Business Statistics

A Guide for BUAD 231

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Introduction

"Whatever you would make habitual, practice it; and if you would not make a thing habitual, do not practice it, but accustom yourself to something else." *Epictetus*

How often do we feel bad about ourselves because we procrastinated, squandered our time, or did not accomplish something meaningful during the day? Making the right decisions takes practice. In this book, I invite you to practice the skills you have learned in BUAD 231 and the skills of focus, dedication, and consistency. Choose a day in the week and start by dedicating some fixed time to these problems (e.g., 15-30 minutes). The idea is to work on consistency (i.e., returning to the book weekly for a given amount of time). Some of us will find that concentrating is challenging. Your next task is to reduce distractions (i.e., the phone, t.v. or even your thoughts about the future). If you keep trying and returning to the book, you will improve at Business Statistics and learn to study with focus and consistency. All it takes is practice. Remember, you are what you practice!

The problems in this book are designed to help you master statistics and its application in R. I recommend reviewing Grolemund (2014) if you need additional help learning R. Finally, I have provided a list of concepts at the beginning of every chapter. Enjoy!

Why R?

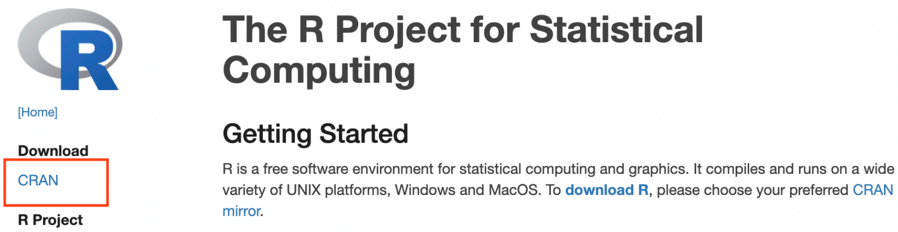
We will be using R to apply the lessons we learn in BUAD 231. R is a language and environment for statistical computing and graphics. There are several advantages to using the R software for statistical analysis and data science. Some of the main benefits include:

- R is a **powerful and flexible programming language** that allows users to manipulate and analyze data in many different ways.
- R has a large and **active community of users**, who have developed a wide range of packages and tools for data analysis and visualization.
- R is **free and open-source**, which makes it accessible to anyone who wants to use it.
- R is **widely used** in academia and industry, which means that there are many resources and tutorials available to help users learn how to use it.
- R is well-suited for working with **large and complex datasets**, and it can handle data from many different sources.
- R can be **easily integrated** with other tools and software, such as databases, visualization tools, and machine learning algorithms.

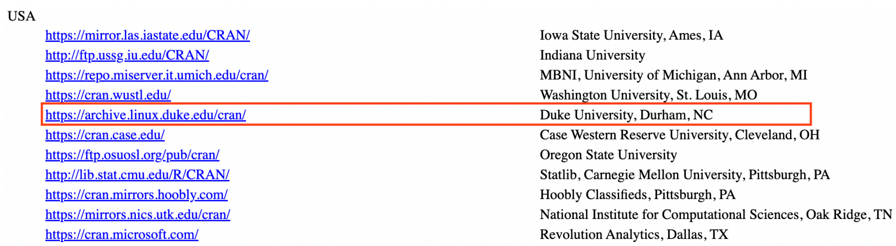
Overall, R is a powerful and versatile tool for data analysis and data science, and it offers many benefits to users who want to work with data.

Installing R

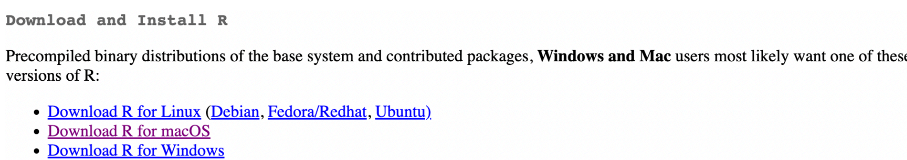
To install R, visit the R webpage at https://www.r-project.org/. Once in the website, click on the CRAN hyperlink.



Here you can select the CRAN mirror. Scroll down until you see USA. You are free to choose any mirror you like, I recommend using the Duke University mirror.



Once you click on the hyperlink, you will be prompted to choose the download for your operating system. Depending on your operating system, choose either a Windows or Macintosh download.



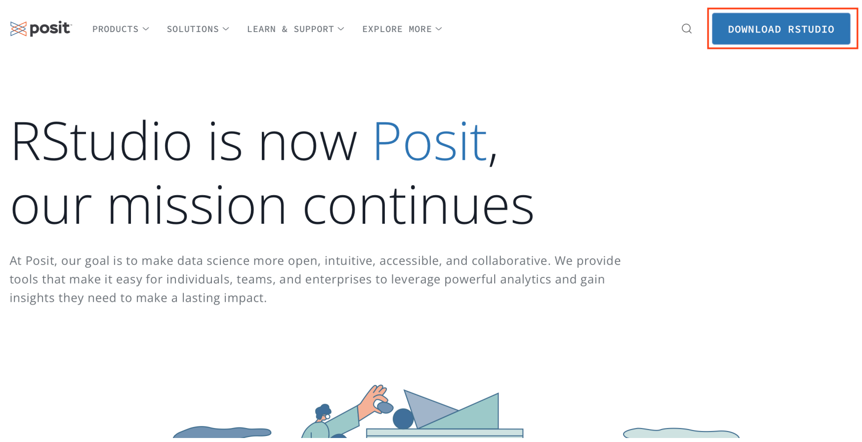
Follow all prompts and complete installation.

