Frequency Response of an Amplifier

In general, any frequency response curve can be split into three regions, namely:

(i) Low-frequency region

(ii) Mid-frequency region

(iii) High-frequency region

In general, the frequency response curve is a plot between magnitudes of gain and logarithmic frequencies. A typical frequency response curve of an RC-coupled amplifier is shown below in figure 1.

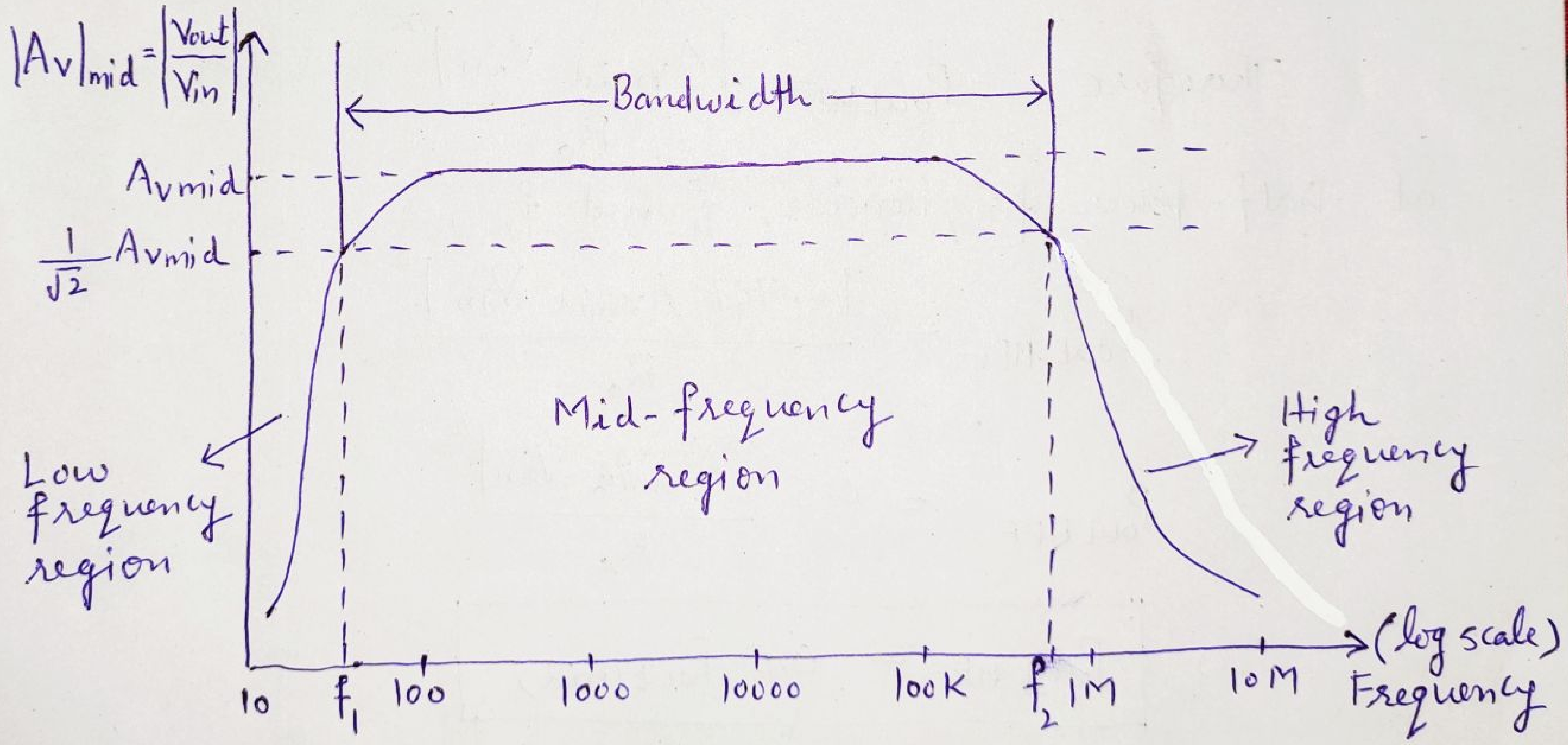

Fig1. Gain Vs frequency (logarithmic scale) of an RC-coupled amplifier

The main reason for the drop in gain at the low-frequency region and the high-frequency region is due to the increase in capacitive reactance in the low-frequency region and due to the parasitic capacitive elements (or) the frequency dependence of the network's gain on the active devices, in the high-frequency region.

