SSY130 - Applied Signal Processing

# Project 2: Group 19

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# 1 Introduction

This document will describe project 2 regarding 'Adaptive Noise Cancellation'. It will provide answers to the exercises, as well as a graphical overview of the workings of the code.

# 2 Empirical Section

## 2.1 Exercise 1

The filter coefficients, magnitude response, and phase response for the sinusoidal filter, $h_{sin}$, are shown in Fig. 1.

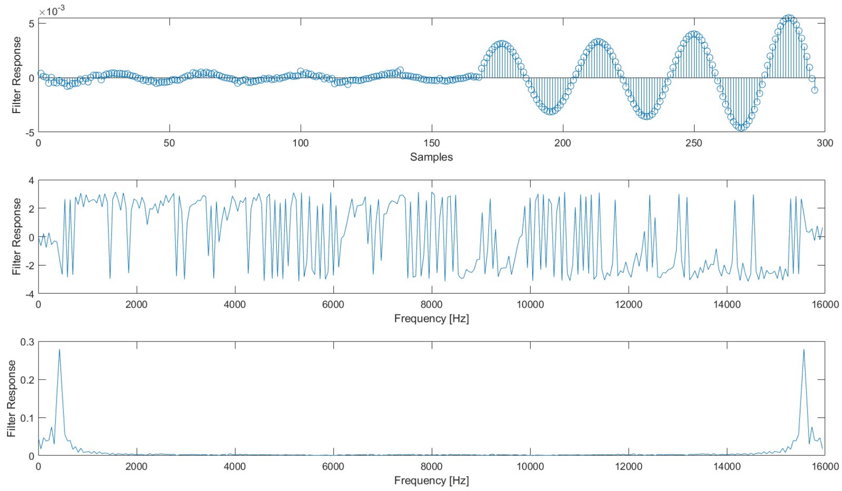


Figure 1: Filter coefficients, magnitude response, and phase response of the sinusoidal filter, $h_{sin}$.

Moreover, the same information for the broad-band filter, $h_{BB}$, is presented in Fig. 2.

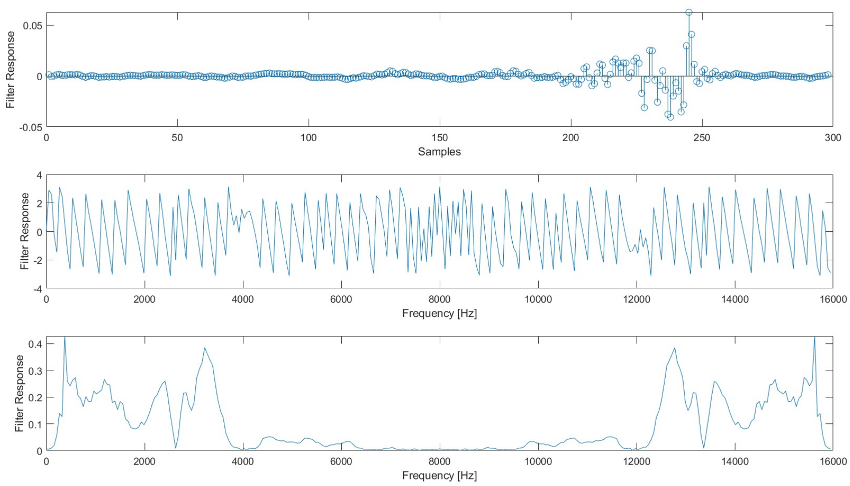


Figure 2: Filter coefficients, magnitude response, and phase response of the broad-band filter, $h_{BB}$

## 2.2 Exercise 2

### 2.2.1 Exercise 2a

From Fig. 3 and Fig. 4 it can be seen that changing the input signal by distance or having a book in between makes the absolute errors go from around zero to increasing absolute errors. This is because we have not changed the filters coefficients, but we have changed the environment for the signals, altering the phase characteristics mainly. The estimated channel is different from the actual channel, which is why we have errors. Adding the book in between generated additional losses, which were detrimental to proper detection of the magnitude.

