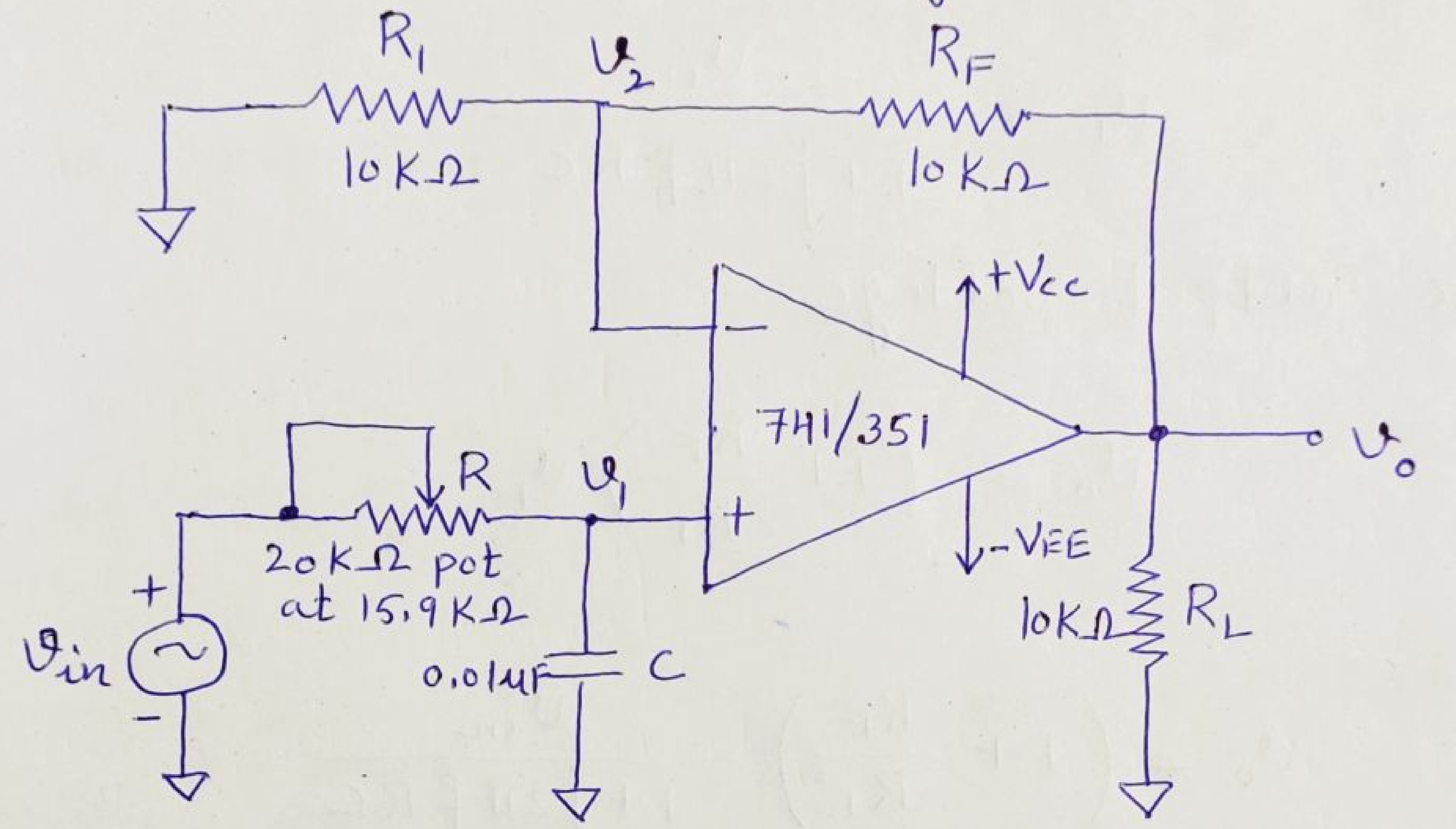
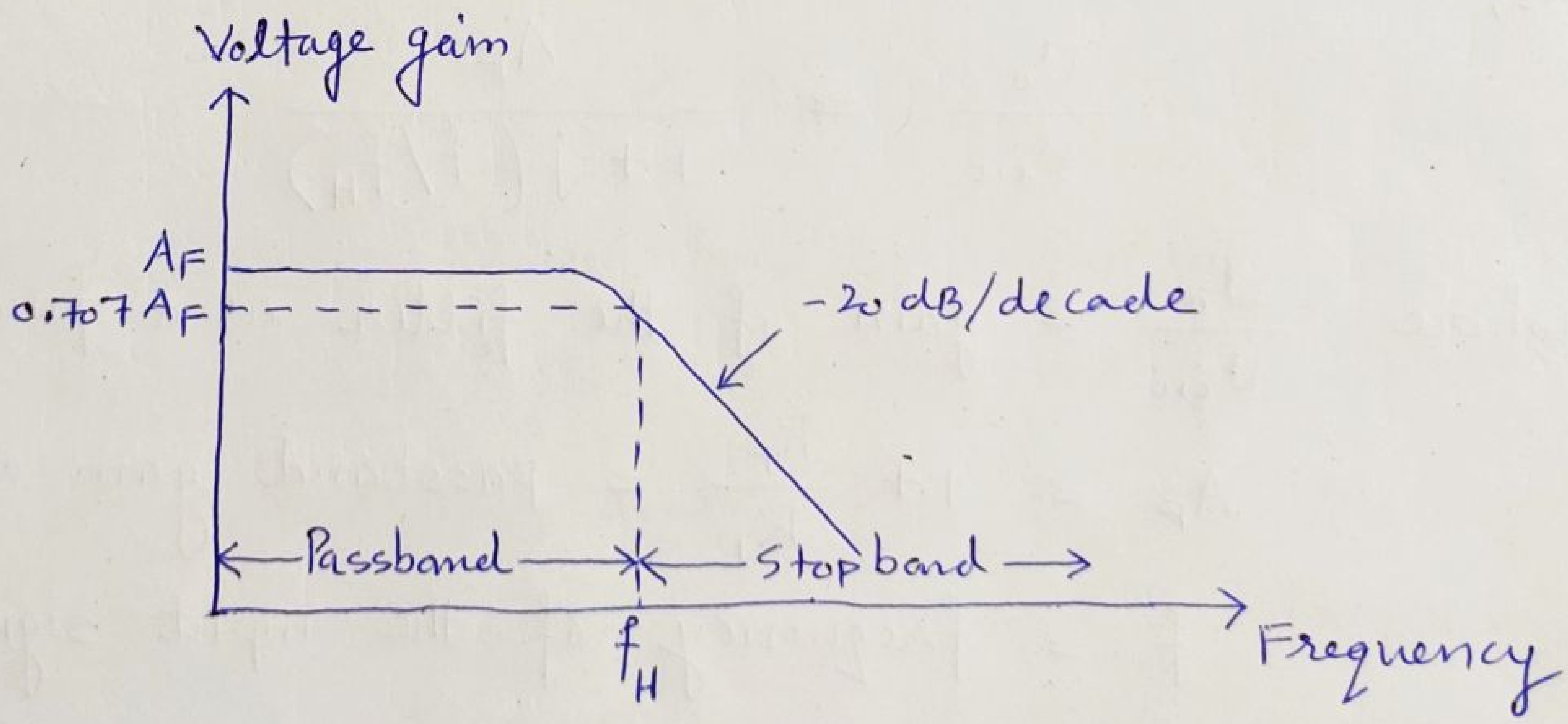
First Order Low-Pass Butterworth Filter

