Contents

1   State-of-the-Art in Group Elicitation Strategies

This section presents the main elicitation or aggregation strategies being used in GRecSys. [@masthoff11group] The purpose of them are to adapt to the group
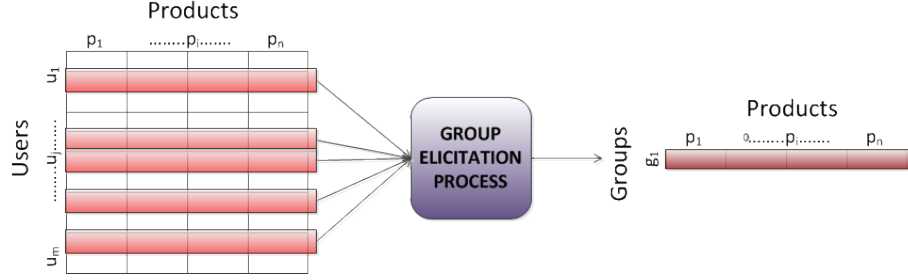
Figure 1: Elicitation Process

as a whole based on information about individual users' likes and dislikes (i.e.: aggregating individual ratings into a group rating).

Veure imatge de [Gartrell, M., Xing, X., Lv, Q., Beach, A., Han, R., Mishra, S., & Seada, K. (2010). Enhancing group recommendation by incorporating social relationship interactions. Proceedings of the 16th ACM international conference on Supporting group work (pp. 97–106). New York, NY, USA: ACM. doi:10.1145/1880071.1880087]

[@Senot:2011:EGP:2283696.2283850]

There exist two main approaches for providing recommendations to a group of users when the "real" group profile is not available. 1. The first combines individual recommendations to generate a list of group recommendations, 2. while the second computes group recommendations using a group profile derived from individual profiles.

- The majority-based strategies use the "most popular" choices (user preferences, item rankings, etc.) among group members.

For example, GroupCast [McCarthy et al., 2001] displays content that suits the intersection of user profiles when the people are close to a public screen.

Plurality Voting, Borda Count, Copeland Rule , Approval Voting

- The consensus-based strategies consider the preferences of all group members.

Examples include the *Utilitarian strategy* which averages the preferences of all the group members, the *Fairness strategy*, or the *Alternated Satisfaction strategy*.

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As an example, MusicFX [McCarthy and Anagnost, 1998] recommends the most relevant music station in a fitness center using a group profile computed by summing the squared individual preferences. By applying this strategy 71% of clients noticed a positive difference (as compared to the absence of the recommendation system).

Median, *Average*, *Additive*, *Average without Misery*, *Fairness* and *Multiplicative*

- The borderline strategies, also called role-based strategies, consider only a subset of items (item categories) in individual profiles, based on user roles or any other relevant criteria.

Least Misery, *Most Pleasure*, *Most respected person*

For example, PolyLens [O'Connor et al., 2001] uses the Least Misery strategy to recommend movies for small user groups based on the MovieLens database (http://www.movielens.org). Their survey showed that 77% of PolyLens users found group recommendations more helpful than individual ones. Yet this system only works with a single strategy.

1.1 Overview

Eleven aggregation strategies inspired by Social Choice Theory are summarized in Table 3 (see [17] for more details).

	A	B	C	D	E	F	G	H	I	J
Peter	10 1	4 9 5	3 8 2	6 9 7	10 7	9 9 8	6 6 5	8 9 6	10 3	8 8 6
Jane	10				9				7	
Mary										

Table 1: Example of individual ratings for ten items (A to J)

Strategy	How it works	Example
Plurality Voting	Uses 'first past the post': repetitively, the item with the most votes is chosen.	A is chosen first, as it has the highest rating for the majority of the group, followed by E (which has the highest rating for the majority when excluding A).
Median	Middle value of individual ratings	B's group rating is `5`, from `(4,9,5)`.
Average or *Utilitarian*	Averages individual ratings	B's group rating is `6`, namely `(4+9+5)/3`.
Additive	Adds individual ratings	B's group rating is `18`, namely `4+9+5`.
Multiplicative	Multiplies individual ratings	B's group rating is `180`, namely `4*9*5`.
Borda Count	Counts points from items' rankings in the individuals' preference lists, with bottom item getting 0 points, next one up getting one point, etc	A's group rating is `17`, namely `0` (last for Jane) `+ 9` (first for Mary) `+ 8` (shared top 3 for Peter)
Copeland Rule	Counts how often an item beats other items (using majority vote) minus how often it looses	F's group rating is `5`, as F beats 7 items (B,C,D,G,H,I,J) and looses from 2 (A,E).
Approval Voting	Counts the individuals with ratings for the item above a approval threshold (e.g. 6)	B's group rating is `1` and F's is `3`.
Least Misery	Takes the minimum of individual ratings	B's group rating is `4`, namely the smallest of `4,9,5`.
Most Pleasure	Takes the maximum of individual ratings	B's group rating is `9`, namely the largest of `4,9,5`.
Average without Misery	Averages individual ratings, after excluding items with individual ratings below a certain threshold (say 4).	J's group rating is `7.3` (the average of `8,8,6`), while A is excluded because Jane hates it.
Fairness	Items are ranked as if individuals are choosing them in turn.	Item E may be chosen first (highest for Peter), followed by F (highest for Jane) and A (highest for Mary).
Most respected person	Uses the rating of the most respected individual.	If Jane is the most respected person, then A's group rating is `1`. If Mary is most respected, then it is `10`.

