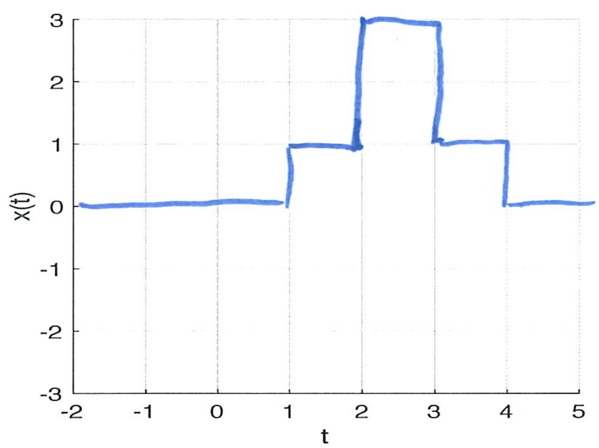
ELEC 2120 Name:________________________________

Test #1: Sample problems.

Part I (25 points): Only a simple instrument of writing is allowed.

1. (a) On the graph provided, neatly sketch the function:

$x(t) = [u(t-1)-u(t-4)]+2[u(t-2)-u(t-3)]$



(b) Is $x(t)$ a discrete-time signal?

no
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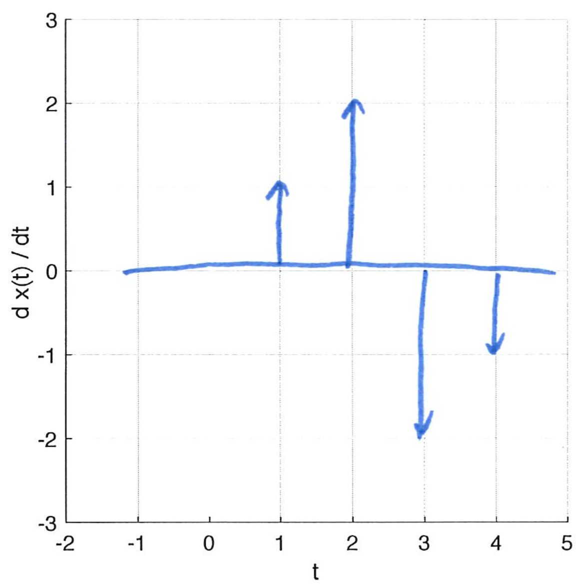
(c) Sketch the derivative, $dx(t)/dt$, in the graph provided.



2. Indicate the time invariance and linearity of the system. (Yes/No answer is sufficient)

System	Time invariant? Yes/No	Linear? Yes/No
$y(t) = \sin(x(t))$	yes	no nonlinear

3. Suppose that the impulse responses of two LTI systems are $h_1(t)$ and $h_2(t)$. If these two systems are connected in series as below, the impulse response of the overall system is

$h_1(t) * h_2(t)$.

$$x(t) \longrightarrow \boxed{h_1(t)} \longrightarrow \boxed{h_2(t)} \longrightarrow y(t)$$

4. Evaluate the following integral:

$$\int_{-2}^{2} t[\delta(t-3) + u(t+1)]dt$$

The impulse $\delta(t-3)$ is not in the duration $-2 \le t \le 2$.

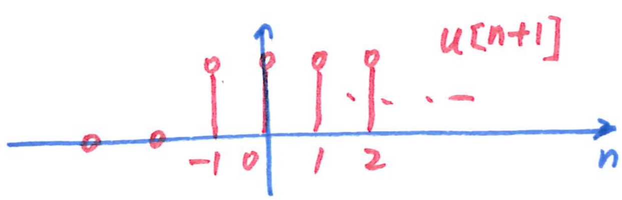
$$\therefore \int_{-2}^{2} t\, \delta(t-3)\, dt = 0$$

$$\int_{-2}^{2} t\, u(t+1)\, dt = \int_{-1}^{2} t\, dt = \frac{1}{2} t^2 \Big|_{-1}^{2} = 2 - \frac{1}{2} = \frac{3}{2}$$

