

. Applications: Warehouse Location Problem

The facility or warehouse location problem is an age-old optimization problem. The goal of the problem is to select the best among potential sites for the construction of a warehouse such that facility costs, geographical demands and transportation distances can be simultaneously optimized. In a more general form the solution to the problem attempts to select a site to minimize cost constraints and maximize the constraints such as distance to opponents, storage capacity etc. In the variation of the problem considered in this paper, a company is looking for a feasible location to establish a warehouse to supply a set of shopping centers distributed across various supermarkets. The company desires to select the best out of several potential sites so it decides to develop a project report based on the evaluation of all possible locations against a set of criteria. Four experts (E_1, E_2, E_3 and E_4) hired from professional consultancy services determine the criteria in consultation with the company's executives. A list of five possible locations (L_1, L_2, L_3, L_4 and L_5) is also elicited by the experts.

The problem addressed in this paper is similar to that presented by ?, so the authors adapt the same set of criteria to illustrate the proposed MCGDM approach. The twelve criteria is identified for the process are as follows: Distance-to-Customers(CDC), Transportation-Cost(CTO), Holding-Cost(CHO), Distance-to-Producers(CDP), Suppliers Distance(CDS), Opponents-Distance (CDO), Delivery-Time (BDT), Storage-Capacity(BSC), Development Rate(BDR), Infrastructure(BIS), Transport-Diversity(BTD) and Climate-Conditions(BCC). The first six criteria are cost (negative), and the last six are benefiting (positive).

Following is the step by step implementation of the proposed approach for the warehouse location problem:

Step 1: Each experts provides a decision matrix and criteria weights using the continuous grey numbers according to Step 1, as shown in Table 3.

