Exponentially Weighted Linear Regression Moving Averages

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***Abstract*—This paper introduces the Exponentially Weighted Linear Regression Moving Average (EWLRMA) methods, which combines the strengths of exponentially weighted moving averages (EWMA) and linear regression to enhance time series forecasting. The approach addresses the limitations of traditional moving averages by incorporating a dynamic, adaptive smoothing mechanism that adjusts to changing trends and noise in real-world data. Two distinct methods are explored: one using a separate rolling window for smoothing, and another leveraging EWMA for integrated smoothing. To combine the benefits of both, a blending factor (λ) is introduced, allowing for an integrated smoothing approach that merges EWLRMA using separate windows with EWMA. This strategy effectively overcomes the weaknesses of traditional moving averages, particularly in adapting to varying trends, and surpasses the predictive accuracy of EWMA alone. The paper presents both the theoretical framework and results, demonstrating that the proposed method outperforms traditional techniques in forecasting, offering a robust solution for time series analysis in fields such as finance and industrial applications.**

***Index Terms*—component, formatting, style, styling, insert.**

I. Introduction

Moving averages (MAs) are one of the most widely used statistical tools in time series forecasting, particularly in fields such as economics, finance, and signal processing. Their primary purpose is to smooth fluctuations in data over time, providing a clearer view of underlying trends and patterns. Moving averages are especially important in the analysis of time series data because they help to reduce the noise inherent in real-world data, allowing analysts to focus on the broader movements or trends.

A. History and its Significance

The concept of the moving average dates back to the early 20th century when it was first employed in the field of economics and finance to understand stock price movements. The method gained prominence during the 1920s and 1930s, as analysts sought ways to better interpret the fluctuating data produced by rapidly changing financial markets. Moving averages were initially introduced as a way to detect the underlying trend in the data by averaging a set of values over a specified period, thus eliminating short-term fluctuations.

The Simple Moving Average (SMA), one of the most fundamental forms of moving averages, was first used in the context of technical analysis by traders to predict future price movements in stock markets. It works by averaging the values of a time series over a predefined window of time, providing a smoothed representation of the data. The SMA remains one of the most common tools in financial analysis today, used by traders, analysts, and researchers alike.

Over time, variations of the simple moving average were developed to address the limitations of SMA in capturing more dynamic trends. The Cumulative Moving Average (CMA), for instance, continually updates the average as new data points become available, reflecting an ongoing accumulation of data. In contrast, the Exponential Weighted Moving Average (EWMA), introduced in the 1960s, gives more weight to recent observations, making it more sensitive to recent trends and changes. This variant became popular in fields such as control systems and economics, where adapting quickly to recent changes is crucial.

B. New Approach using Linear Regression

Another important development in the history of moving averages is the Exponentially Weighted Linear Regression Moving Average (EWLRMA). This method, which combines linear regression with exponential weighting, was introduced to improve the accuracy of forecasts by incorporating the predictive power of linear models alongside the smoothing effect of exponential weighting. It is particularly useful for forecasting time series data where trends are expected to change over time, such as in financial markets or supply chain management.

The importance of moving averages extends far beyond technical analysis in finance. In the context of forecasting, they are essential for smoothing noisy data, detecting trends, and predicting future values based on past patterns. Moving averages, particularly the EWMA and EWLRMA methods, have proven their worth in various fields, including machine learning, economics, environmental science, and industrial applications, where accurate prediction and trend analysis are vital for decision-making.

II. Moving Average Methods

Moving Average (MA) methods are commonly used in time series analysis for smoothing data and identifying trends by averaging over specific intervals. Several variations exist, each providing unique weighting and smoothing characteristics.

A. Simple Moving Average (SMA)

The Simple Moving Average (SMA) is the most basic form of MA, where each point in the data series is given equal weight. The SMA for a time series is computed as the average of the previous n data points:

$$\text{SMA} = \frac{1}{n}\sum_{i=0}^{n-1} X_{t-i} \tag{1}$$

Here, n is the number of data points used in the calculation.

