The Differentiator

Figure 1 shows the differentiator or differentiation amplifier. As its name implies, the circuit performs the mathematical operation of differentiation; that is, the output waveform is the derivative of the input waveform. The differentiator may be constructed from a basic inverting amplifier if an input resistor R_1 is replaced by a capacitor C_1.

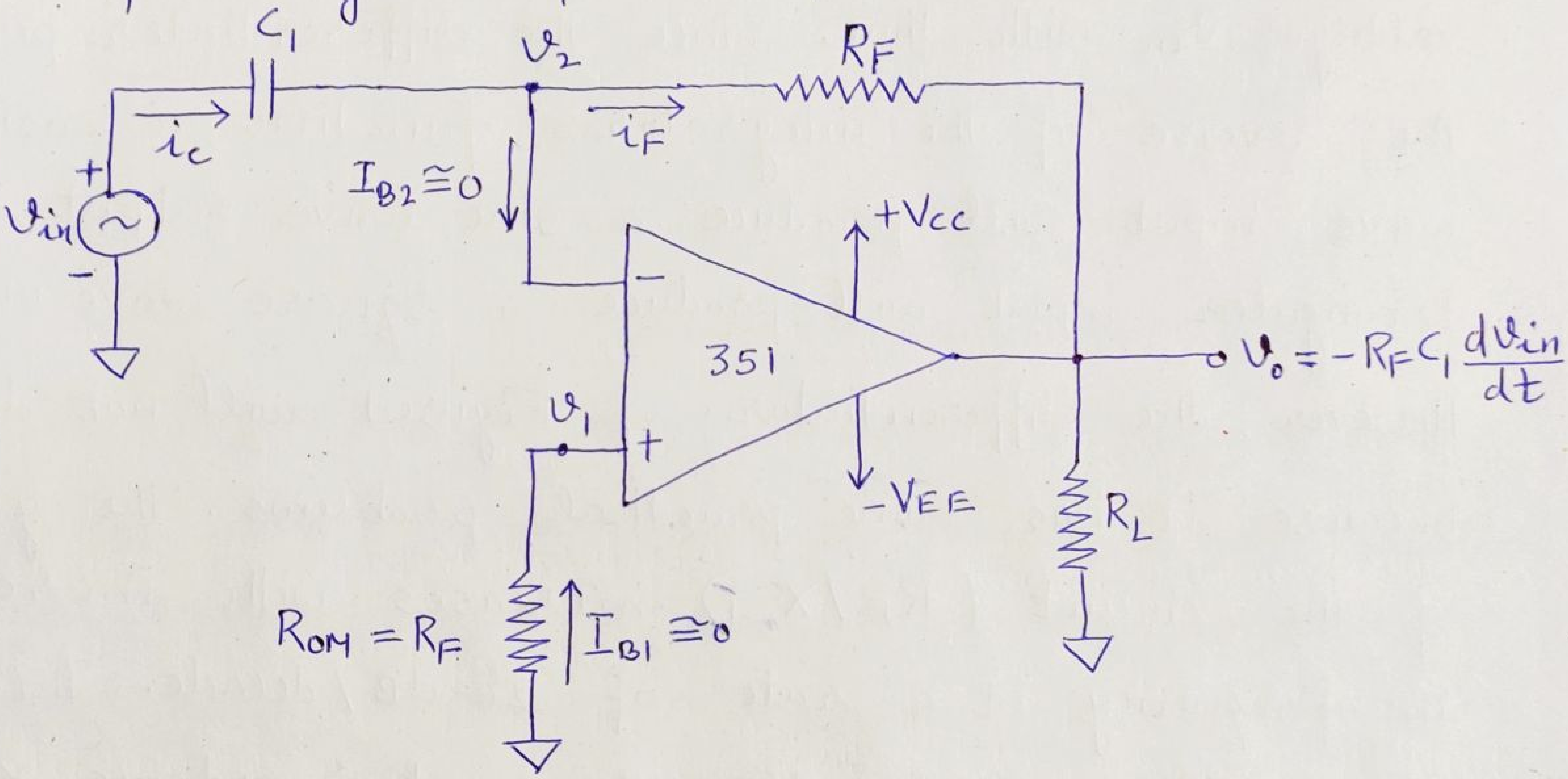

Figure 1. Basic Differentiator Circuit

The expression for the output voltage can be obtained from Kirchhoff's current equation written at node v_2 as follows:

