First-Order High-Pass Butterworth Filter

High-pass filters are often formed simply by interchanging frequency-determining resistors and capacitors in low-pass filters. That is, a first order high-pass filter is formed from a first-order low pass filter by interchanging components R and C.

Figure 1 shows a first-order high-pass Butterworth filter with a low cutoff frequency of f_L. This is the frequency at which the magnitude of the gain is 0.707 times its passband value. Obviously, all frequencies higher than f_L are passband frequencies, with the highest frequency determined by the closed loop bandwidth of the op-amp.

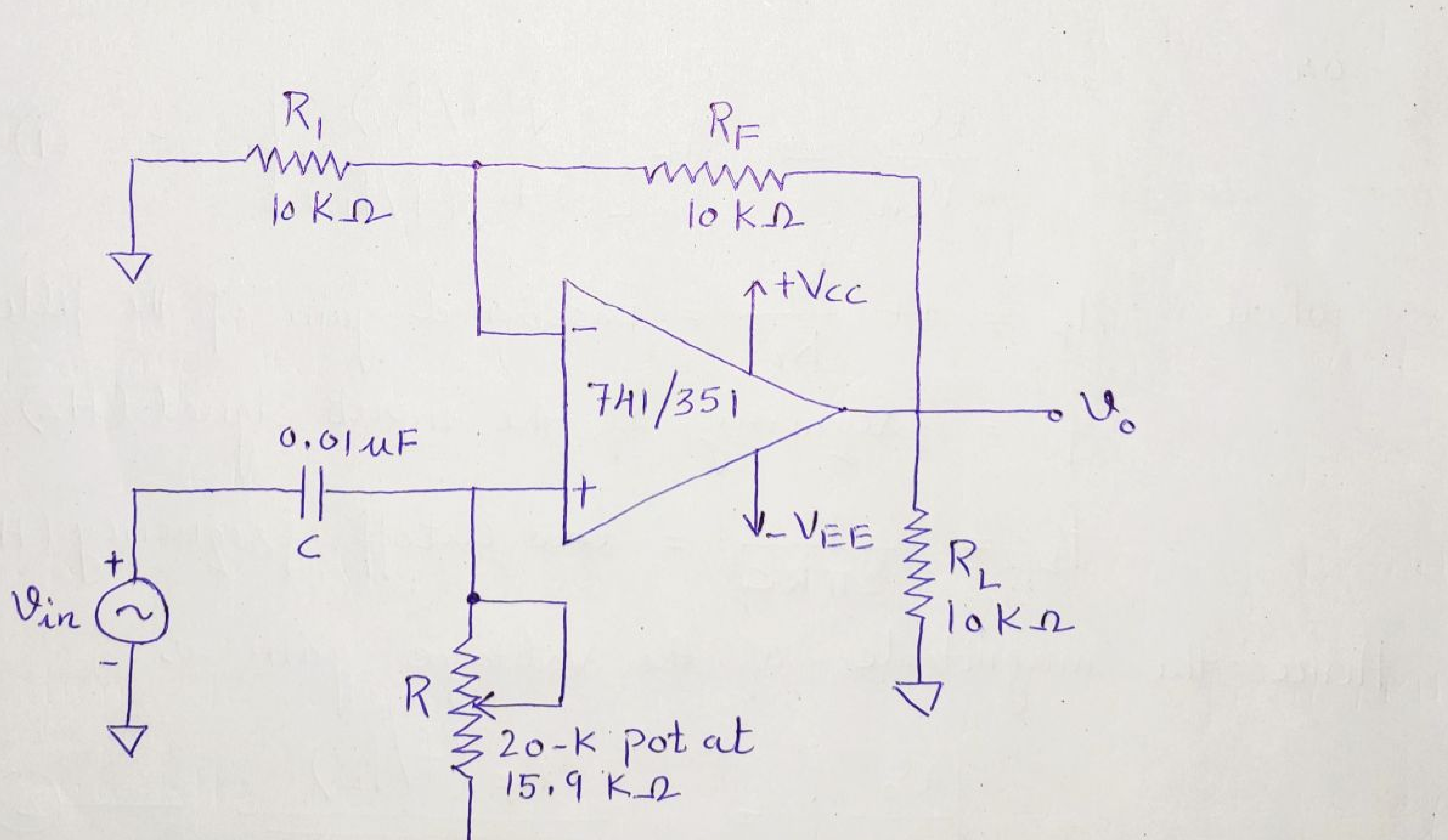

Figure 1. First-order high-pass Butterworth filter.