Table 2: Overview of Aggregation Strategies

Strategy	Example Ranked List
Plurality Voting	$\{p_5, p_6, p_4, p_8, p_{10}, p_1, p_9, p_2, p_7, p_3\}$
Median	$\{p_1, p_5 - p_6, p_8 - p_{10}, p_4 - p_9, p_7, p_2, p_3\}$
Average	$\{p_5 - p_6, p_8, p_4 - p_{10}, p_1, p_9, p_2, p_7, p_3\}$
Additive	$\{p_5 - p_6, p_8, p_4 - p_{10}, p_1, p_9, p_2, p_7, p_3\}$
Multiplicative	$\{p_6, p_5, p_8, p_{10}, p_4, p_9, p_2 - p_7, p_1, p_3\}$
Borda Count	$\{p_6, p_5, p_1, p_4 - p_8, p_9, p_{10}, p_2, p_7, p_3\}$
Copeland Rule	$\{p_5, p_1, p_6, p_9, p_4, p_8, p_{10}, p_2 - p_7, p_3\}$
Approval Voting	$\{p_4 - p_5 - p_6 - p_8 - p_{10}, p_1 - p_7 - p_9, p_2 - p_3\}$
Least Misery	$\{p_6, p_5, p_4 - p_8 - p_{10}, p_7, p_2, p_9, p_3, p_1\}$
Most Pleasure	$\{p_1 - p_5 - p_9, p_2 - p_4 - p_6 - p_8, p_3 - p_{10}, p_7\}$
Average Without Misery	$\{p_5 - p_6, p_8, p_4 - p_{10}, p_2, p_7\}$
Fairness	$\{p_5, p_6, p_4, p_8, p_{10}, p_7, p_1, p_2, p_9, p_3\}$
Purity	$\{p_6, p_5, p_8, p_{10}, p_4, p_9, p_7, p_1, p_2, p_3\}$
Completeness	$\{p_6, p_5, p_8, p_{10}, p_4, p_9, p_1, p_2, p_7, p_3\}$
Logical Sufficiency	$\{p_6, p_5, p_1, p_8, p_4, p_9, p_2, p_{10}, p_3, p_7\}$
Group Sufficiency	$\{p_6, p_7, p_{10}, p_8, p_4, p_5, p_2, p_3, p_9, p_1\}$

Table 3: Overview of Aggregation Strategies

See section 1.2.1.

1.2 Detailed description

The following sections will review theses strategies in detail.

In order to ease the different strategies, a base example is provided:

[@Salamo:2012:GRC:2339097.2339115]

Most well-known strategies used in collaborative filtering algorithms for recommendation generation.

1. The Least misery strategy, as defined in Eq. 13, chooses a product p_i based on the minimum satisfaction of the individual preferences

The rationale is that a group is as satisfied as its least satisfied member. –veure formula

$$least_m isery(p_i, MI) = arg\min$$

$$MSE = \frac{1}{n}\sum_i^n (p_i - a_i)^2$$

2. Most pleasure strategy selects the maximum satisfaction of the individual preferences, see Eq. 14. It considers that at least one member will be maximally satisfied. –veure formula

Previous strategies assume the consensus based on the satisfaction of one individual: the least or the most satisfied. There is no warranty that the recommendations will suit the whole group.

3. Multiplicative strategy multiplies the satisfaction of the individual users, see Eq. 15. With this strategy, it might happen that a member with unique tastes always lose out because their opinion happens to be a minority preference.

[@sweb2012]

Though recommendation approaches have addressed group preference modeling explicitly to a rather limited extent, or in an indirect way in prior work in the computing field, the related issue of social choice (also called group decision making, i.e. deciding what is best for a group given the opinions of individuals) has been studied extensively in Economics, Politics, Sociology, and Mathematics [24, 33].