$$\therefore \int_{-2}^{2} t[\delta(t-3) + u(t+1)]\, dt = \frac{3}{2}$$

5. Convert the discrete-time signal x[n] to the form {a, $\underline{\text{b}}$, c, d, e}

$$x[n] = u[1-n]\ u[n+1] - 2\delta[n-1]$$

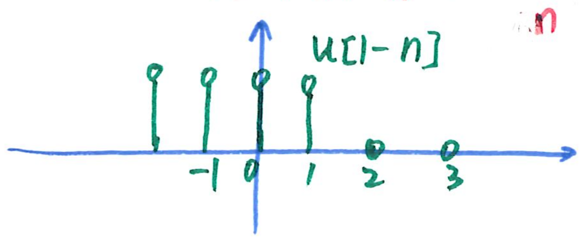


$x[n] = \{1, \underline{1}, -1\}$

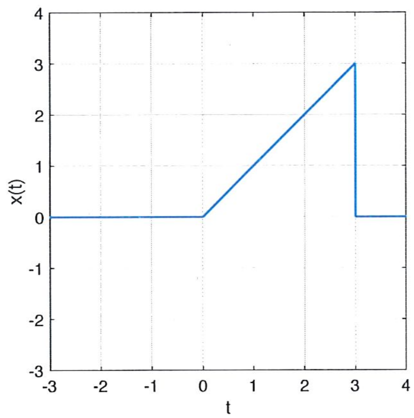
$\because u[k] = 0 \quad k < 0$

$\therefore u[1-n] = 0 \quad 1-n < 0$

$n > 1$



6. Given the function x(t):



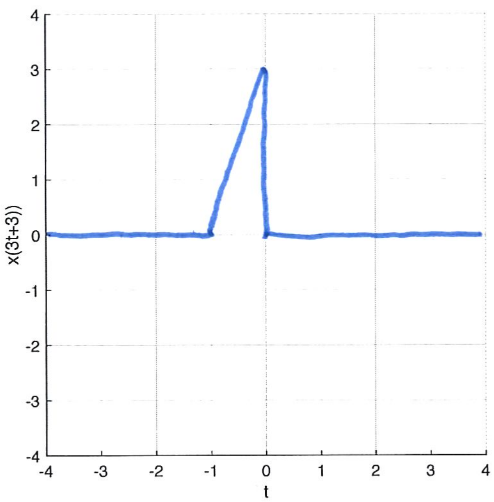
$Z(t) = X(3t)$

$y(t) = X(3(t+1)) = Z(t+1)$

compress by 3 times.

Left shift by 1.

Plot x(3t+3)



verify:

if $t = 0$.

$X(3t+3) = X(3) = 3$.

if $t = -1$

$X(3t+3) = X(0) = 0$.

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Test #1: 2/7/20

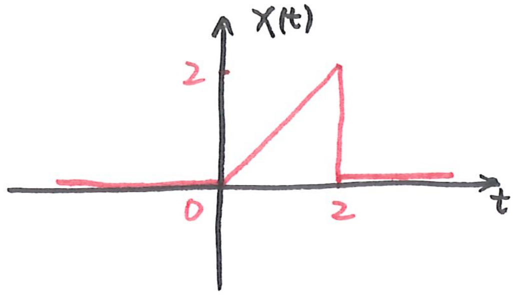
Part II: (20 points each) A 3''x5'' note card is allowed

