ECE305

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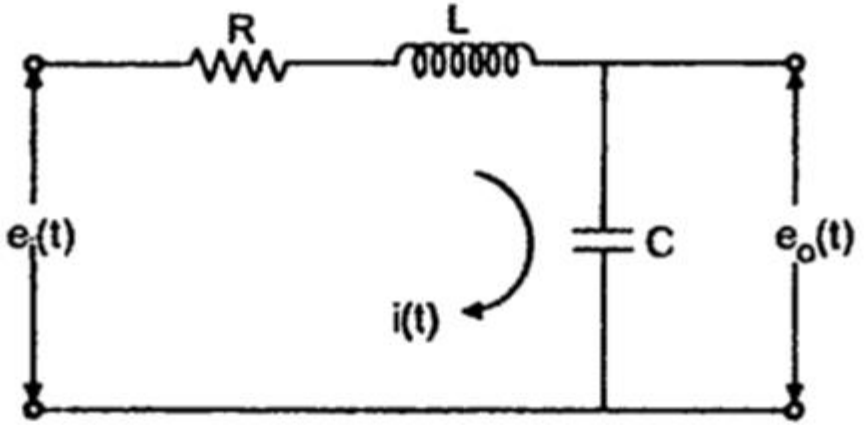
Laplace Transform of Electrical Network

In the use of Laplace in electrical systems, it is always easy to redraw the system by finding Laplace transform of the given network. Electrical network mostly consists of the parameters R, L and C. The various expressions related to these parameters in time domain and Laplace domain are given in the table below.

Element	Time domain expression for voltage	Laplace domain expression for voltage	Laplace domain behaviour
Resistance R	$i(t) \times R$	$I(s)R$	R
Inductance L	$L \frac{d\,i(t)}{dt}$	$sLI(s)$	sL
Capacitance C	$\frac{1}{C}\int i(t)\,dt$	$\frac{1}{sC} I(s)$	$\frac{1}{sC}$

Example : *Find out the T.F. of the given network.*



Laplace transform of $\int F(t)\,dt = \frac{F(s)}{s}$, ... neglecting initial conditions

and Laplace transform of $\frac{df(t)}{dt} = sF(s)$ ... neglecting initial conditions

Take Laplace transform,

$$\therefore \quad E_i(s) = I(s)\left[R + sL + \frac{1}{sC}\right]$$

$$\frac{I(s)}{E_i(s)} = \frac{1}{\left[R + sL + \frac{1}{sC}\right]} \qquad \text{... (2)}$$

Now $$e_o(t) = \frac{1}{C}\int i\,dt \qquad \text{... (3)}$$

$$\therefore \quad E_o(s) = \frac{1}{sC} I(s)$$

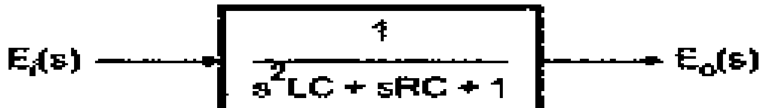
$$\therefore \quad I(s) = sCE_o(s) \qquad \text{... (4)}$$

Substituting value of I(s) in equation (2),

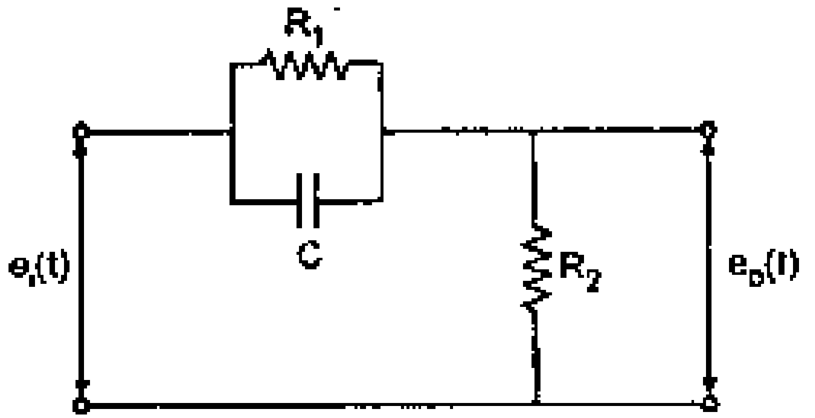
$$\therefore \quad \frac{sCE_o(s)}{E_i(s)} = \frac{1}{\left[R + sL + \frac{1}{sC}\right]}$$

$$\therefore \quad \boxed{\frac{E_o(s)}{E_i(s)} = \frac{1}{sC\left[R + sL + \frac{1}{sC}\right]} = \frac{1}{RsC + s^2 LC + 1}}$$

