Conditions for Oscillations (Barkhausen Criterion)

The noise components are always present in any conductor, tube or transistor. Even when no external signal is applied, the ever-present noise will cause some small signal at the output of the amplifier. When the amplifier is tuned at a particular frequency f_o, the output signal caused by noise signal will be predominant at f_o. If a small fraction (β) of the output signal is fed back to the input with proper phase relation, then this feedback signal will be amplified by the amplifier. If the amplifier has a gain of more than $1/\beta$, then the output increases and thereby the feedback signal becomes larger. This process continues and the output goes on increasing. But as the signal level increases, the gain of the amplifier decreases and at a particular value of output, the gain of the amplifier is reduced exactly equal to $1/\beta$. Then the output voltage remains constant at frequency f_o, called frequency of oscillation. The essential condition for maintaining oscillations are:

1. $|A\beta| = 1$, i.e the magnitude of loop gain must be unity.
2. The total phase shift around the closed loop is zero or 360°.

The block diagram of an oscillator is shown in Figure 1.

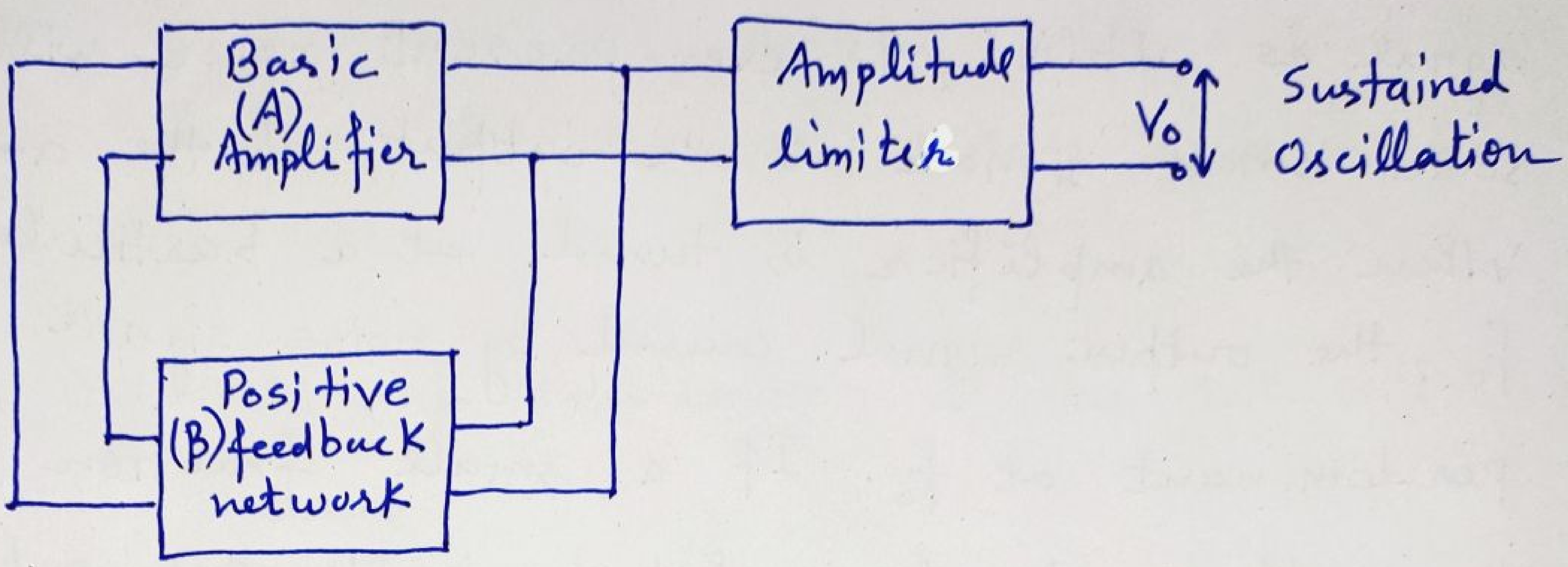

Figure 1. Block diagram of an oscillator

General Form of an LC Oscillator

In the general form of the oscillator shown in Figure 2, any of the active device such as vacuum-tube, transistor, FET and operational amplifier may be used in the amplifier section.

