16720 HW 4 Write-up

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- **Q1.1 Principal points coinciding with origin**

$p_2 F p_1 = 0$

Since coordinate origin coincides with principal point:

$$\begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 0$$

On solving the above equation:
$F_{33} = 0$

- **Q1.2 Pure translation parallel to x-axis**

Because of pure translation:
$C1 = K[I|0]$ and $C2 = K[I|t]$
From the equation for Fundamental matrix: $F = [e']_x K K^{-1} = [e']_x$
If the camera translation is parallel to x axis, then $e' = (1, 0, 0)^T$

$$F = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

If $p_1 = (x_1, y_1, 1)^T and p_2 = (x_2, y_2, 1)^T$ On solving $p_2 F p_1 = 0$,
$y1 = y2$
Equation of epipolar line:

$$l = \begin{bmatrix} [0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0] \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ y1 \end{bmatrix}$$

This means that the epipolar line in the first camera has slope zero and hence is parallel to x-axis. Same cane be proved for the epipolar line in the second camera.
Hence, epipolar lines in the two cameras are parallel to x-axis

- **Q1.3 Frames at different time stamps**

$P \equiv$ 3D world coordinates of the point in the image
$p_1 \equiv$ 2D coordinates on the image plane at time frame i
$p_2 \equiv$ 2D coordinates on the image plane at time frame i+1

World coordinate and image plane can be related by: $P = t_1 + R_1 p_1$
Similarly: $p_2 = R_2^{-1}(P - t_2)$
Combining the above 2 equations:
$p_2 = R_2^{-1}(t_1 + R_1 p_1 - t_2)$
$p_2 = R_2^{-1} R_1 p_1 + R_2^{-1}(t_1 - t_2)$
Comparing this with: $p_2 = R_{rel} p_1 + t_{rel}$

$$R_{rel} = R_2^{-1} R_1$$
$$t_{rel} = R_2^{-1}(t_1 - t_2)$$

Also, from the equations for Essential Matrix and Fundamental Matrix

$$E = [t_{rel}]_x R_{rel}$$
$$F = K^{-T} [t_{rel}]_x R_{rel} K^{-1}$$

- **Q1.4 Camera looking at a point and it's mirror image**

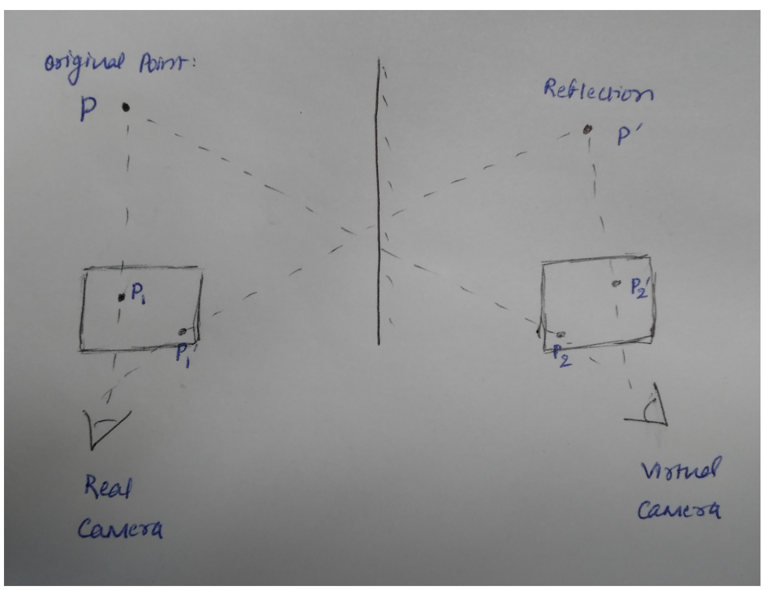


Figure 1: Camera capturing an object and it's mirror image

The case can be considered to have a real camera and a virtual camera which captures the mirror image of the point.
In Fig 1:

$P \equiv$ 3D world coordinates of the original point
$P' \equiv$ 3D world coordinates of the mirror image
$p_1 \equiv$ 2D coordinates corresponding to P on real camera
$p_1' \equiv$ 2D coordinates corresponding to P' on real camera
$p_2 \equiv$ 2D coordinates corresponding to P on virtual camera
$p_2' \equiv$ 2D coordinates corresponding to P' on virtual camera

