SSY130 - Applied Signal Processing

Project 1a: Group 19

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1 Introduction

This document will describe project alternative 1a regarding 'Acoustic Communication - Baseband Equalization'. It will provide answers to the exercises, as well as a graphical overview of the workings of the code.

2 Exercise 1

2.1 Exercise 1a

In the current configuration, it holds that the output, $y(n)$, relates to the OFDM signal, $z(n)$, as

$$y(n) = z(n) = \frac{1}{N} \sum_{k=0}^{N-1} s(k) e^{j2\pi kn/N}, \tag{1}$$

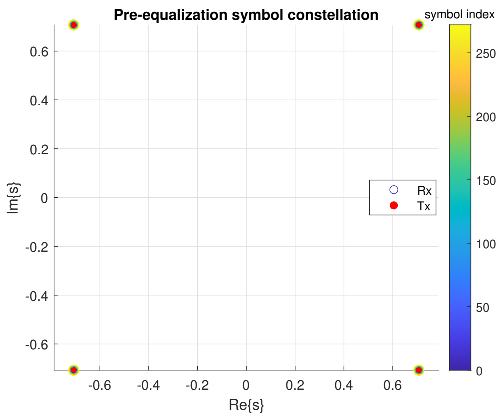
where $s(k)$ are the input symbols. On top of that, the Discrete Fourier Transform (DFT), $r(k)$, is computed as

$$r(k) = \sum_{n=0}^{N-1} y(n) e^{-j2\pi kn/N} = H(k)s(k), \tag{2}$$

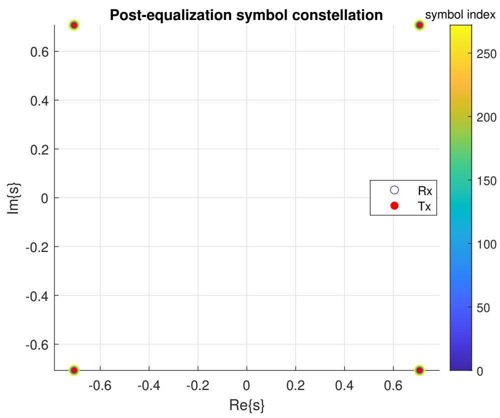
where $H(k)$ is the Fourier transform of the channel impulse response. The equalized symbols, $r_{eq}(k)$ will then be calculated as

$$r_{eq}(k) = \frac{r(k)}{H(k)} = s(k), \tag{3}$$

which corresponds exactly to the input symbols, as can be seen in Fig. 1. This result will yield an Error Vector Magnitude (EVM) of $5.33 \cdot 10^{-14}$ and a Bit Error Rate (BER) of 0. From this we can confirm that there were no errors bigger than machine precision and the transmitted symbols are correct.



Pre-equalization.

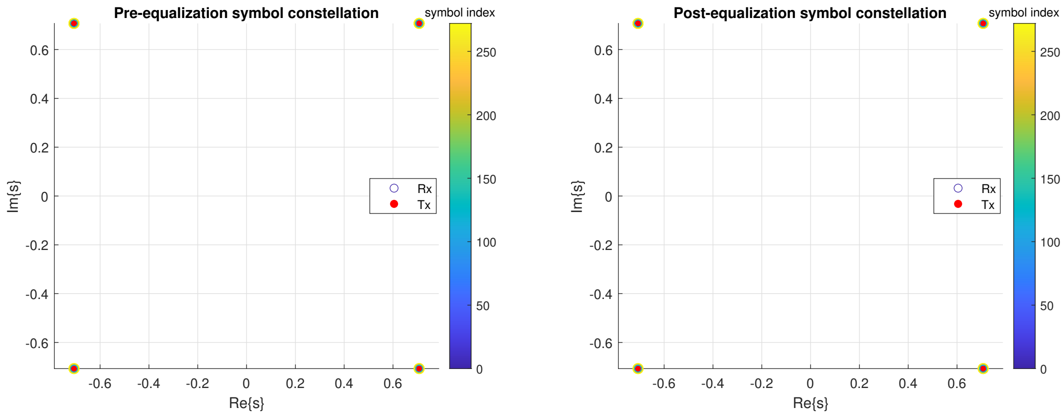


Post-equalization.

