

## Unit IV: Graph Theory I

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**1. A graph is defined as:**

- A) A set of vertices only
- B) A set of edges only
- C) A set of vertices and edges
- D) A matrix

**Answer: C**

**Explanation:** A graph consists of vertices (nodes) and edges (connections).

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**2. In a simple graph, there are no:**

- A) Loops
- B) Multiple edges
- C) Both A and B
- D) Vertices

**Answer: C**

**Explanation:** A simple graph does not allow loops or multiple edges between the same pair of vertices.

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**3. A graph with all vertices having even degree is:**

- A) Eulerian
- B) Hamiltonian
- C) Bipartite
- D) A tree

**Answer: A**

**Explanation:** Eulerian graphs have all vertices of even degree and contain an Eulerian circuit.

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**4. The degree of a vertex in an undirected graph is:**

- A) The number of edges in the graph
- B) The number of vertices adjacent to it
- C) The length of the path
- D) Always even

**Answer: B**

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**5. A graph is said to be connected if:**

- A) There is a path between every pair of vertices
- B) All vertices have even degree
- C) It has no edges
- D) It has loops

**Answer: A**

**6. In a graph with  $n$  vertices, the maximum number of edges in a simple undirected graph is:**

- A)  $n^2$
- B)  $\frac{n(n+1)}{2}$
- C)  $\frac{n(n-1)}{2}$
- D)  $2n$

**Answer: C**

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**7. Two graphs are isomorphic if:**

- A) They have the same number of vertices
- B) Same degree of vertices
- C) Same number of components
- D) Same structure and adjacency relationships

**Answer: D**

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**8. A complete graph with  $n$  vertices has how many edges?**

- A)  $n^2$
- B)  $\frac{n(n-1)}{2}$
- C)  $n(n-1)n(n-1)$
- D)  $n+1$

**Answer: B**

Answered Below

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**9. A path in a graph is:**

- A) A sequence of distinct edges
- B) A sequence of distinct vertices
- C) A circuit
- D) A tree

**Answer: B**

**10. Which of the following is not a type of graph?**

- A) Simple
  - B) Multigraph
  - C) Hypergraph
  - D) Laddergraph
- What is Hyper Graph answered below

**Answer: D**

**11. A cycle in a graph is:**

- A) A path that starts and ends at the same vertex
- B) A path with no repeated edges
- C) A graph with no loops
- D) Always of even length

**Answer: A**

# Understanding Hypergraphs

Bahut badhiya sawaal poocha aapne! (You asked a very good question!) Hypergraphs are fascinating mathematical structures that extend the concept of traditional graphs in a powerful way.

## What Is a Hypergraph?

A hypergraph is a generalization of a graph where an edge (called a hyperedge) can connect any number of vertices, not just two. Formally, a hypergraph  $H$  is defined as a pair  $H = (V, E)$  where:

- $V$  is a set of elements called vertices (or nodes)
- $E$  is a family of subsets of  $V$  called hyperedges

**Real-world example:** Think of a social media platform where users (vertices) can be part of different group chats (hyperedges). Each group chat can include any number of users, not just two people like in a traditional phone call.

## Visualizing Hypergraphs

Unlike regular graphs where edges are lines connecting two points, hyperedges are typically drawn as closed curves or shapes that enclose all vertices they connect. This makes it easy to see which vertices belong to the same hyperedge.

## Types of Hypergraphs

- **k-uniform hypergraphs:** Every hyperedge connects exactly  $k$  vertices
- **Simple hypergraphs:** No repeated edges and no loops (edges with a single vertex)
- **Directed hypergraphs:** Each hyperedge has a "tail" (source vertices) and a "head" (destination vertices)
- **Complete k-uniform hypergraph:** For each possible set of  $k$  vertices, there exists a hyperedge connecting them

**Bold insight:** Traditional graphs are actually just 2-uniform hypergraphs where each edge connects exactly two vertices!