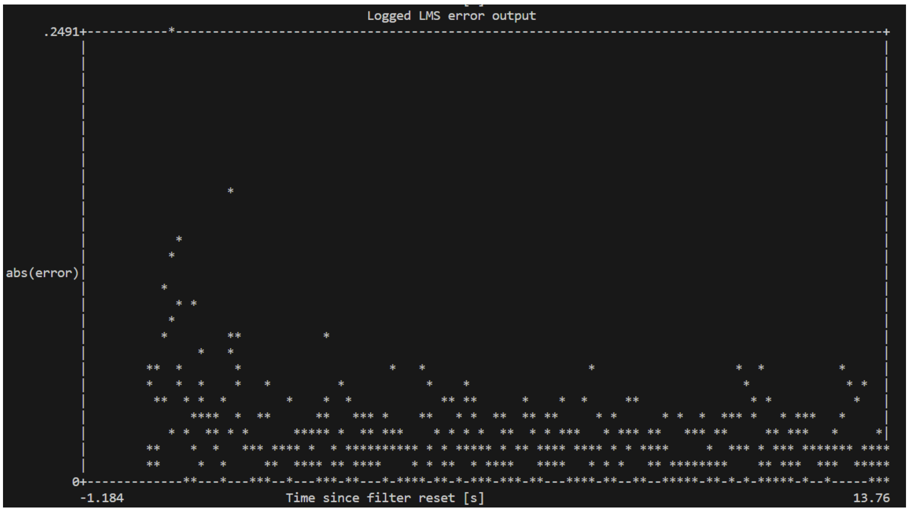


Figure 3: Error at the beginning without the book's presence.

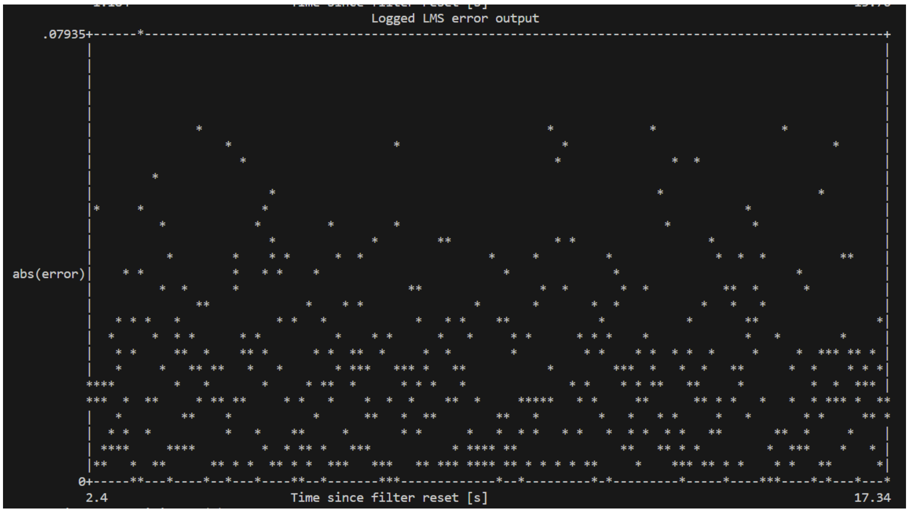


Figure 4: Errors when the book is disturbing.

### 2.2.2 Exercise 2b

From Fig. 5 it is shown that an increased volume means that we increase the amplitude of the sound. If the amplitude is increased, that also means that the errors are increased as well, since the gain of the channel is affected. In Fig. 5, one can see the difference between going from increased to decreased volume. In essence, the channel magnitude is attenuated, causing the error terms of the impulse response.

