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McKay correspondence

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Overview

- Statement of the McKay correspondance
- History and motivation
- examples and proofs

McKay correspondance

The theorem involves $G \leq SL_2(\mathbb{C})$, $S = \mathbb{C}[\![x, y]\!]$, and $R = S^G$. It establishes a correspondence between:

— The irreducible representations of G;

— The indecomposable projective modules of the skew group algebra $S \# G$;

— The indecomposable projective modules of the endomorphism ring $\mathrm{End}_R(S)$;

— The indecomposable MCM modules of R.

History and motivation

— Kleinian singularities, $\mathbb{C}^2/G$

— Resolution graphs ADE Dynkin diagrams

— McKay connected the geometry to representation theory

— Herzog showed that the MCM R-modules are direct summands of S

— Auslander showed the relationship between the projective $S\#G$-modules and the MCM R-modules

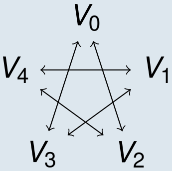
The McKay quiver

Definition

The McKay quiver of G has verticies the irreducible representations of G, and an arrow from W to W' iff W' appears as a direct summand of $V \otimes_{\mathbb{C}} W$, where V is the cannonical representation.

Example

$G = \langle g \rangle \cong \mathbb{Z}/5\mathbb{Z}$, $g = \begin{pmatrix} \omega^2 & 0 \\ 0 & \omega^3 \end{pmatrix}$, $\omega = \exp(2\pi i/5)$,



Non-Dynkin example

Example

$G = \langle \mu, \rho \rangle \cong S_3$, $\mu = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\rho = \begin{pmatrix} \exp(2\pi i/3) & 0 \\ 0 & \exp(-2\pi i/3) \end{pmatrix}$

$$\begin{tikzcd} V_0 \arrow[r, leftrightarrow] & V \arrow[loop above] \arrow[r, leftrightarrow] & V_\mu \end{tikzcd}$$

The skew group algebra

Definition

If S is an algebra and G is a subgroup of $\mathrm{Aut}(S)$, then the *skew group algebra* $S\#G$ is the algebra generated by $s \cdot g$ with $s \in S$ and $g \in G$. The multiplication is given by

$$(s \cdot g)(t \cdot h) = st^g \cdot gh$$

Example

Let $S = \mathbb{C}[\![x, y]\!]$, and $G = \left\{ e := \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, g := \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \right\}$.
Then $x \cdot e + y \cdot g$ and $(x - y) \cdot g$ are in $S\#G$ and their product is

$$(x \cdot e + y \cdot g)((x - y) \cdot g) = x(x - y) \cdot g - y(x + y) \cdot e$$

Main theorem

Definition

$\LaTeX$ makes things easier.

Theorem

This is a theorem

Proof.

*For details, see Flynn [**latex**].* $\square$

References