CHAPTER 5

Logical Agents in AI

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Motivation of Knowledge-Based Agents

- Humans know things and do reasoning, which are important for artificial agents
- Knowledge-based agents benefit from knowledge expressed in general forms, combining information to suit various purposes
- Knowledge and reasoning is important when dealing with partially observable environments
- Understanding natural language requires inferring hidden states
- Flexibility for accepting new tasks

Knowledge Bases

- Knowledge base = set of sentences in a formal language
- Declarative approach to building an agent (or other system):
  - **Tell** it what it needs to know
  - Then it can **Ask** itself what to do - answers should follow from the KB
- Both "Tell" and "Ask" involve inference – deriving new sentences from old

A Simple Knowledge-Based Agent

```
function KB-AGENT(percept) returns an action
    static: KB, a knowledge base
            t, a counter, initially 0, indicating time

    TELL(KB, MAKE-PERCEPT-SENTENCE(percept, t))
    action ← ASK(KB, MAKE-ACTION-QUERY(t))
    TELL(KB, MAKE-ACTION-SENTENCE(action, t))
    t ← t + 1
    return action
```

- Details hidden in three functions
- Similar to agent with internal state in Chapter 2

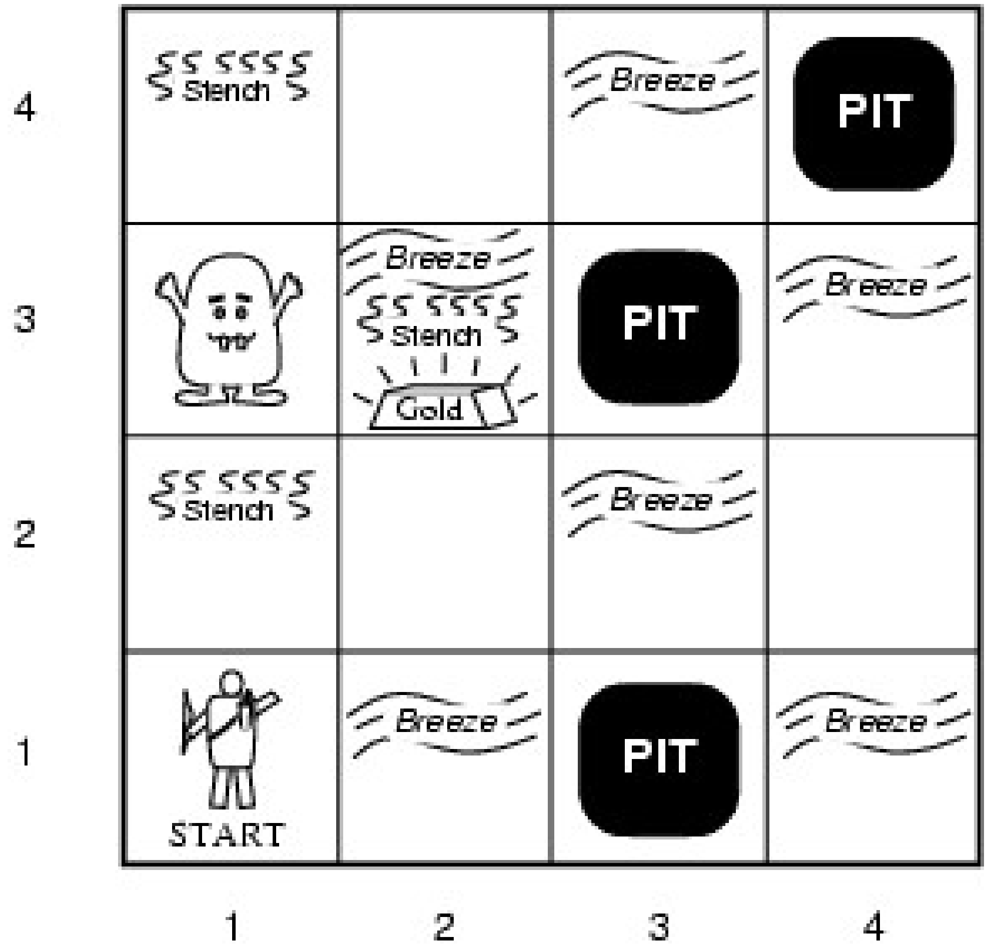
- Agents can be viewed at the knowledge level
  i.e., what they know, regardless of how they are implemented

- Or at the implementation level
  - i.e., data structures in KB and algorithms that manipulate them

Agent's Requirements

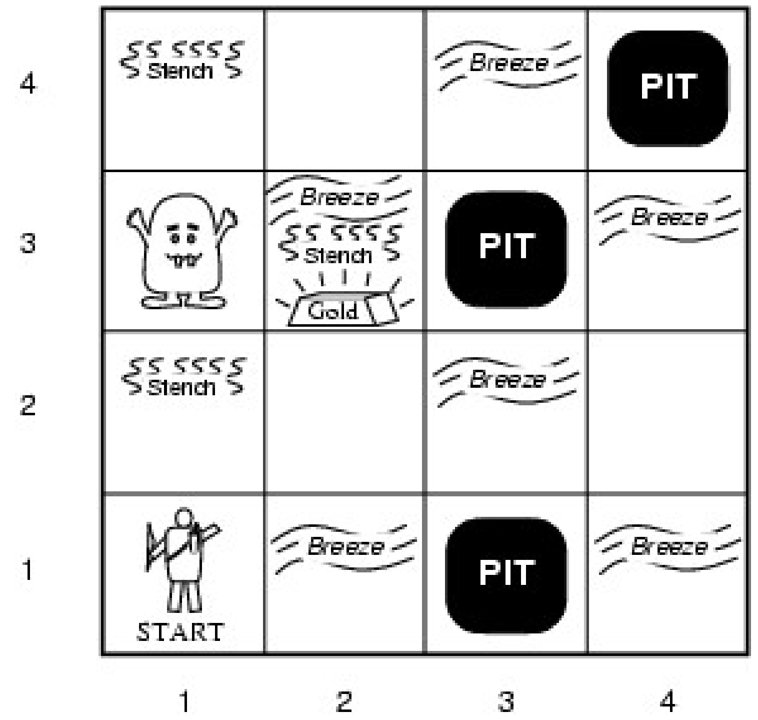
- The agent must be able to:
  - Represent states, actions, etc.
  - Incorporate new percepts
  - Update internal representations of the world
  - Deduce hidden properties of the world
  - Deduce appropriate actions

Our New Toy Problem – Wumpus World



Wumpus World PEAS

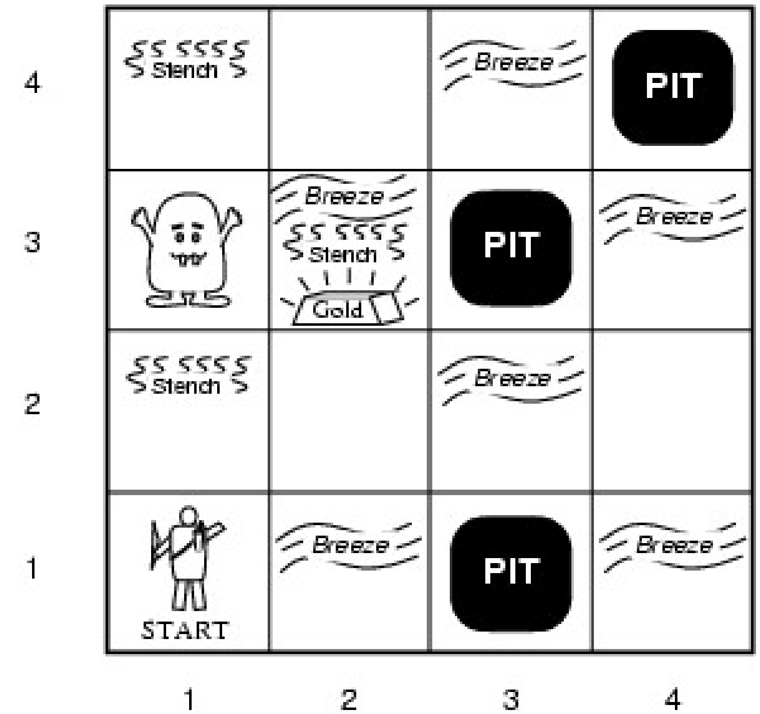
- Performance measure
  - gold +1000, death -1000
  - -1 per step, -10 for using the arrow
- Environment
  - 4x4 grid of rooms
  - Agent always starts at [1,1]
  - Wumpus and gold randomly chosen
  - Each square other than start is pit with 0.2 probability



Wumpus World PEAS

- Actuators: Left turn, Right turn, Forward, Grab, Release, Shoot, Climb
  - Shooting kills wumpus if you are facing it
  - Shooting uses up the only arrow
  - Grabbing picks up gold if in same square
  - Releasing drops the gold in same square
  - Climbing out of the cave from [1, 1]

- Sensors (percepts): Stench, Breeze, Glitter, Bump, Scream
  - Squares adjacent to wumpus are smelly
  - Squares adjacent to pit are breezy
  - Glitter iff gold is in the same square
  - Bump when agent walks into a wall
  - When wumpus is killed, it screams

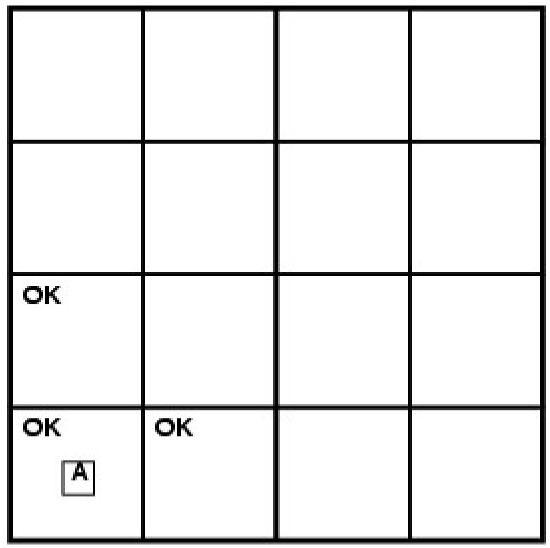


Wumpus World Properties

- Fully observable?
  - No – only local perception
- Deterministic?
  - Yes – outcomes exactly specified
- Episodic?
  - No – sequential at the level of actions
- Static?
  - Yes – Wumpus and Pits do not move
- Discrete?
  - Yes
- Single-agent?
  - Yes – Wumpus is essentially a natural feature

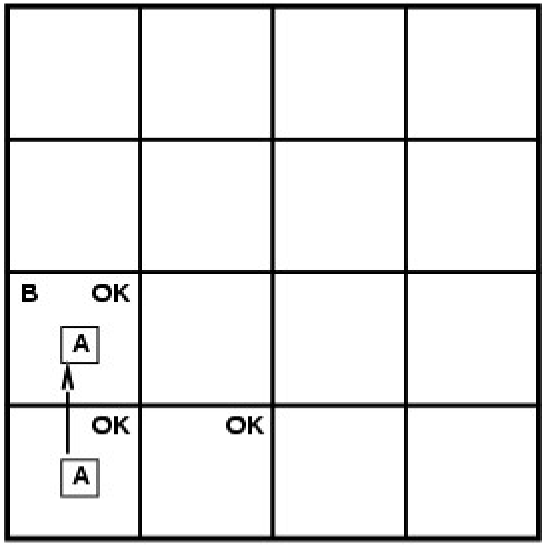
Exploring a Wumpus World

Agent's initial knowledge base contains the rules of the environment: it knows that it's at [1,1] and it's safe at [1,1]



percept: [none, none, none, none, none]