Installing RStudio

Visit the Posit website at https://posit.co. Once on the website, hover to the top right of the screen. You will see a "Download RStudio" blue button.

Next, scroll down until you reach the RStudio desktop section. Click once more on "Download RStudio". You can now just jump to Step 2 since you have already downloaded R. Finally, choose the desired download depending on your operating system.

It is important to note that RStudio will not work if R is not installed. You can think of R as the engine and RStudio as the interface.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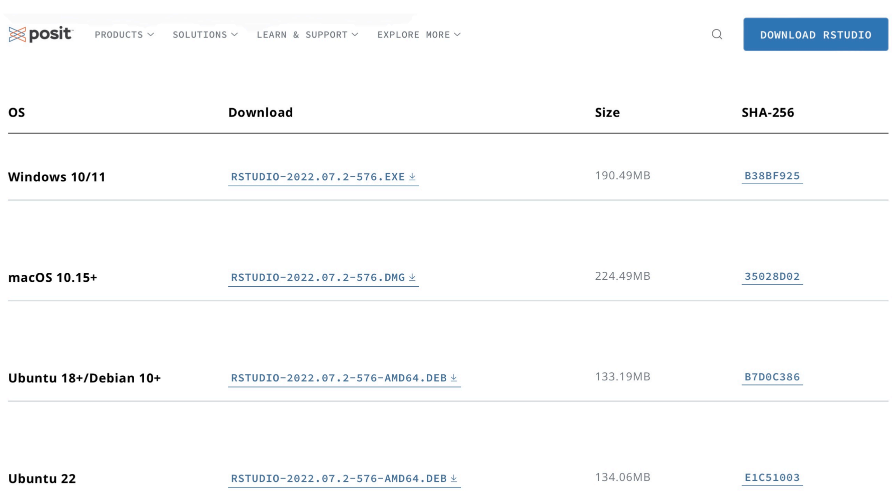
RStudio is now Posit,
our mission continues

At Posit, our goal is to make data science more open, intuitive, accessible, and collaborative. We provide tools that make it easy for individuals, teams, and enterprises to leverage powerful analytics and gain insights they need to make a lasting impact.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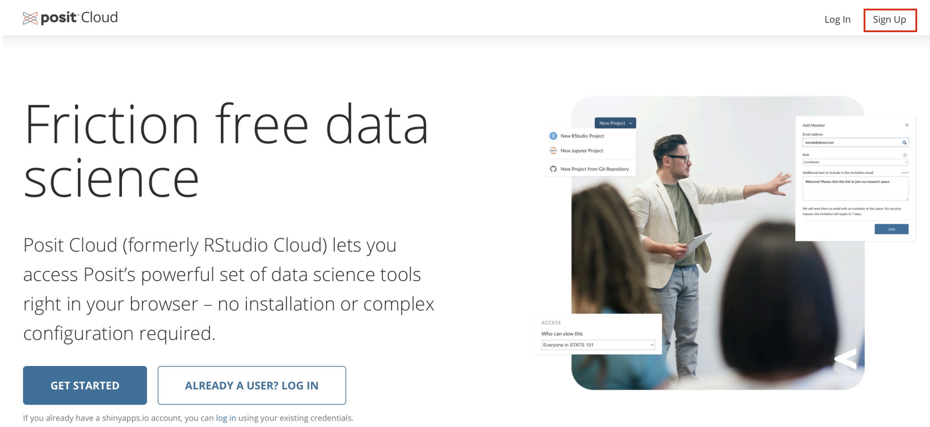
Posit Cloud

If you do not wish to install R, you can always use the cloud version. To do this, visit https://posit.cloud/. On the main page click on the "Sign Up" button.



Choose the "Cloud Free" option and log in using your Google credentials (if you have a Google account) or sign up if you want to create a new account.

Part I

Descriptive Statistics

1 Descriptive Stats I

1.1 Concepts

Data and Types of Data

Data are facts and figures collected, analyzed and summarized for presentation and interpretation. Data can be classified as:

- **Cross Sectional Data** refers to data collected at the same (or approximately the same) point in time. Ex: NFL standings in 1980 or Country GDP in 2015.
- **Time Series Data** refers to data collected over several time periods. Ex: U.S. inflation rate from 2000-2010 or Tesla deliveries from 2016-2022.
- **Structured Data** resides in a predefined row-column format (tidy).
- **Unstructured Data** do not conform to a pre-defined row-column format. Ex: Text, video, and other multimedia.

Data Sets, Variables and Scales of Measurement

A **data set** contains all data collected for a particular study. Data sets are composed of:

- **Elements** are the entities on which data are collected. Ex: Football teams, countries, and individuals.
- **Observations** are the set of measurements obtained for a particular element.
- **Variables** are a set of characteristics collected for each element.

The **scales of measurements** determine the amount and type of information contained in each variable. In general, variables can be classified as **categorical** or **numerical**.