The frequency boundaries of relatively high-gain region is determined by choosing "$\frac{1}{\sqrt{2}} A_{v\,mid}$" to be the gain at the cut-off levels. The frequencies corresponding to such values (f_1 and f_2) are called cut-off frequencies or band frequencies or corner frequencies or half power frequencies. At cut-off frequencies, the output power is half the mid-band power output.

$$P_{out\,(mid)} = \frac{|V_{out}|^2}{R_o}$$

But $$|A_v|_{mid} = \left|\frac{V_{out}}{V_{in}}\right|$$

Therefore, $$P_{out\,(mid)} = \frac{|A_{v\,mid}\, V_{in}|^2}{R_o}$$

at half-power frequencies, f_1 and f_2,

$$P_{out\ HPF} = \frac{|0.707\, A_{v\,mid}\, V_{in}|^2}{R_o}$$

$$P_{out\ HPF} = 0.5\,\frac{|A_{v\,mid}\, V_{in}|^2}{R_o}$$

$$\therefore \quad \boxed{P_{out\ HPF} = 0.5\, P_{out\,(mid)}}$$

The bandwidth of each system is, hence, given by

$$\text{Bandwidth} = f_2 - f_1$$

However, in most applications, it is preferable to have the plot of gains in (dB) and it is also conventional to use a normalized plot for such analysis. A normalized plot of gain vs log frequencies, with the gain values (is a plot) divided by the gain in the mid-frequency ranges (i.e the maximum gain).

