Homework 3

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1. Confidence Intervals

In a group of 20 people, cholesterol was measured first in 1952 and again 10 years later in 1962 (in mg/dl) after enrolling in a long-term health program. (Data loosely based on Dixon WJ and Massey F Jr., Introduction to Statistical Analysis, Fourth Edition, McGraw Hill Book Company, 1983)

```
# code provided
fifties <- c(207, 233, 260, 214, 265,
             272, 284, 255, 290, 298,
             321, 300, 272, 305, 322,
             317, 348, 354, 386, 520)
sixties <- c(209, 239, 258, 202, 261,
             270, 274, 254, 293, 299,
             319, 299, 281, 288, 325,
             303, 339, 338, 385, 525)

chol <- tibble(fifties, sixties, diff = sixties-fifties)
glimpse(chol)
```

```
## Rows: 20
## Columns: 3
## $ fifties <dbl> 207, 233, 260, 214, 265, 272, 284, 255, 290, 298, 321, 300, 27~
## $ sixties <dbl> 209, 239, 258, 202, 261, 270, 274, 254, 293, 299, 319, 299, 28~
## $ diff    <dbl> 2, 6, -2, -12, -4, -2, -10, -1, 3, 1, -2, -1, 9, -17, 3, -14, ~
```

1A. Calculate a 80% confidence interval for the difference in cholesterol over time.

```
chol.t <- broom::tidy(t.test(chol$diff, conf.level = 0.80))

chol.t$conf.low
```

```
## [1] -5.327761
```

```
chol.t$conf.high
```

```
## [1] -0.8722386
```

1B. Provide an interpretation of what the confidence interval you found in 1A means in terms of cholesterol values

> We can be 80% confident that the limits of the CI (-5.3277614, -0.8722386) contain the true mean difference in cholesterol.

1C. Calculate a 95% CI for the difference in cholesterol

```r
chol.t95 <- broom::tidy(t.test(chol$diff, conf.level = 0.95))

chol.t95$conf.low
```

```
## [1] -6.611832
```

```r
chol.t95$conf.high
```

```
## [1] 0.4118319
```

1D. What happened as we went from a 80% CI to a 95% CI. Why did this occur?

> The confidence interval widened from (-5.3277614, -0.8722386) to (-6.6118319, 0.4118319). This happened because increasing the confidence (from 80% to 95%) absent any other changes in the input data (i.e., increased n) inherently requires the range of the interval to increase too.

1E. Does either interval (in 1A or 1C) include 0? Why is the inclusion of a zero important?

> The 80% CI does not include 0, but the 95% CI does. This is important because inclusion of a zero in the confidence interval implies that there is a chance that the mean difference in cholesterol levels is 0.

2. Consider the results of question 1C. Notice that around a quarter of the difference data fit within the 95% confidence interval.

2A. Is this usually the case? What happens to the confidence interval as sample size increases?

There is an inverse square root relationship between confidence intervals and sample sizes.

2B. To help you answer 2A, use the code provided below to increase the sample size to 2000. Re-calculate the 95% confidence interval.

```
# code provided
set.seed(8380)
chol2000 <- chol |>
  select(diff) |>
  rep_sample_n(size = 2000, replace = TRUE, reps = 1)
```

```
chol2000.t <- broom::tidy(t.test(chol2000$diff, conf.level = 0.95))
chol2000.t$conf.low
```

```
## [1] -3.279896
```

```
chol2000.t$conf.high
```

```
## [1] -2.638104
```

2C.To help you answer 2A, use dplyr functions to figure out how many datapoints from chol2000 are in the new 95% CI.

```
# nothing shows up but this code should work?
chol2000 |> filter(between(diff, chol2000.t$conf.low, chol2000.t$conf.high)) |> nrow()
```

```
## [1] 0
```

```
# replaced referenced variables with specific numeric values
# to test if formatting issues were causing the failure
chol2000 |> filter(between(diff, -3.279896, -2.638104)) |> nrow()
```

```
## [1] 0
```

```
# it turns out there are actually no datapoints in the provided dataset
# which equal the only integer found in the 95% CI range (-3)
# this is why the filtering code doesn't return any results
chol2000 |> filter(diff == -3) |> nrow()
```

```
## [1] 0
```

```
# code would work if the 95% CI range was different
# demonstrated by using the 95% CI from the original n=20 dataset
chol2000 |> filter(between(diff, -6.611832, 0.4118319)) |> nrow()
```

```
## [1] 702
```

```
# code works using referenced variables rather than actual numeric values
chol2000 |> filter(between(diff, chol.t95$conf.low, chol.t95$conf.high)) |> nrow()
```

```
## [1] 702
```

For questions 3 and 4, you do not need to run any R code. Please place your answers in the prose portion of the markdown file.

3. Let's examine the relationship between CIs and hypothesis tests.

3A.

You calculate a 95% confidence interval for $\mu_1 - \mu_2$ and come up with (-3, 1). If you test $H_0 : \mu_1 - \mu_2 = 0$ and use alpha = .05, will you reject H_0? Why or why not?

> In this case, you will not reject H_0, because 0 is within the 95% confidence interval.

3B.

Now you calculate a 99% CI for $\mu_1 - \mu_2$ and come up with (-5, -2). If you test $H_0 : \mu_1 - \mu_2 = 0$ and use alpha = .05, will you reject H_0? Why or why not?

> In this case, you will reject H_0, because 0 is outside the 95% confidence interval.

3C.

Finally, you calculate a 95% CI for $\mu_1 - \mu_2$ and come up with (-24, -14). If you test $H_0 : \mu_1 - \mu_2 = -12$ and use alpha = .01, will you reject H_0? Why or why not?

> It is not possible to answer this because we have not determined a 99% CI. Since -12 lies outside the 95% CI, we can't say if it will be within a 99% CI and therefore can't say if $H_0 : \mu_1 - \mu_2 = -12$ will be rejected with alpha = 0.01 or not.

4.

Suppose you test a new medication (H_0: new drug does not work, H_1: new drug works)

4A.

Let's say you reject the null hypothesis (you conclude the drug works). What kind of error could you have made?

> If you reject the null hypothesis, you could make a Type I error (a false positive) in a situation where the drug actually does not work.

4B.

What if you had concluded that the drug does not work (you fail to reject the null hypothesis). What kind of error could you have made?

> If you fail to reject the null hypothesis, you could make a Type II error (a false negative) in a situation where the drug actually does work.