Polar Plot

ECE305

Control Systems

18/3/2020

In polar plot, the magnitude of $G(j\omega)H(j\omega)$ is plotted against the phase angle of $G(j\omega)H(j\omega)$ for various values of ω. In frequency response we have,

$$M = |G(j\omega)H(j\omega)| = \text{Magnitude}$$

$$\phi = \angle G(j\omega)H(j\omega) = \text{Phase}$$

We can obtain the values of M and ϕ by varying the input frequency ω from 0 to ∞. The result can be tabulated as below.

ω	$M = \vert G(j\omega)H(j\omega) \vert$	$\phi = \angle G(j\omega)H(j\omega)$
0	M_0	ϕ_0
ω_1	M_1	ϕ_1
:	:	:
:	:	:
:	:	:
∞	M_∞	ϕ_∞

Example *Consider a system with open loop transfer function as* $G(s)H(s) = \frac{10}{s}$. *Obtain its polar plot .*

Solution : Now to obtain its polar plot, obtain frequency domain transfer function by replacing s by jω.

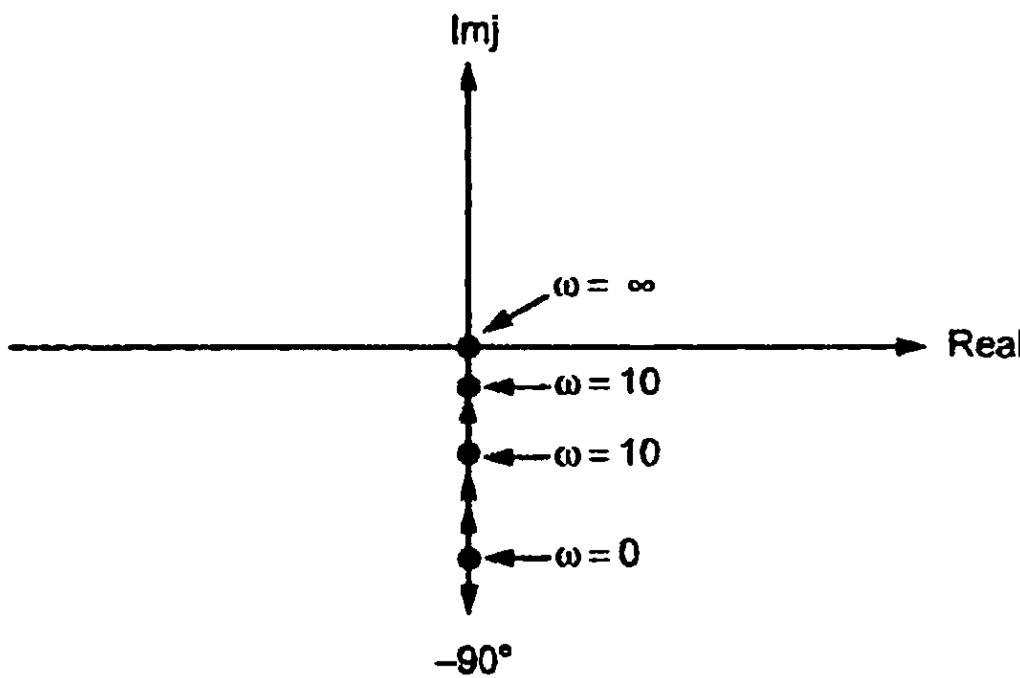
$$G(j\omega)\,H(j\omega) = \frac{10}{j\omega} = \frac{10 + j0}{0 + j\omega}$$

$$\therefore \qquad |G(j\omega)\,H(j\omega)| = M = \frac{10}{\omega}$$

$$\angle G(j\omega)H(j\omega) = \phi = \frac{\tan^{-1}\left(\frac{0}{10}\right)}{\tan^{-1}\left(\frac{\omega}{0}\right)} = \frac{0^\circ}{90^\circ} = -90^\circ$$

For various values of ω, M is changing but angle remains constant as -90°.