The models for the construction of a social welfare function in these works are similar to the group modeling problem we put forward here.

Other areas in which Social Choice Theory has been studied are Collaborative Filtering (CF), Meta-search, and Multi-agent systems.

In CF, preferences of a group of individuals are aggregated to produce a predicted preference for somebody outside the group. Meta-search can be seen –and formulated– as a form of group decision making, where the aggregated inputs are produced by information retrieval systems instead of people. In a meta-search engine, the rankings produced by multiple search engines need to be combined into one single list, forming the well-known problem of rank aggregation in Information Retrieval [3]. Ensemble recommenders combining several recommendation algorithms also involve a particular case of this problem, similarly to

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meta-search except for the absence of a query. Finally, in Multi-agent systems, agents need to take decisions that are not only rational from an individual's point of view, but also from a social point of view.

In all the above fields, different strategies to combine several users' preferences and to aggregate item ranking lists can be applied based on the utilized social welfare function.

These strategies are classified by Senot and colleagues [27] into three categories, namely:

- majority-based strategies, which strength the "most popular" choices (user preferences, item rankings, etc.) among the group, e.g. Borda Count, Copeland Rule, and Plurality Voting strategies;

- consensus-based (or democratic) strategies, which average somehow all the available choices, e.g. Additive Utilitarian, Average without Misery, and Fairness strategies;

- and border-line strategies, also called role-based strategies in [5], which only consider a subset of choices based on user roles or any other relevant criterion, e.g. Dictatorship, Least Misery and Most Pleasure strategies.

In the following, we assume a user has a preference (utility) for each item represented in the form of a numeric 1-10 rating. In all the cases, the greater the rating value, the most useful the item is for the user.

Therefore, following the definitions stated in Annex1, the example considers:

- $U = \{u_1, u_2, u_3\}$ is the set of *users*, where u_i represents the ith user and n the number of users, $|U| = n = 3$.

- $P = \{p_1, \ldots, p_{10}\}$ is the set of *products* or *items* for recommendation, where p_j is the jth product and m the number of items, $|P| = m = 10$.

- $V = \{1, \ldots, 10\}$ is the totally ordered set of *values* representing the possible ratings given by a user for any item, where v_k is the kth value and s the number of items, $|V| = s = 10$.

1.2.1 Plurality voting strategy

Each member votes for his preferred item and the one with the highest votes is selected. In this strategy, the items that were rated highest are combined as follows:

1. A user is randomly selected

2. His L top rated items are taking into account.

   - From them, the item that causes the most utility to the group is chosen for the group profile with a score equal to m, i.e., the number of items.

3. Restart the process in step 1 considering the remaining $m{-}1,m{-}2,etc.$ items and uniformly diminishing to 1 the further assigned scores.

In the final list, the higher score the more influential the item is for the group.

	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9	p_{10}
u_1	10	4	3	6	10	9	6	8	10	8
u_2	1	9	8	9	7	9	6	9	3	8
u_3	10	5	2	7	9	8	5	6	7	6
group	**5**	**3**	**1**	**8**	**10**	**9**	**2**	**7**	**4**	**6**

Table 4: Group choice selection following the *plurality voting strategy*. The ranked list of items for the group would be $\{p_5,p_6,p_4,p_8,p_{10},p_1,p_9,p_2,p_7,p_3\}$ following the user selecting order $\{u_1,u_2,u_3\}$ and $L=3$.

To better understand the strategy, let us explain its first 4 iterations on the example shown in Table [@table].

i. Iteration 0

   1. The user u_1 is selected
   2. Whose $L=3$ top ranked items are p_1, p_5 and p_9.
      - From these items, we choose item p_5 with a score $m=10$. Item p_5 is the one with most votes to all three users, whose votes for items p_1, p_5 and p_9 are respectively 21, 26 and 20.

ii. Iteration 1

   1. The user u_2 is selected
   2. Whose $L=3$ top ranked items are p_2, p_4 and p_6.
      - From these items, we choose item p_6 with a score $m-1=9$. Item p_6 is the one with most votes to all three users, whose votes for items p_2, p_4 and p_6 are respectively 18, 22 and 26.

iii. Iteration 2

    1. The user u_3 is selected

    2. Whose $L = 3$ top ranked items are p_1, p_4 and p_9.

        • From these items, we choose item p_4 with a score $m - 2 = 8$. Item p_4 is the one with most votes to all three users, whose votes for items p_1, p_4 and p_9 are respectively 21, 22 and 20.

iv. Iteration 3

    1. The user u_1 is selected

    2. Whose $L = 3$ top ranked items are p_1, p_9 and p_8.

        • From these items, we choose item p_8 with a score $m - 3 = 7$. Item p_8 is the one with most votes to all three users, whose votes for items p_1, p_9 and p_8 are respectively 21, 20 and 23.

