Mean of geometric variable with parameter λ

$$P(\text{success}) = \lambda \cdot (1-\lambda)^{i-1}$$

$$P(x=i) = \lambda(1-\lambda)^{i-1}$$

$$E(x) = \sum_{i=1}^{\infty} i \cdot P(x=i)$$

$$= \sum_{i=1}^{\infty} i \cdot \lambda(1-\lambda)^{i-1}$$

$$= 1 \cdot \lambda(1-\lambda)^{1-1} + 2 \cdot \lambda(1-\lambda)^{2-1} +$$

$$E(x) = \lambda(1-\lambda)^0 + 2\lambda(1-\lambda)^1 + 3\lambda(1-\lambda)^2 + \quad \text{— } 1$$

multiply both sides by $(1-\lambda)$

$$(1-\lambda)(E(x)) = \lambda(1-\lambda) + 2\lambda(1-\lambda)^2 + 3\lambda(1-\lambda)^3 + \text{---} \quad \text{— } 2$$

$$1 - 2$$

$$E(x) - (1-\lambda)(E(x)) = \lambda - 0 + 2\lambda(1-\lambda) - 2\lambda(1-\lambda) + 3\lambda(1-\lambda)^2 - 2\lambda(1-\lambda)^2 + \text{--}$$

$$E(x)(1-1+\lambda) = \lambda + \lambda(1-\lambda) + \lambda(1-\lambda)^2 + \text{---}$$

$$E(x) = \lambda + \lambda(1-\lambda) + \lambda(1-\lambda)^2 + \text{-}$$

$$\div \lambda$$

$\lambda + 2\lambda(1-\lambda) + \lambda(1-\lambda)^2 + \text{---}$ is a gp

$$= \frac{1-\lambda}{1-(1-\lambda)}$$

$$E(x) = \frac{1-\lambda}{\lambda} \cdot \frac{1}{(1-\lambda)}$$

$$E(x) \quad \frac{1}{\lambda}$$

2) from $C = B \log_2 (1 + SNR)$

$$64 = 3 \log_2 (1 + SNR)$$

$$\frac{64}{3} = \log_2 (1 + SNR)$$

$$2^{\left(64/3\right)} = 1 + SNR$$

$$SNR = 2642244.95$$

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3) Error rate :

Probability for each bit to get an error = e

For each bit group $= \binom{3}{2} e^2 \cdot (1-e)^1$

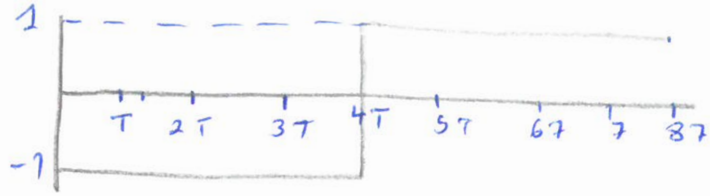
For whole bit stream $= 3 \binom{3}{2} e^2 \cdot (1-e)$

This repitition code improves reliability ~~be~~ since each bit has 3 chances to be correctly read from the repeated bit stream.

ie for a single bit 1, 111 is received. therefore if there is an error say 101, still it would be read as 1 since majority is 1 in repeated string.

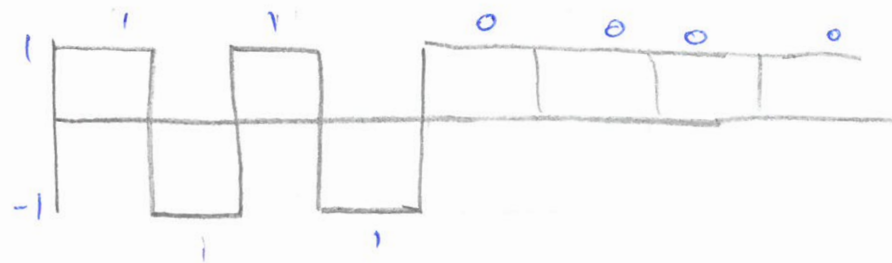
∴ about 33% improvement.

4a)



Synchronization problem is present because a long message of consecutive 0's or 1's will cause the average voltage to stray away from the mid level which can cause errors when decoding / interpreting the received signal.

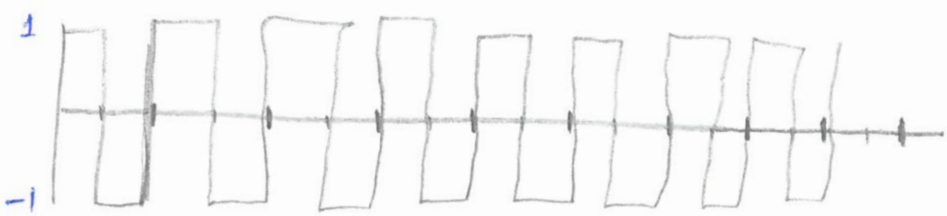
b)



Differential coding

This would also have a slight synchronization problem only if a long ~~str~~ message of consecutive 0's is received.