ω	M	ϕ
0	∞	-90°
10	1	-90°
100	0.1	-90°
:		
:		
∞	0	-90°


Imj
ω = ∞
Real
ω = 10
ω = 10
ω = 0
–90°

Example *Consider a Type 0 system with open loop transfer function*

$$G(s)H(s) = \frac{1}{1+Ts}$$ *where T is constant. Obtain its polar plot.*

Solution : The frequency domain transfer function is,

$$G(j\omega)H(j\omega) = \frac{1}{1+Tj\omega} = \frac{1+j0}{1+j\omega T}$$

$$|G(j\omega)H(j\omega)| = M = \frac{1}{\sqrt{1+\omega^2 T^2}}$$

$$\angle G(j\omega)\, H(j\omega) = \phi = \frac{\tan^{-1}\left(\frac{0}{1}\right)}{\tan^{-1}\left(\frac{\omega T}{1}\right)} = \frac{0^\circ}{(\tan^{-1} \omega T)} = -\tan^{-1}(\omega T)$$

For various values of ω, the result can be tabulated as,

ω	M	ϕ
0	1	0°
$\frac{1}{T}$	$\frac{1}{\sqrt{2}}$	-45°
$\frac{10}{T}$	$\frac{1}{\sqrt{101}}$	-84.2°
:	:	:
:	:	:
∞	0	-90°

Example 12.5 : *Let us add a simple pole and see its effect on polar plot.*

$$G(s)H(s) = \frac{1}{(1+T_1 s)\,(1+T_2 s)}$$

Solution : The frequency domain, transfer function is

$$G(j\omega)H(j\omega) = \frac{1}{(1+T_1\, j\omega)\,(1+T_2\, j\omega)}$$

$$|G(j\omega)H(j\omega)| = M = \frac{1}{\sqrt{1+T_1^2\omega^2} \times \sqrt{1+T_2^2\omega^2}}$$

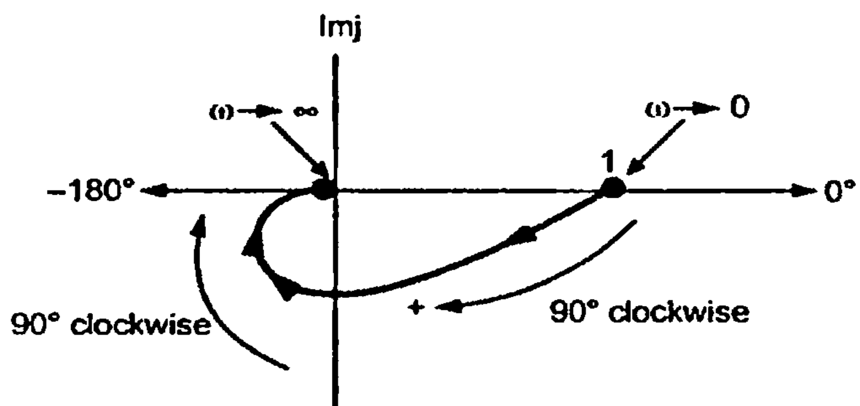
$$\angle G(j\omega)H(j\omega) = \phi = \frac{\tan^{-1}\left(\frac{0}{1}\right)}{\tan^{-1}\left(\frac{\omega T_1}{1}\right)\tan^{-1}\left(\frac{\omega T_2}{1}\right)}$$

$$\therefore \qquad \phi = -\tan^{-1}\omega T_1 - \tan^{-1}\omega T_2$$

Starting point	$\omega \to 0$	$1\angle 0°$	Rotation of plot = $-180°-0° = -180°$ clockwise
Terminating point	$\omega \to \infty$	$0\angle -180°$	

Rotation of plot = $-180°$ i.e. $180°$ in clockwise direction.

So polar plot is as shown in the Fig. 12.8.



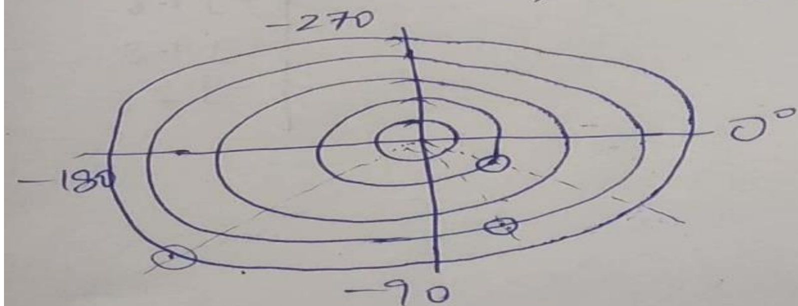
Polar plot is a freq. domain plot. (1)