Table 3. Decision Matrices and Criteria Weights Provided by Experts

Experts	Criteria Locations	CDC	CTO	CHO	CDP	CDS	CDO	BDT	BSC	BDR	BIS	BTD	BCC
E_1	L_1	[16,18]	[20,21]	[34,36]	[45,47]	[51,52]	[64,66]	[6,8]	[7,8]	[5,9]	[9,12]	[11,12]	[8,11]
	L_2	[15,18]	[20,24]	[36,37]	[43,44]	[56,57]	[61,62]	[4,8]	[4,9]	[9,11]	[5,7]	[12,13]	[10,14]
	L_3	[16,18]	[24,25]	[36,37]	[44,45]	[52,55]	[63,66]	[3,4]	[7,8]	[5,9]	[9,10]	[10,11]	[8,9]
	L_4	[13,14]	[21,23]	[35,37]	[41,45]	[54,56]	[65,66]	[2,3]	[6,10]	[6,7]	[5,6]	[10,13]	[11,13]
	L_5	[10,13]	[19,25]	[35,36]	[45,47]	[54,55]	[62,67]	[6,9]	[4,7]	[8,11]	[9,10]	[12,13]	[8,10]
	$\otimes \lambda_j^1$	[2,6]	[4,6]	[4,12]	[6,13]	[2,6]	[4,5]	[3,11]	[2,4]	[5,11]	[1,8]	[3,5]	[5,11]
E_2	L_1	[10,13]	[22,24]	[33,36]	[46,47]	[55,57]	[62,67]	[7,9]	[4,6]	[8,9]	[5,7]	[6,11]	[7,13]
	L_2	[12,14]	[19,22]	[30,31]	[40,45]	[56,57]	[61,65]	[8,9]	[3,8]	[4,6]	[6,9]	[7,12]	[7,13]
	L_3	[11,16]	[21,25]	[35,37]	[41,42]	[51,53]	[61,63]	[3,9]	[4,5]	[5,7]	[9,11]	[6,11]	[10,11]
	L_4	[13,15]	[21,25]	[34,36]	[45,46]	[54,56]	[60,62]	[5,7]	[9,10]	[4,5]	[8,11]	[10,13]	[12,13]
	L_5	[9,17]	[22,23]	[30,33]	[44,45]	[50,51]	[66,67]	[4,6]	[6,9]	[4,9]	[5,10]	[11,13]	[9,13]
	$\otimes \lambda_j^2$	[4,7]	[7,8]	[3,4]	se[4,6]	[5,8]	[4,7]	[6,8]	[7,8]	[4,5]	[2,5]	[5,6]	[4,8]
E_3	L_1	[14,16]	[24,25]	[32,37]	[42,45]	[55,57]	[64,67]	[2,7]	[8,9]	[8,9]	[11,12]	[11,13]	[8,11]
	L_2	[16,18]	[19,23]	[31,35]	[42,46]	[50,54]	[61,66]	[3,6]	[9,10]	[6,10]	[11,12]	[10,13]	[13,14]
	L_3	[14,15]	[21,22]	[32,37]	[45,46]	[56,57]	[61,65]	[8,9]	[6,8]	[10,11]	[7,12]	[12,13]	[7,11]
	L_4	[14,17]	[23,25]	[34,35]	[42,44]	[56,57]	[61,65]	[7,9]	[4,7]	[4,6]	[8,9]	[9,12]	[8,11]
	L_5	[13,14]	[20,25]	[31,32]	[44,46]	[50,53]	[65,67]	[7,9]	[6,7]	[8,9]	[5,10]	[10,11]	[9,13]
	$\otimes \lambda_j^3$	[6,7]	[4,5]	[3,5]	[4,5]	[6,7]	[5,6]	[4,5]	[6,7]	[1,4]	[6,7]	[6,7]	[2,3]
E_4	L_1	[12,16]	[22,23]	[32,36]	[43,47]	[51,55]	[61,63]	[7,9]	[6,10]	[4,10]	[7,9]	[12,13]	[12,14]
	L_2	[16,17]	[22,25]	[36,37]	[42,47]	[53,56]	[62,67]	[5,7]	[6,9]	[8,10]	[11,12]	[10,12]	[13,14]
	L_3	[11,14]	[20,21]	[33,35]	[45,47]	[53,57]	[63,65]	[5,7]	[7,10]	[6,7]	[6,7]	[9,10]	[13,14]
	L_4	[15,16]	[20,21]	[34,36]	[45,47]	[56,57]	[66,67]	[7,8]	[9,10]	[7,8]	[7,8]	[8,12]	[13,14]
	L_5	[11,17]	[19,24]	[31,32]	[43,45]	[52,55]	[62,65]	[6,8]	[7,9]	[10,11]	[8,9]	[9,13]	[7,10]
	$\otimes \lambda_j^4$	[4,5]	[4,7]	[2,8]	[7,8]	[5,6]	[5,8]	[4,8]	[3,6]	[3,7]	[4,7]	[5,7]	[7,8]

Step 2-3: Obtain aggregated decision matrix and normalized aggregated decision matrix using (3) - (7). The two matrices are shown in tables 4 and 5:

Step 4: The initial aggregated criteria weights are obtained using Eq. (8) and are normalized. Table 6 shows the results:

Step 5: In this steps the normalized aggregated decision matrix and initial weight vector are trans-

