail to *X* is delivered.

Q1. (D) If S is the only building receiving priority, mail. Then building S will be the first on priority. Since, mail to W and mail to *X* must be delivered before mail to U is delivered. Then the first four mails in the list are SWXU. Now, according to the point 4 mail T and mail S must be delivered before mail X, so the above list becomes after this condition SWTXVU, which is the correct list.

Q2. (C) Since mail to T and mail to S must be delivered before mail to X, but in this problem T has a priority, so the same elements of the list are T, S, *X* according to point (3) and given priority the list becomes, T, S, X, W, U, V. Hence the correct answer is choice C.

Q3. (B) Clearly V and W is the receiving priority mail. Hence, the correct answer is choice "B".

Q4. (E) If V can be no earlier than fourth in the order, then the receiving priority must be *X*. Because in the list T, S, X, W, U, and V, V replaces with *X*. Hence, the correct answer is choice E.

Solution 5–9

Let *A, B, C,* F, H and *N* represents professors Akhter, Bilal, Chohan, Fraz, Hamid and Noman, respectively. Then, from the given problem, we find the following important points:

1. Chohan will serve as chairman in February.
2. Akhter will serve as chairman before Hamid does. That is A < H.
3. Bilal and Fraz will serve as chairman in consecutive months. That is

$$B << F \text{ and } F << B$$

Q5. (E) Since, Chohan will serve as chairman in February, so Bilal and Fraz could not serve because Bilal and Fraz will serve as chairman in consecutive months. Because, Akhter will serve as chairman before Hamid, now. Hamid could not serve as chairman in February. So Akhter will not serve in January. So, only person Noman is there, which does not violate any of the conditions. Hence, the correct answer is the choice E.

Q6. (E) If Chohan serve as chairman in February, and Noman serves in May, then the possible schedule is given by:

January	February	March	April	May	June
Akhter	Chohan	Bilal	Fraz	Noman	Hamid
Akhter	Chohan	Fraz	Bilal	Noman	Hamid

Hence the correct answer is choice E.

Q7. (E) If Noman serves in April, then the possible schedule is given as:

January	February	March	April	May	June
Akhter	Chohan	Hamid	Noman	Bilal	Fraz
Akhter	Chohan	Hamid	Noman	Fraz	Bilal

From above schedule, it is possible that Akhter can serve as chairman in January, so choice A is not correct choice. It is also clear from above table that Hamid can serve in March, so choice *B* is also not correct choice. Choice, *C* and *D* not correct choices, because Bilal can serve as chiarman both in May and June as shown in the above table. Hence, *C* and *D* are also not correct choices. Only, choice "E" is not possible. Hence the correct answer is choice E.

Q8. (A) If Bilal serve in May, then the possible schedule for Akhter is given as:

January	February	March	April	May	June
January	Chohan	Hamid	Fraz	Bilal	Fraz
Noman	Chohan	Akhter	Hamid	Bilal	Fraz

From above table it is clear that Akhter can serve in January and March. In which, March is the latest month. Hence, the correct choice is choice A.

Q9. (A) The only professors that can serve in January are Akhter and Noman, so one of them must serve in January, and neither in February. So, Akhter and Noman cannot serve as chairman in consecutive months. Hence, the choice A cannot be true. Hence, the correct answer is choice A.

Q10.(B) From the given rules, one of the schedules is given as under:

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Argentine	Senegal	Brazil	Dubai	France	Germany	England

Hence, the correct answer is choice B.

Q11.(A) From the following schedule given in the table, we find that the correct answer is choice A.

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Argentine	Brazil	Dubai	France	Germany	Senegal	England

Q12.(B) If Senegal practiced on Wednesday and the Dubai practiced the next day, the new schedule is given below:

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Argentine	Brazil	Senegal	Dubai	France	Germany	England

So the Brazil will practice on Tuesday. Hence the correct answer is choice B.

Q13.(A) If the Germany practice on Tuesday, then the new schedule may be as:

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Argentine	Dubai	France	Germany	Brazil	Senegal	England

From above table, we find that England and Dubai will practice on Sunday and Tuesday respectively. Hence, the correct answer is choice A.

Q14.(C) If the France practice on Saturday, then the new schedule is given by:

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Argentine	Brazil	Senegal	England	Dubai	France	Germany

From above schedule, under the given rules, if France practice on Saturday, then England must practice on Thursday. Hence the correct answer is choice C.

Solution 15 to 17

From the given problem, we sorted out the following points:

1. The seven party leaders are, *L, M, N, O, P, Q* and R.
2. Prime Minister is in the center.
3. The closer a person to the Prime Minister; the higher is his/her rank.
4. A person sitting on the right of the Prime Minister outranks the one sitting equidistant on the left of the Prime Minister.
5. Q is four places to the left of the Minister of Agriculture, who is two places to the right of N.
6. M's neighbours are *L* and the Minister of Agriculture.
7. R is two places to the left of *O*.
8. The minister of Education, Mining and Culture are seated together, in order from left to right.
9. The remaining Ministers are those of Social Welfare and Defense.

From above points we draw a following sketch:

Culture	Mining	Education	P.M	Minister of Agriculture	Social Welfare	Defense
Q	R	N	O	P	M	L
4	5	6		1	2	3

Q15.(A) From above table, clearly, the fifth rank of the party person is R, who is the Minister of Mining. Hence, correct answer is choice A.

Q16 (E) Including P.M. there are six party leaders outrank the Minister of Education. It is clear from the table.

Q17.(A) From table, it is clear that Minister of Education has the lowest rank. Hence, the correct answer is choice A.

****

III. Verbal Section	No. of Questions = 25
Select the correct answer for each question and blacken the corresponding circle in the answer sheet.	

Instructions (1-10): In this part of test, you have 10 MCQs about English. Each sentence below has one or two blanks, each blank shows that something has been omitted. Choose the correct answer from the four answer choices given with each question, numbered *(A), (B), (C), (D)*.

