Random Final

1.) not symmetric

12.) $Pr(R_1|T_0) = 1 - Pr(R_0|T_0) = 1 - .9 = .1$

13.) $Pr(R_0) = Pr(R_0|T_0)Pr(T_0) + Pr(R_0|T_1)Pr(T_1)$

$= (.9)(1-.42) + (1-.96)(.42)$

$= .5388 = .539$

14.) $Pr(T_1|R_1) = \dfrac{Pr(R_1|T_1)Pr(T_1)}{Pr(R_1) = 1 - Pr(R_0)} = \dfrac{(0.96)(.42)}{1-.539}$

$\boxed{= .8742}$

15.) Pr(4/6 Tx are 0s)

$1 - \binom{6}{6}(\underset{Pr(T_0)}{.58})^6(.42)^0 - \binom{6}{5}(.58)^5(.42)^1$

$= .7965 = \boxed{.797}$

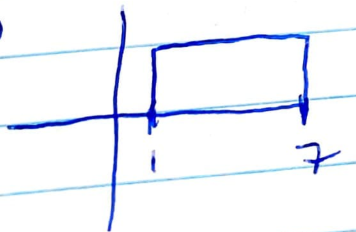
16.) $R_X(\tau) = 4 + 5e^{-3\tau^2}$ find variance σ^2

$\overline{X^2} = R_X(0) = 4 + 5 = 9$ $\quad\sigma^2 = 9 - 4 = 5$

$(\overline{X})^2 = \lim\limits_{\tau \to \infty} R_X(\tau) = 4$

18.) Pr(Sum of 5) $1+4, 4+1, 2+3, 3+2 = \frac{4}{36} = \frac{1}{9}$

19.)



$\frac{1}{2}(x+1)(x+2) = \overline{X} = 4$

$\sigma^2 = \frac{1}{2}[X_2 - X_1]^2$

$\sigma^2 = \overline{X^2} - (\overline{X})^2$

23.) $\overline{X^2} = R_X(0) = 100e^0 = 100 + 5$

24.) $Y(t) = h(t) * x(t)$ $h(t) = \delta(t) + .4\delta(t-2)$

find $\overline{Y}$ $R_X(\tau) = 100e^{-3\tau^2} + 5$

$\overline{X} = \sqrt{5}$

$\overline{Y} = \overline{X} H(0)$

$\int h(t) = 0$

25.) $R_N(\tau) = 3\delta(\tau) + 5$ $\nearrow 0$

$(\overline{N})^2 = 5$ $\overline{N} = \pm\sqrt{5}$

27.) $R_X(\tau) = 50e^{-4\tau^2} + 8$ find $S_X(\omega)$

$16\pi\delta(\omega) + \int_{-\infty}^{\infty} 50e^{-4\tau^2} e^{-j\omega\tau}$

$+50\int_{-\infty}^{\infty} e^{-4\tau^2 - j\omega\tau} \Rightarrow 50\int_{-\infty}^{\infty} e^{-\tau(4\tau - j\omega)}$

29.) $y(t) = h(t) * x(t)$ $h(t) = \delta(t) + .5\delta(t-3)$

find $R_{xy}(\tau)$ $\delta(\lambda) + .5\delta(\lambda - 3)$

$y(t) = h(\lambda) \cdot x(\lambda)$

$y(t) = E\left[x(\lambda)\left[\delta(\lambda) + .5\delta(\lambda - 3 + \tau)\right]\right]$

$R_X(\tau) + .5R_X(\tau - 3)$

30.) Find m $m = \dfrac{\overline{xy} - \overline{x}\,\overline{y}}{(\overline{x^2}) - (\overline{x})^2}$ where T=x any W=y

$\overline{x} = 27.333$

$\overline{y} = 113$ $m = \dfrac{3104.\overline{6} - (27.\overline{3})(113)}{(774.\overline{6}) - (27.\overline{3})^2} = .58064$

$\overline{x^2} = 774.667$

$\overline{xy} = 3104.\overline{6}$

$\overline{y^2} = 12811$

31.) $m = -3.48$ $b = \overline{y} - m\overline{x} \Rightarrow b = 200$

$y = mx + b$

$y - mx = b$

$\overline{y} = 117.667$

$\overline{x} = 23.\overline{6}$

6.) $f_Y(y) = A\int_0^1 2x+y^2\,dx = A\left(x^2+xy^2\Big|_0^1\right) = (1+y^2)A$

$f_X(x) = \frac{4}{3}A(3x+2)$

$f_X(x)\cdot f_Y(y) = \left(4xA+\frac{8}{3}A\right)\left(A+Ay^2\right)$

$= 4xA^2 + A^2 4xy^2 + \frac{8}{3}A^2 + A^2\frac{8}{3}y^2$

$= A^2\left(\frac{4}{3}\right)\left(3x+3xy^2+2+2y^2\right)$

10.) $F(5)-F(0)$

$\Phi\left(\frac{5-3}{4}\right) - \Phi\left(\frac{0-3}{4}\right)$

$\Phi\left(\frac{1}{2}\right) - \Phi\left(-\frac{3}{4}\right)$

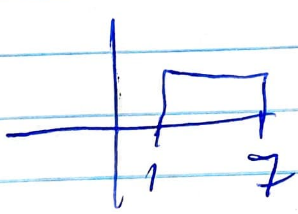
$\Downarrow \qquad \Downarrow$

$0 - 1 - \Phi\left(\frac{3}{4}\right)$

$-1 - .67$

$=$

19.) $\overline{X}^2 = \frac{1^2+7^2}{2} = \frac{49-1=48}{2} =$



mean $=4$ $\quad \int_{-\infty}^{\infty} x^2(x-1)(x-7)\,dx$

$\int_1^7 x^2\,dx = 49-1=48$

$\int_{-\infty}^{\infty} x f(x) = \frac{1}{8}\int_3^{\infty} x e^{-(x-3)/8} = \frac{1}{8}\left(\frac{-1}{\frac{(x-3)}{8}} e^{-(x-3)/8}\Big|_3^{\infty}\right)$

