# HEART: Statistics and Data Science With Networks

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### Outline

- What We've Learned
- Open Problems in Statistical Network Analysis
- Overall Perspective of Data Science

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What We've Learned

- Open Problems in Statistical Network Analysis
- Overall Perspective of Data Science

# Basic Stuff: Probability and Linear Algebra

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- Probability: necessary to understand Bernoulli random variables (e.g. edges of a graph)
- Linear algebra: eigenvectors and eigenvalues (to find graph embeddings)
- Also discussed notions from graph theory (Adjacency matrix, Laplacian matrix, etc.)

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- Degree-Corrected SBM

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What We've Learned

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- Step 4: do something besides cluster!



# Multiple Graphs

Frontier of network literature!

# Multiple Graphs

- Frontier of network literature!
- Ways to aggregate networks:
  - Averaging!
  - Averaging the squared adjacency matrices!
  - Running the whole "pipeline" on one network and then using the singular vectors of the concatenated eigenvectors!

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- Multiple graph models that model real-world networks

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- "Heterophily" versus "homophily"
- Is a community model the "right" model?

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Supervised Learning

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#### Can also study

- Supervised Learning
- Manifold Learning (e.g. here and here)
- Deep learning...