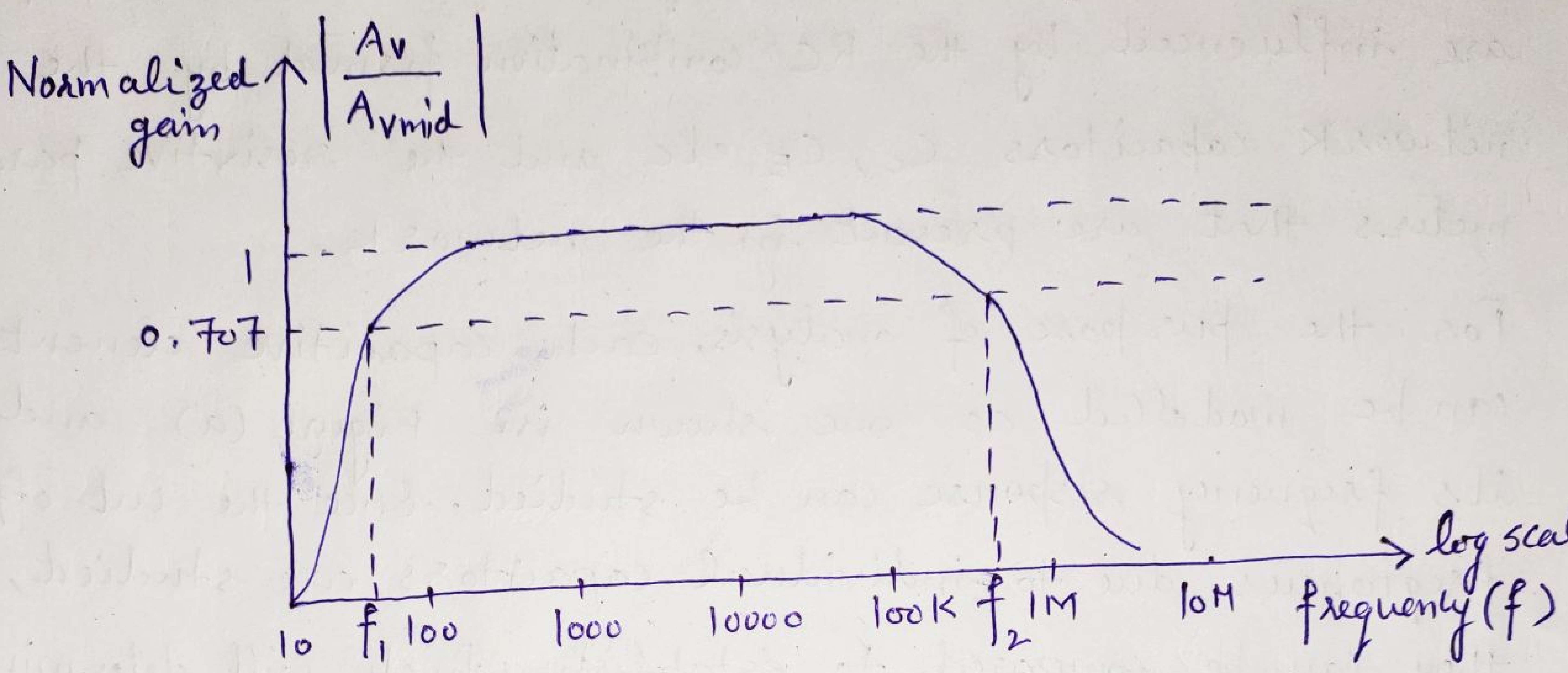

Fig 2. Normalized plot for Figure 1.

A decibel plot can now be obtained by using the following transformation:

$$\left.\frac{A_v}{A_{vmid}}\right|_{dB} = 20 \log_{10} \frac{A_v}{A_{vmid}}$$