1. ***Viruses are invisible through the _____ microscope; but we know that they are there because we can see the _____ they cause.***

(A) Cheap; Damage (B) Elementary; Harm
(C) Simple; Danger (D) Ordinary; Havoc

2. ***The sea was coming after me as high as a great hill and _____ as a _____ enemy.***

(A) Furious; Charging (B) Dreadful; Advancing
(C) Terrible; Charging (D) Angry; Attacking

3. ***Although its publicity has been _____, the film itself is intelligent, well-acted, handsomely produced and altogether _____.***

(A) Tasteless; Respectable (B) Extensive; Moderate
(C) Sophisticated; Moderate (D) Risqué; Crude

4. ***It takes _____ character to _____ the extremities of the arctic region.***

(A) An unflappable; Sustain (B) A dictatorial; Brook
(C) A Spartan; Negotiate (D) An inimitable; Resist

5. ***As a journalist who works to overturn erroneous convictions, Griffin Nicholson was opposed to the court ruling _____ appeals for inmates who might be _____.***

(A) Barring; Culpable (B) Curbing; Exonerated
(C) Encouraging; Innocent (D) Scrutinizing; Eligible

6. *Their conversation was unsettling, for the gravity of their topic contrasted so oddly with the _____ of their tone.*
(A) Uniqueness (B) Rapidity
(C) Lightness (D) Precision

7. *The driver suddenly applied the brakes when he saw a _____ truck ahead of him.*
(A) Stationary (B) Moving
(C) Static (D) Immobile

8. *Salma is much too _____ to have anything to do with that obnoxious affair.*
(A) Noble (B) Proud
(C) Happy (D) Difficult

9. *Roberto Clement was seen as _____ during his life because of both his selflessness on the baseball field and his humanitarian work in his native Nicaragua.*
(A) An individualist (B) A grandstander
(C) A sybarite (D) An altruist

10. *After passing through a great trauma of her husband's death, she _____ hard to achieve mental relaxation.*
(A) struggled (B) struggling
(C) struggle (D) to struggle

Instructions (11-20): Each question below consists of a related pair of words or phrases, followed by four lettered pairs of words or phrases numbered *(A), (B), (C), (D)*. Choose the lettered pair that best expresses a relationship similar to that expressed in the pair given in the question.

11. *CORPULENCE: STOUT::*
(A) Boldness: Hirsute (B) Erudition: Learned
(C) Gauntness: Beautiful (D) Competence: Strict

12. *INDIGENT: WEALTH::*
(A) Emaciated: Nourishment (B) Aristocratic: Stature
(C) Variegated: Variety (D) Contended: Happiness

13. *RUN: RACE::*
(A) Walk: Pogo stick (B) Swim: Boat
(C) Fly: Kite (D) Sink: Bottle

14. *HOBLE: WALK::*
(A) Gallop: Run (B) Stumble: Fall
(C) Sniff: Smell (D) Stammer: Speak

15. *STATIC: MOVEMENT::*
(A) Humdrum: Excitement (B) Chronic: Timeliness
(C) Ecstatic: Decay (D) Diligent: Industry

16. *STICKLER: INSIST::*
(A) Trickster: Risk (B) Braggart: Boast
(C) Laggard: Outlast (D) Mumbler: Enunciate

17. *CONCERT: MUSIC::*
(A) Performance: Artist (B) Exhibition: Art
(C) Play: Actor (D) Operetta: Singer

18. *INFRACTION: LAW::*
(A) Renovation: Structure (B) Punishment: Crime
(C) Enactment: Amendment (D) Interruption: Continuity

19. *TIRADE: ABUSIVE::*
(A) Diatribe: Familial (B) Satire: Pungent
(C) Panegyric: Laudatory (D) Eulogy: Regretful

20. *REHEARSAL: PERFORMANCE::*
(A) Applause: Audience (B) Engagement: Marriage
(C) Entrapment: Game (D) Antidote: Illness

Read the following passages carefully and answer the questions given at its end:

A duty is an obligation. It is something we owe to others as social beings when we live together. We must let others live with us. May right of living implies my duty to my fellowmen to allow them the same conditions of life. Infact, rights and duties are co-related. What is a right in regard to one is a duty in regard to others. Rights and duties are two sides of the same coin. We should always observe from the stand point of others. Thus they are duties. Moral duty is more effective than the legal. A moral duty is that which is upon the people on moral grounds. It is my moral duty to help the poor because of being a member of the society.

I must try to create these conditions which contribute to the welfare of humanity. Similarly, I owe a duty to my parents–to be obedient and respectful to them. This duty originates from the sense of responsibility which is directly related with our conscience. So this maxim is concerned with a moral duty which a man should owe without the legal bondage.

Sense of duty is paramount for the proper development of civilization in the 20th century. Hypocrisy and diplomacy are quite reverse to the sense of duty. Hypocrisy involves wickedness duty involves sincerity and faithfulness.

21. ***Rights and duties according to the passage are:***
- **(A)** Co-related
- **(B)** Two sides of the same coin
- **(C)** Neither of the above
- **(D)** Both *(A)* and *(B)*

22. ***According to the author, the moral duty is:***
- **(A)** Enjoined upon animals
- **(B)** Helping the rich
- **(C)** More effective than the legal duty
- **(D)** Secondary to the legal duty

23. ***Where does the duty originate from ?***
- **(A)** From responsibility but not from conscience
- **(B)** From responsibility which is related with conscience
- **(C)** From legal bondage
- **(D)** From hypocrisy and diplomacy

24. ***Point out the incorrect statement.***
- **(A)** Hypocrisy and diplomacy do not support the sense of duty
- **(B)** Hypocrisy involves wickedness
- **(C)** Duty involves sincerity
- **(D)** Sense of duty is not important for the development of civilization

25. ***The author:***
- **(A)** Has sense of responsibility
- **(B)** Doesn't have sense of duty
- **(C)** Both *(A)* and *(B)*
- **(D)** Neither *(A)* nor *(B)*

ANSWERS

1.	(D)	2.	(A)	3.	(A)	4.	(C)	5.	(C)
6.	(C)	7.	(A)	8.	(A)	9.	(B)	10.	(A)
11.	(B)	12.	(A)	13.	(C)	14.	(D)	15.	(A)
16.	(B)	17.	(B)	18.	(D)	19.	(D)	20.	(B)
21.	(D)	22.	(C)	23.	(B)	24.	(D)	25.	(A)

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NTS TESTS WITH EXPLANATORY ANSWERS

NTS TEST NO. 1

◆ Select the correct answer for each question and blacken the corresponding circle in the answer sheet.

Q1. The area of the circle that is inscribed in a square of area 4, is:

(A) $\frac{\pi}{2}$ (B) π

(C) $\frac{\pi}{3}$ (D) 4π

Q2. If the angles of a five-sided polygon are in the ratio of 2 : 3 : 4 : 4 : 5, what is the measure of the smallest angle?

(A) 30° (B) 60°

(C) 45° (D) 210°

Q3. In a certain city, x gallons of gasoline are needed per month for each car. How long will y gallons last at this rate given that there are z cars in the town?

(A) $\frac{x}{zr}$ (B) $\frac{yz}{x}$

(C) $\frac{z}{xy}$ (D) $\frac{y}{zx}$

Q4. What is fraction of two weeks with 36 minutes?

