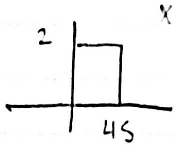
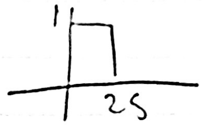
Signals HW 5

$\boxed{2.10}$

b) $A = 2, B = 1, T_1 = 4s, T_2 = 2s$

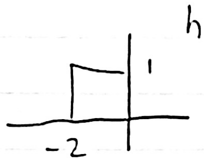
$x(\tau)$ $h(\tau)$

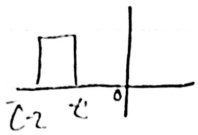




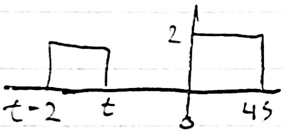
- Time Reversal + Shifting

$h(-\tau)$ $h(t-\tau)$





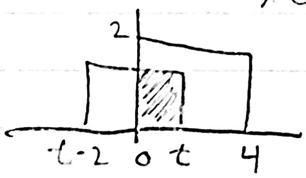
① $t < 0$



no overlaping then $y(t) = 0$

② $0 < t < 2$

$x(\tau)\, h(t-\tau)$



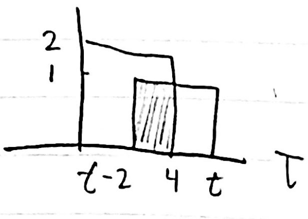
overlap 0 and t

$$y(t) = \int_0^t x(t)\, h(t-T)\, d\tau$$

$$= \int_0^t (1)(2)\, d\tau = (2T)\Big|_0^t = 2(t-0$$

$$y(t) = 2t$$

③ $4 < t < 6$



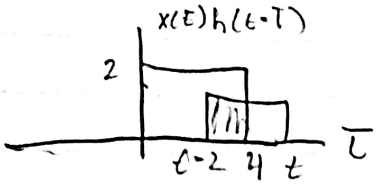
$\Rightarrow$ overlap @ $(t-2 + 4)$

$$y(t) = \int_{t-2}^{4} (1 \cdot 2)\, d\tau = (2\tau)_{t-2}^{4}$$

$$y(t) = 2(t - (t-2)) = 2(\cancel{t-t}+2) = 4$$

(b) continued

(4) $4 < t < 6$



overlap $t-2 \to 4$

$$y(t) = \int_{t-2}^{4} (1 \cdot 2)\, d\tau = (2\tau)\Big|_{t-2}^{4}$$

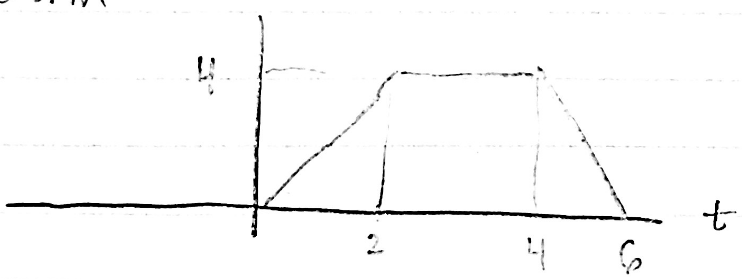
$$y(t) = 2(4-(t-2)) = 2(4-t+2) = 2(6-t)$$

(5) $t > 6$

$y(t) = 0$

$$y(t) = \begin{cases} t < 0, & 0 \\ 0 < t < 2, & 2t \\ 2 < t < 4, & 4 \\ 4 < t < 6, & 2(6-t) \\ t > 6, & 0 \end{cases}$$

Waveform



$\boxed{2.13}$

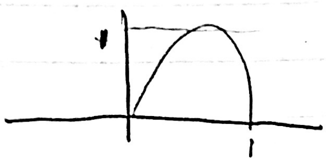
(b) $x(t) = 0 \quad t < 0$

$\sin(\pi t), \quad 0 \le t < 1$

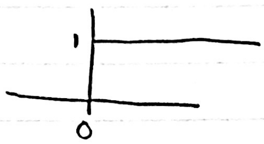
$0 \quad , t \ge 1$

$h(t) = u(t)$

$x(t) =$



$h(t)$



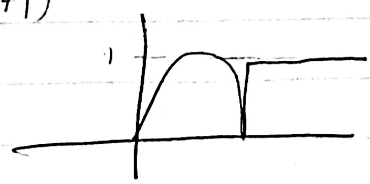
$g(\tau) = x(t)\, h(\tau)$

$\int_0^1 \sin(\pi t)\, dt$

$= \left. \frac{-\cos(\pi t)}{\pi} \right|_0^1 = \frac{1}{\pi}\left[-\cos\pi + \cos 0\right]$

$= \frac{2}{\pi} = 0.63662$

$g(\tau+1) = x(t)\, h(\tau+1)$



no intersection

$\boxed{2.17}$

a) $e^{-t}u(t) \bullet e^{-2t}u(t) = z(t)$

- $x(t) = e^{-t}u(t)$
- $y(t) = e^{-2t}u(t)$

- $z(t) = e^{-t}u(t) \circ e^{-2t}u(t)$

$z(t) = x(t) \circ y(t)$

Laplace Transform

- $Z(s) = X(s) \circ Y(s)$

$x(t) = e^{-t}u(t)$

- $X(S) = \frac{1}{S+1}$

$y(t) = e^{-2t}u(t)$

- $Y(S) = \frac{1}{S+2}$

So, $Z(S) = \frac{1}{(S+1)} \circ \frac{1}{(S+2)}$

$Z(S) = \frac{1}{S+2} - \frac{1}{S+1}$

Inverse Laplace Transform

$z(t) = (e^{-2t} - e^{-t})u(t)$

So, $\Rightarrow e^{-t}u(t) \circ e^{-2t}u(t) = \boxed{(e^{-2t} - e^{-t})u(t)}$