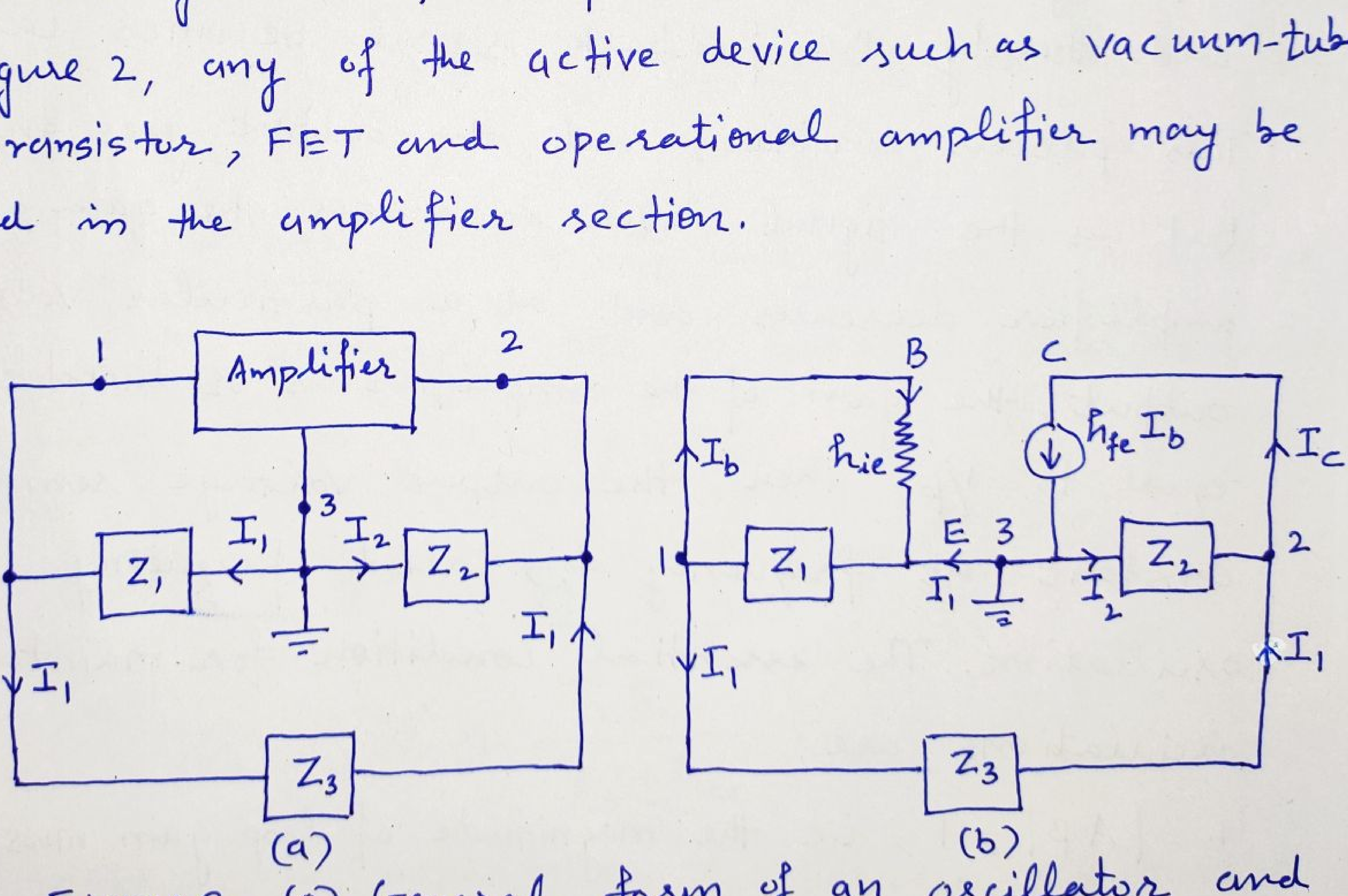

Figure 2. (a) General form of an oscillator and (b) its equivalent circuit.

Z_1, Z_2 and Z_3 are reactive elements constituting the feedback tank circuit which determines the frequency of oscillation. Here, Z_1 and Z_2 serve as an ac voltage divider for the output voltage and feedback signal. Therefore, the voltage across Z_1 is the feedback signal.

