Artificial Intelligence
Chapter 3: Solving Problems by Searching

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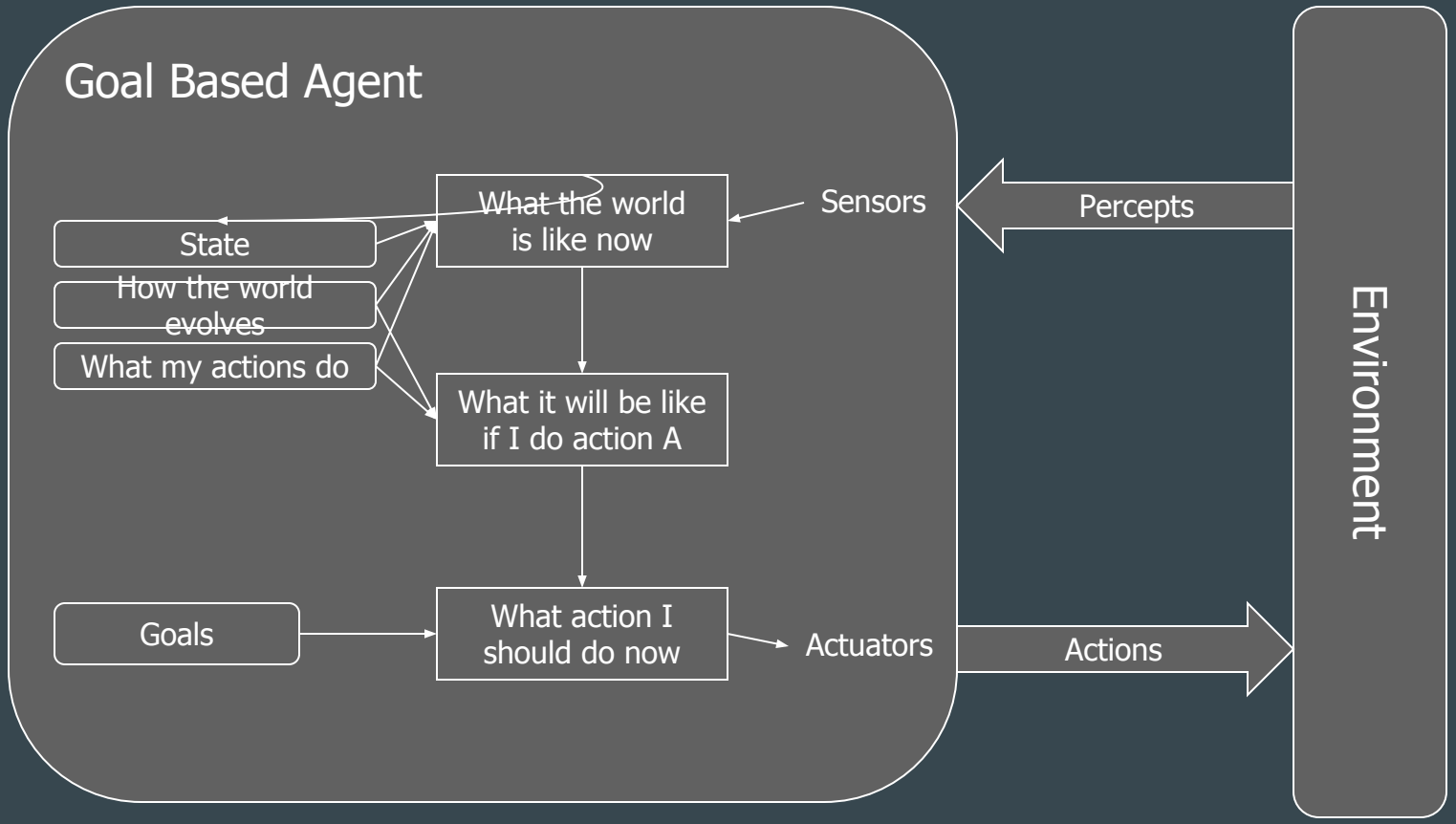
Problem Solving Agents

- Problem solving agent
  - A kind of "goal based" agent
  - Finds sequences of actions that lead to desirable states.

- The algorithms are uninformed
  - No extra information about the problem other than the definition
    - No extra information
    - No heuristics (rules)

Goal Based Agent



Goal Based Agent

```
Function Simple-Problem-Solving-Agent( percept ) returns action

Inputs: percept    a percept
Static: seq     an action sequence initially empty
        state      some description of the current world
        goal       a goal, initially null
        problem    a problem formulation

state <- UPDATE-STATE( state, percept )
if seq is empty then do
    goal <- FORMULATE-GOAL( state )
    problem <- FORMULATE-PROBLEM( state, goal )
    seq <- SEARCH( problem )                # SEARCH
action <- RECOMMENDATION ( seq )            # SOLUTION
seq <- REMAINDER( seq )
return action                          # EXECUTION
```

Goal Based Agents

- Assumes the problem environment is:
  - Static
    - The plan remains the same
  - Observable
    - Agent knows the initial state
  - Discrete
    - Agent can enumerate the choices
  - Deterministic
    - Agent can plan a sequence of actions such that each will lead to an intermediate state
- The agent carries out its plans with its eyes closed
  - Certain of what's going on
  - Open loop system

Well Defined Problems and Solutions

- A problem
  - Initial state
  - Actions and Successor Function
  - Goal test
  - Path cost

Example: Eight Puzzle

- States:
  - Description of the eight tiles and location of the blank tile
- Successor Function:
  - Generates the legal states from trying the four actions {*Left, Right, Up, Down*}
- Goal Test:
  - Checks whether the state matches the goal configuration
- Path Cost:
  - Each step costs 1