Since P' is a transformation of P, they can be related by $P' = TP$. where T is the transformation matrix.
Also, since the virtual camera is obtained by the same transformation over the real camera:
$M_2 = TM_1$
From the properties of fundamental matrix:
$p_2 F p_1' = 0$
Taking transpose: $p_1' F^T p_2 = 0$
Adding the above two equations:
$p_2 F p_1' + p_1' F^T p_2 = 0$
Now, $p_1' = Tp_1$

$$\implies p_2 F T p_1 + T p_1 F^T p_2 = 0 \tag{1}$$

The image coordinates can be related by world coordinates as follows:
$p_2 = M_2 P = TM_1 P$
$p_1 = M_1 P$
Combining this with equation 1:
$TM_1 PFTM_1 P + TM_1 PF^T TM_1 P = 0$

$$\implies M_1 TP(F + F^T) M_1 TP = 0 \tag{2}$$

Since $M_1 TP$ is non-zero, this means $F + F^T = 0$
Hence, $F = -F^T$ and Fundamental matrix is a skew-symmetric matrix

- **Q2.1 The Eight Point Algorithm**

Fundamental Matrix from Eight Point Algorithm:

$$F = \begin{bmatrix} -0.0000 & -0.0000 & 0.0011 \\ -0.0000 & 0.0000 & 0.0000 \\ -0.0010 & 0.0000 & -0.0045 \end{bmatrix}$$

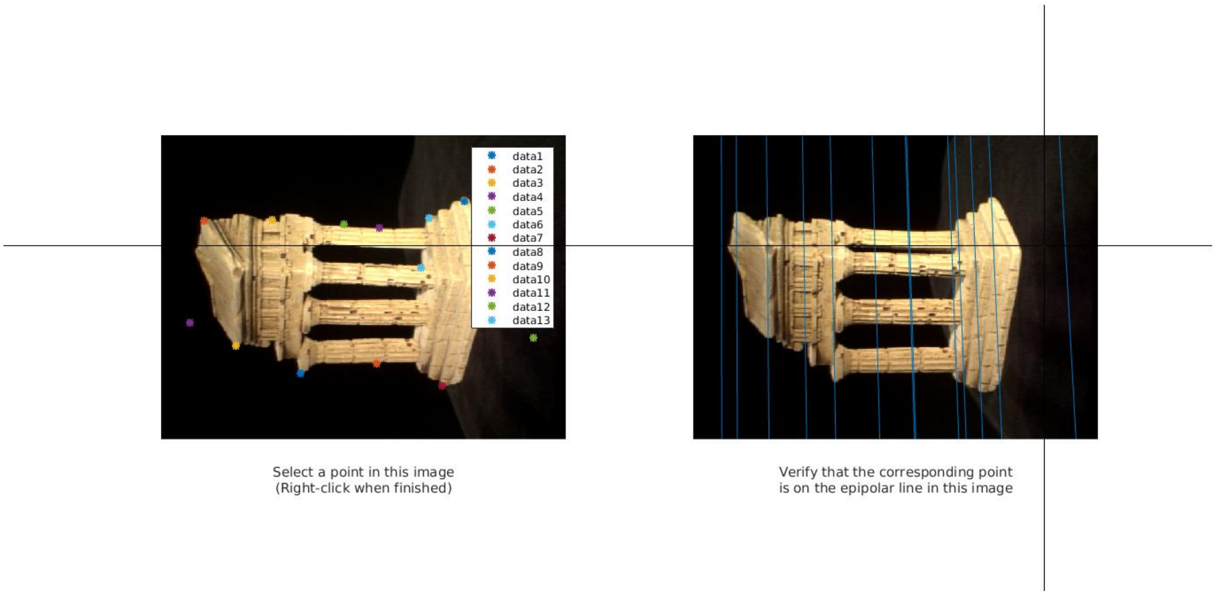


Figure 2: Eight Point Algorithm Output

- **Q2.2 The Seven Point Algorithm**

Fundamental Matrix from Seven Point Algorithm:

$$F = \begin{bmatrix} -0.0000 & 0.0000 & -0.0019 \\ 0.0000 & -0.0000 & -0.0000 \\ 0.0018 & -0.0001 & 0.0280 \end{bmatrix}$$