## Properties of Hypergraphs

- **Order:** The number of vertices in the hypergraph
- **Size:** The number of hyperedges
- **Rank:** The maximum size of any hyperedge
- **Co-rank:** The minimum size of any hyperedge

## Applications of Hypergraphs

Hypergraphs are incredibly useful for modeling complex relationships:

1. **Data clustering:** Grouping related data points together
2. **Computer networks:** Modeling multicast communications where a message is sent to multiple recipients
3. **Chemical reactions:** Representing reactions with multiple reactants and products
4. **Social network analysis:** Studying group affiliations rather than just pairwise connections

## Quick Tips for Working with Hypergraphs

1. **Yaad rakho** (Remember): The power of hypergraphs comes from their ability to represent many-to-many relationships naturally.
2. **Learning shortcut:** When trying to determine if you need a hypergraph, ask yourself: "Do my relationships connect more than two objects at once?"
3. **Visualization trick:** Color-code different hyperedges to make complex hypergraphs easier to understand.

**Bold realization:** Many real-world problems that seem complex with traditional graphs become elegant and straightforward when modeled with hypergraphs!

Hypergraphs extend beyond simple binary relationships to capture the true complexity of interconnected systems in ways that regular graphs simply cannot.



# Maximum Edges in a Simple Undirected Graph

The answer is C:  $n(n-1)/2$

## Why This is Correct

In a simple undirected graph with  $n$  vertices, the maximum number of edges occurs when every vertex is connected to every other vertex - forming what's called a complete graph.

Let's understand why the formula is  $n(n-1)/2$ :

**Real-world example:** Imagine  $n$  people at a party where everyone shakes hands with everyone else exactly once. How many handshakes occur? Each person can shake hands with  $(n-1)$  other people, but if we count this way, we'd count each handshake twice (once from each person's perspective).

## Mathematical Approach

There are two ways to derive this formula:

1. **Degree approach:** In a complete graph, each vertex has edges to all other vertices, giving each vertex a degree of  $(n-1)$ . The sum of degrees equals 2 times the number of edges:
  - Total degrees =  $n \times (n-1)$
  - Number of edges =  $n \times (n-1) \div 2 = n(n-1)/2$
2. **Combination approach:** Each edge connects a pair of vertices, so we're essentially choosing 2 vertices from  $n$  vertices:
  - Number of ways to choose 2 vertices from  $n$  =  $nC2 = n(n-1)/2$

**Bold insight:** This formula represents the maximum number of unique pairs you can form with  $n$  distinct elements - a concept that appears frequently in many combinatorial problems!

## Quick Tip

Yaad rakho (Remember): For simple graphs, we have these maximum edge counts:

- Undirected simple graph:  $n(n-1)/2$
- Directed simple graph:  $n(n-1)$

The key difference is that in directed graphs, we can have edges going both ways between vertices, effectively doubling the maximum.



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**12. A graph with no cycles is called:**

- A) Complete
- B) Connected
- C) Tree
- D) Simple

**Answer: C**

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**13. A Hamiltonian path:**

- A) Visits every edge exactly once
- B) Visits every vertex exactly once
- C) Starts and ends at the same vertex
- D) Exists in all graphs

**Answer: B**

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**14. Which is true about a tree?**

- A) It contains a cycle
- B) It is always disconnected
- C) It has  $n-1$  edges for  $n$  vertices
- D) It must be a complete graph

**Answer: C**

Answered Below

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**15. A graph with  $n$  vertices and  $n$  edges, and no cycles is:**

- A) Tree
- B) Not connected
- C) Has a loop
- D) Not a graph

**Answer: B**

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**16. The minimum number of edges required to make a disconnected graph connected is:**

- A) Number of vertices
- B) Number of components minus 1
- C) Degree of each vertex
- D) Always 1

**Answer: B**

Answered Below

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# Understanding Trees in Graph Theory

Arrey wah! Trees are fascinating structures in graph theory that appear everywhere from computer science to network design. Let's understand why option C is correct!

## Why a Tree Has $n-1$ Edges

A tree is defined as a connected acyclic graph, and this property leads directly to the fact that for  $n$  vertices, a tree must have exactly  $n-1$  edges.