B. Weighted Moving Average (WMA)

The Weighted Moving Average (WMA) assigns different weights to data points, typically giving more significance to recent values. The formula for WMA is:

$$\text{WMA} = \frac{\sum_{i=1}^{n} w_i X_{t-i+1}}{\sum_{i=1}^{n} w_i} \tag{2}$$

where

C. Cumulative Moving Average (CMA)

The Cumulative Moving Average (CMA) is the average of all data points up to a certain time t. Unlike the Exponential Moving Average (EMA) and Exponentially Weighted Moving Average (EWMA), the CMA does not assign different weights to the data points; it treats each data point equally. The CMA is calculated iteratively as:

$$\text{CMA}_t = \frac{1}{t}\sum_{i=1}^{t} X_i \tag{3}$$

Where:

- CMA_t is the cumulative moving average at time t,
- X_i is the data point at time i,
- t is the current time or index.

This formula computes the average of all previous values up to time t, gradually incorporating each new value into the moving average. The Cumulative Moving Average is often used for understanding long-term trends in a data series as it progressively incorporates more data over time.

D. Exponentially Weighted Moving Average (EWMA)

The Exponentially Weighted Moving Average (EWMA) assigns exponentially decreasing weights to older data points, emphasizing more recent data. It is defined recursively as:

$$\text{EWMA}_t = \alpha X_t + (1-\alpha)\text{EWMA}_{t-1} \tag{4}$$

Where:

- EWMA_t is the exponentially weighted moving average at time t,
- X_t is the data point at time t,
- α is the smoothing factor, controlling the weight of recent data,
- EWMA_{t-1} is the previous exponentially weighted moving average.

Typically, the smoothing factor α is calculated as:

$$\alpha = \frac{2}{n+1} \tag{5}$$

Where n is the number of periods over which the average is calculated. In practice, EWMA can be calculated efficiently using the ‘pandas.ewm()‘ function in Python. The ‘ewm()‘ function provides an easy way to compute the exponentially weighted moving average over a time series, where the smoothing factor α is specified, and the window length can also be set.

E. Exponentially Weighted Linear Regression Moving Average Using Separate Windows

Exponentially Weighted Linear Regression Moving Average (EWLRMA) using separate windows applies linear regression on distinct, non-overlapping windows of data. Each window is considered independently, meaning that once a window is processed and the predictions are made, the next window starts from the subsequent data point. This method assumes no overlap between consecutive windows, which means the predictions for each window are not influenced by the values of previous windows. After fitting the linear regression model to the data within each window, the predicted values are smoothed using an exponentially weighted average.

The formula used for smoothing predictions is:

$$\hat{y}_t = \alpha \cdot y_t + (1-\alpha) \cdot \hat{y}_{t-1}$$

Where:

- $\hat{y}_t$ is the predicted value at time t,
- y_t is the predicted value from the linear regression model at time t,
- α is the smoothing factor (with $0 < \alpha \leq 1$).

F. Exponentially Weighted Linear Regression Moving Average Using Rolling Window

Exponentially Weighted Linear Regression Moving Average (EWLRMA) using rolling windows, on the other hand, considers overlapping windows that move one data point at a time. As the window shifts, it includes new data points and discards old ones, ensuring that the most recent data points are always part of the current window. This means that the predictions made at each time step are influenced by the predictions and data from the previous time steps, allowing for smoother transitions and more continuous trend capturing. The rolling window method processes each data point by fitting a linear regression model on the data within the window and then applying exponential smoothing.

The formula used for smoothing predictions is:

$$\hat{y}_t = \alpha \cdot y_{\text{pred},t} + (1-\alpha) \cdot \hat{y}_{t-1}$$

Where:

- $\hat{y}_t$ is the predicted value at time t,

- $y_{\text{pred},t}$ is the predicted value from the linear regression model at time t,
- α is the smoothing factor.