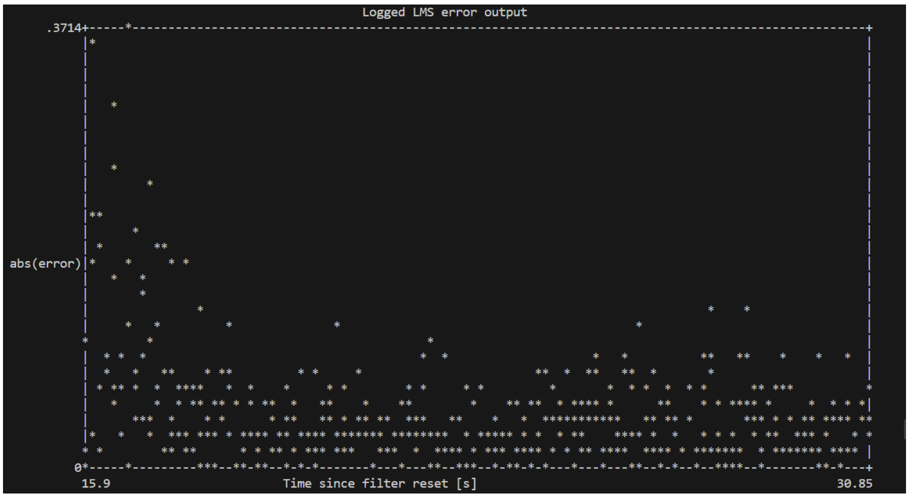


Figure 5: Going from increased volume to decreased.

### 2.2.3 Exercise 2c

Fig. 6 depicts the error terms while playing music and moving the mobile phone around. The error terms are less affected, since the disturbance comes from another source instead of the DSP kit itself. Furthermore, the broad-band filter manages to get rid of most of the frequencies, as it covers a range of frequencies in the audible domain.

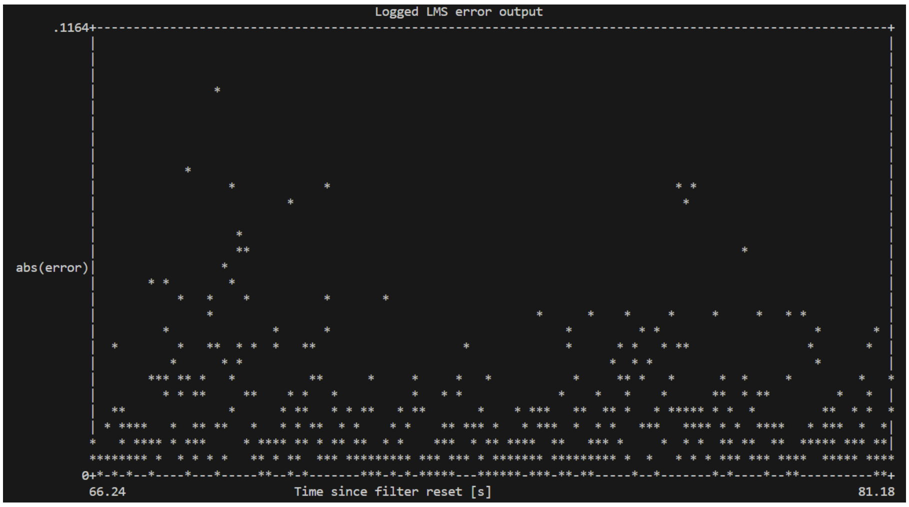


Figure 6: Errors while playing music and moving the mobile phone.

## 2.3 Exercise 3

The step plots with smaller number of elements are just truncated versions of the stem plots with larger number of elements. In the first section, we plotted filter coefficients. A longer filter captures more details and has a longer memory of past inputs. This could be useful for dealing with complex disturbances, but it may introduce more delay. As you reduce the filter length, the filter becomes more focused on recent inputs, potentially losing some ability to capture longer-term dependencies. This reduction in filter length might lead to less computational load, but could impact performance.

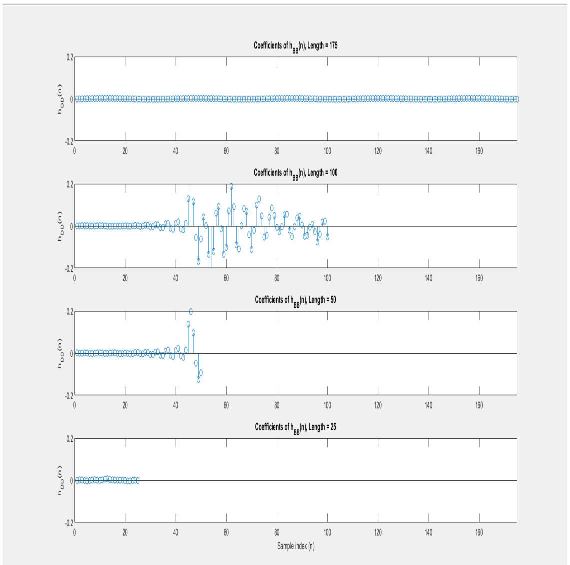


Figure 7: Filter coefficients without reset.