Exploring a Wumpus World



percept: [none, breeze, none, none, none]

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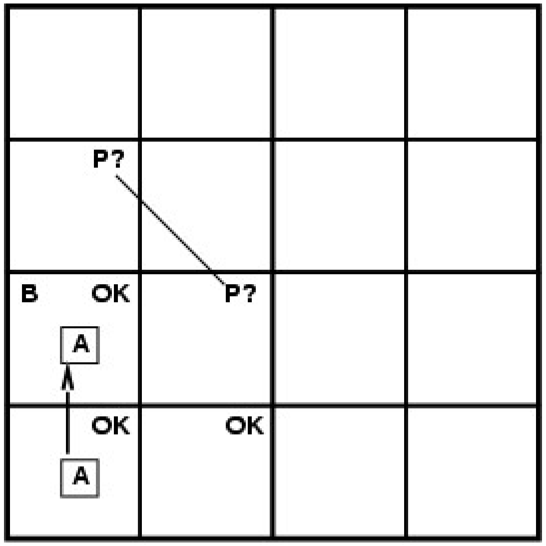
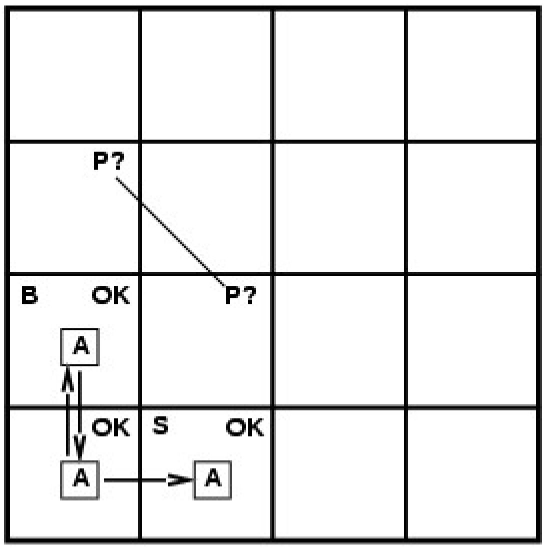


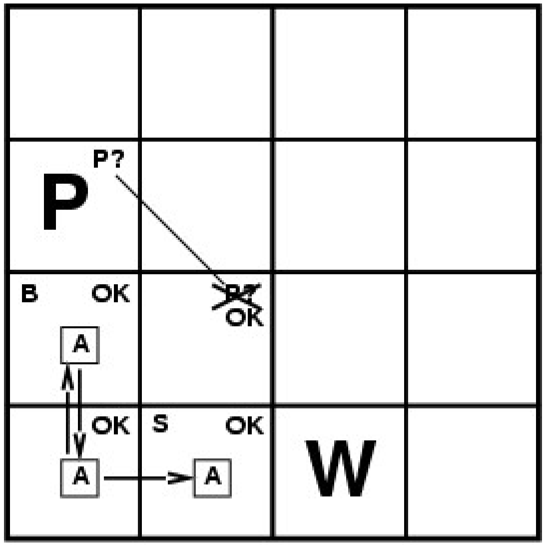
percept: [none, breeze, none, none, none]

Exploring a Wumpus World



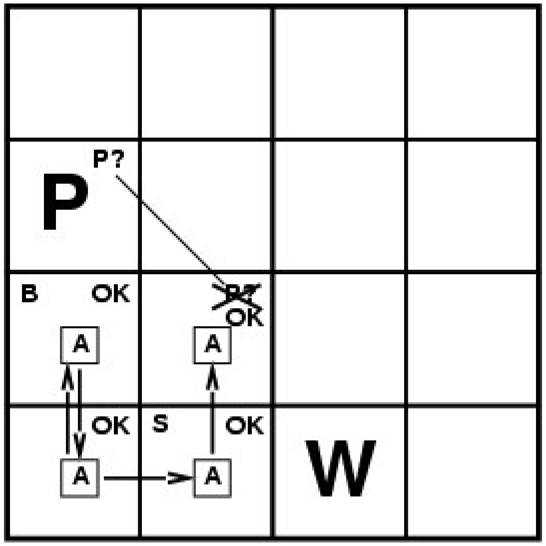
our cautious agent will go back and to [2,1]

Exploring a Wumpus World



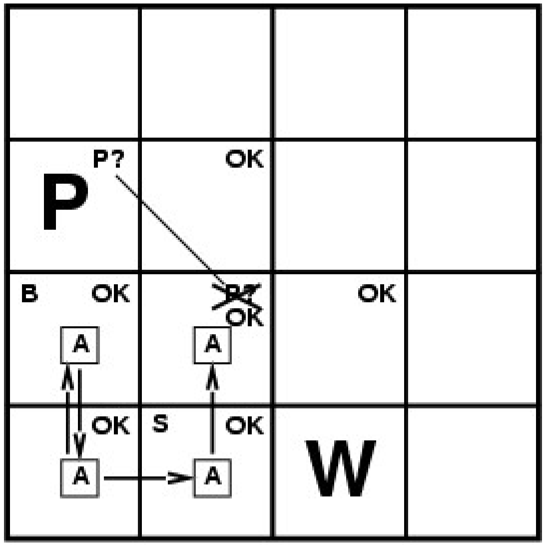
percept: [stench, none, none, none, none]

Exploring a Wumpus World

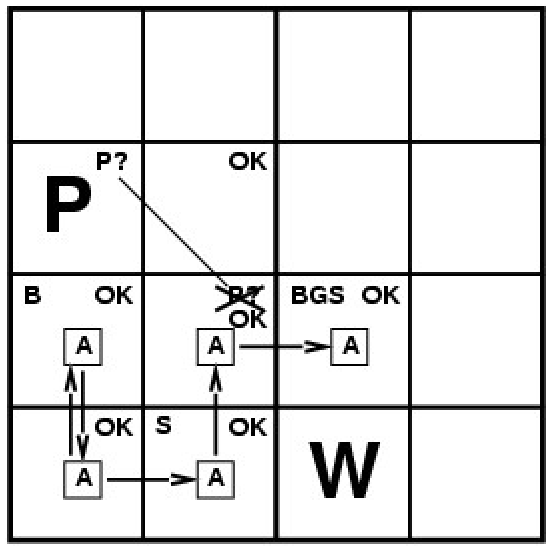


percept: [none, none, none, none, none]

Exploring a Wumpus World



Exploring a Wumpus World



percept: [stench, breeze, glitter, none, none]

Logical Reasoning

- In each case where the agent draws a conclusion from the available information, that conclusion is guaranteed to be correct if the available information is correct

- The above is the fundamental of logical reasoning

Logic in General

- Logics are formal languages for representing information such that conclusions can be drawn
- Syntax defines the sentences in the language
- Semantics define the "meaning" of sentences;
  - i.e., define truth of a sentence in a world

- E.g., the language of arithmetic
  - $x+2 \geq y$ is a sentence; $x2+y > \{\}$ is not a sentence
  - $x+2 \geq y$ is true iff the number $x+2$ is no less than the number y
  - $x+2 \geq y$ is true in a world where $x = 7, y = 1$
  - $x+2 \geq y$ is false in a world where $x = 0, y = 6$

Entailment

- Now, we have a notion of truth, we are ready to study logical reasoning, which involves logical entailment between sentences
- Entailment means that one thing follows from another:

$$KB \models \alpha$$

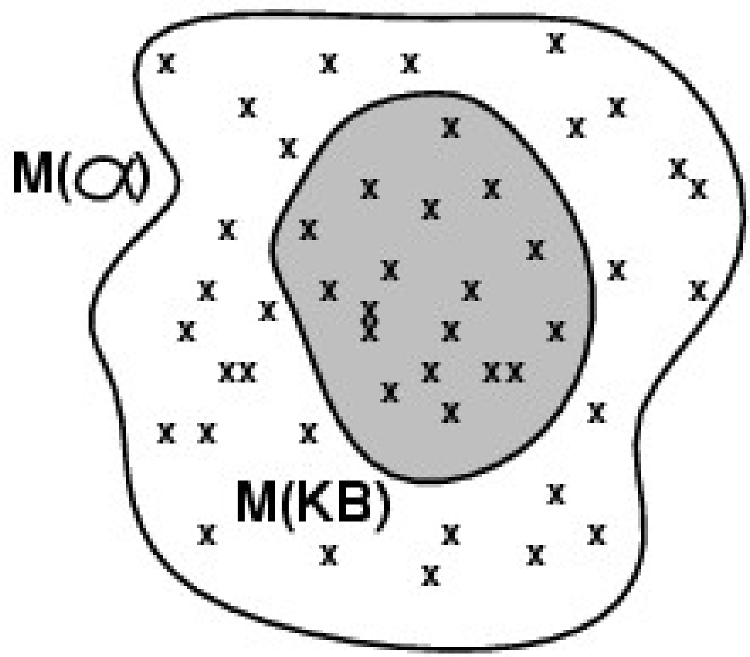
- Knowledge base *KB* entails sentence *α* if and only if *α* is true in all worlds where *KB* is true
  - E.g., the KB containing "the Giants won" and "the Reds won" entails "Either the Giants won or the Reds won"
  - E.g., x+y = 4 entails 4 = x+y
  - Entailment is a relationship between sentences (i.e., syntax) that is based on semantics

Models

- We use the term "model" to represent "possible world"
- We say m is a model of a sentence α if α is true in model m

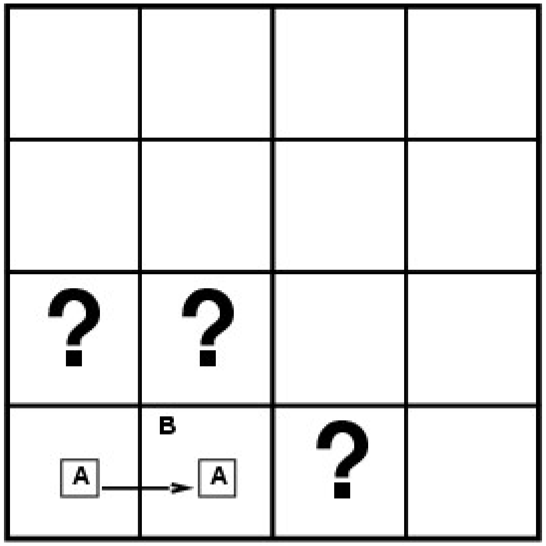
- $M(\alpha)$ is the set of all models of α

- Then KB $\models \alpha$ iff $M(KB) \subseteq M(\alpha)$
  - E.g. KB = Giants won and Reds won, α = Giants won