This important property can be proven several ways:

- By induction (starting with a single vertex and adding vertices one by one)
- By understanding that adding any more edges would create a cycle
- By recognizing that removing any edge would disconnect the graph

**Real-world example:** Think of a company's organizational chart where people are vertices and reporting relationships are edges. If there are 10 people (vertices), there must be exactly 9 reporting relationships (edges) to ensure everyone connects to the organization without any circular reporting structures.

## Why the Other Options Are Wrong

### Option A - "It contains a cycle" ✗

Bahut galat! (Very wrong!) A tree is specifically defined as an acyclic graph - meaning it contains no cycles whatsoever. This is a fundamental property of trees.

### Option B - "It is always disconnected" ✗

Another key property of trees is that they are connected! Every pair of vertices must have a path between them.

### Option D - "It must be a complete graph" ✗

A complete graph with  $n$  vertices has  $n(n-1)/2$  edges, which is more than  $n-1$  for any  $n > 2$ . So trees cannot be complete graphs except for extremely small cases ( $n=1$  or  $n=2$ ).

## Quick Tips for Tree Problems

1. **Visualization trick:** Draw your trees! Start with  $n$  dots and connect them with exactly  $n-1$  lines while ensuring no cycles.
2. **Yaad rakho!** (Remember!) In a tree with  $n$  vertices, adding any edge creates exactly one cycle, and removing any edge disconnects the graph.
3. **Problem-solving approach:** When working with trees, the  $n-1$  edge property is often key to solving many problems.

**Bold insight:** Trees represent the minimal connected structure - they use the fewest possible edges to connect all vertices!





# Connecting Disconnected Graphs: Minimum Edge Requirement

To understand why option B (Number of components minus 1) is correct, let's explore what happens when we add edges to a disconnected graph.

## Understanding the Problem

A disconnected graph consists of multiple connected components that have no paths between them. To make such a graph connected, we need to add enough edges so that there's a path between any two vertices in the graph.

## Minimum Edge Requirement

When adding edges to connect separate components:

- Each new edge can join at most two different components into one
- Each successful connection reduces the total number of components by exactly one

**Real-world example:** Think of islands (components) that need to be connected by bridges (edges). If you have 5 islands, you need at least 4 bridges to ensure people can travel between any two islands.

## Mathematical Proof

If a graph has  $k$  components, then:

- Starting with  $k$  separate components
- To create a single connected component, we need to reduce the number by  $k-1$
- Each edge reduces the component count by at most 1
- Therefore, at least  $k-1$  edges are needed

**Bold insight:** This is similar to how a tree with  $n$  vertices has exactly  $n-1$  edges - when we view each component as a "super vertex," we're essentially creating a tree that connects all components!

## Quick Tip

Bahut aasan tarika! (A very easy method!) To solve such problems, just count the number of separate "islands" in your graph and subtract 1.

For example:

- 2 components  $\rightarrow$  1 edge needed

- 3 components  $\rightarrow$  2 edges needed
- 4 components  $\rightarrow$  3 edges needed

Remember that this formula gives you the *minimum* - you could always add more edges, but you cannot connect the graph with fewer edges than (components - 1).

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**17. An Eulerian circuit must:**

- A) Visit every vertex exactly once
- B) Visit every edge exactly once and return to start
- C) Not repeat any vertex
- D) Be a tree

**Answer: B**

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**18. A graph with 5 vertices and 6 edges cannot be a tree because:**

- A) It is disconnected
- B) It contains a cycle
- C) It is a multigraph
- D) It is bipartite

**Answer: B**

**Explanation:** A tree must have  $n-1$  edges  $\Rightarrow$  5 vertices  $\rightarrow$  max 4 edges for tree.

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**19. If a graph has an odd number of vertices of odd degree:**

- A) It must be Eulerian
- B) It must be connected
- C) It cannot have an Euler path
- D) It must have an odd number of edges

**Answer: C**

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**20. What is the maximum number of edges in a bipartite graph with sets of size  $m$  and  $n$ ?**

