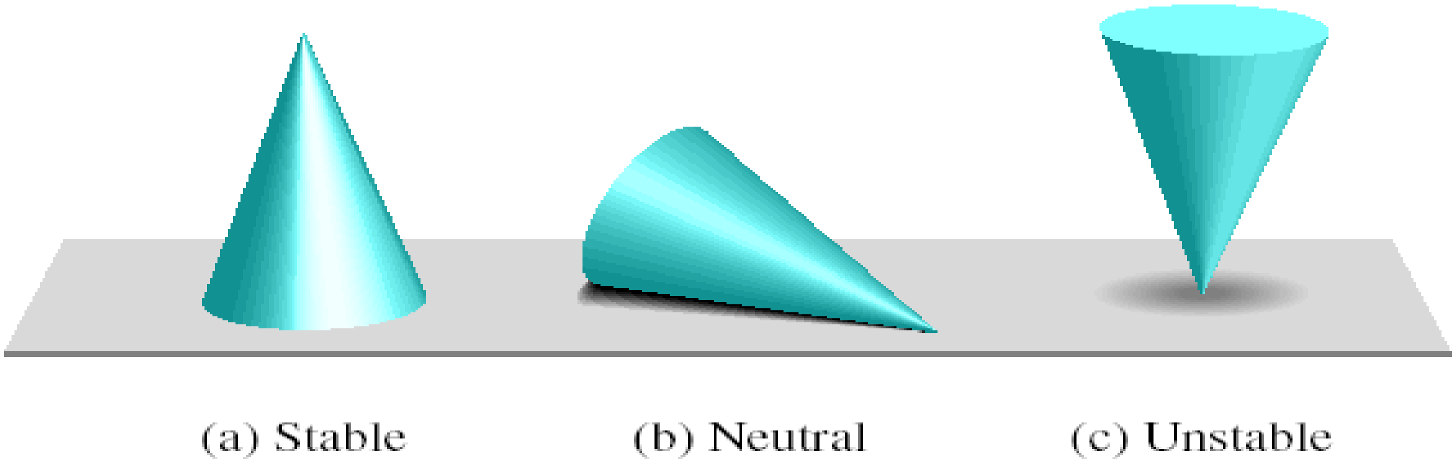
Stability

Specific Objectives

- Appreciate the importance of stability
- Analyze different types of stability
- Apply Routh's stability criterion for stability analysis and solve the numerical.

"Concept of Stability"



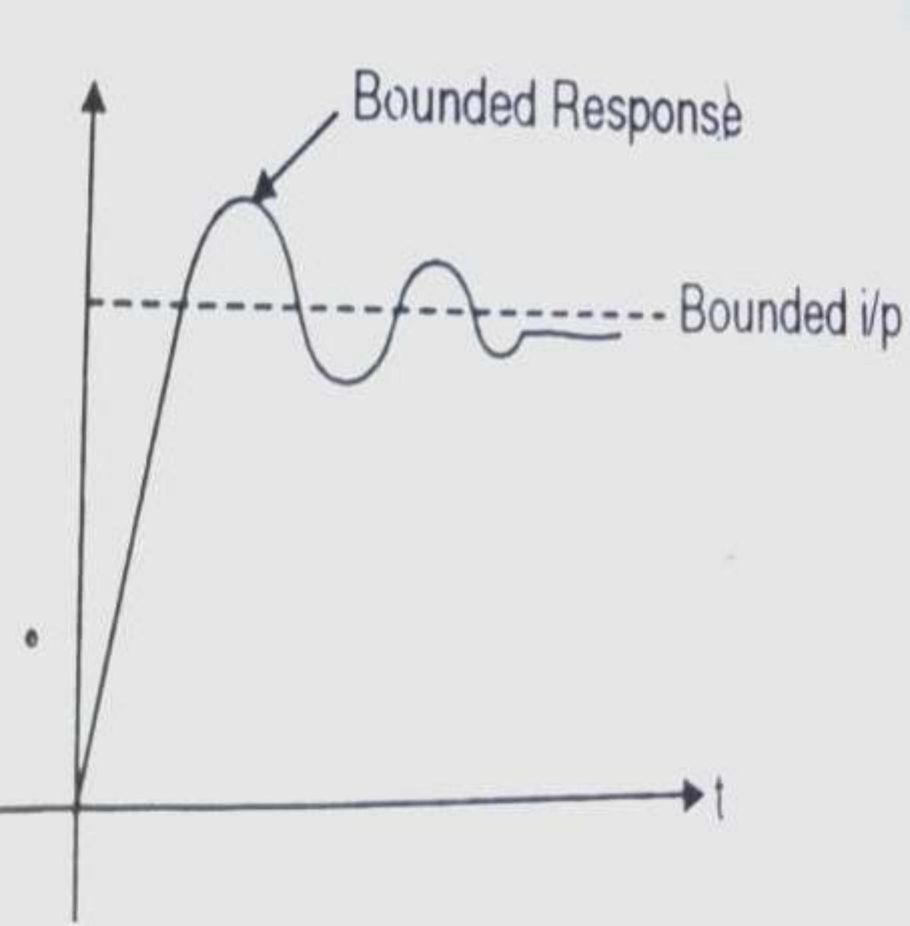
The concept of stability can be illustrated by a cone placed on a plane horizontal surface.

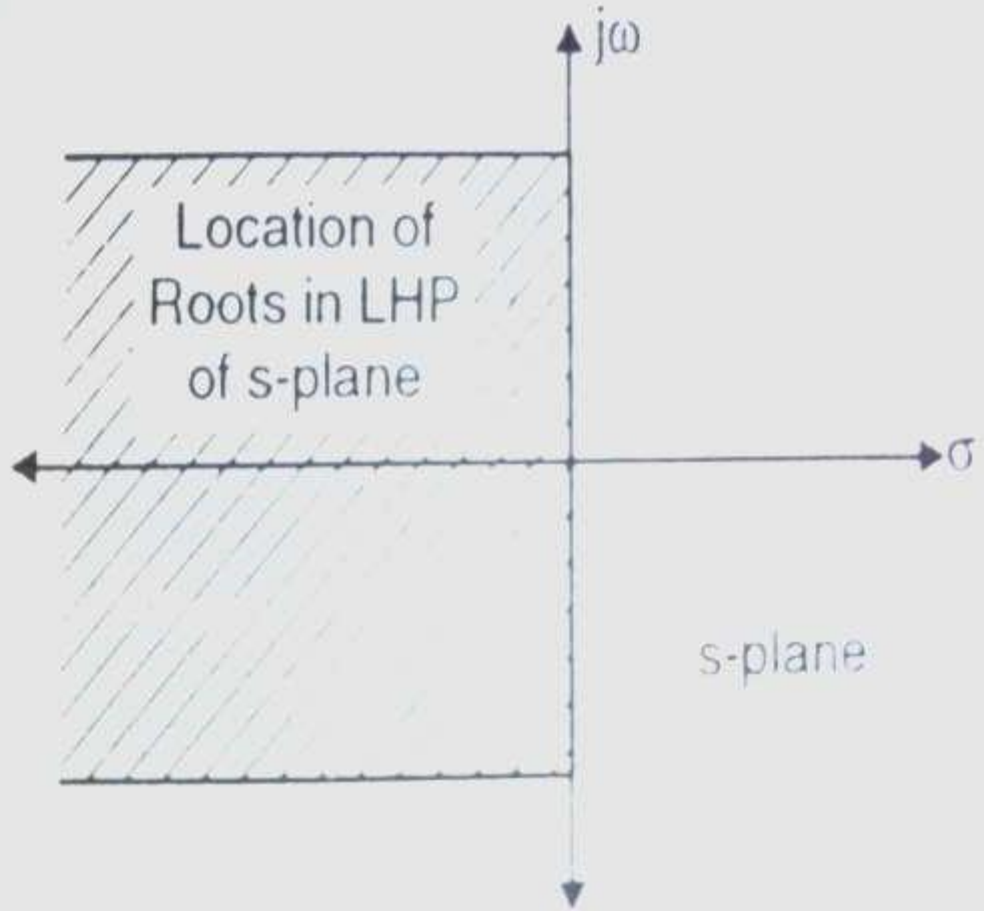
Stable System

A linear time invarient system is stable if following conditions are satisfied:

- A bounded input is given to the system, the response of the system is bounded and controllable.
- In the absence of the inputs, the output should tend to zero as time increases.

