$$\therefore G(s)H(s) \approx K\frac{1}{s^y}$$

On letting, $s = \underset{R\to 0}{Lt}\ Re^{j\theta}$ we get,

$$G(s)H(s)\Big|_{s=\underset{R\to 0}{Lt} Re^{j\theta}} = \frac{K}{\underset{R\to 0}{Lt}\ (Re^{j\theta})^y} = \infty e^{-j\theta y}$$

When $\theta = -\frac{\pi}{2}$, $G(s)H(s) = \infty e^{j\frac{\pi}{2}y}$

When $\theta = \frac{\pi}{2}$, $G(s)H(s) = \infty e^{-j\frac{\pi}{2}y}$

From the above two equations we can conclude that the section C_4 of Nyquist contour in s-plane is mapped as circles/circular arc in G(s)H(s)-plane with origin as centre and infinite radius.

Note :

1. *If there are no poles on the origin then the section C_4 of Nyquist contour will be absent.*
2. *If there are poles on imaginary axis as shown below then the Nyquist contour is divided into the following 8 sections and the mapping is performed sectionwise.*

Section C_1 : $s = j\omega\ ;\ \omega = 0^+ \text{ to } +\omega_1^-$

Section C_2 : $s = \underset{R\to 0}{Lt}\ Re^{j\theta}; \theta = -\frac{\pi}{2} \text{ to} + \frac{\pi}{2}$

Section C_3 : $s = j\omega\ ;\ \omega = +\omega_1^+ \text{ to } + \infty$

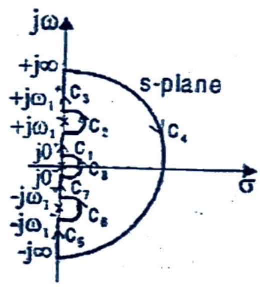
Section C_4 : $s = \underset{R\to \infty}{Lt}\ Re^{j\theta}; \theta = +\frac{\pi}{2} \text{ to } -\frac{\pi}{2}$

Section C_5 : $s = j\omega\ ;\ \omega = -\infty \text{ to } -\omega_1^-$

Section C_6 : $s = \underset{R\to 0}{Lt}\ Re^{j\theta}; \theta = -\frac{\pi}{2} \text{ to} + \frac{\pi}{2}$

Section C_7 : $s = j\omega\ ;\ \omega = -\omega_1^+ \text{ to } 0^-$

Section C_8 : $s = \underset{R\to 0}{Lt}\ Re^{j\theta}; \theta = -\frac{\pi}{2} \text{ to} + \frac{\pi}{2}$



EXAMPLE 4.13

Draw the Nyquist plot for the system whose open loop transfer function is, $G(s)H(s) = \frac{K}{s\,(s+2)\,(s+10)}$.

Determine the range of K for which closed loop system is stable.

SOLUTION

Given that, $G(s)H(s) = \frac{K}{s\,(s+2)\,(s+10)} = \frac{K}{s \times 2\left(\frac{s}{2}+1\right) \times 10\left(\frac{s}{10}+1\right)} = \frac{0.05K}{s\,(1+0.5s)\,(1+0.1s)}$

The open loop transfer function has a pole at origin. Hence choose the Nyquist contour on s-plane enclosing the entire right half plane except the origin as shown in fig 4.13.1.

The Nyquist contour has four sections C_1, C_2, C_3 and C_4. The mapping of each section is performed separately and the overall Nyquist plot is obtained by combining the individual sections.