Table 4. Aggregated Group Decision Matrix

Criteria Locations	CDC	CTO	CHO	CDP	CDS	CDO
L_1	[13, 15.75]	[22, 23.25]	[32.75, 36.25]	[44, 46.5]	[53, 55.25]	[62.75, 65.75]
L_2	[14.75, 16.75]	[20, 23.5]	[33.25, 35]	[41.75, 45.5]	[53.75, 56]	[61.25, 65]
L_3	[13, 15.75]	[21.5, 23.25]	[34, 36.5]	[43.75, 45]	[53, 55.5]	[62, 64.75]
L_4	[13.75, 15.5]	[21.25, 23.5]	[34.25, 36]	[43.25, 45.5]	[55, 56.5]	[63, 65]
L_5	[10.75, 15.25]	[20, 24.25]	[31.75, 33.25]	[44, 45.75]	[51.5, 53.5]	[63.75, 66.5]
Criteria Locations	BDT	BSC	BDR	BIS	BTD	BCC
L_1	[5.5, 8.25]	[6.25, 8.25]	[6.25, 9.25]	[8, 10]	[10, 12.25]	[8.75, 12.25]
L_2	[5, 7.5]	[5.5, 9]	[6.75, 9.25]	[8.25, 10]	[9.75, 12.5]	[10.75, 13.75]
L_3	[4.75, 7.25]	[6, 7.75]	[6.5, 8.5]	[7.75, 10]	[9.25, 11.25]	[9.5, 11.25]
L_4	[5.25, 6.75]	[7, 9.25]	[5.25, 6.5]	[7, 8.5]	[9.25, 12.5]	[11, 12.75]
L_4	[5.75, 8]	[5.75, 8]	[7.5, 10]	[6.75, 9.75]	[10.5, 12.5]	[8.25, 11.5]

Table 5. Normalized Aggregated Group Decision Matrix

Criteria Locations	CDC	CTO	CHO
L_1	[0.682539683,0.826923077]	[0.860215054,0.909090909]	[0.875862069,0.969465649]
L_2	[0.641791045,0.728813559]	[0.85106383,1]	[0.907142857,0.954887218]
L_3	[0.682539683,0.826923077]	[0.860215054,0.930232558]	[0.869863014,0.933823529]
L_4	[0.693548387,0.781818182]	[0.85106383,0.941176471]	[0.881944444,0.927007299]
L_5	[0.704918033,1]	[0.824742268,1]	[0.954887218,1]
Criteria Locations	CDP	CDS	CDO
L_1	[0.897849462,0.948863636]	[0.932126697,0.971698113]	[0.931558935,0.976095618]
L_2	[0.917582418,1]	[0.919642857,0.958139535]	[0.942307692,1]
L_3	[0.927777778,0.954285714]	[0.927927928,0.971698113]	[0.945945946,0.987903226]
L_4	[0.917582418,0.965317919]	[0.911504425,0.936363636]	[0.942307692,0.972222222]
L_5	[0.912568306,0.948863636]	[0.962616822,1]	[0.921052632,0.960784314]
Criteria Locations	BDT	BSC	BDR
L_1	[0.666666667,1]	[0.675675676,0.891891892]	[0.625,0.925]
L_2	[0.606060606,0.909090909]	[0.594594595,0.972972973]	[0.675,0.925]
L_3	[0.575757576,0.878787879]	[0.648648649,0.837837838]	[0.65,0.85]
L_4	[0.636363636,0.818181818]	[0.756756757,1]	[0.525,0.65]
L_5	[0.696969697,0.96969697]	[0.621621622,0.864864865]	[0.75,1]
Criteria Locations	BIS	BTD	BCC
L_1	[0.8,1]	[0.80,0.98]	[0.636363636,0.890909091]
L_2	[0.825,1]	[0.78,1]	[0.781818182,1]
L_3	[0.775,1]	[0.74,0.9]	[0.690909091,0.818181818]
L_4	[0.7,0.85]	[0.74,1]	[0.8, 0.927272727]
L_5	[0.675,0.975]	[0.84,1]	[0.6,0.836363636]

formed into compact forms. Table 7 shows the transformations. Criterion weight is mention under its heading.

Step 6-7: In this step subjective criteria weights are calculated using (9) - (10) and are normalized using (11). Table 8 shows the results. From Table 7 we have,

$$\min(\otimes \lambda_j) = (0.515151515)_{0.242424242}, \quad \max(\otimes \lambda_j) = (0.8125)_{0.375}$$

Using (2), $\delta(\max(\otimes \lambda_j), \min(\otimes \lambda_j)) = 0.387249627$

Step 8-9: This step calculates objective criteria weights using (20). The weights are normalized using (21). Table 9 shows the results.