Stable System





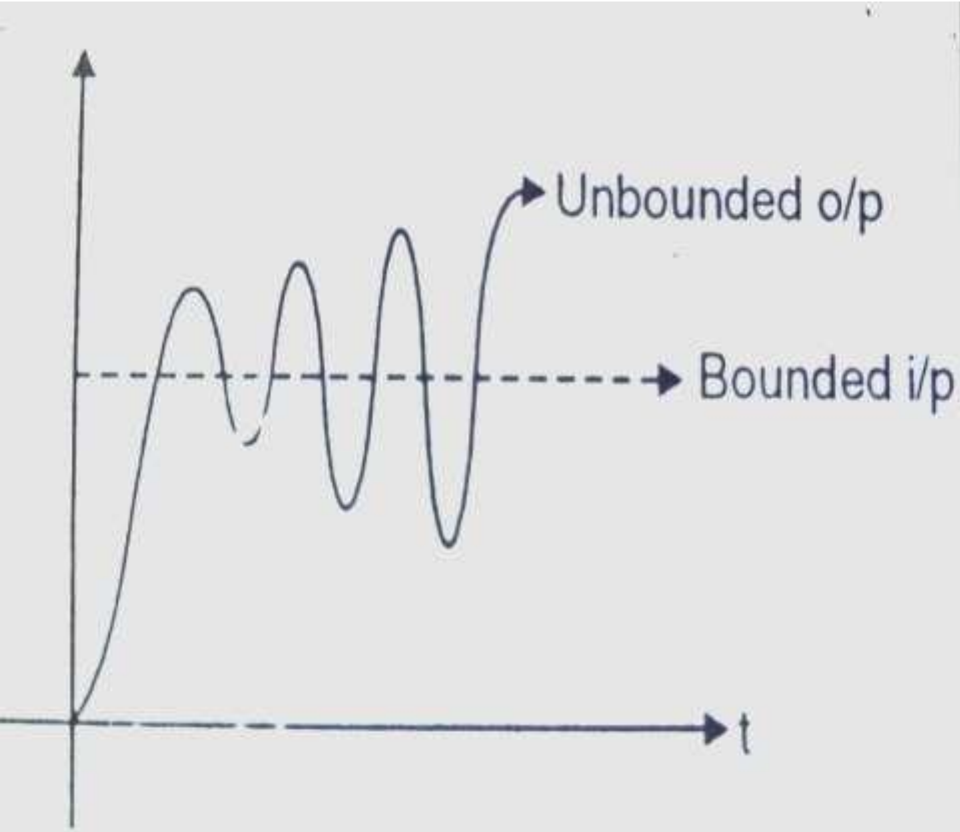
Bounded i/p bounded o/p for stable system

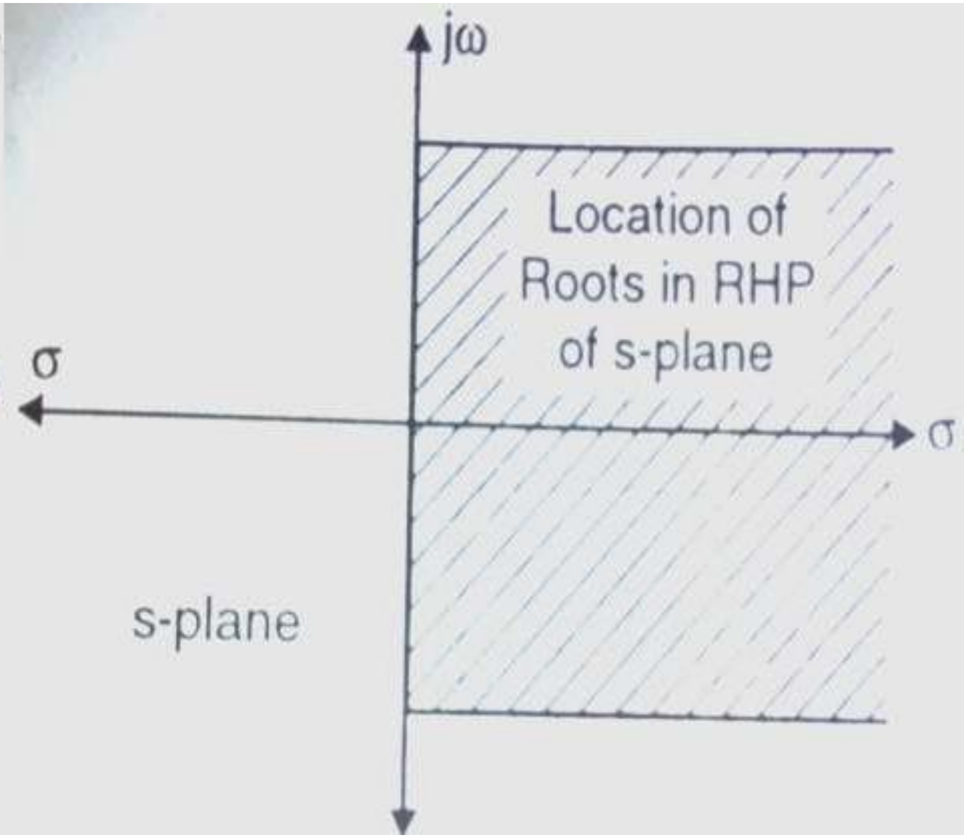
Location of roots for stable system

Unstable System

- A linear time invarient system comes under the class of unstable system if the system is excited by a bounded input, response is unbounded.
- This means once any input is given system output goes on increasing & designer does not have any control on it

Unstable System





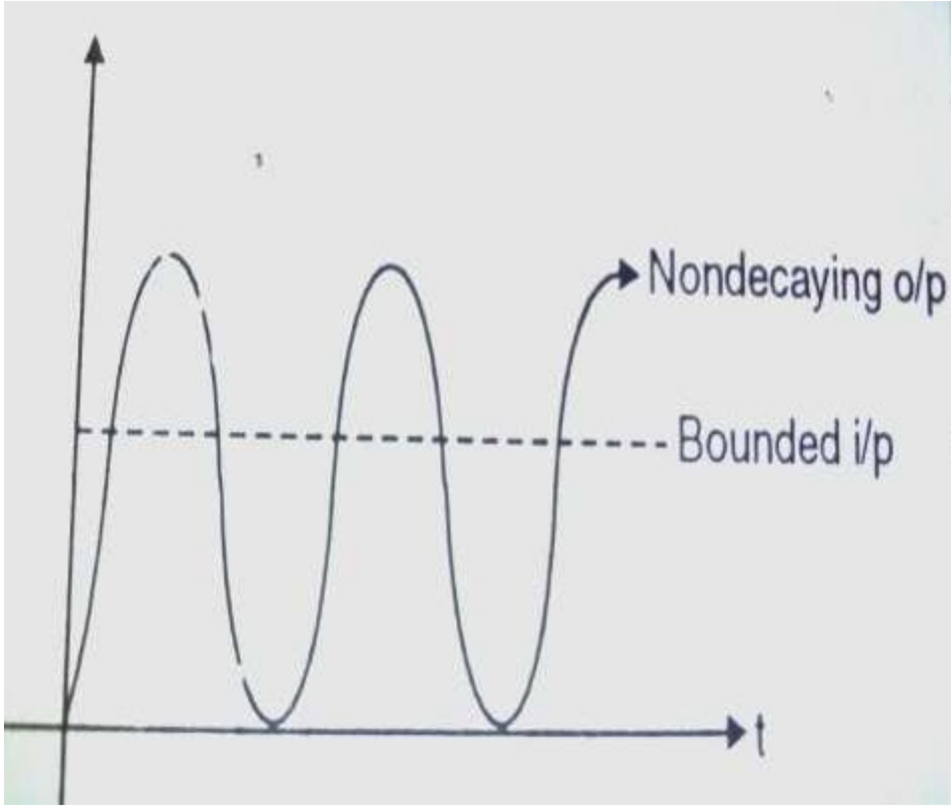
Bounded i/p Unbounded o/p for unstable system

