16720 HW 2 Write-up

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- **Q1.5** Include the image with the detected keypoints in your report.

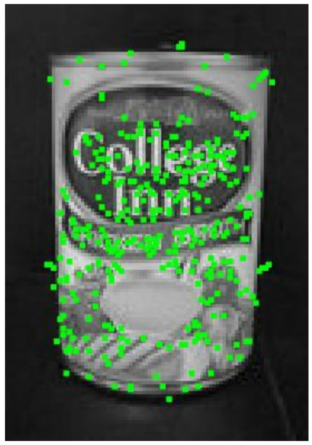

Figure 1: Detected keypoints in the image model chickenbroth.jpg

- **Q2.4** Write a test script testMatch to load two of the chickenbroth images and compute feature matches, present results with the two incline images and with the computer vision textbook cover page. Briefly discuss any cases that perform worse or better

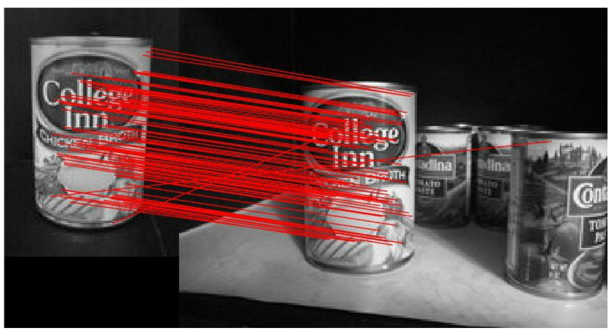

Figure 2: Matches for chickenbroth images

Figure 3: Matches for incline images

Figure 4: Matches for computer vision book images

From observation, the computer vision book has minimum matches and performs the worst. Reason being that one of the image is the rotated version of the other image.
Chickenbroth images perform the best as they have optimum number of matches to visualize. Incline images are too similar and have many interest points. As such, they have too many matches to visualize anything clearly.

- **Q2.5** Take the model chickenbroth.jpg test image and match it to itself while rotating the second image (hint: imrotate) in increments of 10 degrees. Count the number of correct matches at each rotation and construct a bar graph showing rotation angle vs the number of correct matches.

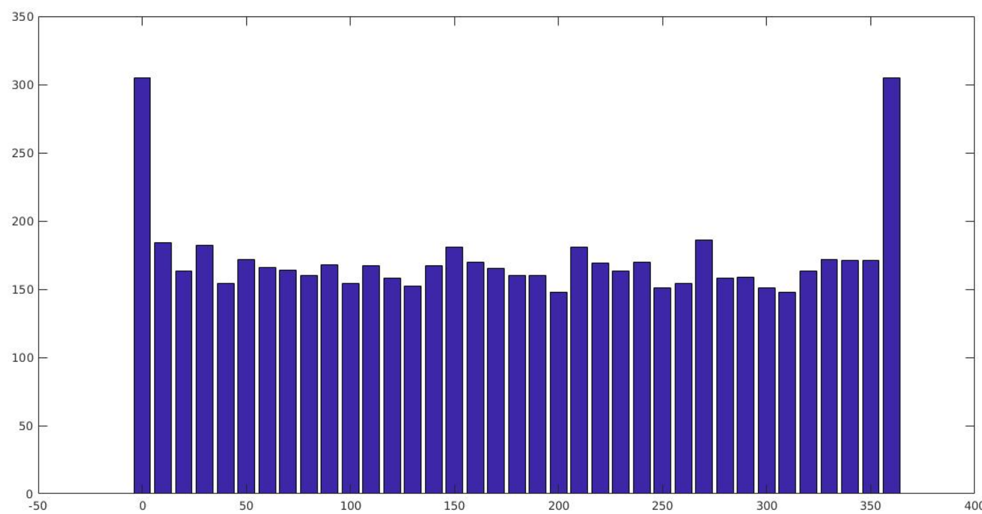

Figure 5: Number of matches in chickenbroth images vs degree of rotation

From observation, the number of matches fall sharply as soon as the image is rotated even with a small angle. The reason for this is that BRIEF descriptor doesn't match well for rotated images. It accounts for variations like change in intensity because of relative comparison. However, that relative comparison fails to generate similar descriptor if the images are rotated.