Table 6. Initial Aggregated Criteria Weights

Experts Criteria	E_1	E_2	E_3	E_4	Aggregated Weights ($\otimes \lambda_j$)	Normalized Aggregated Weights
CDC	[4,7]	[6,7]	[6,7]	[2,4]	[4.5,6.25]	[0.6, 0.83333333]
CTO	[6,8]	[2,6]	[3,5]	[4,5]	[3.75,6]	[0.625,1]
CHO	[4,12]	[7,8]	[5,6]	[2,5]	[4.5,7.75]	[0.483870968, 0.83333333]
CDP	[5,10]	[6,7]	[4,6]	[6,7]	[5.25,7.5]	[0.5, 0.714285714]
CDS	[5,11]	[4,6]	[5,6]	[4,6]	[4.5,7.25]	[0.517241379, 0.83333333]
CDO	[2,3]	[6,7]	[5,6]	[3,8]	[4,6]	[0.625,0.9375]
BDT	[1,10]	[7,8]	[1,5]	[2,5]	[2.75,7]	[0.33333333, 0.848484848]
BSC	[3,10]	[3,6]	[1,5]	[3,7]	[2.5,7]	[0.303030303, 0.848484848]
BDR	[2,6]	[5,6]	[1,3]	[3,8]	[2.75,5.75]	[0.33333333, 0.696969697]
BIS	[2,4]	[3,5]	[2,4]	[6,8]	[3.25,5.25]	[0.393939394, 0.636363636]
BTD	[2,7]	[4,6]	[5,7]	[5,8]	[4,7]	[0.484848485, 0.848484848]
BCC	[5,11]	[5,8]	[3,6]	[6,8]	[4.75,8.25]	[0.575757576, 1]

Table 7. Normalized Aggregated Group Decision Matrix: Compact Form

Criteria Locations	CDC (0.716666667) _{0.233333333}	CTO (0.8125) _{0.375}	CHO (0.658602151) _{0.349462366}
L_1	(0.75473138) _{0.144383394}	(0.884652981) _{0.048875855}	(0.922663859) _{0.09360358}
L_2	(0.685302302) _{0.087022515}	(0.925531915) _{0.14893617}	(0.931015038) _{0.047744361}
L_3	(0.75473138) _{0.144383394}	(0.895223806) _{0.070017504}	(0.901843272) _{0.063960516}
L_4	(0.737683284) _{0.088269795}	(0.89612015) _{0.090112641}	(0.904475872) _{0.045062855}
L_5	(0.852459016) _{0.295081967}	(0.912371134) _{0.175257732}	(0.977443609) _{0.045112782}
Criteria Locations	CDP (0.607142857) _{0.214285714}	CDS (0.675287356) _{0.316091954}	CDO (0.78125) _{0.3125}
L_1	(0.923356549) _{0.051014174}	(0.951912405) _{0.039571416}	(0.953827276) _{0.044536682}
L_2	(0.958791209) _{0.082417582}	(0.938891196) _{0.038496678}	(0.971153846) _{0.057692308}
L_3	(0.941031746) _{0.026507937}	(0.949813021) _{0.043770185}	(0.966924586) _{0.04195728}
L_4	(0.941450168) _{0.047735501}	(0.923934031) _{0.024859212}	(0.957264957) _{0.02991453}
L_5	(0.930715971) _{0.03629533}	(0.981308411) _{0.037383178}	(0.940918473) _{0.039731682}
Criteria Locations	BDT (0.590909091) _{0.515151515}	BSC (0.575757576) _{0.545454545}	BDR (0.515151515) _{0.363636364}
L_1	(0.833333333) _{0.333333333}	(0.783783784) _{0.216216216}	(0.775) _{0.3}
L_2	(0.757575758) _{0.303030303}	(0.783783784) _{0.378378378}	(0.8) _{0.25}
L_3	(0.727272727) _{0.303030303}	(0.743243243) _{0.189189189}	(0.75) _{0.2}
L_4	(0.727272727) _{0.181818182}	(0.878378378) _{0.243243243}	(0.5875) _{0.125}
L_5	(0.833333333) _{0.272727273}	(0.743243243) _{0.243243243}	(0.875) _{0.25}
Criteria Locations	BIS (0.515151515) _{0.242424242}	BTD (0.666666667) _{0.363636364}	BCC (0.787878788) _{0.424242424}
L_1	(0.9) _{0.2}	(0.89) _{0.18}	(0.763636364) _{0.254545455}
L_2	(0.9125) _{0.175}	(0.89) _{0.22}	(0.890909091) _{0.218181818}
L_3	(0.8875) _{0.225}	(0.82) _{0.16}	(0.754545455) _{0.127272727}
L_4	(0.775) _{0.15}	(0.87) _{0.26}	(0.863636364) _{0.127272727}
L_5	(0.825) _{0.3}	(0.92) _{0.16}	(0.718181818) _{0.236363636}