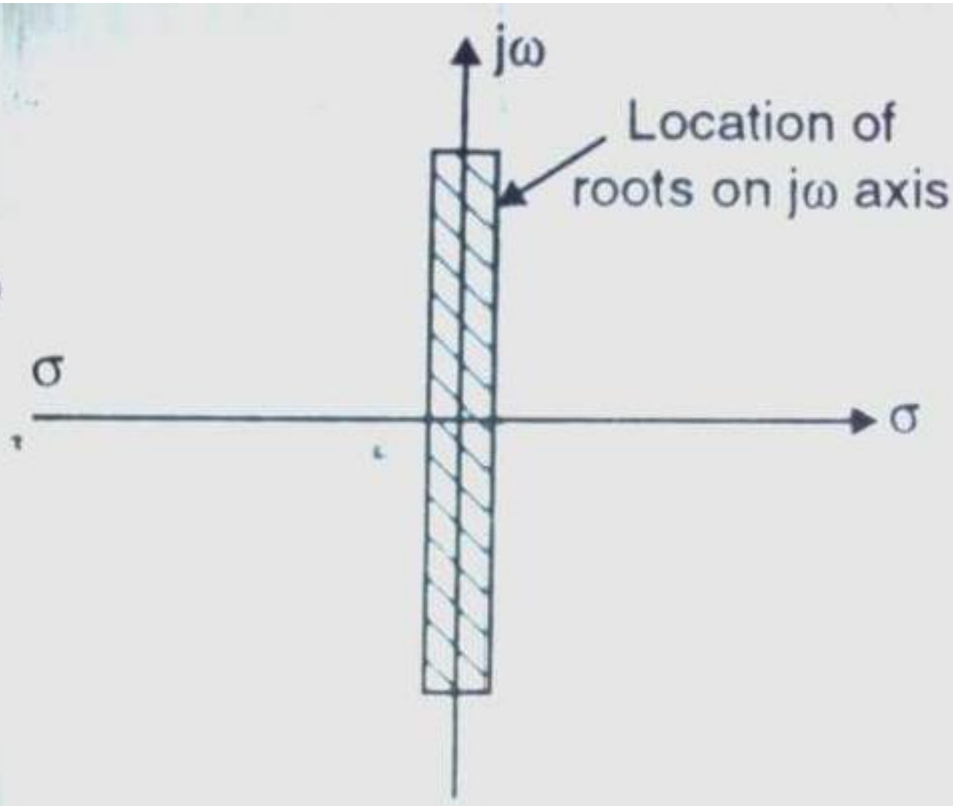
Location of roots for unstable system

Critically Stable System

- When the input is given to a linear time invarient system, for critically stable systems the output does not go on increasing infinitely nor does it go to zero as time increases.
- The output usually oscillates in a finite range or remains steady at some value.
- Such systems are not stable as their response does not decay to zero. Neither they are defined as unstable because their output does not go on increasing infinitely.

Critically stable System





Bounded i/p & o/p response for critically stable system

Location of roots for critically stable system

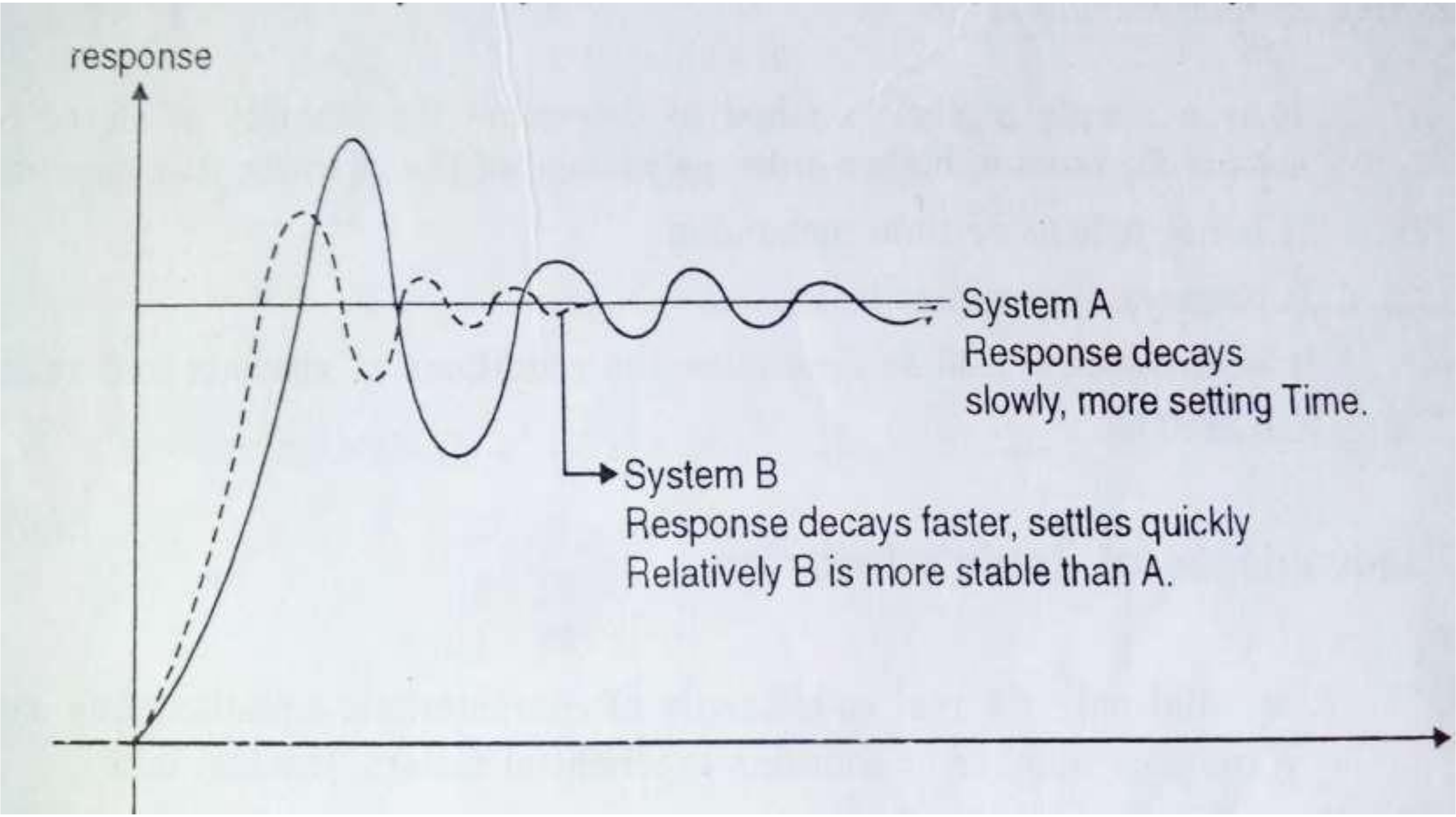
Relative Stability

- A system may be absolutely stable i.e. it may have passed the Routh stability test.
- As a result their response decays to zero under zero input conditions.
- The ratio at which these decay to zero is important to check the concept of “Relative stability”

Relative Stability

- When the poles are located far away from jw axis in LHP of s-plane, the response decays to zero much faster, as compared to the poles close to jw-axis.
- The more the poles are located far away from jw-axis the more is the system relatively stable.

