Signals Test Notes

- $Pr(A|B) = \frac{Pr(A \cap B)}{Pr(B)}$ or $= \frac{Pr(B|A) \cdot Pr(A)}{Pr(B)}$
- $Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A \cap B)$
- $A \subset B$ means A inside of B "Subset"
- $A|B$ means has A if B is true

Bayes Theorum

- $\frac{Pr(A \cap B)}{Pr(B)} = \frac{\cancel{Pr(B|A) \cdot Pr(A)}}{Pr(B)}$ b/c $\frac{Pr(B \cap A)}{Pr(\cancel{A})} \cdot \frac{Pr(\cancel{A})}{Pr(B}$

- $Pr(A \cap B) = Pr(A|B) Pr(B) = Pr(B|A) Pr(A)$

- Compliments

- if given A_0 but not A_1, $A_1 = 1 - A_0$

Ex: (given must be same)

what is $Pr(A_1 | B_1)$ if we have $Pr(A_0 | B_1) = 0.1$

$Pr(A_1 | B_1) = 1 - Pr(A_0 | B_1)$ or $1 - 0.1 = \underline{0.9}$

if not given Any Probability, use total probability

Ex:

Given: $Pr(T_1) = 0.55$, $Pr(R_0 | T_1) = 0.1$, $Pr(R_1 | T_0) = 0.2$

Compliments $\begin{cases} \text{Find: } Pr(T_0) = 1 - Pr(T_1) = 1 - 0.55 = \underline{0.45}, \\ Pr(R_1 | T_1) = 1 - Pr(R_0 | T_1), \quad Pr(R_0 | T_0) = 1 - Pr(R_1 | T_0), \end{cases}$

Find $Pr(R_1)$: it is total probability

$$Pr(R_1) = Pr(R_1 | T_0) Pr(T_0) + Pr(R_1 | T_1) Pr(T_1)$$

words: prob of recieving a 1 is equal to prop of reciving a 1 if transmitting zero times prob of transmitting zero plus prob of recieving one if transmitting one times prob of trans one

$\frac{3}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} = \frac{9}{34}$ $\frac{9}{25}$, $1 - \left(\frac{25}{34}\right)$

- Independance:

If $Pr(A \cap B) = Pr(A)\,Pr(B)$

Not independant is the two sets are not eq

Ex Deck of Cards of 52 A = ace card
B = spade card

Prob of getting Ace of Spade
$Pr(A \cap B) = \frac{1}{52}$ $Pr(A)\,Pr(B) = \frac{4}{52} \cdot \frac{13}{\cancel{52}} = \frac{1}{52}$

two sets are independant

if Independant: $Pr(A \cap B) = Pr(A)\,Pr(B)$
Always: $Pr(A \cap B) = Pr(A|B)\,P(B)$
If Independant: $Pr(A) = Pr(A|B)$

- Combined Experiment based on Independance

- if Pr(A) and the Pr(B) do not affect eachother, then you can find the probability of both happening at the same time b/c they are independant

Ex Pr{A} is 6 on a die
Pr(B) is Heads on a coin

$$Pr(A \cap B) = Pr(A)\,Pr(B) = \frac{1}{6} \cdot \frac{1}{2} = \frac{1}{12}$$

Ex of getting at least one 6 in n throws on a die

= Probability of all other occurances, so 5 out of 6 other numbers can happen, the the n^{th} power. then its the compliment

$1 - \left(\frac{5}{6}\right)^n$ = So for 1 try: $1 - \left(\frac{5}{6}\right)^1 = \frac{1}{6}$ to roll 6

What about rolling 2 die and getting both 6.