From the picture that is shown below we can see that with resetting and without resetting the amount of errors are actually the same. But in the pictures, we can also say that decreasing the tap error will increase. During the experiment, we concluded that by decreasing the tap below 68, noise cancellation decreased sharply. Resetting the filter coefficients to zero after changing the filter length might be beneficial in avoiding artifacts or inconsistencies. It can help the filter to adapt to the new length and prevent any lingering effects from the previous configuration.

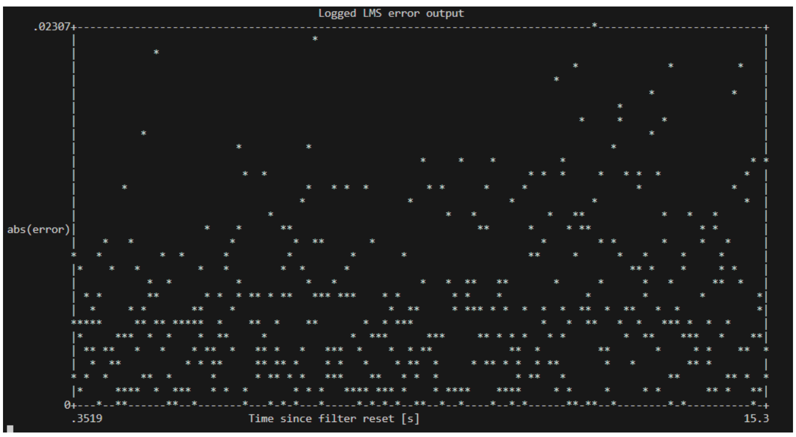


Figure 8: LMS error signal at tap c = 25.

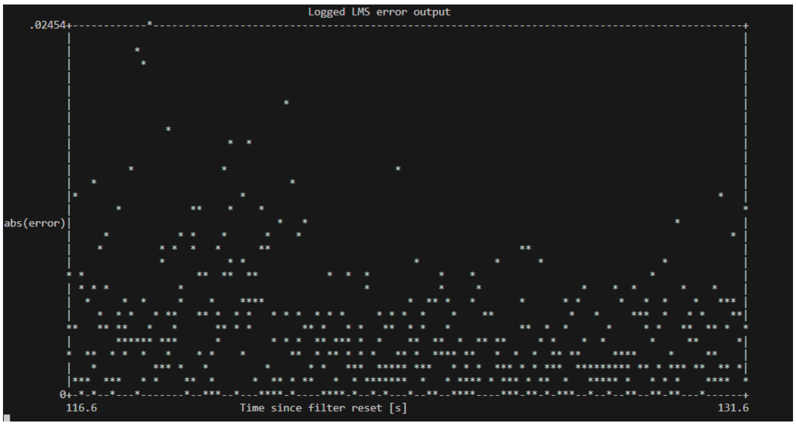


Figure 9: LMS error signal at tap c = 50.


Figure 10: LMS error signal at tap c = 100.

## 2.4 Exercise 4

In the Figures 11 and 12 we changed filter length from 100 to 10, and from 10 to 100 without reset. If the filter is continuously updated, reducing the filter length from 100 to 10 elements may result in a stem plot that emphasizes recent samples. The shorter filter is likely to focus on capturing the characteristics of the sinusoidal disturbance over a shorter time frame. Increasing the filter length from 10 to 100 elements, while continuously updating, may lead to a stem plot that captures longer-term dependencies in the sinusoidal disturbance. The longer filter can provide more context and memory for the periodic nature of the disturbance.

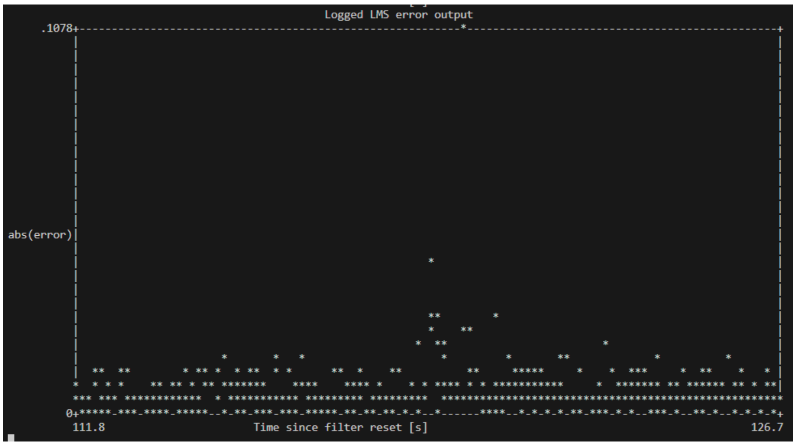


Figure 11: From 100 to 10 without reset.

