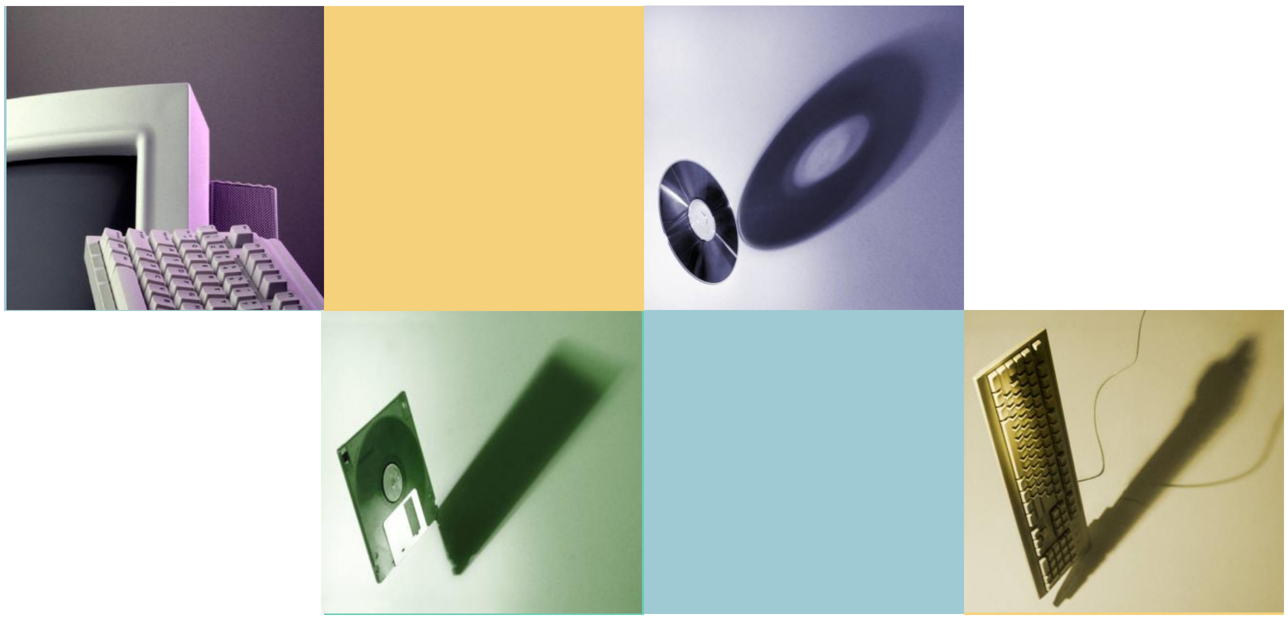
SIGNAL FLOW GRAPH



Dr.Anuj Jain

Outline

- Introduction to Signal Flow Graphs
  - Definitions
  - Terminologies
- Mason's Gain Formula
  - Examples
- Signal Flow Graph from Block Diagrams
- Design Examples

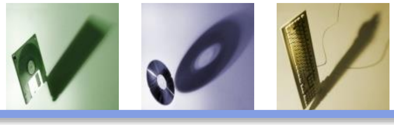
Signal Flow Graph (SFG)

- Alternative method to block diagram representation, developed by Samuel Jefferson Mason.

- Advantage: the availability of a flow graph gain formula, also called Mason's gain formula.

- A signal-flow graph consists of a network in which nodes are connected by directed branches.

- It depicts the flow of signals from one point of a system to another and gives the relationships among the signals.

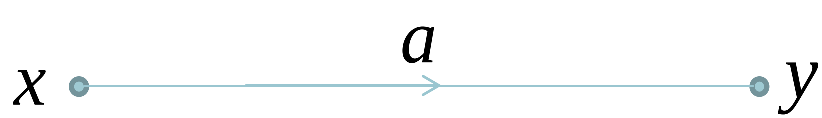


Fundamentals of Signal Flow Graphs

Consider a simple equation below and draw its signal flow graph:

$$y = ax$$

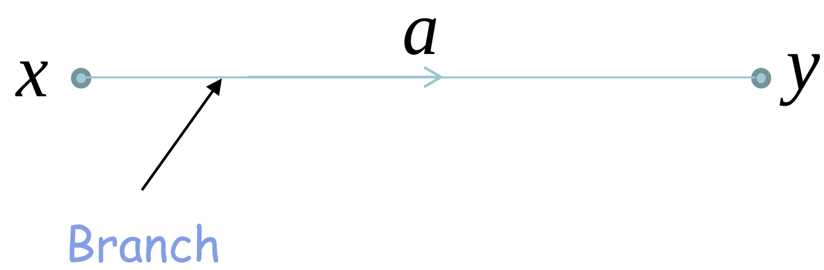
The signal flow graph of the equation is shown below;



Important terminology :

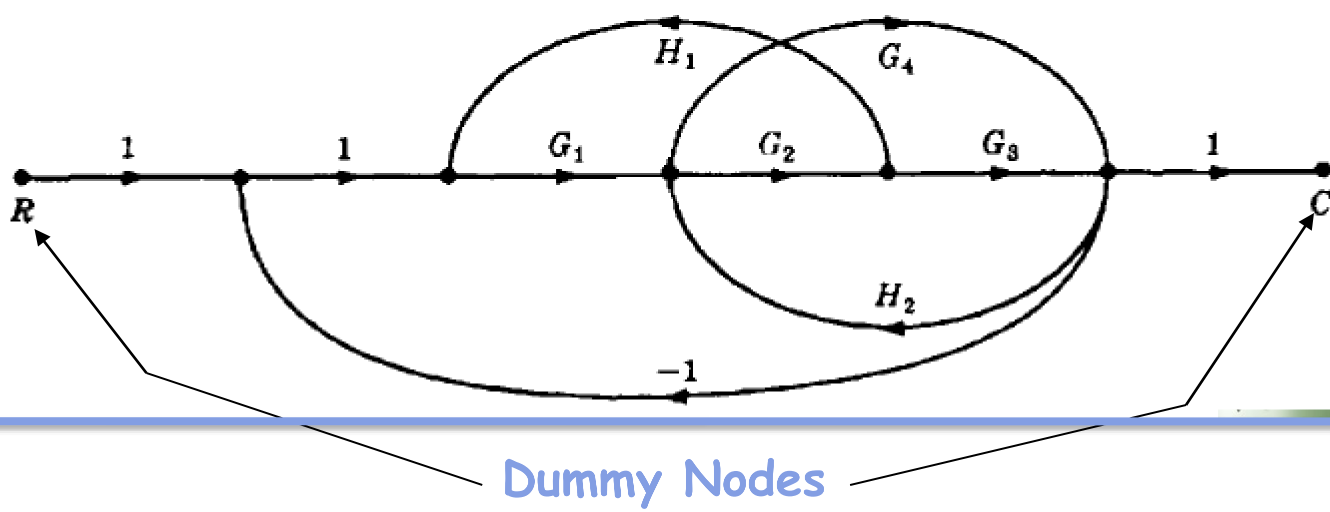
- **Branches :-**

  line joining two nodes is called branch.