- **Categorical** (qualitative) data includes labels or names to identify an attribute of each element. Categorical data can be **nominal** or **ordinal**.
  - With **nominal** data, the order of the categories is arbitrary. Ex: Marital Status, Race/Ethnicity, or NFL division.
  - With **ordinal** data, the order or rank of the categories is meaningful. Ex: Rating, Difficulty Level, or Spice Level.
- **Numerical** (quantitative) include numerical values that indicate how many (discrete) or how much (continuous). The data can be either **interval** or **ratio**.

  - With **interval** data, the distance between values is expressed in terms of a fixed unit of measure. The zero value is arbitrary and does not represent the absence of the characteristic. Ratios are not meaningful. Ex: Temperature or Dates.
  - With **ratio** data, the ratio between values is meaningful. The zero value is not arbitrary and represents the absence of the characteristic. Ex: Prices, Profits, Wins.

Useful R Functions

Base R has some important functions that are helpful when dealing with data. Below is a list that might come handy.

- The `na.omit()` function removes any observations that have a missing value (NA). The resulting data frame has only complete cases.
- The `nrow()` and `ncol()` functions return the number of rows and columns respectively from a data frame.
- The `is.na()` function returns a vector of *True* and *False* that specify if an entry is missing (NA) or not.
- The `summary()` function returns a collection of descriptive statistics from a data frame (or vector). The function also returns whether there are any missing values (NA) in a variable.
- The `as.integer()`, `as.factor()`, `as.double()`, are functions used to coerce your data into a different scale of measurement.

The `dplyr` package has a collection of functions that are useful for data manipulation and transformation. If you are interested in this package you can refer to Wickham (2017). To install, run the following command in the console `install.packages("dplyr")`.

- The `arrange()` function allows you to sort data frames in ascending order. Pair with the `desc()` function to sort the data in descending order.
- The `filter()` function allows you to subset the rows of your data based on a condition.
- The `select()` function allows you to select a subset of variables from your data frame.

1.2 Exercises

The following exercises will help you test your knowledge on the Scales of Measurement. They will also allow you to practice some basic data "wrangling" in R. In these exercises you will:

- Identify numerical and categorical data.
- Classify data according to their scale of measurement.
- Sort and filter data in R.
- Handle missing values (NA's) in R.

Answers are provided below. Try not to peak until you have a formulated your own answer and double checked your work for any mistakes.

Exercise 1

A bookstore has compiled data set on their current inventory. A portion of the data is shown below:

Title	Price	Year Published	Rating
Frankenstein	5.49	1818	4.2
Dracula	7.60	1897	4.0
...	...	...	...
Sleepy Hollow	6.95	1820	3.8

1. Which of the above variables are categorical and which are numerical?
2. What is the measurement scale of each of the above variable?

Exercise 2

A car company tracks the number of deliveries every quarter. A portion of the data is shown below:

Year	Quarter	Deliveries
2016	1	14800
2016	2	14400
...	...	...
2022	3	343840

1. What is the measurement scale of the Year variable? What are the strengths and weaknesses of this type of measurement scale?
2. What is the measurement scale for the Quarter variable? What is the weakness of this type of measurement scale?
3. What is the measurement scale for the Deliveries variable? What are the strengths of this type of measurement scale?

Exercise 3

Use the **airquality** data set included in R for this problem.

1. Sort the data by *Temp*, *Ozone*, and *Wind* all in descending order. What is the day and month of the first observation on the sorted data?
2. Sort the data only by *Temp* in descending order. Of the 10 hottest days, how many of them were in July?
3. How many missing values are there in the data set? What rows have missing values for *Solar.R*?
4. Remove all observations that have a missing values. Create a new object called *CompleteAG*.
5. When using *CompleteAG*, how many days was the temperature at least 60 degrees?
6. When using *CompleteAG*, how many days was the temperature within [55,75] degrees and an *Ozone* below 20?

1.3 Answers

Exercise 1

1. The variables Title and Rating are categorical whereas Price and Year are numerical.
2. The measurement scale is nominal for Title, ordinal for Ratio, ratio for Price, and interval for Year. Recall, that the nominal and ratio scales represent the least and most sophisticated levels of measurement, respectively.

Exercise 2

1. The variable Year is measured on the interval scale because the observations can be ranked, categorized and measured when using this kind of scale. However, there is no true zero point so we cannot calculate meaningful ratios between years.
2. The variable Quarter is measured on the nominal scale, even though it contains numbers. It is the least sophisticated level of measurement because if we are presented with nominal data, all we can do is categorize or group the data.
3. The variable Deliveries is measured on the ratio scale. It is the strongest level of measurement because it allows us to categorize and rank the data as well as find meaningful differences between observations. Also, with a true zero point, we can interpret the ratios between observations.