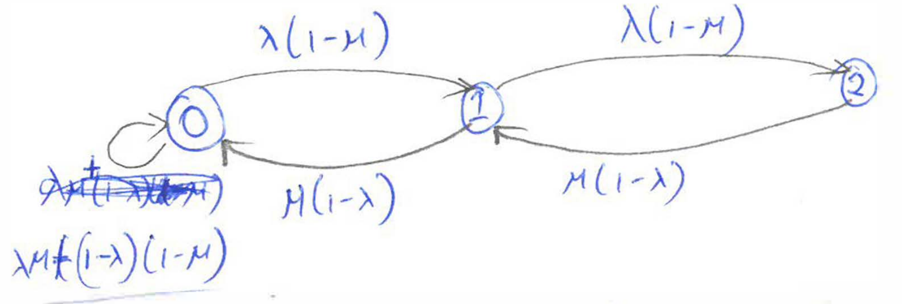
c)



Manchester coding

Synchronization has been improved because there is always a change in polarity whatever the bit being transferred is. Double bandwidth may be needed since the change is always at $T/2$ seconds.

5)



Steady state:

$$L = \sum_{i=0}^{\infty} i \cdot P(L=i)$$

let $P(L=i) = \pi_i$

$$L = \sum_{i=0}^{\infty} i \, \pi_i$$

Total time slots, the queue length increases

$$T \cdot \pi_i \, P(i, i+1)$$

Total time slots queue length decreases

$$T \cdot \pi_{i+1} \, P(i+1, i)$$

At steady state

$$\frac{T \cdot \pi_i \cdot P(i,i+1) - T \cdot \pi_{i+1} P(i+1,i)}{T} \le \frac{1}{T}$$

Since T is $\sim \infty$, $\frac{1}{T} \approx 0$.

$$\pi_i \cdot P(i,i+1) = T \pi_{i+1} P(i+1, i)$$

$$\pi_{i+1} = \frac{\pi_i \, P(i,i+1)}{P(i+1,i)}$$

$$\frac{P_{i,i+1}}{P_{i+1,i}} = \frac{\lambda(1-\mu)}{\mu(1-\lambda)} = \delta$$

$\therefore \pi_{i+1} = \delta \pi_i$

$$\pi_1 = \pi_0 \cdot \delta$$

$$\pi_2 = \pi_1 \delta$$

$$\pi_2 = \pi_0 \delta^2$$

$\therefore \boxed{\pi_i = \pi_0 \delta^i}$ $\quad \pi_i = \delta^i (1-\delta)$

Assumption: $\sum_{i=1}^{\infty} \pi_i = 1$ & $\delta < 1$

$$\sum_{i=0}^{\infty} \pi_0 \delta^i = 1$$

& $\pi_0 \sum_{i=0}^{\infty} \delta^i = 1$

from GP: $\sum_{i=0}^{\infty} \delta^i = \frac{1}{1-\delta}$

$\therefore \frac{\pi_0}{1-\delta} = 1$

$$\boxed{\pi_0 = 1-\delta}$$

$\therefore \pi_0 = 1 - \frac{\lambda(1-\mu)}{\mu(1-\lambda)}$

$$\pi_1 = \left(1 - \frac{\lambda(1-\mu)}{\mu(1-\lambda)}\right) \cdot \frac{\lambda(1-\mu)}{\mu(1-\lambda)}$$

$$\pi_1 = \frac{\lambda(1-\mu)}{\mu(1-\lambda)} - \frac{(\lambda(1-\mu))^2}{(\mu(1-\lambda))^2}$$

c) Probability that packet is rejected because queue is full.

$$= P(L = 3)$$

$$= \pi_3 = \pi_0 \cdot \left(\frac{\lambda(1-\mu)}{\mu(1-\lambda)}\right)^3$$

d) Average number of packets:

$$L = \sum_{i=0}^{\infty} i \cdot P(L=i)$$

$$L = \sum_{i=0}^{\infty} i \cdot \pi_i$$

But from above

$$\pi_i = \pi_0 \delta^i$$

$$\pi_i = \delta^i (1-\delta)$$

$$\cancel{L = \sum_{i=0}^{\infty} i \cdot \pi_0 \cdot \delta^i}$$

$$\cancel{L = \pi_0 \sum_{i=0}^{\infty} \delta^i}$$

$$L = \sum_{i=0}^{\infty} i \cdot \delta^i (1-\delta)$$

$$L = \frac{\delta}{1-\delta}$$

$$= \cancel{\frac{\lambda(1-\mu)}{1-\lambda(1-}} \quad \frac{\lambda(1-\mu)}{\mu(1-\lambda)} \div \left(1 - \frac{\lambda(1-\mu)}{\mu(1-\lambda)}\right)$$

average delay.

from little's law:

$$L = \lambda W$$

$$W = L \div \lambda$$

$$= \cancel{\frac{\lambda(1-\mu)}{\mu(1-\lambda)}} \quad \frac{\delta}{1-\delta} \div \lambda$$

$$= \frac{\delta}{\lambda(1-\delta)}$$