1.2.2 Median strategy

The group rating for a particular item is computed as the middle value of the group members' ratings.

(1)
$$group_j = \underset{1 \leq i \leq n}{median}(r_{i,j})$$

As shown in equation (1)....

	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9	p_{10}
u_1	10	4	3	6	10	9	6	8	10	8
u_2	1	9	8	9	7	9	6	9	3	8
u_3	10	5	2	7	9	8	5	6	7	6
group	**10**	**5**	**3**	**7**	**9**	**9**	**6**	**8**	**7**	**8**

Table 5: Group choice selection following the *median strategy*. The ranked list of items for the group would be $\{p_1, p_5 - p_6, p_8 - p_{10}, p_4 - p_9, p_7, p_2, p_3\}$.

1.2.3 Average strategy

In this strategy, the group rating for a particular item is computed as the average rating over all individuals (Figure 3). Note that if no user or item weighting is

conducted, the ranking list of this strategy is the same as that of the *Utilitarian strategy.*

$$group_j = avg_j = \frac{1}{n} \cdot \sum_{i=1}^{n} r_{i,j}$$

	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9	p_{10}
u_1	10	4	3	6	10	9	6	8	10	8
u_2	1	9	8	9	7	9	6	9	3	8
u_3	10	5	2	7	9	8	5	6	7	6
group	**7**	**6**	**4.3**	**7.3**	**8.7**	**8.7**	**5.7**	**7.7**	**6.7**	**7.3**

Table 6: Group choice selection following the *average strategy*. The ranked list of items for the group would be $\{p_5 - p_6, p_8, p_4 - p_{10}, p_1, p_9, p_2, p_7, p_3\}$.

1.2.4 Additive utilitarian strategy

Preference values from group members are added, and the larger the sum the more influential the item is for the group (Figure 1). Note that the resulting group ranking will be exactly the same as that obtained taking the average of the individual preference values. A potential problem of this strategy is that individuals' opinions tend to be less significant as larger the group is. This strategy could also use a weighted schema, where a weight is attached to individual preferences depending on multiple criteria for single or multiple users.

$$group_j = \sum_{i=1}^{n} r_{i,j}$$

	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9	p_{10}
u_1	10	4	3	6	10	9	6	8	10	8
u_2	1	9	8	9	7	9	6	9	3	8
u_3	10	5	2	7	9	8	5	6	7	6
group	**21**	**18**	**13**	**22**	**26**	**26**	**17**	**23**	**20**	**22**

Table 7: Group choice selection following the *additive strategy*. The ranked list of items for the group would be $\{p_5 - p_6, p_8, p_4 - p_{10}, p_1, p_9, p_2, p_7, p_3\}$.

1.2.5 Multiplicative utilitarian strategy

Instead of adding the preferences, they are multiplied, and the larger the product the more influential the item is for the group (Figure 2). Note that this strategy could be self-defeating: in a small group, the opinion of each individual may have too much impact on the product.

$$group_j = \prod_{i=1}^{n} r_{i,j}$$

	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9	p_{10}
u_1	10	4	3	6	10	9	6	8	10	8
u_2	1	9	8	9	7	9	6	9	3	8
u_3	10	5	2	7	9	8	5	6	7	6
group	**100**	**180**	**48**	**378**	**630**	**648**	**180**	**432**	**210**	**384**

Table 8: Group choice selection following the *multiplicative strategy*. The ranked list of items for the group would be $\{p_6, p_5, p_8, p_{10}, p_4, p_9, p_2 - p_7, p_1, p_3\}$.

1.2.6 Borda count strategy

This strategy is a two-step process:

1. Normalization step

   - Scores are assigned to the items according to their ratings in a user profile: those with the lowest value get zero scores, the next one up one point, and so on.
   - When an individual has multiple preferences with the same rating, the averaged sum of their hypothetical scores are equally distributed to the involved items.

2. Addition step

   - An additive strategy is followed, and the larger the sum the more influential the item is for the group.

	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9	p_{10}
u_1	10	4	3	6	10	9	6	8	10	8
u_2	1	9	8	9	7	9	6	9	3	8
u_3	10	5	2	7	9	8	5	6	7	6

$\Downarrow$

	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9	p_{10}
u_1	8	1	0	2.5	8	6	2.5	4.5	8	4.5
u_2	0	7.5	4.5	7.5	3	7.5	2	7.5	1	4.5
u_3	9	1.5	0	5.5	8	7	1.5	3.5	5.5	3.5
group	**17**	**10**	**4.5**	**15.5**	**19**	**20.5**	**6**	**15.5**	**14.5**	**12.5**

Table 9: Group choice selection following the *borda count strategy*. The ranked list of items for the group would be $\{p_6, p_5, p_1, p_4 - p_8, p_9, p_{10}, p_2, p_7, p_3\}$.