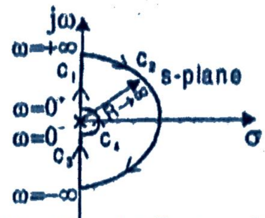


Fig 4.13.1 : *Nyquist Contour in s-plane*

MAPPING OF SECTION C_1

In section C_1, ω varies from 0 to +∞. The mapping of section C_1 is given by the locus of $G(j\omega)H(j\omega)$ as ω is varied from 0 to ∞. This locus is the polar plot of $G(j\omega)H(j\omega)$.

$$G(s)H(s) = \frac{0.05K}{s\,(1+0.5s)\,(1+0.1s)}$$

Let $s = j\omega$.

$$\therefore G(j\omega)H(j\omega) = \frac{0.05K}{j\omega\,(1+j0.5\omega)\,(1+j0.1\omega)} = \frac{0.05K}{j\omega\,(1+j0.6\omega-0.05\omega^2)} = \frac{0.05K}{-0.6\omega^2 + j\omega\,(1-0.05\omega^2)}$$

When the locus of $G(j\omega)H(j\omega)$ crosses real axis the imaginary term will be zero and the corresponding frequency is the phase crossover frequency, ω_{pc}.

$$\therefore \text{At } \omega = \omega_{pc}, \quad \omega_{pc}(1-0.05\omega_{pc}^2) = 0 \quad \Rightarrow \quad 1-0.05\omega_{pc}^2 = 0 \quad \Rightarrow \quad \omega_{pc} = \sqrt{\frac{1}{0.05}} = 4.472 \text{ rad/sec}$$

$$\text{At } \omega = \omega_{pc} = 4.472 \text{ rad/sec}, \qquad G(j\omega)H(j\omega) = \frac{0.05K}{-0.6\omega^2} = -\frac{0.05K}{0.6\times(4.472)^2} = -0.00417K$$

The open loop system is type-1 and third order system. Also it is a minimum phase system with all poles. Hence the polar plot of $G(j\omega)H(j\omega)$ starts at −90° axis at infinity, crosses real axis at −0.00417K and ends at origin in second quadrant. The section C_1 and its mapping are shown in fig 4.13.2. and 4.13.3.

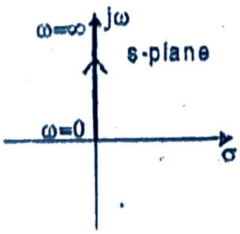


Fig 4.13.2 : Section C_1 in s-plane

⇒

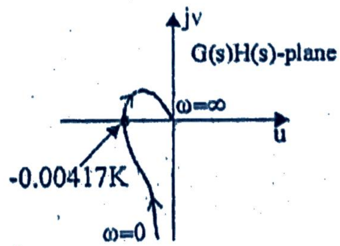


Fig 4.13.3 : *Mapping of section C_1 in G(s)H(s)-plane*

MAPPING OF SECTION C_2

The mapping of section C_2 from s-plane to G(s)H(s)-plane is obtained by letting $s = \underset{R\to\infty}{Lt}\; Re^{j\theta}$ in G(s)H(s) and varying θ from +π/2 to −π/2. Since $s \to Re^{j\theta}$ and $R\to\infty$, the G(s)H(s) can be approximated as shown below, [i.e., $(1+sT) \approx sT$].

$$G(s)\,H(s) = \frac{0.05K}{s\,(1+0.5s)\,(1+0.1s)} \approx \frac{0.05K}{s\times 0.5s\times 0.1s} = \frac{K}{s^3}$$

Let, $s = \underset{R\to\infty}{Lt}\; Re^{j\theta}$.

$$\therefore G(s)H(s)\Bigg|_{s=\underset{R\to\infty}{Lt} Re^{j\theta}} = \frac{K}{s^3}\Bigg|_{s=\underset{R\to\infty}{Lt} Re^{j\theta}} = \frac{K}{\underset{R\to\infty}{Lt}\,(Re^{j\theta})^3} = 0e^{-j3\theta}$$

$$\text{When } \theta = \frac{\pi}{2}, \qquad G(s)H(s) = 0e^{-j3\frac{\pi}{2}} \qquad \text{.....(1)}$$

$$\text{When } \theta = -\frac{\pi}{2}, \qquad G(s)H(s) = 0e^{+j3\frac{\pi}{2}} \qquad \text{.....(2)}$$

From the equations (1) and (2) we can say that section C_2 in s-plane (fig 4.13.4.) is mapped as circular arc of zero radius around origin in G(s)H(s)-plane with argument (phase) varying from $-3\pi/2$ to $+3\pi/2$ as shown in fig 4.13.5.