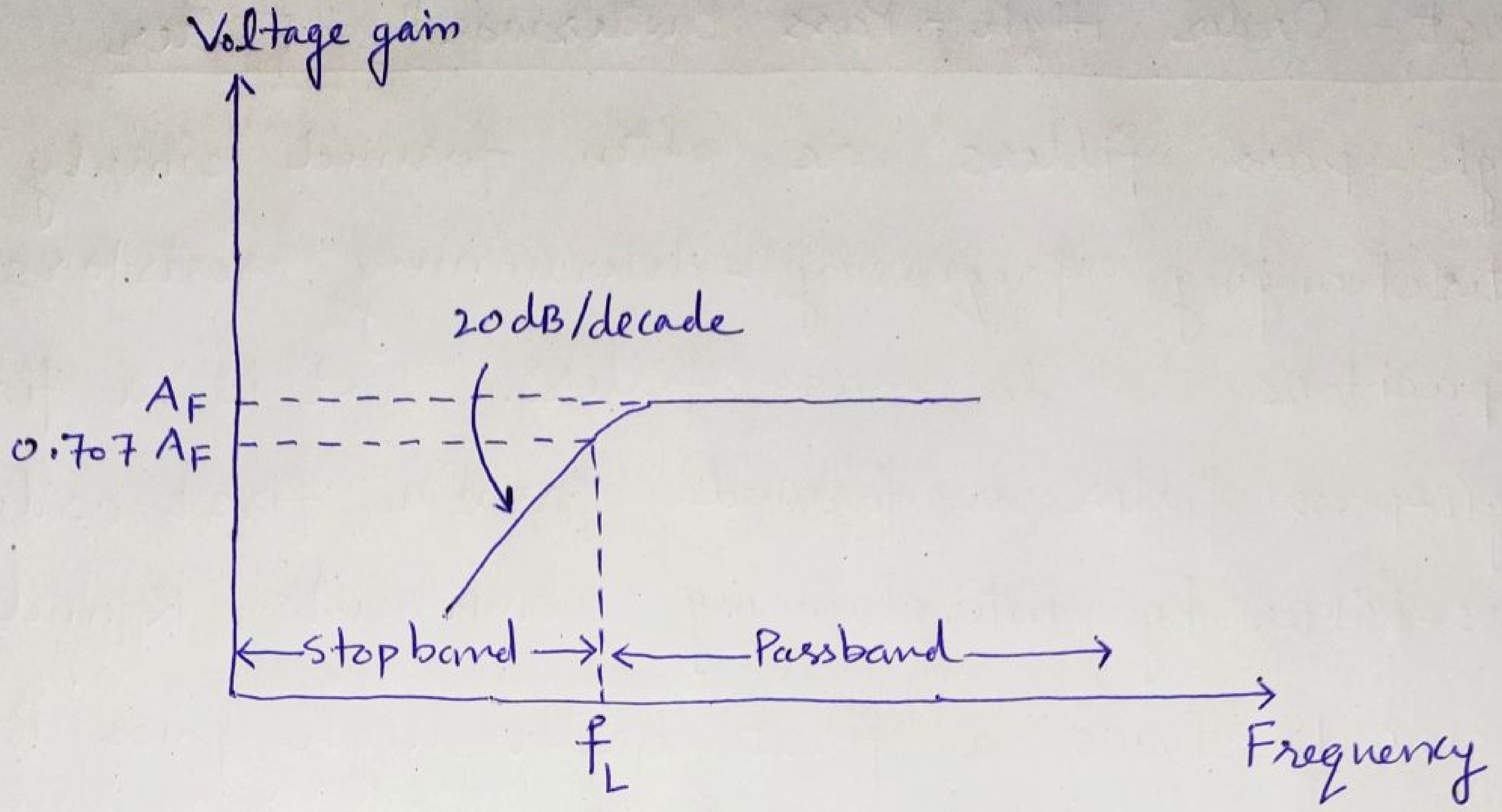

Figure 2. Frequency response of first-order high-pass Butterworth filter.