- **Dummy Nodes:-**

  A branch having one can be added at i/p as well as o/p.



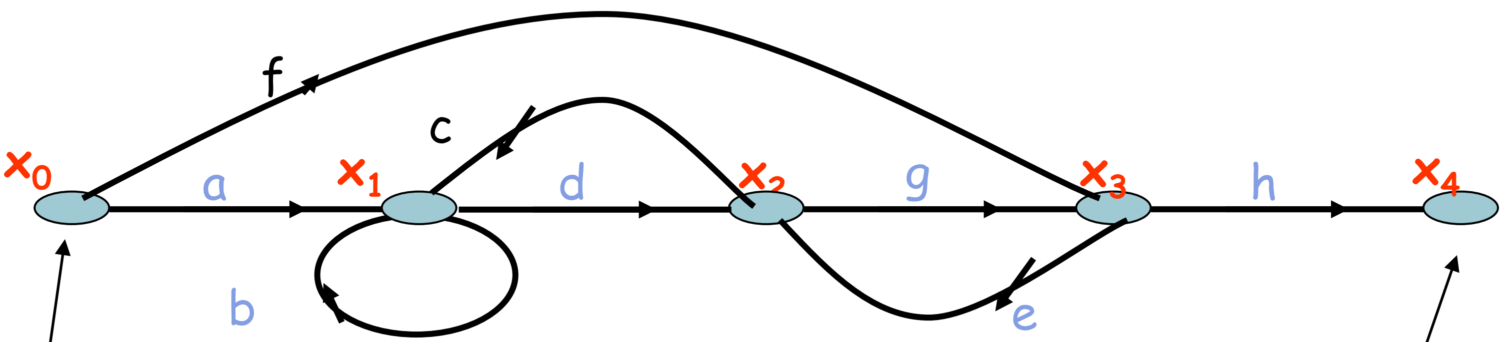
Input & output node

- **Input node:-**

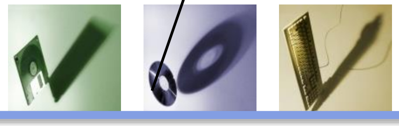
  It is node that has only outgoing branches.

- **Output node:-**

  It is a node that has incoming branches.