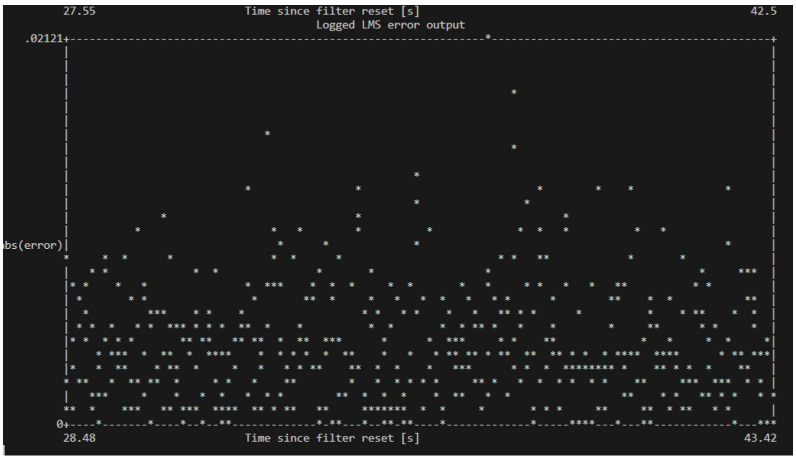


Figure 12: From 10 to 100 without reset.

On the other side, if we reset filter length we get results that are shown on pictures 13. and 14. Whether or not you reset the coefficients after changing the filter length could impact the transition. Resetting may help the filter adapt to the new length, preventing artifacts or lingering effects from the longer filter. Also, resetting the coefficients after changing to a longer filter length can be important to avoid artifacts. It helps the filter adapt to the new length and ensures a smooth transition.

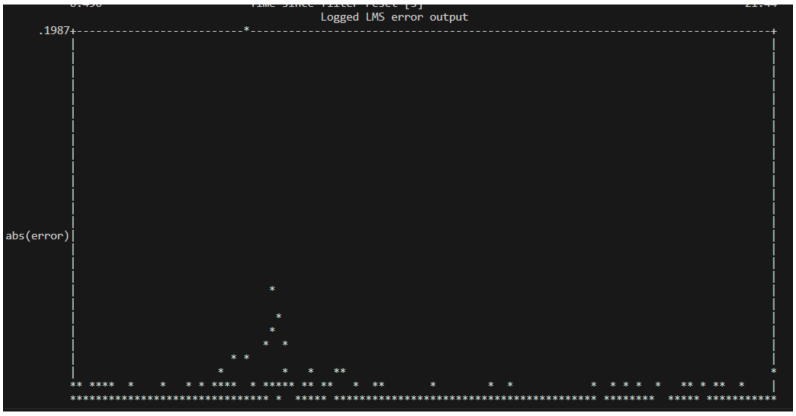


Figure 13: From 100 to 10 with reset.

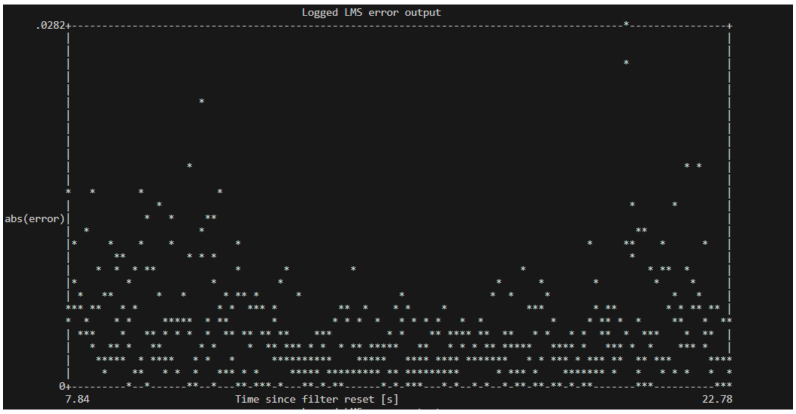


Figure 14: From 10 to 100 with reset.

