Final Exam Signals and Systems

Jacob Howard

$\boxed{1}$ a)

$y(s) = x(s)H(s)$

Step responce $\Rightarrow$ $x(t) = u(t)$

$x(s) = \frac{1}{s}$

$y(s) = \frac{H(s)}{s}$ or $\int_{-\infty}^{t} h(t)\,dt$

(b)

$\int_{-1}^{3} \delta(t-2) + u(t-1)\,dt$

$= 1 + \int_{-1}^{3} u(t-1)\,dt$

$= 1 + \int_{1}^{3} 1 \cdot dt$

$= 1 + [3-1] = \underline{3}$

(c) $y(t) = 4x(t-2)^2$

Is this System linear ___yes___

Is this System time Invariant ___yes___

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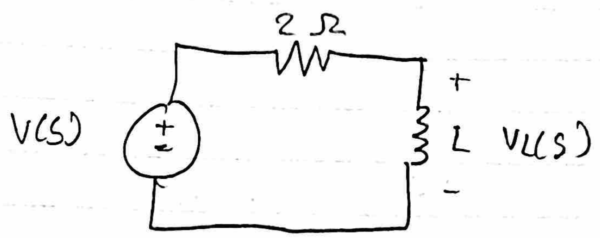
a)



$$V_L(S) = V(S)\left[\frac{S}{S+2}\right]$$

$V_S(t) = \delta(t)\,V$ $\qquad$ $V(S) = 1$

$$V_L(s) = 1\left[\frac{S}{S+2}\right] = \frac{S+2-2}{S+2} = \frac{S+2}{S+2} - \frac{2}{S+2}$$

$$V_L(S) = 1 - \frac{2}{S+1}$$

Inv Laplace $\quad$ $\boxed{V_L(t) = \delta(t) - 2e^{-2t}u(t)}$

(b) $\quad V(t) = u(t)\,V \quad ; \quad V(S) = \frac{1}{S}$

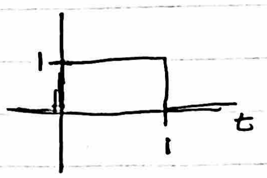
$$V_L(S) = \frac{1}{S}\left[\frac{S}{S+2}\right] = \frac{1}{S+2}$$

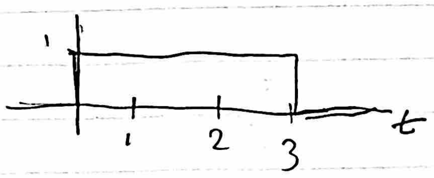
Inv Laplace

$$\boxed{V_L(t) = e^{-2t}u(t)\ V}$$

$\boxed{3}$

(a) $x(t)$





(b) $y(t) = x(t) * h(t)$

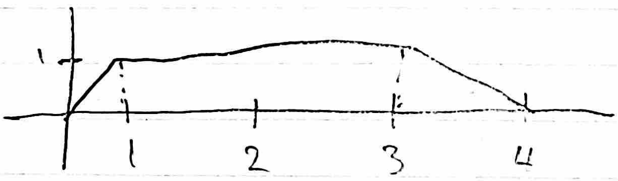
$= (u(t) - u(t-1)) * (u(t) - (u(t-3))$

$= u(t) * u(t) - u(t) * u(t-3) - u(t-1) * u(t) + u(t-1) * u(t-3)$

$= r(t) - r(t-3) - r(t-1) + r(t-4)$

$y(t) = r(t) - r(t-3) - r(t-1) + r(t-4)$

c)



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a) $H(s) = \dfrac{1000}{(1+s)(1000+s)}$

$\dfrac{A}{(1+s)} + \dfrac{A^*}{(1000+s)}$

$A_1 = (1+s)H(s)\,|_{s=-1} \approx 1$

$A_2 = (1000+s)\,H(s)\,|_{s=-1000} \approx -1$

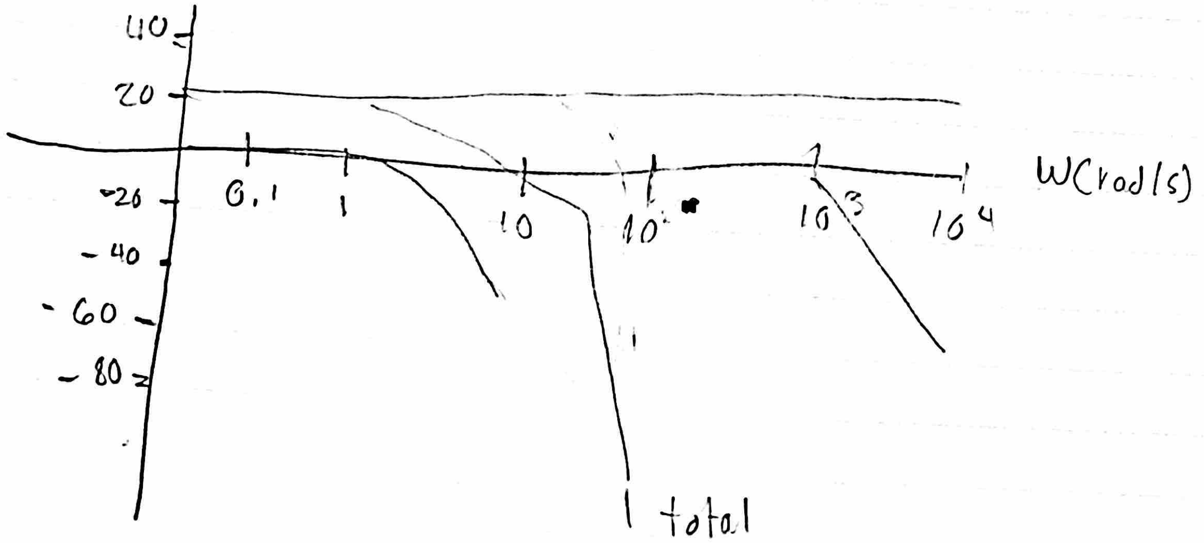
$H(s) = \dfrac{1}{(1+s)} - \dfrac{1}{(1000+s)}$

$H(s) = e^{-t} - e^{-1000t}$

(C) $\cdot s) = \dfrac{1000}{(1+s)(1000+s)} = \dfrac{1}{(1+s)(1+\frac{s}{1000})}$

$H(\omega) = \dfrac{1}{\sqrt{1+\omega^2}\,\sqrt{1+(\frac{\omega}{1000})^2}} \cdot \dfrac{1}{\angle \tan^{-1}(\omega) \quad \angle \tan^{-1}(\frac{\omega}{1000})}$

$H(\omega)_{dB} = 20\log_{10}(1) - 20\log_{10}\sqrt{1+(\frac{d\omega}{1000})^2} - 20\log_{10}\sqrt{1+(\frac{d\omega}{1000})^2}$



$\boxed{5}$ $x(t) = u(t) - u(t-2) + 1$

$\because \quad 1 \xleftrightarrow{FT} 2\pi\delta(\omega)$

$u(t) \xleftrightarrow{FT} \frac{1}{jw}$

$x(t-t_0) \xleftrightarrow{FT} X(jw)e^{-jwt_0}$

$\therefore \quad X(jw) = \frac{1}{jw} - \frac{e^{-j2w}}{jw} + 2\pi\delta(w)$

So $\Rightarrow$ $X(jw) = \frac{1}{jw}(1-e^{-j2w}) + 2\pi\delta(\omega)$