Step 10: The combinative weights are now calculated using (22). Table 10 shows the results.

Step 11-12: The decision score of each alternative is evaluated during this step using (23). Table 11 shows the results. Location with maximum decision score is selected as the best location for establishing the warehouse.

Clearly, the locations order is $L_2 > L_5 > L_1 > L_4 > L_3$ and L_2 is the preferred location for establishing the warehouse.

Table 8. Subjective Criteria Weights

Criteria	Distance ($\delta(\otimes\lambda_j), \min(\otimes\lambda_j)$)	Subjective Weight (ω_j^e)	Normalized Weight Values ($\bar{\omega}_j^e$)
CDC	0.222684	0.575039	0.093944
CTO	0.38725	1	0.16337
CHO	0.196086	0.506356	0.082723
CDP	0.0946	0.244286	0.039909
CDS	0.20442	0.527876	0.086239
CDO	0.325726	0.841127	0.137415
BDT	0.165519	0.427422	0.069828
BSC	0.155188	0.400745	0.06547
BDR	0.031221	0.080623	0.013171
BIS	0	0	0
BTD	0.210285	0.543021	0.088713
BCC	0.37741	0.974592	0.159219

Table 9. Objective Criteria Weights

Criteria	Objective Weights (ω_j^o)	Normalized Weight Values ($\bar{\omega}_j^o$)
CDC	4.177920721	0.065470965
CTO	5.671683544	0.088879282
CHO	5.71471648	0.089553638
CDP	5.80152647	0.090914012
CDS	5.85747095	0.091790702
CDO	6.001982871	0.094055305
BDT	4.903720696	0.076844762
BSC	4.927199274	0.077212688
BDR	4.475379132	0.070132348
BIS	5.660847054	0.088709466
BTD	5.825814676	0.091294625
BCC	4.795074832	0.075142205

Table 10. Combinative Criteria Weights($\psi = 0.5$)

Criteria	Subjective Weights($\bar{\omega}_j^e$)	Objective Weights ($\bar{\omega}_j^o$)	Combinative Weights ($\bar{\omega}_j$)
CDC	0.093943937	0.065471	0.079707451
CTO	0.163369668	0.088879	0.126124475
CHO	0.082723291	0.089554	0.086138465
CDP	0.039908962	0.090914	0.065411487
CDS	0.086238884	0.091791	0.089014793
CDO	0.137414669	0.094055	0.115734987
BDT	0.069827712	0.076845	0.073336237
BSC	0.065469533	0.077213	0.071341111
BDR	0.013171385	0.070132	0.041651867
BIS	0	0.088709	0.044354733
BTD	0.088713154	0.091295	0.09000389
BCC	0.159218805	0.075142	0.117180505

Table 11. Decision Scores of Alternatives

Alternative (Location)	Decision Score	Rankings
L_1	0.865550303	3
L_2	0.880009362	1
L_3	0.847943618	5
L_4	0.858696253	4
L_5	0.87847868	2