7	2	4	
5		6	
8	3	1	

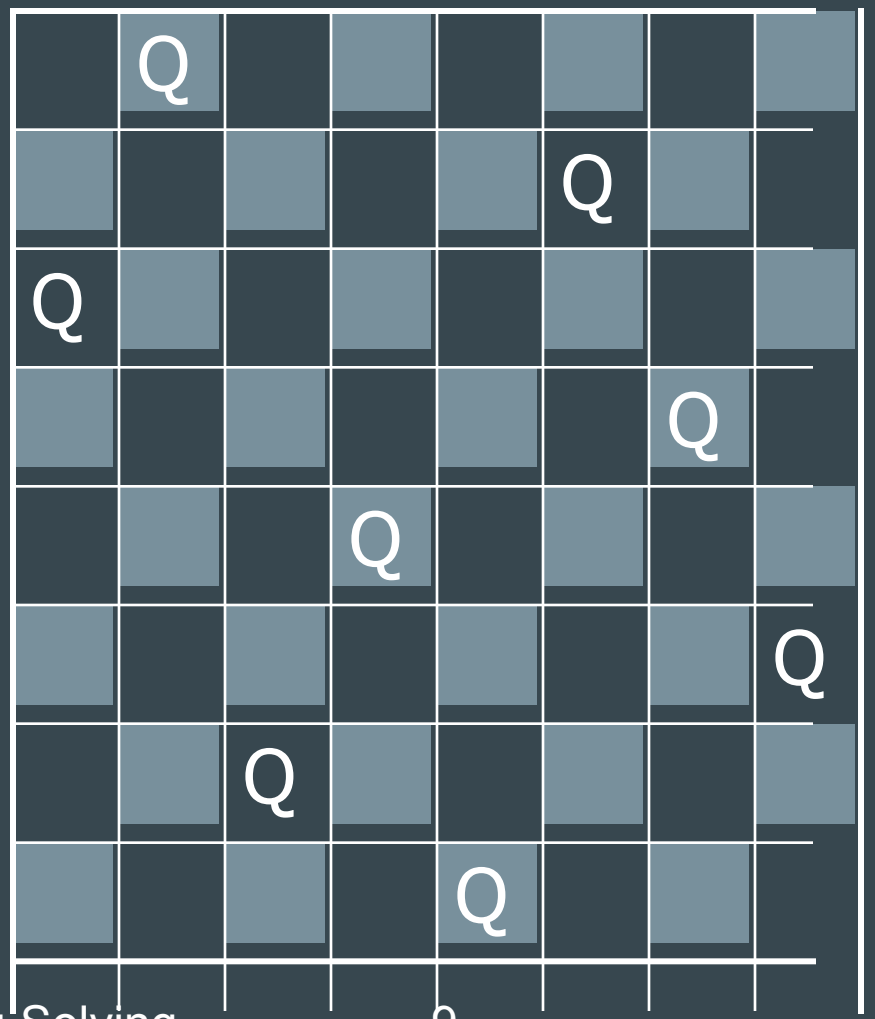
1	2	3	
4	5	6	
7	8		

Example: Eight Puzzle

- Eight puzzle is from a family of "sliding –block puzzles"
  - NP Complete
  - 8 puzzle has 9!/2 = 181440 states
  - 15 puzzle has approx. $1.3*10^{12}$ states
  - 24 puzzle has approx. $1*10^{25}$ states

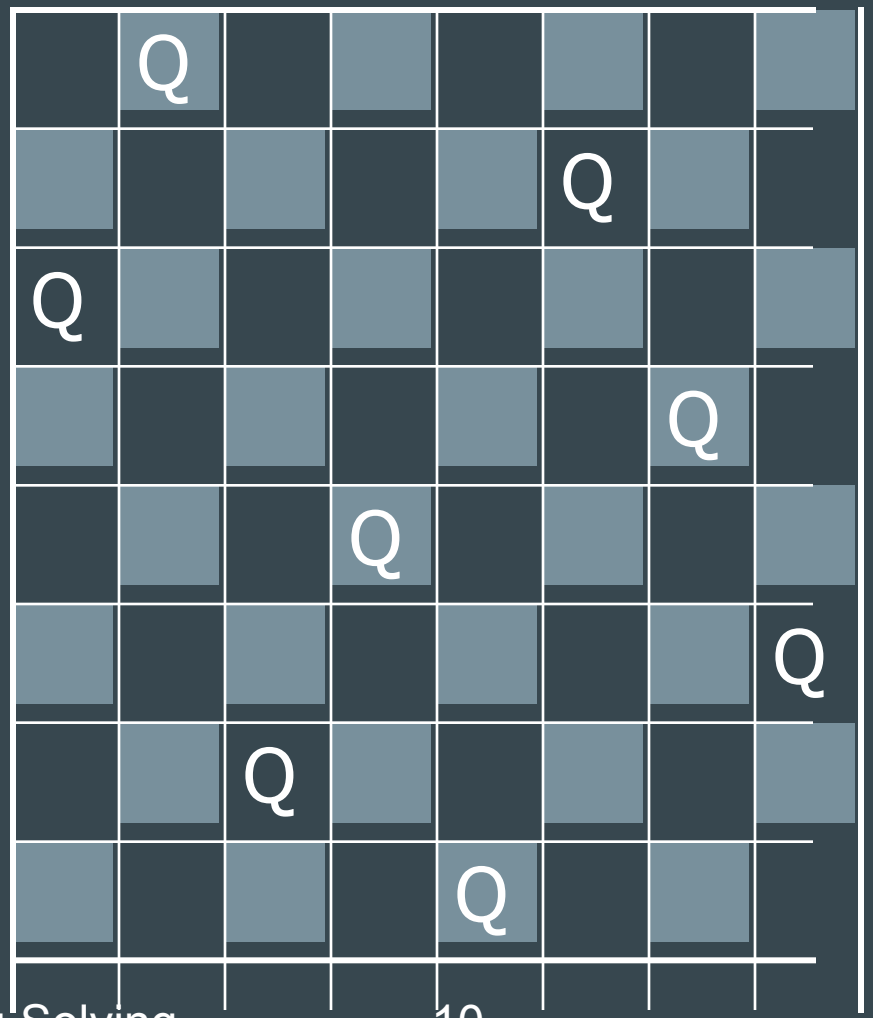
Example: Eight Queens

- Place eight queens on a chess board such that no queen can attack another queen
- No path cost because only the final state counts!
- Incremental formulations
- Complete state formulations



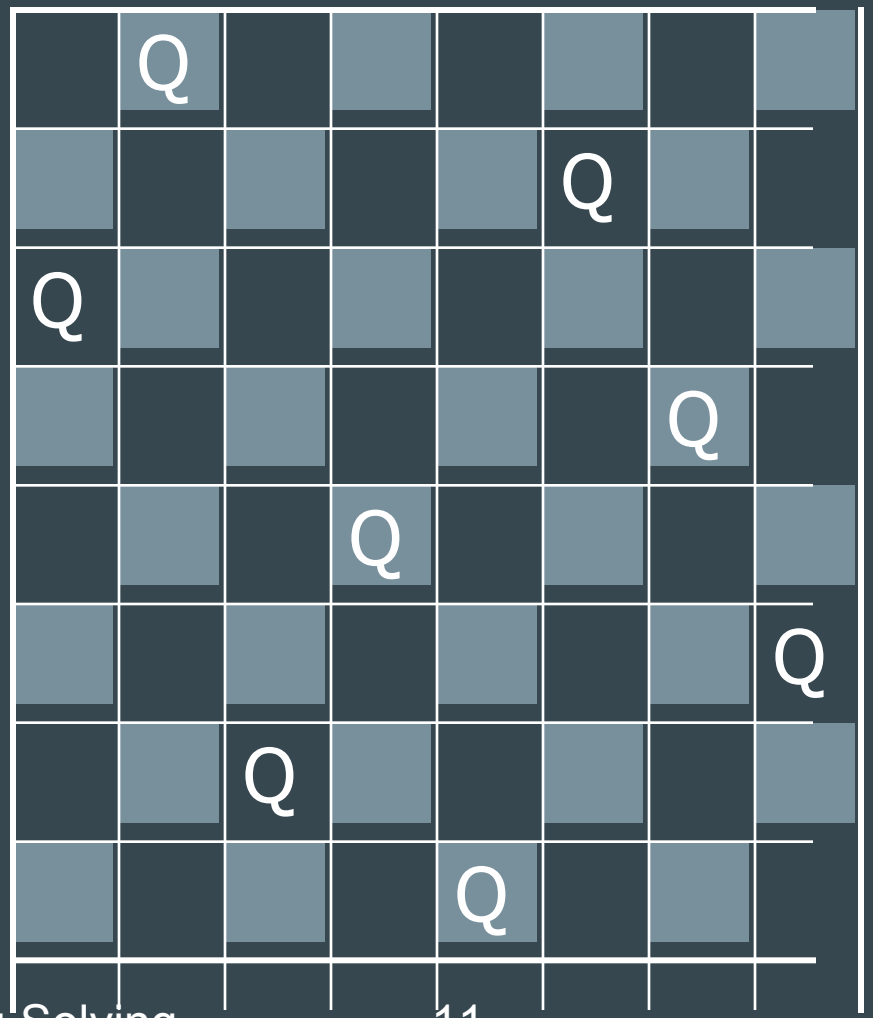
Example: Eight Queens

- States:
  - Any arrangement of 0 to 8 queens on the board
- Initial state:
  - No queens on the board
- Successor function:
  - Add a queen to an empty square
- Goal Test:
  - 8 queens on the board and none are attacked
- $64*63*\ldots*57 = 1.8*10^{14}$ possible sequences
  - Ouch!