The frequency of oscillation of the LC oscillator is

$$f_o = \frac{1}{2\pi\sqrt{LC}}$$

The inductive or capacitive reactances are represented by Z_1, Z_2 and Z_3. In Figure 2 (a), output terminals are 2 and 3, and input terminals are 1 and 3. Figure 2 (b) gives the equivalent circuit of Figure 2 (a).

Load Impedance: Since Z_1 and the input resistance h_{ie} of the transistor are in parallel, their equivalent impedance Z' is given by

$$\frac{1}{Z'} = \frac{1}{Z_1} + \frac{1}{h_{ie}}$$

$$\text{or} \quad Z' = \frac{Z_1 h_{ie}}{Z_1 + h_{ie}}$$

Now, the load impedance Z_L between the output terminals 2 and 3 is the equivalent impedance of Z_2 in parallel with the series combination of Z' and Z_3.

Therefore,
$$\frac{1}{Z_L} = \frac{1}{Z_2} + \frac{1}{Z' + Z_3}$$

$$= \frac{1}{Z_2} + \frac{1}{\frac{Z_1 h_{ie}}{Z_1 + h_{ie}} + Z_3}$$

$$= \frac{1}{Z_2} + \frac{Z_1 + h_{ie}}{Z_1 h_{ie} + Z_1 Z_3 + h_{ie} Z_3}$$

$$= \frac{1}{Z_2} + \frac{Z_1 + h_{ie}}{h_{ie}(Z_1 + Z_3) + Z_1 Z_3}$$

$$= \frac{h_{ie}(Z_1 + Z_3) + Z_1 Z_3 + Z_2(Z_1 + h_{ie})}{Z_2[h_{ie}(Z_1 + Z_3) + Z_1 Z_3]}$$

$$= \frac{h_{ie}(Z_1 + Z_2 + Z_3) + Z_1 Z_2 + Z_1 Z_3}{Z_2[h_{ie}(Z_1 + Z_3) + Z_1 Z_3]}$$

There
$$Z_L = \frac{Z_2[h_{ie}(Z_1 + Z_3) + Z_1 Z_3]}{h_{ie}(Z_1 + Z_2 + Z_3) + Z_1 Z_2 + Z_1 Z_3}$$

Voltage Gain without feedback

This is given by,

$$A_{ve} = -\frac{h_{fe} Z_L}{h_{ie}}$$

Feed back Fraction β : The out put voltage between the terminals 3 and 2 in terms of the current I_1 is given by,

$$V_o = -I_1(Z' + Z_3) = -I_1\left(\frac{Z_1 h_{ie}}{Z_1 + h_{ie}} + Z_3\right)$$

$$V_o = -I_1\left(\frac{h_{ie}(Z_1+Z_3)+Z_1Z_3}{Z_1+h_{ie}}\right)$$

The voltage fed back to the input terminals 3 and 1 is given by

$$V_{fb} = -I_1Z' = -I_1\left(\frac{Z_1 h_{ie}}{Z_1+h_{ie}}\right)$$

Therefore, the feedback ratio β is given by

$$\beta = \frac{V_{fb}}{V_o} = I_1\left(\frac{Z_1 h_{ie}}{Z_1+h_{ie}}\right)\left[\frac{Z_1+h_{ie}}{h_{ie}(Z_1+Z_3)+Z_1Z_3}\right]\frac{1}{I_1}$$

$$\therefore \quad \beta = \frac{Z_1 h_{ie}}{h_{ie}(Z_1+Z_3)+Z_1Z_3}$$

Equation for the Oscillator : For oscillation, we must have,

$$A_{ve}\beta = 1$$

Substituting the values of A_{ve} and β, we get

$$\left(\frac{-h_{fe}Z_L}{h_{ie}}\right)\left[\frac{Z_1 h_{ie}}{h_{ie}(Z_1+Z_3)+Z_1Z_3}\right] = 1$$

$$\left[\frac{h_{fe}Z_2[h_{ie}(Z_1+Z_3)+Z_1Z_3]}{h_{ie}(Z_1+Z_2+Z_3)+Z_1Z_2+Z_1Z_3}\right]\left[\frac{Z_1}{h_{ie}(Z_1+Z_3)+Z_1Z_3}\right] = -1$$

$$\frac{h_{fe}Z_2Z_1}{h_{ie}(Z_1+Z_2+Z_3)+Z_1Z_2+Z_1Z_3} = -1$$

$$h_{ie}(Z_1+Z_2+Z_3)+Z_1Z_2+Z_1Z_3 = -h_{fe}Z_1Z_2$$

or, $h_{ie}(Z_1+Z_2+Z_3)+Z_1Z_2(1+h_{fe})+Z_1Z_3 = 0$

This is the general equation for the oscillator.

