Problem 1

Find the mean of a Geometric random variable with parameter λ.

Problem 2

Suppose we wish to transmit at a rate of 64 kbps over a 3 kHz telephone channel. What is the minimum SNR_{dB} required to accomplish this?

Problem 3

Suppose we are trying to send a series of bits through a channel with a bit error rate ϵ (i.e., each bit has probability ϵ of being received incorrectly). One way to improve the reliability is to use repetition code (3,1). For example, if we want to send the bits '011', we actually send '000111111'. The receiver looks at the bits in groups of three, and decodes each group to the bit that occurs most often in the group. What is the error rate by using repetition code (3,1) (i.e., the probability that a group of three repeated bits will be decoded incorrectly given ϵ)? How much does this repetition code improve reliability?

Problem 4

Most digital transmission systems are "self-clocking" in that they derive the bit synchronization from the signal itself. To do this, the systems use the transitions between positive and negative voltage levels. These transitions help define the boundaries of the bit intervals.

- The nonreturn-to-zero (NRZ) signaling method transmits a 0 with a +1 voltage of duration T, and a 1 with a -1 voltage of duration T. Plot the signal for the sequence 4 consecutive 1s followed by 4 consecutive 0s. Explain why this code has a synchronization problem.
- In differential coding the sequence of 0s and 1s induces changes in the polarity of the signal; a binary 0 results in no change in polarity, and a binary 1 results in a changes in polarity. Repeat part (a). Does this scheme have a synchronization problem?
- The Manchester signaling method transmits a 0 as a +1 voltage for $T/2$ seconds followed by a -1 for $T/2$ seconds; a 1 is transmitted as a -1 voltage for $T/2$ seconds followed by a +1 for $T/2$ seconds. Repeat part (a) and explain how the synchronization problem has been addressed. What is the cost in bandwidth in going from NRZ to Manchester coding?

Problem 5

Consider the single-server queue with geometrically distributed inter-arrival times and service times, with parameters λ and μ respectively, as discussed in class. Now assume that the system can store a maximum of 3 packets (including the one in service).

- Draw the state transition diagram of the queueing system.
- Write down the balance equations of each state.
- Solve the steady-state probabilities.
- What is the probability that a new arrival will be rejected because the queue is full?

Problem 5 continued on next page...

- What is the average number of packets in the system?
- What is the average packet delay in the system?