Exercise 3

1. The day and month of the first observation is August 28th.

The easiest way to sort in R is by using the `dplyr` package. Specifically, the `arrange()` function within the package. Let's also use the `desc()` function to make sure that the data is sorted in descending order. We can use indexing to retrieve the first row of the sorted data set.

```
library(dplyr)
SortedAQ<-arrange(airquality,desc(Temp),desc(Ozone),desc(Wind))
SortedAQ[1,]
```

```
  Ozone Solar.R Wind Temp Month Day
1    76     203  9.7   97     8  28
```

2. Of the 10 hottest days only two were in July.

We can use the `arrange()` function one more time for this question. Then we can use indexing to retrieve the top 10 observations.

```
SortedAQ2<-arrange(airquality,desc(Temp))
SortedAQ2[1:10,]
```

```
   Ozone Solar.R Wind Temp Month Day
1     76     203  9.7   97     8  28
2     84     237  6.3   96     8  30
3    118     225  2.3   94     8  29
4     85     188  6.3   94     8  31
5     NA     259 10.9   93     6  11
6     73     183  2.8   93     9   3
7     91     189  4.6   93     9   4
8     NA     250  9.2   92     6  12
9     97     267  6.3   92     7   8
10    97     272  5.7   92     7   9
```

3. There are a total of 44 missing values. *Ozone* has 37 and *Solar.R* has 7. Rows 5, 6, 11, 27, 96, 97, 98 are missing for *Solar.R*.

We can easily identify missing values with the `summary()` function.

```r
summary(airquality)
```

```
     Ozone           Solar.R           Wind             Temp      
 Min.   :  1.00   Min.   :  7.0   Min.   : 1.700   Min.   :56.00  
 1st Qu.: 18.00   1st Qu.:115.8   1st Qu.: 7.400   1st Qu.:72.00  
 Median : 31.50   Median :205.0   Median : 9.700   Median :79.00  
 Mean   : 42.13   Mean   :185.9   Mean   : 9.958   Mean   :77.88  
 3rd Qu.: 63.25   3rd Qu.:258.8   3rd Qu.:11.500   3rd Qu.:85.00  
 Max.   :168.00   Max.   :334.0   Max.   :20.700   Max.   :97.00  
 NA's   :37       NA's   :7                                       
     Month            Day      
 Min.   :5.000   Min.   : 1.0  
 1st Qu.:6.000   1st Qu.: 8.0  
 Median :7.000   Median :16.0  
 Mean   :6.993   Mean   :15.8  
 3rd Qu.:8.000   3rd Qu.:23.0  
 Max.   :9.000   Max.   :31.0  
```

To view the rows that have NA's in them, we can use the `is.na()` function and indexing. Below we see that 7 values are missing for the *Solar.R* variable in the months 5 and 8 combined.

```r
airquality[is.na(airquality$Solar.R),]
```

```
   Ozone Solar.R Wind Temp Month Day
5     NA      NA 14.3   56     5   5
6     28      NA 14.9   66     5   6
11     7      NA  6.9   74     5  11
27    NA      NA  8.0   57     5  27
```

```
96     78        NA   6.9    86       8    4
97     35        NA   7.4    85       8    5
98     66        NA   4.6    87       8    6
```

4. To create the new object of complete observations we can use the `na.omit()` function.

```
CompleteAQ<-na.omit(airquality)
```

5. There were 107 days where the temperature was at least 60.

Using base R we have:

```
nrow(CompleteAQ[CompleteAQ$Temp>=60,])
```

```
[1] 107
```

We can also use `dplyr` for this question. Specifically, using the `filter()` and `nrow()` functions we get:

```
nrow(filter(CompleteAQ,Temp>=60))
```

```
[1] 107
```

6. There were 24 days where the temperature was between 55 and 75 and the ozone level was below 20.

Using base R we have:

```
nrow(CompleteAQ[CompleteAQ$Temp>55 & CompleteAQ$Temp<75 & CompleteAQ$Ozone<20,])
```

```
[1] 24
```

Using the `filter()` function once more we get:

```
nrow(filter(CompleteAQ,Temp>55,Temp<75,Ozone<20))
```

```
[1] 24
```

2 Descriptive Stats II

2.1 Concepts

Frequency

A **frequency distribution** is a tabular summary of data showing the number of items in each of several non-overlapping classes.

- The **relative frequency** is calculated by f_i/n, where f_i is the frequency of class i and n is the total frequency.
- The **cumulative frequency** shows the number of data items with values less than or equal to the upper class limit of each class.
- The **cumulative relative frequency** is given by cf_i/n, where cf_i is the cumulative frequency of class i.

Plots

A **bar plot** illustrates the frequency distribution of qualitative data.

- Is an illustration for qualitative data.
- Includes the classes in the horizontal axis and frequencies or relative frequencies in the vertical axis.
- Has gaps between each bar.

A **histogram** illustrates the frequency distribution of quantitative data.

- Is an illustration for quantitative data.
- There are no gaps between the bars.
- The **number**, **width** and **limits** of each class must be determined.
  - The **number** of classes can be determined by the 2^k rule: select k such that 2^k is greater than the number of observations by the smallest amount.
  - The **width** of the class is approximately *range/(# of Classes)*. The value should be rounded up.
  - The **limits** should be chosen so that each point belongs to only one class.

Useful R Functions

The `table()` command generates frequency distributions or contingency tables if two variables are used.

The `prop.table()` command generates relative frequency distributions from an object that contains a table.

The `cut()` function generates class limits and bins used in frequency distributions (and histograms) for quantitative data.