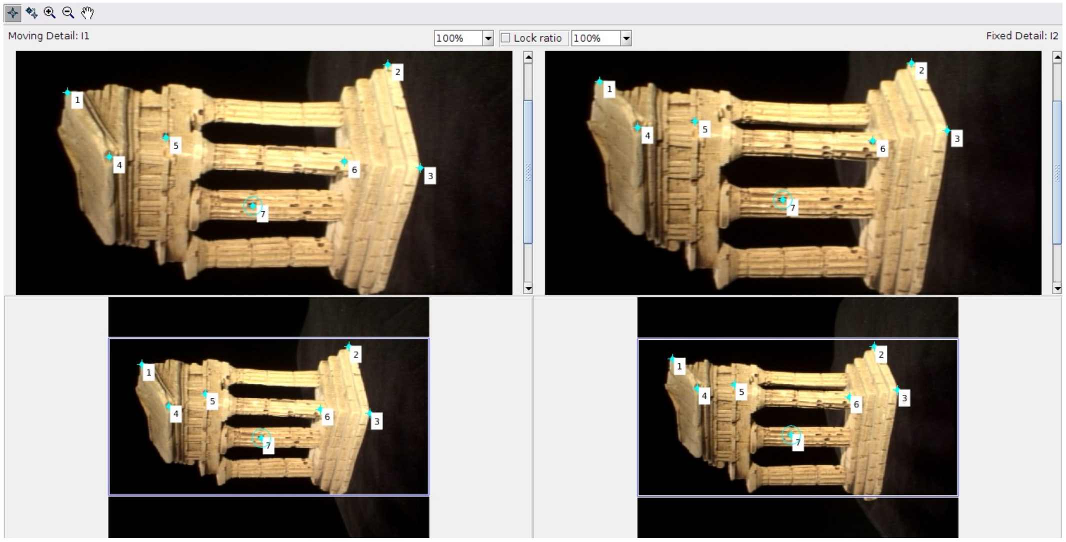


Figure 3: Seven points used for the Algorithm

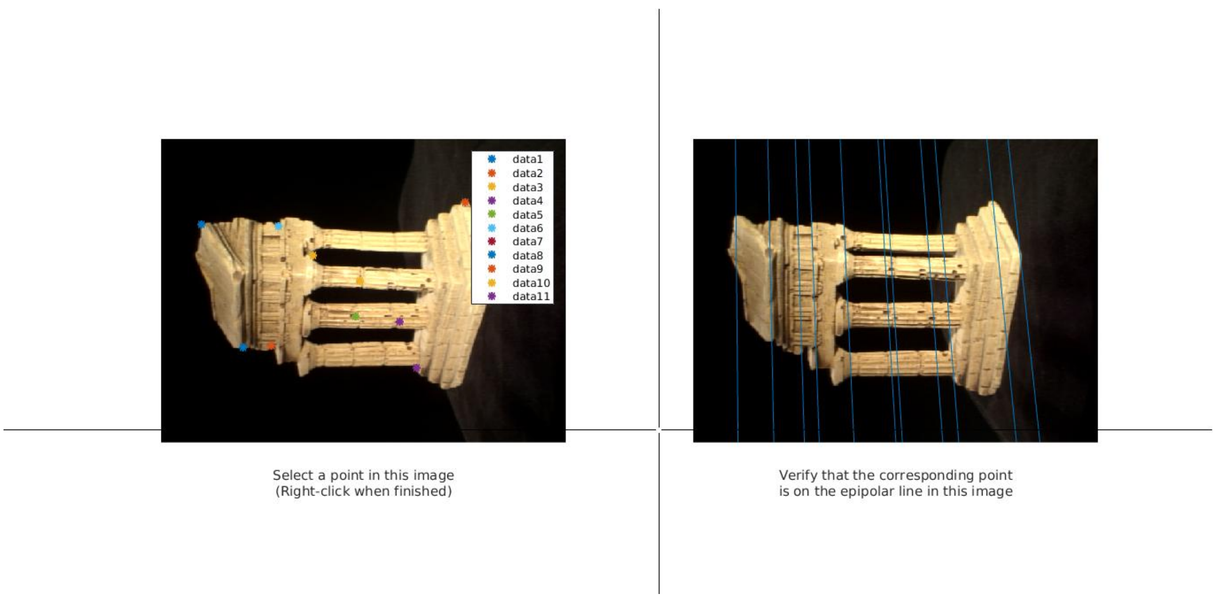


Figure 4: Seven Point Algorithm Output

- **Q3.1 Essential Matrix**

Essential Matrix using F from Eight Point Algorithm:

$$E = \begin{bmatrix} -0.0043 & -0.1804 & 1.6318 \\ -0.2463 & 0.0030 & -0.0433 \\ -1.6367 & -0.0119 & -0.0006 \end{bmatrix}$$