(A) $\frac{1}{840}$ (B) $\frac{1}{120}$

(C) $\frac{1}{20160}$ (D) $\frac{1}{560}$

Q5. Let T = total area of five circles of radius r and let S = total area of three circles of radius u. If $T = S$, then $\frac{r}{u} =$

(A) $\sqrt{\frac{5}{3}}$ (B) $\frac{3}{5}$

(C) $\frac{\sqrt{3}}{\sqrt{5}}$ (D) 3π

Q6. A sum of Rs. 7000 is divided among A, B and C in such a way that shares of A and B are in the ratio 2 : 3 and those of B and C are in the ratio 4 : 5. The amount received by C is:

(A) Rs. 3900 (B) Rs. 3000

(C) Rs. 2800 (D) Rs. 2600

Q7. If $x : y = 2 : 5$, then $(3x + 4y) : (4x + 5y) =$

(A) 6 : 20 (B) 9 : 21

(C) 26 : 33 (D) 16 : 20

Q8. If the postal charges for a package are 62 cents for the first five ounces and 8 cents for each additional ounce, what is the weight of a package for which the charges are \$1.66? (16 ounces = one pound approx):

(A) 1.125 pounds (B) 1.1 pounds

(C) 1.3 pounds (D) 0.8 pounds

Q9. Which of the following must be true?

I. $(36 - 81) = (6 - 9)(6 + 9)$

II. $5(16 + 7) = 5(16) + 5(7)$

III. $5 \div (10 - 1) = (5 \div 10) - (5 \div 1)$

(A) I only (B) II only

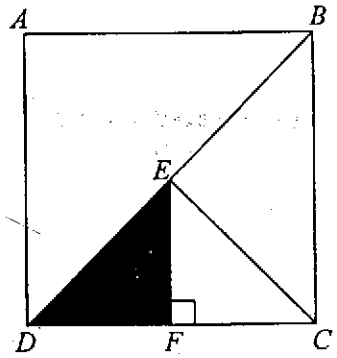
(C) I and II only (D) II and III only

Q10. Two candidates contested an election. One got 65% of the votes and won by 300 votes. The total number of votes polled is:

(A) 1200 (B) 1000

(C) 800 (D) 600

Q11. In square ABCD below, if DE = EB and DF = FC, then, what is the fraction of the area of the shaded region to the area of square region ABCD?



(A) $\frac{1}{3}$ (B) $\frac{1}{8}$

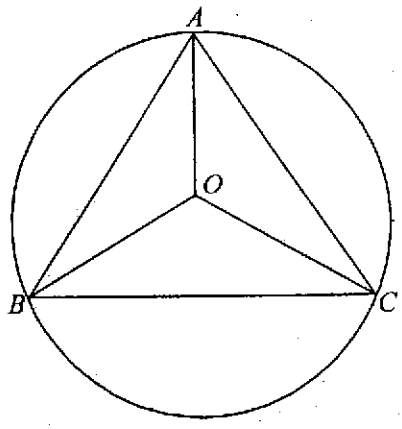
(C) $\frac{1}{16}$ (D) $\frac{1}{6}$

Q12. In a certain pizza, two straight cuts of different radii succeed in removing $\frac{4}{15}$ of the total pizza. What is the central angle in degrees of the piece cut?

(A) 85 (B) 92

(C) 96 (D) 60

Q13. In the figure below, *O* is the centre of the circle. If $\angle OBC = 25^\circ$, then $\angle BAC$ is equal to:



(A) 150° (B) 65°

(C) 30° (D) 25°

Q14. If $x : y$ is 7 : 6 and $3y : 2z$ is 2 : 3, what is $\frac{z}{x}$?

(A) $\frac{14}{27}$ (B) $\frac{27}{14}$

(C) $\frac{7}{9}$ (D) $\frac{6}{7}$

Q15. Fatima is having a party, at 7:00 P.M., guests begin arriving at a uniform rate of 8 people every 15 minutes. If this pattern continues, how many guests will have arrived at 9:00 P.M.?

(A) 80 (B) 64

(C) 40 (D) 20

Q16. For positive integers x and y, if $x^2 + 2y^2 = 41$, and $2x^2 + y^2 = 34$, then $x^2 =$

(A) $\frac{41}{2}$ (B) 9

(C) $\frac{27}{2}$ (D) 20

Q17. The value of $\left[\frac{(0.05)^2 + (0.41)^2 + (0.073)^2}{(0.005)^2 + (0.041)^2 + (0.0073)^2}\right]$ is:

(A) 0.1 (B) 10

(C) 100 (D) 1000

Q18. Aslam sells a chair at a gain of $7\frac{1}{2}$%. If he had bought it at a $12\frac{1}{2}$% less and sold it for Rs. 5 more, he would have gained 30%. The cost price of the chair is:

(A) Rs. 72 (B) Rs. 80

(C) Rs. 88 (D) Rs. 96

Q19. For developing pictures, Modern Photo Lab., charges a service fee of \$6 for every order it receives in addition to a printing fee. If the order consists of 12 pictures or less, the printing fee per picture is \$0.24. If the order consists of more than 12 pictures, the printing fee per picture is \$0.16. What is the total cost per picture for an order consisting of 30 pictures.

(A) \$0.44 (B) \$0.16

(C) \$4.8 (D) \$0.36

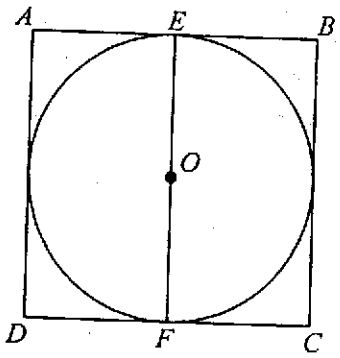
Q20. If the interior angle of a regular polygon is 11 times its exterior angle, the number of sides of the polygon is:

(A) 11 (B) 18

(C) 24 (D) 22

Explanatory Answers

Q1. (B) First of all, we draw a diagram



Since the area of square ABCD is 4

$\therefore$ AD = 2. Also, diameter EF = 2 and

radius OE = OF = $\frac{2}{2} = 1$

Area of the circle = $\pi r^2 = \pi(1)^2 = \pi$

Hence the correct answer is choice B.

Q2. (B) The sum of the angles of a five-sided polygon is $(5 - 2) \times 180 = 3 \times 180 = 540$.

Now, $540 = 2x + 3x + 4x + 4x + 5x \Rightarrow 18x = 540$

$\Rightarrow x = 30$

The measure of the smallest angle is $= 2 \times 30 = 60^\circ$

Hence the correct answer is choice B.

Q3. (D) Each car needed gasoline = x gallons

z cars needed gasoline = zx gallons

y gallons required at this rate = $\frac{y}{zx}$

Hence the correct answer is choice D.