Prob of : rolling a 6 is $\frac{1}{6}$. Prob of rolling 2 $\frac{1}{6}$ is

$\left(\frac{1}{6} \cdot \frac{1}{6}\right) \Rightarrow \frac{1}{36}$ so $\frac{35}{36}$ times is the prob you will not roll a 6 twice

So prob of rolling two '6' once for n times it is $\boxed{1 - \left(\frac{35}{36}\right)^n}$

$\frac{25}{36}$

- Counting Prob w/ random object :

Arange items in different sequence

Ex {1, 2, 3, 3} how many differant arrangement can there be of these 4 numbers

$\frac{n!}{m!}$ ← number of total objects / ← number of same objects $\frac{4!}{2!} = 12$

Combinations : order does not matter

permutation : order does matter

$\frac{n!}{(n-k)!\,k!}$ How many Possible lab groups can be formed out of 20 people of Labgroup size 5

$\frac{20!}{(20-5)!\,5!} =$ 15,504 combinations

How many dif. groups could you end up in $\frac{19!}{(19-4)!\,4!} = 3876$

~~$Pr(E)$ $Pr(B_0|E_0) + Pr(B_1|E_1)$~~

~~$Pr(B_0|E_0)Pr(E_0) +$~~

- Repeated Trials: ~ Bernoulli trials ~

~~probability $= p^K \cdot q^{n-K}$~~ $\times$ (# of combinations)

So Ex prob of error of reading 1 bit = 0.001
what is the prob of getting one error in 8 bits

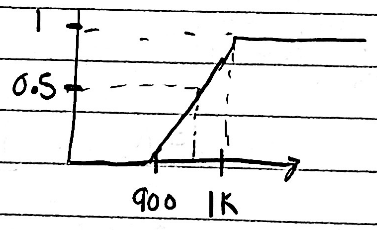
$\binom{n}{K} p^K q^{n-K}$

$\binom{8}{1} (0.001)^1 (0.999)^7$ $n = 8$ bits, $K = 1$ error

$\frac{8!}{(8-1)!\,1!} \cdot (0.001)^1 (0.999)^7 = 7.94 \times 10^{-3}$

- Distribution Function:

$$F_X(x) = Pr(X \le x)$$ if x is in range of 900 to 1K



$Pr(X \le 10{,}000) = 1$

properties of CDF

1) $0 \le F_X(x) \le 1$, $-\infty < x < \infty$
2) $F_X(-\infty) = 0$ $F_X(\infty) = 1$
3) $F_X(x)$ non-decreasing as x increases
4) $Pr(x_1 < X \le x_2) = F_X(x_2) - F_X(x_1)$
5) $Pr(X > x) = 1 - F_X(x)$

- $Pr(X = x) = 0$

Probability Density Function: (lower case f) $f_X(x)$
PDF

$$\Pr(x_1 < X \le x_2) = \int_{x_1}^{x_2} f_X(x)\,dx$$

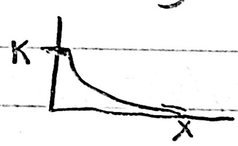
→ Integrate PDF to compute probability that x is in a given range

properties of PDF

- $f_X(x) \ge 0 \quad -\infty < x < \infty$
- $\int_{-\infty}^{\infty} f_X(x)\,dx = 1$
- $F_X(x) = \int_{-\infty}^{x} f_X(u)\,du$
- $\int_{x_1}^{x_2} f_X(x)\,dx = \Pr(x_1 < X \le x_2)$

- Ex: Find Value of Height K of Density function

$$f_X(x) = Ke^{-2x}u(x)$$



- Value of K: $1 = \int_{-\infty}^{\infty} Ke^{-2x}u(x)\,dx = \int_{0}^{\infty} Ke^{-2x}dx$

$$1 = -\frac{K}{2}e^{-2x}\Big|_0^{\infty} = 0 - \left(-\frac{K}{2}\right) = \frac{K}{2} \quad \text{So } 1 = \frac{K}{2} \text{ or } \underline{K = 2}$$

$$\Pr(X > 1) = \int_{1}^{\infty} 2e^{-2x}u(x)\,dx = \int_{1}^{\infty} 2e^{-2x}dx = -e^{-2x}\Big|_1^{\infty} = \underline{e^{-2}}$$

$$\Pr(X \le 0.5) = \int_{-\infty}^{0.5} 2e^{-2x}u(x)\,dx = \int_{0}^{0.5} 2e^{-2x} = e^{-2x}\Big|_0^{0.5} = \underline{-e^{-2(0.5)} + 1}$$