- A)  $m \cdot n$
- B)  $m+n$
- C)  $\frac{(m+n)^2}{2}$
- D)  $\min(m, n)$

**Answer: A**

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**21. Which of the following is not necessarily true for an isomorphic graph?**

- A) Same number of vertices
- B) Same degree for corresponding vertices
- C) Same number of components
- D) Same adjacency matrix

**Answer: D**

**Explanation:** Isomorphic graphs may have different adjacency matrices due to vertex labeling.

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**22. A graph is regular if:**

- A) All vertices have the same degree
- B) It has no cycles
- C) It is complete
- D) All edges have same length

**Answer: A**

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**23. What is the number of edges in a complete bipartite graph  $K_{3,4}$ ?**

- A) 7
- B) 12
- C) 10
- D) 14

**Answer: B**

**Explanation:**  $m \times n = 3 \times 4 = 12$

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**24. A connected graph with  $n$  vertices and  $n$  edges always contains:**

- A) A tree
- B) A cycle
- C) A complete subgraph
- D) A pendant vertex

**Answer: B**

**Explanation:** A tree has  $n-1$  edges, so  $n$  edges  $\Rightarrow$  must contain a cycle.

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**25. A loop in a graph contributes how much to the degree of a vertex?**

- A) 1
- B) 2
- C) 0
- D) Infinite

**Answer: B**

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**26. The degree sum formula for undirected graphs states:**

- A) Sum of degrees = number of edges
- B) Sum of degrees = twice the number of edges
- C) Sum of degrees = number of vertices
- D) Always even

**Answer: B**

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**27. A simple undirected graph with 6 vertices can have at most how many edges?**

- A) 12
- B) 15
- C) 18
- D) 21

**Answer: B**

**Explanation:**  $6 \cdot 5 / 2 = 15$

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**28. What is the minimum degree for a graph with 10 vertices to be connected?**

- A) 1
- B) 2
- C) 9
- D) At least  $n-1$

**Answer: A**

**Explanation:** Minimum is 1 edge connecting isolated vertex.

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**29. A graph where every vertex is connected to every other vertex is called:**

- A) Bipartite graph
- B) Tree
- C) Regular graph
- D) Complete graph

**Answer: D**

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**30. Which of the following cannot be the degree sequence of a simple graph?**

- A) 3, 3, 2, 2
- B) 4, 3, 2, 1
- C) 3, 3, 3, 3
- D) 2, 2, 2, 2, 2

**Answer: D**

**Explanation:** Degree sum must be even, but no graph with 5 vertices can have all degree 2 without forming multiple cycles (violates simple graph).

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**31. A graph invariant is a property that:**

- A) Is preserved under edge deletion
- B) Changes if vertices are relabeled
- C) Remains unchanged under graph isomorphism
- D) Is only for directed graphs

**Answer: C**

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**32. The complement of a graph has:**

- A) Same vertices, inverted edge connections
- B) All loops removed
- C) Same edges, different vertices
- D) One edge per vertex

**Answer: A**

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**33. A graph with all degrees even and connected:**

- A) Must be Hamiltonian
- B) Must be a tree
- C) Must have Eulerian circuit
- D) Must be bipartite

**Answer: C**

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**34. Which of the following graphs is both Eulerian and Hamiltonian?**

- A)  $K_5$
- B) Cycle  $C_4$
- C) Star graph
- D) Path graph

**Answer: B**

**Explanation:** Even degrees and connected  $\Rightarrow$  Eulerian; 4-cycle also has Hamiltonian circuit.

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**35. A Hamiltonian circuit:**

- A) May repeat edges
- B) May repeat vertices
- C) Visits each vertex once and returns to start
- D) Always exists

**Answer: C**

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**36. Which graph is not a tree?**

- A) A connected graph with 5 vertices and 4 edges
- B) A graph with 6 vertices and 5 edges with no cycles
- C) A graph with a cycle
- D) A graph with 5 vertices and no cycles

**Answer: C**

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**37. A graph with two vertices of odd degree:**

- A) Cannot exist
- B) Has Eulerian path but not circuit
- C) Has Eulerian circuit
- D) Is a tree

