Signals and Systems HW3

Jacob Howard

$\boxed{2.1}$

a)

i) • Consider input $x_1(t)$

output: $y_1(t) = 3x_1(t) + 1$

• Consider input $x_2(t)$

output: $y_2(t) = 3x_2(t) + 1$

• Consider input $x_3(t) = x_1(t) + x_2(t)$

Output: $y_3(t) = 3x_3(t) + 1$

$= 3(x_1(t) + x_2(t)) + 1$

$= 3x_1(t) + 3x_2(t) + 1$

$y_3(t) \neq y_1(t) + y_2(t)$

So, $\boxed{\text{Non linear}}$

ii) Apply delay t_0 to input

find output of $x(t-t_0)$

$y_1(t) = 3x(t-t_0) + 1$

$\Rightarrow$ $y(t) = 3x(t) + 1$ (apply delay)

$= y(t-t_0) = 3x(t-t_0) + 1$

So, $y_1(t) = y(t-t_0)$

$\boxed{\text{Time invariant}}$

$\boxed{2.1}$

c) i) $\frac{dy(t)}{dt} + ty(t) = x(t)$

Concider input $x_1(t)$

Output: $\frac{dy_1(t)}{dt} + ty_1(t) = x_1(t)$

Concider input $x_2(t)$

output: $\frac{dy_2(t)}{dt} = ty_2(t) = x_2(t)$

add equasions

$$\frac{dy_1(t)}{dt} + ty_1(t) + \frac{dy_2(t)}{dt} + ty_2(t) = x_1(t) + x_2(t)$$

$$\Rightarrow \frac{d}{dt}\left(y_1(t) + y_2(t)\right) + t\left(y_1(t) + y_2(t)\right) = x_1(t) + x_2(t)$$

Compair:

$y(t) = y_1(t) + y_2(t)$

$x(t) = x_1(t) + x_2(t)$

There for $\boxed{\text{linear}}$

$\boxed{2.1}$

c) ii) Appy delay t_0

Input: $x(t-t_0)$

Output: $\frac{dy(t)}{dt} + ty(t) = x(t-t_0)$

the DE is: $\frac{dy(t)}{dt} + ty(t) = x(t)$

Apply delay: $\frac{dy(t)}{dt} + (t-t_0)y(t) = x(t-t_0)$

Equasions are not Equal so $\boxed{\text{Not time invariant}}$

$\boxed{2.2}$ a) i) $y(t) = 3x(t-1)$

apply $x_1 = 3x_1(t-1) \rightarrow y_1(t)$

apply $2x_1 = 3(2x_1(t-1)) \rightarrow 2y_1(t)$

apply $x_2 = 3x_2(t-1) \rightarrow y_2(t)$

$\Rightarrow y_1(t) + y_2(t) = 3x_1(t-1) + 3x_2(t-1)$

additivity holds

$2x_1 \rightarrow 2(3x_1(t-1)) \rightarrow 2y_1(t)$

homogenity holds

$\boxed{\text{So linear}}$

$\boxed{2.2}$ a) ii) $y(t) = 3x(t-1)$

$x_1(t-1) = x_2(t)$

$y_1(t-1) = y_2(t)$

$3x_2(t-1-1) = 3x_2(t-2)$

$y_2(t-1) = 3x_2(t-1-1) = 3x_2(t-2)$

$\boxed{\text{So Time Invariant}}$

$\boxed{2.2}$

c) i) $\frac{dy}{dt} + y(t-1) = x(t)$

appy $x_1 \Rightarrow \frac{dy_1}{dt} + y_1(t-1) = x_1(t)$

appy $ax_1 \Rightarrow \frac{dy_1}{dt} + y_1(t-1) = ax_1(t)$

apply $ay_1 \Rightarrow \frac{d(ay)}{dt} + ay_1(t-1) = x_1(t)$

apply $x_2 \Rightarrow \frac{dy_2}{dt} + y_2(t-1) = x_2(t)$

both additivity + homoginity hold

$\boxed{\text{So linear}}$

$\boxed{2.2}$ c) ii)

$$x_2(t) = x(t-1)$$

$$\Rightarrow \frac{dy}{dt} + y(t-1) = x(t-1)$$

$$y_2(t) = y(t-1)$$

$$\Rightarrow \frac{dy}{dt} + y(t-2) = x(t-1)$$

$\boxed{\text{So Time variant}}$