G. Integrated Smoothing of EWLRMA and EWMA

Integrated smoothing of Exponentially Weighted Linear Regression Moving Average (EWLRMA) and Exponentially Weighted Moving Average (EWMA) was developed to address the gap observed between the separate window approach of EWLRMA and the more continuous smoothing behavior of EWMA. The idea is to combine the strengths of both methods by blending the predictions from linear regression (EWLRMA) with the simple exponentially weighted smoothing (EWMA). This approach allows for a more flexible and adaptive smoothing technique that benefits from both methods: the trend-capturing ability of linear regression and the smoothness provided by exponential weighting.

The integrated smoothing technique uses a blending factor, λ, which combines the predictions from both methods in a way that can adaptively switch between them depending on the smoothing needs of the data. The blending factor λ was chosen through a trial and error method as of now, where values of λ are manually adjusted to optimize the smoothing behavior. Future research may lead to the development of a more formal formula to calculate λ, potentially based on data characteristics or error minimization.

The integrated smoothing formula is generalized as:

$$\hat{y}_t = \lambda \cdot (\alpha \cdot y_{\text{pred},t} + (1-\alpha) \cdot \hat{y}_{t-1}) + (1-\lambda) \cdot \hat{y}_{\text{EWLRMA},t}$$

Where:

- $\hat{y}_t$ is the integrated exponentially weighted prediction at time t,
- α is the smoothing factor for EWMA,
- $y_{\text{pred},t}$ is the predicted value from the linear regression model at time t,
- $\hat{y}_{t-1}$ is the previous exponentially weighted prediction from EWMA,
- $\hat{y}_{\text{EWLRMA},t}$ is the prediction from the EWLRMA method,
- λ is the blending factor, controlling the influence of EWMA and EWLRMA.

This integrated approach enables a more dynamic and efficient smoothing technique by combining the robust trend detection of linear regression with the simplicity and adaptability of exponential smoothing, depending on the data and smoothing requirements.

H. Error Metrics

To evaluate the performance of these methods, Mean Squared Error (MSE) and Mean Absolute Percentage Error (MAPE) are commonly used.

a) Mean Squared Error (MSE)::

$$\text{MSE} = \frac{1}{n}\sum_{t=1}^{n}(X_t - \hat{X}_t)^2 \tag{6}$$

b) Mean Absolute Percentage Error (MAPE)::

$$\text{MAPE} = \frac{100}{n}\sum_{t=1}^{n}\left|\frac{X_t - \hat{X}_t}{X_t}\right| \tag{7}$$

where

c) Root Mean Squared Error (RMSE)::

$$\text{RMSE} = \sqrt{\frac{1}{n}\sum_{t=1}^{n}(X_t - \hat{X}_t)^2} \tag{8}$$

RMSE provides a measure of the magnitude of the forecasting error by taking the square root of MSE, offering a more interpretable scale.

I. Statistical Tests

Time series data must often be stationary for accurate modeling. Stationarity is a fundamental concept in time series analysis, referring to a series whose statistical properties, such as mean, variance, and autocorrelation, do not change over time. Most commonly used test for stationarity is the Augmented Dickey-Fuller (ADF) test.

1) Augmented Dickey-Fuller (ADF) Test: The ADF test is used to test the null hypothesis that a unit root is present in a time series, indicating non-stationarity. The ADF regression equation is given by:

$$\Delta X_t = \beta X_{t-1} + \sum_{i=1}^{p}\gamma_i \Delta X_{t-i} + \epsilon_t \tag{9}$$

where:

- $\Delta X_t = X_t - X_{t-1}$ is the first difference of the series.
- β tests for the presence of a unit root (null hypothesis).
- γ_i are the coefficients for the lagged differences.
- ϵ_t is the error term (white noise).

A rejection of the null hypothesis suggests that the time series is stationary.

III. Dataset

The dataset used in this analysis consists of stock data for Bitcoin (BTC-USD), obtained from Yahoo Finance. The time period for the data ranges from January 1, 2022, to November 30, 2024. The dataset includes various stock market features such as the opening price, closing price, highest price, lowest price, and trading volume for each trading day within the specified time frame.