Hartley Oscillator

In the Hartley oscillator shown in Figure 3, Z_1 and Z_2 are inductors and Z_3 is a capacitor. Resistors R_1, R_2 and R_E provide the necessary dc bias to the transistor. C_E is a bypass capacitor. C_{C1} and C_{C2} are coupling capacitors. The feedback network consisting of inductors L_1 and L_2, and capacitor C determines the frequency of the oscillator.

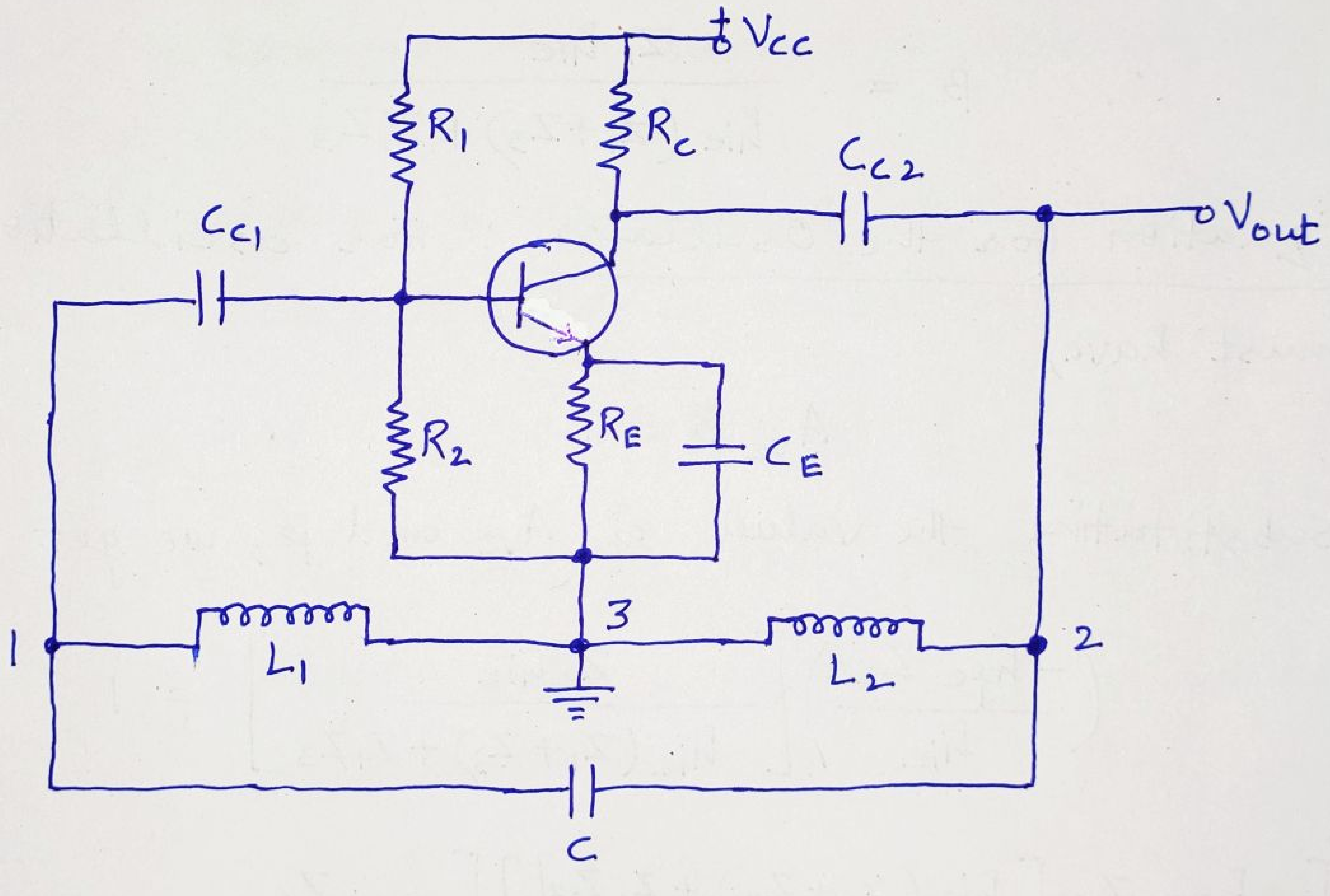

Figure 3. Hartley Oscillator

When the supply voltage +Vcc is switched ON, a transient current is produced in the tank circuit and consequently, damped harmonic oscillations are set up in the circuit. The oscillatory current in the tank circuit produces ac voltage across L_1 and L_2. As terminal 3 is earthed, it is at zero potential.

If terminal 1 is at a positive potential with respect to 3 at any instant, then terminal 2 will be at a negative potential with respect to 3 at the same instant. Thus the phase difference between the terminals 1 and 2 is always 180°.

In CE mode, the transistor provides the phase difference of 180° between the input and output. Therefore, the total phase shift is 360°. Thus, at the frequency determined for the tank circuit, the necessary condition for sustained oscillations is satisfied.

If the feedback is adjusted so that the loop gain $A\beta = 1$, the circuit acts as an oscillator.

The frequency of oscillation is $f_o = \frac{1}{2\pi\sqrt{LC}}$, where $L = L_1 + L_2 + 2M$, and M is the value of mutual inductance between coils L_1 and L_2. The condition for sustained oscillation is

$$h_{fe} \geq \frac{L_1 + M}{L_2 + M}$$

Analysis : In the Hartley oscillator, Z_1 and Z_2 are inductive rectances and Z_3 is the capacitive reactance. Suppose M is the mutual inductance between the inductors, then

$$Z_1 = j\omega L_1 + j\omega M$$