Hence, the gain at half-power point becomes

$$20 \log_{10}\left(\frac{1}{\sqrt{2}}\right) = -3 \text{ dB}$$

The plot is shown in Figure 3.

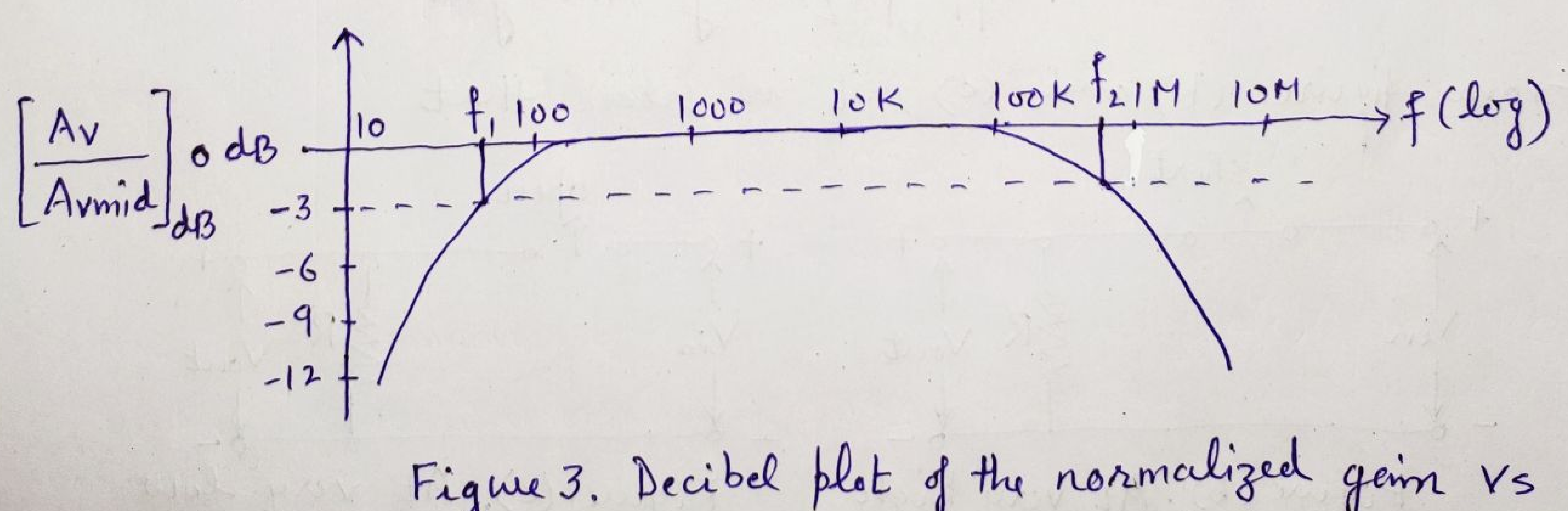

Figure 3. Decibel plot of the normalized gain vs frequency (logarithmic) of Figure 2.