- **Q3.2 Triangulation**

Expression of A$_i$ in the equation $A_i P_i = 0$

$$A_i = \begin{bmatrix} x1 * C_1(3,1) - C_1(1,1) & x1 * C_1(3,2) - C_1(1,2) & x1 * C_1(3,3) - C_1(1,3) \\ y1 * C_1(3,1) - C_1(2,1) & y1 * C_1(3,2) - C_1(2,2) & y1 * C_1(3,3) - C_1(2,3) \\ x2 * C_2(3,1) - C_2(1,1) & x2 * C_2(3,2) - C_2(1,2) & x2 * C_2(3,3) - C_2(1,3) \\ y2 * C_2(3,1) - C_2(2,1) & y2 * C_2(3,2) - C_2(2,2) & y2 * C_2(3,3) - C_2(2,3) \end{bmatrix}$$

- **Q4.1 Epipolar Correspondence**

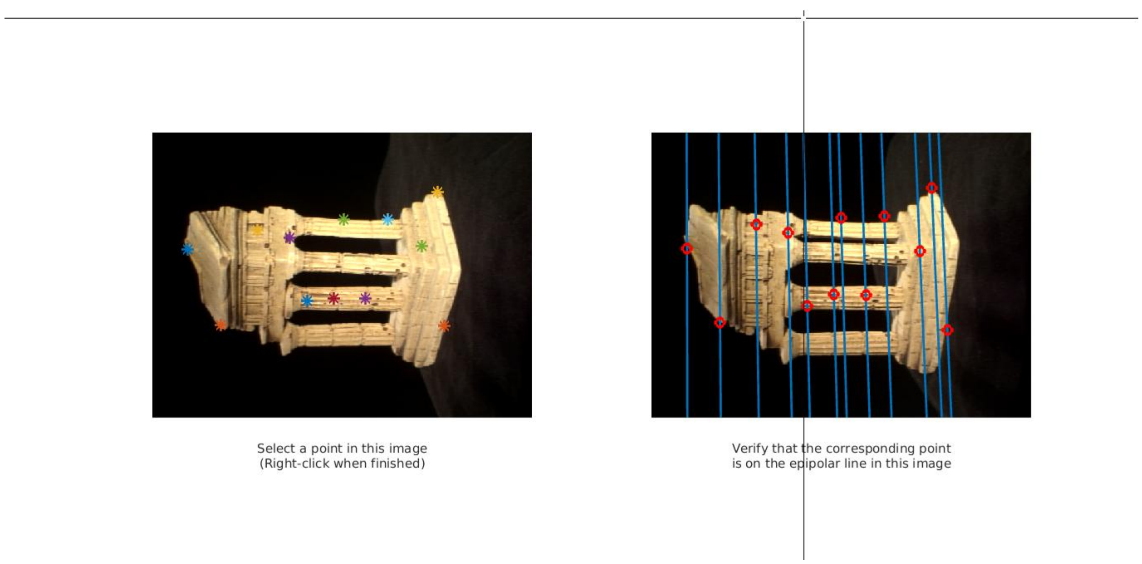
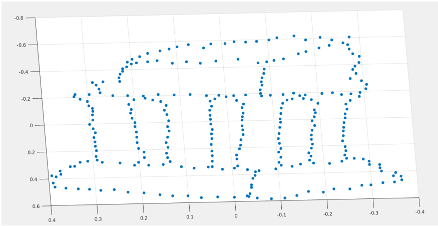
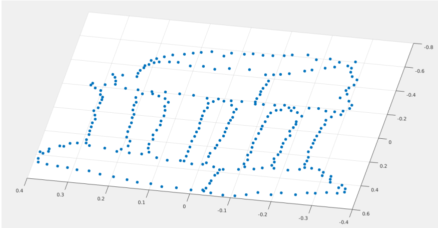


Figure 5: Points selected in image 1 and the corresponding points found by Epipolar Correspondence

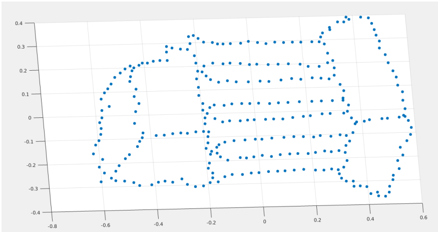
Q4.2 3D Visualization



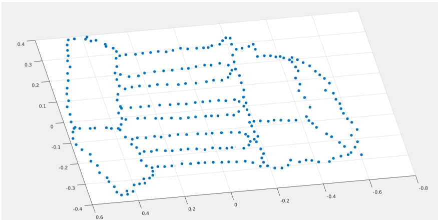
(a)



(b)



(c)



(d)

Figure 6: Point cloud of the 3D reconstruction