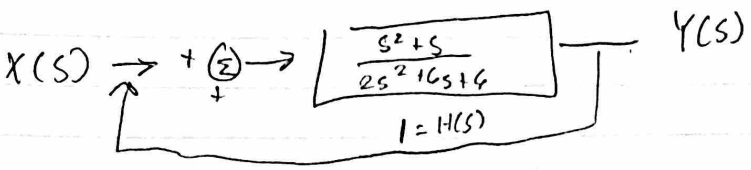
Signals and Systems Test 2

$\boxed{1}$



Let $G(s) = \dfrac{s^2+s}{2s^2+6s+6}$ $\qquad H(s) = 1$

$$H(s) = \frac{G(s)}{1-G(s)H(s)} \qquad H(s) = \frac{Y(s)}{X(s)} = \frac{\frac{s^2+s}{2s^2+6s+6}}{1-\left(\frac{s^2+s}{2s^2+6s+6}\right)(1)}$$

$$\Rightarrow = \frac{s^2+s}{2s^2+6s+6-s^2-s} \Rightarrow \frac{s^2+s}{s^2+5s+6} = \frac{s^2+s}{s^2+2s+3s+6}$$

$$\Rightarrow = \frac{s(s+1)}{(s+2)(s+3)} \Rightarrow \boxed{H(s) = \frac{s(s+1)}{(s+2)(s+3)}}$$

b) poles

$(s+2)(s+3) = 0 \Rightarrow s = -2, -3$

zeros $s(s+1) = 0 \Rightarrow s = 0, -1$

poles
$-2, -3$

zeros
$0, -1$

(2)

$x(t) = e^{-t} u(t)$

$x(t-2) = e^{-t-2} u(t-2)$

$= \mathcal{L}\{x(t-2)\} = \boxed{\dfrac{e^{-2s}}{s+1}}$

$t\,x(t-2) = t\,e^{-t-2} u(t-2)$

$\mathcal{L}\{t\,x(t-2)\} = \boxed{\dfrac{e^{-2s}}{(s+1)^2}}$

3

(a) $s^2+9=0$ so $s=\sqrt{-9}$

$2(s+1)=0 \Rightarrow s=-1$

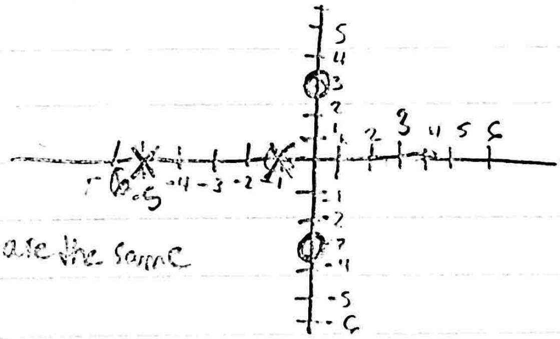
$s^2+6s+8=0$

$$\Rightarrow s = \frac{-6 \pm \sqrt{6^2-4(8)}}{2} = \frac{-6 \pm \sqrt{36-32}}{2}$$

$\Rightarrow -3 \pm 2$ so $s=-1,-5$

a)

Zeros	Poles
$\pm 3i$,	$-1, -5$



b) No, Degree of numerator + denominator are the same

c) Yes, 0

$$\lim_{s \to 0} \frac{s(s^2+9)}{2(s+1)(s^2+6s+8)}$$

$$\lim_{s \to \infty} \frac{s(s^2+9)}{2(s+1)(s^2+6s+8)}$$

4

a) $H(s) = \dfrac{2s+4}{s^2+4s+8}$

$H(s) = \dfrac{2s+6}{(s+2)^2+(2)^2} = \dfrac{2s}{(s+2)^2+(2)^2} + \dfrac{6}{(s+2)^2+(2)^2}$

$= \dfrac{2(s+2-2)}{(s+2)^2+(2)^2} + \dfrac{6}{(s+2)^2+(2)^2}$

$= \dfrac{2(s+2)-4}{(s+2)^2+(2)^2} + \dfrac{6}{(s+2)^2+(2)^2} = \dfrac{2(s+2)}{(s+2)^2+(2)^2} + \dfrac{2}{(s+2)^2+(2)^2}$

$\cos \omega t \Rightarrow \dfrac{s}{s^2+\omega^2}$ $\qquad e^{-at} \cos \omega t \Rightarrow \dfrac{s+a}{(s+a)^2+\omega^2}$

$\therefore$ Inv Laplace

so, $\dfrac{2(s+2}{(s+2)^2+(2)^2} = 2e^{-2t} \cos 2t$

Inv Laplace $\dfrac{2}{(s+2)^2+(2)^2} = e^{-2t} \sin 2t$

so $\Rightarrow$ $\boxed{h(t) = 2e^{-2t}\cos 2t + e^{-2t}\sin 2t}$

(b) on next page

#4 (b) Input $x(t) = e^{-3t}u(t)$

$$X(s) = \frac{1}{s+3}$$

$$H(s) = \frac{2s+6}{s^2+4s+8}$$

output $Y(s) = H(s) \cdot X(s)$

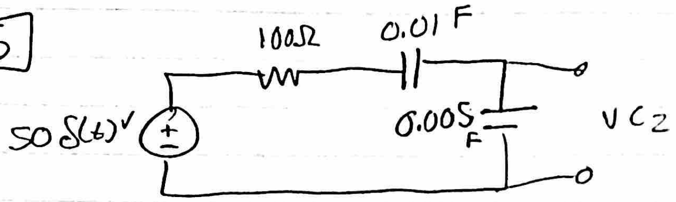
$$\frac{2\cancel{(s+3)}}{s^2+4s+8} \cdot \frac{1}{\cancel{(s+3)}}$$

$$Y(s) = \frac{2}{s^2+4s+8} \Rightarrow \frac{2}{(s+2)^2+(2)^2}$$

Inv Laplace

$$Y(s) = \frac{2}{(s+2)^2+(2)^2} \Rightarrow \boxed{y(t) = e^{-2t}\sin 2t}$$

5



$\delta(t) = 1$

$\Rightarrow \left[100 + \frac{1}{0.01s} + \frac{1}{0.005s}\right] I(s)$

$= \left[100 + \frac{100}{s} + \frac{1000}{5s}\right] I(s) \Rightarrow \boxed{I(s) = \frac{50s}{100s+300}}$

$V_{c2}(s) = I(s) \cdot \frac{1}{C_2 s}$

$= \left[\frac{50s}{100s+300}\right] \cdot \frac{1}{0.005s} = \frac{50 \cdot s}{100[s+3]} \cdot \frac{200}{s}$

$V_{c2}(s) = \frac{100}{s+3}$

Inv Laplace : $V_{c2}(t) = LT^{-1}\left[\frac{100}{s+3}\right]$

$\boxed{V_{c2}(t) = 100e^{-3t}\ V}$