Figure 1: 1a - Pre- and post-equalization.

2.2 Exercise 1b

The cyclic prefix is needed since the transmitter is not turned on at $t = -\infty$, and to eliminate potential Inter-Symbol Interference (ISI). A periodic signal is desired for a finite length impulse response, and thus the cyclic prefix length, N_{cp}, should be greater or equal to the impulse response length, N_h, given as $N_{cp} \geq N_h$. In this way, the interference caused by the channel convolution of the last block will be entirely absorbed by the prefix samples, that will be discarded later by the receiver. Result of this is in a near-zero EVM of $5.3306 \cdot 10^{-14}$ and a BER of 0, as shown in Fig. 2.

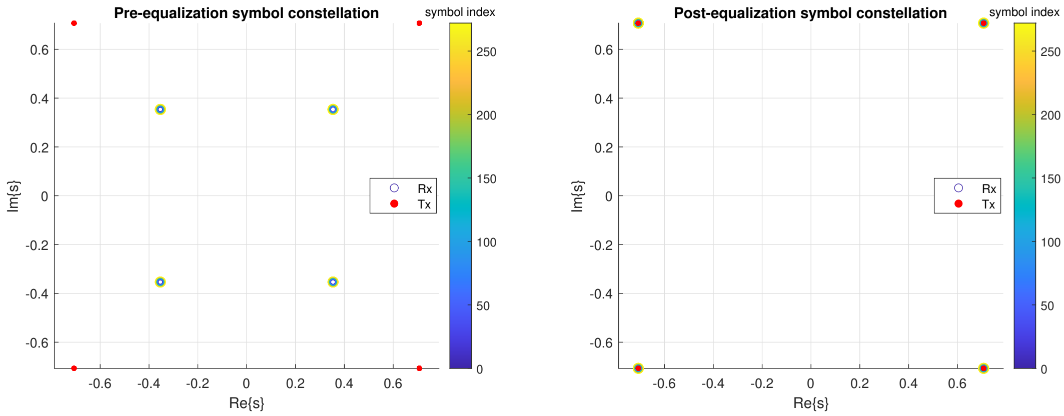


Pre-equalization. Post-equalization.

Figure 2: 1b - Pre- and post-equalization.

2.3 Exercise 1c

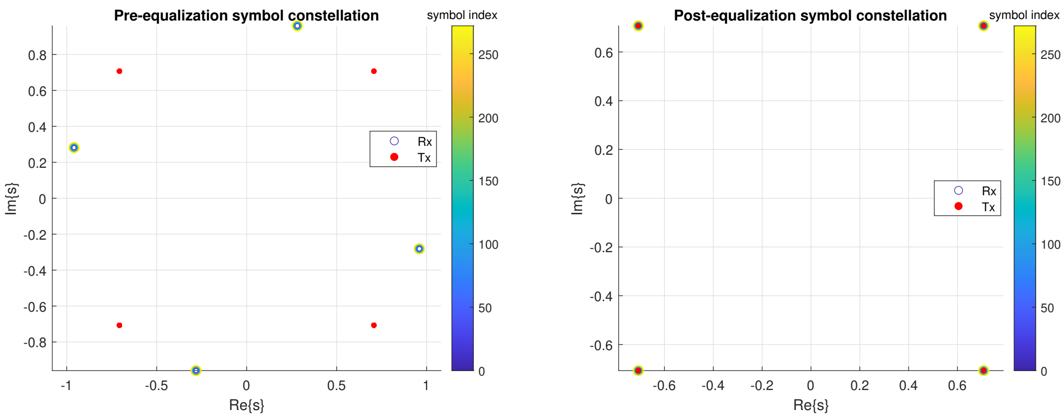
With magnitude scaling (impulse response h_2) the value for α is 0.5, where it can be seen that the constellation moves closer to the center before equalization, as presented in Fig. 3. The EVM before equalization increases significantly, since the distance between the received and transmitted symbols is larger. The BER will remain unaffected since the received points before equalization are still in the correct quadrants. After equalization, the EVM will be very small again.