## 2.5 Exercise 5

When we initially trained the filter for hBB and then transitioned to hsin, the error performance remained consistent. This suggests that hBB effectively mitigated sinusoidal disturbances. In contrast, when we trained the filter for a sinusoidal disturbance and later switched to hBB, the noise level experienced a significant change, indicating that hsin struggled to suppress broad-band noise.

The disparity in performance between these two scenarios can be attributed to the nature of the training data. When the system was trained for hBB, which includes multiple frequencies, the transition to a sinusoidal disturbance was seamless because the specific frequency was already accounted for in hBB. However, in the second scenario, training for a sinusoidal disturbance and then switching to hBB disrupted the system's performance. This mismatch occurred because the system was trained for a single frequency but was now presented with multiple frequencies, resulting in elevated errors compared to the previous case.

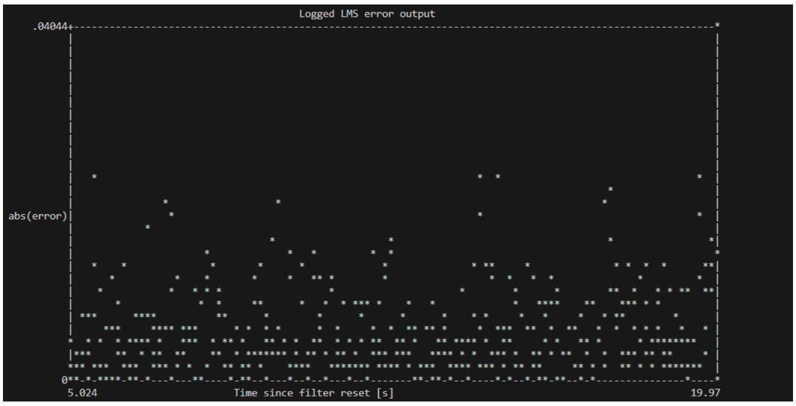


Figure 15: hBB in sin noise.

## 2.6 Exercise 6

Exercise 6 demonstrates the LMS algorithm's limitations when encountering non-linear phenomena such as channel saturation due to high-volume output from a cellphone speaker. This saturation, or clipping, invalidates the assumption of linearity essential for the LMS algorithm to function correctly. The LMS algorithm strives to reduce the error by applying linear filtering methods. However, the presence of non-linearities from the clipping means that these linear methods are ineffective, leading to persistently high error rates which are shown in Figure 13 and 14.