Q4. (D) There are 24 hours in a day, and 24×14

= 336 hours in two weeks, also 336×60

= 20,160 minutes in two weeks. Thus the fraction with 36 minutes is $\frac{36}{20,160} = \frac{1}{560}$.

Hence the correct answer is choice D.

Q5. (C) Since T is the total area of the 5 circles,

therefore, $T = 5(\pi r^2)$

Now, S is the total area of the 3 circles of radius u, therefore $S = 3(\pi u^2)$

$\because T = S$ (given)

$5\pi r^2 = 3\pi u^2$

$\Rightarrow \frac{r^2}{u^2} = \frac{3\pi}{5\pi} \Rightarrow \frac{r^2}{u^2} = \frac{3}{5}$

$\Rightarrow \frac{r}{u} = \sqrt{\frac{3}{5}} = \frac{\sqrt{3}}{\sqrt{5}}$

Hence the correct answer is choice C.

Q6. (B) A : B = 2 : 3 and B : C = 4 : 5

$= \frac{3}{4} \times 4 : \frac{3}{4} \times 5 = 3 : \frac{15}{4}$

$\therefore A : B : C = 2 : 3 : \frac{15}{4} = 8 : 12 : 15$

$\therefore$ C's share = Rs. $\left[7000 \times \frac{15}{35}\right]$ = Rs. 3000

Hence the correct answer is choice B.

Q7. (C) Given that $\frac{x}{y} = \frac{2}{5}$

Now, $\frac{3x+4y}{4x+5y} = \frac{3\left(\frac{x}{y}\right)+4}{4\left(\frac{x}{y}\right)+5}$

$= \frac{3\times\frac{2}{5}+4}{4\times\frac{2}{5}+5} = \frac{\frac{6}{5}+4}{\frac{8}{5}+5}$

$= \frac{\frac{6+20}{5}}{\frac{8+25}{5}} = \frac{26}{33}$

Hence, $(3x + 4y) : (4x + 5y) = 26 : 33$

Thus, correct answer is choice C.

Q8. (A) Since charges on first five ounces = 62 cents. Now, 1.66 dollars = 100 + 66 = 166 cents. Thus, first 62 cents are charged for five ounces, then by remaining cents will be 166 – 62 = 104. Since 8 cents are received for additional ounce. Therefore, 104 is received for $\frac{104}{8}$ = 13 ounces. The total package is

$13 + 5 = 18$ ounces.

$\because$ 16 ounces = 1 pound

1 ounces $= \frac{1}{16}$ pound

18 ounces $= \frac{1}{16} \times 18$ pounds

$= 1.125$ pounds

Hence the correct answer is choice "A".

Q9. (C) Take I, $(36 - 81) = (6 - 9)(6 + 9)$ is true because $[(6)^2 - (9)^2] = (6 - 9)(6 + 9)$, as $a^2 - b^2 = (a + b)(a - b)$

Take II, $5(16 + 7) = 5(16) + 5(7)$ is also true, because according to distributive property of multiplication over addition

$x(a + b) = xa + xb$

Now, take III, $5 \div (10 - 1) = (5 \div 10) - (1 \div 5)$ is not a true statement. Hence the correct answer is choice C.

Q10.(B) Let the total number of votes polled be x.

Then 65% of x – 35% of x = 300

$\Rightarrow$ 30% of x = 300

$\therefore \frac{30}{100} \times x = 300$ or $x = \frac{300 \times 100}{30} = 1000$

Hence the correct answer is choice B.

Q11.(B) $\because$ DE = EB and DF = FC, the area of the shaded region is one-fourth the area of triangular region BCD, since BD divides square ABCD into two equal triangular regions, the shaded region is $\left(\frac{1}{2}\right)\left(\frac{1}{4}\right)$, or $\frac{1}{8}$, of the area of square region ABCD. Hence, the best answer is B.

Q12.(C) Let x be the total prize and let y be the required angle then setting the proportion, we have

$x : 360 :: \frac{4}{15}x : y$

$\frac{x}{360} = \frac{4x}{15} \quad \Rightarrow xy = \frac{4x}{15} \times 360$

$\Rightarrow y = 4 \times 24 = 96^\circ$

Hence the correct answer is choice C.

Q13.(B) OB = OC $\Rightarrow \angle OCB = \angle OBC = 25^\circ$

So, $\angle BOC = [180 - (25 + 25)] = 130^\circ$

$\therefore \angle BAC = \frac{1}{2}\angle BOC$

$= 65^\circ$

Hence the correct answer is choice B.

Q14.(B) $x : y = 7 : 6 \quad \Rightarrow \frac{x}{y} = \frac{7}{6}$ ...(i)

also $3y : 2z = 2 : 3 \Rightarrow \frac{3y}{2z} = \frac{2}{3}$

$\Rightarrow \frac{y}{z} = \frac{4}{9}$ ...(ii)

$\Rightarrow \frac{z}{y} = \frac{9}{4}$ ...(iii)

dividing equation (iii) by (i), we get

$$\frac{\frac{z}{y}}{\frac{x}{y}} = \frac{\frac{9}{4}}{\frac{7}{6}}$$

$\Rightarrow \frac{z}{y} \times \frac{y}{x} = \frac{9}{4} \times \frac{6}{7}$

$\Rightarrow \boxed{\frac{z}{x} = \frac{27}{14}}$

Hence the correct answer is choice B.

Q15.(B) Guests will arrive according to the following schedule:

7 : 15	_____	8
7 : 30	_____	8
7 : 45	_____	8
8 : 00	_____	8
8 : 15	_____	8
8 : 30	_____	8
8 : 45	_____	8
9 : 00	_____	8
Total guests		64

Hence the correct answer is choice B.

Q16.(B) $x^2 + 2y^2 = 41$ ...(i)

$2x^2 + y^2 = 34$ ...(ii)

Subtracting equation (i) from (ii), we get

$2x^2 + y^2 = 34$

$$\underline{x^2 + 2y^2 = 41}$$
$$x^2 - y^2 = -7 \quad \ldots(iii)$$

adding (ii) and (iii), we get

$$2x^2 + y^2 = 34$$
$$x^2 - y^2 = -7$$
$$3x^2 = 27$$
$$x^2 = \frac{27}{3}$$
$$\Rightarrow \boxed{x^2 = 9}$$

Hence the correct answer is choice B.

Q17.(C) Given that, Let $a = .05$, $b = 0.41$ and $c = .073$

$$= \frac{a^2 + b^2 + c^2}{\left(\frac{a}{10}\right)^2 + \left(\frac{b}{10}\right)^2 + \left(\frac{c}{10}\right)^2}$$
$$= \frac{100(a^2 + b^2 + c^2)}{(a^2 + b^2 + c^2)} = 100$$

Hence the correct answer is choice C.