- **Q3 Planar Homographies: Theory**
  - a Given the N correspondences across the two views and using Equation 8, derive a set of 2N independent linear equations in the form: Ah = 0

2D Homogeneous coordinates: { $x'_1, x'_2, ..., x'_N \} and \{u'_1, u'_2, ..., u'_N\}$
$\Lambda x'_i = Hu'_n, i = 1 : N$
$To Derive : Ah = 0$

Let H = $\begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix}$

Considering a sample coordinate: $\lambda x'_i = Hu'_i$

$$\lambda \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} u_i \\ v_i \\ 1 \end{bmatrix}$$

Breaking down into equations
$\lambda x_i = h_{11}u_i + h_{12}v_i + h13$
$\lambda y_i = h_{21}u_i + h_{22}v_i + h23$
$\lambda = h_{31}u_i + h_{32}v_i + h33$

Rearranging
$x_i h_{31}u_i + h_{32}v_i + h33 = h_{11}u_i + h_{12}v_i + h13$
$y_i h_{31}u_i + h_{32}v_i + h33 = h_{21}u_i + h_{22}v_i + h23$

Rearranging:

$$\begin{bmatrix} 0 & 0 & 0 & -u_i & -v_i & -1 & y_iu_i & y_iv_i & y_i \\ u_i & v_i & 1 & 0 & 0 & 0 & -x_iu_i & -x_iv_i & -x_i \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \\ h_{33} \end{bmatrix} = 0$$

i=1:N

$$\begin{bmatrix} 0 & 0 & 0 & -u_1 & -v_1 & -1 & y_1u_1 & y_1v_1 & y_1 \\ u_1 & v_1 & 1 & 0 & 0 & 0 & -x_1u_1 & -x_1v_1 & -x_1 \\ 0 & 0 & 0 & -u_2 & -v_2 & -1 & y_2u_2 & y_2v_2 & y_2 \\ u_2 & v_2 & 1 & 0 & 0 & 0 & -x_2u_2 & -x_2v_2 & -x_2 \\ \cdots & & & & & & & & \\ \cdots & & & & & & & & \\ 0 & 0 & 0 & -u_N & -v_N & -1 & y_Nu_N & y_Nv_N & y_N \\ u_N & v_N & 1 & 0 & 0 & 0 & -x_Nu_N & -x_Nv_N & -x_N \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \\ h_{33} \end{bmatrix} = 0$$

This will give 2N linear equations of the form:
Ah = 0

– b. How many elements are there in h?

There are 9 elements in h. However, degree of freedom of h is 8 because it is independent up to a scale factor. As such, we can force one of the elements to be 1 and hence there are only 8 elements left in h.

– c. How many point pairs (correspondences) are required to solve this system?

h has 8 degrees of freedom (1 dof out of 9 is removed because of scaling factor. As such, 4 point pair correspondences are enough to solve this problem, provided no 3 of them are collinear.

– d. Show how to estimate the elements in h to find a solution to minimize this homogeneous linear least squares system.

We are required to minimize the sum of squared error (S) of $\mathbf{Ah} = \mathbf{0}$. It can be written as:
S $= \|Ah - 0\|^2 \implies S = \|Ah\|^2 \implies S =$(Ah)^{T}Ah
$\implies S =$(h$^T A^T Ah$)

To minimize S, taking differentiation wrt h

S' $= 0 \implies$ 2A$^T Ah = 0 \implies$ A$^T Ah = 0$

From the equation, h is the eigen vector that will correspond to 0 eigen value of $A^T A$. In presence of noise, it will correspond to the smallest eigen value of $A^T A$ matrix. To find h, find the eigen vectors V of $A^T A$. Assuming first column corresponds to the smallest eigen value, Vector h is the first column of V.

- **Q6.1 Image Stitching**

Showing un-blended image to highlight the 2 images

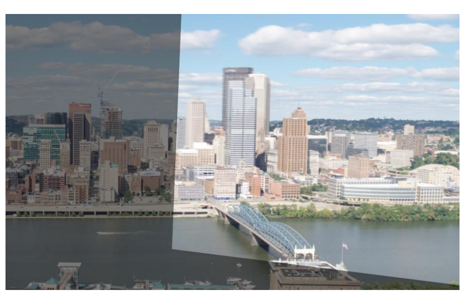

Figure 6: Image 2 warped on image 1 reference frame

- **Q6.2 Image Stitching with No Clipping**

  H is computed by RANSAC

Figure 7: Image 1 and Image 2 warped on a 3rd reference frame

- **Q6.3 Generate Panorama**

  H is computed by RANSAC

Figure 8: Image 1 and Image 2 warped on a 3rd reference frame