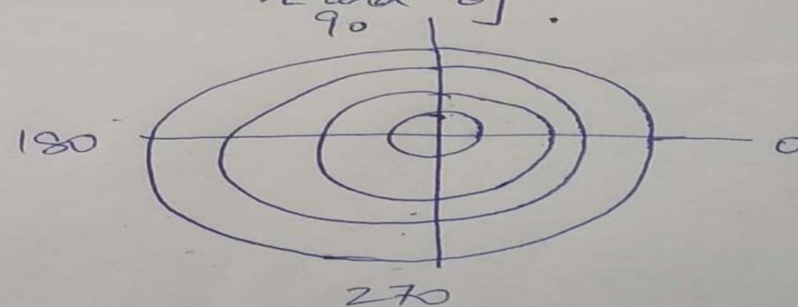
The polar plot of a sinusoidal transfer function $G(j\omega)H(j\omega)$ is a plot of the magnitude $G(j\omega)H(j\omega)$ versus the phase angle of $G(j\omega)H(j\omega)$ on polar coordinate as ω is varied from 0 to ∞. The polar plot therefore is the locus of vector $|G(j\omega)| \angle G(j\omega)$ as ω is varied from 0 to ∞.

In polar plot the magnitude of $G(j\omega)H(j\omega)$ is plotted as the distance from the origin while phase angle is measured from +ve real axis.

+ve phase angle is measured anticlockwise while −ve phase angle is measured clockwise from real axis.

$|G(j\omega)| = r$, $\angle G(j\omega) = \theta$ [polar coordinates, r and θ].





Steps Method-I

1. Put $s = j\omega$ in open loop transfer function. $G(j\omega)H(j\omega)$.

2. Calculate $|G(j\omega)H(j\omega)|$ and $\angle G(j\omega)H(j\omega)$

3. Make a table for different values of ω; varying from 0 to ∞.

ω	0.1	0.2	- -	- —
$\lvert G(j\omega)\rvert$				
$\angle G(j\omega)$				

4. Convert $|G(j\omega)| = r$, $\angle G(j\omega) = \theta$ to Cartesian coordinates.

$$x = r\cos\theta , \; y = r\sin\theta .$$

ω	0.1	0.2	- - - .
x			
y			

5. $$GM = \left.\frac{1}{|G(j\omega)H(j\omega)|}\right|_{\omega = \omega_{pc}}$$

$$GM\Big|_{dB} = 20 \log GM$$

6. $PM = 180° + \phi_{gc}$ [For calculating ϕ_{gc}, find the point where ~~polar plot intersects~~ unity circle