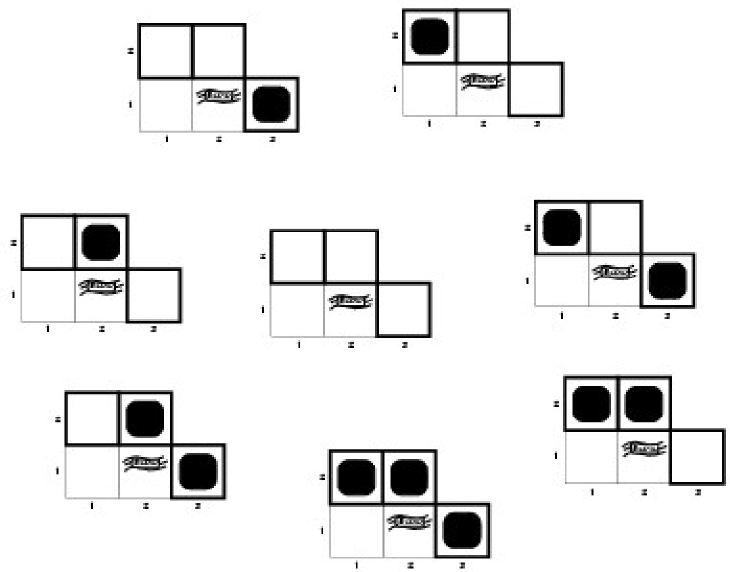


Entailment in the Wumpus World

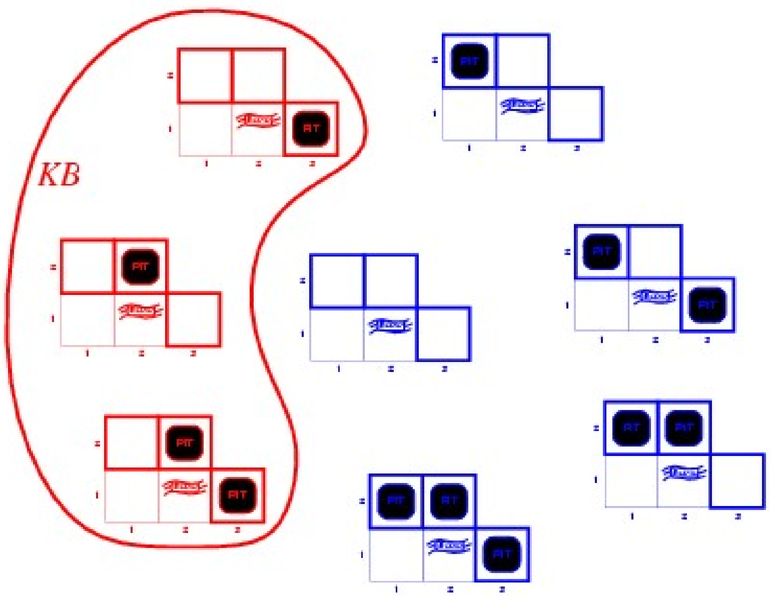
- Situation after detecting nothing in [1,1], moving right, breeze in [2,1]
- Consider possible models for *KB* assuming only pits
- 3 Boolean choices $\Rightarrow$ 8 possible models



Wumpus Models

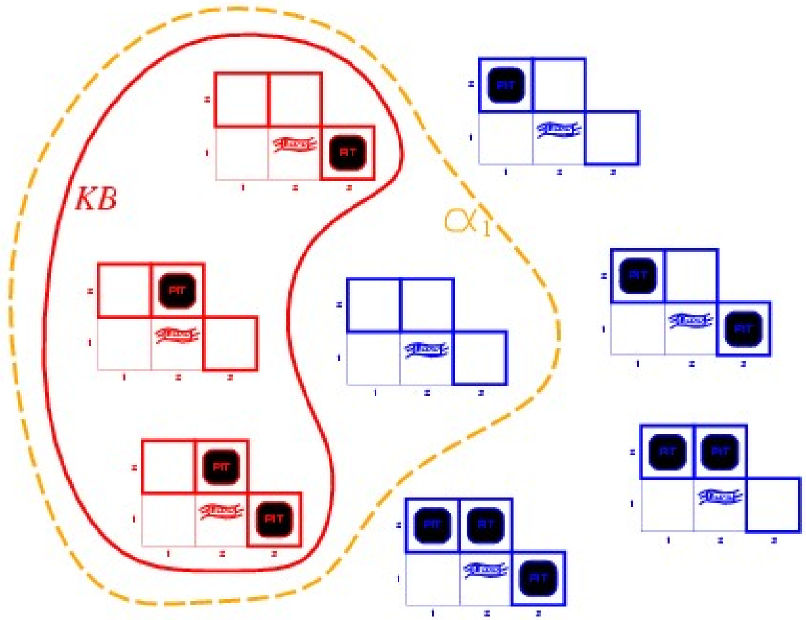


Wumpus Models



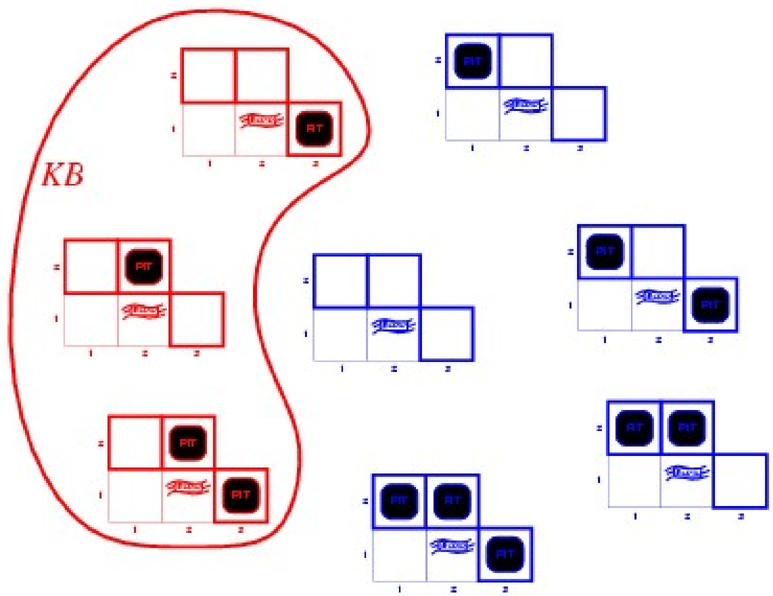
- *KB* = wumpus-world rules + observations

Wumpus Models



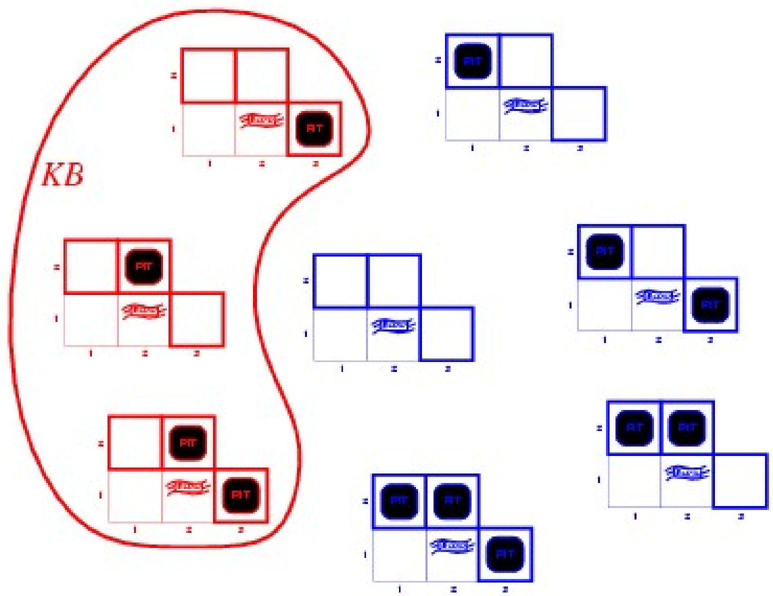
- *KB* = wumpus-world rules + observations
- α_1 = "[1,2] is safe", $KB \models \alpha_1$, proved by model checking
  - enumerate all possible models to check that α is true in all models in which KB is true

Wumpus Models



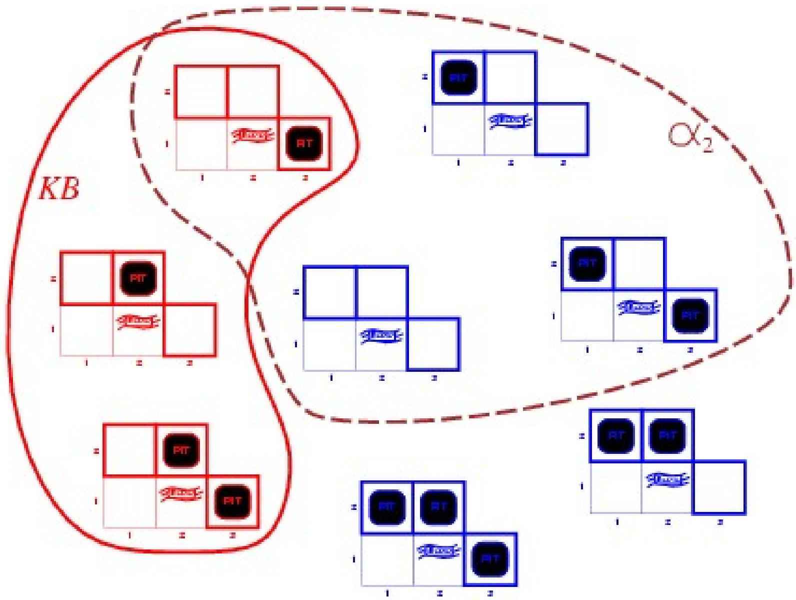
- *KB* = wumpus-world rules + observations

Wumpus Models



- *KB* = wumpus-world rules + observations
- α_2 = "[2,2] is safe", $KB \models \alpha_2$?

Wumpus Models



- *KB* = wumpus-world rules + observations
- α_2 = "[2,2] is safe", $KB \not\models \alpha_2$

Soundness and Completeness

- An inference algorithm that derives only entailed sentences is called sound
- An inference algorithm is complete if it can derive any sentence that is entailed
- If KB is true in the real world, then any sentence derived from KB by a sound inference procedure is also true in the real world

Grounding Problem

- Grounding: how do we know that KB is true in the real world?
  - Agent's sensors create the connection
    - Meaning and truth of percept sentences through sensing and sentence construction
  - General rules
    - From perceptual experience

Propositional Logic: Syntax

- Propositional logic is the simplest logic – illustrates basic ideas

- The proposition symbols S_1, S_2 etc are sentences
  - If S is a sentence, $\neg S$ is a sentence (negation)
  - If S_1 and S_2 are sentences, $S_1 \wedge S_2$ is a sentence (conjunction)
  - If S_1 and S_2 are sentences, $S_1 \vee S_2$ is a sentence (disjunction)
  - If S_1 and S_2 are sentences, $S_1 \Rightarrow S_2$ is a sentence (implication)
  - If S_1 and S_2 are sentences, $S_1 \Leftrightarrow S_2$ is a sentence (biconditional)

Propositional Logic: Semantics

Each model specifies true/false for each proposition symbol

E.g.	$P_{1,2}$	$P_{2,2}$	$P_{3,1}$
	false	true	false

With these symbols, 8 possible models, can be enumerated automatically.

Rules for evaluating truth with respect to a model *m*:

$\neg S$	is true iff	S is false		
$S_1 \wedge S_2$	is true iff	S_1 is true	and	S_2 is true
$S_1 \vee S_2$	is true iff	S_1 is true	or	S_2 is true
$S_1 \Rightarrow S_2$	is true iff	S_1 is false	or	S_2 is true
i.e.,	is false iff	S_1 is true	and	S_2 is false
$S_1 \Leftrightarrow S_2$	is true iff	$S_1 \Rightarrow S_2$ is true	and	$S_2 \Rightarrow S_1$ is true

Simple recursive process evaluates an arbitrary sentence, e.g.,

$$\neg P_{1,2} \wedge (P_{2,2} \vee P_{3,1}) = \mathit{true} \wedge (\mathit{true} \vee \mathit{false}) = \mathit{true} \wedge \mathit{true} = \mathit{true}$$

Truth Tables for Connectives

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
$false$	$false$	$true$	$false$	$false$	$true$	$true$
$false$	$true$	$true$	$false$	$true$	$true$	$false$
$true$	$false$	$false$	$false$	$true$	$false$	$false$
$true$	$true$	$false$	$true$	$true$	$true$	$true$

Wumpus World Sentences

- Let $P_{i,j}$ be true if there is a pit in [i, j]
- Let $B_{i,j}$ be true if there is a breeze in [i, j]
  - R1: $\neg P_{1,1}$
  - R2: $\neg B_{1,1}$
  - R3: $B_{2,1}$
- "Pits cause breezes in adjacent squares"
  - R4: $B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$
  - R5: $B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$
- The KB consists of the above 5 sentences. It can also be considered as a single sentence – the conjunction R1 $\wedge$ R2 $\wedge$ R3 $\wedge$ R4 $\wedge$ R5

Inference

- The aim of logical inference is to decide whether KB $\models \alpha$ for some sentence α.

