

Vector

vector can be represented as a **one-dimensional array**, either as a column vector (vertical array) or a row vector (horizontal array) [02:01].

It represent direction & length(also called Magnitude).

$$V1 = (x_1, x_2, x_3 \dots x_n). \quad V2 = (y_1, y_2, y_3 \dots y_n)$$

$$V1 = [1, 2, 3] \quad v2 = [4, 5, 6]$$

$$V1 + v2 = [5, 7, 9]$$

$$V1 - v2 = [3, 3, 3]$$

Addition & subtraction

$$V1 + V2 = (x_1 + y_1, x_2 + y_2, \dots, x_n + y_n)$$

Dot product (multiplication)

$$V1 \cdot V2 = x_1 y_1 + x_2 y_2 + x_3 y_3 \dots x_n y_n). \quad \sum_{i=1}^n x_i y_i$$

$$V1 \cdot V2 = v1 \cdot v2 = [4, 10, 18] = 4 + 10 + 18 = 32$$

Vector Algebra

$$v_1 = (x_1, x_2, \dots, x_n) \in \mathbb{R}^n$$

$$v_2 = (y_1, y_2, \dots, y_n) \in \mathbb{R}^n$$

$$v_1 \cdot v_2 = x_1 y_1 + x_2 y_2 + \dots + x_n y_n$$

$$= \sum_{i=1}^n x_i y_i$$

➤ Addition/Subtraction

✓ Dot Product

➤ Length/Magnitude

➤ Angle between two vectors

Length (Magnitude) \rightarrow
 $|v|$

$$\text{Length } v1 = \text{squareroot}[1, 2, 3] = 1^2 + 2^2 + 3^2 = 1 + 4 + 9 = \text{squareroot}(14)$$

$$\text{Length: } \vec{v} = (x_1, x_2, \dots, x_n)$$

$$|\vec{v}| = v = \sqrt{\vec{v} \cdot \vec{v}} = \sqrt{x_1^2 + x_2^2 + \dots + x_n^2}$$

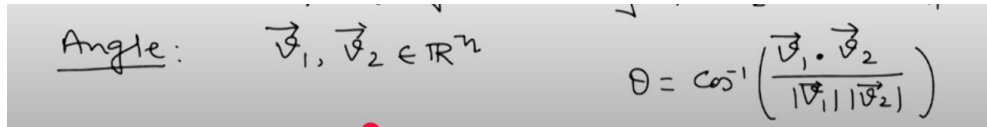
$$\text{Ex: } \vec{v} = (1, -1, 2)$$

$$|\vec{v}| = \sqrt{1^2 + (-1)^2 + 2^2}$$

$$= \sqrt{1 + 1 + 4}$$

$$= \sqrt{6}$$

Angle between 2 vector



Angle: $\vec{v}_1, \vec{v}_2 \in \mathbb{R}^n$

$$\theta = \cos^{-1} \left(\frac{|\vec{v}_1 \cdot \vec{v}_2|}{|\vec{v}_1| |\vec{v}_2|} \right)$$

linear combination of vectors:

Example:

If you have two vectors, $v_1 =$ and $v_2 =$, and scalars $c_1 = 2$ and $c_2 = -1$, the linear combination would be:

- $v_1 = [2, 1]$

- $v_2 = [-1, 3]$

$$2 * v_1 + (-1) * v_2 = 2(2, 1) + -1(-1, 3) = (4+1, 2-3) = [5, -1]$$

linearly independent(LI): if linear combination(Addition) of vectors is equal to Zero ($2v_1 + (-1)v_2 = 0$), if and if c_1, c_2 scalar(2, -1) should zero.

Ex. $C_1 v_1 [1, 2], c_2 v_2 [-1, -2] = 0$ but c_1, c_2 not zero, So, vector linear dependent.

linearly dependent(LD): if linear combination of vectors is equal to non-Zero ($2v_1 + (-1)v_2 = [5, -1]$)

Orthogonal Vector : if vector v_1, v_2 . Their dot product is zero.

$$v_1 \cdot v_2 = 0 \quad v_1 * v_2 = 0$$

if $u = (u_1, u_2, \dots, u_n)$ and $v = (v_1, v_2, \dots, v_n)$, the dot product is: $u \cdot v = u_1 v_1 + u_2 v_2 + \dots + u_n v_n = 0$

Orthonormal vector : 1. if vector is orthogonal i.e. dot product is zero. And

2, length or magnitude is 1.

Example in \mathbb{R}^2

$$u = [1, 0], v = [0, 1]$$

- **Dot product:**

$$u \cdot v = (1)(0) + (0)(1) = 0 + 0 = 0$$

- **Magnitudes:**

$$\|u\| = \|v\| = 1,$$

$$\text{Ex, } u = \sqrt{1^2 + 0^2} = 1$$

$$V = \sqrt{0^2 + 1^2} = 1$$

- $u = (1, 0)$
- $v = (0, 1)$

Check Orthogonality:

- Dot product: $u \cdot v = 1 \cdot 0 + 0 \cdot 1 = 0$. They are orthogonal.

Check Norm (Magnitude):

- For u : $\|u\| = \sqrt{1^2 + 0^2} = \sqrt{1} = 1$
- For v : $\|v\| = \sqrt{0^2 + 1^2} = \sqrt{1} = 1$

Both vectors have a magnitude of 1 and are orthogonal, so $\{(1, 0), (0, 1)\}$ is an orthonormal set.

LI & orthogonal : -

LI is addition is zero & in orthogonal dot product is zero

Exercise:

<https://colab.research.google.com/>.

```
import numpy as np

v=np.array([1,-1,2])
u=np.array([2,5,2])

print("add two vector",v+u)
print("subtract two vector",v-u)
print("dot product:",np.dot(v,u))

#scalar product
print("scalar product",3*v)

#find magnitude(length of vector)
print("length of vector :",np.linalg.norm(v), " u:",np.linalg.norm(u))
```