Example: Eight Queens

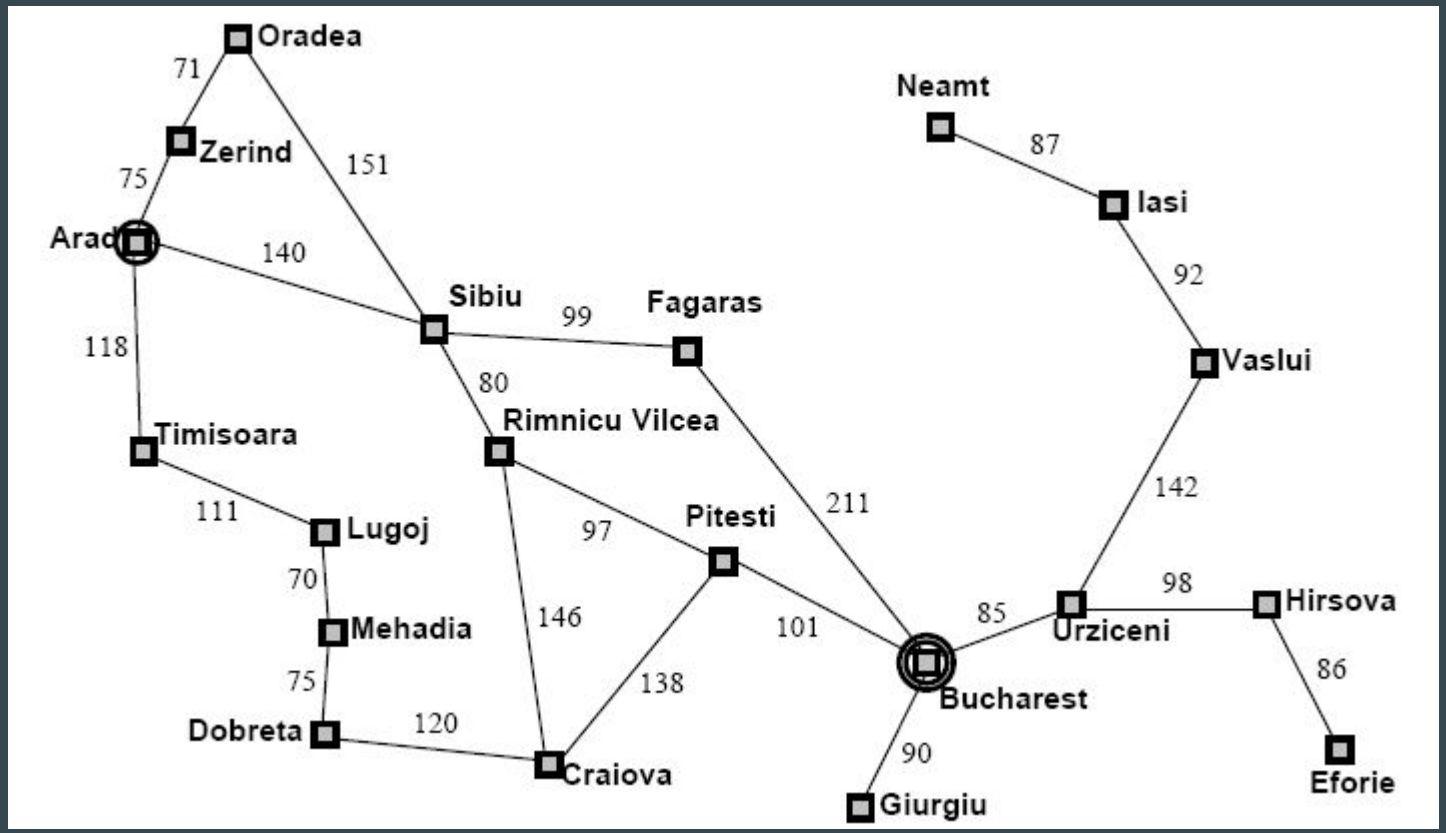
- States:
  - Arrangements of n queens, one per column in the leftmost n columns, with no queen attacking another are states
- Successor function:
  - Add a queen to any square in the leftmost empty column such that it is not attacked by any other queen.
- 2057 sequences to investigate



Other Toy Examples

- Another Example: Jug Fill
- Another Example: Black White Marbles
- Another Example: Row Boat Problem
- Another Example: Sliding Blocks
- Another Example: Triangle Tee

Example: Map Planning



Searching For Solutions

- Initial State
  - e.g. "At Arad"
- Successor Function
  - A set of action state pairs
  - S(Arad) = {(Arad->Zerind, Zerind), …}
- Goal Test
  - e.g. x = "at Bucharest"
- Path Cost
  - sum of the distances traveled

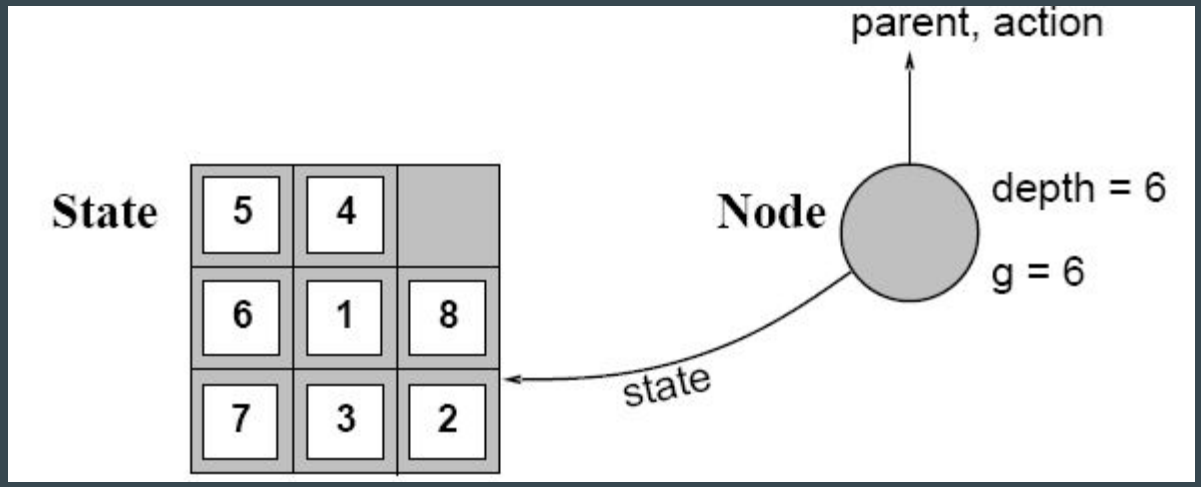
Searching For Solutions

- Having formulated some problems…how do we solve them?
- Search through a state space
- Use a search tree that is generated with an initial state and successor functions that define the state space

Searching For Solutions

- A ***state*** is (a representation of) a physical configuration
- A ***node*** is a data structure constituting part of a search tree
  - Includes parent, children, depth, path cost
- States do not have children, depth, or path cost
- The EXPAND function creates new nodes, filling in the various fields and using the SUCCESSOR function of the problem to create the corresponding states

