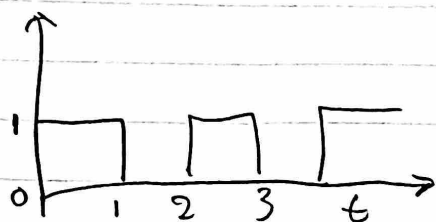


HW 13 Signals & Systems

11



Time period = 2

$$a_0 = \frac{1}{T} \int_{-T}^T f(t) dt = \frac{1}{2} \int_0^2 f(t) dt$$

$$= \frac{1}{2} \left[\int_0^1 (1) dt + \int_1^2 (0) dt \right] = \frac{1}{2} \times \int_0^1 1 \times dt$$

$$= \frac{1}{2} \times \int_0^1 1 \times dt = \frac{1}{2} \times 1 \Rightarrow \boxed{a_0 = 1/2}$$

$$a_k = \frac{2}{T} \int_{-T}^T f(t) \cos n \omega_0 t dt$$

$$= \frac{2}{T} \int_0^T f(t) \cos n \omega_0 t dt$$

$$= \frac{2}{2} \int_0^2 f(t) \cos n \omega_0 t dt = \int_0^1 (1) \cos n \omega_0 t dt$$

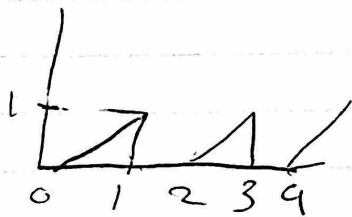
$$= \frac{\sin n \omega_0 t}{n \omega_0} \Big|_0^1 = \frac{1}{n \omega_0} \times \sin n \omega_0$$

$$\Rightarrow \boxed{a_k = \frac{\sin n \pi}{n \pi}}$$

$$\begin{aligned}
 b_k &= \frac{2}{T} \int_{-T}^T f(t) \sin n \omega_0 t dt \\
 &= 0 \int_0^2 (1) \sin n \omega_0 t dt \\
 &= \frac{\cos n \omega_0 t}{n \omega_0} \Big|_0^1
 \end{aligned}$$

$$\Rightarrow b_k = \frac{\cos n \omega_0 - 1}{n \omega_0}$$

2



$T=2$

$$\begin{aligned}
 a_0 &= \frac{1}{T} \int_{-T}^T f(t) dt \\
 &= \frac{1}{2} \int_0^2 f(t) dt = \frac{1}{2} \left[\int_0^1 0 dt + \int_1^2 t dt \right] = \frac{1}{2} \left[\frac{t^2}{2} \right]_1^2
 \end{aligned}$$

$$a_0 = \frac{1}{4}$$

$$a_k = \frac{1}{T} \int_{-T}^T f(t) \cos n\omega_0 t dt$$

$$= \frac{2}{2} \int_0^2 t \cos n\omega_0 t dt$$

$$= \int_0^2 [t \cdot \cos n\omega_0 t] dt$$

$$= t \cdot \frac{\sin n\omega_0 t}{n\omega_0} - \int 1 \times \frac{\sin n\omega_0 t}{n\omega_0} dt$$

$$= \frac{t}{n\omega_0} \sin n\omega_0 t - \frac{1}{n\omega_0} \frac{\cos n\omega_0 t}{n\omega_0} \Big|_0^2$$

$$a_k = \frac{0 \cdot \sin n\omega_0}{n\omega_0} - \frac{1}{(n\omega_0)^2} [\cos n\omega_0 - 1]$$

$$a_k = \frac{1}{(n\omega_0)^2} (\cos n\omega_0 - 1)$$

$$b_k = \frac{2}{T} \int_0^2 f(t) \sin n\omega_0 t dt$$

$$= \frac{2}{2} \int_0^2 f(t) \sin n\omega_0 t dt$$

$$= t \times \frac{\cos n\omega_0 t}{n\omega_0} - \int 1 \times \frac{\cos n\omega_0 t}{n\omega_0} dt$$

$\Rightarrow \Delta$

next page

$$= \frac{t}{n\omega_0} \cosh n\omega_0 t - \frac{1}{n\omega_0} \frac{\sin n\omega_0 t}{(n\omega_0)} \Big|_0^1$$

$$= \frac{t \cosh n\omega_0}{n\omega_0} \Big|_0^1 - \frac{1}{(n\omega_0)^2} \sin n\omega_0 t \Big|_0^1$$

$$= \frac{\cosh n\omega_0}{n\omega_0} - \frac{\sin n\omega_0}{(n\omega_0)^2}$$

$$b_k = \frac{\cosh n \cdot \pi}{n \pi} - \frac{\sin n \pi}{(n \pi)^2}$$