Base R has the `barplot()` function for categorical variable, `histogram()` function for numerical data, and the `plot()` function for line charts or scatter plots. Below are some arguments that are helpful when plotting.

- *main*: used to set the plot's title. The title should be entered as a character.
- *col*: used to set the color of the plot. Hex and RGB values are allowed as inputs. The color should be entered as a character.
- *xlab* and *ylab*: are used to set the labels for the x and y axis respectively. The labels should be entered as characters.
- `legend()` is a function to customize the legend of a graph. This argument may be used with the `plot()`, `barplot()` or `histogram()` functions.
  - x: used to set the location of the legend in the plotting area. Ex: "bottomleft".
  - *legend*: a vector specifying the legend names to be included.
  - *col*: a vector specifying the color of each item in the legend.

2.2 Exercises

The following exercises will help you practice summarizing data with tables and simple graphs. In particular, the exercises work on:

- Developing frequency distributions for both categorical and numerical data.
- Constructing bar charts, histograms, and line charts.
- Creating contingency tables.

Answers are provided below. Try not to peak until you have a formulated your own answer and double checked your work for any mistakes.

Exercise 1

Install the `ISLR2` package in R. You will need the **BrainCancer** data set to answer this question.

1. Construct a frequency and relative frequency table of the *Diagnosis* variable. What was the most common diagnosis? What percentage of the sample had this diagnosis?
2. Construct a bar chart. Summarize the findings.

3. Construct a contingency table that shows the *Diagnosis* along with the *Status*. Which diagnosis had the highest number of non-survivals (0)? What was the survival rate of this diagnosis?
4. Construct a stacked column chart. Which two *Diagnosis* and *Status* combinations are the most frequent?

Exercise 2

You will need the **airquality** data set (in base R) to answer this question.

1. Construct a frequency distribution for *Temp*. Use five intervals with widths of $50 < x \leq 60$; $60 < x \leq 70$; etc. Which interval had the highest frequency? How many times was the temperature between 50 and 60 degrees?
2. Construct a relative frequency, cumulative frequency and the relative cumulative frequency distributions. What proportion of the time was *Temp* between 50 and 60 degrees? How many times was the *Temp* 70 degrees or less? What proportion of the time was *Temp* more than 70 degrees?
3. Construct the histogram. Is the distribution symmetric? If not, is it skewed to the left or right?

Exercise 3

You will need the **Portfolio** data set from the `ISLR2` package to answer this question.

1. Construct a line chart that shows the returns over time for each portfolio (X and Y) by using two lines each with a unique color. Assume the data is for the period 1901 to 2000. Include also a legend that matches colors to portfolios.

2.3 Answers

Exercise 1

1. The most common diagnosis is Meningioma, a slow-growing tumor that forms from the membranous layers surrounding the brain and spinal cord. The diagnosis represents about 48.28% of the sample.

Start by loading the `ISLR2` package. To construct the frequency distribution table, use the `table()` function.

```
library(ISLR2)
table(BrainCancer$diagnosis)
```

```
Meningioma  LG glioma  HG glioma      Other 
        42          9         22         14 
```

The relative frequency distribution can be easily retrieved by saving the frequency table in an object and then using the `prop.table()` function.

```
freq<-table(BrainCancer$diagnosis)
prop.table(freq)
```

```
Meningioma  LG glioma  HG glioma      Other 
 0.4827586  0.1034483  0.2528736  0.1609195 
```

2. The majority of diagnosis are Meningioma. Low grade glioma is the least common of diagnosis. High grade glioma and other diagnosis have about the same frequency.

To construct the bar chart use the `barplot()` function in R.

```
barplot(freq, col = "#F5F5F5", ylim=c(0,50))
```

3. 33 people did not survive Meningioma. The survival rate of Meningioma is only 21.43%.

Use the `table()` function one more time to create the contingency table for the two variables.

```
(freq2<-table(BrainCancer$status,BrainCancer$diagnosis))
```

```
   Meningioma LG glioma HG glioma Other
  0         33         5         5     9
  1          9         4        17     5
```

To get the survival rates, we can use the `prop.table()` function once again.

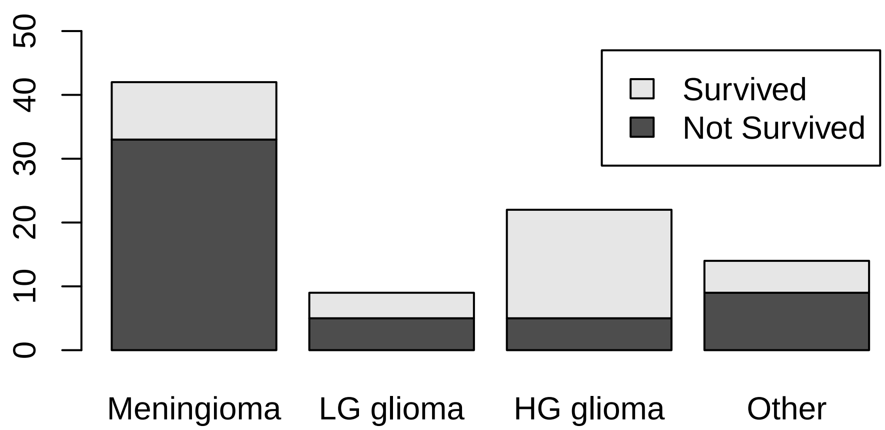
```
prop.table(freq2,margin = 2)
```

```
   Meningioma LG glioma HG glioma     Other
0   0.7857143 0.5555556 0.2272727 0.6428571
1   0.2142857 0.4444444 0.7727273 0.3571429
```

4. Meningioma and not surviving is the most common with 33 occurrences. High grade glioma and surviving is the the second most common.