Searching For Solutions



Searching For Solutions

```
function TREE-SEARCH( problem, strategy) returns a solution, or failure
    initialize the search tree using the initial state of problem
    loop do
        if there are no candidates for expansion then return failure
        choose a leaf node for expansion according to strategy
        if the node contains a goal state then return the corresponding solution
        else expand the node and add the resulting nodes to the search tree
    end
```

Searching For Solutions

```
function TREE-SEARCH( problem, fringe) returns a solution, or failure
    fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
    loop do
        if fringe is empty then return failure
        node ← REMOVE-FRONT(fringe)
        if GOAL-TEST[problem] applied to STATE(node) succeeds return node
        fringe ← INSERTALL(EXPAND(node, problem), fringe)
```

```
function EXPAND( node, problem) returns a set of nodes
    successors ← the empty set
    for each action, result in SUCCESSOR-FN[problem](STATE[node]) do
        s ← a new NODE
        PARENT-NODE[s] ← node;  ACTION[s] ← action;  STATE[s] ← result
        PATH-COST[s] ← PATH-COST[node] + STEP-COST(node, action, s)
        DEPTH[s] ← DEPTH[node] + 1
        add s to successors
    return successors
```

Uninformed Search Strategies

- ***Uninformed*** strategies use only the information available in the problem definition
  - Also known as blind searching

- Breadth-first search
- Uniform-cost search
- Depth-first search
- Depth-limited search
- Iterative deepening search

Comparing Uninformed Search Strategies

- Completeness
  - Will a solution always be found if one exists?
- Time
  - How long does it take to find the solution?
  - Often represented as the number of nodes searched
- Space
  - How much memory is needed to perform the search?
  - Often represented as the maximum number of nodes stored at once
- Optimal
  - Will the optimal (least cost) solution be found?

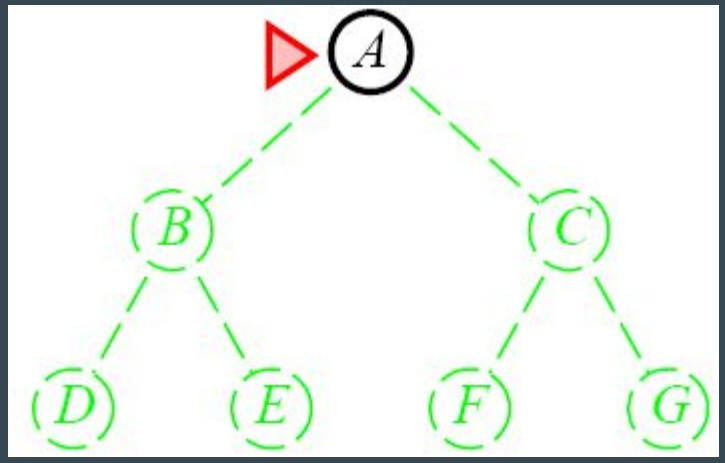
Comparing Uninformed Search Strategies

- Time and space complexity are measured in
  - b – maximum branching factor of the search tree
  - m – maximum depth of the state space
  - d – depth of the least cost solution

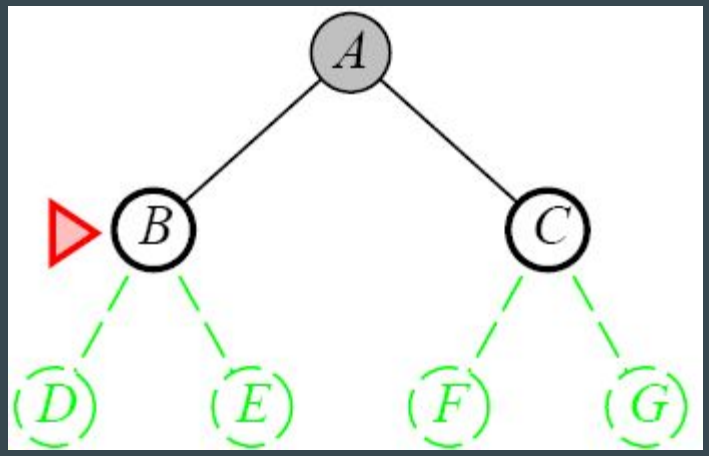
Breadth-First Search

- Recall from Data Structures the basic algorithm for a breadth-first search on a graph or tree
- Expand the ***shallowest*** unexpanded node
- Place all new successors at the end of a FIFO queue

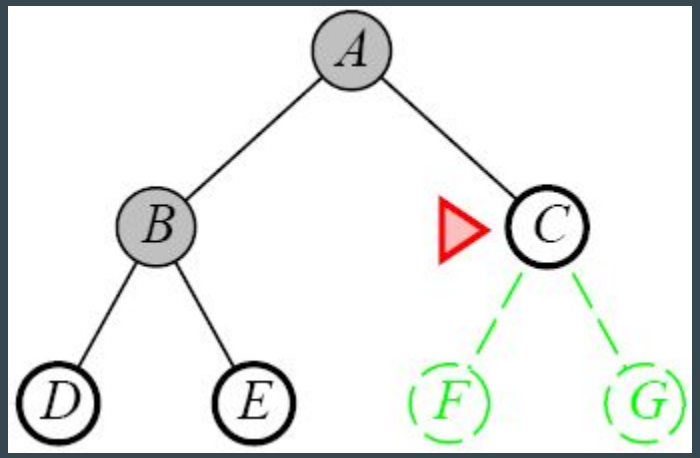
Breadth-First Search



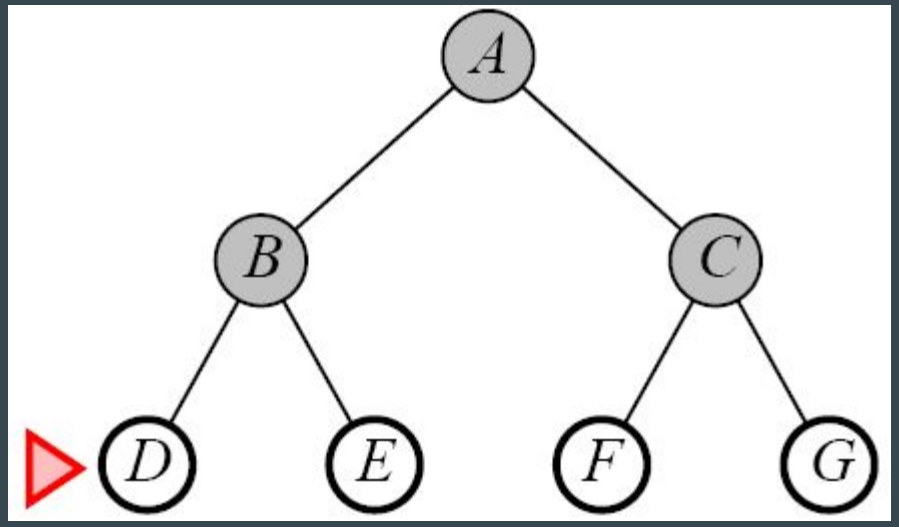
Breadth-First Search



Breadth-First Search



Breadth-First Search



Properties of Breadth-First Search

- Complete
  - Yes if b (max branching factor) is finite
- Time
  - $1 + b + b^2 + \ldots + b^d + b(b^d-1) = O(b^{d+1})$
  - exponential in d
- Space
  - $O(b^{d+1})$
  - Keeps every node in memory
  - This is the big problem; an agent that generates nodes at 10 MB/sec will produce 860 MB in 24 hours
- Optimal
  - Yes (if cost is 1 per step); not optimal in general

Lessons From Breadth First Search

- The memory requirements are a bigger problem for breadth-first search than is execution time

- Exponential-complexity search problems cannot be solved by uniformed methods for any but the smallest instances