- Is $P_{2,2}$ entailed?

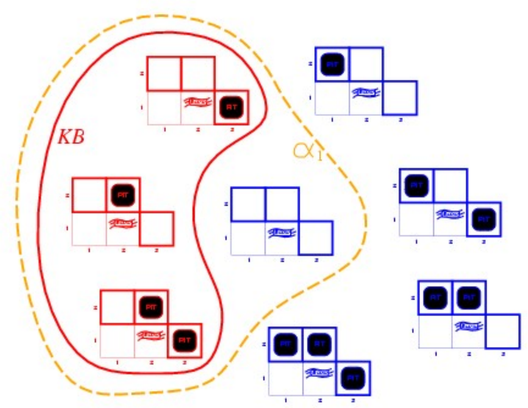
- Our first algorithm for inference will be a direct implementation of the definition of entailment:
  - Enumerate the models, and check that α is true in every model in which KB is true

Truth Tables for Inference

$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	KB	α_1
false	*false*	*false*	*false*	*false*	*false*	*false*	*false*	*true*
false	*false*	*false*	*false*	*false*	*false*	*true*	*false*	*true*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
false	*true*	*false*	*false*	*false*	*false*	*false*	*false*	*true*
false	*true*	*false*	*false*	*false*	*false*	*true*	*true*	*true*
false	*true*	*false*	*false*	*false*	*true*	*false*	*true*	*true*
false	*true*	*false*	*false*	*false*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*true*	*false*	*false*	*false*	*true*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
true	*true*	*true*	*true*	*true*	*true*	*true*	*false*	*false*

$P_{1,2}$ is false, cannot decide on $p_{2,2}$

α_1 = "[1,2] is safe"



Inference by Enumeration

- Depth-first enumeration of all models is sound and complete

```
function TT-ENTAILS?(KB, α) returns true or false
    symbols ← a list of the proposition symbols in KB and α
    return TT-CHECK-ALL(KB, α, symbols, [])
---------------------------------------------------------------
function TT-CHECK-ALL(KB, α, symbols, model) returns true or false
    if EMPTY?(symbols) then
        if PL-TRUE?(KB, model) then return PL-TRUE?(α, model)
        else return true
    else do
        P ← FIRST(symbols); rest ← REST(symbols)
        return TT-CHECK-ALL(KB, α, rest, EXTEND(P, true, model)) and
                   TT-CHECK-ALL(KB, α, rest, EXTEND(P, false, model))
```

- For n symbols, time complexity is $O(2^n)$, space complexity is $O(n)$

  PL-TRUE? returns true if a sentence holds within a model.

Proof Methods In General

- Proof methods divide into (roughly) two kinds:
  - Model checking
    - truth table enumeration (always exponential in *n*)
    - improved backtracking, e.g., Davis--Putnam-Logemann-Loveland (DPLL)
    - heuristic search in model space (sound but incomplete)
      - e.g., min-conflicts-like hill-climbing algorithms
  - Application of inference rules
    - Legitimate (sound) generation of new sentences from old
    - Proof = a sequence of inference rule applications
    - Can use inference rules as operators in a standard search algorithm
    - Typically require transformation of sentences into a normal form

Validity and Satisfiability

- A sentence is valid if it is true in all models, also known as tautology
  - e.g., *True*, $A \vee \neg A$, $A \Rightarrow A$, $(A \wedge (A \Rightarrow B)) \Rightarrow B$
- Validity is connected to inference via the Deduction Theorem
  - $KB \models \alpha$ if and only if ($KB \Rightarrow \alpha$) is valid
- A sentence is satisfiable if it is true in some model
  - e.g., $A \vee B$, C
- A sentence is unsatisfiable if it is true in no models
  - e.g., $A \wedge \neg A$
- Satisfiability is connected to inference via the following:
  - $KB \models \alpha$ if and only if ($KB \wedge \neg\alpha$) is unsatisfiable
  - Thus proof by contradiction

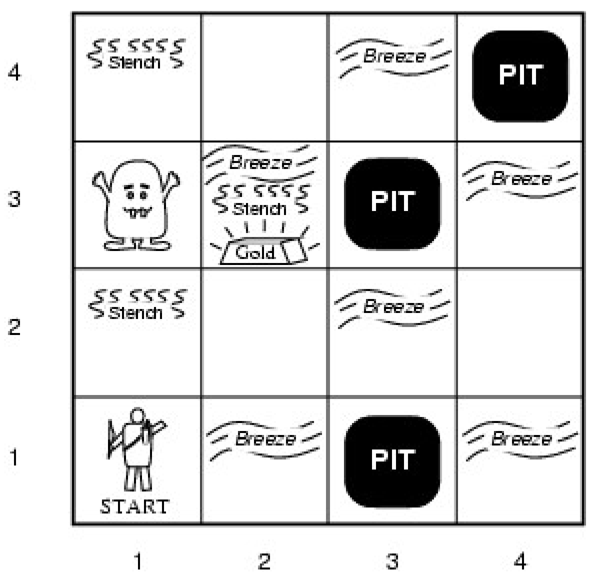
Equivalence Rules

- Two sentences are logically equivalent iff they are true in the same models: $\alpha \equiv \beta$ iff $\alpha \models \beta$ and $\beta \models \alpha$

$$
\begin{aligned}
(\alpha \land \beta) &\equiv (\beta \land \alpha) \quad \text{commutativity of } \land \\
(\alpha \lor \beta) &\equiv (\beta \lor \alpha) \quad \text{commutativity of } \lor \\
((\alpha \land \beta) \land \gamma) &\equiv (\alpha \land (\beta \land \gamma)) \quad \text{associativity of } \land \\
((\alpha \lor \beta) \lor \gamma) &\equiv (\alpha \lor (\beta \lor \gamma)) \quad \text{associativity of } \lor \\
\neg(\neg\alpha) &\equiv \alpha \quad \text{double-negation elimination} \\
(\alpha \Rightarrow \beta) &\equiv (\neg\beta \Rightarrow \neg\alpha) \quad \text{contraposition} \\
(\alpha \Rightarrow \beta) &\equiv (\neg\alpha \lor \beta) \quad \text{implication elimination} \\
(\alpha \Leftrightarrow \beta) &\equiv ((\alpha \Rightarrow \beta) \land (\beta \Rightarrow \alpha)) \quad \text{biconditional elimination} \\
\neg(\alpha \land \beta) &\equiv (\neg\alpha \lor \neg\beta) \quad \text{de Morgan} \\
\neg(\alpha \lor \beta) &\equiv (\neg\alpha \land \neg\beta) \quad \text{de Morgan} \\
(\alpha \land (\beta \lor \gamma)) &\equiv ((\alpha \land \beta) \lor (\alpha \land \gamma)) \quad \text{distributivity of } \land \text{ over } \lor \\
(\alpha \lor (\beta \land \gamma)) &\equiv ((\alpha \lor \beta) \land (\alpha \lor \gamma)) \quad \text{distributivity of } \lor \text{ over } \land
\end{aligned}
$$

Inference Rules

From	Can Derive	Abbreviation for rule
$R, R \rightarrow S$	S	Modus Ponens- **mp**
$R \rightarrow S, S'$	R'	Modus Tollens- **mt**
$R, \; S$	$R \wedge S$	Conjunction-**con**
$R \wedge S$	R, S	Simplification- **sim**
R	$R \vee S$	Addition- **add**



- R1: $\neg P_{1,1}$
- R2: $B_{1,1} \Leftrightarrow P_{1,2} \vee P_{2,1}$
- R3: $B_{2,1} \Leftrightarrow P_{1,1} \vee P_{2,2} \vee P_{3,1}$
- R4: $\neg B_{1,1}$
- R5: $B_{2,1}$

- Can we prove $\neg P_{1,2}$?

Now, Your Turn

From	Can Derive	Abbreviation for rule
R, R → S	S	Modus Ponens- **mp**
R → S, S′	R′	Modus Tollens- **mt**
R, S	R ∧ S	Conjunction-**con**
R ∧ S	R, S	Simplification- **sim**
R	R ∨ S	Addition- **add**

- Prove $\neg P_{2,1}$ and $P_{3,1}$ given:

$R1: \neg P_{1,1}$

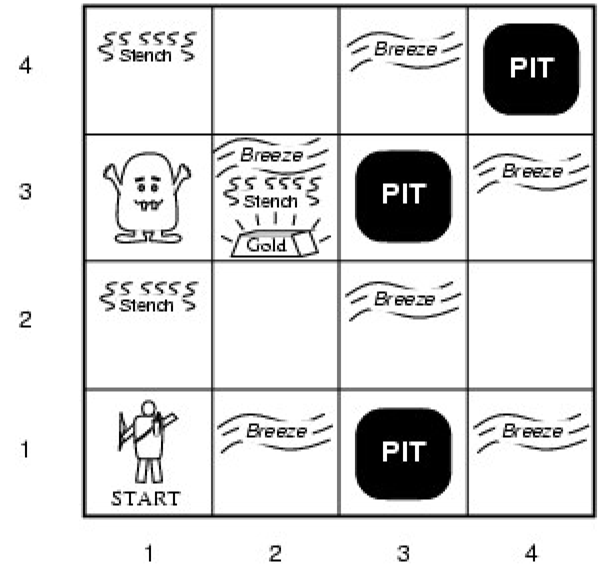
$R2: B_{1,1} \Leftrightarrow P_{1,2} \vee P_{2,1}$

$R3: B_{2,1} \Leftrightarrow P_{1,1} \vee P_{2,2} \vee P_{3,1}$

$R4: \neg B_{1,1}$

$R5: B_{2,1}$

$R6: \neg P_{2,2}$



Proof Problem as Search Problem

- Initial state: initial knowledge base
- Actions:
  - set of actions consists of all the inference rules applied to all the sentences that match the left half of the inference rule
- Result: add the sentence in the right half of the inference rule
- Goal: the goal is a state that contains the sentence we are trying to prove

- Practically, finding a proof can be more efficient because the proof can ignore irrelevant propositions, no matter how many of them there are

Resolution

- The inference rules can be applied whenever suitable premises are found in the KB
- The conclusion of the rule must follow regardless of what else is in the KB
- The above rules are sound, but if the available inference rules are inadequate, then it's not complete
- Now, we introduce a single inference rule, **resolution**, that gives a complete inference algorithm when coupled with any complete search algorithm

Example

- Recall our previous proof of $P_{3,1}$ given:

$R1: \neg P_{1,1}$

$R2: B_{1,1} \Leftrightarrow P_{1,2} \vee P_{2,1}$

$R3: B_{2,1} \Leftrightarrow P_{1,1} \vee P_{2,2} \vee P_{3,1}$

$R4: \neg B_{1,1}$

$R5: B_{2,1}$

$R6: \neg P_{2,2}$

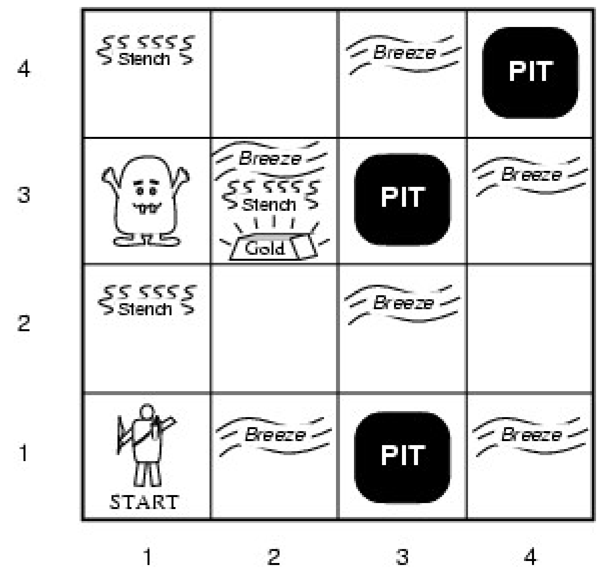
...