Low frequency response of a RC-coupled stage

The cut-off frequencies of single stage BJT/FET amplifiers are influenced by the RC combination formed by the network capacitors C_c, C_E etc and the resistive parameters that are present in the network.

For the purpose of analysis, each capacitive element can be modelled as one shown in Figure (a) and its frequency response can be studied. Once the cut-off frequencies due to individual capacitors are studied, they can be compared to establish, which will determine the cut-off frequency (lower cut-off) for the system. In this topic, a methodology for determining the lower cut-off frequency (f_1) is presented.

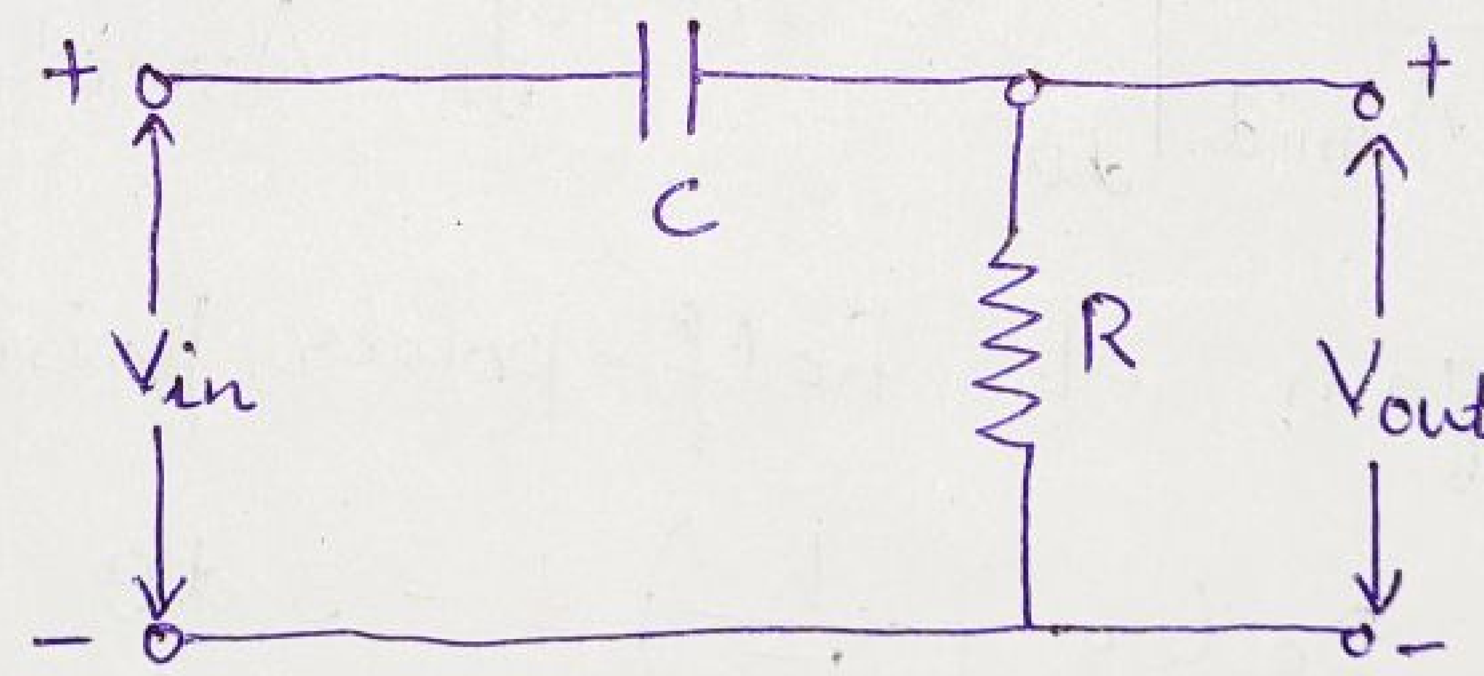

Figure (a). Equivalent circuit for lower cut-off frequency analysis

From figure (b) and (c), it is seen that

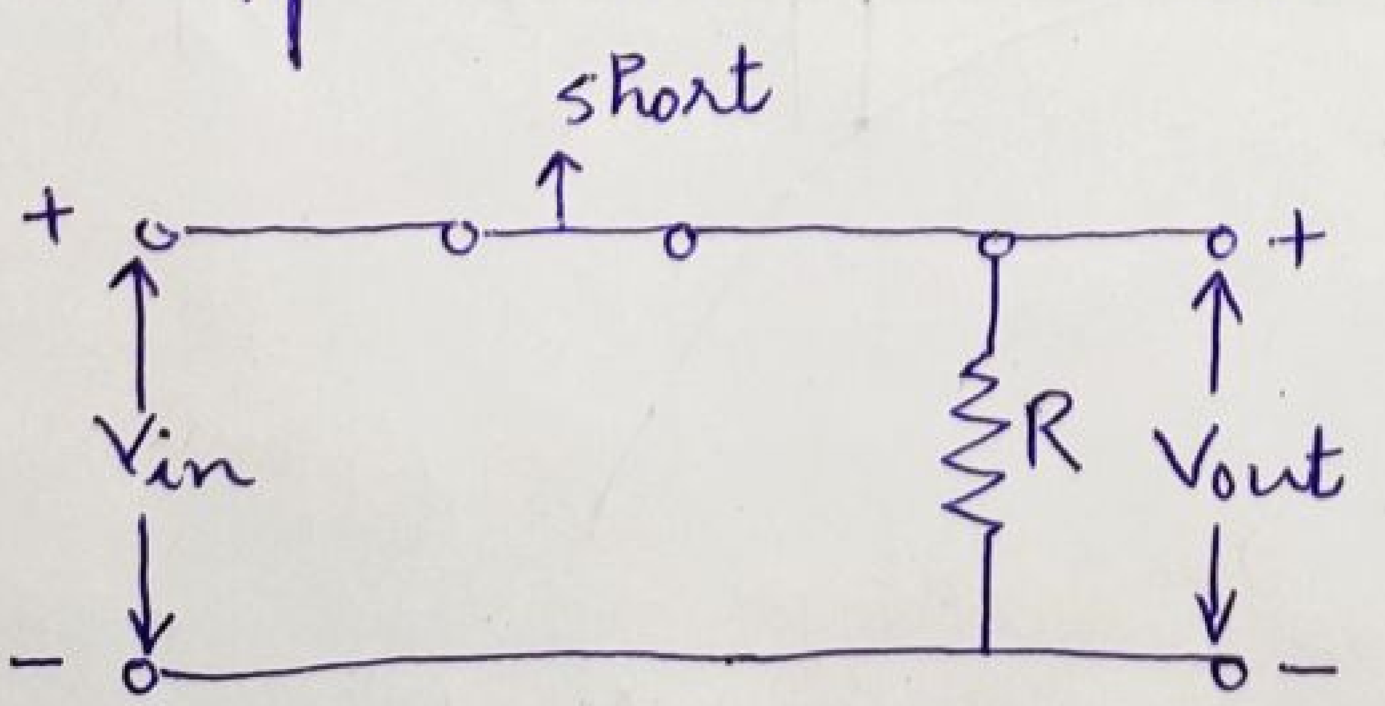

Figure (b). At very high frequencies

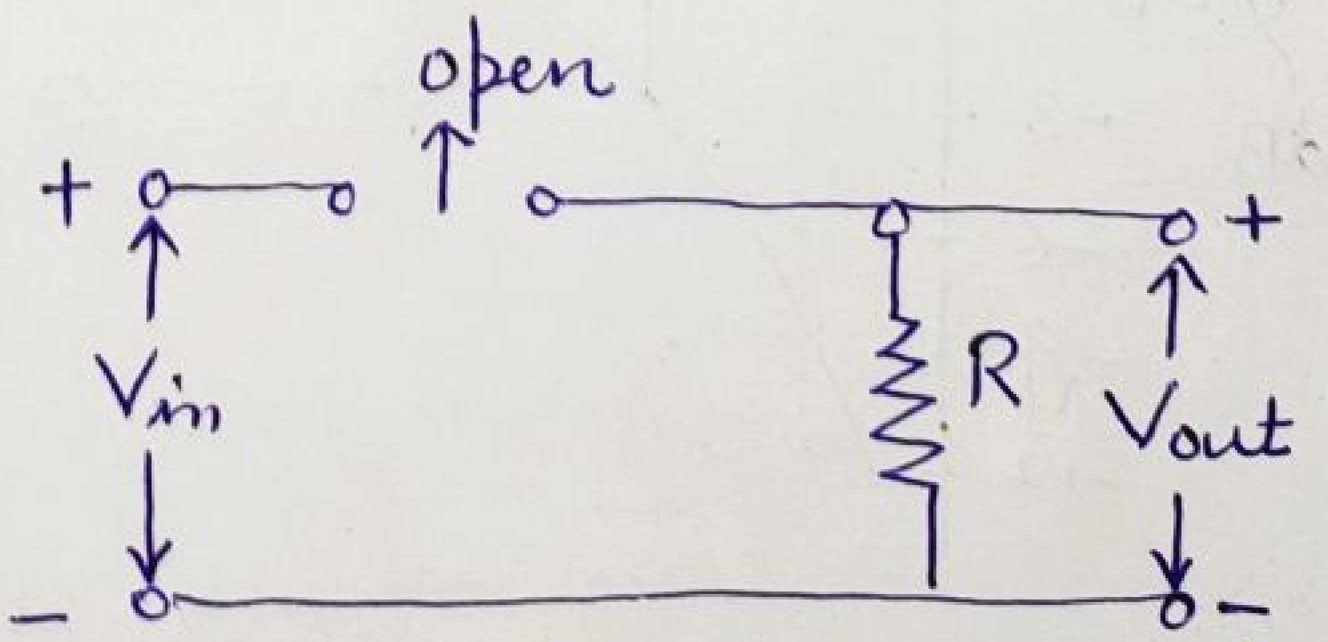

Figure (c). At very low frequencies

at high frequencies, $X_c = \frac{1}{2\pi f C} \approx 0\,\Omega$

at low frequencies, $X_c = \frac{1}{2\pi f C} \approx \infty\,\Omega$

A typical frequency response between the above two extremes is show in Figure (d).