$$i_c = i_B + i_F$$

Since $I_B \cong 0$,

$$i_c = i_F$$

$$C_1 \frac{d}{dt}(v_{in} - v_2) = \frac{v_2 - v_0}{R_F}$$

But $v_1 = v_2 \cong 0$ V, because A is very large. Therefore

$$C_1 \frac{dv_{in}}{dt} = -\frac{v_o}{R_F}$$

or

$$v_o = -R_F C_1 \frac{dv_{in}}{dt} \qquad (1)$$

Thus, the output v_o is equal to the $R_F C_1$ times the negative instantaneous rate of change of the input voltage v_{in} with time. Since the differentiator performs the reverse of the integrator's function, a cosine wave input will produce a sine wave output, or a triangular input will produce a square wave output. However, the differentiator of Figure 1 will not do this because it has some practical problems. The gain of the circuit (R_F / X_{C1}) increases with increase in frequency at a rate of 20 dB/decade. This makes the circuit unstable. Also, the input impedance X_{C1} decreases with increase in frequency, which makes the circuit very susceptible to high-frequency noise. When amplified, this noise can completely override the differentiated output signal. The frequency response of the basic differentiator is shown in Figure 2. In this Figure, f_a is the frequency at which the gain is 0 dB and is given by

$$f_a = \frac{1}{2\pi R_F C_1} \qquad (2)$$

Also, f_c is the unity gain bandwidth of the op-amp, and f is some relative operating frequency.