Q) Draw the polar plot for the following transfer function. ②

$$G(s) = \frac{1}{s(s+1)(2s+1)}$$

Sol) put $s = j\omega$ Method I

$$G(j\omega) = \frac{1}{j\omega(1+j\omega)(1+2j\omega)}$$

$$|G(j\omega)| = \frac{1}{\omega\sqrt{1+\omega^2}\sqrt{1+4\omega^2}}$$

$$\angle G(j\omega) = -90^\circ - \tan^{-1}\omega - \tan^{-1}2\omega$$

Method - I

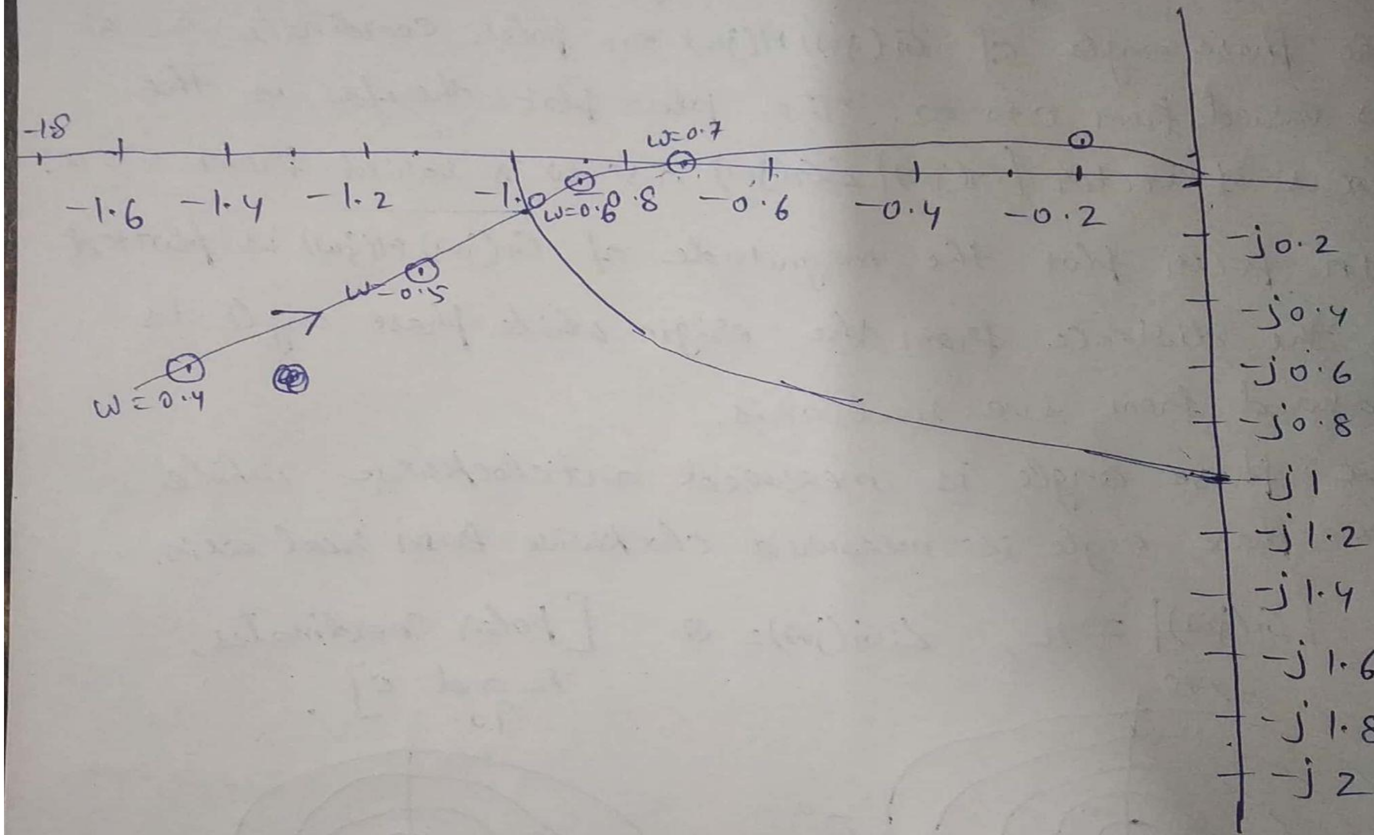
ω	0.1	0.2	0.3	0.4	0.5	0.6	0.7	1.0
$\lvert G(j\omega)\rvert$	9.75	4.55	2.7	1.8	1.26	0.9	0.7	0.3
$\angle G(j\omega)$	-107°	-123.11	-137.66	-150.46	-161.56	-171	-179.5	-198

convert polar coordinates to rectangular coordinates

$x = |G(j\omega)| \cos \angle G(j\omega)$

$y = |G(j\omega)| \sin \angle G(j\omega)$

ω	0.1	0.2	0.3	0.4	0.5	0.6	0.7	1
x	-2.8	-2.4	-1.99	-1.5	-1.19	-0.88	-0.69	-0.28
y	-9.32	-3.8	-1.81	-0.8	-0.39	-0.14	-0.006	0.09



-1.8
-1.6
-1.4
-1.2
-1.0
-0.8
-0.6
-0.4
-0.2
ω=0.7
ω=0.6
ω=0.5
ω=0.4
-j0.2
-j0.4
-j0.6
-j0.8
-j1
-j1.2
-j1.4
-j1.6
-j1.8
-j2

$$\text{Gain Margin} = \frac{1}{|G(j\omega_{pc})|} = \frac{1}{0.7}$$

$$= 1.428$$

$$\text{Phase Margin} = 180^\circ + \phi_{gc}$$

$$= 180 - 168^\circ$$

$$= 12^\circ$$