Unit resolution

$R9: P_{1,1} \vee P_{2,2} \vee P_{3,1}$

$R10: P_{2,2} \vee P_{3,1}$ Resolution rule: $\neg P_{1,1}$ resolves with $P_{1,1}$ in R9

$R11: P_{3,1}$ Resolution rule: $\neg P_{2,2}$ resolves with $P_{2,2}$ in R10



Resolution Rules

- Unit resolution
  - l_i and m are complementary literals

$$\frac{\ell_1 \vee \ldots \vee \ell_k, \qquad m}{\ell_1 \vee \ldots \vee \ell_{i-1} \vee \ell_{i+1} \vee \ldots \vee \ell_k}$$

- Generalized resolution rule

$$\frac{\ell_1 \vee \ldots \vee \ell_k, \qquad m_1 \vee \ldots \vee m_n}{\ell_1 \vee \ldots \vee \ell_{i-1} \vee \ell_{i+1} \vee \ldots \vee \ell_k \vee m_1 \vee \ldots \vee m_{j-1} \vee m_{j+1} \vee \ldots \vee m_n}$$

  - where l_i and m_j are complementary literals

Soundness and Completeness

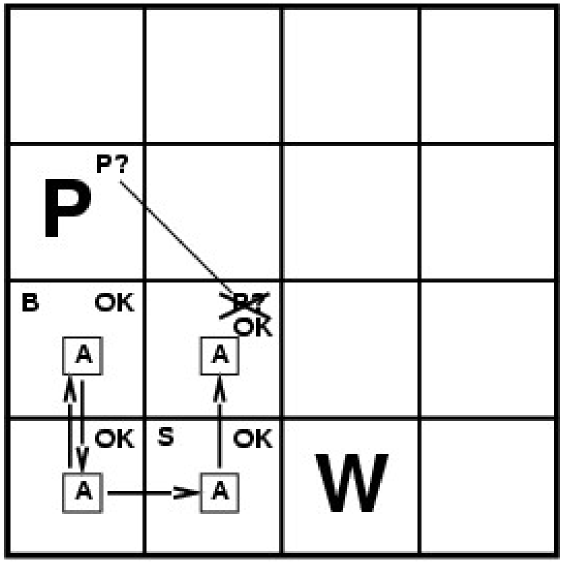
- Soundness of resolution inference rule:
  - if l_i is true, then m_j is false
    - hence $(m_1 \vee \ldots \vee m_{j-1} \vee m_{j+1} \vee \ldots \vee m_n)$ must be true
  - if l_i is false, then
    - $(l_1 \vee \ldots \vee l_{i-1} \vee l_{i+1} \vee \ldots \vee l_k)$ must be true
  - no matter what
    - $(l_1 \vee \ldots \vee l_{i-1} \vee l_{i+1} \vee \ldots \vee l_k) \vee (m_1 \vee \ldots \vee m_{j-1} \vee m_{j+1} \vee \ldots \vee m_n)$ is true

- *Any complete search algorithm, applying only the resolution rule, can derive any conclusion entailed by any knowledge base in propositional logic*

- A resolution-based theorem prover can, for any sentences α and β in propositional logic, decide whether $\alpha \models \beta$

Resolution and CNF

- Resolution rule applies only to disjunctions of literals
- *Every sentence of propositional logic is logically equivalent to a conjunction of disjunctions of literals*
- Conjunctive Normal Form (CNF):
  - conjunction of disjunctions of literals clauses
  - E.g., $(A \vee \neg B) \wedge (B \vee \neg C \vee \neg D)$



$$\frac{P_{1,3} \vee P_{2,2}, \qquad \neg P_{2,2}}{P_{1,3}}$$

Conversion to CNF

$B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$

1. Eliminate $\Leftrightarrow$, replacing $\alpha \Leftrightarrow \beta$ with $(\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)$.

$(B_{1,1} \Rightarrow (P_{1,2} \vee P_{2,1})) \wedge ((P_{1,2} \vee P_{2,1}) \Rightarrow B_{1,1})$

2. Eliminate $\Rightarrow$, replacing $\alpha \Rightarrow \beta$ with $\neg\alpha \vee \beta$.

$(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg(P_{1,2} \vee P_{2,1}) \vee B_{1,1})$

3. Move $\neg$ inwards using de Morgan's rules and double-negation:

$(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge ((\neg P_{1,2} \wedge \neg P_{2,1}) \vee B_{1,1})$

4. Apply distributive law ($\wedge$ over $\vee$) and flatten:

$(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg P_{1,2} \vee B_{1,1}) \wedge (\neg P_{2,1} \vee B_{1,1})$

Resolution Algorithm

- To show that KB $\models \alpha$, we show that (KB ^ $\neg\alpha$) is unsatisfiable, proof by contradiction

- First, (KB ^ $\neg\alpha$) is converted into CNF
- Then the resolution rule is applied to the resulting clauses
  - Each pair that contains complementary literals is resolved to produce a new clause, which is added to the set if it's not present
- Process continues until:
  - no new clauses can be added, thus KB does not entail α
  - two clauses resolve to yield {}, thus KB entails α

- {} is a disjunction of no disjuncts is equivalent to False, thus the contradiction

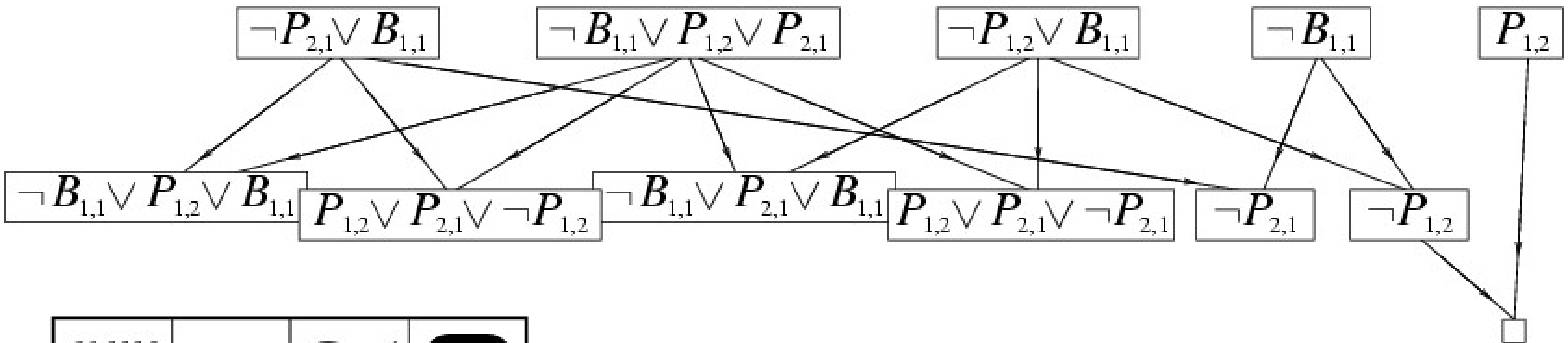
Resolution Algorithm

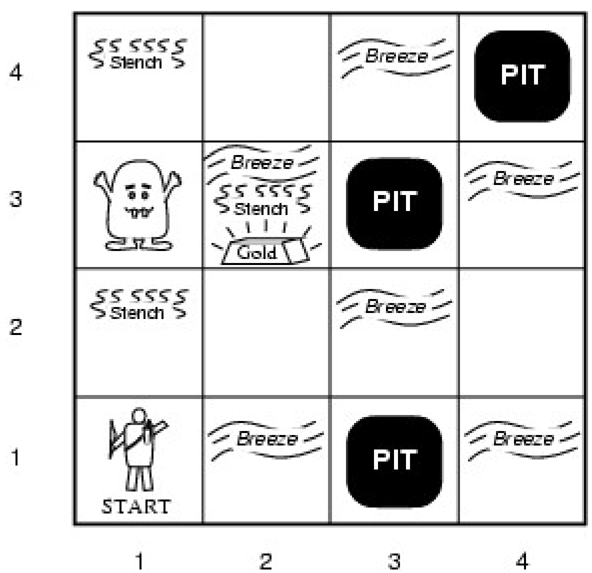
- Proof by contradiction, i.e., show that $KB \wedge \neg\alpha$ is unsatisfiable

```
function PL-RESOLUTION(KB, α) returns true or false
   clauses ← the set of clauses in the CNF representation of KB ∧ ¬α
   new ← { }
   loop do
      for each C_i, C_j in clauses do
         resolvents ← PL-RESOLVE(C_i, C_j)
         if resolvents contains the empty clause then return true
         new ← new ∪ resolvents
      if new ⊆ clauses then return false
      clauses ← clauses ∪ new
```

Resolution Example

- $KB = (B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})) \wedge \neg B_{1,1}$ $\alpha = \neg P_{1,2}$





Horn Clauses

- Why horn clauses?
  - In many practical situations, however, the full power of resolution is not needed
  - Real-world KBs often contain some restricted clauses – Horn clauses

- Horn Form (restricted)
  - KB = conjunction of Horn clauses
  - Horn clause = disjunction of literals of which at most one is positive
    - E.g., $(\neg L_{1,1} \vee \neg Breeze \vee B_{1,1})$

Properties of Horn Clauses

- Every horn clause can be written as an implication whose premise is a conjunction of positive literals and whose conclusion is a single positive literal
  - E.g., ($L_{1,1}$ ^ Breeze) $\Rightarrow B_{1,1}$

- Modus Ponens (for Horn Form): complete for Horn KBs

$$\frac{\alpha_1, \ldots, \alpha_n, \qquad \alpha_1 \wedge \ldots \wedge \alpha_n \Rightarrow \beta}{\beta}$$

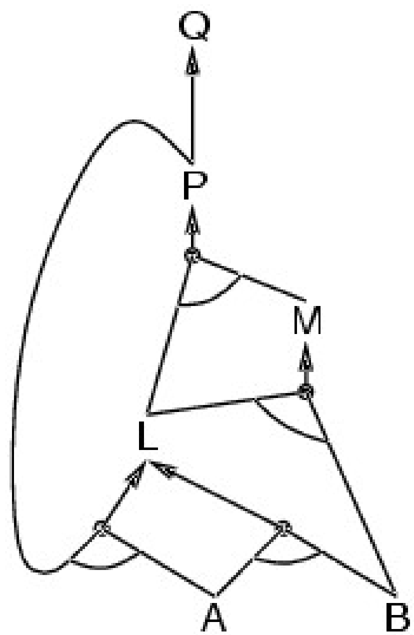
- Definite clause: horn clauses with exactly one positive literal
  - The positive literal is called the **head** and the negative literals form the **body**
  - Definite clauses form the basis for logic programming