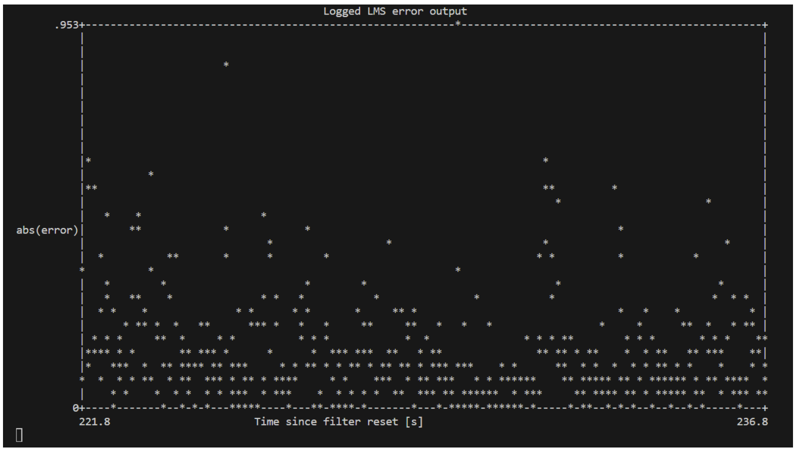


Figure 16: Results from the filter in saturated mode

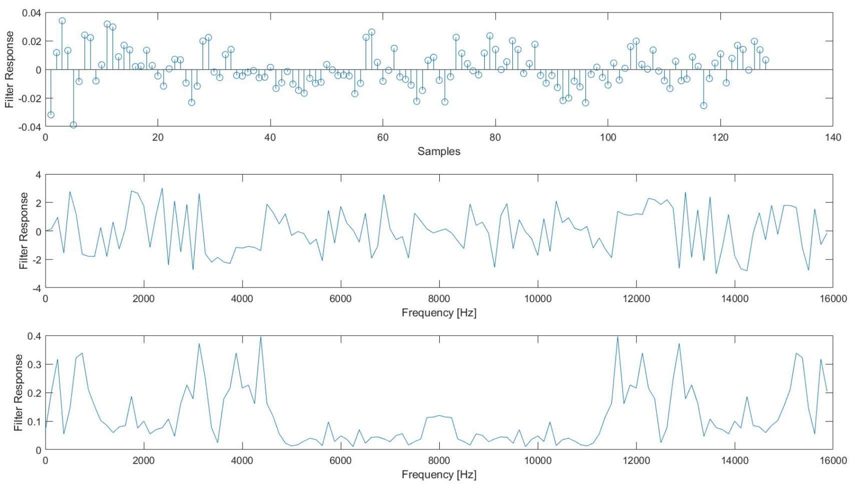


Figure 17: Plots of hbb,sat

# 3 Analytical Section

## 3.1 Exercise 1a

A large step size, $\mu$, indicates that the filter adapts faster to the channel, while a smaller step-size shows that the filter takes time to learn from and adapt to the channel. However, a large step-size might mess up the the convergence, since this parameter will determine the rate of convergence to the desired global minimum, which might not be found when it is too large. This property follows from Eq. 13 of the lab manual, where the influence of the step size on the Least-Mean Squares (LMS) algorithm can be seen.

## 3.2 Exercise 1b

A large value of $\mu$ will cause the estimated channel to diverge because, in order to calculate the estimated channel, we take the product of the error signal and step size to compute the gradient. So if the value of $\mu$ is too large, the algorithm will diverge from the optimum value.

## 3.3 Exercise 1c

After convergence, the step size is still important to consider, since we cannot fully verify whether the error terms have been eliminated to zero. If the step size would suddenly be increased after convergence, the filter response might still diverge due to the remaining (very small) error terms.

## 3.4 Exercise 2

For the broad-band noise condition, doing a reset of the filter and changing the length of the filter does not change the way the estimated filter behaves. It looks like it behaves as an infinite impulse response. However, for the sinusoidal noise condition, if the channel is reset and the length of the channel is increased, we see a lot of points on the figure which are close to 0. Hence, it looks like the channel has a finite impulse response, which becomes more clear in Exercise 5, which shows that the sinusoidal impulse response has a theoretical minimum length of 2.

## 3.5 Exercise 3

From Fig. 18 it can be seen that the sinusoidal filter cannot fully eliminate the broad-band disturbance as it covers a single frequency. Hence, the sinusoidal filter is unable to filter out all the broad-band noise, since it is a single tone, which cannot cover the whole broad-band frequency spectrum. The overall filter response response does not change compared to $h_{BB}$.

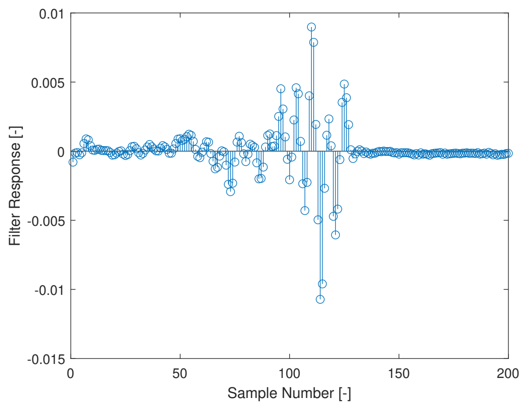


Figure 18: Filter coefficients for sinusoidal trained filter switched to broad-band filter.

On the contrary, when the broad-band filter is switched to the sinusoidal filter, it can be seen that the sinusoidal impulse response merges with the broad-band impulse response, as shown in Fig. 19. This ensures that the filter can fully get rid of the sinusoidal noise, as well as the broad-band noise, since it covers a whole range of frequencies. All in all, the result for $h_{BB,sin}$ will not be the same as the result for $h_{sin}$. This will change the overall filter response, merging the responses of $h_{sin}$ and $h_{BB}$.