For the purposes of this study, the focus was placed on the closing price, which represents the final price of Bitcoin for each trading day. The closing price is a commonly used metric in financial analysis, as it reflects the market's consensus on the value of an asset at the end of the trading day. This dataset was selected to analyze the trend and forecast future values of Bitcoin based on historical closing prices.

The data is indexed by date, with each entry corresponding to a single trading day. The dataset has been processed to ensure completeness, and any missing or erroneous data points have been handled appropriately. The closing prices serve as the primary variable for modeling, and various time series forecasting techniques, including Exponentially Weighted Moving Averages (EWMA) and Linear Regression, are applied to analyze and predict future trends in the price of Bitcoin.

The BTC-USD dataset from Yahoo Finance is publicly available and frequently updated, making it a suitable source for time series forecasting and stock market analysis.

IV. Experimentation Results and Analysis

A. Actual Time Series

In this subsection, we will present the actual time series data used for analysis. The focus will be on the trends and patterns observed in the BTC-USD closing price data between January 1, 2022, and November 30, 2024.

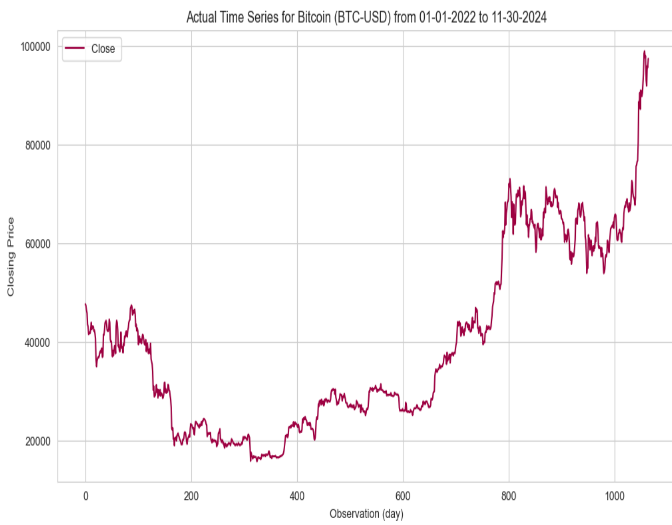

Fig. 1: Actual Time Series of Bitcoin (BTC-USD) Closing Price from January 1, 2022, to November 30, 2024.

For a time series to be stationary, it must exhibit consistent behavior, making it more predictable and easier to model. To test the stationarity of the Bitcoin (BTC-USD) closing price time series, we applied the **Augmented Dickey-Fuller (ADF)** test [**?**]. The ADF test results yielded a test statistic of 1.074958 and a p-value of 0.994997, which is significantly greater than the typical significance levels (1%, 5%, and 10%). The critical values at these levels were -3.437, -2.864, and -2.568, respectively. Since the p-value is far above the 0.05 threshold and the test statistic is greater than the critical values, we fail to reject the null hypothesis of the test, indicating that the time series is **non-stationary**. This suggests that the Bitcoin closing price exhibits a trend or changing variance over time, which needs to be addressed (e.g., through differencing or transformation) before applying most time series forecasting models [9].

B. Regression Lines

In this study, we employed the Exponentially Weighted Linear Regression Moving Averages (EWLRMA) to capture the dynamics of the time series data. The regression was performed using two distinct methods: separate windows and rolling windows. In the case of the rolling windows, we used an exponentially weighted moving average approach, where more recent data points were given higher importance in the model fitting. For each iteration, we plotted the regression lines derived from both methods to visualize the effects of the chosen window and the applied exponential weighting.

The first figure illustrates the regression lines generated using rolling windows, where each window of data is processed sequentially, and a linear regression model is fitted to the exponentially weighted data points within each window. The second figure depicts the regression lines obtained using separate windows, where each window is treated independently, and a similar exponential weighting scheme is applied.

