COMP 3270 Introduction to Algorithms

Homework 2

1. Use the Master Method to solve the following three recurrence relations and state the complexity orders of the corresponding recursive algorithms.

(a) $T\ n \qquad 2T\ 99n/100 \qquad 100n$

T(n) = 2T(99n/100) + 100n
T(n) = 2T(n/(100/99)) + 100n
a = 2, b = 100/99
n^(logb a) = n^(log1o0/99^2)= n^(68.97) n^(logb a) = n^(log100/99^2) = n^(68-97) > f (n) = 100n Time complexity is 0(n^68.97)

(b) $T\ n \qquad 16T\ n/2 \qquad n\ lgn$

T(n) = 16T(n/2) + n^3 log n a = 16, b = 2 , n^(logb a) = n(logv2 16) = n^4 > f(n)= n^3 log n , Time complexity is θ(n4)

(c) $T\ n \qquad 16T\ n/4 \qquad n$

T(n) = 16T(n/4) + n^2
a = 16, b = 4, n^(logb a)= n^(log4 16) = n^2, n^(logb a) = n^(log4 16) = n^2 same as f(n) = n2 , so, time complexity is θ(f(n) log n) = θ(n^2 log n)

2. Use the Substitution Method to solve the following recurrence relation. Give an exact solution:

$$T\ n\ Tn \qquad 1\ n/2$$

$T(n) = \{1 \quad n = 0,$
$T(n-1) + n/2 \quad n > 0$

$T(n) = T(n-1) + n/2$

$T(n) = T(n-2) + (n-1)/2 + n/2$

$T(n) = T(n-3) + (n-2)/2 + (n-1)/2 + n/2$

$T(n) = T(n-4) + (n-3)/2 + (n-2)/2 + (n-1)/2 + n/2$

$T(n) = T(n-k) + (n-(k-1))/2 + (n-(k-2))/2 + (n-(k-3))/2 + \ldots\ldots + (n-1)/2 + n/2$

$T(n) = T(n-n) + (n-(n-1))/2 + (n-(n-2))/2 + (n-(n-3))/2 + \ldots\ldots\ldots\ldots\ldots + (n-1)/2 + n/2$

$T(n) = T(0) + (1)/2 + (2)/2 + (3)/2 + (4)/2 + \ldots\ldots\ldots\ldots\ldots + (n-1)/2 + n/2$

$T(n) = 1 + (1/2 + 2/2 + 3/2 + 4/2 + \ldots\ldots\ldots\ldots\ldots + (n-1)/2 + n/2)$

$T(n) = 1 + (1 + 2 + 3 + 4 + \ldots\ldots\ldots\ldots + (n-1) + n)/2$

$T(n) = 1 + (n(n+1)/2)/2$

$T(n) = 1 + (n^2 + n)/4$

3. Use Strassen's algorithm to compute the matrix product. Show detailed procedure of your work.
:

1	3	6	8
7	5	4	2

M1 = (A11 + A22)(B11 +B22) = 6x8=48
M2 = (A21+A22)(B1)
M3=A11(B12-B22)
M4=A22(B21-B11)
M5=(A11+A12)B22
M6=(A21+A11)(B11+B12)
M7=(A12-A22)(B21+B22)

C11=M1+M4-M5+M7 = 48-10-8+(-12)=28
C12=M3+M5

C21=M2+m4

C22=M1-M2+m3+m6

C = (18   14)
(62   66)

4. Using pages 4-16 of the slides (which can be found under the file section on Canvas) of Chapter 4 as a model, illustrate the operation of PARTITIONon the array *A* = [13, 19, 9, 5, 12, 8, 7, 4, 21, 2, 6, 11].

[13,19,9,5,12,8,7,4,21,2,6,11]

[13,19,9,5,12,8,7,4,21,2,6,11]

[13,19,9,5,12,8,7,4,21,2,6,11]

[9,19,13,5,12,8,7,4,21,2,6,11]

[9,5,13,19,12,8,7,4,21,2,6,11]

[9,5,13,19,12,8,7,4,21,2,6,11]

[9,5,8,19,12,13,7,4,21,2,6,11]

[9,5,8,7,12,13,19,4,21,2,6,11]

[9,5,8,7,4,13,19,12,21,2,6,11]

[9,5,8,7,4,13,19,12,21,2,6,11]

[9,5,8,7,4,2,19,12,21,13,6,11]

[9,5,8,7,4,2,6,12,21,13,19,11]

[9,5,8,7,4,2,6,11,21,13,19,12]

5. Show that the running time of QUICKSORT is $\Theta\ n$ when the array A contains distinct elements and is sorted in decreasing order.

so $T(n) = T(n-1) + n + c$

$T(n) = T(n-1) + n$

$T(n-1) = T(n-2) + n-1$

$T(n-2) = T(n-3) + n-2$
$T(n) = T(n-2) + n-1 + n$

$T(n) = T(n-3) + n-2 + n-1 + n$

$T(n) = T(n-k) + kn - k(k-1)/2$
For base case:

$n - k = 1$ so we can get $T(1)$
$\Rightarrow k = n - 1$

$T(n) = T(1) + (n-1)n - (n-1)(n-2)/2$
So $n^2 \Rightarrow \Theta\left(n^2\right)$.