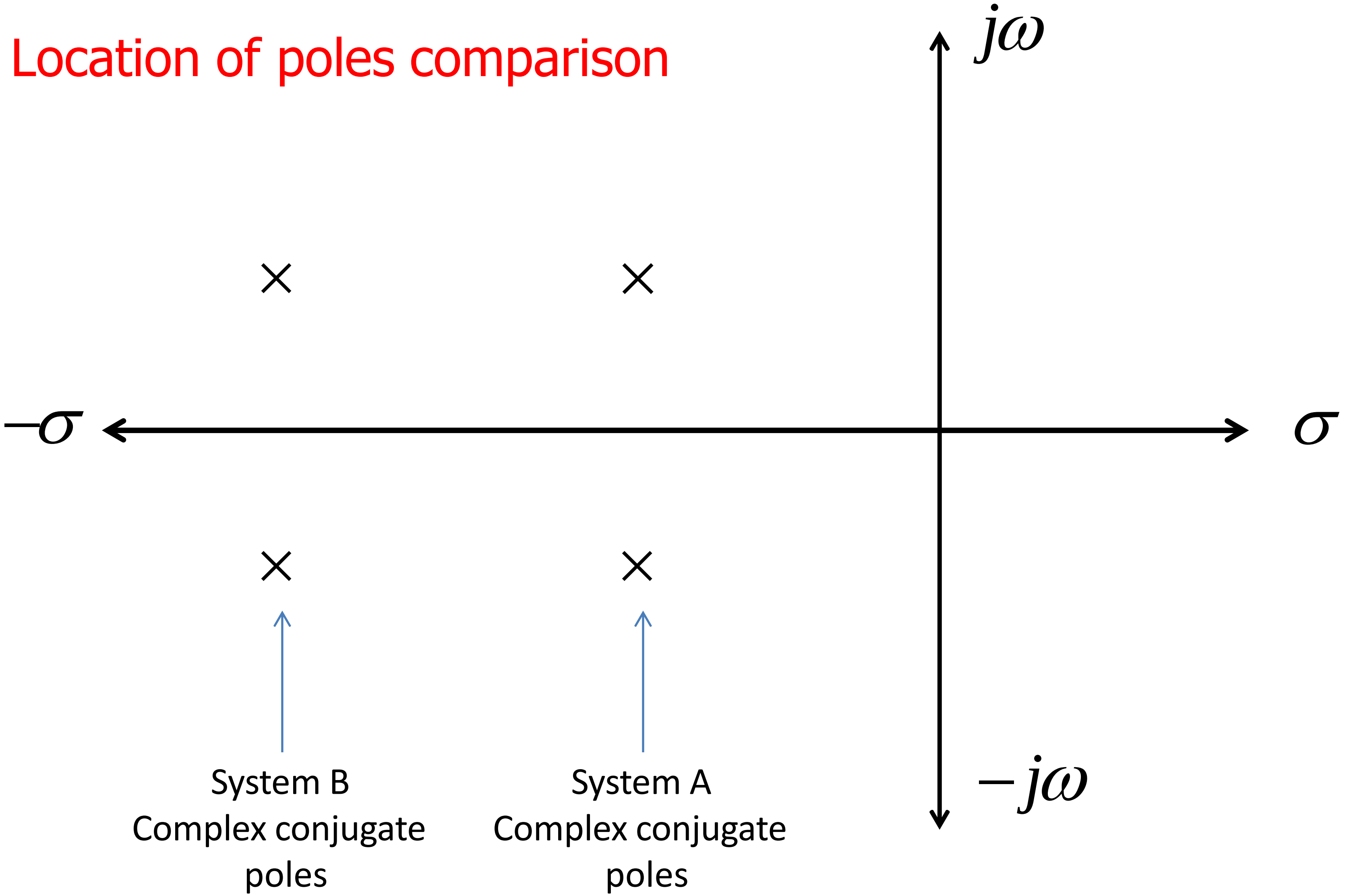
Relative Stability



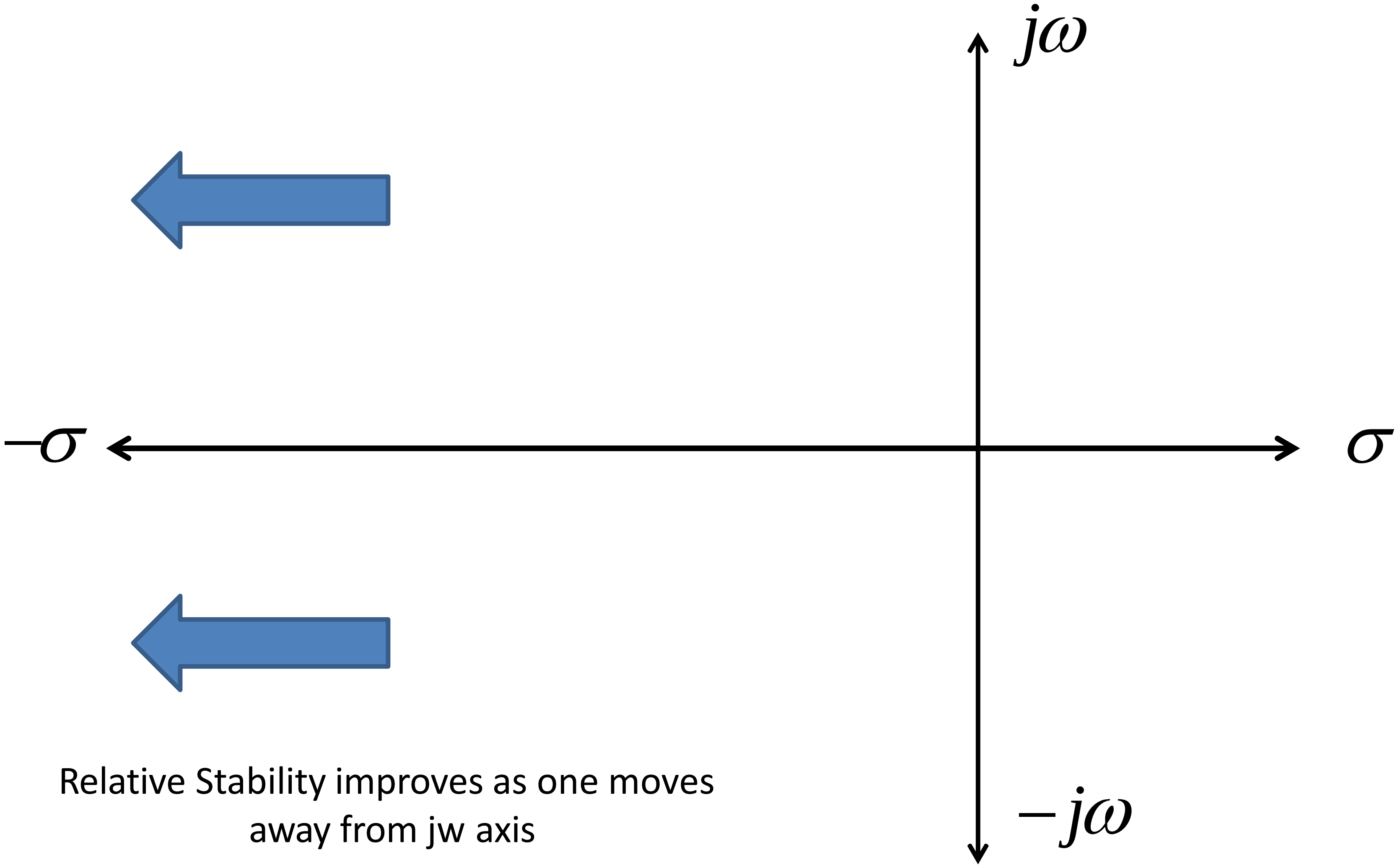
Response comparison

Relative Stability

Location of poles comparison



Relative Stability



Relative Stability improves as one moves away from jw axis

Routh’s Stability Criterion

For the transfer function;

$$\frac{C(s)}{R(s)} = \frac{b_0 s^m + b_1 s^{m-1} + \ldots\ldots\ldots\ldots\ldots + b_m}{\boxed{a_0 s^n + a_1 s^{n-1} + \ldots\ldots\ldots\ldots\ldots + a_n}}$$

In this criterion, the coefficients of denominator are arranged in an Array called “Routh’s Array”;

$$a_0 s^n + a_1 s^{n-1} + \ldots\ldots\ldots\ldots\ldots + a_n = 0$$

The coefficients of s^n and s^{n-1} row are directly written from the given equation.