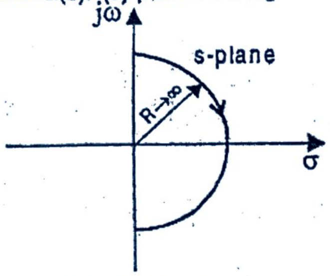


Fig 4.13.4 : Section C_2 in s-plane

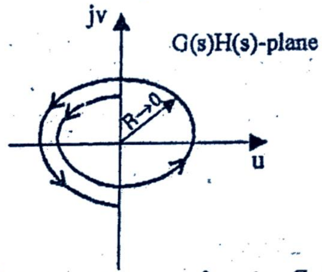


Fig 4.13.5 : *Mapping of section C_2 in G(s)H(s)-plane*

MAPPING OF SECTION C_3

In section C_3, ω varies from $-\infty$ to 0. The mapping of section C_3 is given by the locus of G(jω)H(jω) as ω is varied from $-\infty$ to 0. This locus is the inverse polar plot of G(jω)H(jω).

The inverse polar plot is given by the mirror image of polar plot with respect to real axis. The section C_3 in s-plane and its corresponding contour in G(s)H(s) plane are shown in fig 4.13.6 and fig 4.13.7.

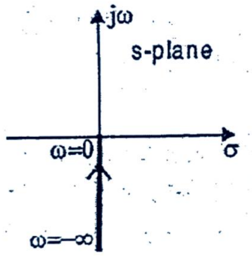


Fig 4.13.6 : Section C_3 in s-plane

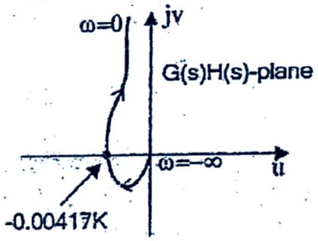


Fig 4.13.7 : *Mapping of section C_3 in G(s)H(s)-plane*

MAPPING OF SECTION C_4

The mapping of section C_4 from s-plane to G(s)H(s)-plane is obtained by letting $s = \underset{R\to 0}{Lt}\ R\,e^{j\theta}$ in G(s)H(s) and varying θ from $-\pi/2$ to $+\pi/2$. Since $s \to R\,e^{j\theta}$ and $R \to 0$, the G(s) H(s) can be approximated as shown below, [i.e., $(1+sT) \approx 1$].

$$G(s)H(s) = \frac{0.05K}{s\,(1+0.5s)\,(1+0.1s)} \approx \frac{0.05K}{s \times 1 \times 1} = \frac{0.05K}{s}$$

Let $s = \underset{R\to 0}{Lt}\ Re^{j\theta}$.

$$\therefore\ G(s)H(s)\Big|_{s=\underset{R\to 0}{Lt} Re^{j\theta}} = \frac{0.05K}{s}\Big|_{s=\underset{R\to 0}{Lt} Re^{j\theta}} = \frac{0.05K}{\underset{R\to 0}{Lt}(Re^{j\theta})} = \infty e^{-j\theta}$$

$$\text{When } \theta = -\frac{\pi}{2},\quad G(s)H(s) = \infty e^{+j\frac{\pi}{2}} \qquad .....(3)$$

$$\text{When } \theta = \frac{\pi}{2},\quad G(s)H(s) = \infty e^{-j\frac{\pi}{2}} \qquad .....(4)$$

From the equations (3) and (4) we can say that section C_4 in s-plane (fig 4.13.8.) is mapped as a circular arc of infinite radius with argument (phase) varying from $+\pi/2$ to $-\pi/2$ as shown in fig 4.13.9.