As an example, we show how the first step of the process is performed for user u_1. That is, how ratings are normalized according to their relative relevance within the users' preferences.

The items sequence in increasing rating value for user u_1 are $\{p_3, p_2, p_4 - p_7, p_8 - p_{10}, p_6, p_1 - p_5 - p_9\}$.

1. For item p_3 a *0* score is assigned to $r_{1,3}$.

2. Item p_2 receives a score of value *1* to $r_{1,2}$.

3. The next score to be assigned would be *2*. In this case, the next two items with lowest rating value, $\{p_4 - p_7\}$, have the same rating. Therefore, the two next scores *2* and *3* are considered, and the average of them (i.e., $(2 + 3)/2 = 2.5$) is assigned to both items to $r_{1,4}$ and $r_{1,7}$.

4. The next score to be assigned would be *4*. In this case, the next two items with lowest rating value, $\{p_8 - p_{10}\}$, have the same rating. Therefore, the two next scores *4* and *5* are considered, and the average of them (i.e., $(4 + 5)/2 = 4.5$) is assigned to both items to $r_{1,8}$ and $r_{1,10}$.

5. Item p_6 receives a score of value *6* to $r_{1,6}$.

6. The next score to be assigned would be *7*. In this case, the next three items with lowest rating value, $\{p_1 - p_5 - p_9\}$, have the

same rating. Therefore, the next three scores *7*, *8* and *9* are considered, and the average of them (i.e., $(7 + 8 + 9)/3 = 8$) is assigned to corresponding items to $r_{1,1}$, $r_{1,5}$ and $r_{1,9}$.

1.2.7 Copeland rule strategy

Being a form of majority voting, this strategy sorts the items according to their *Copeland index*:

> The difference between the number of times an item beats (has higher ratings) the rest of the items and the number of times it loses

In the bottom table, a $+/-$ symbol in the ij-th cell (i for rows, and j for columns) means that item at j-th column was rated higher/lower than item at i-th row by the majority of the users. A zero value in a cell means that the corresponding items were rated with the same number of "beats" and "looses".

	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9	p_{10}
u_1	10	4	3	6	10	9	6	8	10	8
u_2	1	9	8	9	7	9	6	9	3	8
u_3	10	5	2	7	9	8	5	6	7	6

The *Copeland* corresponding matrix is:

	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9	p_{10}
p_1	0	−	−	−	0	−	−	−	0	−
p_2	+	0	−	+	+	+	0	+	+	+
p_3	+	+	0	+	+	+	+	+	+	+
p_4	+	−	−	0	+	+	−	0	0	−
p_5	0	−	−	−	0	−	−	−	−	−
p_6	+	−	−	−	+	0	−	−	−	−
p_7	+	0	−	+	+	+	0	+	+	+
p_8	+	−	−	0	+	+	−	0	+	−
p_9	0	−	−	0	+	+	−	−	0	−
p_{10}	+	−	−	+	+	+	−	+	+	0
group	**+7**	**-6**	**-9**	**+1**	**+8**	**+5**	**-6**	**0**	**+3**	**-3**

Table 10: Group choice selection following the *Copeland rule strategy*. The ranked list of items for the group would be $\{p_5, p_1, p_6, p_9, p_4, p_8, p_{10}, p_2 - p_7, p_3\}$.

To better understand the strategy, let's explain 3 cells with different symbol values:

- *Cell 2,1*: + symbol. Means that item p_1 was rated higher than item p_2 by the majority of the users.

users	p_1	p_2
u_1	10	4
u_2	1	9
u_3	10	5
beats (B)	*2*	*1*
loses (L)	*1*	*2*
B-L	**1**	**-1**

- *Cell 5,1*: 0 symbol. Means means that the item p_1 and item p_5 were rated with the same number of "beats" and "looses".

users	p_1	p_5
u_1	10	10
u_2	1	7
u_3	10	9
beats (B)	*1*	*1*
loses (L)	*1*	*1*
B-L	**0**	**0**

- *Cell 5,3*: $-$ symbol. Means that item p_3 was rated lower than item p_5 by the majority of the users.

users	p_3	p_5
u_1	3	10
u_2	8	7
u_3	2	9
beats (B)	*1*	*2*
loses (L)	*2*	*1*
B-L	**-1**	**1**

1.2.8 Approval voting strategy

A threshold θ is considered for the item ratings:

- Only those ratings greater or equal than the threshold value are taking into account for the profile combination.

