Extended Kalman Filter for State Estimation of a Micro Aerial Vehicle

Methods and Results

Jagennath Hari – jh7454

Robot Localization and Navigation - Project 1

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Dr. Loianno
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1) Introduction

In this project, I implemented an Extended Kalman filter to estimate the position, velocity, orientation, and sensor biases of a Micro Aerial Vehicle. There are two parts in the project. In the first part, the measurement update is given by the position and orientation from the Vicon. In the second part, the measurement update is given by the Velocity only.

Since the difference in both parts only lies in the measurement models, I will first discuss my approach to deriving the process model for the prediction step. The measurement models and results sections will each be divided in two for both the parts.

2) Process model and Prediction model

The state x, is a 15x1 vector and is defined by the following –

$$X = \begin{bmatrix} \mathbf{p} \\ \mathbf{q} \\ \dot{\mathbf{p}} \\ \mathbf{b}_g \\ \mathbf{b}_a \end{bmatrix}$$

Where p is the position, q is the orientation, ṗ is the velocity of the VICON, b_g and b_a are gyro and accelerometer bias respectively.

2.1) Ẋ Vector

The gyro measurements, ω_n is represented in the body frame and is written as –

$$\omega_n = \omega + b_g + n_g$$

n_g is the additive gaussian white noise. To change the frames, we know the following procedure –

G(q) is the Euler rates, and we know this to be –

$$G = \begin{pmatrix} \cos(q_y)\cos(q_z) & -\sin(q_z) & 0 \\ \cos(q_y)\sin(q_z) & \cos(q_z) & 0 \\ -\sin(q_y) & 0 & 1 \end{pmatrix}$$

We know –

$$\omega = G(q)\dot{q}$$

Hence –

$$\dot{q} = G(x_2)^{-1}(\omega_n - b_g - n_g)$$

a_m is the accelerometer measurement so we can write it as –

$$a_m = R(q)^T(\ddot{p} - g) + b_a + n_a$$

Note that that g, the gravity component in the world frame. R(q) is used to change the frames. Following that we know –

$$\ddot{p} = g + R(q)(a_m - b_a - n_a)$$

From the above we can find $\dot{X}$

$$\dot{\mathbf{x}} = \begin{bmatrix} \mathbf{x}_3 \\ G(\mathbf{x}_2)^{-1}(\omega_m - \mathbf{x}_4 - \mathbf{n}_g) \\ \mathbf{g} + R(\mathbf{x}_2)(\mathbf{a}_m - \mathbf{x}_5 - \mathbf{n}_a) \\ \mathbf{n}_{bg} \\ \mathbf{n}_{ba} \end{bmatrix}$$

2.2) Prediction Step

From the above we can model the prediction step –

$$\bar{\mu}_t = \mu_{t-1} + \delta t\, f(\mu_{t-1}, u_t, 0)$$
$$\bar{\Sigma}_t = F_t\, \Sigma_{t-1}\, F_t^T + V_t\, Q_d\, V_t^T$$

By assuming these –

$$\left.\begin{array}{l} \circ\ \dot{x} = f(x, u, n) \\ \circ\ n \sim N(0, Q) \end{array}\right\} \text{Assumptions}$$

We get –

- $A_t = \frac{\partial f}{\partial x}\Big|_{\mu_{t-1}, u_t, 0}$
- $U_t = \frac{\partial f}{\partial n}\Big|_{\mu_{t-1}, u_t, 0}$

Linearization

- $F_t = I + \delta t \, A_t$
- $V_t = U_t$
- $Q_d = Q \, \delta t$

Discretization

3) Measurement model and Update Step

In the first part we are provided with measurements of the position and orientation for the update. In the second part, only velocity is provided. The following equations are used for the update step:

- $\mu_t = \bar{\mu}_t + K_t \, (z_t - g(\bar{\mu}_t, 0))$
- $\Sigma_t = \bar{\Sigma}_t - K_t \, C_t \, \bar{\Sigma}_t$
- $K_t = \bar{\Sigma}_t \, C_t^T \, (C_t \, \bar{\Sigma}_t \, C_t^T + W_t \, R \, W_t^T)^{-1}$

With the following –

- $z = g(x, v)$
- $v \sim N(0, R)$

Assumptions

- $C_t = \frac{\partial g}{\partial x}\Big|_{\bar{\mu}_t, 0}$
- $W_t = \frac{\partial g}{\partial v}\Big|_{\bar{\mu}_t, 0}$

Linearization

3.1) Update model for part 1

$$z = \begin{bmatrix} p \\ q \end{bmatrix} + v = \begin{bmatrix} I & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} p \\ q \\ \dot{p} \\ b_g \\ b_a \end{bmatrix} + v = C\boldsymbol{X} + \boldsymbol{v}$$

In this case, C is a 6x15 matrix.