The first-order high-pass filter of Figure 1 has the output voltage given by,

$$v_o = \left(1 + \frac{R_F}{R_1}\right) \frac{j2\pi f RC}{1 + j2\pi f RC}\, v_{in}$$

or

$$\frac{v_o}{v_{in}} = A_F \left[\frac{j(f/f_L)}{1 + j(f/f_L)}\right] \quad \text{——} (1)$$

where $A_F = 1 + \dfrac{R_F}{R_1}$ = passband gain of the filter

f = frequency of the input signal (Hz)

$f_L = \dfrac{1}{2\pi RC}$ = low cutoff frequency (Hz)

Hence the magnitude of the voltage gain is

$$\left|\frac{v_o}{v_{in}}\right| = \frac{A_F (f/f_L)}{\sqrt{1 + (f/f_L)^2}} \quad \text{——} (2)$$

Since high-pass filters are formed from low-pass filters simply by interchanging R's and C's, the design and frequency scaling procedure of the low-pass filters are also applicable to the high-pass filters.

Q1. Design a high-pass filter at a cutoff frequency of 1 KHz with a passband gain of 2.

Solution: Follow the design steps.

1. $f_L = 1$ KHz

2. Let $C = 0.01\ \mu F$

3. Then $R = \dfrac{1}{2\pi f_L C} = \dfrac{1}{2\pi (10^3)(10^{-8})} = 15.9\ K\Omega$

4. Since the passband gain is 2, R_1 and R_F must be equal. Therefore, let $R_1 = R_F = 10\ K\Omega$. The complete circuit is shown in Figure 1 along with the component values.

Band Pass Filter

A band-pass filter has a passband between two cutoff frequencies f_H and f_L such that $f_H > f_L$. Any input frequency outside this passband is attenuated.

Basically, there are two types of band-pass filters: (1) wide band pass and (2) narrow band pass. We will define a filter as wide band pass if its quality factor $Q < 10$. On the other hand, if $Q > 10$, we will call the filter a narrow band-pass filter. The relationship between Q, the 3-dB bandwidth and the center frequency f_c is given by

$$Q = \frac{f_c}{BW} = \frac{f_c}{f_H - f_L} \quad \text{--- (3)}$$

For the wide band-pass filter, the center frequency f_c can be defined as

$$f_c = \sqrt{f_H f_L} \quad \text{--- (4)}$$

where f_H = high cutoff frequency (Hz)

f_L = low cutoff frequency of the wide band pass filter (Hz)

In a narrow band-pass filter, the output voltage peaks at the center frequency.

Wide Band Pass Filter

A wide band-pass filter can be formed by simply cascading high-pass and low pass sections. The order of the band pass filter depends on the order of the high pass and low-pass filter section.

Figure 3 shows the ± 20dB/decade wide band-pass filter, which is composed of first-order high pass and first-order low-pass filters. To realize a band-pass response, f_H must be larger than f_L.

The voltage gain magnitude of the band-pass filter is equal to the product of the voltage gain magnitudes of the high pass and low-pass filters.