- An item receives a vote for each user profile that has its rating surpassing the established threshold value.

The larger the number of votes the more influential the item is for the group. This strategy intends to promote the election of moderate alternatives: those that are not strongly disliked.

	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9	p_{10}
u_1	10	4	3	6	10	9	6	8	10	8
u_2	1	9	8	9	7	9	6	9	3	8
u_3	10	5	2	7	9	8	5	6	7	6

For a threshold $\theta = 5$, we get the following votes:

	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9	p_{10}
u_1	1	-	-	1	1	1	1	1	1	1
u_2	-	1	1	1	1	1	1	1	-	1
u_3	1	-	-	1	1	1	-	1	1	1
group	**2**	**1**	**1**	**3**	**3**	**3**	**2**	**3**	**2**	**3**

Table 11: Group choice selection following the *approval voting strategy*. The ranked list of items for the group would be $\{p_4 - p_5 - p_6 - p_8 - p_{10}, p_1 - p_7 - p_9, p_2 - p_3\}$.

See table 11. See equation 0. See figure 1. Arabic of table 11

1.2.9 Least misery strategy

The score of an item in the group profile is the minimum of its ratings in the user profiles. The lower rating, the less influential the item is for the group. Thus, a group is as satisfied as its least satisfied member (Figure 5).

Note that a minority of the group could dictate the opinion of the group: although many members like a certain item, if one member really hates it, the preferences associated to it will not appear in the group profile.

$$group_j = \min_{1 \leq i \leq n} (r_{i,j})$$

	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9	p_{10}
u_1	10	4	3	6	10	9	6	8	10	8
u_2	1	9	8	9	7	9	6	9	3	8
u_3	10	5	2	7	9	8	5	6	7	6
group	**1**	**4**	**2**	**6**	**7**	**8**	**5**	**6**	**3**	**6**

Table 12: Group choice selection following the *least misery strategy*. The ranked list of items for the group would be $\{p_6, p_5, p_4 - p_8 - p_{10}, p_7, p_2, p_9, p_3, p_1\}$.

1.2.10 Most pleasure strategy

It works as the *least misery strategy*, but instead of considering for an item the smallest ratings of the users, it selects the greatest ones. The higher rating the more influential the item is for the group, as shown in Figure 6.

$$group_j = \max_{1 \leq i \leq n} (r_{i,j})$$

	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9	p_{10}
u_1	10	4	3	6	10	9	6	8	10	8
u_2	1	9	8	9	7	9	6	9	3	8
u_3	10	5	2	7	9	8	5	6	7	6
group	**10**	**9**	**8**	**9**	**10**	**9**	**6**	**9**	**10**	**8**

Table 13: Group choice selection following the *most pleasure strategy*. The ranked list of items for the group would be $\{p_1 - p_5 - p_9, p_2 - p_4 - p_6 - p_8, p_3 - p_{10}, p_7\}$.

1.2.11 Average without misery strategy

As the average strategy, this one assigns an item the average of its ratings in the individual profiles. The difference here is that those items which have a rating under a certain threshold θ will not be considered in the group recommendations. Figure 4 shows an example of group formation following this strategy with a threshold value of 3.

$$group_j = \begin{cases} -, & \text{if } \exists_i : r_{i,j} \leqslant \theta \\ avg_j, & \text{otherwise} \end{cases}$$

	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9	p_{10}
u_1	10	4	3	6	10	9	6	8	10	8
u_2	1	9	8	9	7	9	6	9	3	8
u_3	10	5	2	7	9	8	5	6	7	6
group	**-**	**6**	**-**	**7.3**	**8.7**	**8.7**	**5.7**	**7.7**	**-**	**7.3**

Table 14: Group choice selection following the *average without misery strategy*. The ranked list of items for the group would be $\{p_5 - p_6, p_8, p_4 - p_{10}, p_2, p_7\}$.

1.2.12 Fairness strategy

In this strategy, the items that were rated highest and cause less misery to all the users of the group are combined as follows:

1. A user is randomly selected

2. His L top rated items are taking into account.

   - From them, the item that less misery causes to the group (that from the worst alternatives that has the highest rating) is chosen for the group profile with a score equal to m, i.e., the number of items.

3. Restart the process in step 1 considering the remaining $m–1,m–2,etc.$ items and uniformly diminishing to 1 the further assigned scores.

In the final list, the higher score the more influential the item is for the group.

Note that this list would be different if we let other users to choose first.