3.2) Update model for part 2

$$\mathbf{z} = \begin{bmatrix} \mathbf{p} \\ \mathbf{q} \\ \dot{\mathbf{p}} \end{bmatrix} + \mathbf{v} = \begin{bmatrix} I & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{p} \\ \mathbf{q} \\ \dot{\mathbf{p}} \\ \mathbf{b}_g \\ \mathbf{b}_a \end{bmatrix} + \mathbf{v} = C\,\mathbf{x} + \mathbf{v}$$

In this case, C is a 9x15 matrix.

4) Results Part 1

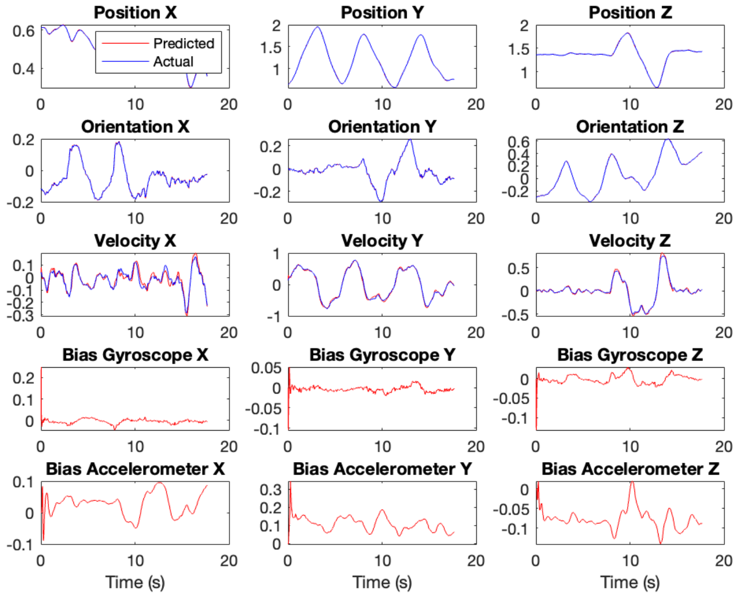

Figure 1: Dataset 1

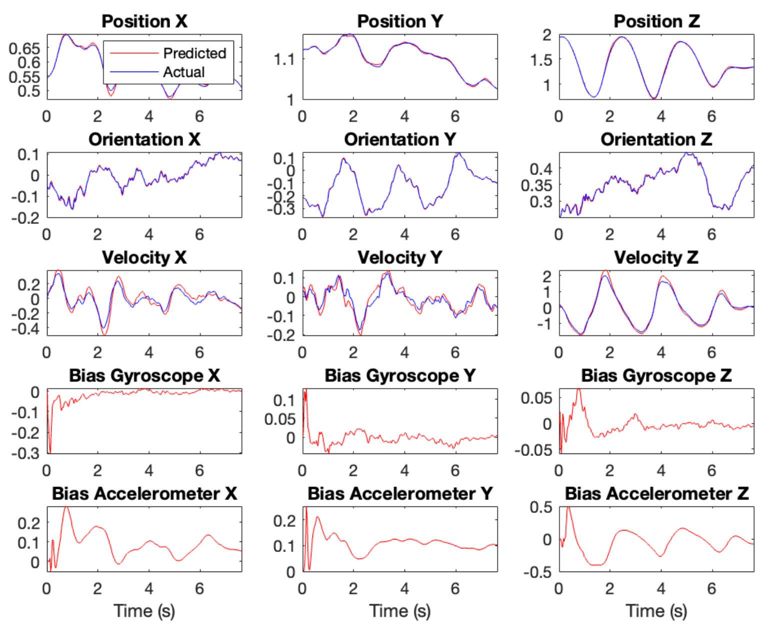

Figure 2: Dataset 4

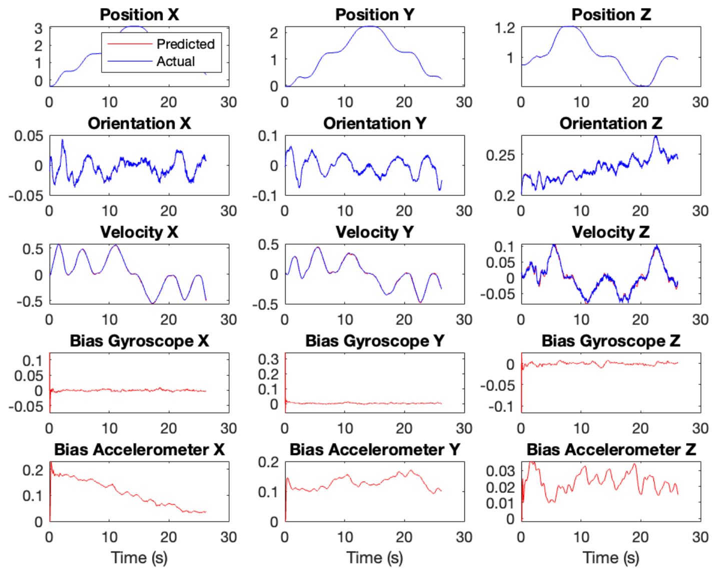

Figure 3: Dataset 9

5) Results Part 2

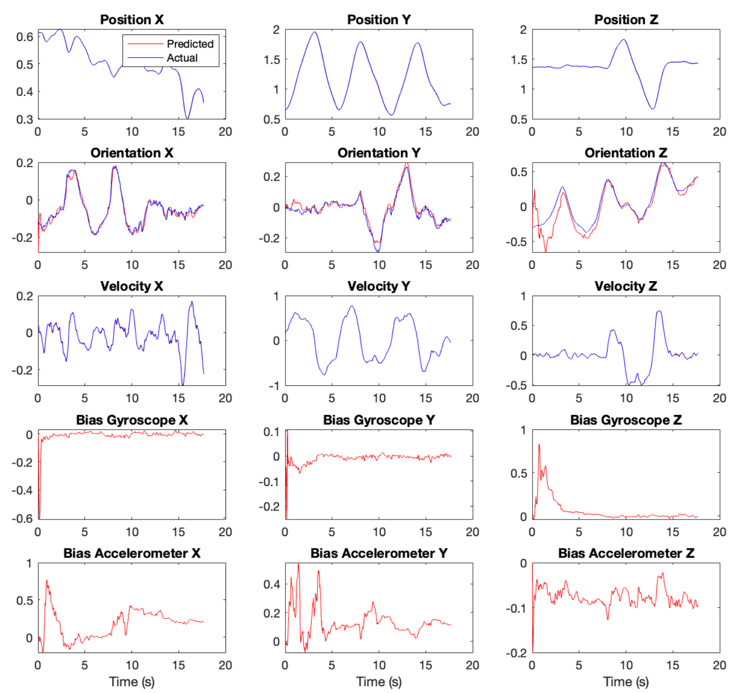

Figure 4: Dataset 1

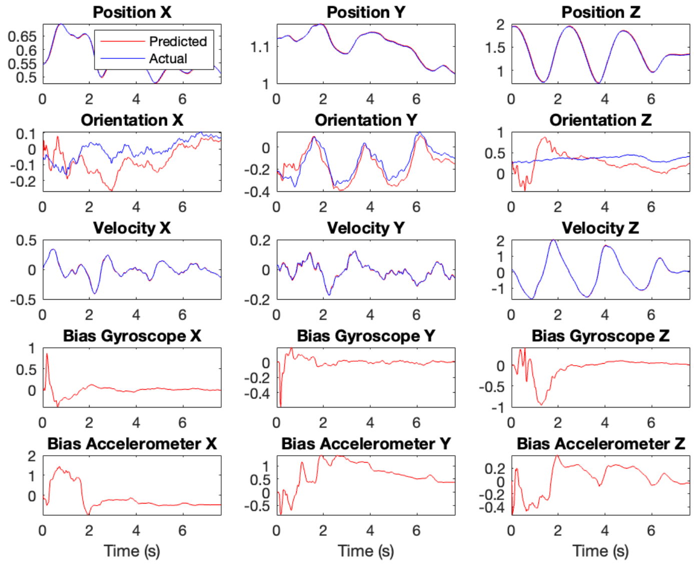

Figure 5: Dataset 4

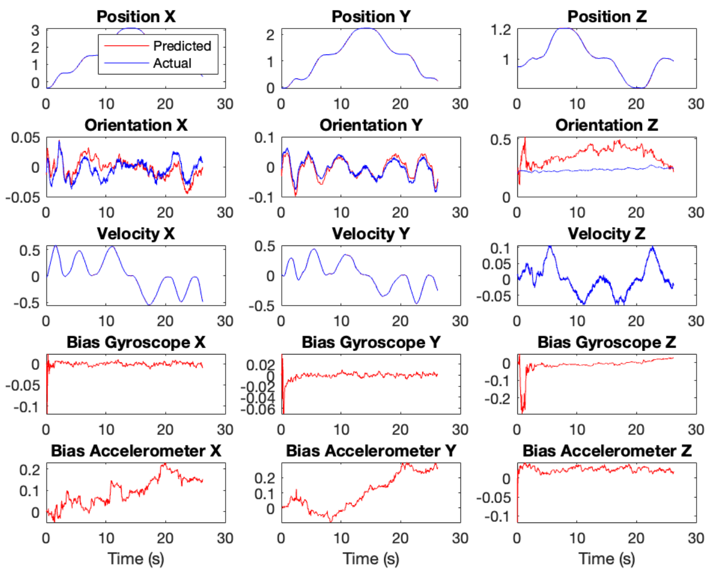

Figure 6: Dataset 9