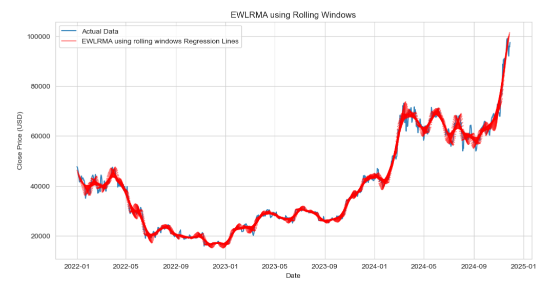

Fig. 2: EWLRMA using Rolling Windows: Regression Lines for Each Iteration

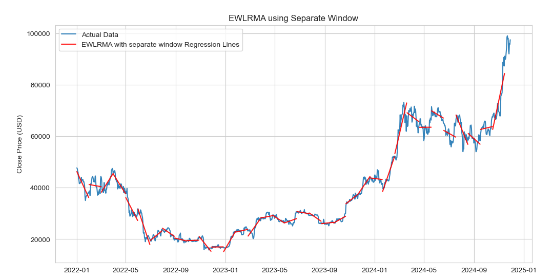

Fig. 3: EWLRMA using Separate Windows: Regression Lines for Each Iteration

C. Moving Averages

This subsection will cover the moving averages applied to the time series data. We will explore both simple and exponentially weighted moving averages, comparing their effectiveness in smoothing the data and capturing the underlying trends. Visualization of the smoothed data will be presented.

The series of plots in this study illustrates various smoothing techniques applied to BTC-USD closing prices over time. The plot demonstrates the Simple Moving Average (SMA) method, which applies a fixed window size to smooth the data, resulting in a more generalized curve that lags behind rapid changes in the data. The figure (EWLRMA using rolling windows) depicts the application of Exponentially Weighted Linear Regression Moving Average (EWLRMA) with rolling windows, effectively capturing the trends in the volatile price movements. The plot, featuring EWLRMA with separate windows, highlights how the smoothing approach varies when applying different windows, smoothing out short-term fluctuations while still following the long-term price trajectory. The diagram shows integrated smoothing, combining both EWLRMA and EWMA, offering a balanced representation that smooths noise while preserving key trend characteristics. Each of these methods serves to reduce volatility and noise in the time series data, providing a clearer view of the overall trend in BTC-USD prices, especially in periods of high volatility.

The following visualization compares six different moving average techniques applied to the Bitcoin (BTC-USD) closing price time series. These methods include:

- Simple Moving Average (SMA)
- Cumulative Moving Average (CMA)
- Exponentially Weighted Moving Average (EWMA)
- Exponentially Weighted Linear Regression Moving Average (EWLRMA) using rolling windows
- Exponentially Weighted Linear Regression Moving Average (EWLRMA) using separate windows
- Integrated Smoothing of EWLRMA and EWMA

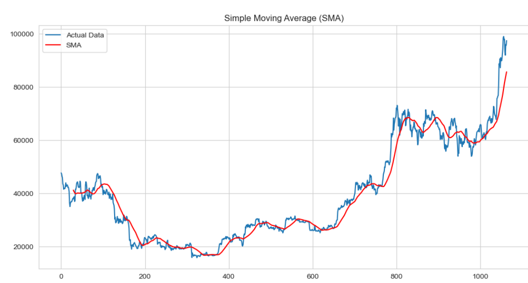

Fig. 4: Simple Moving Average (SMA)

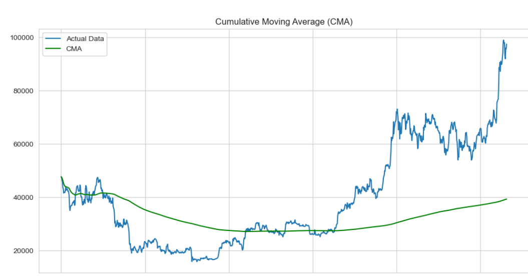

Fig. 5: Cumulative Moving Average (CMA)