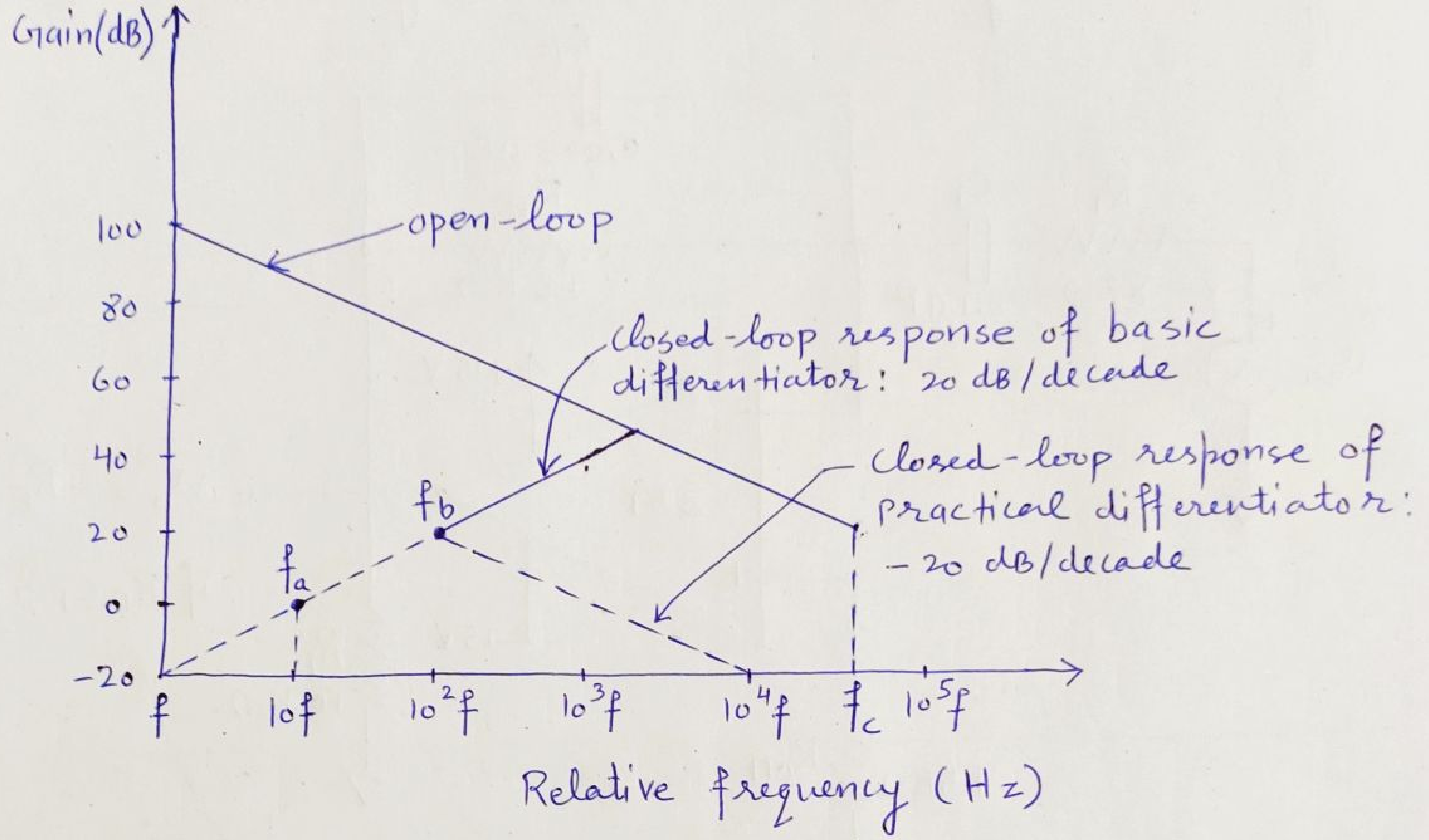

Figure 2. Frequency response of differentiator

Both the stability and the high-frequency noise problems can be corrected by the addition of two components: R_1 and C_F, as shown in Figure 3. This circuit is a practical differentiator, the frequency response of which is shown in Figure 2 by a dashed line. From frequency f to f_b, the gain increases at 20 dB/decade. However, after f_b the gain decreases at 20 dB/decade. This 40 dB change in gain is caused by the R_1C_1 and R_FC_F combinations. The gain limiting frequency f_b is given by,

$$f_b = \frac{1}{2\pi R_1 C_1} \quad \text{——— (3)}$$

where $R_1C_1 = R_FC_F$.

Thus R_1C_1 and R_FC_F help to reduce significantly the effect of high frequency input, amplifier noise, and offsets. Above all, it makes the circuit more stable by preventing the increase in gain with frequency. (4)