$$a_0 s^n + a_1 s^{n-1} + \ldots\ldots\ldots\ldots\ldots + a_n = 0$$

The Routh's array as below;

$$\begin{array}{c|cccc}
s^n & a_0 & a_2 & a_4 & a_6 \\
s^{n-1} & a_1 & a_3 & a_5 & a_7 \\
s^{n-2} & b_1 & & & \\
s^{n-3} & & & & \\
 & & & & \\
s^0 & & & &
\end{array}$$

For next row i.e. s^{n-2} ;

$$b_1 = \frac{a_1.a_2 - a_3 a_0}{a_1}$$

The coefficients of s^n and s^{n-1} row are directly written from the given equation.

$$a_0 s^n + a_1 s^{n-1} + \ldots\ldots\ldots\ldots\ldots + a_n = 0$$

The Routh's array as below;

s^n	a_0	a_2	a_4	a_6
s^{n-1}	a_1	a_3	a_5	a_7
s^{n-2}	b_1	b_2		
s^{n-3}				
s^0				

For next row i.e. s^{n-2} ;

$$b_1 = \frac{a_1.a_2 - a_3 a_0}{a_1}$$

$$b_2 = \frac{a_1.a_4 - a_0.a_5}{a_1}$$

The coefficients of s^n and s^{n-1} row are directly written from the given equation.

$$a_0 s^n + a_1 s^{n-1} + \ldots\ldots\ldots\ldots\ldots + a_n = 0$$

The Routh's array as below;

$$\begin{array}{c|cccc}
s^n & a_0 & a_2 & a_4 & a_6 \\
s^{n-1} & a_1 & a_3 & a_5 & a_7 \\
s^{n-2} & b_1 & b_2 & b_3 & \\
s^{n-3} & & & & \\
\vdots & & & & \\
s^0 & & & &
\end{array}$$

For next row i.e. s^{n-2} ;

$$b_1 = \frac{a_1 . a_2 - a_3 a_0}{a_1}$$

$$b_2 = \frac{a_1 . a_4 - a_0 a_5}{a_1}$$

$$b_3 = \frac{a_1 . a_6 - a_0 . a_7}{a_1}$$

Now the same technique is used, for the next row i.e. s^{n-3} row, but only previous two rows are used i.e. s^{n-1} and s^{n-2}

The Routh's array as below;

$$
\begin{array}{c|cccc}
s^{n} & a_0 & a_2 & a_4 & a_6 \\
s^{n-1} & a_1 & a_3 & a_5 & a_7 \\
s^{n-2} & b_1 & b_2 & b_3 & \\
s^{n-3} & c_1 & & & \\
\vdots & & & & \\
s^{0} & & & &
\end{array}
$$

For next row i.e. s^{n-2} ;

$$b_1 = \frac{a_1 . a_2 - a_3 a_0}{a_1}$$

$$b_2 = \frac{a_1 . a_4 - a_0 . a_5}{a_1}$$

$$b_3 = \frac{a_1 . a_6 - a_0 . a_7}{a_1}$$

For next row i.e. s^{n-3} ;

$$c_1 = \frac{b_1 . a_3 - b_2 a_1}{b_1}$$

Now the same technique is used, for the next row i.e. s^{n-3} row, but only previous two rows are used i.e. s^{n-1} and s^{n-2}

The Routh's array as below;

s^n	a_0	a_2	a_4	a_6
s^{n-1}	a_1	a_3	a_5	a_7
s^{n-2}	b_1	b_2	b_3	
s^{n-3}	c_1	c_2		
s^0				

For next row i.e. s^{n-2} ;

$$b_1 = \frac{a_1.a_2 - a_3 a_0}{a_1}$$

$$b_2 = \frac{a_1.a_4 - a_0.a_5}{a_1}$$

$$b_3 = \frac{a_1.a_6 - a_0.a_7}{a_1}$$

For next row i.e. s^{n-3} ;

$$c_1 = \frac{b_1.a_3 - b_2 a_1}{b_1}$$

$$c_2 = \frac{b_1.a_5 - b_3 a_1}{b_1}$$

- Each column will reduce by one as we move down the array.

- This process is obtained till last row is obtained.

$$
\begin{array}{c|cccc}
s^n & a_0 & a_2 & a_4 & a_6 \\
s^{n-1} & a_1 & a_3 & a_5 & a_7 \\
s^{n-2} & b_1 & b_2 & b_3 & \\
s^{n-3} & c_1 & c_2 & & \\
 & & & & \\
s^0 & a_n & & &
\end{array}
$$

Routh's Criterion

- The necessary & sufficient conditions for a system to be stable is all terms in the first column at Routh's Array should have same sign.
- There should not be any sign change in first column.

Must have same sign

$$
\begin{array}{c|cccc}
s^{n} & a_0 & a_2 & a_4 & a_6 \\
s^{n-1} & a_1 & a_3 & a_5 & a_7 \\
s^{n-2} & b_1 & b_2 & b_3 & \\
s^{n-3} & c_1 & c_2 & & \\
 & & & & \\
s^{0} & a_n & & &
\end{array}
$$

Routh's Criterion

- When there are sign changes in the first column of Routh's array then the system is unstable.
- There are roots in RHP.
- The number of sign changes equal the number of roots in RHP.

$$
\begin{array}{c|cccc}
s^{n} & +a_0 & a_2 & a_4 & a_6 \\
s^{n-1} & +a_1 & a_3 & a_5 & a_7 \\
s^{n-2} & -b_1 & b_2 & b_3 & \\
s^{n-3} & +c_1 & c_2 & & \\
 & & & & \\
s^{0} & a_n & & &
\end{array}
$$

Example 1

$$s^3 + 6s^2 + 12s + 8 = 0$$

$$a_0 \quad a_1 \quad a_2 \quad a_3$$

s^3	1	12
s^2	6	8
s^1	b_1	b_2
s^0	a_n	

s^3	1	12
s^2	6	8
s^1	b_1	b_2
s^0	a_n	

$$b_1 = \frac{a_1.a_2 - a_3.a_0}{a_1}$$

Example 1 cont......

s^3	1	12
s^2	6	8
s^1	b_1	b_2
s^0	a_n	

$$b_1 = \frac{(6\times 12)-(8\times 1)}{6}$$

Example 1 cont……

s^3	1	12	0
s^2	6	8	0
s^1	10.67	b_2	
s^0	a_n		

$$b_2 = \frac{a_1.a_4 - a_0.a_5}{a_1}$$

Example 1 cont……

s^3	1	12	0
s^2	6	8	0
s^1	10.67	b_2	
s^0	a_n		

$$b_2 = \frac{1\times 0 - 6\times 0}{6}$$

s^3	1	12	0
s^2	6	8	0
s^1	10.67	0	
s^0	a_n		

$$b_2 = \frac{10.67 \times 8 - 6 \times 0}{10.67}$$

Example 1 cont......

$$s^3 + 6s^2 + 12s + 8 = 0$$

s^3	1	12	0
s^2	6	8	0
s^1	10.67	0	
s^0	8		

Elements in first column are of same sign

s^3	1	12	0
s^2	6	8	0
s^1	10.67	0	
s^0	8		

As all elements in first column are of same sign. Hence system is stable

Example 2

Comment on stability. $s^4+2s^3+6s^2+10s+3=0$

$$
\begin{array}{c|ccc}
s^4 & 1 & 6 & 3 \\
s^3 & 2 & 10 & 0 \\
s^2 & b_1 & b_2 & \\
s^1 & c_1 & & \\
s^0 & a_n & &
\end{array}
$$

$$b_1 = \frac{a_1.a_2 - a_3.a_0}{a_1}$$

$$b_1 = \frac{(2\times 6)-(1\times 10)}{2}$$

$$b_1 = 1$$

s^4	1	6	3
s^3	2	10	0
s^2	1	b_2	
s^1	c_1		
s^0	a_n		

$$b_2 = \frac{a_1.a_4 - a_0.a_5}{a_1}$$

$$b_2 = \frac{2 \times 3 - 1 \times 0}{2}$$

$$b_2 = 3$$

Example 2 cont....

