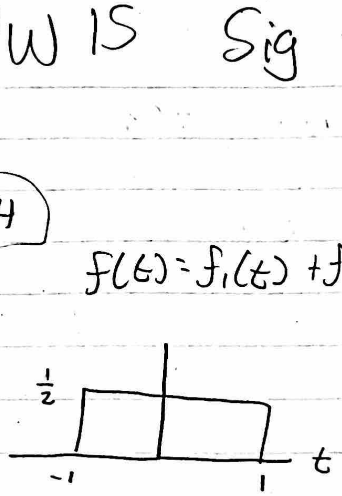
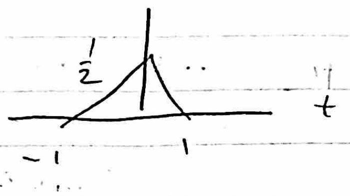
HW 15 Sig + Sys

$\boxed{5.44}$

$f(t) = f_1(t) + f_2(t)$

$f_2(t)$

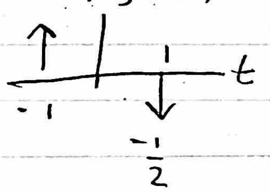


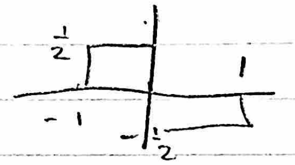


differentiation

$f'(t)$

$f_2'(t)$





$A\delta(t) \xrightarrow{F(t)} A$

$\delta(t+T) \longrightarrow A e^{-j\omega t}$

$\delta(t-T) \longrightarrow A e^{+j\omega t}$

$f_2''(t) = \frac{1}{2}\delta(t+1)$
$-1\delta(t) + \frac{1}{2}(t-1)$

Fourier transform

$F_1'(w) = \frac{1}{2}e^{jw} - \frac{1}{2}e^{-jw}$

$F_1'(w) = \frac{1}{2}e^{jw} - \frac{1}{2}e^{-jw}$

$F_2''(w) = \frac{1}{2}e^{jw}$
$-1 + \frac{1}{2}e^{-jw}$

$F''(w) = -1 + \frac{1}{2}e^{jw} + \frac{1}{2}e^{-jw}$

$\sin\theta = \frac{1}{2j}e^{+j\theta} - \frac{1}{2j}e^{-j\theta}$

$\cos\theta = \frac{1}{2}e^{j\theta} + \frac{1}{2}e^{-j\theta}$

$F'_1(\omega) = j\sin(\omega)$

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$F''(\omega) = -1 + \cos(\omega)$

$= -1(1 - \cos(\omega))$

$= -1(1-(-1-2\sin^2(\frac{\omega}{2}))$

$F''(\omega) = -2\sin^2(\frac{\omega}{2})$

$f(t) \xrightarrow{FT} F(\omega)$

$f'(t) \xrightarrow{FT} (j\omega)F(\omega)$

$f''(t) \xrightarrow{FT} (j\omega)^2 F(\omega)$

$(j\omega)F_1(\omega) = j\sin(\omega)$

$F_1(\omega) = \frac{\cancel{j}\sin(\omega)}{\cancel{j}\omega}$

$F_1(\omega) = \frac{\sin\omega}{\omega}$

$\frac{\sin\theta}{\theta} = \operatorname{sinc}\theta$

$(j\omega)^2 F_2(\omega) = -2\sin^2(\frac{\omega}{2})$

$F_2(\omega) = \frac{-2\sin^2(\omega/2)}{(+j\omega)^2}$

$F_2(\omega) = \frac{2\sin^2(\omega/2)}{\omega^2}$

$\frac{\sin^2 a\omega}{(a\omega)^2} = \operatorname{sinc}^2(a\omega)$

$F_1(\omega) = \operatorname{Sinc}(\omega)$

$F_1(\omega) = \operatorname{sinc}(\omega)$

$F_2(\omega) = \frac{1}{2}\frac{\sin^2(\omega/2)}{(\omega/2)^2}$

$F_2(\omega) = \frac{1}{2}\operatorname{sinc}^2(\frac{\omega}{2})$

$f(t) = f_1(t) + f_2(t) \xrightarrow{FT} F(\omega) = F_1(\omega) + F_2(\omega)$

$F(\omega) = F_1(\omega) + F_2(\omega)$

$$\boxed{F(\omega) = \operatorname{sinc}(\omega) + \frac{1}{2}\operatorname{sinc}^2(\frac{\omega}{2})}$$

5.48

a) $f(t) = A\cos(w_0 t - \theta_0)$; $-\infty < t < \infty$

$$F(w) = \int_{-\infty}^{\infty} f(t)\, e^{-jwt}\, dt$$

$$= \int_{-\infty}^{\infty} \frac{A}{2}\left[e^{j(w_0 t - \phi)} + e^{-j(w_0 t - t)}\right] e^{-jwt}\, dt$$

$$\Rightarrow \left[F(w) = A\pi\left[\delta(w - w_0)\, e^{-j\phi} + \delta(w + w_0)\, e^{j\phi}\right]\right]$$

$$F(w) = 2\pi\left[\delta(w - 5)\, e^{-j\pi/5} + \delta(w + 5)\, e^{j\pi/5}\right]$$

b) $g(t) = e^{-\alpha t}\cos(w_0 t)\, u(t)$

$$\Rightarrow g(t) \rightarrow G(w) = \pi\left[\delta(w + w_0 + \alpha) + \delta(w - w_0 + \alpha)\right]$$

So, $\Rightarrow \left[G(w) = \pi\left[\delta(w + 5 + 0.5) + \delta(w + 0.5 - 5)\right]\right]$

5.52

a) $f(3t-2)$

$$F(at-b) \rightarrow \frac{1}{|a|} e^{-j\frac{b}{a}w} \cdot X\left(\frac{w}{a}\right)$$

$$F(3w-2) \rightarrow \frac{1}{3} e^{-j\frac{3}{2}w} X\left(\frac{w}{3}\right)$$

$$F(3w-2) = \frac{1}{3} e^{-j\frac{3}{2}w} \times \frac{5}{2+j\frac{w}{3}}$$

$$= e^{-j\frac{3}{2}w} \times \frac{5}{6+jw}$$

b) $tF(t) \longrightarrow -j\frac{dX(w)}{dw}$

$$-j\frac{dF(w)}{dw} = -j\frac{d\frac{5}{2+jw}}{dw} = \frac{5}{(2+jw)^2}$$

$$tF(t) = \frac{5}{(2+jw)^2}$$

c) $\frac{dF(t)}{dt}$

$F(t) \longrightarrow F(w)$

$\frac{dF(t)}{dt} \longrightarrow jwF(w)$

$$\frac{dF(t)}{dt} = jwF(w) = \frac{jw \times 5}{(2+jw)} = \frac{j5w}{2+jw}$$