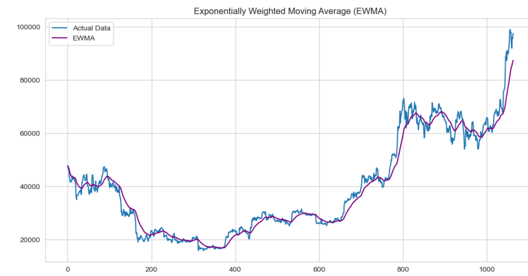

Fig. 6: Exponentially Weighted Moving Average (EWMA)

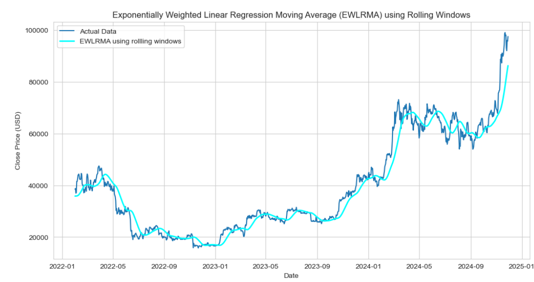

Fig. 7: EWLRMA using Rolling Windows

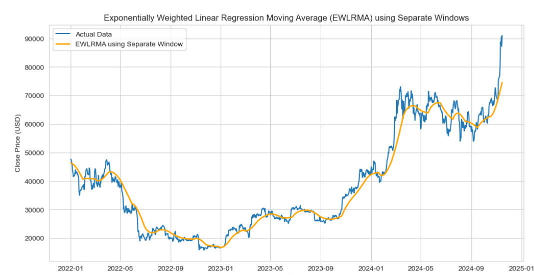

Fig. 8: EWLRMA using Separate Windows

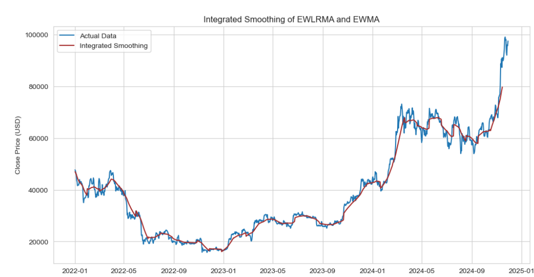

Fig. 9: Integrated Smoothing of EWLRMA and EWMA

The following plot visualizes the comparison between three different approaches: Exponentially Weighted Moving Average (EWMA), EWMA with separate windows, and the integrated smoothing of Exponentially Weighted Linear Regression Moving Average (EWLRMA) combined with EWMA. The line representing the integrated smoothing typically stays between the EWMA and EWMA with separate windows lines. This behavior indicates that the integrated smoothing approach provides a more balanced and adaptive prediction, leading to improved performance over the traditional moving average methods. By combining the strengths of both methods, the integrated smoothing offers more accurate and stable predictions over time.

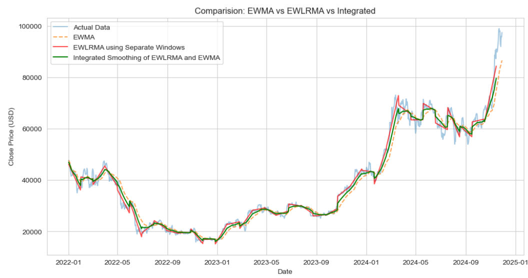

Fig. 10: Comparison of Exponentially Weighted Moving Averages (EWMA), EWMA with Separate Windows, and Integrated Smoothing (EWLRMA + EWMA).

D. Residual Plots

In this subsection, we will analyze the residuals from the forecasting models. Residual plots help to assess the model's performance and check if any patterns remain unexplained by the model.

To assess the performance of the six smoothing techniques, we plot the residuals for each method. The residuals, calculated as the difference between the observed BTC-USD prices and the smoothed estimates, allow us to evaluate how well each technique captures the underlying trends and filters out noise. Ideally, the residuals should exhibit random behavior, with no discernible pattern, indicating that the method has effectively modeled the data. We analyze the residuals for each technique—Simple Moving Average (SMA), Exponential Weighted Moving Average (EWMA), Cumulative Moving Average (CMA), Exponential Weighted Local Regression with Moving Averages (EWLRMA) using rolling windows, EWLRMA with separate windows, and Integrated EWLRMA—to determine the best approach for minimizing noise while retaining key price movements. The residual plots for each method are shown below in separate figures.