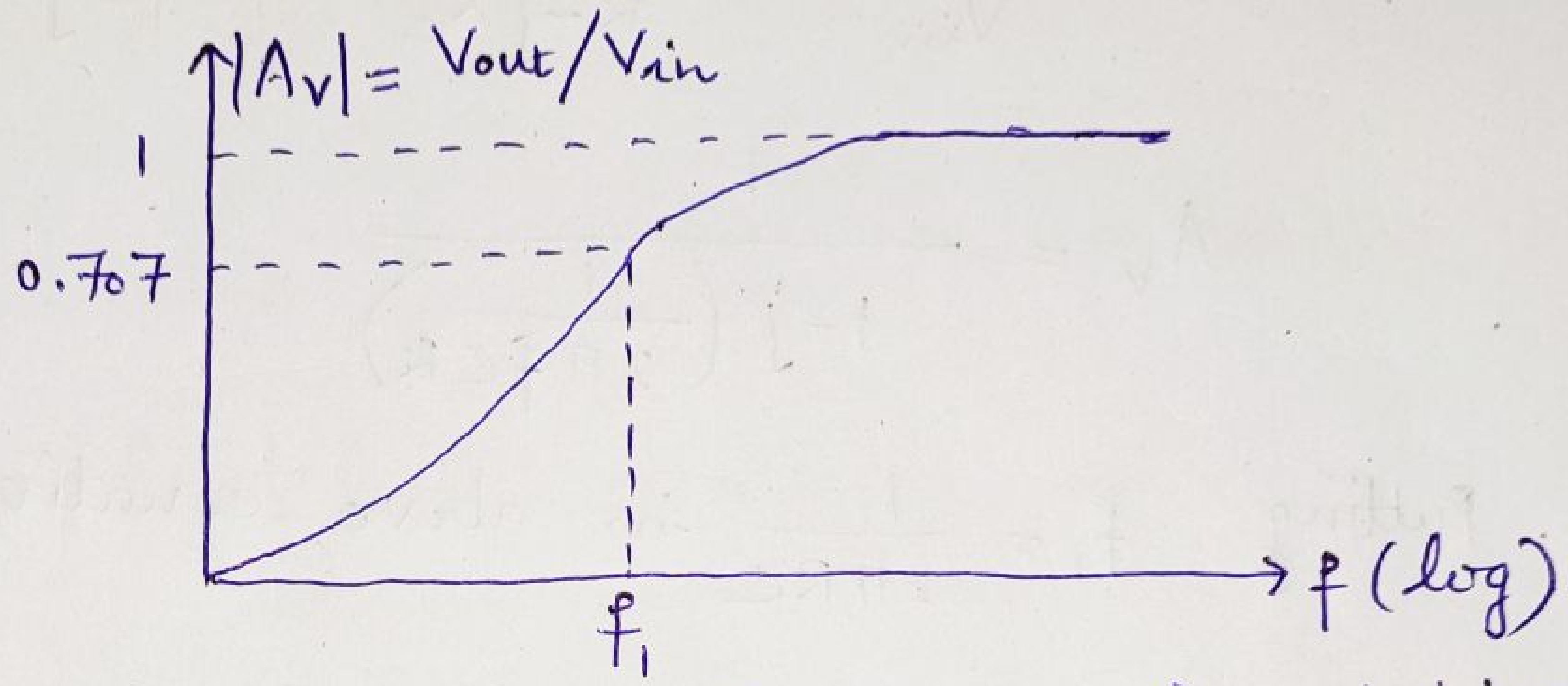

Figure (d). A_v vs frequency of a typical RC Network.

Applying the voltage divider rule in Figure (a),

$$V_{out} = \frac{R\,V_{in}}{R - jX_c} \quad \text{———} \quad (*)$$

where magnitude of V_{out} is given by

$$|V_{out}| = \frac{R\,V_{in}}{\sqrt{R^2 + X_c^2}}$$

Case: When $X_c = R$

$$|V_{out}| = \frac{R\,V_{in}}{\sqrt{R^2 + R^2}}$$

$$|V_{out}| = \frac{1}{\sqrt{2}} V_{in}$$

Therefore, $|A_v| = \frac{V_{out}}{V_{in}} = \frac{1}{\sqrt{2}} \cdot \frac{V_{in}}{V_{in}} = 0.707$

Now, as per assumption

$$X_c = R$$

$$\frac{1}{2\pi f_1 C} = R$$

$$\therefore \quad f_1 = \frac{1}{2\pi RC}$$

In terms of dB, 0.707 corresponds to 3 dB from equation (*).

$$A_V = \frac{V_{out}}{V_{in}} = \frac{R}{R - jX_C} = \frac{1}{1 - j\frac{X_C}{R}}$$

$$A_V = \frac{1}{1 - j\left(\frac{1}{2\pi fCR}\right)}$$

Putting $f_1 = \frac{1}{2\pi RC}$ in above equation,

$$A_V = \frac{1}{1 - j(f_1/f)}$$

when $f = f_1$,

$$A_V = \frac{1}{\sqrt{1^2 + 1^2}} = \frac{1}{\sqrt{2}} = 0.707,$$ that

corresponds to (− 3 dB).