Q18.(B) Let C.P = Rs. 100. Then, first S.P = Rs. 107.50

New C.P = Rs. 87.50 and gain on it = 30%

$\therefore$ Second S.P = 130% of 87.50 = 113.75

Difference in two selling prices = Rs. (113.75 – 107.50)

= Rs. 6.25

$\therefore$ Actual S.P $= \text{Rs.}\left[\frac{100}{6.25} \times 5\right]$

= Rs. 80

Hence the correct answer is choice B.

Q19.(D) Service fee = \$6

Charges of 30 pictures @, \$0.16 per picture = 30 × 0.16 = 4.8

Total charges = \$(6 + 4.8) = \$10.8

Cost per picture $= \frac{10.8}{30} = \$0.36$

Hence the correct answer is choice D.

Q20.(C) Each exterior angle $= \frac{360}{n}$, and

Each interior angle $= \left[180 - \frac{360}{n}\right]$

$$\therefore \quad 180 - \frac{360}{n} = 11 \times \frac{360}{n}$$
$$\Rightarrow \quad \frac{180n - 360}{n} = \frac{11 \times 360}{n}$$
$$\Rightarrow \quad 180n - 360 = 3960$$
$$\Rightarrow \quad 180n = 3960 + 380 \Rightarrow n = \frac{4340}{180}$$
$$\Rightarrow \quad \boxed{n = 24}$$

NTS TEST NO. 2

◆ Select the correct answer for each question and blacken the corresponding circle in the answer sheet.

Q1. The sum of the lengths of all the edges of a cube is 4 centimeters. What is the volume in cubic centimeters of the cube?

(A) $\frac{1}{8}$ (B) $\frac{1}{27}$

(C) 64 (D) 27

Q2. A 6-foot long cylindrical pipe has an inner diameter of 8 feet and outer diameter of 10 feet. If the total surface area (including inside, outside and ends) is $x\pi$, what is the value of x?

(A) 20 (B) 118

(C) 100 (D) 109

Q3. November is the busiest month at Panorama centre, Lahore, where sale in November is 40 percent higher than the average. If sale in March is typically 20 percent lower than the average, what is the ratio of the March sale to November sale?

(A) 2 : 3 (B) 2 : 1

(C) 3 : 2 (D) 4 : 7

Q4. How many 3-digit numbers are there that consist of only odd digits?

(A) 125 (B) 625

(C) 12500 (D) 225

Q5. If $(x+y) : (x-y) = 1 : 5$, then $(x^2 - y^2) : (x^2 + y^2)$ equals:

(A) 2 : 3 (B) 1 : 2

(C) 5 : 13 (D) 1 : 10

Q6. What annual payment will discharge a debt of Rs. 580 due in 5 years, the rate being 8% per annum?

(A) Rs. 120 (B) Rs. 100

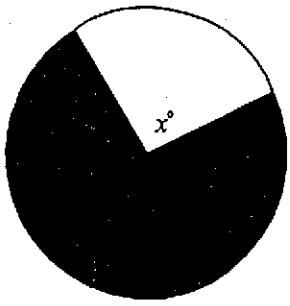
(C) Rs. 65.60 (D) Rs. 166.40

Q7. Amina found an easy way to add up a sequence of positive even integers with an even number of terms. She forms pairs of equal sums by adding the first integer to the last, the second integer to the next-to-last, and so on. She then computed the total by adding these equal sums. If the total that Amina obtained was 930, how many terms were there in the sequence of positive even integers if the sequence started with the number 2?

(A) 39 (B) 24

(C) 30 (D) 25

Q8. In the following figure, the area of the shaded sector is 75% of the area of the entire circle, what is the value of x?



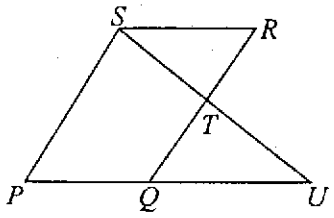
(A) 25 (B) 90°

(C) 270 (D) 45

Q9. **Ali goes to a park and runs in the following manner:**
From the starting point, he goes west 25 *m*, then to north 60m, then to east 80 *m* and finally to south 12 *m*. The distance between the starting point and the finishing point is:

(A) 73 *m* (B) 83 m
(C) 103 *m* (D) 177 m

Q10. **PQRS is a parallelogram and T is the midpoint of QR. ST and PQ when produced meet at U. Then**



(A) $PU = \frac{3}{2}PQ$ (B) $PU = 2PQ$
(C) $PU = 3PQ$ (D) $PU^2 = 2PQ^2$

Q11. **The diameter of a circle is 105 cms less than the circumference. The radius of the circle is:**

(A) 24.5 cm (B) 24 cm
(C) 23 cm (D) 22 cm

Q12. **The perimeter of a square whose area is equal to that of a circle with perimeter $2\pi x$ is:**

(A) $4\sqrt{\pi}\,x$ (B) $4\sqrt{\pi x}$
(C) $\sqrt{\pi}\,x$ (D) $2\pi x$

Q13. **A can do a piece of work in 35 days while B can complete it in 45 days. They start the work together, but A drops out after 7 days. In how many days will B take to finish the remaining work?**

(A) 29 (B) 36
(C) 45 (D) None of these

Q14. **If $\frac{3}{5}$ of the employees in Dogar Publishers are not college graduates, what is the ratio of the number of college graduates to those who are not college graduates?**

(A) 1 : 3 (B) 3 : 2
(C) 1 : 5 (D) 2 : 5

Q15. **If $\frac{3a-1}{25} = \frac{a+5}{11}$, what is the value of a?**

(A) 13 (B) 136
(C) $\frac{5}{6}$ (D) 17

Q16. **The sum of the six consecutive integers is T, what is the largest of those integers in terms of T?**

(A) $\frac{T+10}{5}$ (B) $\frac{T+6}{5}$
(C) $\frac{T+5}{6}$ (D) $\frac{T+15}{6}$

Q17. **If $\frac{1}{2}y$ years ago, Rizwan was 10, and $\frac{1}{2}y$ years from now he will be $2y$ years old, how old will he be $3y$ years from now?**

(A) 54 (B) 60

(C) 34 (D) 30

Q18. If xyz = 1, then $\left(\frac{1}{1+x+y^{-1}}+\frac{1}{1+y+z^{-1}}+\frac{1}{1+z+x^{-1}}\right)$ is equal to:

(A) 0 (B) $\frac{1}{xy}$

(C) xy (D) 1

Q19. A man spends 75% of his income. His income is increased by 20% and he increased his expenditure by 10%. His savings are increased by:

(A) 10% (B) 25%

(C) 37.5% (D) 50%

Q20. Electricity tax is increased by 20% and its consumption is decreased by 20%. The change in the expenditure is:

(A) nil (B) 5% decrease

(C) 4% increase (D) 4% decrease

Explanatory Answers

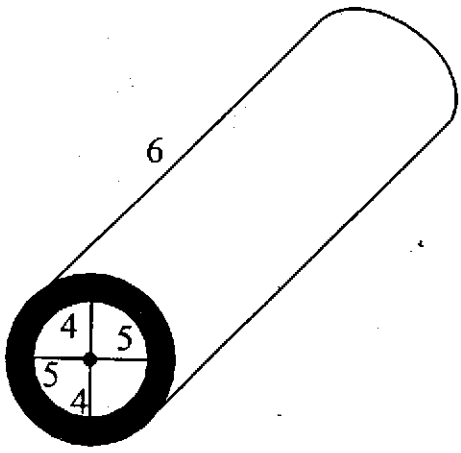
Q1. (B) Since a cube has 12 edges, therefore

$12e = 4 \Rightarrow e = \frac{1}{3}$

As $v = e^3 \Rightarrow v = \left(\frac{1}{3}\right)^3$

$\Rightarrow v = \frac{1}{27}$

Q2. (B)



According to above diagram, surface area of the cylinder is, A = 2πrh. Thus the area of the outer edge is = 2π(5)(6) = 60π, and the surface area of the inner side = 2π(4)(5) = 40π. Now

Area of the shaded end = Area of the outer circle – the area of the inner circle

$= \pi r^2 - \pi r^2 = \pi(5)^2 - \pi(4)^2$

$= 25\pi - 16\pi = 9\pi$

Now the total surface area $= 60\pi + 40\pi + 9\pi + 9\pi$

$= 118\pi = x\pi$

$\boxed{\Rightarrow x = 118}$

Hence the correct answer is choice B.

Q3. (D) Let the average sale be 100, then sale in November = 100 + 40 = 140 and the sale in March = 100 – 20 = 80. Then ratio between the sale of March to November is 80 : 140 ⇒ 4 : 7

Q4. (A) At unit place, there could be only 5 odd digits, *i.e.*, 1, 3, 5, 7 and 9.

Also, at tenth place there could be only 5 odd numbers and at last, at 100th place, there could be 5 odd integers. Then by-product rule

$$5 \times 5 \times 5 = 125$$

Hence the correct answer is choice A.

Q5. (C) $\frac{x+y}{x-y} = \frac{1}{5} \Leftrightarrow \frac{x+y+x-y}{x+y-x+y} = \frac{1+5}{1-5}$

(By Commend & Divendo)

$$\Rightarrow \frac{x}{y} = \frac{6}{-4} \qquad \Rightarrow \frac{x}{y} = \frac{-3}{2}$$

$$\Rightarrow \frac{x^2}{y^2} = \frac{9}{4} \Rightarrow \frac{x^2-y^2}{x^2+y^2} = \frac{9-4}{9+4}$$

$$\Rightarrow \frac{x^2-y^2}{x^2+y^2} = \frac{5}{17}$$

Hence the correct answer is choice C.

Q6. (B) Let the annual payment be Rs. x. Then (Amount of x for 4 years) + (Amount of x for 3 years) + (Amount of x for 2 years) + (Amount of x for 1 year) + x = 580

$$\Rightarrow \left(x + \frac{x \times 4 \times 8}{100}\right) + \left(x + \frac{x \times 3 \times 8}{100}\right) + \left(x + \frac{x \times 2 \times 8}{100}\right) + \left(x + \frac{x \times 1 \times 8}{100}\right) + x = 580$$

$$\Rightarrow \quad 5x + \frac{32x + 24x + 16x + 8x}{100} = 580$$

$$\Rightarrow \quad \frac{500x + 80x}{100} = 580$$

$$\Rightarrow \quad \frac{580x}{100} = 580$$

$$\Rightarrow \quad \boxed{x = 100}$$

Hence the correct answer is choice B.

Q7. (C) The sequence of the even numbers are in the form of

2 + 4 + 6 + ...

Using the formula for sum of integers

$$Sn = \frac{n}{2}\{2a + (n-1)d\}$$

Here n = ?, Sn = 930, $d = 4 - 2 = 2$

and a = 2 *i.e.*, first term. Now

$$930 = \frac{n}{2}\{2 \times 2 + (n-1)2\}$$

$$\Rightarrow \quad 930 \times 2 = n\{4 + 2n - 2\}$$

$$\Rightarrow \quad 1860 = 2n^2 + 2n$$

$\Rightarrow \quad 930 = n^2 + n$

$\Rightarrow \quad n^2 + n - 930 = 0$

$\Rightarrow \quad n^2 + 31n - 30n - 930 = 0$ (Factorizing)

$\Rightarrow n(n + 31) - 30(n + 31) = 0$

$\Rightarrow \quad (n - 30)(n + 31) = 0$

$\Rightarrow \quad n = 30$ or -31

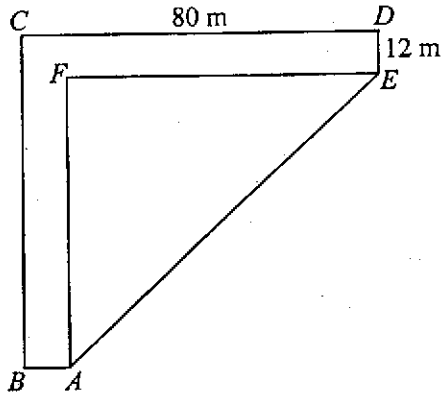
Since n is always positive so the correct answer is choice C.

Q8. (B) Since the shaded area of the given circle is 75% of the entire circle, thus the white area is 25% (100 – 75) of the circle. Now x is the 25% of 360°.

$$\Rightarrow x = 360 \times \frac{25}{100} = 90^\circ$$

Hence, the correct answer is choice B.

Q9. (A) Here, we draw the following diagram:



$$\begin{aligned} AE &= \sqrt{FE^2 + AF^2} \\ &= \sqrt{(80-25)^2 + (60-12)^2} \\ &= \sqrt{(55)^2 + (48)^2} \\ &= 73\text{m} \end{aligned}$$

Hence the correct answer is choice A.