Pre-equalization. Post-equalization.

Figure 3: 1c - Pre- and post-equalization with h_2.

With a phase offset (impulse response h_3) the value of α is $e^{j0.5}$, resulting in a counter-clockwise phase rotation before equalization, as shown in Fig. 4. Again, before equalization the EVM will be worse, but the BER should be zero, since the neighbouring quadrant is not yet entered. After equalization, the EVM will reduce significantly again.



Pre-equalization. Post-equalization.

Figure 4: 1c - Pre- and post-equalization with h_3.

2.4 Exercise 1d

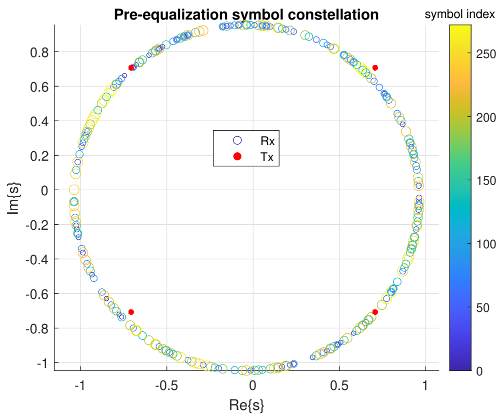
For a synchronization error of e.g ± 1 the constellation will show a donut shape, following from a random phase rotation applied to each individual symbol, as presented in Fig. 5 and Fig. 6. The BER will be at maximum 0.5, corresponding to all symbols being received wrongly, while the EVM will significantly increased due to the scattered points in the constellation. With a synchronization error of ± 1 the beginning and end of the message can still be properly decoded, while the middle is scrambled. For a synchronization error of ± 2, the beginning, middle, and end of the message can be decoded correctly, while the intermediate sections are scrambled. Synchronization error is equal to an unexpected delay of the signal. In frequency perspective that implies:

$$x(n-k) \rightarrow X(w) \cdot \exp\left(-j \cdot \frac{w}{w_s} \cdot 2\pi \cdot k\right) \tag{4}$$

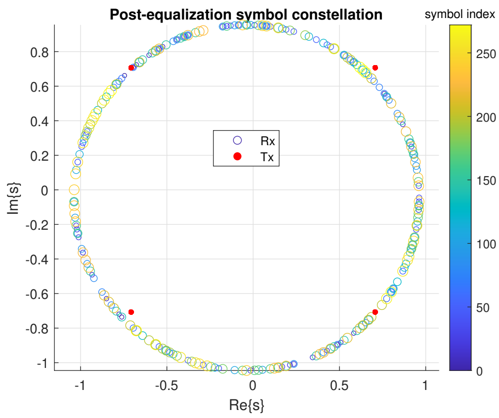
where k is the delay in time step units. As can be noticed, this delay introduced a distortion in the frequency domain, shifting the phase of X(w) to

$$-\left(\frac{w}{w_s} \cdot 2\pi \cdot k\right) \text{ rad} \tag{5}$$

When this equation is equal to zero we will have very little effect.

