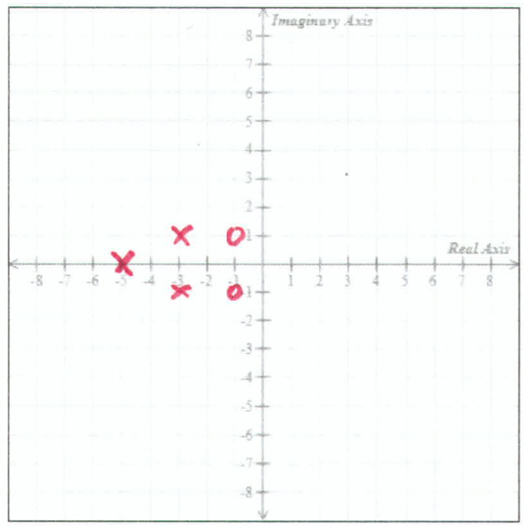
ELEC 2120 Name: Key

Test #2: 3/23/20, 9:50 am – 11:00 am

Open-book test. Formula sheet provided on Canvas.

1. (20 points) Consider $X(s) = \dfrac{s^2+2s+2}{5(s+5)(s^2+6s+10)}$

(a) Indicate the poles and zeros for $X(s)$ on the graph provided see figure.



(b) Does the initial value $x(0^+)$ exist? yes

If not, why not?

If so, calculate $x(0^+)$ and record it here:

$x(0^+) = \frac{1}{5}$

(c) Does the initial value $x(\infty)$ exist? yes

If not, why not?

If so, calculate $x(\infty)$ and record it here:

$x(\infty) = 0$

(a) zeros: $s^2+2s+2=0$. $s = \dfrac{-2 \pm \sqrt{4-8}}{2} = \dfrac{-2 \pm 2j}{2} = -1 \pm j$.

poles: $s^2+6s+10=0$ $(s+3)^2+1=0$ $\therefore$ $s+3 = \pm j$

$s = -3 \pm j$.

poles: $s = -5, -3 \pm j$.

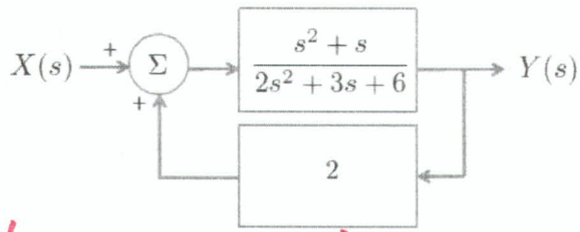
(b) $x(0^+) = \lim_{s\to\infty} sX(s) = \lim_{s\to\infty} \dfrac{s(s^2+2s+2)}{5(s+5)(s^2+6s+10)} = \dfrac{1}{5}$.

it exists.

(c) $x(\infty) = \lim_{s\to 0} sX(s) = \lim_{s\to 0} \dfrac{s(s^2+2s+2)}{5(s+5)(s^2+6s+10)} = 0$.

it exists. because all poles are on the left side of the imaginary axis (i.e., $Re(s) < 0$)

2. (20 points) (a) Find the transfer function $H(s) = \dfrac{Y(s)}{X(s)}$ of the following feedback system:



$$\left(X(s) + 2Y(s)\right)\frac{s^2+s}{2s^2+3s+6} = Y(s)$$

$$\therefore \quad X(s)\frac{s^2+s}{2s^2+3s+6} = Y(s)\left[1 - \frac{2(s^2+s)}{2s^2+3s+6}\right]$$

$$X(s)\frac{s^2+s}{2s^2+3s+6} = Y(s)\frac{2s^2+3s+6-2s^2-2s}{2s^2+3s+6} = Y(s)\frac{s+6}{2s^2+3s+6}$$

(b) Find the zeros and poles of $H(s)$

$$\therefore \quad X(s)(s^2+s) = Y(s)(s+6)$$

$$\therefore \quad H(s) = \frac{Y(s)}{X(s)} = \frac{s^2+s}{s+6}$$

Zeros of $H(s) = -1, 0$.

poles of $H(s) = -6$.

3. (20 points) Use Laplace transforms to find the solution to the differential equation

$$\frac{dy(t)}{dt} + y(t) = e^{-t}u(t), \text{ with } y(0^-) = 5$$

$$\mathcal{L}\left\{\frac{dy(t)}{dt}\right\} = sY(s) - y(0^-) = sY(s) - 5.$$

$$\mathcal{L}\{y(t)\} = Y(s)$$

$$\mathcal{L}\{e^{-t}u(t)\} = \frac{1}{s+1}$$

$$\therefore \quad sY(s) + Y(s) - 5 = \frac{1}{s+1}$$

$$\therefore \quad (s+1)Y(s) = 5 + \frac{1}{s+1}$$

$$\therefore \quad Y(s) = \frac{5}{s+1} + \frac{1}{(s+1)^2}$$

$$\therefore \quad y(t) = \mathcal{L}^{-1}\{Y(s)\}$$

$$= 5e^{-t}u(t) + te^{-t}u(t).$$

4. (20 points) Find the inverse Laplace transform.

$$X(s) = 1 + \frac{2}{s+1} + \frac{s+4}{s^2+2s+4}$$

$$X(s) = 1 + \frac{2}{s+1} + \frac{s+1+3}{(s+1)^2+(\sqrt{3})^2}$$

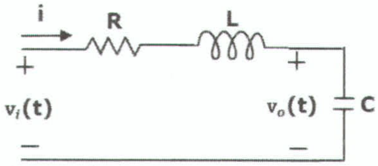
$$= 1 + \frac{2}{s+1} + \frac{s+1}{(s+1)^2+(\sqrt{3})^2} + \frac{\sqrt{3}\cdot\sqrt{3}}{(s+1)^2+(\sqrt{3})^2}$$

$$x(t) = \mathcal{L}^{-1}\{X(s)\}$$

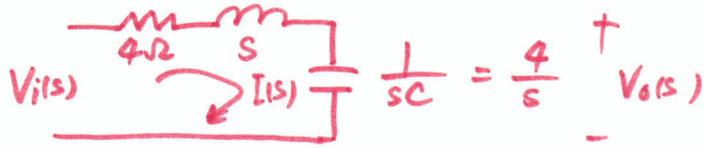
$$= \delta(t) + 2e^{-t}u(t) + e^{-t}\cos\sqrt{3}t\, u(t)$$

$$+ \sqrt{3}\, e^{-t}\sin\sqrt{3}t\, u(t).$$

5. (20 points) Determine $v_o(t)$ in the circuit below, given that $v_i(t) = 5\delta(t)$ Volts, $R = 4$ Ohms, $L = 1$ Henry, and $C = 0.25$ Farads.



s-domain circuit model:



$$V_i(s) = I(s)\left(R + sL + \frac{1}{sC}\right)$$

$$V_o(s) = I(s)\frac{1}{sC}$$

$$\therefore V_o(s) = \frac{V_i(s)}{R + sL + \frac{1}{sC}} \cdot \frac{1}{sC}$$

$$= \frac{V_i(s)}{LCs^2 + RCs + 1}$$

$V_i(s) = \mathcal{L}\{5\delta(t)\} = 5.$

$$= \frac{5}{\frac{1}{4}s^2 + s + 1}$$

$$= \frac{20}{s^2 + 4s + 4}$$

$$= \frac{20}{(s+2)^2}$$

$$\therefore v_o(t) = \mathcal{L}^{-1}\{V_o(s)\}$$

$$= 20te^{-2t}u(t) \text{ V}$$