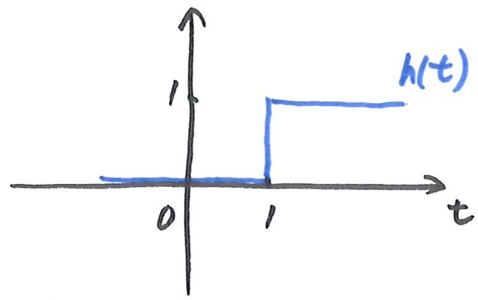
1. Given the signal $x(t) = t[u(t)-u(t-2)]$ and impulse response $h(t) = u(t-1)$,

a. neatly plot $x(t)$ and $h(t)$

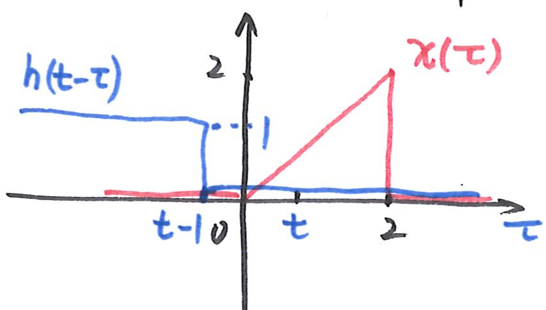
b. compute the convolution $y(t) = x(t)*h(t)$.

c. neatly plot the convolution $y(t) = x(t)*h(t)$.



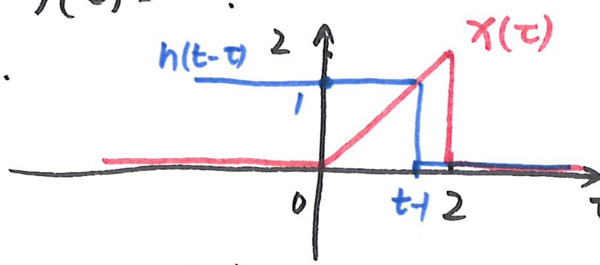


① $t<1$.



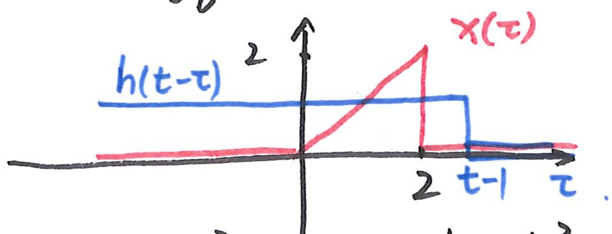
$y(t) = 0$.

② $1 \le t < 3$.



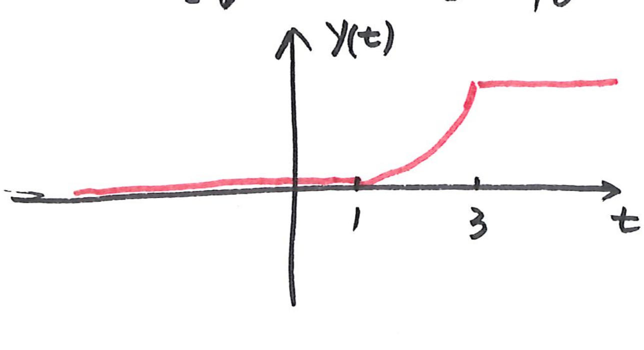
$$y(t) = \int_0^{t-1} \tau \cdot 1 \, d\tau = \frac{1}{2}\tau^2 \Big|_0^{t-1} = \frac{1}{2}(t-1)^2$$

③ $t>3$.



$$y(t) = \int_0^2 \tau \, d\tau = \frac{1}{2}\tau^2 \Big|_0^2 = 2.$$

$$y(t) = \begin{cases} 0 & t<1 \\ \frac{(t-1)^2}{2} & 1 \le t < 3 \\ 2 & t \ge 3 \end{cases}$$



Part II. Problem 1.

Method 2.