**Answer: B**

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**38. How many edges does a complete graph  $K_7$  have?**

- A) 21
- B) 42
- C) 14
- D) 28

**Answer: A**

**Explanation:**  $\binom{7}{2} = \frac{7 \cdot 6}{2} = 21$

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**39. A bipartite graph does not contain:**

- A) Odd-length cycles
- B) Loops
- C) Hamiltonian paths
- D) Paths

**Answer: A**

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**40. The number of edges in a complete undirected graph with  $n$  vertices is:**

- A)  $n(n-1)n(n-1)n(n-1)$
- B)  $\frac{n(n+1)}{2}$
- C)  $\frac{n(n-1)}{2}$
- D)  $n!$

**Answer: C**

**41. A tree with  $n$  vertices has how many edges?**

- A)  $n$
- B)  $n-1$
- C)  $n+1$
- D)  $2n$

**Answer: B**

**Explanation:** Trees always have  $n-1$  edges.

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# Complete Graph K7 Edge Count

*Arrey wah, graph theory bahut mast subject hai! (Wow, graph theory is an awesome subject!)*

In a complete graph, every vertex is connected to every other vertex with exactly one edge. For  $K_7$ , this means each of the 7 vertices connects to all other 6 vertices.

## Why the Answer is 21 Edges

To find the number of edges in a complete graph  $K_7$ :

- **Direct counting approach:** Each of the 7 vertices connects to 6 other vertices, giving  $7 \times 6 = 42$  connections. But wait! This counts each edge twice (once from each end), so we divide by 2:  $42 \div 2 = 21$  edges.
- **Formula approach:** For any complete graph  $K_n$ , the number of edges is  $n(n-1)/2$ . With  $n = 7$ :  $7(7-1)/2 = 7 \times 6/2 = 21$  edges.
- **Combination approach:** The number of edges equals the number of ways to choose 2 vertices from  $n$  vertices, which is  $\binom{n}{2}$  or  $\binom{7}{2}$ . So  $\binom{7}{2} = \frac{7!}{2!(7-2)!} = \frac{7 \times 6}{2 \times 1} = 21$  edges.

**Real-world example:** Think of 7 people in a social network where everyone is friends with everyone else. The number of friendship connections would be 21 total relationships.

*Tension mat lo! (Don't stress!)* **Complete graphs have a beautiful pattern to their edge counts:**  
 **$K_1$ : 0,  $K_2$ : 1,  $K_3$ : 3,  $K_4$ : 6,  $K_5$ : 10,  $K_6$ : 15,  $K_7$ : 21...**

**Quick tip:** When solving graph problems, always start by identifying the type of graph (directed/undirected, complete/bipartite, etc.) as this immediately tells you which formulas to apply!





**42. The number of labeled trees with  $n$  vertices is:**

- A)  $2n^{2n}$
- B)  $n^{n^n}$
- C)  $n^{n-2} n^{n-2}$
- D)  $n!n!$

**Answer: C**

**Explanation:** Cayley's formula:  $n^{n-2}$  labeled trees.

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**43. Which of the following graphs is bipartite?**

- A)  $K_3$
- B)  $C_5$
- C)  $K_{2,3}$
- D)  $K_4$

**Answer: C**

**Explanation:**  $K_{2,3}$  is a complete bipartite graph.

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**44. A planar graph is one that:**

- A) Can be drawn in a plane without edge crossings
- B) Is always connected
- C) Has loops
- D) Cannot contain a cycle

**Answer: A**

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**45. Euler's formula for planar graphs is:**

- A)  $V - E + F = 2$
- B)  $E = V + F$
- C)  $V = E - F$
- D)  $V + E + F = 2$

**Answer: A**

**Explanation:** Classic formula for planar graphs.

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**46. A graph is acyclic if:**

- A) It contains exactly one cycle
- B) It contains no cycles
- C) It is a complete graph
- D) All degrees are 2

**Answer: B**

The Correct Formula

Euler's formula establishes a beautiful relationship between the key elements of any connected planar graph:

$$V - E + F = 2$$

Where:

$V$  = number of vertices

$E$  = number of edges

$F$  = number of faces (including the outer face)

This elegant formula works for any connected planar graph, regardless of how it's drawn (as long as no edges cross).

