Signals and Systems HW 7

3.01 (a) waveform $x_1(t) = 4[U(t) - U(t-1)] - 2[U(t-1) - U(t-2)]$

Laplace: $X_1(s) = 4\left[\frac{1}{s} - \frac{e^{-s}}{s}\right] - 2\left[\frac{e^{-s}}{s} - \frac{e^{-2s}}{s}\right]$

(f) $x_6(t) = -5t\,u(t) + 10(t-2)u(t-2) - 10(t-6)u(t-6) + 5(t-8)u(t-8)$

Laplace transform

$$X_6(s) = \frac{-5}{s^2} + 10\cdot\frac{e^{-2s}}{s^2} - 10\cdot\frac{e^{-6s}}{s^2} + 5\cdot\frac{e^{-8s}}{s^2}$$

3.03 (a) $x_1(t) = -4t\,e^{-2t}u(t)$

let $g(t) = e^{-2t}u(t)$

$G(s) = \mathcal{L}[e^{-2t}u(t)] = \frac{1}{s+2}$

$X_1(s) = \frac{d}{ds}G(s) = \frac{-4}{(s+2)^2}$

(c) $x_3(t) = 12\,e^{-3(t-4)}u(t-4)$

$\Rightarrow \mathcal{L}[e^{-a(t-T)}\cdot u(t-T)] \to \frac{e^{-Ts}}{s+a}$

$$X_3(s) = \frac{e^{-4s}}{s+3}$$

3.5 (a) $x_1(t) = 16e^{-2t}\cos 4t\, u(t)$

$\cos \omega_0 t \xrightarrow{\mathcal{L}} \dfrac{s}{s^2+\omega_0^2}$

$e^{s_0 t}\, x(t) \longrightarrow X(s-s_0)$

$\cos 4t\, u(t) \longrightarrow \dfrac{s}{s^2+(4)^2}$

$e^{-2t}\cos 4(t)\, u(t) \longrightarrow \dfrac{s+2}{(s+2)^2+(4)^2}$

$x(t) = 16e^{-2t}\cos 4t\, u(t)$

$Y(s) = 16\,\dfrac{s+2}{(s+2)^2+16}$

(c) $x_3(t) = 10\cdot e^{-3t}\, u(t-4)$

$= 10\, e^{-3t-12+12}\, u(t-4) = 10e^{-12}e^{-3(t-9)}\, u(t-4)$

$e^{-at}\, u(t) \longrightarrow \dfrac{1}{s+a}$

$x(t-t_0) \longrightarrow e^{-st_0}X(s)$

$e^{-3t}\, u(t) \longrightarrow \dfrac{1}{s+3}$

$e^{-3(t-4)}\, u(t-4) \longrightarrow \dfrac{e^{-4s}}{s+3}$

$x_3(t) = 10\, e^{-12}\, e^{-3(t-4)}\, u(t-4)$

$X_3(s) = \dfrac{10e^{-12}e^{-4s}}{s+3}$

3.6 (a) $x_1(t) = 30(e^{-3t} + e^{3t})u(t)$

$e^{-at}u(t) \rightarrow \frac{1}{s+a}$

$$X_1(s) = \mathcal{L}[x_1(t)] = 30\left[\frac{1}{s+3} + \frac{1}{s-3}\right]$$

$$= 30\left[\frac{(s-3)+(s+3)}{(s+3)(s-3)}\right]$$

$$= 30\left(\frac{2s}{s^2-9}\right)$$

$$\boxed{X_1(s) = \frac{60s}{s^2-9}}$$