Use the `barplot()` function one more time to construct the stacked column chart.

```
barplot(table(BrainCancer$status,BrainCancer$diagnosis),
        legend.text = c("Not Survived","Survived"), ylim=c(0,50))
```



Exercise 2

1. The highest frequency is in the $80 < x \leq 90$ bin. 8 temperatures were between $50 < x \leq 60$ degrees.

Create a vector containing the intervals desired by using the `seq()` function.

```
intervals <- seq(50, 100, by=10)
```

Next use the `cut()` function to create the cuts for the histogram.

```r
intervals.cut <- cut(airquality$Temp, intervals, left=FALSE, right=TRUE)
```

The frequency distribution can be obtained by using the `table()` function on the *interval.cut* object created above.

```r
table(intervals.cut)
```

```
intervals.cut
 (50,60]  (60,70]  (70,80]  (80,90] (90,100]
       8       25       52       54       14
```

2. The temperature was 5.22% of the time between 50 and 60; The temperature was 70 or less 33 times; The temperature was above 70, 78.43% of the time.

To get the relative frequency table, start by saving the proportion table into an object.Then you can use the `prop.table()` function.

```r
freq<-table(intervals.cut)
prop.table(freq)
```

```
intervals.cut
   (50,60]    (60,70]    (70,80]    (80,90]   (90,100]
0.05228758 0.16339869 0.33986928 0.35294118 0.09150327
```

For the cumulative distribution you can use the `cumsum()` function on the frequency distribution.

```r
cumulfreq<-cumsum(freq)
cumulfreq
```

```
 (50,60]  (60,70]  (70,80]  (80,90] (90,100]
       8       33       85      139      153
```

Lastly, for the relative cumulative distribution table, you can use the `cumsum()` function on the relative frequency table.

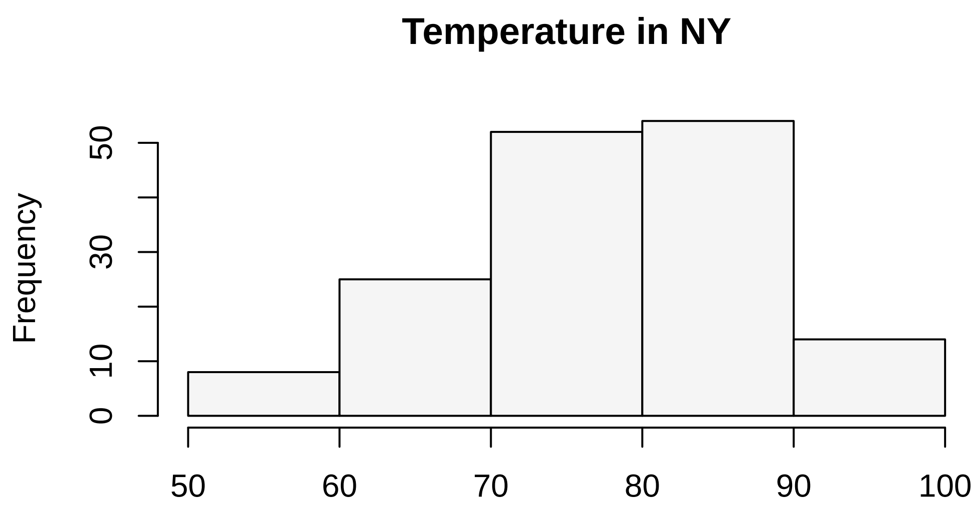
```r
cumsum(prop.table(freq))
```

```
   (50,60]    (60,70]    (70,80]    (80,90]   (90,100]
0.05228758 0.21568627 0.55555556 0.90849673 1.00000000
```

3. The distribution is not perfectly symmetric. It is skewed slightly to the left (see histogram.)

Use the `hist()` function to create the histogram.

```
hist(airquality$Temp, breaks=intervals,
     right=TRUE,col="#F5F5F5", main="Temperature in NY", xlab="")
```