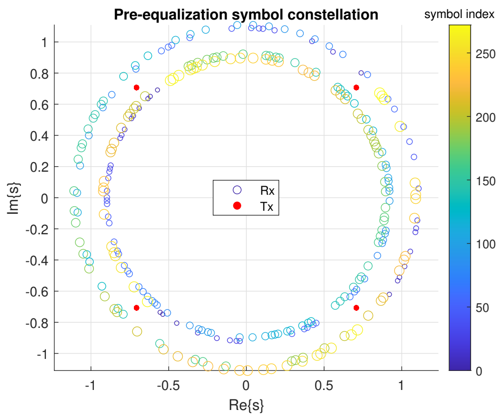


Pre-equalization.

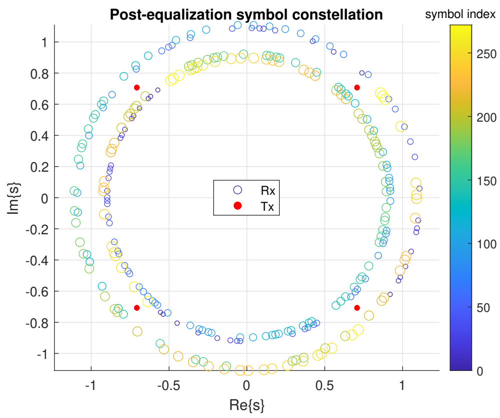


Post-equalization.

Figure 5: 1d - Pre- and post-equalization with a synchronization error of -1.



Pre-equalization.

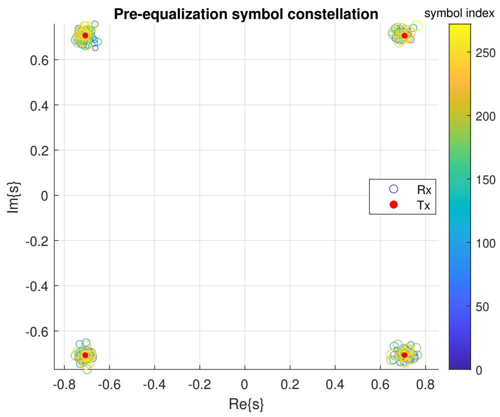


Post-equalization.

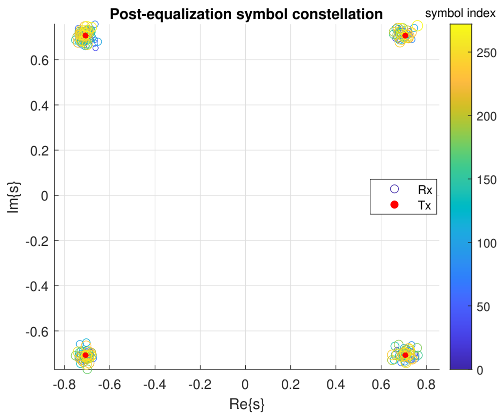
Figure 6: 1d - Pre- and post-equalization with a synchronization error of 1.

2.5 Exercise 1e

When the Signal-to-Noise Ratio (SNR) is decreased from infinity to 30 dB, the received constellations will show more clutter around the desired transmitted symbols, shown in Fig. 7. This is due to the noise floor closing in on the signal. The EVM will increase to 0.0293, but the BER will remain 0.



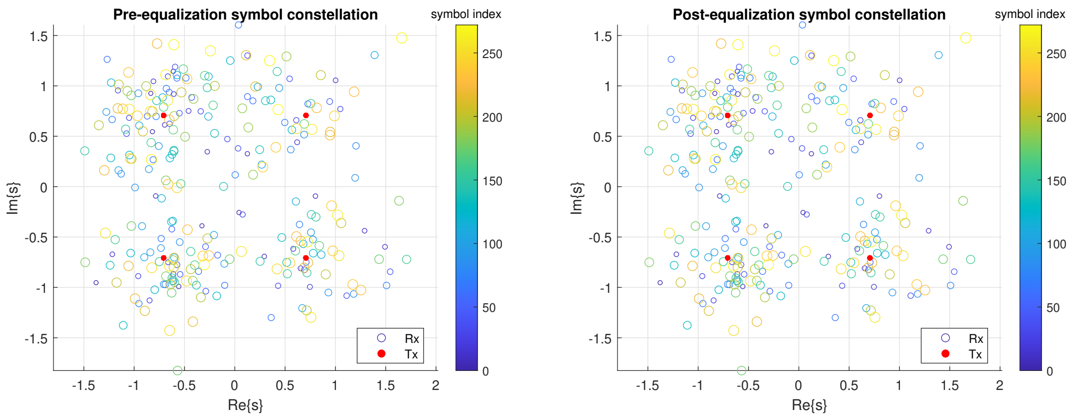
Pre-equalization.