Q10.(B) In ΔTRS and ΔTUQ, we have

$\angle SRT \cong \angle TQU$ ($\because$ alternative angles)

and $\angle STR \cong \angle UTQ$ (vertical opposite angles)

and $TR = QT$

$\therefore \quad \Delta TRS \cong TUQ$, so $SR = QU$

$\therefore \quad PU = PQ + QU$

$= PQ + SR$

$\Rightarrow \quad PU = 2PQ$ ($\because$ in || gram $PQ = RS$)

Hence the correct answer is choice B.

Q11.(A) Here,

$2\pi r - 2r = 105$

$\Rightarrow 2r(\pi - 1) = 105$

$\Rightarrow r = 105 \times \frac{7}{15} \times \frac{1}{2}$

$= 24.5$ cm

Hence the correct choice is choice A.

Q12.(A) Since, given area $= 2\pi x$

But, the area of a circle $= 2\pi r$

$\Rightarrow$ radius, r $= 2\pi$

$\therefore$ Area of square = Area of circle $= \pi x^2$

$\therefore$ Perimeter $= 4 \times$ side

$= 4\sqrt{\pi x^2}$

$= 4x\sqrt{\pi}$

Hence, the correct answer is choice A.

Q13.(A) (A + B)'s 7 days work $= 7 \times \left(\frac{1}{35} + \frac{1}{45}\right)$

$= \frac{16}{45}$

Remaining work $= 1 - \frac{16}{45} = \frac{29}{45}$

B will finish $\frac{29}{45}$ of the work in $\frac{29}{45} \times 45$ days *i.e.,* in 29 days.

Q14.(B) Since for every 5 employees, three are not college graduates, and (5 – 3) = 2 are college graduates. So the ratio of the graduates to non-graduates is 2 : 3.

Hence the correct answer is choice B.

Q15.(D) $\frac{3a-1}{25} = \frac{a+5}{11}$

$11(3a-1) = 25(a+5)$

$33a - 11 = 25a + 125$

$33a - 25a = 125 + 11$

$\Rightarrow 8a = 136$

$\Rightarrow \boxed{a = 17}$

Q16.(D) Let the six consecutive integers be $n, n+1, n+2, n+3, n+4, n+5$. Then

$T = n + (n+1) + (n+2) + (n+3) + (n+4) + (n+5)$

$T = 6n + 15 \Rightarrow 6n = T - 15$

$\Rightarrow n = \frac{T-15}{6}$

Since n is the smallest of the integers, the largest is

$n + 5 = \frac{T-15}{6} + 5$

$= \frac{T - 15 + 30}{6}$

$\Rightarrow n + 5 = \frac{T+15}{6}$

Hence the correct answer is choice D.

Q17.(D) Because, $\frac{1}{2}y$ years ago, Rizwan was 10, he is now $10 + \frac{1}{2}y$. Now, $\frac{1}{2}y$ years from now, he will be 10 +

$\frac{1}{2}y + \frac{1}{2}y = 10 + y$. But given that at that time he will be $2y$ years old. Thus, $10 + x = 2x \Rightarrow x = 5$. Thus he is now, 10 + 5 = 15, and $3y$ or 15 years from now he will be 15 + 15 = 30 years. Hence the correct answer is choice D.

Q18.(D) Given that, $\frac{y}{y+xy+1} + \frac{1}{1+y+xy} + \frac{xy}{xy+1+y}$ $(\because z = \frac{1}{xy})$

$$= \left(\frac{y+1+xy}{y+1+xy}\right) = 1$$

Hence the correct answer is choice D.

Q19.(D) Let income = 100. Then, expenditure = 75 and saving = 25.

New income = 120, new expenditure = 110% of 75

= 82.5

New saving = (120 – 82.5) = 37.5

Increase in saving $= \left(\frac{12.5}{25} \times 100\right)\%$

= 50%

Q20.(D) Let Tax = Rs. 100 per unit and consumption = 100 units

Original Expenditure = Rs. (100 × 100) = Rs. 10000

New Tax = Rs. 120 per unit and New consumption = 80 units

∴ New expenditure = Rs. (120 × 80) = Rs. 9600

∴ Decrease in expenditure $= \left(\frac{400}{10000} \times 100\right)\%$

= 4%

****

NTS TEST NO. 3

◆ **Select the correct answer for each question and blacken the corresponding circle in the answer sheet.**

Instructions (1-10): In this part of test, you have 10 MCQs about English. Each sentence below has one or two blanks, each blank shows that something has been omitted. Choose the correct answer from the four answer choices given with each question, numbered *(A), (B), (C), (D)*.

1. ***He laboured _____ the hill; sat watching the city; then ran _____ the hill.***

(A) Along; Towards (B) Down; Up
(C) Up; Down (D) Towards; From

2. ***Put the milk _____ the table and cover it _____ a cloth.***

(A) On; With (B) Near; By
(C) On; By (D) In; With

3. ***Consumers refused to buy meat products from the company because of rumors that the water supply at the meat processing plant was _____; the rumors, however, were quite _____, with no hard evidence to back them up.***

(A) Un-inspected, reckless
(B) Contaminated, unsubstantiated
(C) Impure, damaging
(D) Misdirected, scandalous

4. ***Consumption of red meat has _____ because its fat content has become a worrisome and _____ matter.***

(A) Abated, Dubious (B) Skyrocketed, Stressful

(C) Abounded, Divisive (D) Declined, Controversial

5. *Florence Nightingale was _____ in the development of modern medicine, _____ such practices as sanitization of hospital wards and isolation of actively infected patients.*
   (A) A collaborator, Rejecting
   (B) A maverick, Protesting
   (C) An innovator, Initiating
   (D) A pioneer, Criticizing

6. *Sofia _____ collect stamps, but now she has other interests.*
   (A) Used to (B) Was used to
   (C) Used to be (D) Using to

7. *Salman finished _____ two of his published compositions before his twelfth birthday.*
   (A) Written (B) Writing
   (C) To write (D) Wrote

8. *Throughout the animal kingdom, _____ bigger than the elephant.*
   (A) Whale is only the (B) Only the whale is
   (C) Is the whale only (D) Only whale is the

9. *Linda Greenhouse's articles for the New York Times are an outstanding example of _____, capsulizing prose into a necessarily limited space.*
   (A) Callousness (B) Brevity
   (C) Intuition (D) Propriety

10. *One of the least effective ways of sorting information is learning _____ it.*
   (A) Repeat (B) Repeating
   (C) To repeat (D) How repeat

Instructions (11-20): Each question below consists of a related pair of words or phrases, followed by four lettered pairs of words or phrases numbered *(A), (B), (C), (D)*. Choose the lettered pair that best expresses a relationship similar to that expressed in the pair given in the question.