24.) $y(t) = h(t) * x(t)$

34.) $C.I = 99\%$ $Z_c = 2.33$

$$Z = \frac{\bar{x} - \bar{X}}{\sigma/\sqrt{n}} = \frac{396-400}{30/\sqrt{80}} = -1.19257$$

we want to accept H_0 if $Z_c + 400 < Z$

$$400 \pm \frac{(2.33)(\tilde{5})}{\sqrt{n}}$$

22.) mean of $f_x = \frac{1}{8} e^{-(x-3)/8} u(x-3)$

$$\frac{1}{8}\int_0^3 e^{-(x-3)/8} = \frac{1}{8}\left(\frac{1}{\frac{-(x-3)}{8}} e^{-(x-3)/8}\Big|_0^3\right)$$

$$\int_3^\infty \frac{1}{8} e^{-(x-3)/8} \qquad = \frac{1}{8}\left(\frac{1}{0}\right) \quad \frac{1}{8} - \frac{1}{3/8} \quad \frac{1}{8} - \frac{8}{3} = \frac{1}{3}$$

17.) $S_x(\omega) = 14\cancel{\pi}\delta(\omega) + 26\pi e^{-2\omega^2}$ find $(\bar{X})^2$

$$(\bar{X})^2 = \frac{1}{2\pi} S_0 = \frac{1}{2\pi}\left[14\cancel{\pi} + 26\pi\right] = \frac{7}{\cancel{\pi}} \cancel{+13} = 7$$

$$R_x(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_x(\omega) e^{j\omega\tau} d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} 14\delta(\omega) + 26\pi e^{-2\omega^2} d\omega$$

$$\frac{7}{\pi} + \sqrt{\frac{\pi}{2}}$$

8.) $f_x = \frac{1}{\sqrt{32\pi}} e^{-\frac{(x-3)^2}{32}}$

$\bar{X} = 3$, $2\sigma_x^2 = 32$ $\sigma_x^2 = 16$ $\sigma = \pm 4$

10.) $Pr(0 \le X \le 5) = F(5) - F(0)$

$Q(1/4) = 0.67$

$$= \phi\left(\frac{5-3}{4}\right) - \phi\left(\frac{0-3}{4}\right) \quad 1 - 0 - (1-$$

$0 - (1 - .67)$ $\quad (1 - Q(\frac{1}{2}))$

$\phi(x) = 1 - Q(x)$ $\quad = .33$ $\quad = \phi\left(\frac{1}{2}\right) - \phi\left(\frac{-3}{4}\right)$

7.) $f(x,y) = A(2xy+y^2)$ $\quad 0\le x\le 1, -1\le y<1$

find $f(y|x)$ where $f_X(x) = \frac{2}{3}A$ for $0\le x<1$

$f_Y(y) = A(y+y^2)$ for $-1\le y<1$

$f(y|x) = \frac{f(x,y)\, f_Y(y)}{f_X(x)} = \frac{A(2xy+y^2)[A(y+y^2)]}{\frac{2}{3}A}$

$(2xyA+y^2A)(Ay+y^2A) = A^2 2xy^2 + A^2 2xy^3 + A^2y^3 + A^2y^4$

$= \frac{A^{\cancel{2}}(y^4+y^3+2xy^3+2xy^2)}{\frac{2}{3}\cancel{A}} \cdot \frac{3}{2}$

$y^2(y^2+y+2xy+2x)$

$f(y|x) = \frac{f(x,y)}{f_X(x)} = \frac{A2xy+y^2A}{2/3\,A} = \frac{3}{2}(2xy+y^2)$

$= \frac{3}{2}y(2x+y)$

4.) $1 = A\int_0^1\int_0^2 2x+y^2\,dy\,dx = A\int_0^1 \left(2xy+\frac{y^3}{3}\Big|_0^2\right) = A\int_0^1 4x+\frac{8}{3}\,dx$

$= 4A\int_0^1 x+\frac{2}{3}\,dx = 4A\left(\frac{x^2}{2}+\frac{2}{3}x\Big|_0^1\right) = 4A\left(\frac{3}{6}+\frac{4}{6}\right)=1$

$A = 6/28 = 3/14 \quad 4A\left(\frac{7}{6}\right)=1$

$A=3/14$

5.) $f_X(x) = A\int_0^2 2x+y^2\,dy = A\left(2xy+\frac{y^3}{3}\right)\Big|_0^2 = A\left(4x+\frac{8}{3}\right)$

$\frac{4}{3}A(3x+2) = \frac{12x}{3}+\frac{8}{3} = 4x+\frac{8}{3}$ ✓