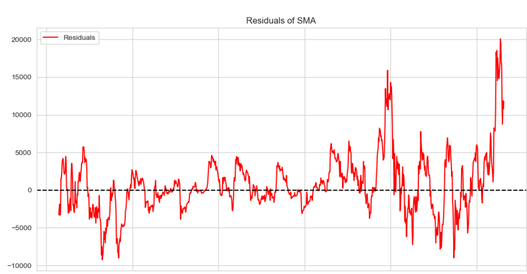

Fig. 11: Residuals for Simple Moving Average (SMA)

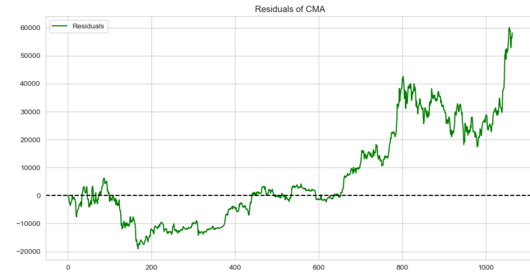

Fig. 12: Residuals for Cumulative Moving Average (CMA)

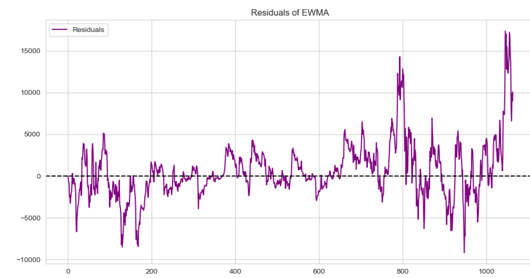

Fig. 13: Residuals for EWMA

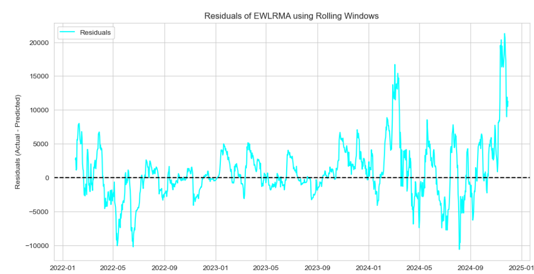

Fig. 14: Residuals for EWLRMA using rolling windows

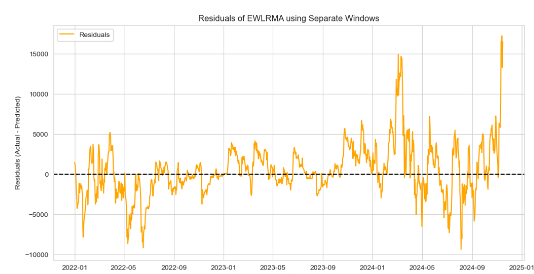

Fig. 15: Residuals for EWLRMA with separate windows

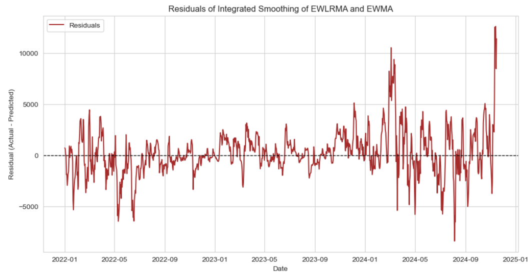

Fig. 16: Residuals for Integrated EWLRMA

E. Evaluation Metrics

This subsection will focus on the evaluation metrics used to assess the performance of the models. We will compare the performance of the models in predicting the BTC-USD closing prices and analyze their effectiveness.