Post-equalization.

Figure 7: 1e - Pre- and post-equalization with an SNR of 30 dB.

Further decreasing the SNR to 5 dB will show an almost unintelligible received symbols, as depicted in Fig. 8. The signal is now only 5 dB above the noise floor, and hence, the EVM increased significantly to 0.52 and the BER has increased to 0.0478. To conclude if the noise affects the received symbols and they still remain in their right quadrant, final message will not be corrupted.



Pre-equalization. Post-equalization.

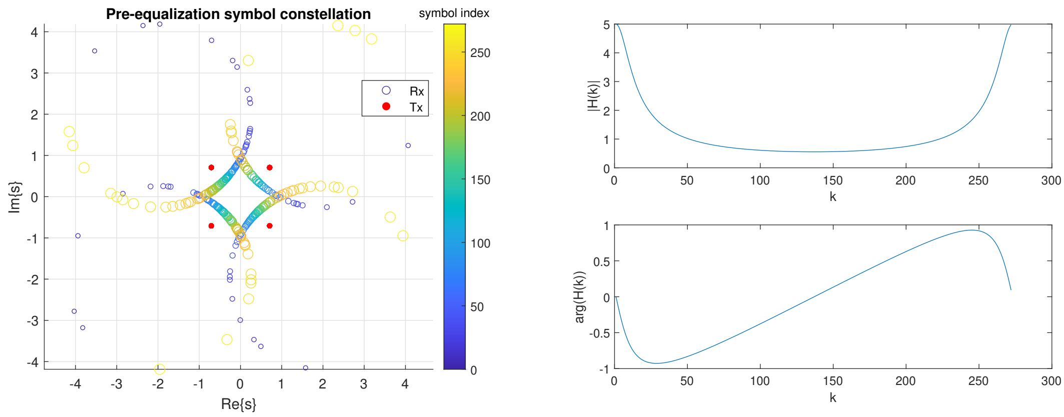
Figure 8: 1e - Pre- and post-equalization with an SNR of 5 dB.

2.6 Exercise 2a

Now, it is given that

$$r(k) = H(k)z(k), \tag{6}$$

and the impulse response has a low-pass behaviour, as presented in Fig. 9. The impulse response has few high magnitude points, and lots of small magnitude points, which can be seen accordingly in the constellation diagram. Furthermore, the phase offset is minimum for the beginning, middle, and end of the impulse response, which can be confirmed with how the constellation looks like at those samples. When $H(1) = 5$, the first constellation symbol will be found at $5 \cdot \sqrt{\frac{1}{2}}(\pm 1 \pm j)$.

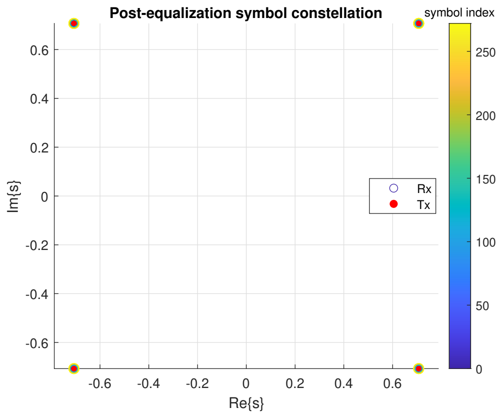


Pre-equalization. Filter response of h_4.

