Characterisation of the ISP of Santa's Sleigh

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Specific impulse (ISP) is an indication of the efficiency of a rocket engine and is defined as $I_{sp} = \frac{F}{\dot{m} g_0}$ [2]. In order to estimate this for Santa's sleigh we must assume several things:

- Santa must visit 200 million children in 75 million houses separated by an average of 2.62km meaning he has to travel an average of 8.2×10^6km/s between each house and leaves each house instantly after arrival[3]
- Every child receives 0.5kg of presents so a total of 100 million kg of presents. The mass of the sleigh, Santa and the reindeer can therefore be neglected as they are $\sim$ 2000kg
- The reindeer receive 2 carrots at each house, each weigh 150kg and have a base metabolic rate of 2W/kg[1] and all other energy is converted into acceleration
- Each carrot weighs 0.06kg and provides 25kcal (10.46kJ)
- Air resistance is neglected

Since there is no air resistance and if it is assumed that the acceleration is constant (i.e. the sleigh accelerates at a constant rate for half the flight time between houses and then decelerates at the same constant rate for the second half of the flight) so:

$$t = \frac{1}{2}\frac{d}{v} = \frac{1}{2}\frac{2620}{8.2 \times 10^9} = 1.6 \times 10^{-7}\text{s}$$

$$s = \frac{1}{2}at^2 \rightarrow a = \frac{2s}{t^2} = \frac{2620}{(1.6 \times 10^{-7})^2} = 1 \times 10^{17}\text{m/s}$$

$$F = ma \rightarrow F = 1 \times 10^{25} N$$

If the reindeer get the 75000000×2 carrots evenly over a 24 hour period then they consume $\sim$ 1700 carrots per second (18×10^6W so the energy they consume to stay alive can be neglected). This gives 102kg/s. Plugging these back into the formula for ISP gives:

$$I_{sp} = \frac{F}{\dot{m} g_0} = \frac{1 \times 10^{23}}{9.81 \times 102} = 10^{20}\text{s}^{-1}$$

This is 3333333333333333 times more efficient than the F1 engine which took people to the moon which isn't surprising when you consider the fact that it consumes 200 times less fuel per second but the sleigh weighs 33 times more.

References

[1] K. J. Nilssen, J. A. Sundsfjord, and A. S. Blix. "Regulation of metabolic rate in Svalbard and Norwegian reindeer". In: *American Journal of Physiology-Regulatory, Integrative and Comparative Physiology* 247.5 (1984). DOI: 10.1152/ajpregu.1984.247.5.r837.

[2] *Specific Impulse*. URL: https://www.grc.nasa.gov/WWW/k-12/airplane/specimp.html.

[3] Abigail Wise. *FYI: How Long Would It Take Santa To Deliver Presents To Every Kid On Earth?* URL: https://www.popsci.com/science/article/2012-12/fyi-how-long-would-it-take-santa-deliver-presents-every-kid-earth/.