$$Z_2 = j\omega L_2 + j\omega M$$

$$Z_3 = \frac{1}{j\omega C} = \frac{-j}{\omega C}$$

Substituting the values of Z_1, Z_2 and Z_3 in general equation for the oscillator,

i.e $h_{ie}(Z_1+Z_2+Z_3) + Z_1Z_2(1+h_{fe}) + Z_1Z_3 = 0$

or $j\omega h_{ie}\left[L_1+L_2+2M-\frac{1}{\omega^2 C}\right] - \omega^2(L_1+M)\left[(L_2+M)(1+h_{fe}) - \frac{1}{\omega^2 C}\right] = 0$

The frequency of oscillation $f_o = \frac{\omega_o}{2\pi}$ can be determined by equating the imaginary part of above equation to zero.

Therefore $\left[L_1+L_2+2M-\frac{1}{\omega_o^2 C}\right] = 0$

Simplifying this equation, we obtain

$$f_o = \frac{\omega_o}{2\pi} = \frac{1}{2\pi\sqrt{(L_1+L_2+2M)C}}$$

$$\therefore \quad f_o = \frac{1}{2\pi\sqrt{(L_1+L_2+2M)C}} = \frac{\omega_o}{2\pi}$$

The condition for maintenance of oscillation is obtained by substituting above equation in general equation for oscillator,

$$\left[(L_2+M)(1+h_{fe}) - \frac{1}{\omega_o^2 C}\right] = 0$$

$$h_{fe} = \frac{L_1+M}{L_2+M}$$

In Hartley oscillator, if loading effect of the base is ignored, then the feedback fraction becomes $\beta = \frac{L_1}{L_2}$. For oscillations to occur, the voltage gain A_v must be

$$A_v > 1/\beta \quad \text{or} \quad A_v > L_2/L_1.$$

MCQ

Q1. For Practical oscillator, which law has to be obeyed?

(a) Faraday law (b) Hertz law

(c) Fleming law (d) Barkhausen law

Q2. Which of the following expressions depicts Barkhausen criteria?

(a) $|A\beta| = 1$ (b) $A\beta = 0$

(c) $A\beta < 1 < A\beta$ (d) $A\beta < 1$

Q3. Barkhausen criteria states phase of loop gain must be o for a self sustaining oscillator.

(a) True (b) False

Q4. What are Oscillators?

(a) Switching circuits (b) Converts dc to ac

(c) Converts ac to dc (d) Filter circuits.

Q5. An Oscillator requires an input voltage of high amplitude.

(a) True (b) False.