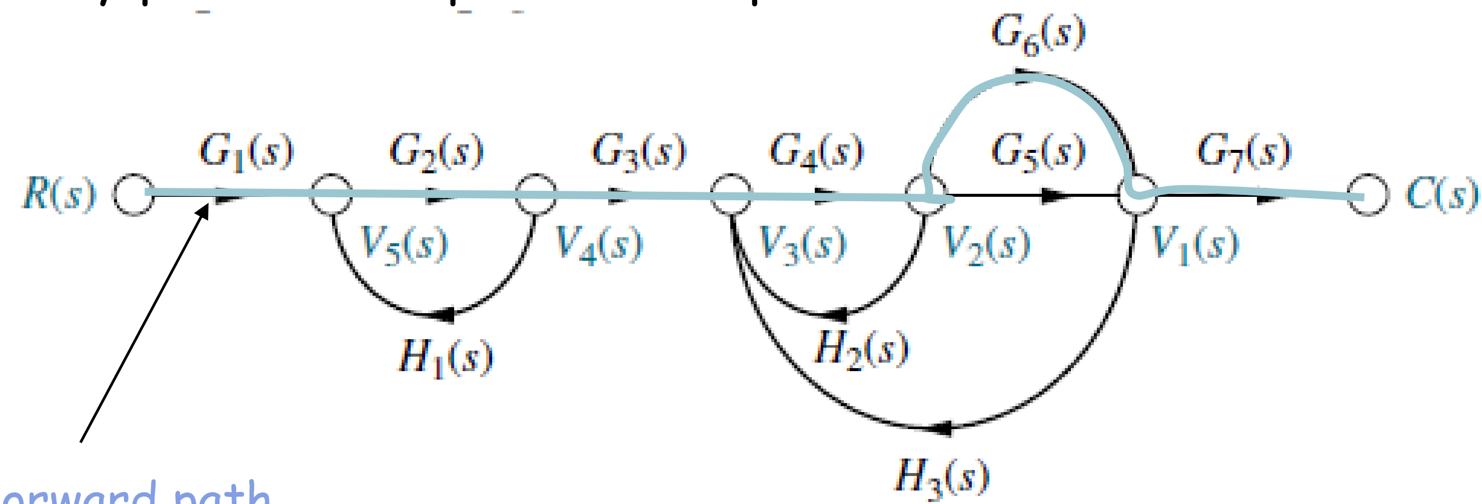
Input node



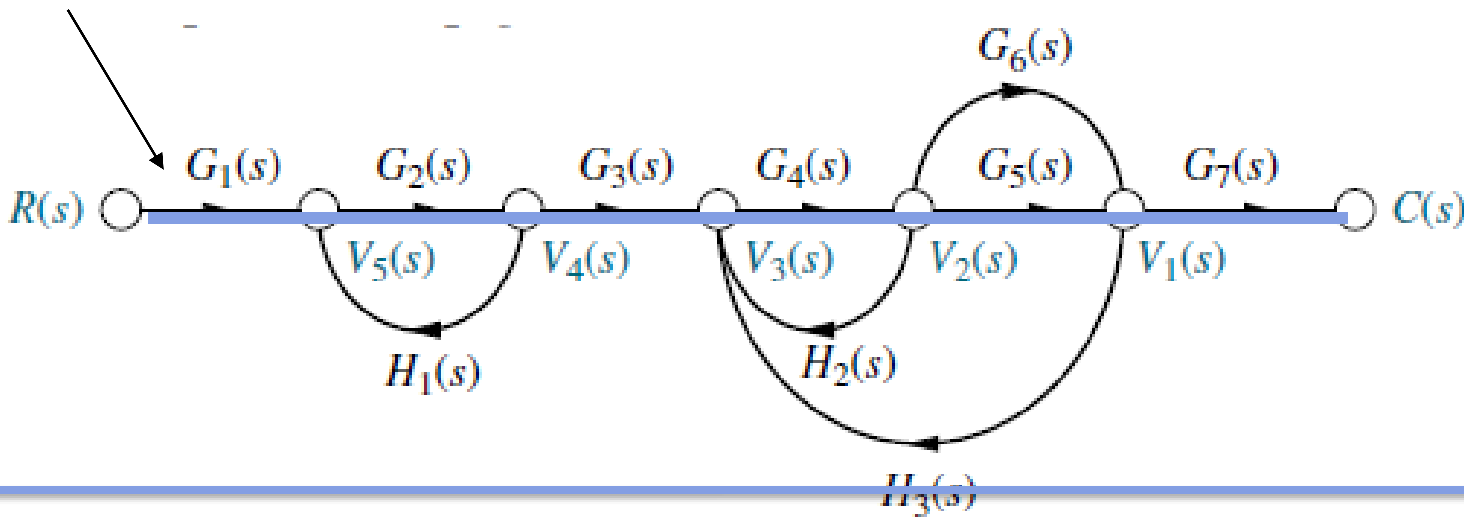
Out put node

Forward path:-

- Any path from i/p node to o/p node.

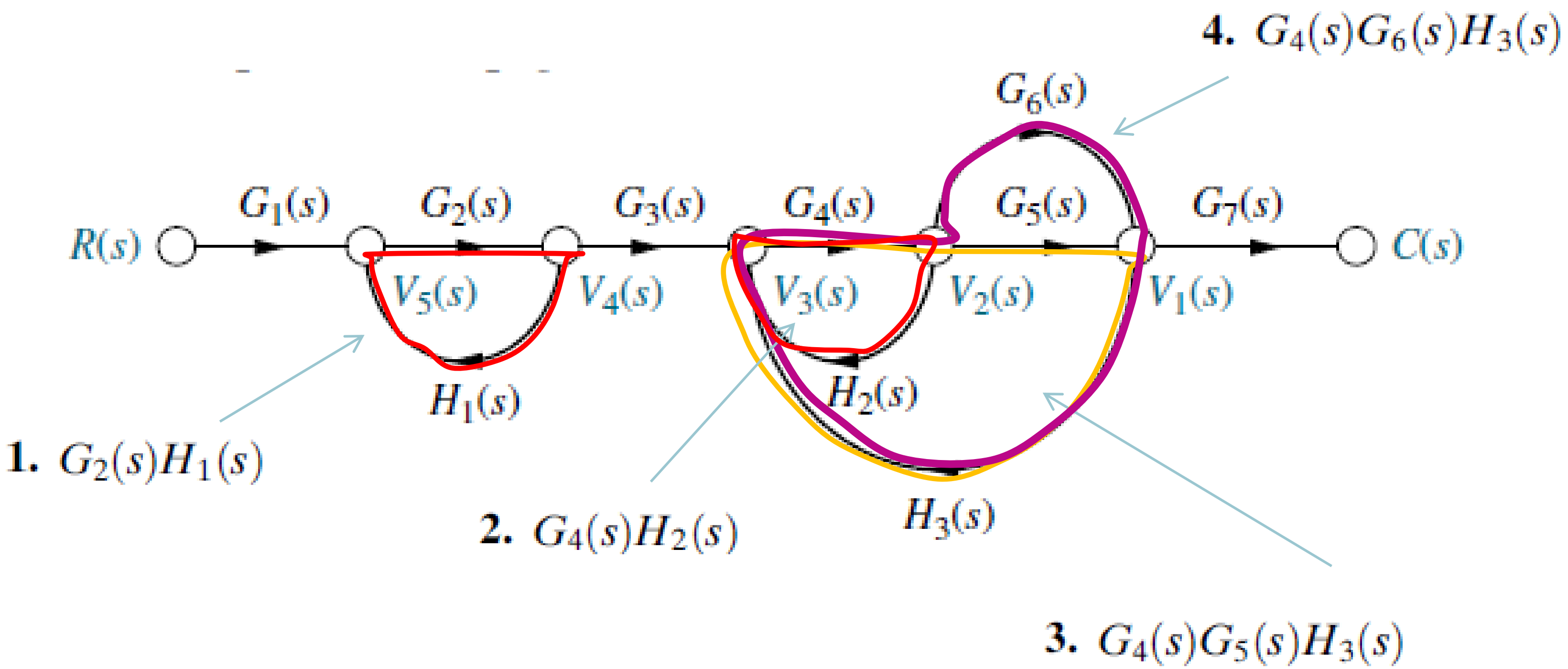


Forward path



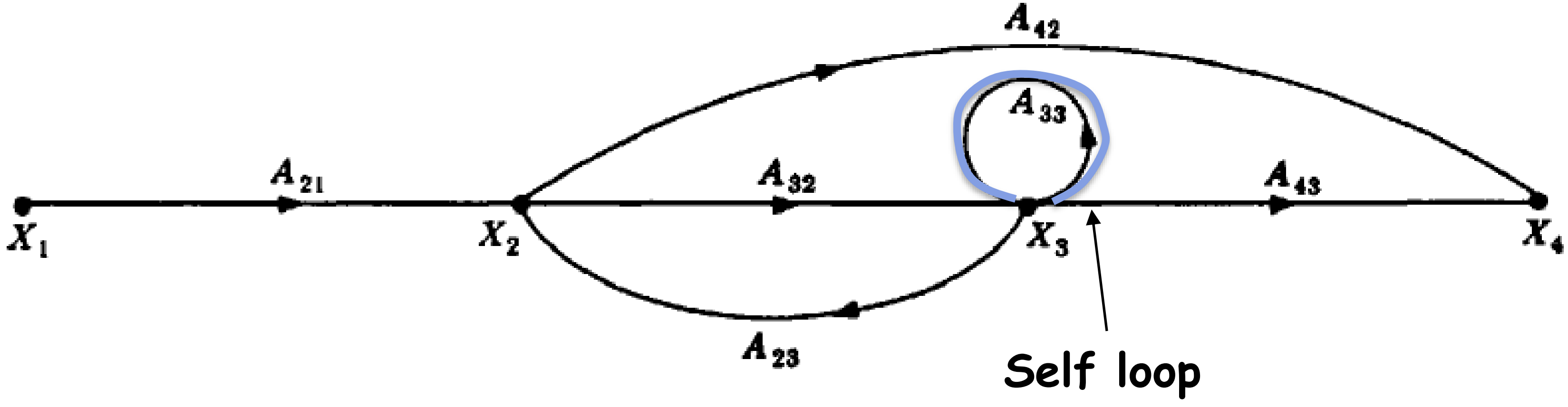
Loop :-

- A closed path from a node to the same node is called loop.