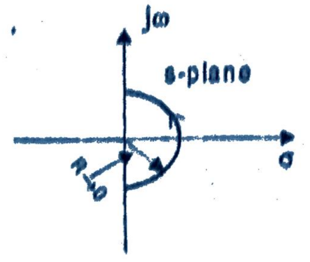


Fig 4.13.8 : Section C_4 in s-plane

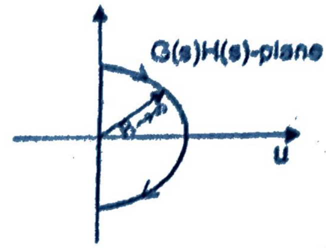


Fig 4.13.9 : *Mapping of section C_4 in G(s)H(s)-plane*

COMPLETE NYQUIST PLOT

The entire Nyquist plot in G(s)H(s)-plane can be obtained by combining the mappings of individual sections, as shown in fig 4.13.10.

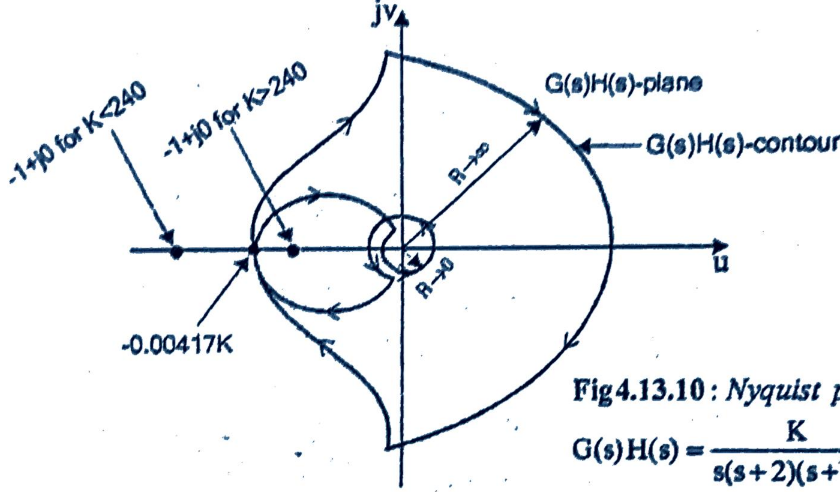


Fig4.13.10 : *Nyquist plot of* $G(s)H(s) = \dfrac{K}{s(s+2)(s+10)}$

STABILITY ANALYSIS

When, $-0.00417K = -1$, the contour passes through $(-1+j0)$ point and corresponding value of K is the limiting value of K for stability.

$$\therefore \text{Limiting value of } K = \frac{1}{0.00417} = 240$$

When $K < 240$

When K is less than 240, the contour crosses real axis at a point between 0 and $-1+j0$. On travelling through Nyquist plot along the indicated direction it is found that the point $-1+j0$ is not encircled. Also the open loop transfer function has no poles on the right half of s-plane. Therefore the closed loop system is stable.

When $K > 240$

When K is greater than 240, the contour crosses real axis at a point between $-1+j0$ and $-\infty$. On travelling through Nyquist plot along the indicated direction it is found that the point $-1+j0$ is encircled in clockwise direction two times. [Since there are two clockwise encirclement and no right half open loop poles, the closed loop system has two poles on right half of s-plane]. Therefore the closed loop system is unstable.

RESULT

The value of K for stability is $0 < K < 240$

EXAMPLE 4.14

Construct the Nyquist plot for a system whose open loop transfer function is given by $G(s)H(s) = \dfrac{K(1+s)^2}{s^3}$. Find the range of K for stability.

SOLUTION

Given that, $G(s)H(s) = \frac{K(1+s)^2}{s^3}$

The open loop transfer function has three poles at origin. Hence choose the Nyquist contour on s-plane enclosing the entire right half plane except the origin as shown in fig 4.14.1.

The Nyquist contour has four sections C_1, C_2, C_3 and C_4. The mapping of each section is performed separately and the overall Nyquist plot is obtained by combining the individual sections.

