Signals and Systems Test 3

1

a) Parsevals Theorum

FS $\Rightarrow$ $\frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2 = \sum_{n=-\infty}^{\infty} |c_n|^2$

FT $\Rightarrow$ $\int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega$

b) $x(t) = 2[u(t) - u(t-2)]$

$u(t) - u(t-2) = \begin{cases} 1, 0 \le t \le 2 \\ 0, \text{otherwise} \end{cases}$

$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$

$= 2\int_0^2 (1) dt = 2(t)_0^2$

$= 2 \times 2 = 4$ watts

2 a) even signal

period of signal is 4

(b) Even $b_K = 0$

$$a_K = \frac{2}{T}\int_0^T x(t)\cos(k\omega_0 t)\,dt$$

$$= \frac{4}{T}\int_0^{T/2} x(t)\cos(k\omega_0 t)\,dt$$

$$1\int_0^{T/2} -t\cos(k\omega_0 t)$$

$$= -\frac{t}{k\omega_0}\sin(k\omega_0 t)\Big|_0^2 + \int_0^2 \frac{1}{k\omega_0}\sin(k\omega_0 t)\,dt$$

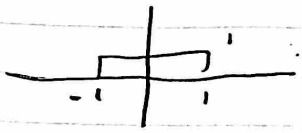
zero

$$= \frac{-2}{2k\pi}(\sin k\pi) + \frac{1}{k^2\omega_0^2}\cos(k\omega_0 t)\Big|_0^2$$

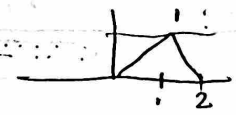
$$= \frac{T^2}{4k^2\pi^2}\left(-\cos\left(\frac{4k\pi}{2}\right)+1\right)$$

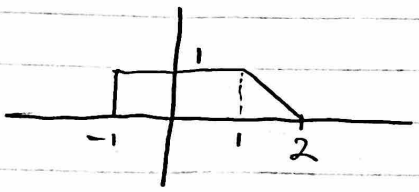
3 (a) Sketch $x(t) = \text{rect}(t/2) + \text{tri}(t-1/2)$

rect



tri





(b) $\mathcal{F}[x(t)] = 2\,\text{sinc}(w) + \text{sinc}^2\left(\frac{w}{2}\right)e^{jw}$

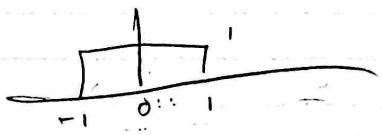
$\mathcal{F}\{\text{rect}\left(\frac{t}{T}\right)\} = 2\,\text{sinc}(w)$

$\mathcal{F}\{\text{tri}((t-1)/2)\}$

$= \frac{2}{2}\,\text{sinc}^2\left(\frac{w}{4}\right) \cdot e^{jw} = \text{sinc}^2\left(\frac{w}{2}\right)e^{jw}$

4 a)

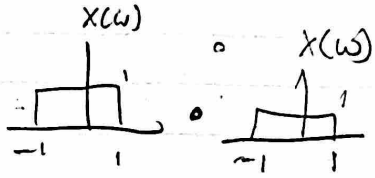
$$X(w) = u(w+1) - u(w-1)$$



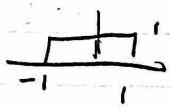
$$y(t) = x(t) * x(t)$$

$$y(w) = X(w) \times X(w) \Rightarrow y(w) = [X(w)]^2$$

$$y(w) = [u(w+1) - u(w-1)]$$



$$= y(w)$$



$$y(w) = X(w) \Rightarrow$$

$$y(w) = u(w+1) - u(w-1)$$

(b) $z(t) = x(t) * x(t)$

$$z(w) = \frac{1}{2\pi}[X(w) * X(w)]$$

$$z(w) = \frac{1}{2\pi}[[u(w+1) - u(w-1)] * [u(w+1) - u(w-1)]]$$

$$\Rightarrow u(w) * u(w) = \delta(w)$$

$$u(w-w_1) * u(w-w_2) = \delta(w-w_1-w$$

$$z(w) = \frac{1}{2\pi}[u(w+1) * u(w+1) = u(w-1) * u(w+1) - u(w-1) * u(w+1) + u(w-1) * u(w-1)]$$

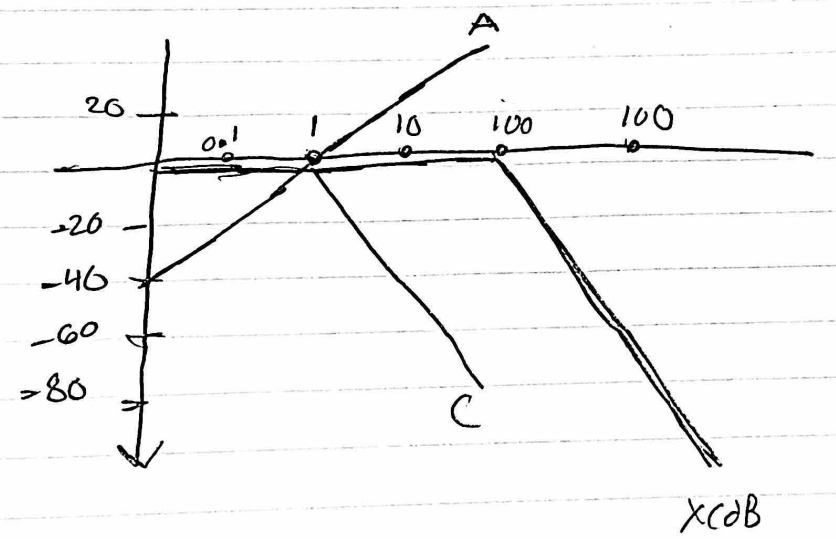
$$= \frac{1}{2\pi}[\delta(w+2) - \delta(w) - \delta(w) + \delta(w-2)]$$

$$z(w) = \frac{1}{2\pi}[\delta(w+2) - 2\delta(w) + \delta(w-2)]$$

5 $X(s) = \frac{100s}{(s+1)(s+100)} = \frac{s}{(s+\frac{1}{100})(s+1)}$

$X(\omega) = \frac{j\omega}{(j\omega+\frac{1}{100})(j\omega+1)}$

$$X(dB) = \underbrace{20\log_{10}(\omega)}_{A} - \underbrace{20\log_{10}\sqrt{1+(\frac{\omega}{100})^2}}_{B} - \underbrace{20\log_{10}\sqrt{1+(\frac{\omega}{1})^2}}_{C}$$



6

I will submit teaching evaluation

I thought this class was taught very well online. Thank You!