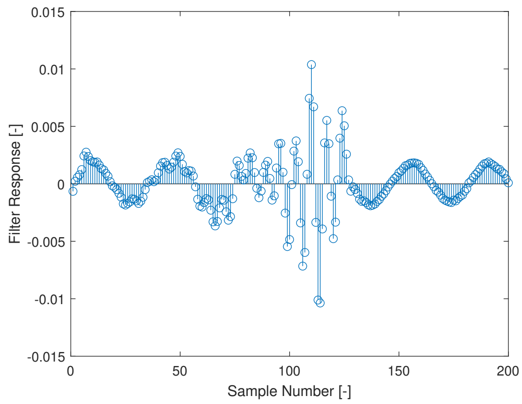


Figure 19: Filter coefficients for broad-band trained filter switched to sinusoidal filter.

### 3.6 Exercise 4

Considering the magnitude and phase response of $H_{sin}(f_0)$ at $f_0 = 440$ Hz, it shows clear similarities in both the magnitude and phase at the specific frequency of 440 Hz. However, when checking frequencies around $f_0$, the magnitude and phase tend to drift to completely different values. Similarly, the magnitude and phase response of $H_{BB}(f_0)$ are almost exactly the same at $f_0 = 440$ Hz, showing drifting values when going away from this specific frequency point. It is expected that the estimate for $H_{sin}(f_0)$ will give more accurate results, as it only a single tone, while $H_{BB}(f_0)$ covers a broad range of frequencies.

### 3.7 Exercise 5

Considering a sinusoidal disturbance, the minimum theoretical length of the filter response can easily be computed, since the filter response needs to have the same phase and magnitude response as the disturbance. It is known that the sinusoidal signal only has a single term in the frequency domain, which is a delta function, $\delta(x)$, which the filter should eliminate. Moreover, the DTFT of the sinusoidal filter, $H_{sin}(\omega)$, is given by

$$H_{sin}(\omega) = \sum_{-\infty}^{\infty} h_{sin}[n] e^{-jn\omega\Delta t}, \tag{1}$$

where $h_{sin}$ is the sinusoidal filter impulse response, $n$ is the sample index, $\omega$ is the angular frequency, and $\Delta t$ are the timing instants. Hence, due to symmetry around 0, the DTFT can be rewritten as

$$H_{sin}(\omega) = \sum_{0}^{N_{h_{sin}}-1} h_{sin}[n] e^{-jn\omega\Delta t}, \tag{2}$$

where the value for $H_{sin}(0)$ only contains a real part, which cannot get rid of the phase response of the noise term. Ultimately, another term is needed for the filter response, namely $H_{sin}(1)$, which will have a magnitude and phase term, and can thus get rid of the sinusoidal disturbance. In total, the minimal theoretical filter length is then 2, having $h_{sin} = [h_{sin(0)}, h_{sin(1)}]$.

### 3.8 Exercise 6

For an increased filter length of $h_{sin}$, the sinusoidal filter will have a distinctive sinusoidal response, following from Eq. 13 of the lab manual, which gives

$$\hat{\mathbf{h_{sin}}}(n) = \hat{\mathbf{h_{sin}}}(n-1) + 2\mu\mathbf{y}(n = \left(x(n) - \hat{\mathbf{h_{sin}}}^T(n-1)\mathbf{y}(n)\right), \tag{3}$$

where it shows that $\hat{\mathbf{h_{sin}}}(n)$ is being repeatedly updated using past values, while the frequency is fixed. Hence, $\hat{\mathbf{h_{sin}}}(n)$ will have the same frequency, but a varying phase and magnitude response respectively. In essence, it is just a superposition of sinusoidal terms with a fixed frequency, but different magnitude and/or phase scaling.

# 4 Appendix

## 4.1 LMS function

```
for (int n=0; n<block_size; n++)
{
    float* y_book = &lms_state[n];
    arm_dot_prod_f32(lms_coeffs,y_book,lms_taps, &xhat[n]);
    e[n] = x[n] - xhat[n];
    for (int k=0; k<lms_taps; k++)
    {
    lms_coeffs[k] += 2 * lms_mu * y_book[k] * e[n];
    }
}
```

Figure 20: Code snipped for LMS algorithm