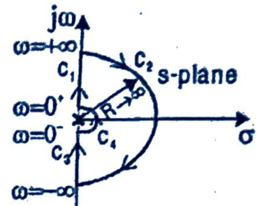


Fig 4.14.1 : *Nyquist Contour in s-plane*

MAPPING OF SECTION C_1

In section C_1, ω varies from 0 to $+\infty$. The mapping of section C_1 is given by the locus of $G(j\omega)H(j\omega)$ as ω is varied from 0 to ∞. This locus is the polar plot of $G(j\omega)H(j\omega)$.

$$G(s)H(s) = \frac{K(1+s)^2}{s^3}$$

Let $s = j\omega$.

$$\therefore G(j\omega)H(j\omega) = \frac{K(1+j\omega)^2}{(j\omega)^3} = \frac{K(1-\omega^2+2j\omega)}{-j\omega^3} = \frac{K(1-\omega^2)}{-j\omega^3} + \frac{K\,2j\omega}{-j\omega^3} = -\frac{2K}{\omega^2} + j\frac{K(1-\omega^2)}{\omega^3}$$

When the $G(j\omega)H(j\omega)$ locus crosses real axis the imaginary term will be zero and the corresponding frequency is the phase crossover frequency, ω_{pc}.

$$\therefore \text{At } \omega = \omega_{pc}, \quad K(1-\omega_{pc}^2) = 0 \quad \Rightarrow \quad 1-\omega_{pc}^2 = 0 \quad \Rightarrow \quad \omega_{pc} = 1\text{ rad/sec}$$

At $\omega = \omega_{pc} = 1$ rad/sec,

$$G(j\omega)H(j\omega) = -\frac{2K}{\omega^2} = -\frac{2K}{1^2} = -2K \qquad .....(1)$$

$$G(j\omega)H(j\omega) = \frac{K(1+j\omega)^2}{(j\omega)^3} = \frac{K\sqrt{1+\omega^2}\,\angle\tan^{-1}\omega\,\sqrt{1+\omega^2}\,\angle\tan^{-1}\omega}{\omega^3\angle 270^\circ} = \frac{K(1+\omega^2)}{\omega^3}\angle(2\tan^{-1}\omega - 270^\circ)$$

$$\text{As } \omega \to 0, \quad G(j\omega)H(j\omega) \to \infty\angle -270^\circ \qquad .....(2)$$

$$\text{As } \omega \to \infty, \quad G(j\omega)H(j\omega) \to 0\angle -90^\circ \qquad .....(3)$$

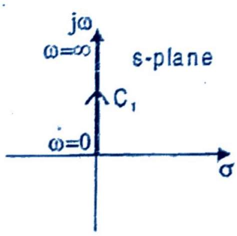


Fig 4.14.2 : *Section C_1 in s-plane*

$\Rightarrow$

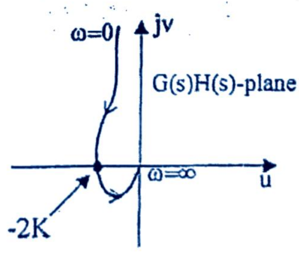


Fig 4.14.3 : *Mapping of section C_1 in G(s)H(s)-plane*

From equations (1), (2) and (3) we can say that the polar plot starts at $-270°$ axis at infinity, crosses real axis at $-2K$ and ends at origin in third quadrant. The section C_1 and its mapping are shown in fig 2 and 3.

MAPPING OF SECTION C_2

The mapping of section C_2 from s-plane to G(s) H(s)-plane is obtained by letting $s = \underset{R\to\infty}{Lt} Re^{j\theta}$ in G(s)H(s) and varying θ from $+\pi/2$ to $-\pi/2$. Since $s \to Re^{j\theta}$ and $R\to\infty$, the G(s) H(s) can be approximated as shown below, [i.e., $(1+sT) \approx sT$].