So we can represent the system as in the Fig. 3.6.



Example *Obtain the transfer function of the lead network shown in the Fig.*



The parallel combination of R_1 and $\frac{1}{sC}$ gives impedance of,

$$Z = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{R_1 \times \frac{1}{sC}}{R_1 + \frac{1}{sC}} = \frac{R_1}{1 + s R_1 C}$$

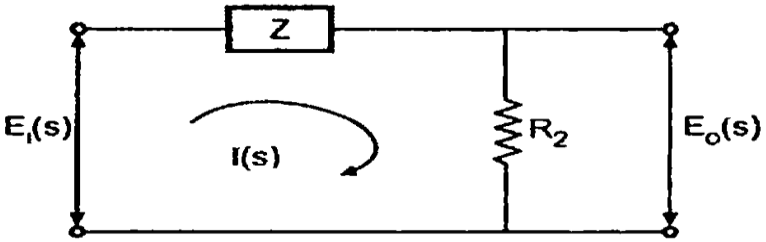


Fig. 3.17

Applying KVL to the circuit,

$$E_i(s) = Z\,I(s) + I(s)\,R_2 \qquad \ldots (1)$$

$$E_o(s) = I(s)\,R_2 \qquad \ldots (2)$$

$$\therefore \quad I(s) = \frac{E_o(s)}{R_2} \qquad \text{from (2)}$$

Substituting in (1) we get,

$$E_i(s) = I(s)\,[Z + R_2] = \frac{E_o(s)}{R_2}[Z + R_2]$$

$$\therefore \quad \frac{E_o(s)}{E_i(s)} = \frac{R_2}{Z + R_2}$$

Substituting Z, T. F. $= \dfrac{R_2}{\dfrac{R_1}{1 + s R_1 C} + R_2} = \dfrac{R_2 (1 + s R_1 C)}{R_1 + R_2 (1 + s R_1 C)}$

$$= \frac{s R_1 R_2 C + R_2}{R_1 + s R_1 R_2 C + R_2} = \frac{s + \alpha}{s + \beta}$$

where $\alpha = \dfrac{1}{R_1 C}, \quad \beta = \dfrac{(R_1 + R_2)}{R_1 R_2 C}$

This circuit is also called **lead compensator.**

Example : *The transfer function of a system is given by,*

$$T(s) = \frac{K(s+6)}{s\,(s+2)\,(s+5)\,(s^2+7s+12)}$$

Determine i) Poles ii) Zeros iii) Characteristic equation and iv) Pole-zero plot in s-plane

Solution :

i) Poles are the roots of the equation obtained by equating denominator to zero i.e. roots of,

$$s(s+2)(s+5)(s^2+7s+12) = 0$$

i.e. $s(s+2)(s+5)(s+3)(s+4) = 0$

So there are 5 poles located at $s = 0, -2, -5, -3$ and -4

ii) Zeros are the roots of the equation obtained by equating numerator to zero i.e. roots of $K(s+6) = 0$

i.e. $s = -6$

There is only one zero.

iii) Characteristic equation is one, whose roots are the poles of the transfer function. So it is,