	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9	p_{10}
u_1	10	4	3	6	10	9	6	8	10	8
u_2	1	9	8	9	7	9	6	9	3	8
u_3	10	5	2	7	9	8	5	6	7	6
group	**4**	**3**	**1**	**8**	**10**	**9**	**5**	**7**	**2**	**6**

Table 15: Group choice selection following the *fairness strategy*. The ranked list of items for the group would be $\{p_5, p_6, p_4, p_8, p_{10}, p_7, p_1, p_2, p_9, p_3\}$ following the user selecting order $\{u_1, u_2, u_3\}$ and $L = 3$.

To better understand the strategy, let us explain its first 3 iterations on the example shown in Table [@table].

i. Iteration 0

1. The user u_1 is selected
2. Whose $L = 3$ top ranked items are p_1, p_5 and p_9.
   - From these items, we choose item p_5 with a score $m = 10$. Item p_5 is the one that less misery causes to users u_2 and u_3, whose lowest ratings for items p_1, p_5 and p_9 are respectively 1, 7 and 3.

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ii. Iteration 1

    1. The user u_2 is selected

    2. Whose $L = 3$ top ranked items are p_2, p_4 and p_6.

        • From these items, we choose item p_6 with a score $m - 1 = 9$. Item p_6 is the one that less misery causes to users u_1 and u_3, whose lowest ratings for items p_2, p_4 and p_6 are respectively 4, 6 and 8.

iii. Iteration 2

    1. The user u_3 is selected

    2. Whose $L = 3$ top ranked items are p_1, p_4 and p_9.

        • From these items, we choose item p_4 with a score $m - 2 = 8$. Item p_4 is the one that less misery causes to users u_1 and u_2, whose lowest ratings for items p_1, p_4 and p_9 are respectively 1, 6 and 3.

1.2.13 Purity strategy

The objective is to measure how many preferences are covered (i.e., satisfied) by a product while considering all the preferences of the group. The *deviation* is included in the equation to denote the dispersion of the satisfaction.

- A value $purity_i = 1$, means that all members of the group satisfy their preferences

- Whereas a value $purity_i = 0$ denotes that none of the preferences of the group are satisfied.

$$dev_j = \sqrt{\frac{1}{n-1} \cdot \sum_{i=1}^{n}(r_{i,j} - avg_j)^2}$$

$$group_j = \frac{\sum_{i=1}^{n} r_{i,j} - dev_j}{n \cdot v_s}$$

	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9	p_{10}
u_1	10	4	3	6	10	9	6	8	10	8
u_2	1	9	8	9	7	9	6	9	3	8
u_3	10	5	2	7	9	8	5	6	7	6
dev	**5,196**	**2,646**	**3,215**	**1,528**	**1,528**	**0,577**	**0,577**	**1,528**	**3,512**	**1,155**
group	**0,527**	**0,512**	**0,326**	**0,682**	**0,816**	**0,847**	**0,547**	**0,716**	**0,550**	**0,695**

Table 16: Group choice selection following the *purity strategy*. The ranked list of items for the group would be $\{p_6, p_5, p_8, p_{10}, p_4, p_9, p_7, p_1, p_2, p_3\}$.

1.2.14 Completeness strategy

The objective of the *completeness strategy* is to favor high scores while penalizing big differences between members.

$$group_j = \frac{\sum_{i=1}^{n} w_i \cdot \sqrt{r_{i,j}}}{n \cdot \sqrt{v_s}}$$

where w_j is a weighting factor used to tune the strategy—i.e., to focus more on one of the user's preferences. Also, must comply?

$$\begin{cases} \sum_{i=1}^{n} w_i = 1 \\ w_i \geq 0 \end{cases}$$

	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9	p_{10}
u_1	10	4	3	6	10	9	6	8	10	8
u_2	1	9	8	9	7	9	6	9	3	8
u_3	10	5	2	7	9	8	5	6	7	6
group	**0,193**	**0,191**	**0,157**	**0,213**	**0,232**	**0,233**	**0,188**	**0,218**	**0,199**	**0,214**

Table 17: Group choice selection following the *completeness strategy*. The ranked list of items for the group would be $\{p_6, p_5, p_8, p_{10}, p_4, p_9, p_1, p_2, p_7, p_3\}$.

1.2.15 Logical Sufficiency strategy

The degree of logical sufficiency (i.e, in short the *ls* measure) is a standard likelihood ratio statistic, which has been applied to measure rule quality of rule

induction systems. This measure divides the proportion of positive examples that are covered by a rule by the proportion of negative examples.

$$group_j = \frac{\sum_{i=1}^{n} ls_{i,j}}{s}$$

$$ls_{i,j} = \frac{r_{i,j}}{v_s - r_{i,j}}$$

Note that in the extreme $ls_{i,j}$ approaches to infinity and, in this case, we restrict the value to $ls_{i,j} = r_{i,j}$. On the other hand, if $ls_{i,j}$ is much less than unity, then the product i is discouraging for user j.