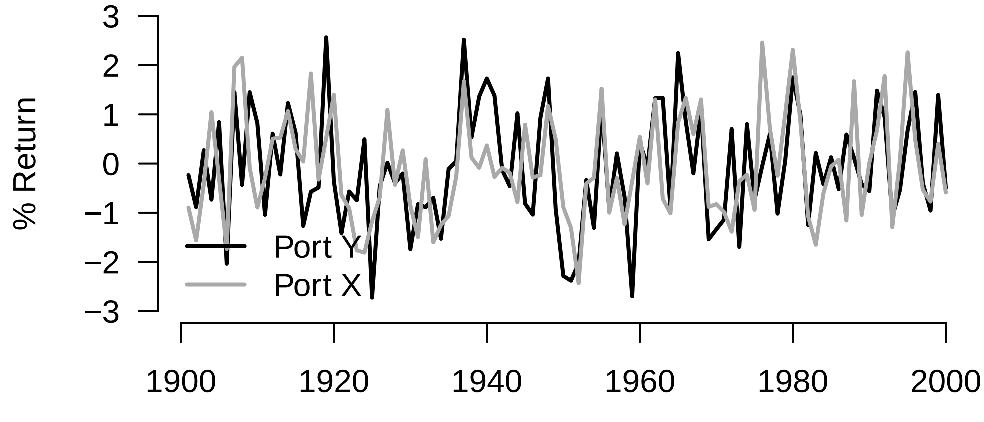


Exercise 3

1. From 1901 through 2000, both portfolios have behaved very similarly. Returns are between -3% and 3%, there is no trend, and positive (negative) returns for X seem to match with positive (negative) returns of Y.

You can use the `plot()` function to create a plot of Portfolio Y. The line for Portfolio X can be added with the `lines()` function.

```
plot(Portfolio$Y,
     x=seq(1901,2000), type="l",
     col="black", xlab="", ylab="% Return", ylim=c(-3,3),
     xlim=c(1901,2000), lwd=2, axes = F)
axis(side=1, labels=TRUE, font=1,las=1)
axis(side=2, labels=TRUE, font=1,las=1)
lines(Portfolio$X, x=seq(1901,2000), type="l",
      col="darkgrey", lwd=2)
legend(x = "bottomleft",
       legend = c("Port Y", "Port X"),
       lty = c(1, 1),
       col = c("black", "darkgrey"),
       lwd = 2,
       bty="n")
```



% Return
3
2
1
0
−1
−2
−3
Port Y
Port X
1900
1920
1940
1960
1980
2000

3 Descriptive Statistics III

3.1 Concepts

Measures of Central Location

Measures of Central Location determine where the center of a distribution lies.

- The **mean** is the average value for a numerical variable. The sample statistic is estimated by $\bar{x} = \sum x_i/n$, where x_i is observation i, and n is the number of observations. The population parameter is defined as $\mu = \sum x_i/N$.
- The **median** is the value in the middle when data is organized in ascending order. When n is even, the median is the average between the two middle values.
- The **mode** is the value with highest frequency from a set of observations.
- The **weighted mean** uses weights to determine the importance of each data point of a variable. It is calculated by $\frac{\sum w_i x_i}{\sum w_i}$, where w_i are the weights associated to the values x_i.
- The **geometric mean** is a multiplicative average that is less sensitive to outliers. It is used to average growth rates or rated of return. It is calculated by $\sqrt[n]{(1+r_1)*(1+r_2)...(1+r_n)}-1$, where $\sqrt[n]{}$ is the n_{th} root, and r_i are the returns or growth rates.

Useful R functions

Base R has a collection of functions that calculate measures of central location.

- The `mean()` function calculates the average of a vector of values.
- The `median()` function returns the median of a vector of values.
- The `table()` function provides us with a frequency distribution. We can then identify the mode(s) of the vector provided.
- The `summary()` function returns a collection of descriptive statistics for a vector or data frame.

3.2 Exercises

The following exercises will help you practice the measures of central location. In particular, the exercises work on:

- Calculating the mean, median, and the mode.
- Calculating the weighted average.
- Applying the geometric mean for growth rates and returns.

Answers are provided below. Try not to peak until you have a formulated your own answer and double checked your work for any mistakes.

Exercise 1

For the following exercises, make your calculations by hand and verify results using R functions when possible.

1. Use the following observations to calculate the mean, the median, and the mode.

8	10	9	12	12

2. Use following observations to calculate the mean, the median, and the mode.

-4	0	-6	1	-3	-4

3. Use the following observations, calculate the mean, the median, and the mode.

20	15	25	20	10	15	25	20	15

Exercise 2

Download the `ISLR2` package. You will need the **OJ** data set to answer this question.

1. Find the mean price for Country Hill (*PriceCH*) and Minute Maid (*PriceMM*).
2. Find the mean price of Country Hill (*PriceCH*) in store 1 and store 2 (*StoreID*). Which store had the better price?
3. Find the mean price paid by Country Hill (*PriceCH*) purchasers (*Purchase*) in store 1 (*StoreID*)? How about store 2? Which store had the better price?

Exercise 3

1. Over the past year an investor bought TSLA. She made these purchases on three occasions at the prices shown in the table below. Calculate the average price per share.

Date	Price Per Share	Number of Shares
February	250.34	80
April	234.59	120
Aug	270.45	50

2. What would have been the average price per share if the investor would have bought equal amounts of shares each month?

Exercise 4

1. Consider the following observations for the consumer price index (CPI). Calculate the inflation rate (Growth Rate of the CPI) for each period.

1.0	1.3	1.6	1.8	2.1

2. Suppose that you want to invest $1000 dollars in a stock that is predicted to yield the following returns in the next four years. Calculate both the arithmetic mean and the geometric mean. Use the geometric mean to estimate how much money you would have by the end of year 4.

Year	Annual Return
1	17.3
2	19.6
3	6.8
4	8.2

3.3 Answers

Exercise 1

1. To find the mean we will use the following formula ($\frac{1}{n}\sum_{i=i}^{n} x_i$). The summation of the values is 51 and the number of observations is 5. The mean is $51/5 = 10.2$.

