HW 9

3.13

a) $X_1(s) = \frac{6}{(s+2)(s+4)}$

$= \frac{A_1}{s+2} + \frac{A_2}{s+4}$

$A_1 = (s+2)X_1(s)|_{s=-2} = \frac{6}{s+4}\Big|_{s=-2} = \frac{6}{-2+4} = 3$

$A_2 = (s+4)X_1(s)|_{s=-4} = \frac{6}{s+2}\Big|_{s=-4} = \frac{6}{-4+2} = -3$

$$X_1(s) = \frac{3}{s+2} - \frac{3}{s+4}$$

$$x_1(t) = 3(e^{-2t} - e^{-4t})u(t)$$

c) $x_3(s) = \frac{3s^3 + 36s^2 + 131s + 144}{s(s+4)(s^2+6s+9)}$

$= \frac{A_1}{s} + \frac{A_2}{s+4} + \frac{B_2}{(s+3)^2} + \frac{B_1}{s+3}$

$A_1 = sX_3(s)|_{s=0} = 4$

$A_2 = (s+4)X(s)|_{s=-4} = -1$

$B_2 = (s+3)^2 X(s)|_{s=-3} = 2$

$B_1 = \frac{d}{ds}\left[\frac{3s^3 + 36s^2 + 131s + 144}{s(s+4}\right]\Big|_{s=-3} = 0$

So $X_3(s) = \frac{4}{s} - \frac{1}{s+4} + \frac{2}{s+3)^2}$

$$x_3(t) = [4 - e^{-4t} + 2te^{-3t}]u(t)$$

3.14

(b) $X_2(s) = \dfrac{2s^2+10s+16}{(s+2)(s^2+6s+10)}$

roots $s_1 = \dfrac{-6-\sqrt{36-40}}{2} = -3-j1$

$s_2 = -3+j1$

So $X_2(s) = \dfrac{A_1}{s+2} + \dfrac{B_1}{s+3+j1} + \dfrac{B_i}{s+3-j1}$

$A_1 = (s+2)\,X_2(s)\big|_{s=-2} = 2$

$B_1 = (s+3+j1)\,X_2(s)\big|_{s=-3-j1} = -1j$

So $X_2(s) = \dfrac{2}{s+2} - \dfrac{j}{s+3+j1} + \dfrac{j}{s+3-j1}$

$x_2(t) = [2e^{-2t} - 2e^{-3t}\sin(t)]\,u(t)$

3.15

(c) $X_3(s) = \dfrac{\sqrt{2}(s+1)}{s^2+6s+13}$

$s^2+6s+13 = 0 \quad \therefore \quad s_1 = -3-j2$

$s_2 = -3+j2$

So $X_3(s) = \dfrac{B}{s+3+j2} + \dfrac{B^*}{s+3-j2}$

$B = (s+3+j2)\,X_3(s)\big|_{s=-3-j2} = \dfrac{\sqrt{2}}{2}(1-j) = e^{-j45^\circ}$

So $X_3(s) = \dfrac{e^{-j45^\circ}}{s+3+j2} + \dfrac{e^{j45^\circ}}{s+3-j2}$

$x_3(t) = 2e^{-3t}\cos(2t+45^\circ)\,u(t)$

3.17

(c) $X_3(s) = \frac{(s+5)e^{-2s}}{(s+1)(s+3)} = X_c(s)e^{-2s}$

$$X_c(s) = \frac{s+5}{(s+1)(s+3)}$$

$$= \frac{A_1}{s+1} + \frac{A_2}{s+3}$$

$A_1 = (s+1)X_c(s)\big|_{s=-1} = 2$

$A_2 = (s+3)X_c(s)\big|_{s=-3} = -1$

So $X_3(s) = \frac{2e^{-2s}}{s+1} - \frac{e^{-2s}}{s+3}$

$x_3(t) = [2e^{-(t-2)} - e^{-3(t-2)}]u(t-2)$

3.21

$$v_s(t) = 20\,u(t) \quad \Rightarrow \quad V_s = \frac{2}{s}$$

$$L \Rightarrow sL$$

$$C \Rightarrow \frac{1}{sC}$$

$$\frac{V - V_s}{R_1} + \frac{V}{\frac{1}{sC}} + \frac{V}{R_2 + sL} = 0$$

$$V = \frac{2(R_2 + sL)}{s[R_1 LCs^2 + (L + R_1 R_2 C)s + (R_1 + R_2)]}$$

$$= \frac{24(3 + 0.2s)}{s(s^2 + 16s + 48)}$$

$$V = \frac{A_1}{s} + \frac{A_2}{s+4} + \frac{A_3}{s+12}$$

$$A_1 = sV\big|_{s=0} = 1.5$$

$$A_2 = (s+4)V\big|_{s=-4} = -1.56$$

$$A_3 = (s+12)V\big|_{s=-12} = 0.072$$

$$V = \frac{1.5}{s} = \frac{1.56}{s+4} + \frac{0.072}{s+12}$$

$$v(t) = [1.5 - 1.56e^{-4t} + 0.072e^{-12t}]\,u(t)$$