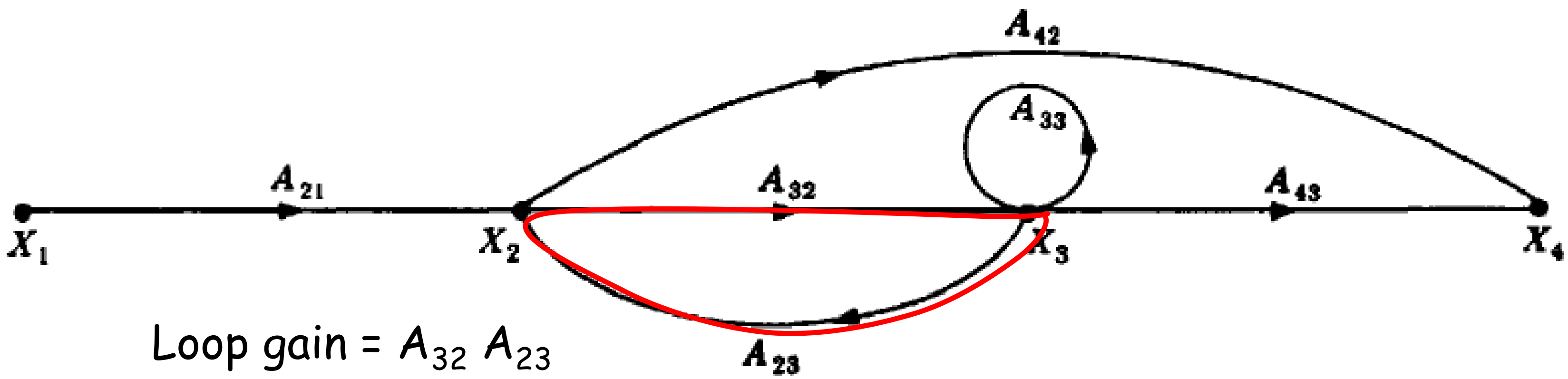
Self loop:-

• A feedback loop that contains of only one node is called self loop.



Loop gain:-

The product of all the gains forming a loop



Loop gain = $A_{32}\ A_{23}$

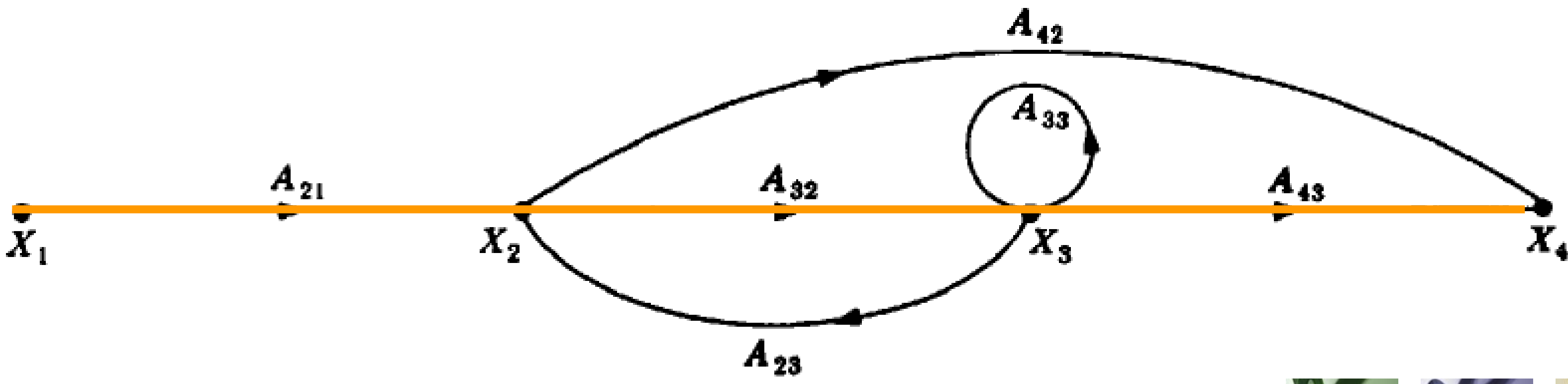
Path & path gain

Path:-

A path is a traversal of connected branches in the direction of branch arrow.