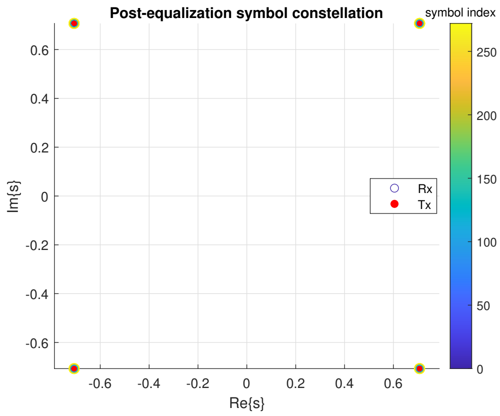
Figure 9: 2a - Pre-equalization and filter response.

2.7 Exercise 2b

The BER of the setup from Exercise 2a is 0, and the EVM is $8.7007 \cdot 10^{-14}$. When the cyclic prefix length is increased to e.g. 90, the EVM and BER do not change at all. However, when the cyclic prefix length is reduced to e.g. 30, the EVM will rather quickly increase, yielding an EVM of $6.4485 \cdot 10^{-5}$. In both cases, the BER is still 0, as shown in Fig. 10.
The magic number that makes EVM almost 0 is any number greater than or equal to 60 which is the length of the impulse filter. The value of BER doesn't change with change in length of cyclic prefix.



Post-equalization with $N_{cp} = 30$.

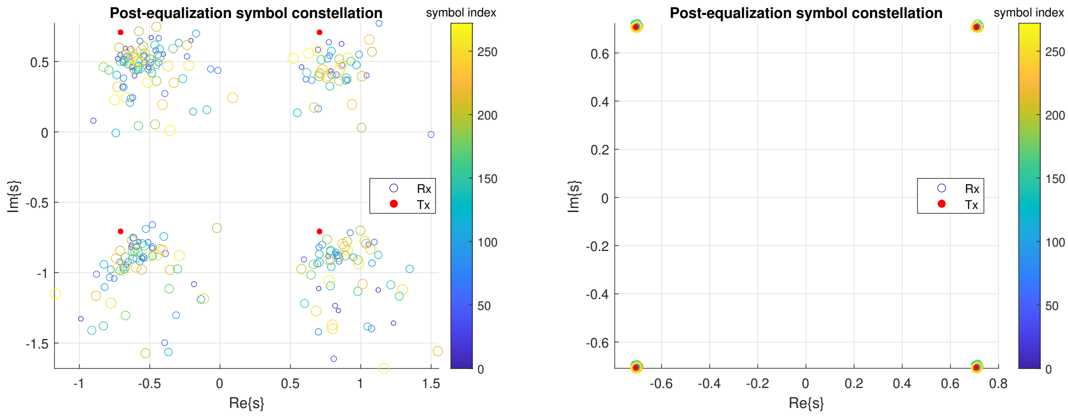


Post-equalization with $N_{cp} = 90$.

Figure 10: 2b - Post-equalization for different N_{cp}.

2.8 Exercise 2c

With a modified low-pass filter as impulse response, it is much more crucial to properly choose the cyclic prefix length, since a decreased cyclic prefix length will result in an increasing BER. When $N_{cp} \leq 29$ it will give BER > 0.0051. Moreover, when $N_{cp} \leq 58$ the noise will quickly increase, presented in Fig. 11.



Post-equalization with $N_{cp} = 29$. Post-equalization with $N_{cp} = 58$.

Figure 11: 2c - Post-equalization for different N_{cp}.

If we do not use cyclic prefix using this channel, we will get errors, because we need the cyclic prefix to make the finite transmitted signal looks like being periodic.