Figure 1 shows a first-order low-pass Butterworth filter that uses an RC network for filtering. Note that the op-amp is used in the noninverting configuration; hence it does not load down the RC network. Resistors R_1 and R_F determine the gain of the filter.



(a)



(b)

Figure 1. First order low pass Butterworth filter (a) circuit (b) Frequency response.

According to the voltage-divider rule, the voltage at the non inverting terminal (across capacitor C) is

$$v_1 = \frac{-jX_C}{R - jX_C} v_{in} \quad \text{——— (1)}$$

where

$$j = \sqrt{-1} \quad \text{and} \quad -jX_C = \frac{1}{j2\pi fC}$$

Simplifying equation (1), we get

$$v_1 = \frac{v_{in}}{1 + j2\pi fRC}$$

and the output voltage

$$v_o = \left(1 + \frac{R_F}{R_1}\right) v_1$$

That is,

$$v_o = \left(1 + \frac{R_F}{R_1}\right) \frac{v_{in}}{1 + j2\pi fRC}$$

or,

$$\frac{v_o}{v_{in}} = \frac{A_F}{1 + j(f/f_H)} \quad \text{——— (2)}$$

where $\frac{v_o}{v_{in}}$ = gain of the filter as a function of frequency

$A_F = 1 + \frac{R_F}{R_1}$ = passband gain of the filter

f = frequency of the input signal

$f_H = \frac{1}{2\pi RC}$ = high cutoff frequency of the filter

The gain magnitude and phase angle equations of the low-pass filter can be obtained by converting equation (2) into its equivalent polar form, as follows:

$$\left|\frac{v_o}{v_{in}}\right| = \frac{A_F}{\sqrt{1+(f/f_H)^2}} \quad \text{--- (3)}$$

$$\phi = -\tan^{-1}\left(\frac{f}{f_H}\right) \quad \text{--- (4)}$$

where ϕ is the phase angle in degrees.