   The median is found by locating the middle value when data is sorted in ascending order. The median in this example is 10.

   The mode is the value with the highest frequency. In this example the mode is 12 since it is repeated twice and all other numbers appear only once.

The mean can be easily verified in R by using the `mean()` function:

```
mean(c(8,10,9,12,12))
```

```
[1] 10.2
```

Similarly, the median is easily verified by using the `median()` function:

```
median(c(8,10,9,12,12))
```

```
[1] 10
```

We can use the `table()` function to calculate frequencies and easily identify the mode.

```
table(c(8,10,9,12,12))
```

```

 8  9 10 12 
 1  1  1  2 
```

2. The mean is -2.67, the median is -3.5, the mode is -4.

These mean is verified in R:

```
mean(c(-4,0,-6,1,-3,-4))
```

```
[1] -2.666667
```

The median in R:

```
median(c(-4,0,-6,1,-3,-4))
```

```
[1] -3.5
```

Finally, the mode in R:

```
table(c(-4,0,-6,1,-3,-4))
```

```

-6 -4 -3  0  1 
 1  2  1  1  1 
```

3. The mean is 18.33, the median is 20, the data is bimodal with both 15 and 20 being modes.

These mean is verified in R:

```r
mean(c(20,15,25,20,10,15,25,20,15))
```

```
[1] 18.33333
```

The median in R:

```r
median(c(20,15,25,20,10,15,25,20,15))
```

```
[1] 20
```

The frequency distribution identifies the modes:

```r
table(c(20,15,25,20,10,15,25,20,15))
```

```

10 15 20 25
 1  3  3  2
```

Exercise 2

1. The mean price for Country Hill is 1.87. The mean price for Minute Maid is 2.09.

The means can be easily found with the `mean()` function:

```r
library(ISLR2)
mean(OJ$PriceCH)
```

```
[1] 1.867421
```

```r
mean(OJ$PriceMM)
```

```
[1] 2.085411
```

2. The mean price at store 1 for Country Hill is 1.80 vs. 1.84 for store 2. The juice is cheaper at store 1.

The means for each store can be found by using indexing and a logical statement. The Country Hill mean price at store 1 is given by:

```r
mean(OJ$PriceCH[OJ$StoreID==1])
```

```
[1] 1.803758
```

The Country Hill mean price at store 2 is given by:

```r
mean(OJ$PriceCH[OJ$StoreID==2])
```

```
[1] 1.841216
```

3. Purchasers of Country Hill at store 1 paid and average of 1.80 for Country Hill juice. At store 2 they paid 1.86. Once again the average price was lower at store 1.

The mean for Country Hill purchasers at store 1 is given by:

```r
mean(OJ$PriceCH[OJ$StoreID==1 & OJ$Purchase=="CH"])
```

```
[1] 1.797176
```

The mean for Country Hill purchasers at store 2 is:

```r
mean(OJ$PriceCH[OJ$StoreID==2 & OJ$Purchase=="CH"])
```

```
[1] 1.857383
```

Exercise 3

1. The average price of sale is found by using the weighted average formula. $\frac{\sum w_i x_i}{\sum w_i}$ The weights (w_i) are given by the number of shares bought and the values (x_i) are the prices. The weighted average is 246.802.

In R you can create two vectors. One holds the share price and the other one the number of shares bought.

```r
PricePerShare<-c(250.34,234.59,270.45)
NumberOfShares<-c(80,120,50)
```

Next, you can multiply the *PricePerShare* and *NumberOfShares* vectors to find the numerator and then use `sum()` function to find the denominator. The weighted average is:

```r
(WeightedAverage<-
  sum(PricePerShare*NumberOfShares)/sum(NumberOfShares))
```

```
[1] 246.802
```

2. The average if equal shares were bought would be 251.7933.

In R you can use the `mean()` function on the PricePerShare vector.

```r
(Average<-mean(PricePerShare))
```

```
[1] 251.7933
```

Exercise 4

1. The inflation rate for each period is shown in the table below:

30%	23.08%	12.5%	16.67%

In R create an object to store the values of the CPI:

```
CPI<-c(1,1.3,1.6,1.8,2.1)
```

Next use the `diff()` function to find the difference between the end value and start value. Divide the result by a vector of starting value and multiply times 100.

```
(Inflation<-100*diff(CPI)/CPI[1:4])
```

```
[1] 30.00000 23.07692 12.50000 16.66667
```

2. At the end of 4 years it is predicted that you would have 1621.17 dollars. Each year you would have gained 12.84% on average.

In R include the annual rates in a vector:

```
growth<-c(0.173,0.196,0.068,0.082)
```

The arithmetic mean is:

```
100*mean(growth)
```

```
[1] 12.975
```

The geometric mean is:

```
(geom<-((prod(1+growth))^(1/4)-1)*100)
```

```
[1] 12.8384
```

At the end of the four years we would have:

```
1000*(1+geom/100)^4
```

```
[1] 1621.167
```

References

Grolemund, Garret. 2014. "Hands-on Programming with r." https://jjallaire.github.io/hopr/.
Wickham, Hadley. 2017. "R for Data Science." https://r4ds.hadley.nz.