quiz 1.

1. $E[X] = \sum_i i \cdot P(X=i)$

$= 1 \cdot P(X=1) + 2 \cdot P(X=2) + 4 \cdot P(X=4) + 5 \cdot P(X=5)$

$= 1 \times 0.2 + 2 \times 0.1 + 4 \times 0.5 + 5 \times 0.2$

$= 3.4$

2. P(0 is received)

= P(1 is sent and received incorrectly)

+ P(0 is sent and received correctly)

$= \frac{1}{3} \times (1 - 0.8) + \frac{2}{3} \times 0.9$

$= \frac{2}{3}$.

3. define random variable X the number of corrupted bit in the received packet. X is Binomial (n, p)

a. $P(X=1) = \binom{n}{1} p^1 (1-p)^{n-1} = np(1-p)^{n-1}$

b $P(X=0) = \binom{n}{0} p^0 (1-p)^{n-0} = (1-p)^n$

c. $P(X>0) = 1 - P(X=0)$

$= 1 - (1-p)^n$

4. Define random variable X the number of successful servers in a cluster.

Two-server cluster : $X \sim$ Binomial $(N=2, p)$

$P($ two-server cluster is successful $)$

$= P(X=1) + P(X=2)$

$= \binom{2}{1} p^1 (1-p)^1 + \binom{2}{2} p^2 (1-p)^0$

$= 2p(1-p) + p^2$

$= 2p - p^2$

Four-server cluster : $X \sim$ Binomial $(N=4, p)$

$P($ four-server cluster is successful $)$

$= P(X=2) + P(X=3) + P(X=4)$

$= \binom{4}{2} p^2 (1-p)^2 + \binom{4}{3} p^3 (1-p) + \binom{4}{4} p^4 (1-p)^0$

$= 6p^2(1-p)^2 + 4p^3(1-p) + p^4$

The four-server cluster is preferable if :

$$6p^2(1-p)^2 + 4p^3(1-p) + p^4 \geq 2p - p^2$$

$$6p(1-p)^2 + 4p^2(1-p) + p^3 \geq 2 - p$$

$$3p^3 - 8p^2 + 7p - 2 \geq 0$$

$$(p-1)^2(3p-2) \geq 0$$

$\Leftrightarrow$ $3p-2 \geq 0$ or $p \geq \frac{2}{3}$.