The operation of the low-pass filter can be verified from the gain magnitude equation (3);

1. At very low frequencies, that is $f < f_H$,

$$\left|\frac{v_o}{v_{in}}\right| \cong A_F$$

2. At $f = f_H$,

$$\left|\frac{v_o}{v_{in}}\right| = \frac{A_F}{\sqrt{2}} = 0.707\, A_F$$

3. At $f > f_H$,

$$\left|\frac{v_o}{v_{in}}\right| < A_F$$

Thus the low-pass filter has a constant gain A_F from 0 Hz to the high cutoff frequency f_H. At f_H the gain is $0.707\, A_F$, and after f_H, it decreases at a constant rate with an increase in frequency. That is, when the frequency is increased ten fold (one decade), the voltage gain is divided by 10. In other words, the gain decreases 20 dB ($= 20 \log 10$) each time the frequency is increased by 10.

The frequency $f = f_H$ is called the cutoff frequency because the gain of the filter at this frequency is down by 3 dB (= 20 log 0.707) from 0 Hz.

Filter Design

A low-pass filter can be designed by implementing the following steps:

1. Choose a value of high cutoff frequency f_H.
2. Select a value of C less than or equal to 1 μF.
3. Calculate the value of R using

$$R = \frac{1}{2\pi f_H C}$$

4. Finally, select values of R_1 and R_F dependent on the desired passband gain A_F using

$$A_F = 1 + \frac{R_F}{R_1}$$

Frequency Scaling

Once a filter is designed, there may sometimes be a need to change its cutoff frequency. The procedure used to convert an original cutoff frequency f_H to a new cutoff frequency f_H' is called frequency scaling. Frequency scaling is accomplished as follows. To change a high cutoff frequency, multiply R or C, but not both, by the ratio of the original cutoff frequency to the new cutoff frequency. A variable capacitor C is not commonly used. Therefore, choose a standard value of capacitor, and then calculate the value of resistor for a desired cutoff frequency. This is because for a nonstandard value of resistor a potentiometer can be used.

Q1. Design a low-pass filter at a cutoff frequency of 1 KHz with a passband gain of 2.

Solution: Follow the preceding design steps.

1. $f_H = 1 \text{ KHz}$.

2. Let $C = 0.01 \, \mu F$

3. Then $R = \dfrac{1}{2\pi (10^3)(10^{-8})} = 15.9 \text{ K}\Omega$ (use a $20 \text{ K}\Omega$ potentiometer)

4. Since the passband gain is 2, R_1 and R_F must be equal. i.e.

$$A_F = 1 + \frac{R_F}{R_1} = 2$$

$$\frac{R_F}{R_1} = 1 \quad \text{or} \quad R_F = R_1$$

Therefore, let $R_1 = R_F = 10 \text{ K}\Omega$. The complete circuit is shown in Figure 1 (a).

Q2. Using the frequency scaling technique, convert the 1-KHz cutoff frequency of the low-pass filter of Q1 to a cutoff frequency of 1.6 KHz.

Solution: To change a cutoff frequency from 1 KHz to 1.6 KHz, we multiply the 15.9 KΩ resistor by

$$\frac{\text{original cutoff frequency}}{\text{new cutoff frequency}} = \frac{1\,KHz}{1.6\,KHz} = 0.625$$

Therefore, new resistor $R = (15.9\,K\Omega)(0.625) = 9.94\,K\Omega$.

However, $9.94\,K\Omega$ is not a standard value. Therefore, use $R = 10\,K\Omega$ potentiometer and adjust it to $9.94\,K\Omega$. Thus the new cutoff frequency is,

$$f_H = \frac{1}{(2\pi)(0.01\,\mu F)(9.94\,K\Omega)}$$

$$f_H = \frac{1}{(2\pi)(0.01 \times 10^{-6})(9.94 \times 10^{3})}$$

$$f_H = 1.6\,KHz$$