s^4	1	6	3
s^3	2	10	0
s^2	1	3	
s^1	c_1		
s^0	a_n		

$$c_1 = \frac{b_1.a_3 - b_2 a_1}{b_1}$$

$$c_1 = \frac{1\times 10 - 2\times 3}{1}$$

$$c_1 = 4$$

s^4	1	6	3
s^3	2	10	0
s^2	1	3	
s^1	4		
s^0	a_n		

$$a_n = \frac{c_1.b_2 - b_1c_2}{c_1}$$

$$a_n = \frac{4\times 3 - 1\times 0}{4}$$

$$a_n = 3$$

Example 2 cont….

s^4	1	6	3
s^3	2	10	0
s^2	1	3	
s^1	4		
s^0	3		

As no sign change in first column; system is stable

Example 3

Comment on stability. $2s^3 + 4s^2 + 4s + 12 = 0$

s^3	2	4	0
s^2	4	12	
s^1	b_1	b_2	
s^0	a_n		

$$b_1 = \frac{a_1.a_2 - a_3.a_0}{a_1}$$

$$b_1 = \frac{(4\times4)-(2\times12)}{4}$$

$$b_1 = -2$$

Example 3 cont....

s^3	2	4	0
s^2	4	12	
s^1	-2	b_2	
s^0	a_n		

$$b_2 = \frac{(4\times 0)-(2\times 0)}{4}$$

$$b_2 = 0$$

Example 3 cont....

s^3	2	4	0
s^2	4	12	
s^1	-2	0	
s^0	a_n		

$$a_n = \frac{(-2\times 12)-(4\times 0)}{-2}$$

$$a_n = 12$$

s^3	2	4	0
s^2	4	12	
s^1	-2	0	
s^0	12		

There are two sign changes +4 to -2 and -2 to +12. Hence two roots are in RHP S-plane and system is unstable

Example 4

Comment on stability. $s^5+2s^4+4s^3+6s^2+2s+5=0$

s^5	1	4	2
s^4	2	6	5
s^3	b_1	b_2	
s^2	c_1	c_2	
s^1	d_1		
s^0	a_n		

$$b_1=\frac{(2\times4)-(6\times1)}{2}$$

$$b_1 = 1$$

$$b_2=\frac{(2\times2)-(5\times1)}{2}$$

$$b_2 = -0.5$$

s^5	1	4	2
s^4	2	6	5
s^3	1	-0.5	
s^2	c_1	c_2	
s^1	d_1		
s^0	a_n		

$$c_1 = \frac{(1\times 6)-(-0.5\times 2)}{1}$$

$$c_1 = 7$$

$$c_2 = \frac{(1\times 5)-(0\times 2)}{1}$$

$$c_2 = 5$$

Example 4 cont.....

s^5	1	4	2
s^4	2	6	5
s^3	1	-0.5	
s^2	7	5	
s^1	d_1		
s^0	a_n		

$$d_1 = \frac{(7\times -0.5)-(5\times 1)}{7}$$

$$d_1 = -1.21$$

Example 4 cont.....

s^5	1	4	2
s^4	2	6	5
s^3	1	-0.5	
s^2	7	5	
s^1	-1.21		
s^0	a_n		

$$a_n = \frac{(5 \times -1.21) - (7 \times 0)}{-1.21}$$

$$a_n = 5$$

Example 4 cont.....

s^5	1	4	2
s^4	2	6	5
s^3	1	-0.5	
s^2	7	5	
s^1	-1.21		
s^0	5		

There are two sign changes +7 to -1.21 and -1.21 to +5. Hence two roots are in RHP S-plane and system is unstable

Example 5

Comment on stability. $s^5+s^4+2s^3+2s^2+3s+5=0$

s^5	1	2	3
s^4	1	2	5
s^3	b_1	b_2	
s^2	c_1	c_2	
s^1	d_1		
s^0	a_n		

$$b_1=\frac{(1\times 2)-(2\times 1)}{1}$$

$$b_1 = 0$$

$$b_2=\frac{(1\times 3)-(5\times 1)}{1}$$

$$b_2 = -2$$

Example 5 cont.....

Comment on stability. $s^5+s^4+2s^3+2s^2+3s+5=0$

s^5	1	2	3
s^4	1	2	5
s^3	0	-2	
s^2	c_1	c_2	
s^1	d_1		
s^0	a_n		

Case I – "When the first of any row is zero and the rest of the row is non-zero."

Routh's Criterion Special Cases

<u>Case I</u> – "When the first of any row is zero and the rest of the row is non-zero." Here the next row cannot be formed as division by 0 will take place.

<u>Method to Overcome:</u> A method to overcome above problem is to replace s by $\frac{1}{z}$ and complete the Routh's test for z.

Example 5 cont.....

Replace s by (1/z)

$$\left(\frac{1}{z}\right)^5 + \left(\frac{1}{z}\right)^4 + 2\left(\frac{1}{z}\right)^3 + 2\left(\frac{1}{z}\right)^2 + 3\left(\frac{1}{z}\right) + 5 = 0$$

Take L.C.M

$$\frac{1 + z + 2z^2 + 2z^3 + 3z^4 + 5z^5}{z^5} = 0$$

$$1 + z + 2z^2 + 2z^3 + 3z^4 + 5z^5 = 0$$

$$5z^5 + 3z^4 + 2z^3 + 2z^2 + z + 1 = 0$$

Use above characteristics equation and complete Routh's Test

Example 5 cont.....

$$
\begin{array}{c|ccc}
z^5 & 5 & 2 & 1 \\
z^4 & 3 & 2 & 1 \\
z^3 & -\frac{4}{3} & -\frac{2}{3} & \\
z^2 & \frac{1}{2} & 1 & \\
z^1 & 2 & & \\
z^0 & 1 & &
\end{array}
$$

There are two sign changes in first column.
Hence two roots are in RHP S-plane and system is unstable

Example 6

Comment on stability. $s^4+6s^3+11s^2+6s+10=0$

s^4	1	11	10
s^3	6	6	0
s^2	b_1	b_2	
s^1	c_1		
s^0	a_n		

$$b_1=\frac{(6\times 11)-(1\times 6)}{6}$$

$$b_1=10$$

$$b_2=\frac{(6\times 10)-(1\times 0)}{6}$$

$$b_1=10$$

Example 6 cont....

s^4	1	11	10
s^3	6	6	0
s^2	10	10	
s^1	c_1		
s^0	a_n		

$$c_1 = \frac{(10\times6)-(10\times6)}{10}$$

$$c_1 = 0$$

s^4	1	11	10
s^3	6	6	0
s^2	10	10	
s^1	0		
s^0	a_n		

$$c_1 = \frac{(10\times6)-(10\times6)}{10}$$

$$c_1 = 0$$

Case II – "When all elements in any one row is zero."

Routh's Criterion Special Cases

<u>Case II</u> – "When all elements in any one row is zero."

<u>Method to Overcome:</u>

- ✓ Here form an auxillary equation with the help of the coefficients of the coefficients of the row just above the row of zeros.
- ✓ Take the derivative of this equation and replace it's coefficients in the present row of zeros.
- ✓ Then proceed for Routh's test.

Example 6 cont....

s^4	1	11	10
s^3	6	6	0
s^2	10	10	
s^1	0		
s^0	a_n		

Here s^1 row breaks down. Hence write auxiliary equation for s^2 .

$$A(s) = 10s^2 + 10$$

(Note each term of next column differs by degree of 2)

Take derivative of auxiliary equation

$$\frac{d}{ds}A(s) = 20s$$

Use these for s row coefficients.