	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9	p_{10}
u_1	10	4	3	6	10	9	6	8	10	8
u_2	1	9	8	9	7	9	6	9	3	8
u_3	10	5	2	7	9	8	5	6	7	6

$\Downarrow$

	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9	p_{10}
u_1	10	0,67	0,43	1,50	10	9	1,50	4	10	4
u_2	0,11	9	4	9	2,33	9	1,50	9	0,43	4
u_3	10	1	0,25	2,33	9	4	1	1,50	2,33	1,50
sum	**20,11**	**10,67**	**4,68**	**12,83**	**21,33**	**22,00**	**4,00**	**14,50**	**12,76**	**9,50**
group	**0,670**	**0,356**	**0,156**	**0,428**	**0,711**	**0,733**	**0,133**	**0,483**	**0,425**	**0,317**

Table 18: Group choice selection following the *logical sufficiency strategy*. The ranked list of items for the group would be $\{p_6, p_5, p_1, p_8, p_4, p_9, p_2, p_{10}, p_3, p_7\}$.

1.2.16 Group Sufficiency strategy

gs measures the satisfaction of a member in relation to the satisfied preferences for the rest of the group and to the unsatisfied preferences for the individual in comparison with the whole group.

The intuition behind this strategy comes from the fact that the satisfaction of an individual is likely to depend on that of other individual in the group (emotional contagion).

$$group_j = \frac{\sum_{i=1}^{n} gs_{i,j}}{n}$$

$$gs_{i,j} = \frac{\frac{r_{i,j}+1}{\sum_{j=1}^{n} r_{i,j}+2}}{\frac{v_s - r_{i,j}+1}{\sum_{j=1}^{n} v_s - r_{i,j}+2}}$$

This equation estimates as the best product the one that minimizes gs.

	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9	p_{10}
u_1	10	4	3	6	10	9	6	8	10	8
u_2	1	9	8	9	7	9	6	9	3	8
u_3	10	5	2	7	9	8	5	6	7	6

$\Downarrow$

	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9	p_{10}
u_1	5,26	0,50	0,63	0,58	2,36	1,07	1,11	1,08	6,00	1,25
u_2	0,10	3,50	3,80	2,08	0,43	1,07	1,11	1,80	0,27	1,25
u_3	5,26	0,70	0,42	0,83	1,07	0,64	0,79	0,50	1,09	0,58
sum	**10,62**	**4,70**	**4,86**	**3,50**	**3,86**	**2,79**	**3,00**	**3,38**	**7,36**	**3,08**
group	**3,539**	**1,567**	**1,619**	**1,167**	**1,286**	**0,929**	**1,000**	**1,128**	**2,455**	**1,028**

Table 19: Group choice selection following the *group sufficiency strategy*. The ranked list of items for the group would be $\{p_6, p_7, p_{10}, p_8, p_4, p_5, p_2, p_3, p_9, p_1\}$.

+NOTA: veure fórmules a [Gartrell, M., Xing, X., Lv, Q., Beach, A., Han, R., Mishra, S., & Seada, K. (2010). Enhancing group recommendation by incorporating social relationship interactions. Proceedings of the 16th ACM international conference on Supporting group work (pp. 97–106). New York, NY, USA: ACM. doi:10.1145/1880071.1880087] [Jameson, A., & Smyth, B. (2007). Recommendation to Groups. The Adaptive Web: Methods and Strategies of Web Personalization, 4321, 596–627.]

1.3 Existing Group RecSys stategies

1.3.1 Average strategy

Travel Decision Forum [15] implements multiple group modeling strategies, including the *average strategy* and the median strategy.

In [36], Yu and colleagues present a TV program recommender that performs a variation of the average strategy, where the group preference vector minimizes its distance compared to the individual members' preference vectors.

1.3.2 Additive utilitarian strategy

For example, in INTRIGUE [2], weights are assigned to particular users' ratings depending on the number of people in the group, and the group's members' relevance (children and disabled have a higher relevance).

1.3.3 Least misery strategy

PolyLens [23] uses this strategy, assuming a group of people going to watch a movie together tends to be small, and the group is as happy as its least happy member.

1.3.4 Average without misery strategy

MusicFX [21], which chooses a radio station for background music in a fitness center, follows an average without misery strategy, and a weighted random selection is made from the top stations in order to avoid starvation and always picking the same station. CATS system [22] helps users to choose a joint holiday based on individuals' critiques on holiday package features, and applying the misery aspect.