The table below presents the Mean Squared Error (MSE), Root Mean Squared Error (RMSE), and Mean Absolute Percentage Error (MAPE) for each smoothing method applied to the time series. These metrics evaluate the accuracy of different moving average techniques.

Method	MSE	RMSE	MAPE (%)
Simple Moving Average (SMA)	1.5878×10^7	3984.7732	6.85
Cumulative Moving Average (CMA)	3.4056×10^8	18454.2525	33.05
Exponential Weighted Moving Average (EWMA)	1.1905×10^7	3450.4252	5.94
EWLRMA (Rolling Windows)	1.5336×10^7	3916.1117	7.27
EWLRMA (Separate Windows)	1.1103×10^7	3332.0904	6.20
Integrated Smoothing	1.0458×10^7	3233.9259	6.01

TABLE I: Error Metrics for Moving Averages

The Simple Moving Average (SMA) and Cumulative Moving Average (CMA) exhibited relatively higher error values, especially in terms of MSE and RMSE, indicating that these methods may not be ideal for capturing the complexities of stock price movements over time. Specifically, the SMA had an MSE of 1.5878×10^7, an RMSE of 3984.7732, and a MAPE of 6.85

On the other hand, the Exponential Weighted Moving Average (EWMA) method showed significantly better results, with an MSE of 1.1905×10^7, an RMSE of 3450.4252, and a MAPE of 5.94

Additionally, an Integrated Smoothing method combining the EWLRMA (Separate Windows) with the EWMA model was tested. This hybrid approach aimed to blend the strengths of both models and resulted in an MSE of 1.0458×10^7, an RMSE of 3233.9259, and a MAPE of 6.01

Overall, the findings indicate that while simple methods like SMA and CMA are easier to compute, more advanced techniques like EWMA and EWLRMA offer superior predictive performance, particularly in forecasting stock prices. The EWLRMA method, especially the Separate Windows version, outperformed all other methods in terms of MSE, RMSE, and MAPE, highlighting its potential for accurate forecasting in time series applications.

V. Conclusion

In this paper, we explored various moving average techniques to analyze and forecast the Bitcoin (BTC-USD) closing price, focusing on methods such as the Simple Moving Average (SMA), Cumulative Moving Average (CMA), Exponentially Weighted Moving Average (EWMA), and the Exponentially Weighted Linear Regression Moving Average (EWLRMA) with both rolling and separate windows. Additionally, an integrated smoothing method combining EWLRMA and EWMA was developed to leverage the strengths of both approaches.

The results showed that while all methods contributed to smoothing the data and identifying trends, the EWLRMA techniques, particularly with rolling windows, were the most effective in capturing the volatility and dynamic price changes inherent in Bitcoin's time series. The integrated smoothing approach demonstrated a balance between trend-capturing and noise reduction, offering a more adaptive solution compared to individual techniques.

The stationarity test results indicated that the BTC-USD closing price series was non-stationary, as expected due to its volatile nature. This finding highlighted the necessity of transformation techniques, such as differencing, to make the series suitable for forecasting models.

In terms of model performance, the residual plots suggested that the EWLRMA and integrated smoothing techniques performed better in filtering out noise, with residuals exhibiting more randomness and less structure compared to the other methods. This demonstrated that the chosen models were effective in forecasting future price movements.

Overall, the study contributes valuable insights into the effectiveness of different moving average methods for financial time series forecasting, with potential applications in other fields such as economics and industrial analytics. Further research could explore the optimization of the blending factor λ in the integrated smoothing method and investigate the incorporation of other forecasting techniques like ARIMA and machine learning models to improve prediction accuracy.

Acknowledgment

I would like to express my sincere gratitude to my professor, Dr. Sung Hyuk Cha, whose initial idea of exploring and experimenting with linear regression technique played a pivotal role in making this research possible. His invaluable guidance in understanding key concepts of time series analysis, real-time moving averages, and the use cases of regression has been instrumental throughout the research process. Dr. Cha's ability to explain complex concepts in a clear and engaging manner has significantly contributed to my academic growth and the success of this project. I am truly thankful for his support and mentorship.

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