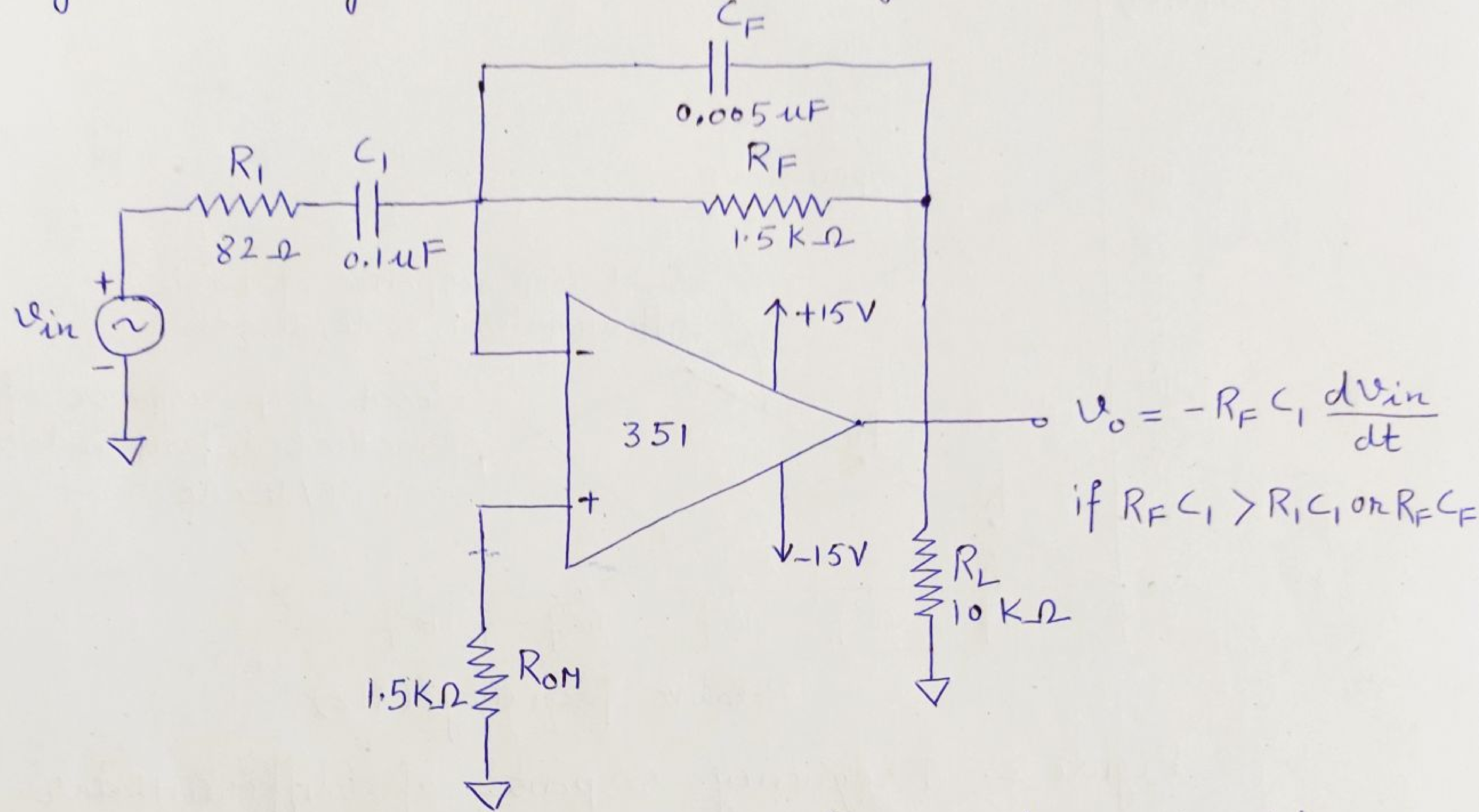

Figure 3. Practical differentiator circuit.

Generally, the value of f_b and in turn $R_1 C_1$ and $R_F C_F$ values should be selected such that

$$f_a < f_b < f_c$$

where

$$f_a = \frac{1}{2\pi R_F C_1}$$

$$f_b = \frac{1}{2\pi R_1 C_1} = \frac{1}{2\pi R_F C_F}$$

f_c = unity gain-bandwidth

The input signal will be differentiated properly if the time period T of the input signal is larger than or equal to $R_F C_1$. That is,

$$T \geq R_F C_1$$

Figure 4 shows the sine wave and square wave inputs and resulting differentiated outputs, respectively, for the practical differentiator.