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**47. Which condition is necessary for a graph to have an Eulerian path but not a circuit?**

- A) All vertices even degree
- B) Exactly two vertices of odd degree
- C) Disconnected graph
- D) All degrees odd

**Answer: B**

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**48. Which of the following must be true for a Hamiltonian graph?**

- A) Must be complete
- B) Must be bipartite
- C) Each vertex has degree at least  $\frac{n}{2}$
- D) Must be regular

**Answer: C**

**Explanation:** Dirac's Theorem gives a sufficient condition.

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**49. A connected acyclic graph is:**

- A) A cycle
- B) A tree
- C) A path
- D) A star

**Answer: B**

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**50. Which of the following is always true in a tree?**

- A) Number of edges = number of vertices
- B) There is exactly one path between any pair of vertices
- C) All degrees are equal
- D) There is a cycle

**Answer: B**

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**51. Which of these is not a tree?**

- A) A 3-vertex path
- B) A 3-vertex cycle
- C) A star graph
- D) A connected graph with 4 vertices and 3 edges

**Answer: B**

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**52. The diameter of a tree is:**

- A) Number of leaves
- B) Number of edges
- C) Length of the longest path
- D) Maximum degree

**Answer: C**

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**53. A spanning tree of a graph:**

- A) Contains all vertices and forms a tree
- B) Has cycles
- C) Must be complete
- D) Must have the minimum number of vertices

**Answer: A**

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**54. The number of spanning trees of  $K_n$  is:**

- A)  $n^{n-1}$
- B)  $n^{n-2}$
- C)  $\binom{n-1}{2}$
- D)  $2^{n-1}$

**Answer: B**

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**55. A graph isomorphism must preserve:**

- A) Number of vertices only
- B) Number of edges only
- C) Degree sequence and adjacency
- D) Labeling

**Answer: C**

**56. A graph with all degrees 1 is:**

- A) Cycle
- B) Tree
- C) Disjoint set of edges
- D) Complete

**Answer: C**

**57. The minimum number of edges in a connected graph with 7 vertices:**

- A) 6
- B) 7
- C) 5
- D) 10

**Answer: A**

# Understanding Graphs with All Degree 1 Vertices

*Arey wah! Graph theory ka maza aata hai!* (Wow! Graph theory is so enjoyable!)

When we have a graph where every vertex has exactly degree 1, what kind of structure does this create? Let's think through this:

If every vertex has degree 1, then each vertex is connected to exactly one other vertex - no more, no less. This creates a very specific type of graph structure.

## Why Option C is Correct

A graph where all vertices have degree 1 must be a **disjoint set of edges**. This means:

- Each component consists of exactly two vertices connected by a single edge
- The graph consists of multiple disconnected pairs of vertices
- No vertex can be connected to more than one other vertex

**Real-world example:** Think of a dance where everyone must hold hands with exactly one partner. The resulting arrangement would be pairs of people scattered across the room, with no chains or groups larger than 2 people.

## Why Other Options Are Incorrect

- **Cycle (A):** In a cycle, every vertex must have degree 2 (connected to two neighbors)
- **Tree (B):** A tree with more than 2 vertices must have some vertices with degree greater than 1
- **Complete graph (D):** In a complete graph with  $n$  vertices, each vertex has degree  $n-1$

**Quick tip:** When analyzing degree sequences, try drawing small examples to visualize what the graph must look like. For degree-1 vertices, each must terminate a path or be part of an isolated edge.

*Bahut badhiya! Ab aap samajh gaye honge!* (Very good! Now you must have understood!)

**Interesting fact:** The total number of vertices in a graph where all degrees are 1 must be even, because each edge accounts for exactly two degree-1 vertices.