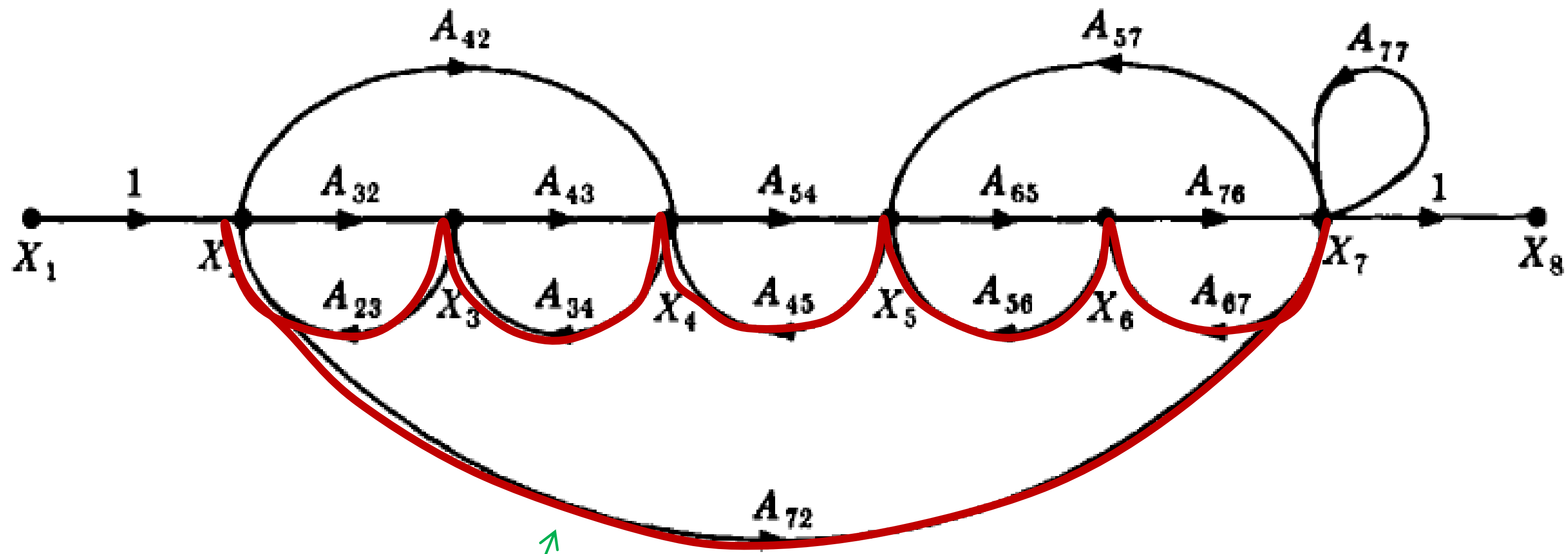
Path gain:-

The product of all branch gains while going through the forward path it is called as path gain.



Feedback path or loop :-

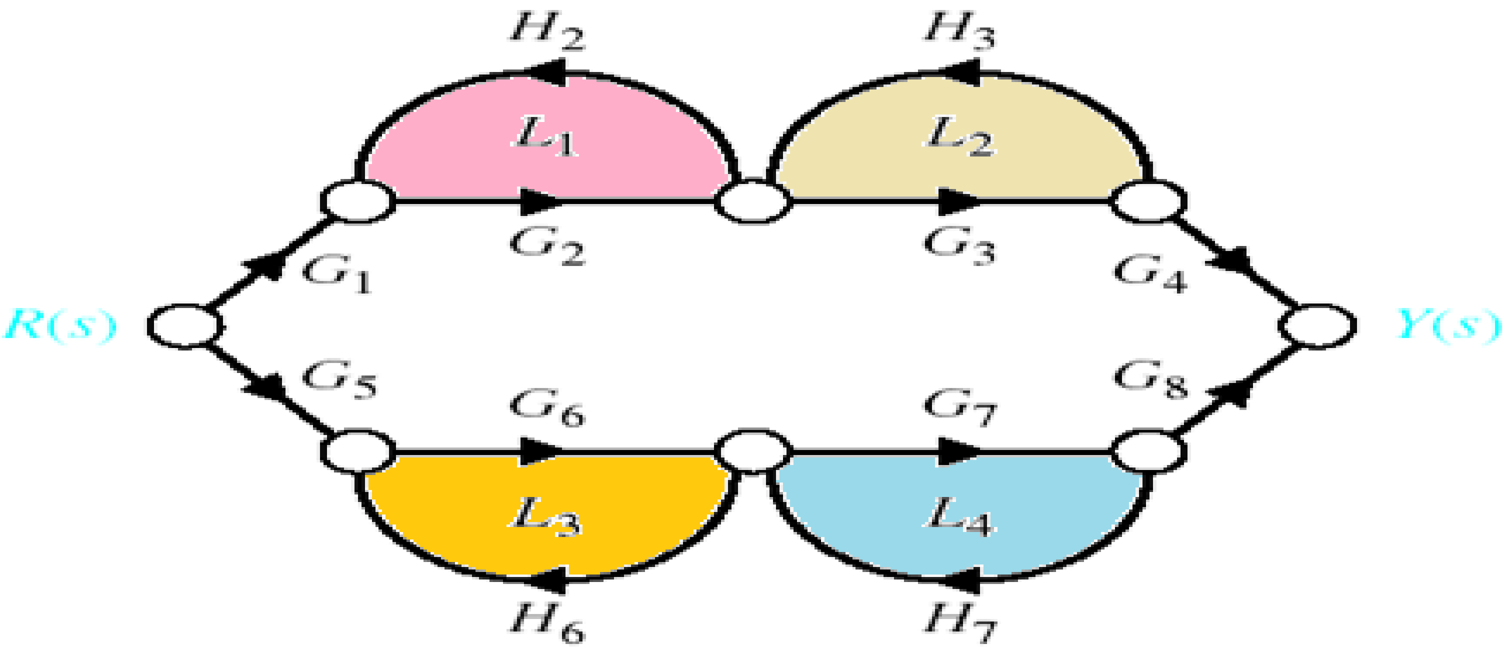
- it is a path to o/p node to i/p node.