Example 6 cont....

s^4	1	11	10
s^3	6	6	0
s^2	10	10	
s^1	20		
s^0	a_n		

$$a_n = \frac{(20 \times 10) - (10 \times 0)}{20}$$

$$a_n = 10$$

Example 6 cont....

s^4	1	11	10
s^3	6	6	0
s^2	10	10	
s^1	20		
s^0	10		

As no sign change in first column; system is stable

Example 7

Comment on stability. $s^6+3s^5+5s^4+9s^3+8s^2+6s+4=0$

s^6	1	5	8	4
s^5	3	9	6	
s^4	b_1	b_2	b_3	
s^3	c_1	c_2		
s^2	d_1			
s^1	e_1			
s^0	a_n			

$$b_1=\frac{(3\times 5)-(9\times 1)}{3}$$

$$b_1 = 2$$

$$b_2=\frac{(3\times 8)-(6\times 1)}{3}$$

$$b_2 = 6$$

$$b_3=\frac{(3\times 4)-(0\times 1)}{3}$$

$$b_3 = 4$$

Example 7 cont....

s^6	1	5	8	4
s^5	3	9	6	
s^4	2	6	4	
s^3	c_1	c_2		
s^2	d_1			
s^1	e_1			
s^0	a_n			

$$c_1 = \frac{(2\times 9)-(3\times 6)}{2}$$

$$c_1 = 0$$

$$c_2 = \frac{(2\times 6)-(4\times 3)}{2}$$

$$c_2 = 0$$

Example 7 cont....

s^6	1	5	8	4
s^5	3	9	6	
s^4	2	6	4	
s^3	0	0		
s^2	d_1			
s^1	e_1			
s^0	a_n			

Here s^3 row breaks down.
Hence write auxiliary equation for s^4 .

$$A(s) = 2s^4 + 6s^2 + 4$$

(Note each term of next column differs by degree of 2)

Take derivative of auxiliary equation

$$\frac{d}{ds} A(s) = 8s^3 + 12s$$

Use these for s^3 row coefficients.

Example 7 cont....

s^6	1	5	8	4
s^5	3	9	6	
s^4	2	6	4	
s^3	8	12		
s^2	d_1	d_2		
s^1	e_1			
s^0	a_n			

$$d_1 = \frac{(8\times 6)-(12\times 2)}{8}$$

$$d_1 = 3$$

$$d_2 = \frac{(8\times 4)-(0\times 2)}{8}$$

$$d_2 = 4$$

Example 7 cont....

s^6	1	5	8	4
s^5	3	9	6	
s^4	2	6	4	
s^3	8	12		
s^2	3	4		
s^1	e_1			
s^0	a_n			

$$e_1 = \frac{(3\times 12) - (8\times 4)}{3}$$

$$e_1 = 4$$

Example 7 cont....

s^6	1	5	8	4
s^5	3	9	6	
s^4	2	6	4	
s^3	8	12		
s^2	3	4		
s^1	4			
s^0	a_n			

$$a_n = \frac{(4\times 4)-(3\times 0)}{4}$$

$$a_n = 4$$

s^6	1	5	8	4
s^5	3	9	6	
s^4	2	6	4	
s^3	8	12		
s^2	3	4		
s^1	4			
s^0	4			

As no sign change in first column; system is stable

Example 8

Comment on stability. $s^6+2s^5+8s^4+12s^3+20s^2+16s+16=0$

s^6	1	8	20	16
s^5	2	12	16	
s^4	b_1	b_2	b_3	
s^3	c_1	c_2		
s^2	d_1			
s^1	e_1			
s^0	a_n			

$$b_1=\frac{(2\times 8)-(12\times 1)}{2}$$

$$b_1 = 2$$

$$b_2=\frac{(2\times 20)-(16\times 1)}{2}$$

$$b_2 = 12$$

$$b_3=\frac{(2\times 16)-(0\times 1)}{2}$$

$$b_3 = 16$$

Example 8 cont.....

s^6	1	8	20	16
s^5	2	12	16	
s^4	2	12	16	
s^3	c_1	c_2	c_3	
s^2	d_1			
s^1	e_1			
s^0	a_n			

$$c_1 = \frac{(2\times 12)-(2\times 12)}{2}$$

$$c_1 = 0$$

$$c_2 = \frac{(2\times 16)-(2\times 16)}{2}$$

$$c_2 = 0$$

$$c_3 = \frac{(2\times 0)-(2\times 0)}{2}$$

$$c_3 = 0$$

Example 8 cont.....

s^6	1	8	20	16
s^5	2	12	16	
s^4	2	12	16	
s^3	0	0	0	
s^2	d_1			
s^1	e_1			
s^0	a_n			

Here s^3 row breaks down.
Hence write auxiliary equation for s^4 .

$$A(s) = 2s^4 + 12s^2 + 16$$

(Note each term of next column differs by degree of 2)

Take derivative of auxiliary equation

$$\frac{d}{ds}A(s) = 8s^3 + 24s$$

Use these for s^3 row coefficients.

Example 8 cont.....

s^6	1	8	20	16
s^5	2	12	16	
s^4	2	12	16	
s^3	8	24	0	
s^2	d_1	d_2		
s^1	e_1			
s^0	a_n			

$$d_1 = \frac{(8\times12)-(24\times2)}{8}$$

$$d_1 = 6$$

$$d_2 = \frac{(8\times16)-(0\times2)}{8}$$

$$d_2 = 16$$

s^6	1	8	20	16
s^5	2	12	16	
s^4	2	12	16	
s^3	8	24	0	
s^2	6	16		
s^1	e_1			
s^0	a_n			

$$e_1 = \frac{(6\times 24)-(8\times 16)}{6}$$

$$e_1 = 2.67$$

Example 8 cont.....

s^6	1	8	20	16
s^5	2	12	16	
s^4	2	12	16	
s^3	8	24	0	
s^2	6	16		
s^1	2.67			
s^0	a_n			

$$a_n = \frac{(2.67 \times 16) - (6 \times 0)}{2.67}$$

$$a_n = 16$$

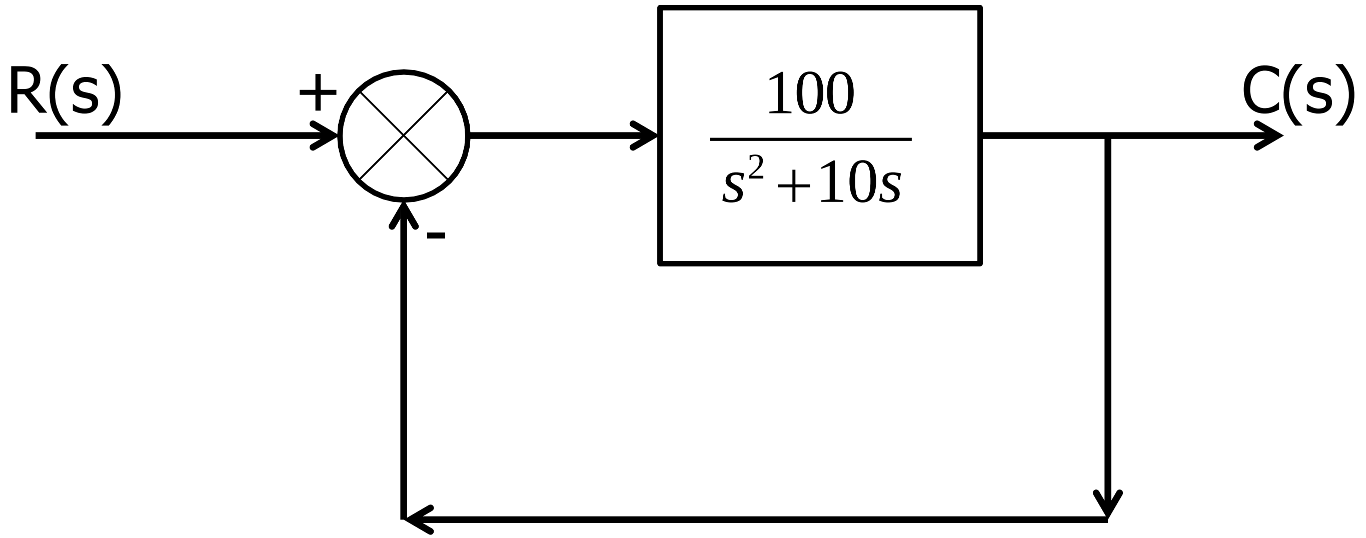
Example 8 cont.....

s^6	1	8	20	16
s^5	2	12	16	
s^4	2	12	16	
s^3	8	24	0	
s^2	6	16		
s^1	2.67			
s^0	16			

As no sign change in first column; system is stable

Example 9

Problem: Using routh's criteria find the stability for given figure.