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**58. A cut-vertex is a vertex that:**

- A) Is connected to all others
- B) Has highest degree
- C) On removal increases number of components
- D) Has degree 1

**Answer: C**

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**59. A bridge in a graph is:**

- A) An edge that, if removed, increases the number of connected components
- B) A loop
- C) A multi-edge
- D) A cycle

**Answer: A**

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**60. A graph traversal method that uses a stack is:**

- A) BFS
- B) DFS
- C) Dijkstra's
- D) Kruskal's

**Answer: B**

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**61. A graph traversal method that uses a queue is:**

- A) DFS
- B) BFS
- C) Prim's
- D) Floyd's

**Answer: B**

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**62. The maximum number of edges in a simple directed graph with  $n$  vertices:**

- A)  $n(n-1)n(n-1)n(n-1)$
- B)  $n^2n^2n^2$
- C)  $n(n-1)2\frac{n(n-1)}{2}2n(n-1)$
- D)  $nnn$

**Answer: A**

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**63. A self-loop in a graph:**

- A) Increases degree by 1
- B) Increases degree by 2
- C) Doesn't affect degree
- D) Is not allowed in any graph

**Answer: B**

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**64. If a graph has 6 vertices of degree 2, how many edges does it have?**

- A) 6
- B) 12
- C) 3
- D) 9

**Answer: A**

**Explanation:** Degree sum = 12  $\Rightarrow$   $12/2 = 6$  edges

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**65. The adjacency matrix of a simple undirected graph is:**

- A) Always symmetric
- B) Diagonal
- C) Asymmetric
- D) Triangular

**Answer: A**

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**66. A complete bipartite graph  $K_{m,n}$  is:**

- A) Simple
- B) Not regular
- C) Not Hamiltonian
- D) Bipartite with all possible edges

**Answer: D**

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**67. If a graph has odd number of vertices with odd degree, it is:**

- A) Not possible
- B) Eulerian
- C) Not Eulerian
- D) Hamiltonian

**Answer: A**

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**68. Which traversal visits all vertices at a given level before going deeper?**

- A) DFS
- B) BFS
- C) Preorder
- D) Inorder

**Answer: B**

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**69. Which is a minimum spanning tree algorithm?**

- A) Dijkstra's
- B) Bellman-Ford
- C) Prim's
- D) DFS

**Answer: C**

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**70. A graph with  $n$  vertices and  $\binom{n}{2}$  edges is:**

- A) Complete
- B) Eulerian
- C) Tree
- D) Bipartite

**Answer: A**

---

**71. How many edges in  $K_{4,5}$ ?**

- A) 20
- B) 9
- C) 16
- D) 36

**Answer: A ( $4 \times 5$ )**

**72. The chromatic number of a bipartite graph is:**

- A) 1
- B) 2
- C) 3
- D) Depends on size

**Answer: B**

**73. A graph with no isolated vertex and all even degrees:**

- A) Is a tree
- B) Has Euler circuit
- C) Has Euler path
- D) Is complete

**Answer: B**

---

**74. Which is true about Hamiltonian cycles?**

- A) Exists in all complete graphs
- B) Exists in all bipartite graphs
- C) Exists only in trees
- D) Exists in graphs with odd degrees

**Answer: A**

---

**75. A star graph has:**

- A) One central vertex connected to all others
- B) All vertices connected to each other
- C) No edges
- D) Equal degree

**Answer: A**

---

**76. If all degrees in a graph are even:**

- A) Euler circuit exists
- B) Hamiltonian path exists
- C) Tree
- D) Bipartite

**Answer: A**

---

**77. A tree with 1 internal node and 3 leaves has:**

- A) 4 vertices
- B) 5 vertices
- C) 3 edges
- D) Both A and C

**Answer: D**

---

**78. A cycle graph with even number of vertices is:**

- A) Eulerian
- B) Hamiltonian
- C) Bipartite
- D) All of the above

**Answer: D**

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**cqs79. Number of edges in a tree with 15 vertices:**

- A) 14
- B) 15
- C) 13
- D) 30

**Answer: A**

---

**80. Final: A graph is connected if:**

- A) Every pair of vertices is joined by a path
- B) It has no loops
- C) It is complete
- D) Its degree sum is even

**Answer: A**