Uniform-Cost Search

- Same idea as the algorithm for breadth-first search…but…
  - Expand the ***least-cost*** unexpanded node
  - FIFO queue is ordered by cost
  - Equivalent to regular breadth-first search if all step costs are equal

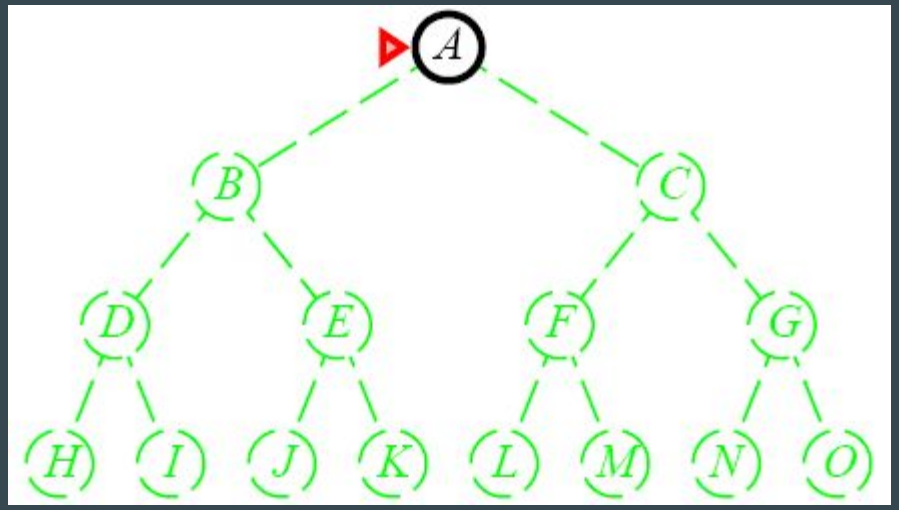
Uniform-Cost Search

- Complete
  - Yes if the cost is greater than some threshold
  - step cost >= ε
- Time
  - Complexity cannot be determined easily by d or d
  - Let C* be the cost of the optimal solution
  - $O(b^{ceil(C^*/ \varepsilon)})$
- Space
  - $O(b^{ceil(C^*/ \varepsilon)})$
- Optimal
  - Yes, Nodes are expanded in increasing order

Depth-First Search

- Recall from Data Structures the basic algorithm for a depth-first search on a graph or tree
- Expand the ***deepest*** unexpanded node
- Unexplored successors are placed on a stack until fully explored

Depth-First Search



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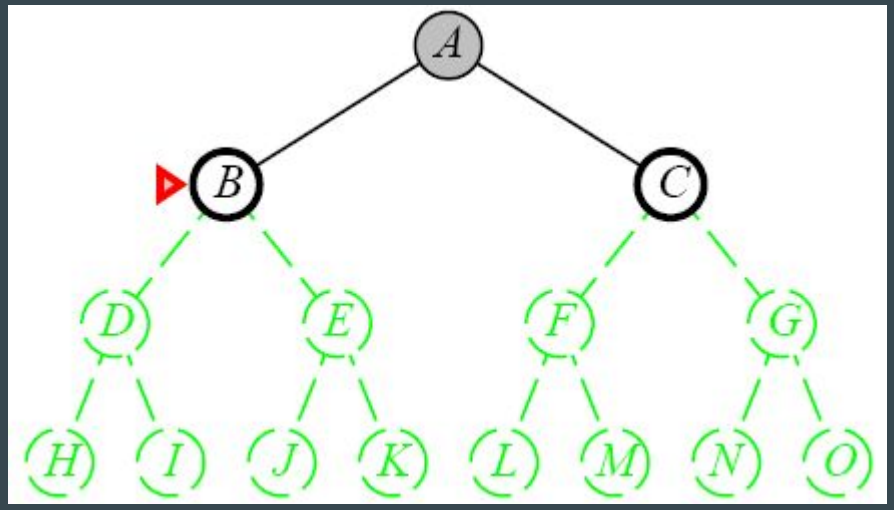

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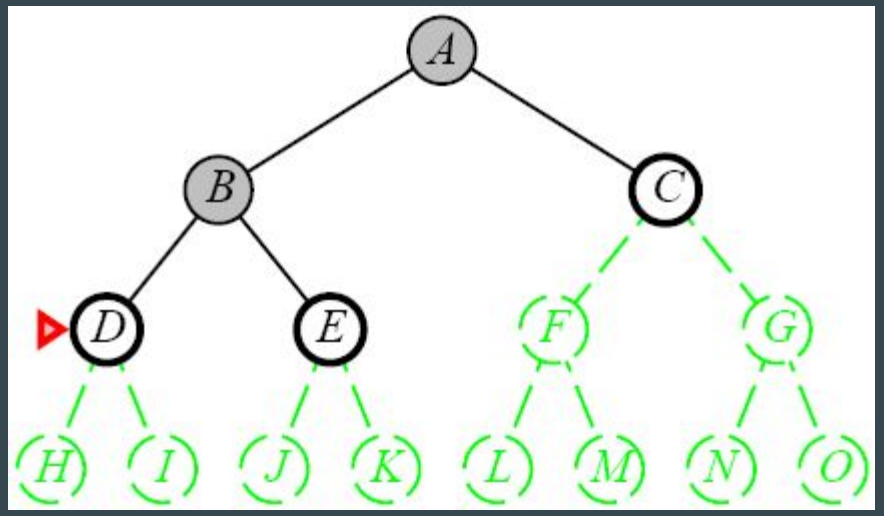
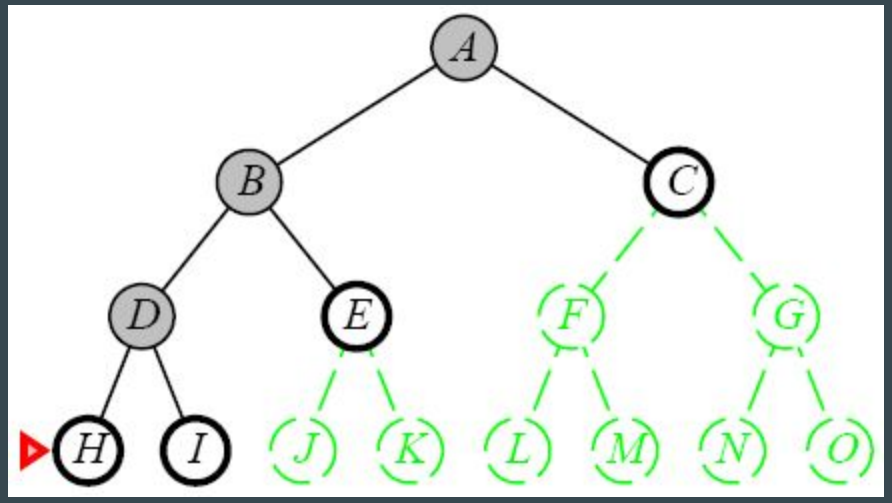