X_2 to X_7 to X_6 to X_5 to X_4 to X_3 to X_2

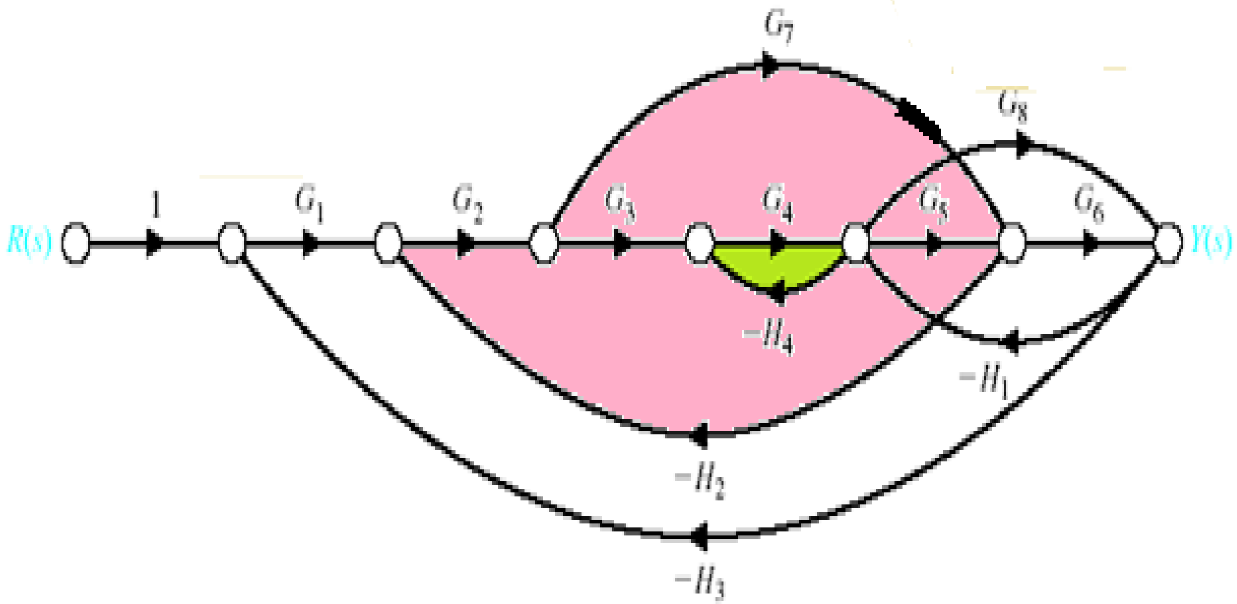
Touching loops:-

- when the loops are having the common node that the loops are called touching loops.

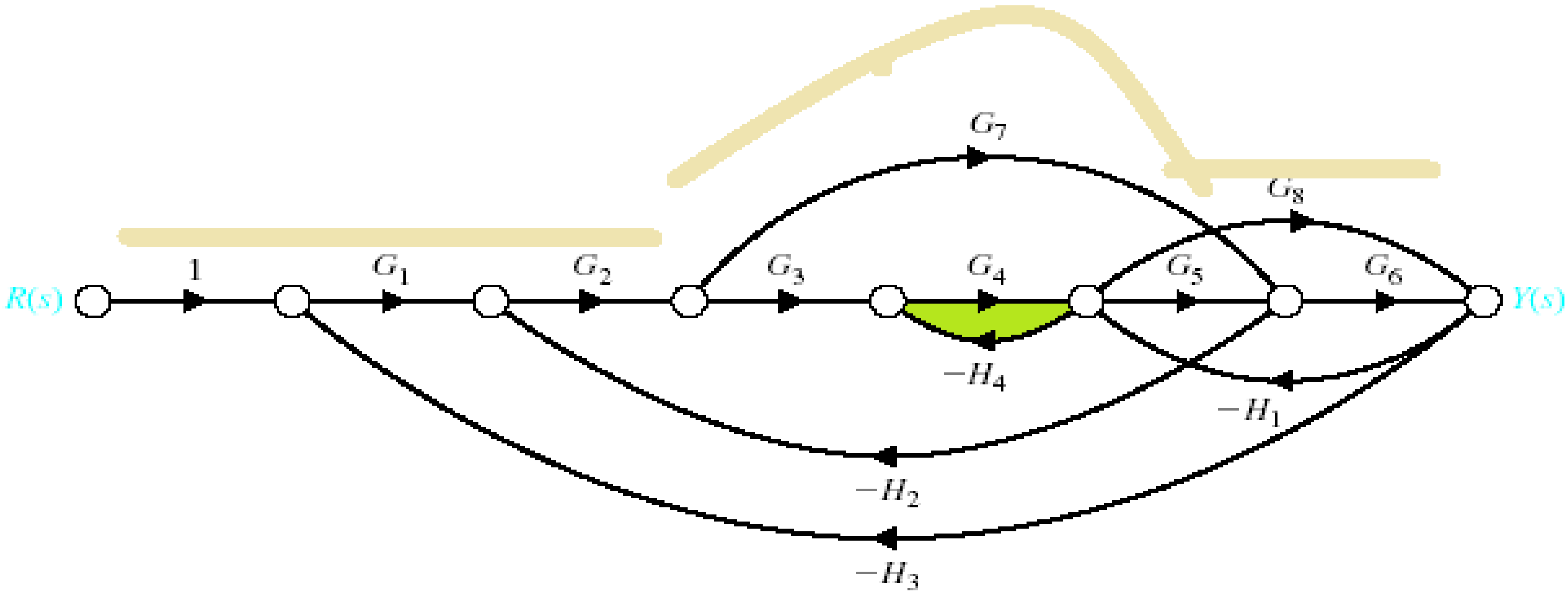


Non touching loops:-

- when the loops are not having any common node between them that are called as non- touching loops.