- Can be used with forward chaining or backward chaining
- These algorithms are very natural and run in linear time

Forward Chaining

- Idea:
  - fire any rule whose premises are satisfied in the *KB*
  - add its conclusion to the *KB*, until query is found

$$P \Rightarrow Q$$
$$L \wedge M \Rightarrow P$$
$$B \wedge L \Rightarrow M$$
$$A \wedge P \Rightarrow L$$
$$A \wedge B \Rightarrow L$$
$$A$$
$$B$$



and-or graph

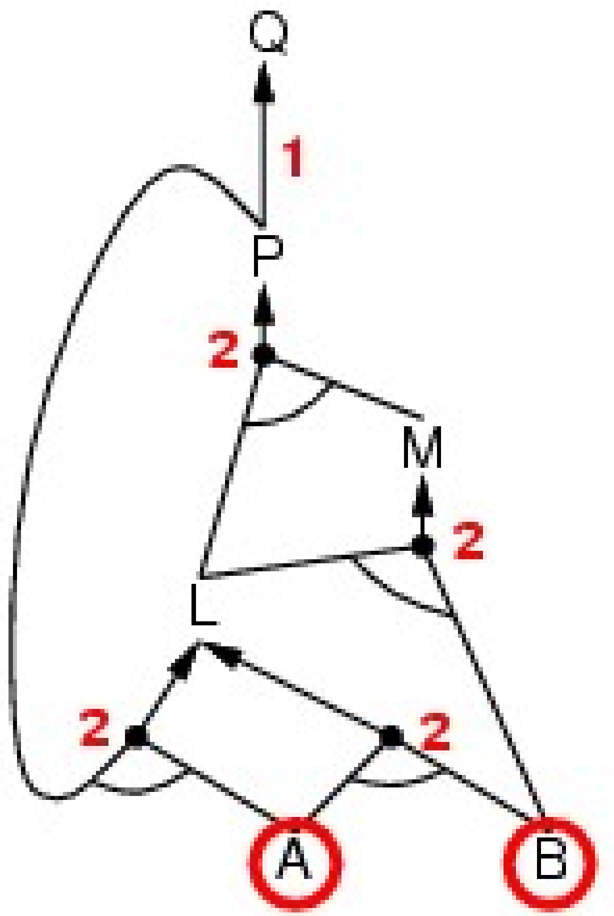
Forward Chaining Algorithm

```
function PL-FC-ENTAILS?(KB, q) returns true or false
  local variables: count, a table, indexed by clause, initially the number of premises
                   inferred, a table, indexed by symbol, each entry initially false
                   agenda, a list of symbols, initially the symbols known to be true

  while agenda is not empty do
      p ← POP(agenda)
      if p = q then return true
      if inferred[p] = false then
         inferred[p] = true
         for each clause c in KB where p is in c.Premise do
            decrement count[c]
            if count[c] = 0 then add c.Conclusion to agenda

  return false
```

- Forward chaining is sound and complete for Horn KB

Forward Chaining Example

$$P \Rightarrow Q$$
$$L \wedge M \Rightarrow P$$
$$B \wedge L \Rightarrow M$$
$$A \wedge P \Rightarrow L$$
$$A \wedge B \Rightarrow L$$
$$A$$
$$B$$



Forward Chaining Example

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A
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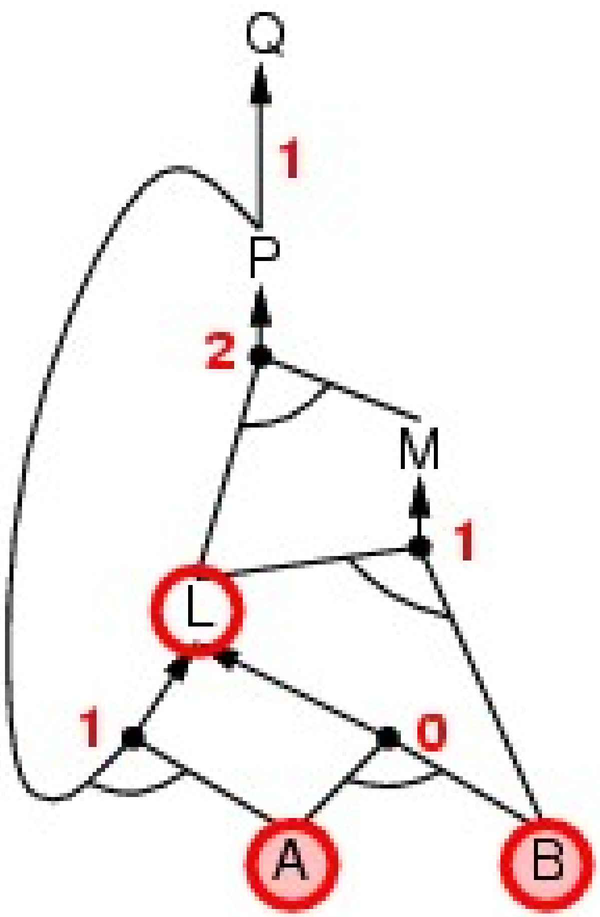

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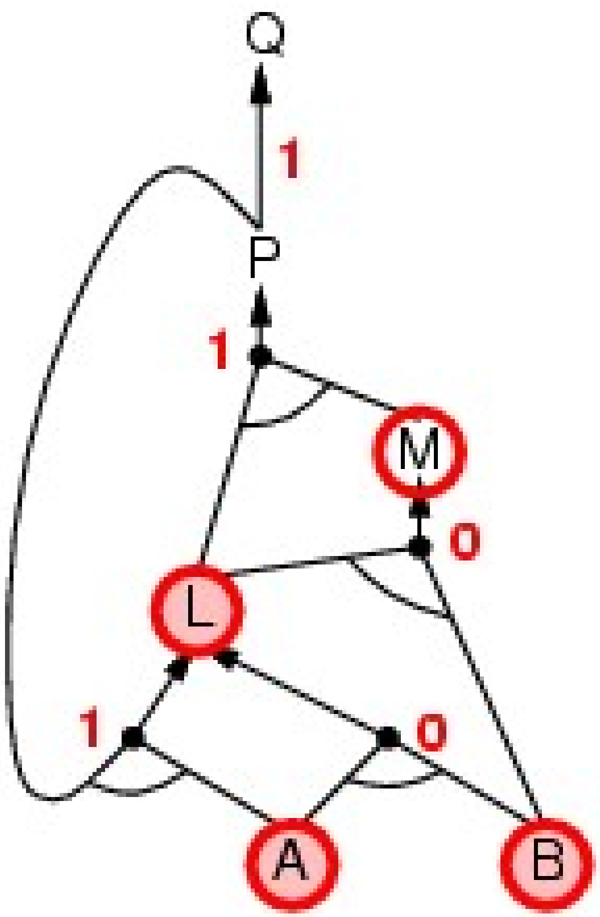

Forward Chaining Example

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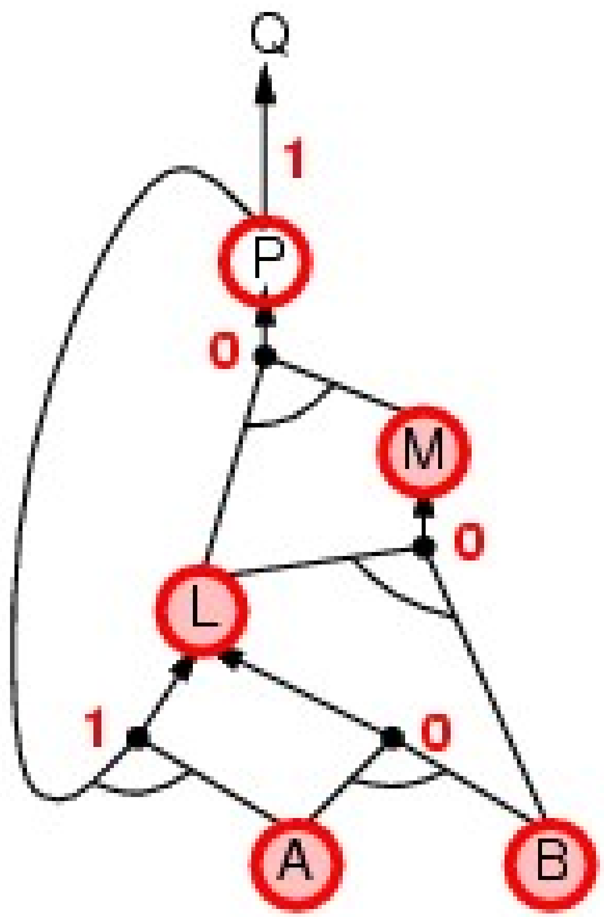

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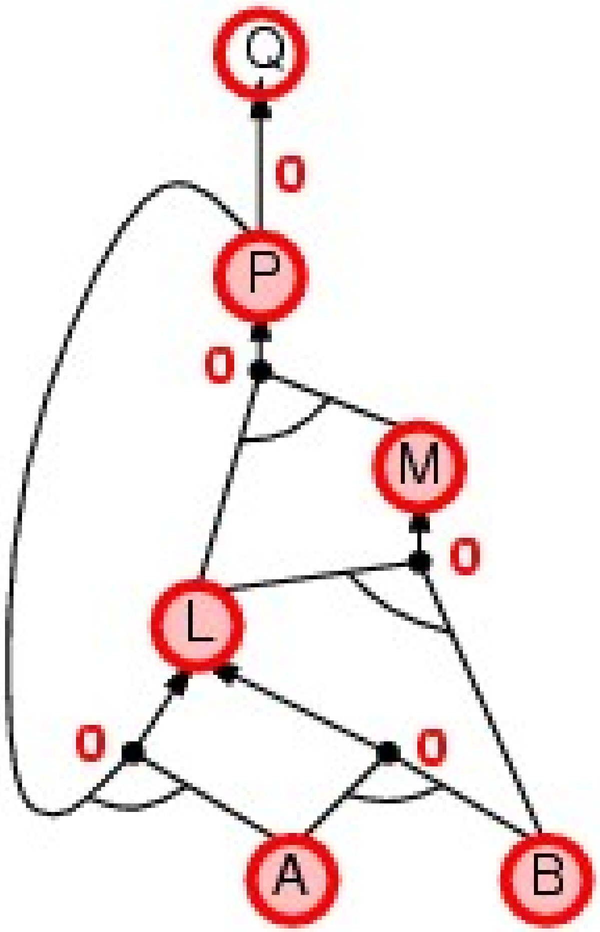

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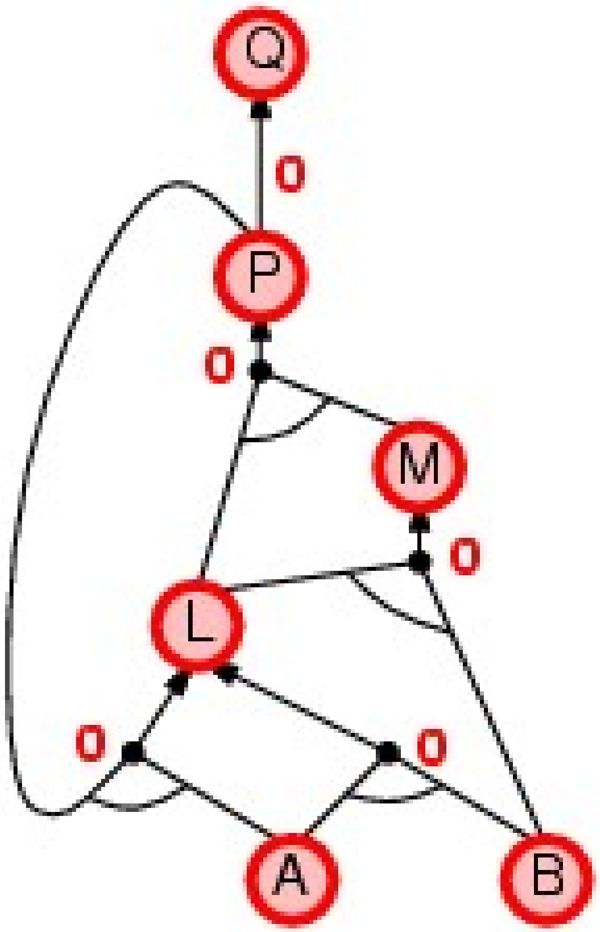

