Signals and Systems HW 11

$\boxed{3.42}$ $h(t) = \delta(t) + 4e^{-3t}\cos 2(t) \cdot u(t)$

$h(t) \rightarrow H(t)$ $\qquad$ $\delta(t) \rightarrow 1$

$$4e^{-3t}\cos(2t)u(t) \xrightarrow{LT} \left[\frac{4(s)}{s^2+2^2}\right]_{s \to s+3} = \frac{4(s+3)}{(s+3)^2+4}$$

$$H(s) = 1 + \frac{4(s+3)}{(s+3)^2+4} = \frac{s^2+9+6s+4s+12}{(s+3)^2+4} = \frac{s^2+10s+25}{(s+3)^2+4}$$

$$H(s) = \frac{(s+5)^2}{(s+3)^2+4}$$

(a) $\quad s = j\omega \qquad Re(s) = \sigma = 0$

$$H(s)\big|_{s=j\omega} = H(\omega) = \frac{(j\omega+5)^2}{(j\omega+3^2)+4} = \frac{-\omega^2+25+j10\omega}{-\omega^2+9+j6\omega+4}$$

$$\Rightarrow H(\omega) = \boxed{\frac{\omega^2 - j10\omega - 25}{\omega^2 - j6\omega - 13}}$$

(b) $\quad H(s) = \dfrac{(s+5)^2}{(s+3)^2+4}$

Zeros

$(s+5)^2 = 0$

$s = 5, -5$

Poles

$(s+3)^2+4 = 0$

$(s+3)^2 = -4 \Rightarrow s+3 = \pm j2$

$s = -3 \pm j2$

$\underline{s = -3+j2}$ + $\underline{s = -3-j2}$

3.42

(d) $x(t) \rightarrow \boxed{h(t)} \longrightarrow y(t)$

$$y(t) = x(t) \otimes h(t)$$

$$Y(S) = X(S) \cdot H(S)$$

input: $x(t) = 2te^{-5t}u(t)$

$$X(s) = \frac{2}{(s+5)^2}$$

$$Y(s) = \frac{2}{(s+5)^2} \times \frac{(s+5)^2}{(s+3)^2+4} \Rightarrow Y(s) = \frac{2}{(s+3)^2+2^2}$$

Inv Laplace: $y(t) = e^{-3t} \cdot \sin(2t) \cdot u(t)$

4.8

$v_C(0^-) = 24V$

$v_C(t) = v_C(0^-)e^{-\frac{t}{\tau}} = 24e^{-t/\tau}$

$s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0 \qquad \alpha = \frac{1}{2RC} \qquad w_0^2 = \frac{1}{LC}$

$\alpha = \frac{1}{2(1)(0.25)} = 2 \qquad w_0^2 = \frac{1}{(0.8)(0.25)} = 5$

$\alpha < w_0^2 \qquad w_d = \sqrt{w_0^2 - \alpha^2} = \sqrt{5-4} = 1$

$v_C(0) = e^{-\alpha(0)}(B_1 \cos w_d t + B_2 \sin w_d t)$

At $t=0$

$v_C(0) = e^{-\alpha(0)}(B_1 \cos w_d(0) + B_2 \sin w_d(0))$

$24 = 1(B_1(1) + 0) \Rightarrow B_1 = 24$

$v_C(t) = e^{-\alpha t}(B_1 \cos w_d t + B_2 \sin w_d t)$

$B_2 = \frac{\alpha}{w_d}B_1 - \frac{v_C(0)}{w_d RC} - \frac{i_L(0)}{w_d C} = \frac{2}{1}(24) - \frac{24}{1 \cdot 1 \cdot 0.25} - \frac{0}{w_d C} = 48 - 96 = -48$

$v_C(t) = e^{-2t}(24\cos - 48\sin t) \qquad i_L = \frac{1}{L}\int_0^t v_C(t)\,dt$

$= \frac{1}{0.8}\int_0^t \{e^{-2t}(24\cos t - 48\sin t)\}\,dt$

$i_L = -12e^{-2t}(-2\sin t - \cos t) + 6e^{-2t}(-2\cos t + \sin t)$

4.13

$i_s(t) = [10\,u(t) + 20\,\delta(t)]\,mA$

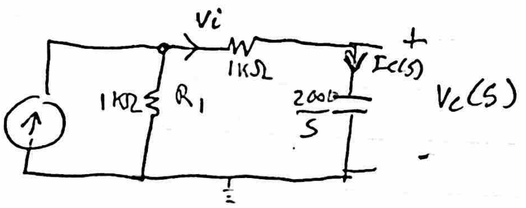
$\mathcal{L}[i_s(t)] = \mathcal{L}[10\,u(t) + 20\,\delta(t)]$

$i_s(s) = \frac{10}{s} + 20 \cdot 1$

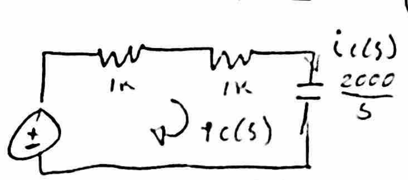
$i_s(s) = \frac{10}{s} + 20\,mA$

$c = 0.5\,mf$

$\frac{1}{sC} = \frac{1}{s \cdot 0.5} = \frac{2 \times 10^3}{s}$



$V(s) = i(s) \cdot 1K\Omega = \left(\frac{10}{s} + 20\right) \times 10^{-3} \times 1K \Rightarrow V(s) = \left(\frac{10}{s} + 20\right) V$



KVL: $2k\,i_c(s) + \frac{2000}{s} i_c(s) = \frac{10}{s} + 20$

$i_c(s) = \frac{10 + 20s}{2ks + 2000}$

$i_c(s) = \frac{10 + 20s}{2k(s+1)}$

$V_c(s) = \frac{10 + 20s}{s(s+1)}$

$\frac{10 + 20s}{s(s+1)} = \frac{A}{s} + \frac{B}{s+1}$ $\quad A = \frac{10+20s}{s+1}\Big/_{s=0} = 10$

$B = \frac{10+20s}{s}\Big/_{s=-1} = 10$

$V_c(s) = \frac{10}{s} + \frac{10}{s+1}$

Inv Laplace $= V_c(t) = 10\,u(t) + 10\,e^{-t}\,u(t)$