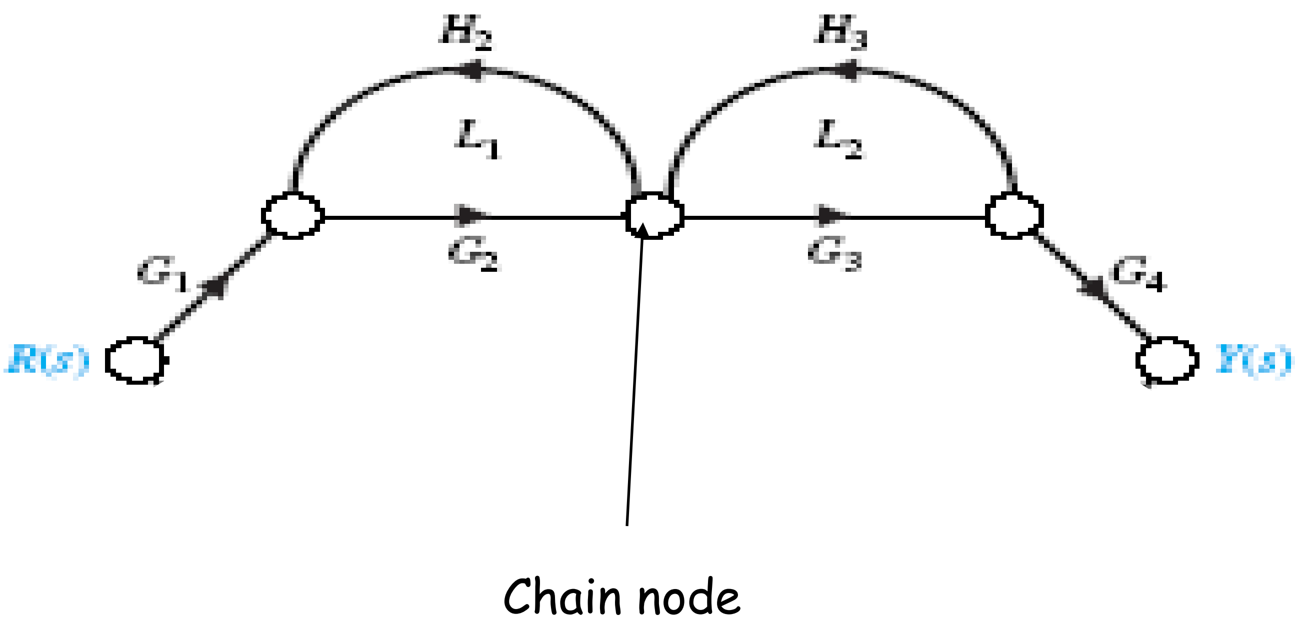


Non-touching loops for forward paths

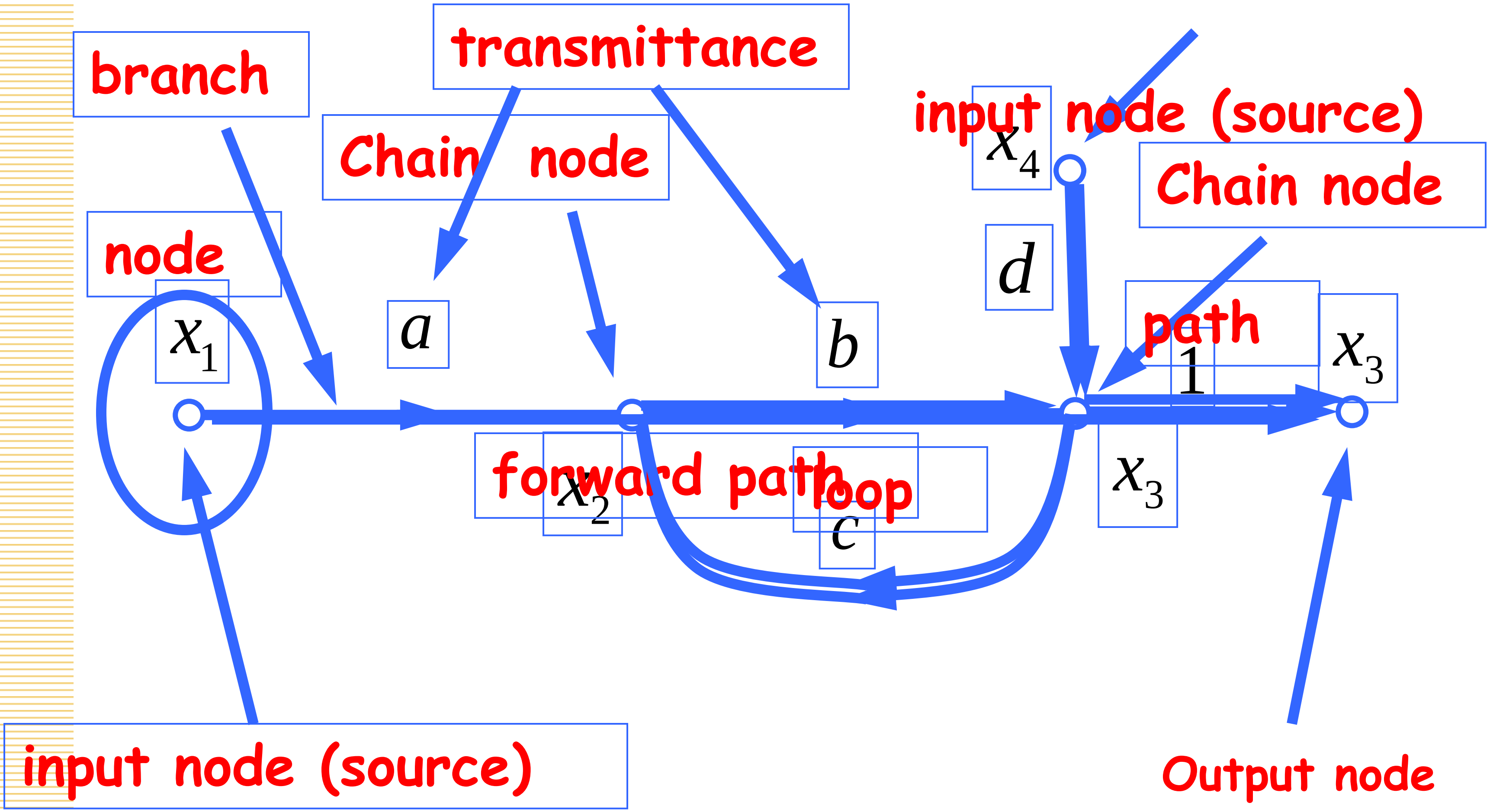


Chain Node :-

- it is a node that has incoming as well as outgoing branches.



SFG terms representation

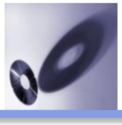


Mason's Rule (Mason, 1953)

- The block diagram reduction technique requires successive application of fundamental relationships in order to arrive at the system transfer function.

- On the other hand, Mason's rule for reducing a signal-flow graph to a single transfer function requires the application of one formula.

- The formula was derived by S. J. Mason when he related the signal-flow graph to the simultaneous equations that can be written from the graph.


Mason's Rule :-

- The transfer function, *C(s)/R(s)*, of a system represented by a signal-flow graph is;

$$\frac{C(s)}{R(s)} = \frac{\sum_{i=1}^{n} P_i \Delta_i}{\Delta}$$

- Where

- n = number of forward paths.
- Pi = the i th forward-path gain.
- Δ = Determinant of the system
- Δi = Determinant of the ith forward path