$$x(t) = r(t) - r(t-2) - 2u(t-2).$$

$$h(t) = u(t-1).$$

$$z(t) = r(t) * u(t) = \int_0^t \tau \cdot 1 \, d\tau \cdot u(t)$$

$$= \int_0^t d\left(\frac{\tau^2}{2}\right) \cdot u(t)$$

$$= \frac{t^2}{2} u(t).$$

$$\therefore \; y(t) = x(t) * h(t)$$

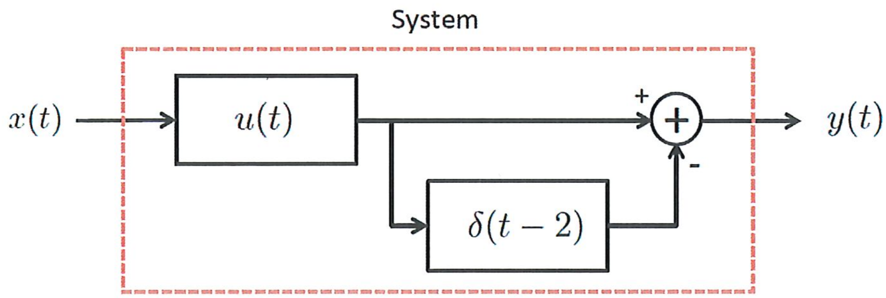
$$= [r(t) - r(t-2) - 2u(t-2)] * u(t-1)$$

$$= z(t-1) - z(t-3) - 2r(t-3)$$

$$= \frac{(t-1)^2}{2} u(t-1) - \frac{(t-3)^2}{2} u(t-3) - 2(t-3)u(t-3)$$

$$= \frac{(t-1)^2}{2} [u(t-1) - u(t-3)] + 2\, u(t-3)$$

2. (a) Find the impulse response of the following system.

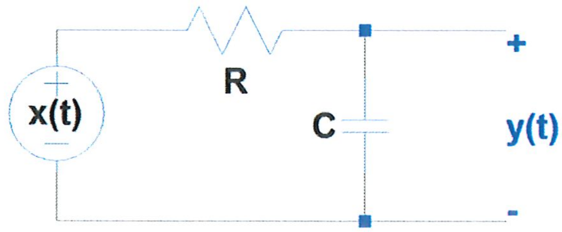


$$h(t) = u(t) * [\delta(t) - \delta(t-2)]$$
$$= u(t) - u(t-2)$$

(c) Find the step response of the above system.

$$y_{step}(t) = u(t) * h(t)$$
$$= u(t) * [u(t) - u(t-2)]$$
$$= r(t) - r(t-2)$$

3. For the circuit shown, R = 1 MΩ and C = 1 μF such that the RC time constant is 1 second. The impulse response for this circuit is:



$$h(t) = e^{-t}u(t)$$

And the step response is

$$y_{step}(t) = (1 - e^{-t})u(t)$$

a. Use the simplest approach you can think of to derive the ramp response.

if $t > 0$, $y_{ramp}(t) = \int_{-\infty}^{t} y_{step}(\tau)\,d\tau$

$= \int_{-\infty}^{t} (1 - e^{-\tau})\,u(\tau)\,d\tau$

$= \int_{0}^{t} (1 - e^{-\tau})\,d\tau$

$= \int_0^t d\tau + \int_0^t e^{-\tau}\,d(-\tau) = t + e^{-\tau}\Big|_0^t$

$= t + e^{-t} - 1$

if $t < 0$ $y_{ramp}(t) = \int_{-\infty}^{t} y_{step}(\tau)\,d\tau = \int_{-\infty}^{t} 0\,d\tau = 0$

$\therefore\ y_{ramp}(t) = (t + e^{-t} - 1)\,u(t)$.

b. Suppose x(*t*) = t[u(*t*) – u(*t*-1)]. Find the response y(*t*).

$x(t) = t\,u(t) - t\,u(t-1)$

$= r(t) - u(t-1) - (t-1)\,u(t-1)$

$= r(t) - u(t-1) - r(t-1)$

$y(t) = y_{ramp}(t) - y_{ramp}(t-1) - y_{step}(t-1)$

$\cancel{= (t + 2e^{-t} - 2)\,u(t) - [(t-1) + e^{-(t-1)} - 1]\,u(t-1)}$.

$= (t + e^{-t} - 1)\,u(t) - (t-1)\,u(t-1)$

c. Evaluate the value of y(*t*) at *t* = 1 second.

$\cancel{y(1) = (1 + 2e^{-1} - 2) = 2e^{-1} - 1}$.

$y(1) = (1 + e^{-1} - 1) = e^{-1}$