Depth-First Search



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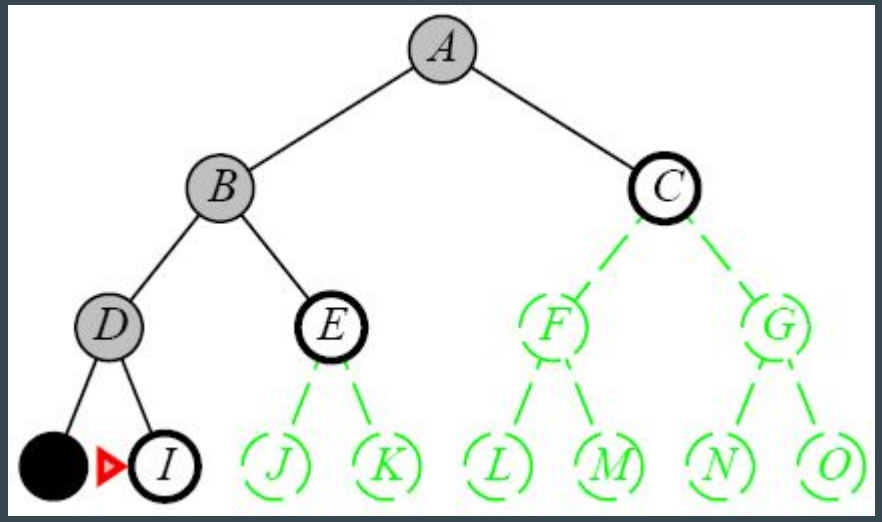

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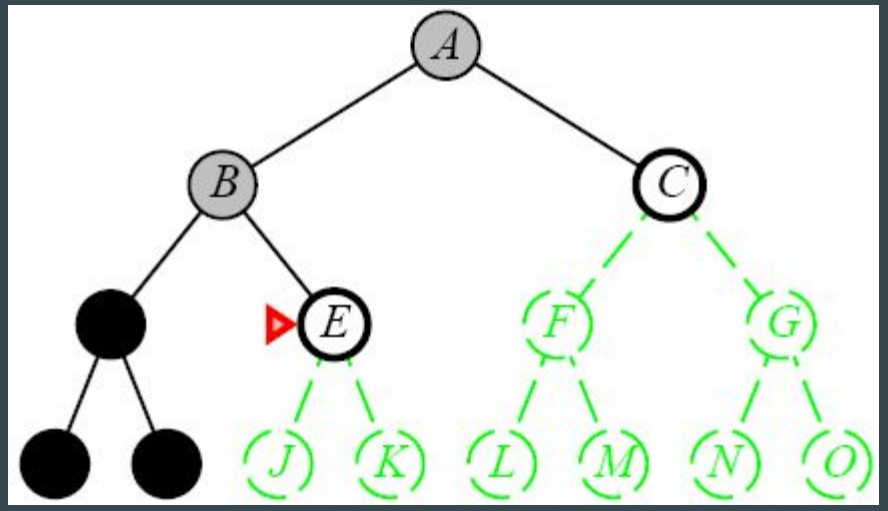

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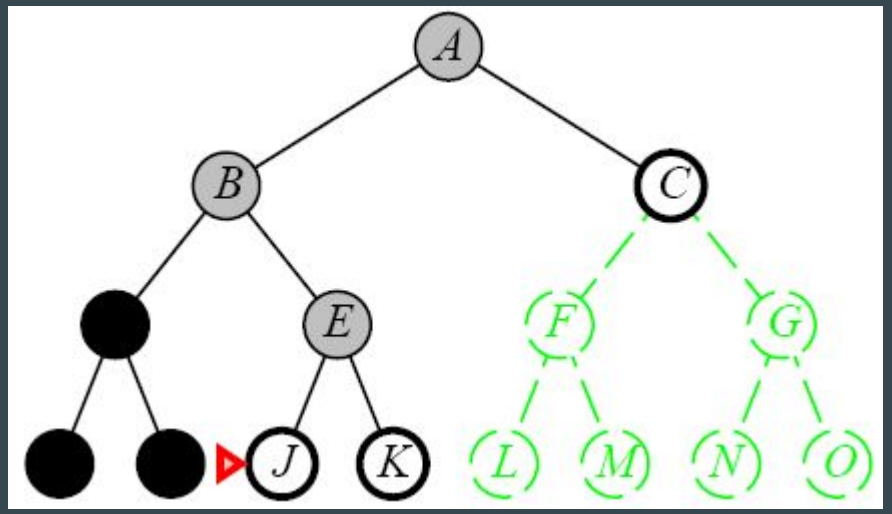

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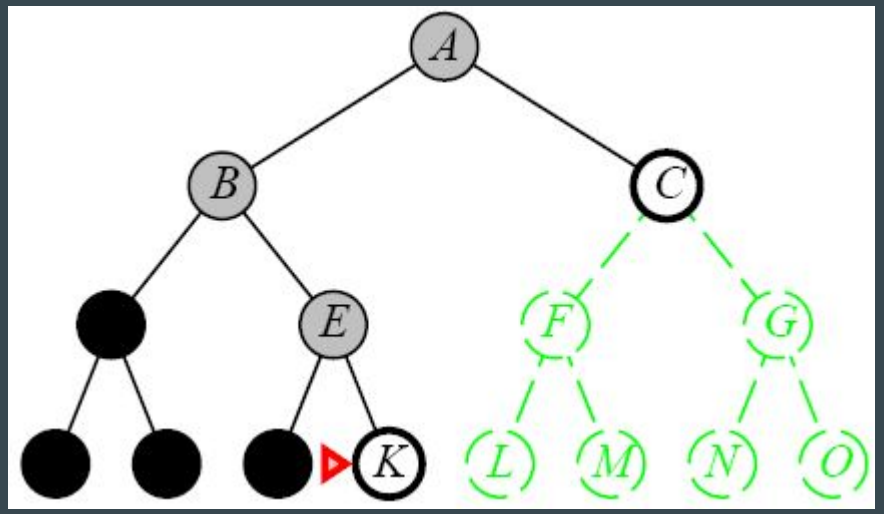

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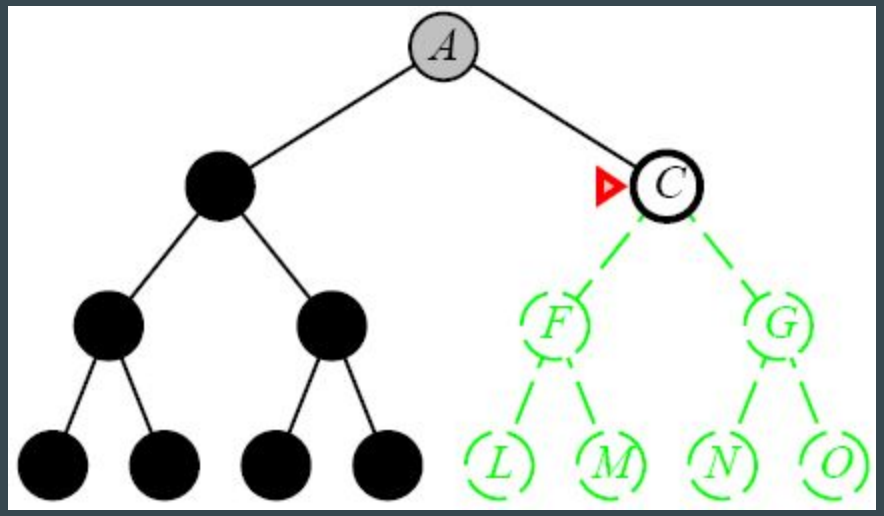