2.9 Exercise 3a

From now on it is assumed that the channel is unknown, and thus the channel gain has to be calculated as

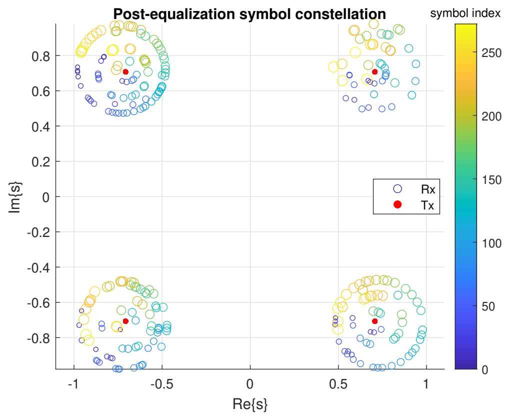
$$H(k) = \frac{r_p(k)}{s_p(k)}, \tag{7}$$

where $r_p(k)$ is the DFT of the pilot symbols and $s_p(k)$ are the input pilot symbols. Afterwards, the equalization is performed as

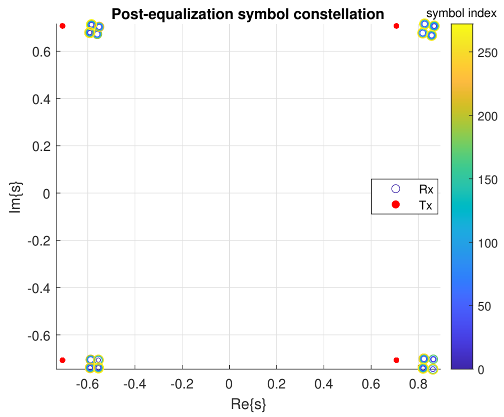
$$r_{eq}(k) = \frac{r_d(k)}{H(k)}, \tag{8}$$

where $r_d(k)$ is the DFT of the data symbols. The constellation diagrams for synchronization errors of ± 1 are given in Fig. 12.

In this case, since the channel is unknown, on the receiver side, we need to perform synchronization to account for any changes in the phase and angle of the incoming signal. Therefore, the pre-equalization figure in Fig. 12 shows some noise/distortion while the post-equalization figure is relatively better. However, in the previous scenario, when the channel is known, we did not have to perform synchronization. So both the pre- and post-equalization figures in Fig. 1 are the same and are much better than when the channel is unknown.



Post-equalization with a synchronization error of -1.



Post-equalization with a synchronization error of 1.

Figure 12: 3a - Post-equalization for different synchronization errors.

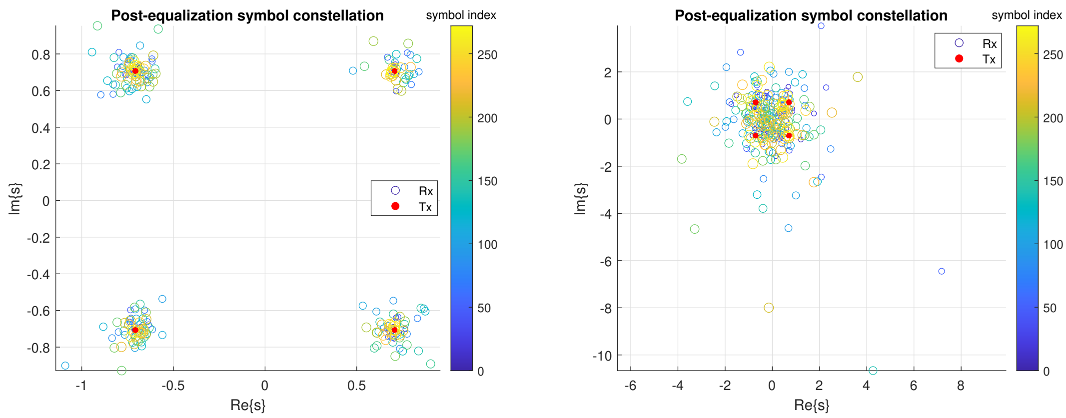
2.10 Exercise 3b

With the low-pass filter implementation, the cyclic prefix length will have to be be equal or greater than 60, which is the impulse response length. For $N_{cp} = 60$ the EVM is $1.1106 \cdot 10^{-13}$ and the BER is 0.