Note:

In general, A_V can be written in magnitude and phase form:

$$A_V = \frac{V_{out}}{V_{in}} = \frac{\tan^{-1}(f_1/f)}{\sqrt{1 + (f_1/f)^2}}$$

In the logarithmic form (dB)

$$|A_V|_{dB} = 20 \log_{10} \frac{1}{\sqrt{1 + \left(\frac{f_1}{f}\right)^2}}$$

$$|A_V|_{dB} = -10 \log_{10}\left[1 + \left(\frac{f_1}{f}\right)^2\right]$$

If $f_1 >> f$, $(f_1/f)^2 >> 1$,

$$|A_v|_{dB} = -10 \log_{10}\left(\frac{f_1}{f}\right)^2$$

$$\boxed{|A_v|_{dB} = -20 \log_{10}\left(\frac{f_1}{f}\right)}$$

e.g. when $f = f_1$; $\frac{f_1}{f} = 1$

$$\therefore \quad |A_v|_{dB} = -20 \log_{10}(1) = 0 \text{ dB}$$

when $f = \frac{1}{2} f_1$; $\frac{f_1}{f} = 2$

$$\therefore \quad |A_v|_{dB} = -20 \log_{10}(2) = -6 \text{ dB}$$

Therefore,

(i) A change in frequency by a factor of '2' results in a 6 dB change in the ratio.

(ii) For a 10:1 change (i.e $f_1 \leftrightarrow f_1/10$) in frequency, there is a 20 dB change in the ratio.

FET Model at High Frequency

In the high-frequency model of FET, the capacitances between nodes have to be added in the low frequency model. The resulting equivalent circuit is shown in figure below:

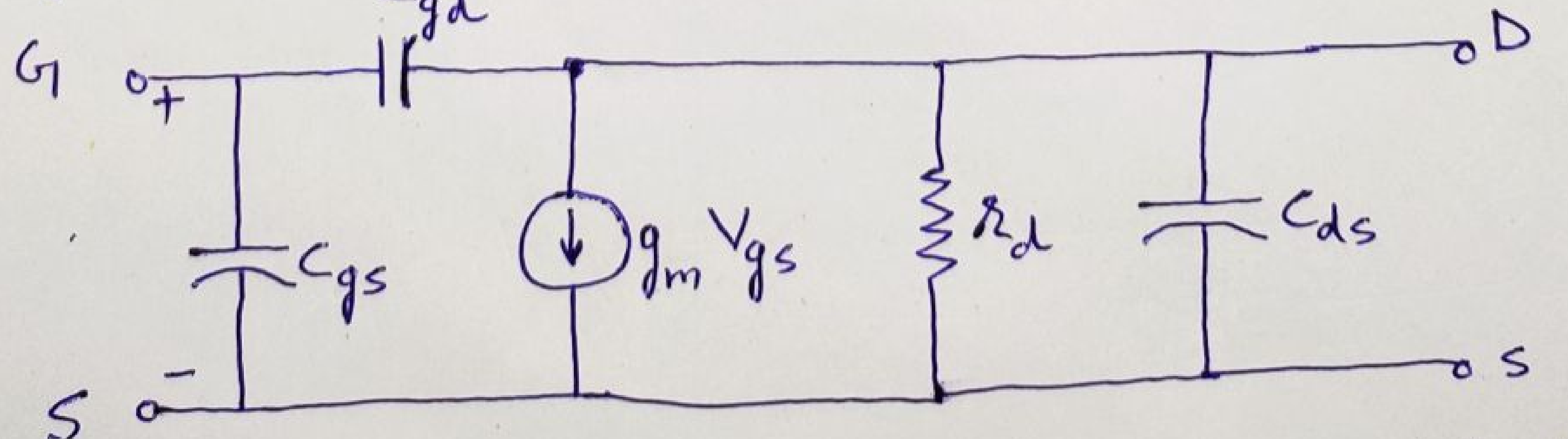

Figure. The high frequency model of FET.

Here, C_{gs} is the barrier capacitance between gate and source. C_{gd} is the barrier capacitance between gate and drain. C_{ds} is the drain to source capacitance of the channel. These capacitances produce the following effects: (i) feedback results between the output circuit and the input circuit and (ii) voltage gain drops rapidly as the frequency increases.