Proof of Soundness

- Every inference is an application of Modus Ponens

Proof of Completeness

- FC derives every atomic sentence that is entailed by *KB*
  1. FC reaches a fixed point where no new atomic sentences are derived
  2. Consider the final state as a model *m*, assigning true/false to symbols; true for each symbol inferred, false otherwise
  3. Every definite clause in the original *KB* is true in *m*

     $a_1 \wedge \ldots \wedge a_k \Rightarrow b$, assuming the opposite, then it contradicts "fixed point"
  4. Hence, every entailed atomic sentence will be derived

Backward Chaining

- Idea: work backwards from the query *q*:
  - to prove *q* by BC
    - check if *q* is known already, or
    - prove by BC all premises of some rule concluding *q*

- Avoid loops: check if new subgoal is already on the goal stack

- Avoid repeated work: check if new subgoal
  - has already been proved true, or
  - has already failed

Backward Chaining Example

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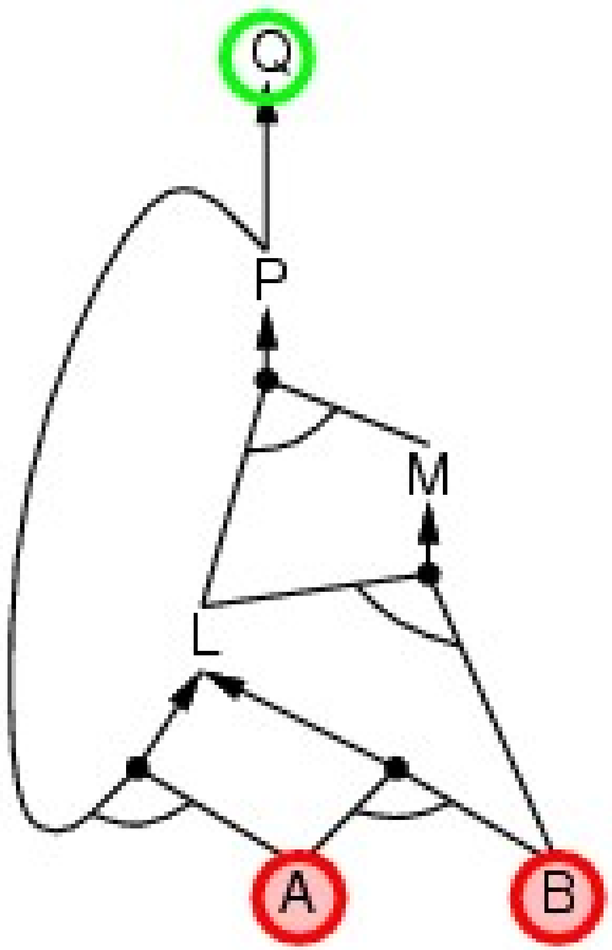
$L \wedge M \Rightarrow P$
$B \wedge L \Rightarrow M$
$A \wedge P \Rightarrow L$
$A \wedge B \Rightarrow L$
A
B



Backward Chaining Example

$P \Rightarrow Q$

$L \wedge M \Rightarrow P$

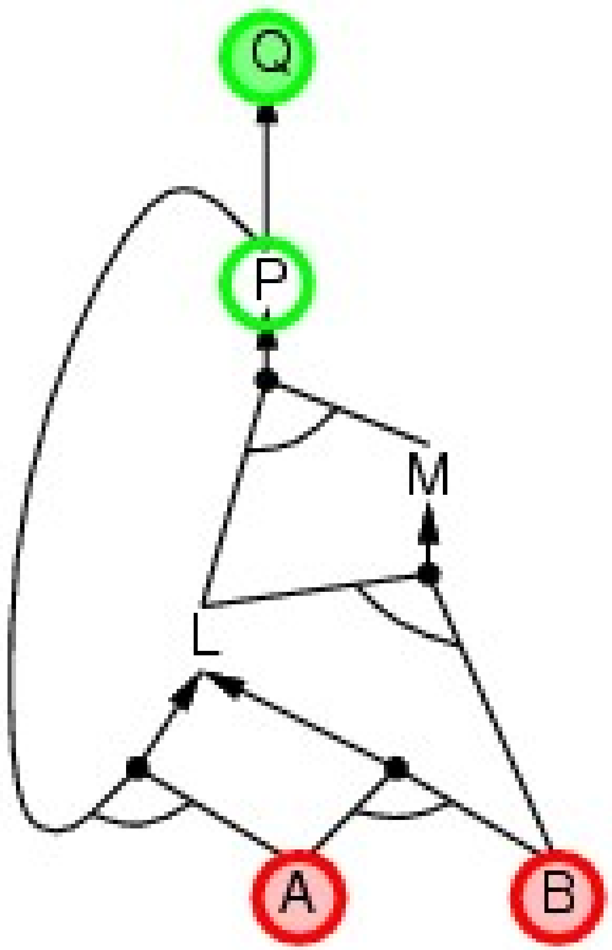
$B \wedge L \Rightarrow M$

$A \wedge P \Rightarrow L$

$A \wedge B \Rightarrow L$

A

B



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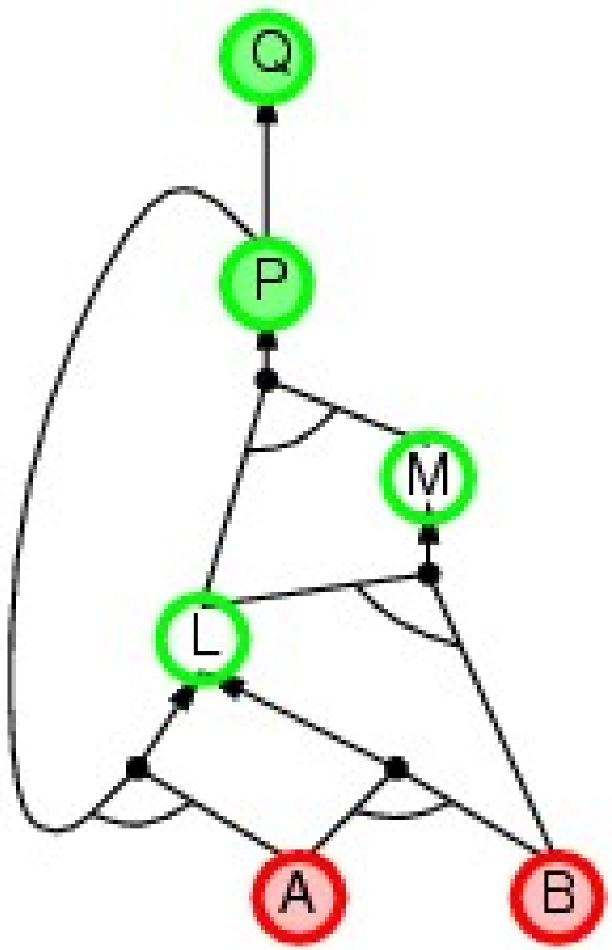

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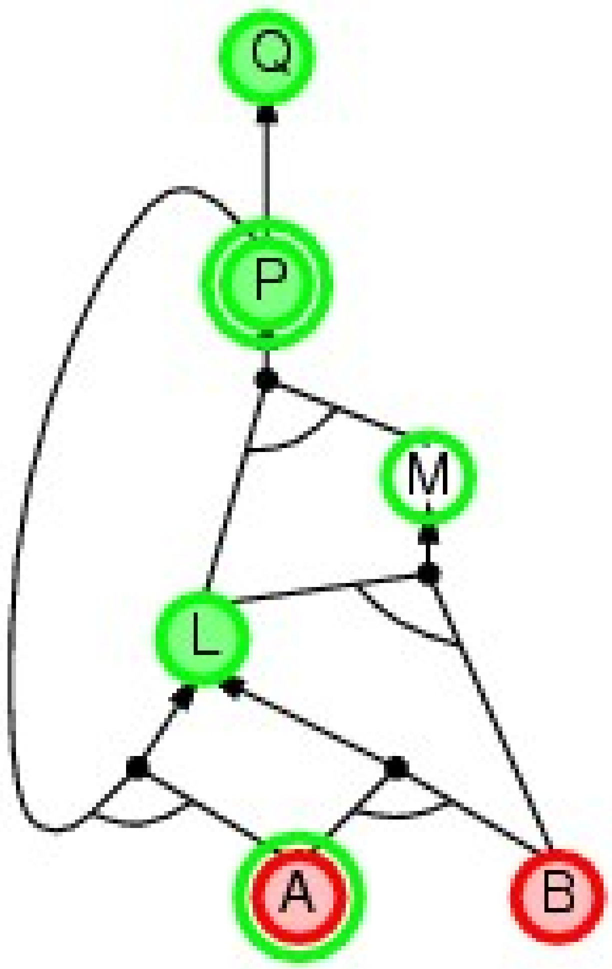

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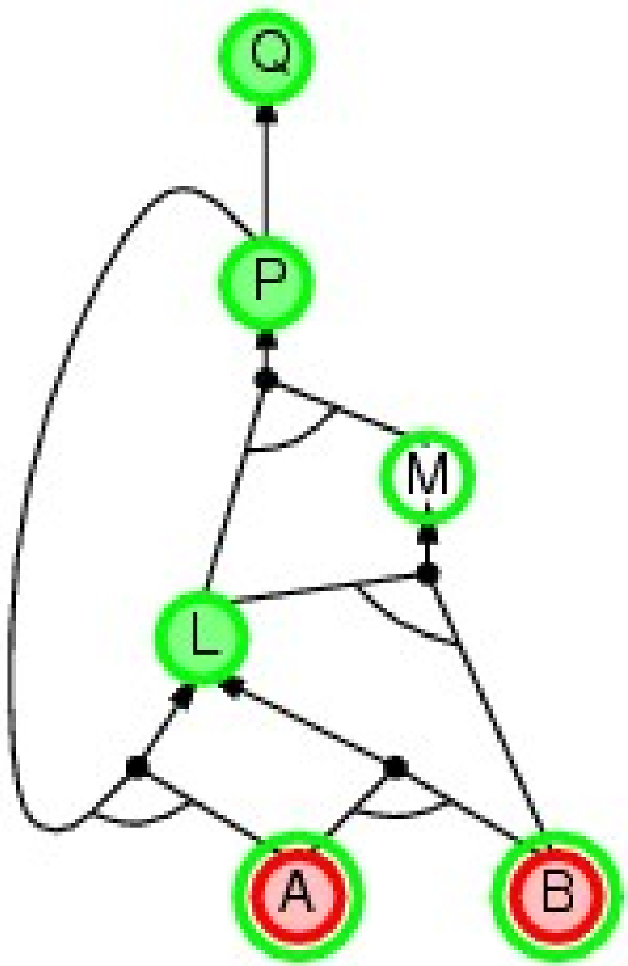