5

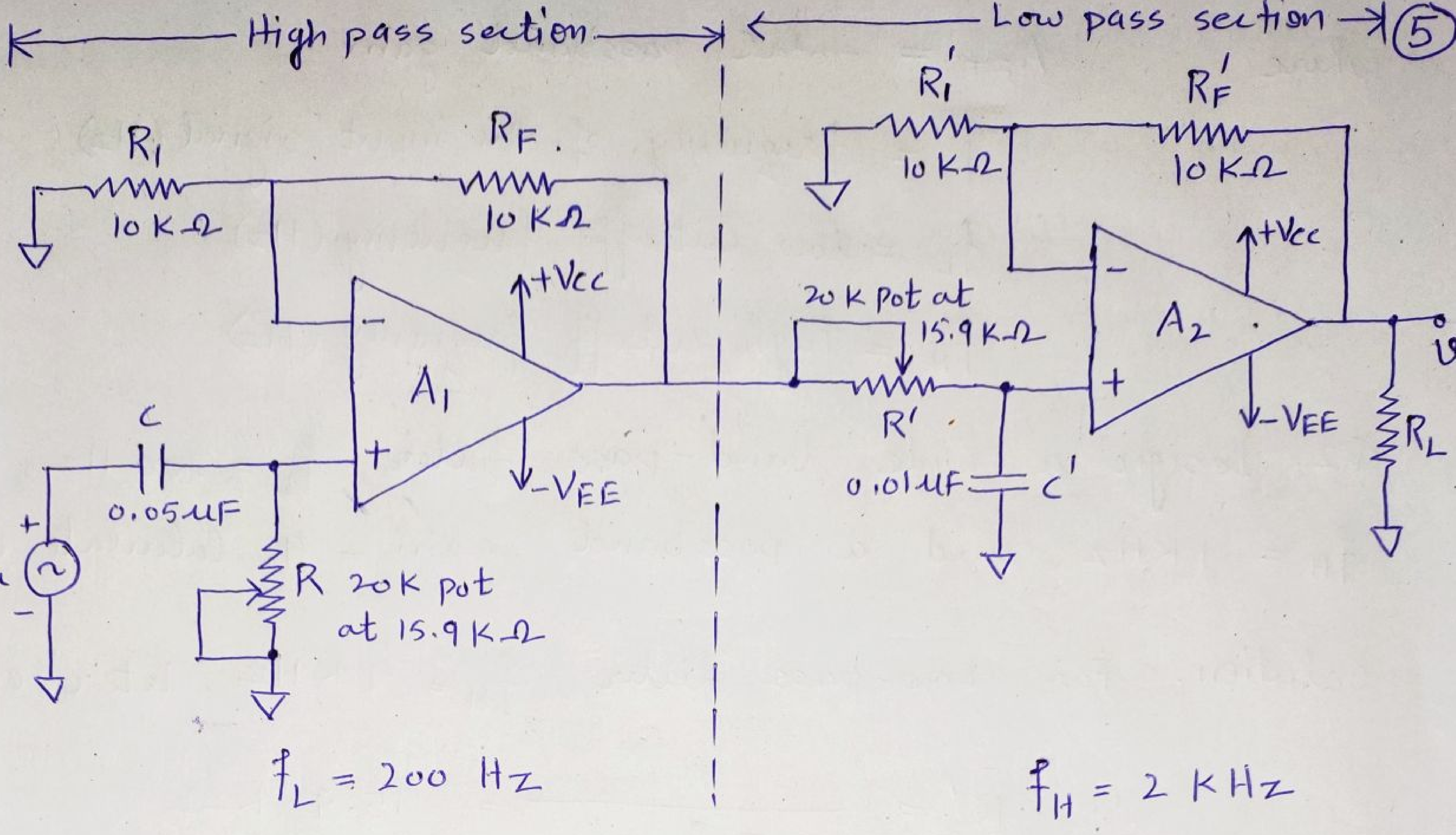

Figure 3. ± 20 dB/decade - wide band-pass filter.