Example 9 cont......

$$G(s) = \frac{100}{s^2 + 10s} \qquad \mathrm{H(s)} = 1$$

$$\therefore \frac{\mathrm{C(s)}}{R(\mathrm{s})} = \frac{G(\mathrm{s})}{1 + G(\mathrm{s})H(\mathrm{s})}$$

$$\therefore \frac{\mathrm{C(s)}}{R(\mathrm{s})} = \frac{\dfrac{100}{s^2 + 10s}}{1 + \dfrac{100}{s^2 + 10s}} = \frac{\dfrac{100}{s^2 + 10s}}{\dfrac{s^2 + 10s + 100}{s^2 + 10s}} = \frac{100}{s^2 + 10s + 100}$$

Characteristics equation is the denominator of the CLTF

Characteristics equation $\qquad s^2 + 10s + 100 = 0$

$$s^2 + 10s + 100 = 0$$

s^2	1	100
s^1	10	
s^0	a_n	

$$a_n = \frac{(10 \times 100) - (1 \times 0)}{10}$$

$$a_n = 100$$

$$s^2 + 10s + 100 = 0$$

s^2	1	100
s^1	10	
s^0	100	

As no sign change in first column; system is stable

Example 10

Comment on stability. $s^6+3s^5+4s^4+6s^3+5s^2+3s+2=0$

s^6	1	4	5	2
s^5	3	6	3	
s^4	b_1	b_2	b_3	
s^3	c_1	c_2		
s^2	d_1			
s^1	e_1			
s^0	a_n			

$$b_1=\frac{(3\times 4)-(1\times 6)}{3}$$

$$b_1 = 2$$

$$b_2=\frac{(3\times 5)-(1\times 3)}{3}$$

$$b_2 = 4$$

$$b_3=\frac{(3\times 2)-(0\times 1)}{3}$$

$$b_3 = 2$$

Example 10 cont.....

s^6	1	4	5	2
s^5	3	6	3	
s^4	2	4	2	
s^3	c_1	c_2	c_3	
s^2	d_1			
s^1	e_1			
s^0	a_n			

$$c_1 = \frac{(2\times 6)-(3\times 4)}{2}$$

$$c_1 = 0$$

$$c_2 = \frac{(2\times 3)-(2\times 3)}{2}$$

$$c_2 = 0$$

$$c_3 = \frac{(2\times 0)-(2\times 0)}{2}$$

$$c_3 = 0$$

s^6	1	4	5	2
s^5	3	6	3	
s^4	2	4	2	
s^3	0	0	0	
s^2	d_1	d_2		
s^1	e_1			
s^0	a_n			

Here s^3 row breaks down.
Hence write auxiliary equation for s^4 .

$$A(s) = 2s^4 + 4s^2 + 2$$

$$A(s) = s^4 + 2s^2 + 1$$

(Note each term of next column differs by degree of 2)

Take derivative of auxiliary equation

$$\frac{d}{ds} A(s) = 4s^3 + 4s$$

Use these for s^3 row coefficients.

s^6	1	4	5	2
s^5	3	6	3	
s^4	2	4	2	
s^3	4	4	0	
s^2	d_1	d_2		
s^1	e_1			
s^0	a_n			

$$d_1 = \frac{(4\times4)-(4\times2)}{4}$$

$$d_1 = 2$$

$$d_2 = \frac{(4\times2)-(0\times2)}{4}$$

$$d_2 = 2$$

Example 10 cont.....

s^6	1	4	5	2
s^5	3	6	3	
s^4	2	4	2	
s^3	4	4	0	
s^2	2	2		
s^1	e_1	e_2		
s^0	a_n			

$$e_1 = \frac{(2\times4)-(2\times4)}{2}$$

$$e_1 = 0$$

$$e_2 = \frac{(2\times0)-(0\times4)}{2}$$

$$e_2 = 0$$

Example 10 cont.....

s^6	1	4	5	2
s^5	3	6	3	
s^4	2	4	2	
s^3	4	4	0	
s^2	2	2		
s^1	0	0		
s^0	a_n			

Here s^1 row breaks down. Hence write auxiliary equation for s^2 .

$$A(s) = 2s^2 + 2$$

(Note each term of next column differs by degree of 2)

Take derivative of auxiliary equation

$$\frac{d}{ds} A(s) = 4s$$

Use these for s^1 row coefficients.

Example 10 cont......

s^6	1	4	5	2
s^5	3	6	3	
s^4	2	4	2	
s^3	4	4	0	
s^2	2	2		
s^1	4	0		
s^0	a_n			

$$a_n = \frac{(4\times 2)-(2\times 0)}{4}$$

$$a_n = 2$$

s^6	1	4	5	2
s^5	3	6	3	
s^4	2	4	2	
s^3	4	4	0	
s^2	2	2		
s^1	4	0		
s^0	2			

As no sign change in first column; system is stable

Application of Routh's Criterion

- ✓ The gain is kept in terms of k and Routh's array is solved to find k for stable operation.

Example 11

Determine the range of k for stable system. $s^4 + 5s^3 + 5s^2 + 4s + k = 0$

$$
\begin{array}{c|ccc}
s^4 & 1 & 5 & k \\
s^3 & 5 & 4 & 0 \\
s^2 & 4.2 & k & \\
s^1 & \dfrac{16.8-5k}{4.2} & & \\
s^0 & k & &
\end{array}
$$

Example 11 cont......

For stability all elements of first column 1 should be positive

i.e. $k > 0$ For s^0 row ------------------------(1)

and $\dfrac{16.8 - 5k}{4.2} > 0$ For s^1 row

i.e. $16.8 > 5k$ or $k < \dfrac{16.8}{5}$

$k < 3.36$ ------------------------(2)

Thus combining equations (1) and (2), $0 < k < 3.36$

This is the range of k stable operation.

Example 12

Determine the range of k for stable system. $s^4 + 4s^3 + 4s^2 + 3s + k = 0$

s^4	1	4	k
s^3	4	3	0
s^2	$\frac{13}{4}$	k	
s^1	$\frac{39-16k}{13}$		
s^0	k		

Example 12 cont......

For stability,

i.e. $k > 0$ and $\dfrac{39-16k}{13} > 0$

i.e. $39-16k > 0$ or $k < \dfrac{39}{16}$

$$k < 2.43$$

Thus $0 < k < 2.43$

This is the range of k stable operation.

Example 13

Determine the range of k for stable system. $s^3 + s^2(2+k) + 30sk + 200k = 0$

s^3	1	$30k$
s^2	$2+k$	$200k$
s^1	$\dfrac{30k(2+k)-200k}{(2+k)}$	
s^0	k	

Example 13 cont……

For stability,

i.e. $k > 0$

i.e. $k + 2 > 0$

$$k > -2$$

and $\dfrac{30k(k+2) - 200k}{2+k} > 0$

$$k > 4.67$$

Thus $4.67 < k < \infty$

This is the range of k stable operation.

Advantages of Routh's Criterion

- It is a simple algebraic method to determine the stability of closed loop without solving for roots of higher order polynomial of the characteristics equation.
- It is not tedious or time consuming.
- It progress systematically.
- It is frequently used to determine the conditions of absolute & relative stability of a system.
- It can determine range of k for stable operation.

Disadvantages of Routh's Criterion

- It is valid only for real coefficients of characteristics equation. Any coefficient that is a complex number or contains exponential factors, the test fails.
- It is applicable only to the linear systems.
- Exact location of poles is not known.
- Only idea is obtained about stability. A method to stabilize the system is not suggested.

Thank You

Dr.Anuj Jain