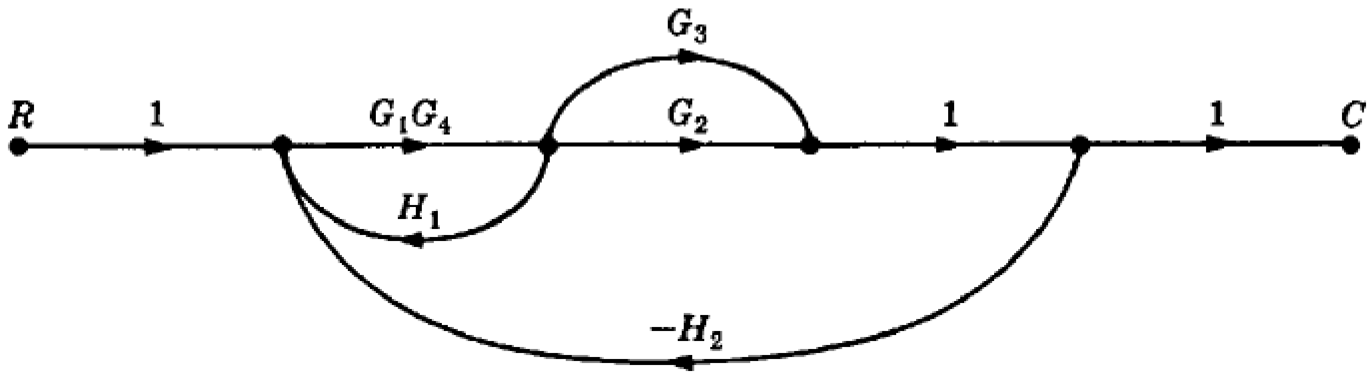
Δ is called the signal flow graph determinant or characteristic function. Since Δ=0 is the system characteristic equation.

$$\frac{C(s)}{R(s)} = \frac{\sum_{i=1}^{n} P_i \Delta_i}{\Delta}$$

Δ = 1- (sum of all individual loop gains) + (sum of the products of the gains of all possible two loops that do not touch each other) – (sum of the products of the gains of all possible three loops that do not touch each other) + … and so forth with sums of higher number of non-touching loop gains

Δ*i* = value of Δ for the part of the block diagram that does not touch the i-th forward path (Δi = 1 if there are no non-touching loops to the i-th path.)

Example1: Apply Mason's Rule to calculate the transfer function of the system represented by following Signal Flow Graph



There are two forward paths:

$P_1 = G_1G_2G_4 \qquad P_2 = G_1G_3G_4$

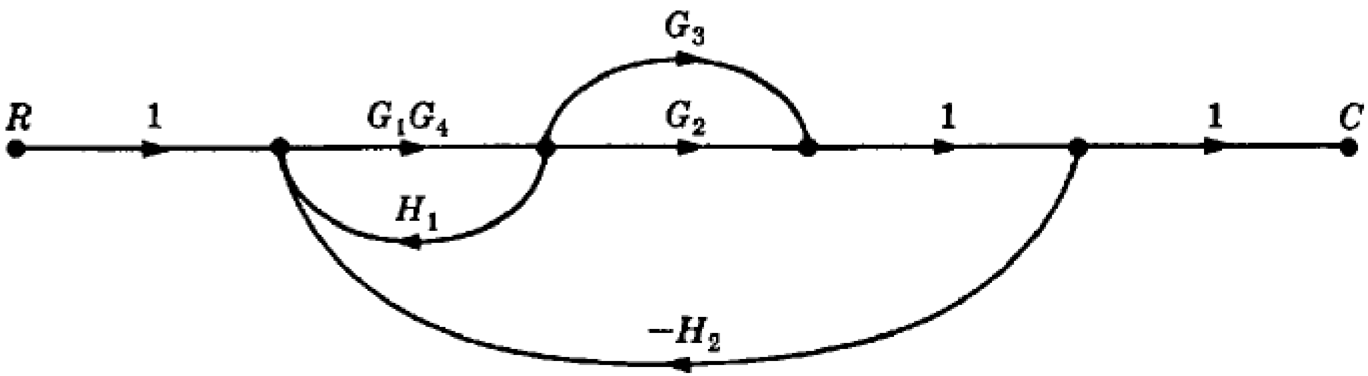
Therefore,

$$\frac{C}{R} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

There are three feedback loops

$$L_1 = G_1G_4H_1, \quad L_2 = -G_1G_2G_4H_2, \quad L_3 = -G_1G_3G_4H_2$$

Continue……



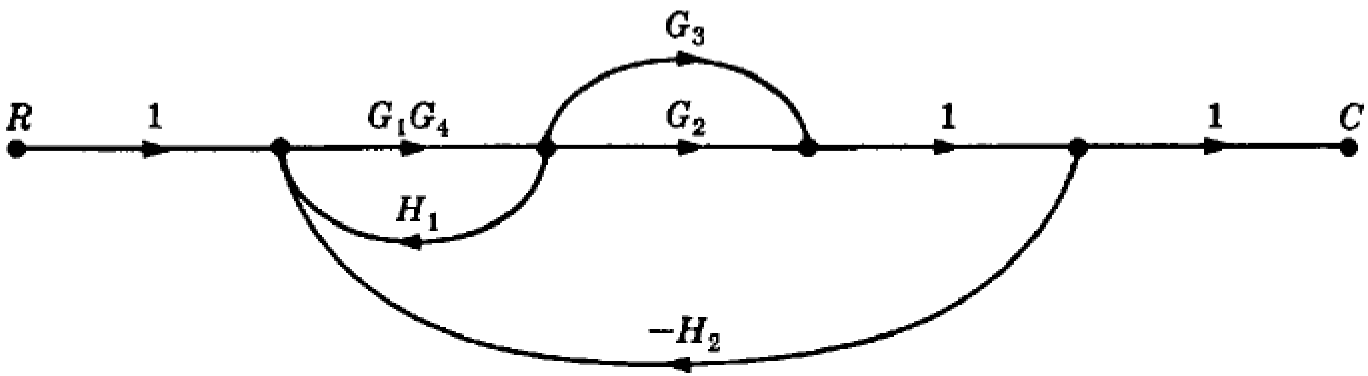
There are no non-touching loops, therefore

Δ = 1- (sum of all individual loop gains)

$$\Delta = 1 - \left(L_1 + L_2 + L_3\right)$$

$$\Delta = 1 - \left(G_1G_4H_1 - G_1G_2G_4H_2 - G_1G_3G_4H_2\right)$$

Continue……



Eliminate forward path-1

Δ_1 = 1- (sum of all individual loop gains)+...

Δ_1 = 1

Eliminate forward path-2

Δ_2 = 1- (sum of all individual loop gains)+...

Δ_2 = 1

Continue……

$$\frac{C}{R} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}.$$

$$= \frac{G_1G_2G_4 + G_1G_3G_4}{1 - G_1G_4H_1 + G_1G_2G_4H_2 + G_1G_3G_4H_2}$$

$$= \frac{G_1G_4(G_2 + G_3)}{1 - G_1G_4H_1 + G_1G_2G_4H_2 + G_1G_3G_4H_2}$$