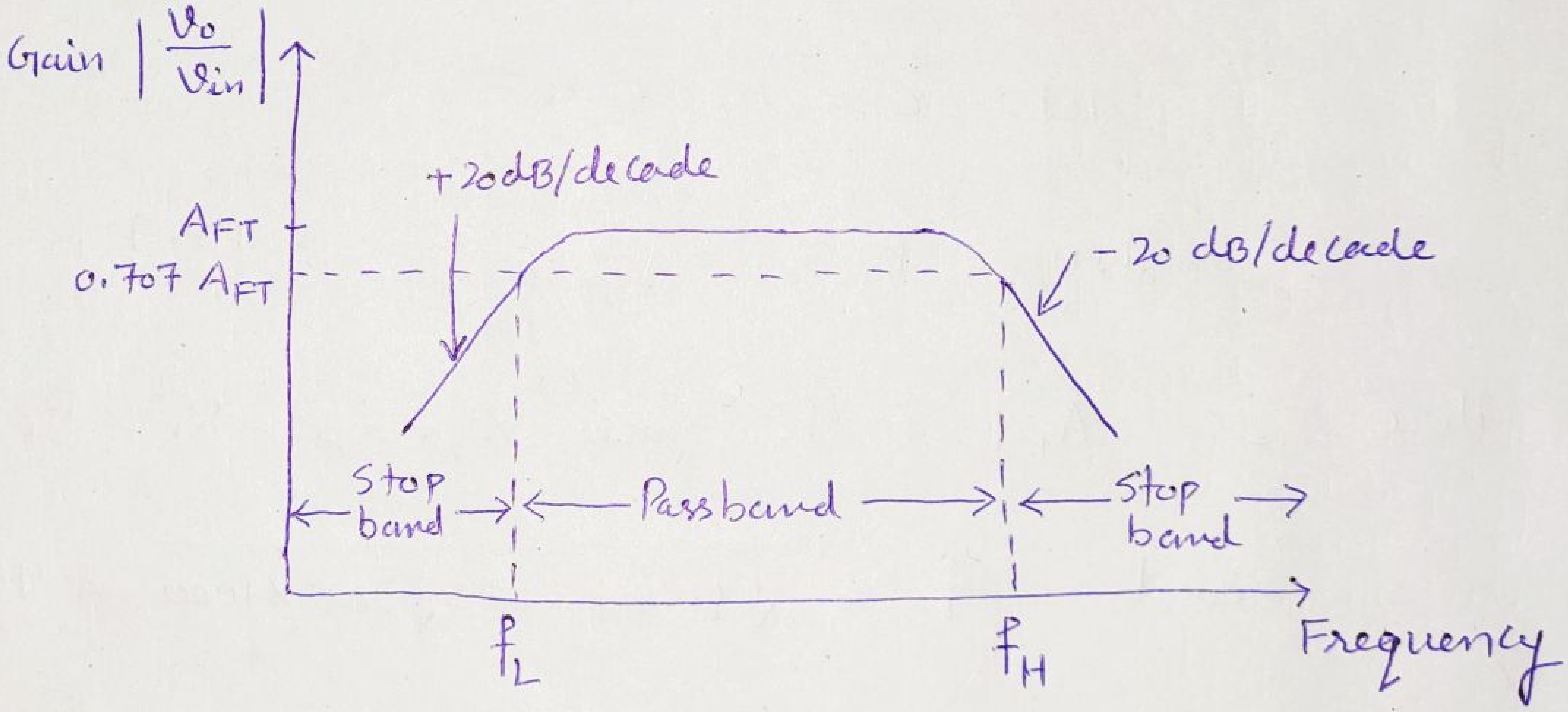

Figure 4. Frequency response of wide band-pass filter.

The frequency response of wide band-pass filter is shown in Figure 4. The gain (voltage) magnitude of wide band-pass filter is given by

$$\left|\frac{v_o}{v_{in}}\right| = \frac{A_{FT}\,(f/f_L)}{\sqrt{\left[1+(f/f_L)^2\right]\left[1+(f/f_H)^2\right]}} \quad -(5)$$

where A_{FT} = total passband gain

f = frequency of the input signal (Hz)

f_L = low cutoff frequency (Hz)

f_H = high cutoff frequency (Hz)

Q2. Design a wide band-pass filter $f_L = 200$ Hz, $f_H = 1$ KHz, and a passband gain = 4. Calulate Q.

Solution: For Low-pass filter, $f_H = 1$ KHz, let $C' = 0.01\,\mu F$.

$$\therefore \quad R' = \frac{1}{2\pi f_H C'} = 15.9\ K\Omega$$

In the case of high-pass filter, $f_L = 200$ Hz.

let $C = 0.05\,\mu F$

then
$$R = \frac{1}{2\pi f_L C} = 15.9\ K\Omega$$

Here $A_{FT} = 4$ since $R_1 = R_F = R_1' = R_F' = 10\ K\Omega$

and
$$f_c = \sqrt{f_L f_H} = \sqrt{200 \times 1000} = 447.2\ Hz$$

$$\therefore \quad Q = \frac{447.2}{1000 - 200} = 0.56 \text{ (Answer)}$$