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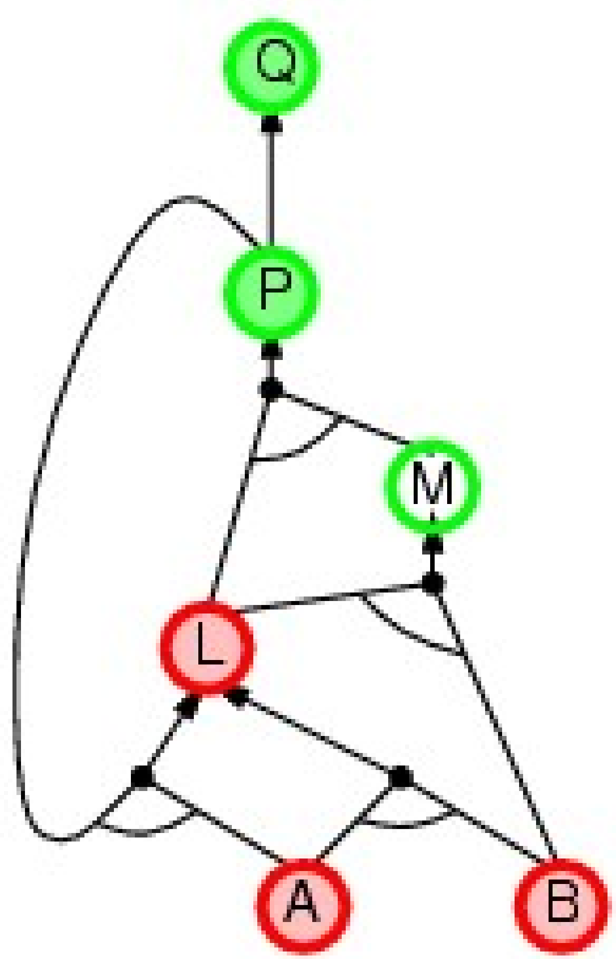

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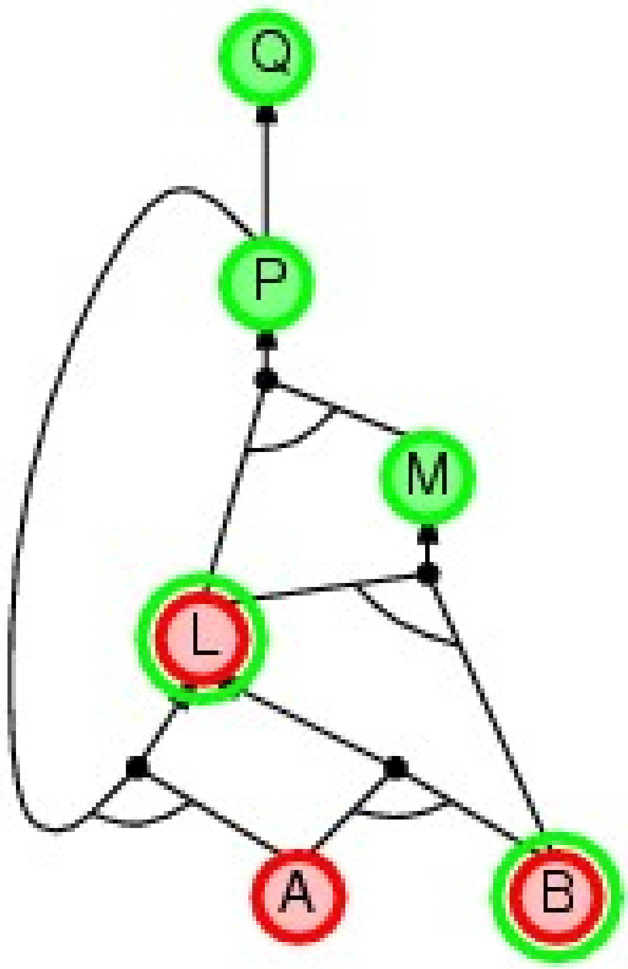

Backward Chaining Example

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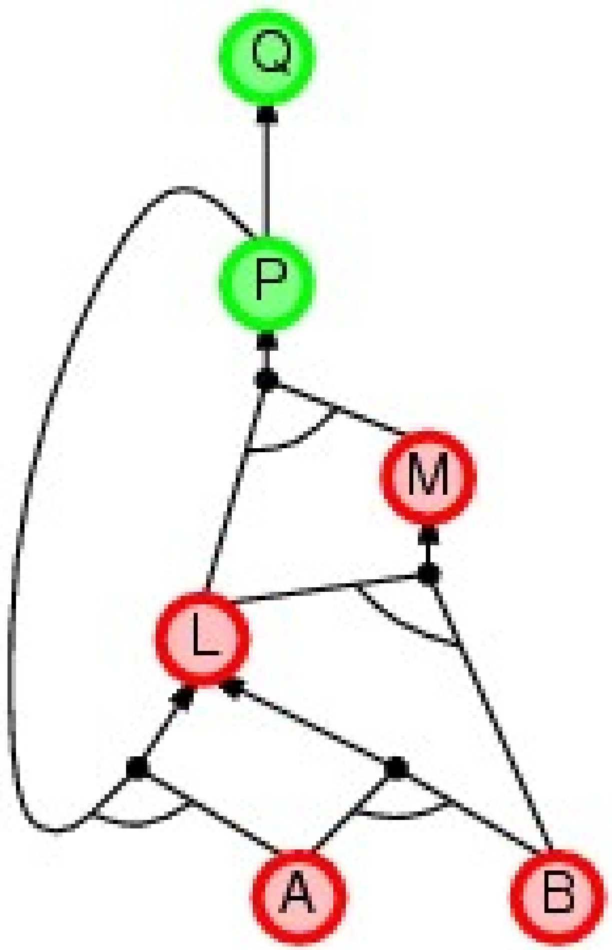

Backward Chaining Example

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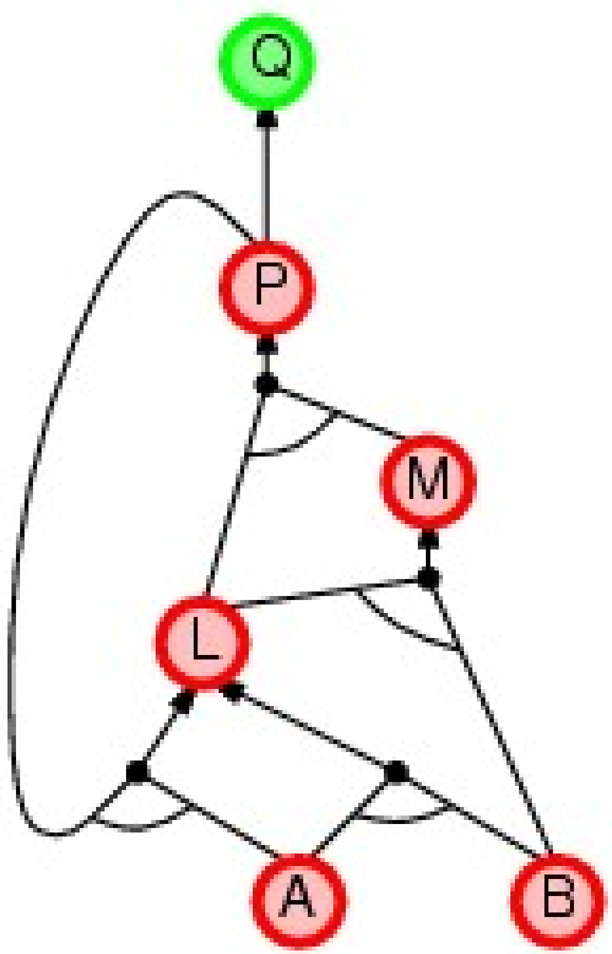

Backward Chaining Example

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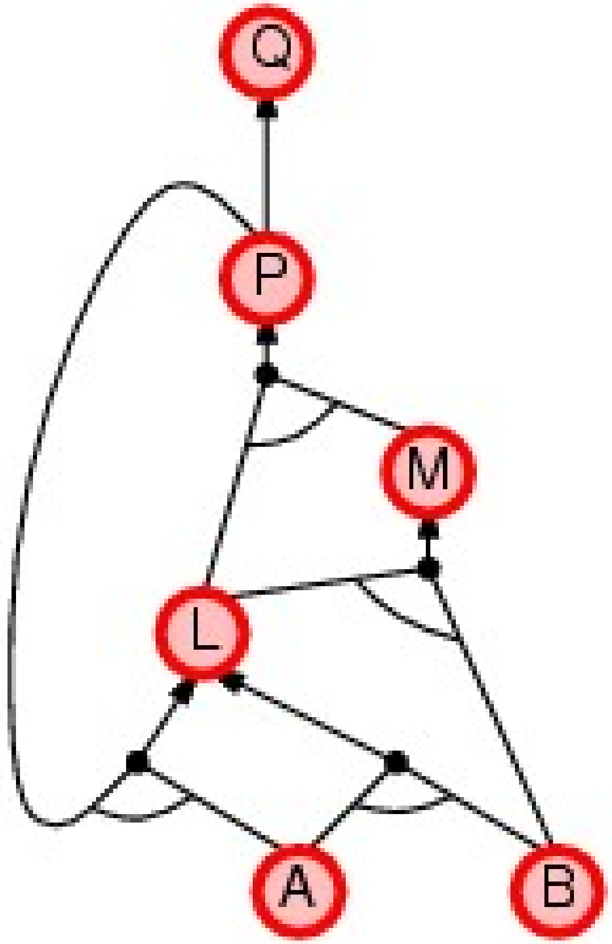

Forward vs. Backward Chaining

- FC is data-driven, automatic, unconscious processing
  - e.g., object recognition, routine decisions
- May do lots of work that is irrelevant to the goal
- BC is goal-driven, appropriate for problem-solving,
  - e.g., Where are my keys? How do I get into a PhD program?
- Complexity of BC can be much less than linear in size of KB

Inference-Based Agents in the Wumpus World

- A wumpus-world agent using propositional logic:

$\neg P_{1,1}$

$\neg W_{1,1}$

$B_{x,y} \Leftrightarrow (P_{x,y+1} \vee P_{x,y-1} \vee P_{x+1,y} \vee P_{x-1,y})$

$S_{x,y} \Leftrightarrow (W_{x,y+1} \vee W_{x,y-1} \vee W_{x+1,y} \vee W_{x-1,y})$

$W_{1,1} \vee W_{1,2} \vee \ldots \vee W_{4,4}$

$\neg W_{1,1} \vee \neg W_{1,2}$

$\neg W_{1,1} \vee \neg W_{1,3}$

…

$\Rightarrow$ 64 distinct proposition symbols, 155 sentences

Expressiveness Limitation

- KB contains "physics" sentences for every single square

- For every time t and every location [x,y],
  $L_{x,y} \wedge \mathit{FacingRight}^{\,t} \wedge \mathit{Forward}^{\,t} \Rightarrow L_{x+1,y}$

- Rapid proliferation of clauses

Summary

- Logical agents apply inference to a knowledge base to derive new information and make decisions
- Basic concepts of logic:
  - syntax: formal structure of sentences
  - semantics: truth of sentences based on models
  - entailment: necessary truth of one sentence given another
  - inference: deriving sentences from other sentences
  - soundness: derivations produce only entailed sentences
  - completeness: derivations can produce all entailed sentences
- Wumpus world requires the ability to represent partial and negated information, reason by cases, etc.
- Resolution is complete for propositional logic
  Forward, backward chaining are linear-time, complete for Horn clauses
- Propositional logic lacks expressive power