11. *ELM: TREE::*
   (A) Whale: Mammal (B) Cart: Horse
   (C) Cloud: Rain (D) Painting: Artist

12. *GULLIBLE: DUPED::*
   (A) Myopic: Misled (B) Careful: Cautioned
   (C) Malleable: Molded (D) Credible: Cheated

13. *IMPLICATE: COMPLICATE::*
   (A) Vitality: Inevitable (B) Empathy: Sympathy
   (C) Importune: Construct (D) Imply: Simplify

14. *AFTERNOON: DUSK::*
   (A) Breakfast: Dinner (B) Yesterday: Tomorrow
   (C) Sunday: Saturday (D) Night: Dawn

15. *MINISTER: PULPIT::*
   (A) Doctor: Patient (B) Judge: Bench
   (C) Student: Teacher (D) Programmer: Logic

16. *AUGER: CARPENTER::*
   (A) Cement: Mason (B) Apron: Chef
   (C) Awl: Cobbler (D) Studio: Sculptor

17. *CONTROVERSY: ARBITRATOR::*
   (A) Peacemaker: Conflict (B) Artifact: Anthropologist
   (C) Game: Referee (D) Dispute: Mediator

18. *Condemnation: Disapproval::*
   (A) Ignorance: Patience (B) Sorrow: Intention
   (C) Optimism: Insight (D) Blasphemy: Irreverence

19. *ASYLUM: SHELTER*
   (A) Harbor: Concealment (B) Palisade: Display

(C) Stronghold: Defense (D) Cloister: Storage

20. *CAPTAIN: SHOAL::*

(A) Soldier: Ambush (B) Lawyer: Litigation
(C) Corporal: Sergeant (D) Pilot: Radar

Read the following passages carefully and answer the questions given at its end:

Passage: The purpose of education is to make the student an expert in his subject. This must be clearly understood, and mere muddling through lessons and lectures and books and passing examinations are relegated to secondary importance as means to end-which is excellence in the field chosen.

But there are so many fields, and no man can become an expert in all the fields. It is necessary to decide which fields are the important ones that a man should know well.

It is clear that one's own work is the most important. This has been realised and modern civilization has accordingly provided vocational education. It is now possible to acquire high professional skill in the various fields, medicine, engineering production, commerce and so on–but with good and bad mixed together, and no standard for guidance.

21. ***The purpose of education is to make the student:***

(A) An expert in all fields
(B) An expert in his subject
(C) Only capable of earning
(D) Confident only

22. ***What, according to the writer, is the end ?***

(A) Excellence in the field chosen
(B) Passing the examination
(C) Earning more and more money
(D) Cramming lectures and books

23. ***The modern civilization has provided:***

(A) Vocational education (B) Art of conversation
(C) Adult education (D) Higher education

Passage: According to Aristotle, the most important question in the physical world was the search for happiness. He was not an idealist preaching impossible ideals and counsels of perfection. His scientific training made him a realist who believed that happiness was the aim of life and every activity, even ethics, was subordinate to it. He tries to explore the nature of happiness and explains it by differentiating man from other animals in that he possesses the thinking faculty by which he masters the earth and surpasses all living beings and its development will give him fulfilment and happiness.

24. ***In the given paragraph:***

(A) Aristotle surpasses the earth and all living beings
(B) Aristotle tries to explore the nature of physical world
(C) Aristotle speaks about all living beings
(D) Aristotle tries to explore the nature of happiness

25. ***Man differs from other animals in having:***

(A) A brain (B) Speaking faculty
(C) Thinking faculty (D) Eating faculty

ANSWERS

1.	(C)	2.	(A)	3.	(B)	4.	(D)	5.	(C)
6.	(A)	7.	(B)	8.	(B)	9.	(B)	10.	(C)
11.	(A)	12.	(C)	13.	(B)	14.	(D)	15.	(B)
16.	(C)	17.	(D)	18.	(D)	19.	(C)	20.	(A)
21.	(B)	22.	(A)	23.	(A)	24.	(D)	25.	(C)

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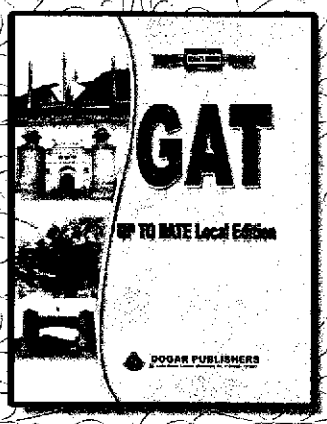

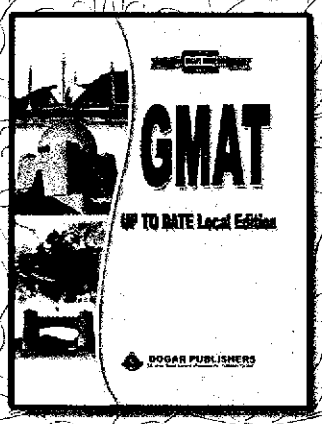

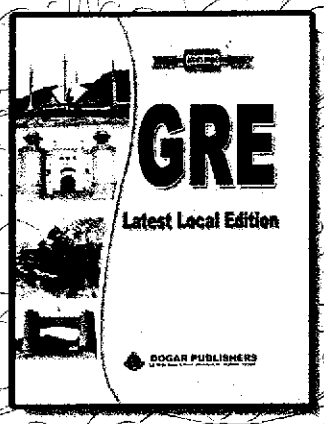

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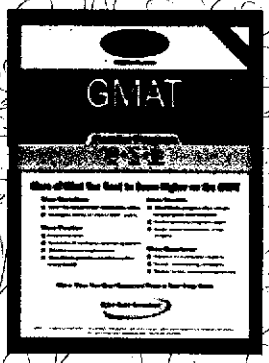

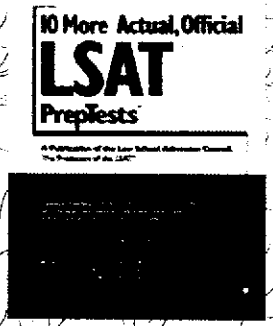

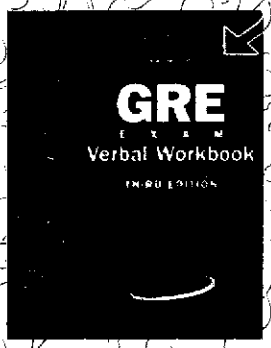

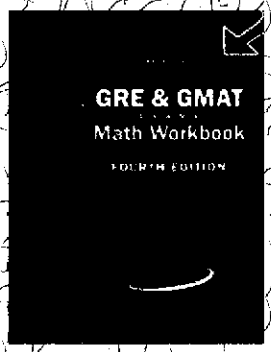

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