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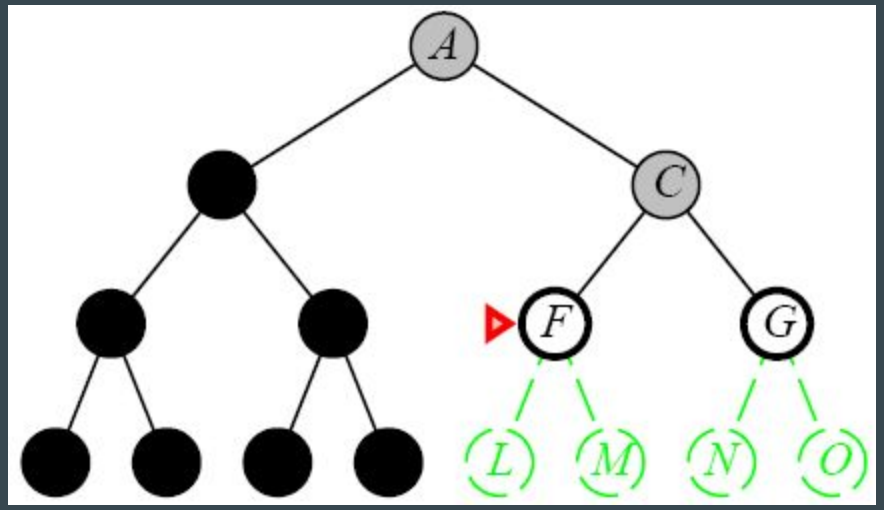

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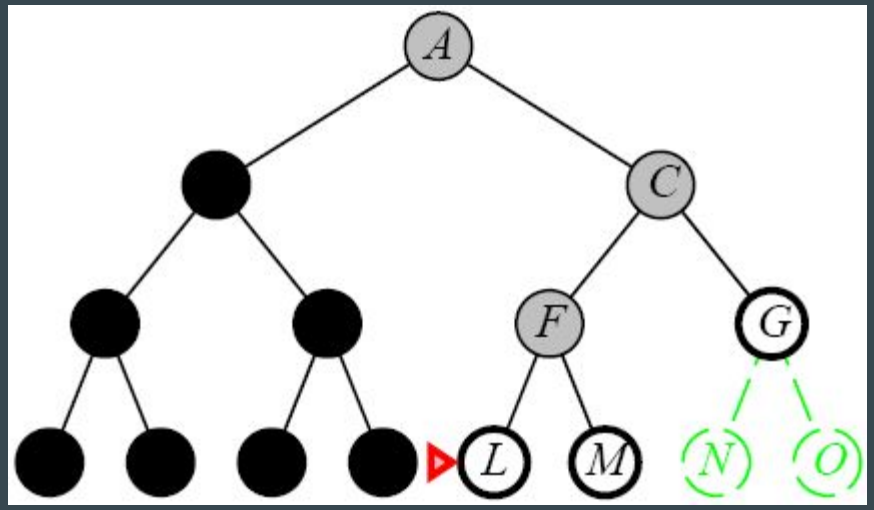

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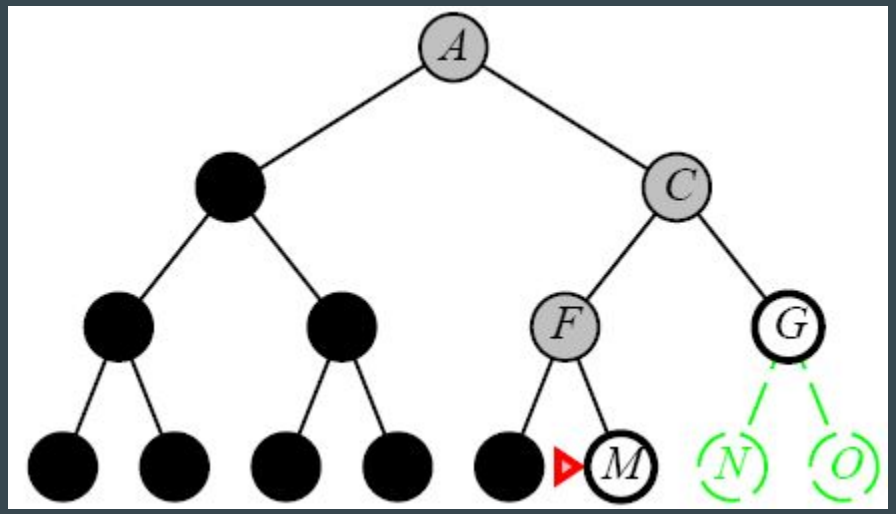


Depth-First Search

- Complete
  - No: fails in infinite-depth spaces, spaces with loops
    - Modify to avoid repeated spaces along path
  - Yes: in finite spaces
- Time
  - $O(b^m)$
  - Not great if m is much larger than d
  - But if the solutions are dense, this may be faster than breadth-first search
- Space
  - O(bm)…linear space
- Optimal
  - No

Depth-Limited Search

- A variation of depth-first search that uses a depth limit
  - Alleviates the problem of unbounded trees
  - Search to a predetermined depth **l** ("ell")
  - Nodes at depth l have no successors

- Same as depth-first search if $l = \infty$
- Can terminate for failure and cutoff

Depth-Limited Search

Recursive implementation:

```
function DEPTH-LIMITED-SEARCH( problem, limit) returns soln/fail/cutoff
    RECURSIVE-DLS(MAKE-NODE(INITIAL-STATE[problem]), problem, limit)

function RECURSIVE-DLS(node, problem, limit) returns soln/fail/cutoff
    cutoff-occurred? ← false
    if GOAL-TEST[problem](STATE[node]) then return node
    else if DEPTH[node] = limit then return cutoff
    else for each successor in EXPAND(node, problem) do
        result ← RECURSIVE-DLS(successor, problem, limit)
        if result = cutoff then cutoff-occurred? ← true
        else if result ≠ failure then return result
    if cutoff-occurred? then return cutoff else return failure
```

Depth-Limited Search

- Complete
  - Yes if l < d
- Time
  - $O(b^l)$
- Space
  - O(bl)
- Optimal
  - No if l > d

Iterative Deepening Search

- Iterative deepening depth-first search
  - Uses depth-first search
  - Finds the best depth limit
    - Gradually increases the depth limit; 0, 1, 2, … until a goal is found

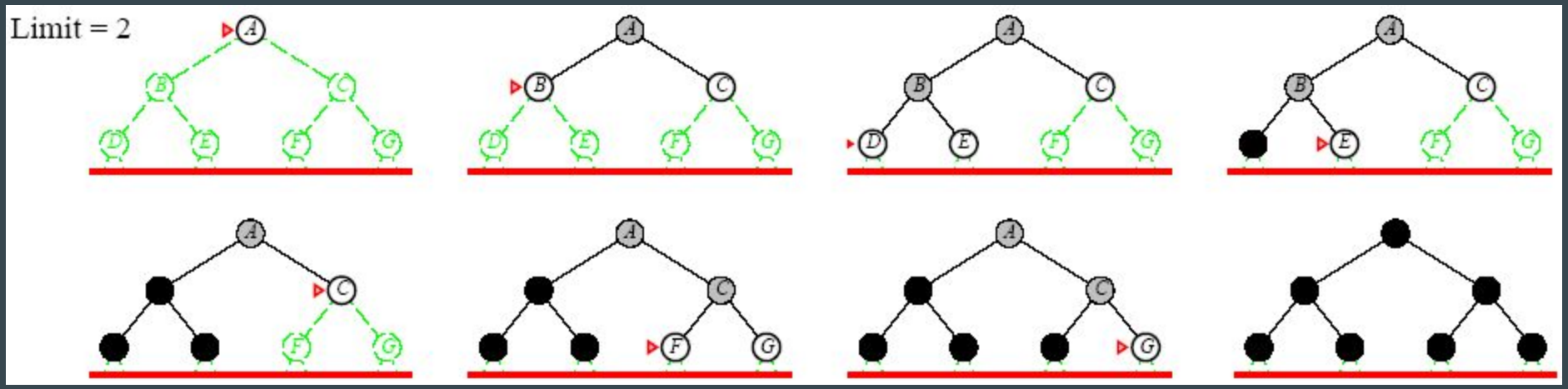
Iterative Deepening Search

```
function ITERATIVE-DEEPENING-SEARCH( problem) returns a solution
    inputs: problem, a problem
    for depth ← 0 to ∞ do
        result ← DEPTH-LIMITED-SEARCH( problem, depth)
        if result ≠ cutoff then return result
    end
```