As observed, the minimum length of cyclic response (length of $h_4(t)$) remains the same as observed before (length of $h_1(t)$). This does not change in case of the low-pass filter impulse response ($h_4(t)$) because it also has a length of 60. The cyclic prefix only depends on the length of the impulse response.

When the SNR is decreased from ∞ dB to e.g. 30 dB, it can be seen that the EVM will get slightly worse due to the signal being closer to the noise floor. Decreasing the SNR further to 5 dB will result in a very noisy constellation, as depicted in Fig. 13, having an EVM of 1.77 due to very far outliers, as well as a BER of 0.244.

This setup is more sensitive to noise because as the channel is unknown, there can be changes in the channel during transmission which cannot be predicted. These changes can add on to the interference, noise and distortions. However, if the channel is known, it is easy to identify changes in the channel and fix them on the receiver side.



Post-equalization with an SNR of 30 dB.

Post-equalization with an SNR of 5 dB.

Figure 13: 3b - Post-equalization for different SNR values.

2.11 Exercise 3c

When a multi-path channel is used, given by

$$h_5 = [0.5, 0, 0, 0, 0, 0, 0, 0, 0.5, 0, ...], \tag{9}$$

the cyclic prefix length has to be set to $N_{cp} = 8$ to maintain periodicity for the given impulse response. Hence, the value 0.5 then repeats itself, alluding to a periodic response. When $N_{cp} \geq 8$, the EVM is 0.9129 and the BER is 0.0092, as presented in Fig. 14.

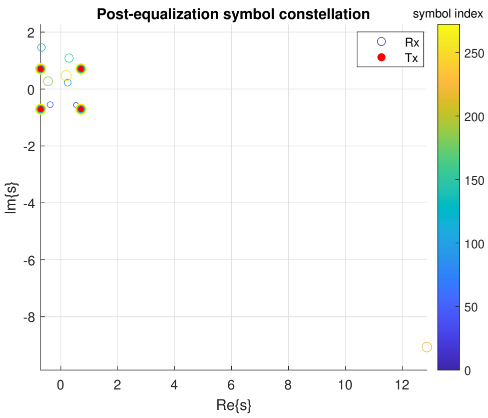
In this scenario, the channel is unknown. This presents a variety of uncertainties which could cause bit errors. It could be caused because of quantization errors while converting analog to discrete signals, error in timing of the signal despite synchronization or some minor ISI.

With an alternative multi-path channel, defined as

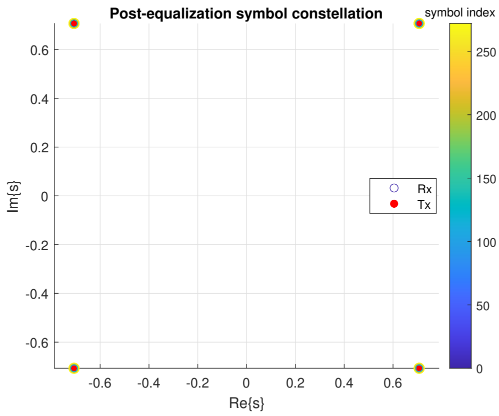
$$h'_5 = [0.5, 0, 0, 0, 0, 0, 0, 0, 0.4, 0, ...], \tag{10}$$

the EVM is $1.3151 \cdot 10^{-13}$ and the BER is 0.

The value of $|H_5|$ is 1 at time 0 and it reaches 1 at constant intervals of time. However, the value $|H'_5|$ at 0 is less than 1 and it is never equal to 1, always less than 1.



Post-equalization.



Post-equalization with alternative filter.

Figure 14: 3c - Post-equalization for different filters with $N_{cp} = 8$.