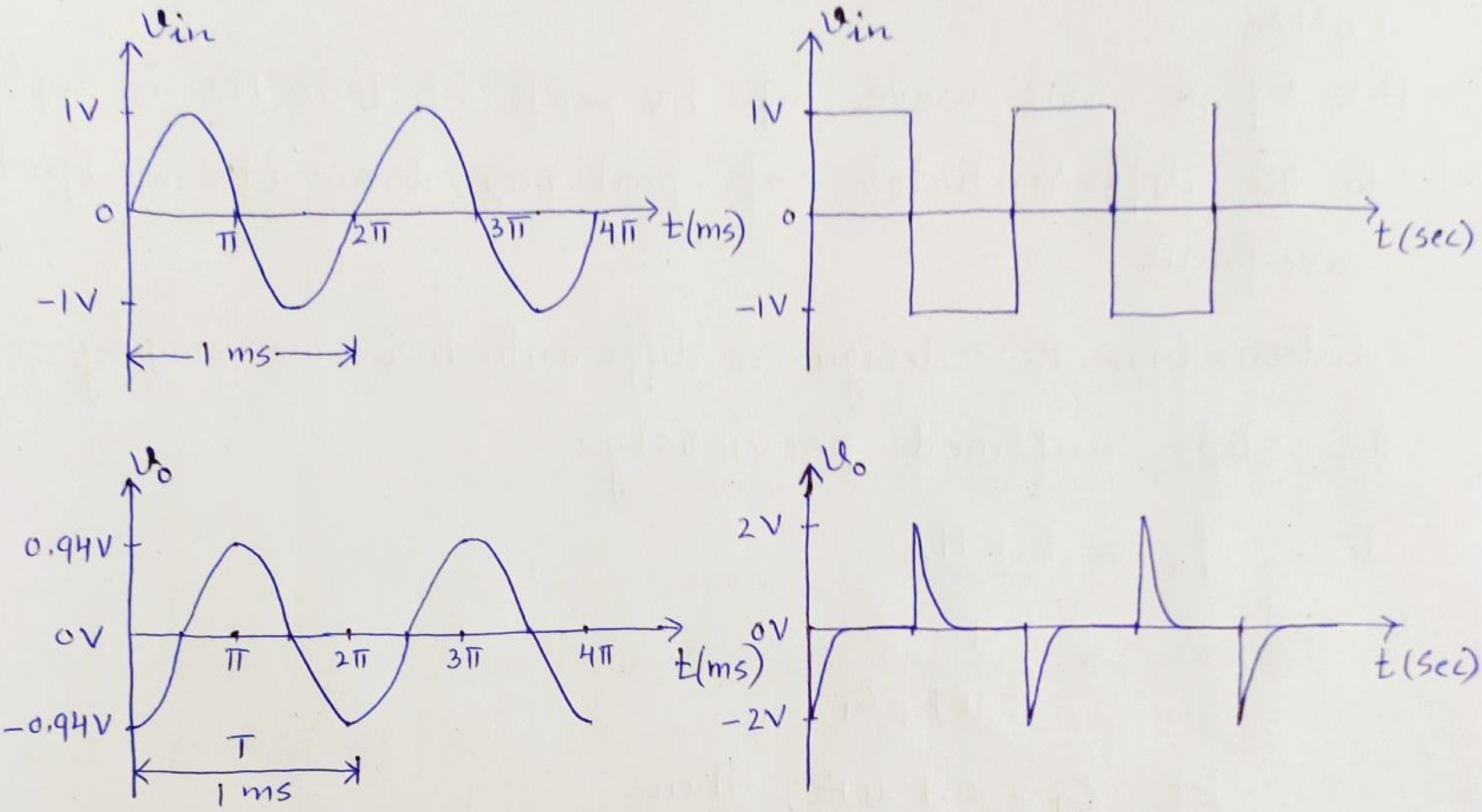

Figure 4. Sine wave and square wave input and resulting cosine wave and spike output respectively.

A workable differentiator can be designed by implementing the following steps:

1. Select f_a equal to the highest frequency of the input signal to be differentiated. Then, assuming a value of $C_1 < 1\,\mu F$, calulate the value of R_F.

2. Choose $f_b = 20 f_a$ and calculate the values of R_1 and C_F so that $R_1 C_1 = R_F C_F$.

The differentiator is most commonly used in waveshaping circuits to detect high frequency components in an input signal and also as a rate-of-change detector in FM modulators.

Q 1(a). Design a differentiator to differentiate an input signal that varies in frequency from 10 Hz to about 1 KHz.

(b). If a sine wave of 1 V peak at 1000 Hz is applied to the differentiator of part (a), draw its output waveform.

Solution: (a). To design a differentiator, we simply follow the steps outlined previously:

1. $f_a = 1 \text{ KHz}$

$$= \frac{1}{2\pi R_F C_1}$$

Let $C_1 = 0.1 \mu F$; then

$$R_F = \frac{1}{(2\pi)(10^3)(10^{-7})} = 1.59 \text{ K}\Omega$$

Let R_F be $1.5 \text{ K}\Omega$.

2. $f_b = 20 \text{ KHz}$

$$= \frac{1}{2\pi R_1 C_1}$$

Hence, $R_1 = \frac{1}{(2\pi)(2)(10^4)(10^{-7})} = 79.5 \ \Omega$

Let R_1 be $82 \ \Omega$. Since $R_1 C_1 = R_F C_F$

$$\therefore \quad C_F = \frac{(82)(10^{-7})}{1.5 \text{ K}\Omega} \cong 0.0055 \mu F$$

Let C_F be $0.005 \mu F$. Finally, $R_{OM} = R_F \cong 1.5 \text{K}\Omega$.

The complete circuit with component values is shown in Figure 3.

(b). Since $V_P = 1V$ and $f = 1000$ Hz, the input voltage is,

$$v_{in} = V_P \sin \omega t$$

$$v_{in} = \sin (2\pi)(10^3) t$$

Hence, from equation 1,

$$v_o = -R_F C_1 \frac{dv_{in}}{dt}$$

$$v_o = -(1.5\,K\Omega)(0.1\,\mu F) \frac{d}{dt}\left[\sin(2\pi)(10^3)t\right]$$

$$v_o = -(1.5\,K\Omega)(0.1\,\mu F)(2\pi)(10^3) \cos\left[(2\pi)(10^3)t\right]$$

$$v_o = -0.94 \cos\left[(2\pi)(10^3)t\right]$$

The input and differentiated output waveforms are shown in Figure 4.

Active Filters

An electric filter is often a frequency-selective circuit that passes a specified band of frequencies and blocks or attenuates signals of frequencies outside this band. Filters may be classified in a number of ways:

1. Analog or digital
2. Passive or active
3. Audio (AF) or radio frequency (RF)

Analog filters are designed to process analog signals, while digital filters process analog signals using digital techniques. Elements used in passive filters are resistors, capacitors, and inductors. Active filters, on the other hand, employ transistors or op-amp in addition to the resistors and capacitors. RC filters are commonly used for audio or low frequency operation, whereas LC or crystal filters are employed at RF or high frequencies.

An active filter offers the following advantages over a passive filter:

1. Gain and frequency adjustment flexibility

Since the op-amp is capable of providing a gain, the input signal is not attenuated as it is in a passive filter. In addition, the active filter is easier to tune or adjust.

2. No loading Problem

Because of the high input resistance and low output resistance of the op-amp, the active filter does not cause loading of the source or load.

3. Cost

Typically, active filters are more economical than passive filters. This is because of the variety of cheaper op-amps and the absence of inductors.

The most commonly used filters are:

1. Low-pass filter
2. High-pass filter
3. Band-pass filter
4. Band-reject filter
5. All-pass filter