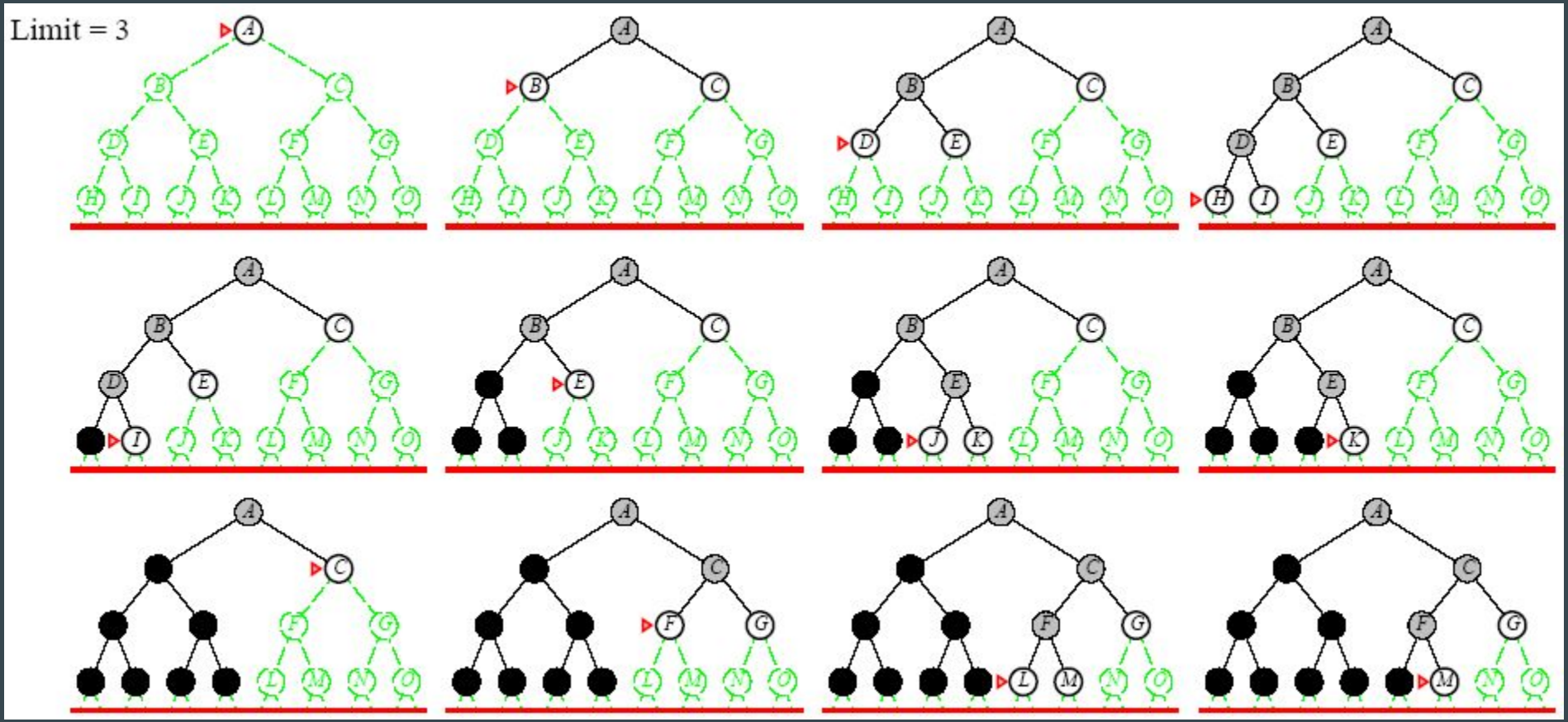
Iterative Deepening Search

Iterative Deepening Search

Iterative Deepening Search



Iterative Deepening Search



Iterative Deepening Search

- Complete
  - Yes
- Time
  - $O(b^d)$
- Space
  - O(bd)
- Optimal
  - Yes if step cost = 1
  - Can be modified to explore uniform cost tree

Lessons From Iterative Deepening Search

- Faster than BFS even though IDS generates repeated states
  - BFS generates nodes up to level d+1
  - IDS only generates nodes up to level d

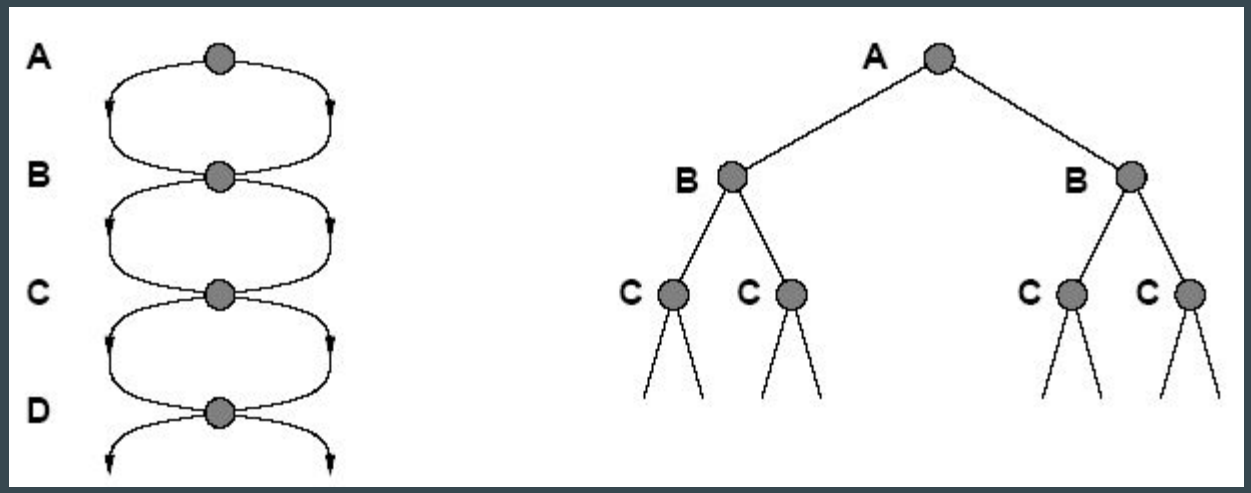
- In general, iterative deepening search is the preferred uninformed search method when there is a large search space and the depth of the solution is not known

Avoiding Repeated States

- Complication of wasting time by expanding states that have already been encountered and expanded before
  - Failure to detect repeated states can turn a linear problem into an exponential one

- Sometimes, repeated states are unavoidable
  - Problems where the actions are reversable
    - Route finding
    - Sliding blocks puzzles

Avoiding Repeated States



State Space

Search Tree

Avoiding Repeated States

```
function GRAPH-SEARCH( problem, fringe) returns a solution, or failure

    closed ← an empty set
    fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
    loop do
        if fringe is empty then return failure
        node ← REMOVE-FRONT(fringe)
        if GOAL-TEST[problem](STATE[node]) then return node
        if STATE[node] is not in closed then
            add STATE[node] to closed
            fringe ← INSERTALL(EXPAND(node, problem), fringe)
    end
```