COMP 3270 Introduction to Algorithms

Homework 2

1. Use the Master Method to solve the following three recurrence relations and state the complexity orders of the corresponding recursive algorithms.

(a) $T(n) = 2T(99n/100) + 100n$

T(n) = 2T(99n/100) + 100n
T(n) = 2T(n/(100/99)) + 100n
a = 2, b = 100/99
n^(logb a) = n^(log1o0/99^2)= n^(68.97) n^(logb a) = n^(log100/99^2) = n^(68-97) > f (n) = 100n Time complexity is 0(n^68.97)

(b) $T(n) = 16T(n/2) + n^3 lgn$
T(n) = 16T(n/2) + n^3 log n a = 16, b = 2 , n^(logb a) = n(logv2 16) = n^4 > f(n)= n^3 log n , Time complexity is θ(n4)

(c) $T(n) = 16T(n/4) + n^2$
T(n) = 16T(n/4) + n^2
a = 16, b = 4, n^(logb a)= n^(log4 16) = n^2, n^(logb a) = n^(log4 16) = n^2 same as f(n) = n2 , so, time complexity is θ(f(n) log n) = θ(n^2 log n)

2. Use the Substitution Method to solve the following recurrence relation. Give an exact solution:

$$T(n) = T(n-1) + n/2$$

3. Use Strassen's algorithm to compute the matrix product. Show detailed procedure of your work.

$\begin{pmatrix} 1 & 3 \\ 7 & 5 \end{pmatrix}\begin{pmatrix} 6 & 8 \\ 4 & 2 \end{pmatrix}$.

4. Using pages 4-16 of the slides (which can be found under the file section on Canvas) of Chapter 4 as a model, illustrate the operation of PARTITION on the array A = [13, 19, 9, 5, 12, 8, 7, 4, 21, 2, 6, 11].

5. Show that the running time of QUICKSORT is $\Theta(n^2)$ when the array A contains distinct elements and is sorted in decreasing order.