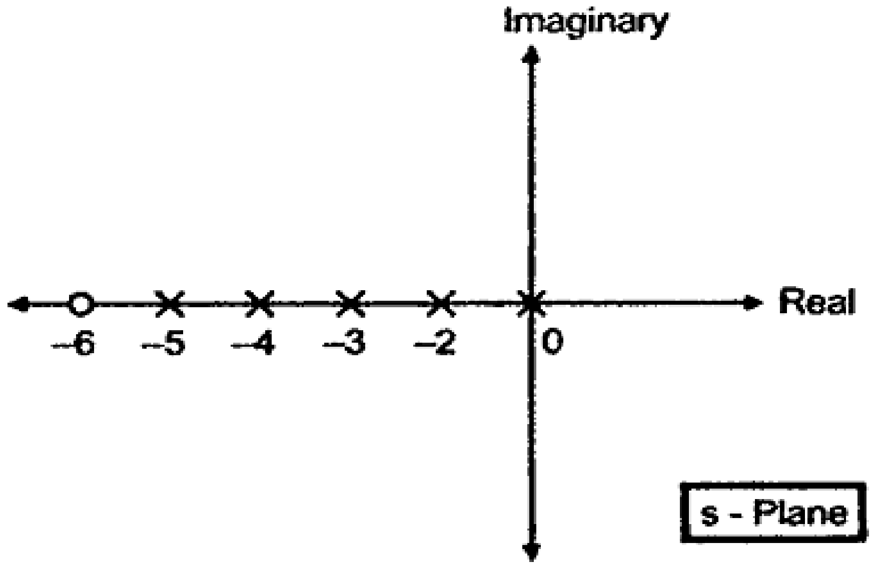
$$s(s+2)(s+5)(s^2+7s+12) = 0$$

i.e. $s(s^2+7s+10)(s^2+7s+12) = 0$

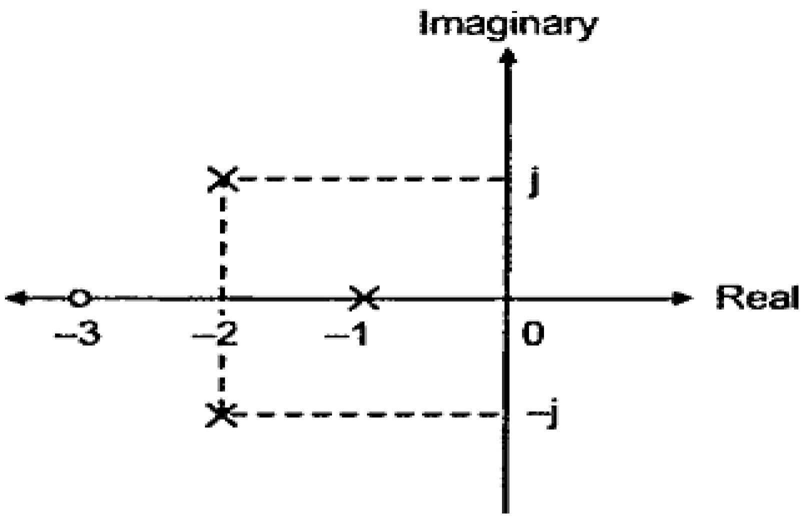
i.e $s^5+14s^4+71s^3+154s^2+120s = 0$

iv) Pole-zero plot

This is shown in the Fig.



Example *Determine the transfer function if the d.c. gain is equal to 10 for the system whose pole-zero plot is shown below.*



Solution : From pole-zero plot given, the transfer function has 3 poles at s = –1, –2+j and –2–j. And it has one zero at s = –3.

$$\therefore \qquad T(s) = \frac{K(s+3)}{(s+1)(s+2+j)(s+2-j)} = \frac{K(s+3)}{(s+1)[(s+2)^2-(j)^2]}$$

$$= \frac{K(s+3)}{(s+1)[s^2+4s+5]}$$

Now d.c. gain is value of T(s) at s = 0 which is given as 10.

$$\therefore \qquad \text{d.c. gain} = T(s)|_{\text{at } s=0}$$

$$\therefore \qquad 10 = \frac{K \times 3}{1 \times 5}$$

$$\therefore \qquad K = \frac{50}{3} = 16.667$$

$$\therefore \qquad \boxed{T(s) = \frac{16.667\,(s+3)}{(s+1)\,(s^2+4s+5)}}$$

This is the required transfer function.