Project report
Group 7

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January 12, 2024

1 Hard vs. Soft Receiver

1.1

Task I aims to evaluate the performance of the a system which has encoder ε_2 along with QPSK for both hard and soft receiver. This can be observed in the figure attached below:

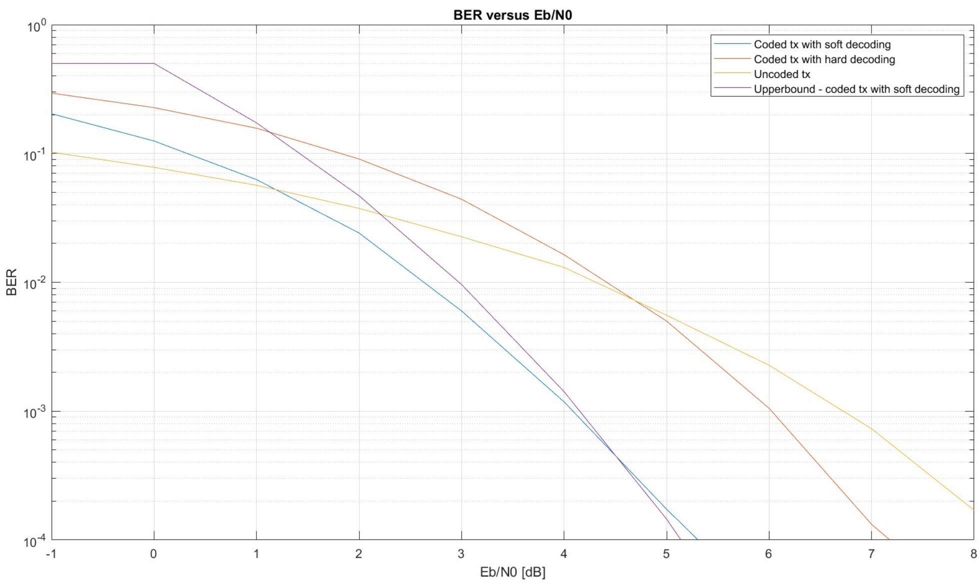

Figure 1: BER vs Eb/N0 for E2 with QPSK using hard and soft decoder

In order to calculate the coding gain for both hard and soft receiver, we just calculate the difference, (in dB), between the uncoded transmission with coded hard-transmission and coded soft-transmission.

For asymptotic gain however, we use the formula:

$$G_\infty = 10\log_{10}(R_c d_{min})$$

From the given data, we know that the Rate(R_c is 1/2 and minimum hamming distance (d_{min} can be calculated. We calculate the asymptotic gain only for the hard receiver. Lastly, to calculate the $\frac{Eb}{N0}_{min}$ we can just note where the plots for the coded (hard/soft) coincide with the plot for uncoded transmission. We can also use the formula -

$$\frac{Eb}{N0}_{min} = \frac{2^R - 1}{R}$$

The table below is a compilation of all our results -

Receiver	**Coding gain**	**Asymptotic gain**	**Min.** E_b/N_0
Hard	1.145	-	4.601
Soft	3.15	3.8278	0.8284

From the above table, we can see that there is almost a 2dB difference in the coding gains between hard and soft receivers. Despite having better performance, (as seen in the table and plot), the downside is that soft decoding is hard to implement in most cases.

2 Encoder Comparison

The performance of the three encoders ε_1,ε_2,ε_3 for X_{QPSK} with the Gray mapping is compared in this section. Under low E_b/N_0 conditions, the performance of the uncoded system surpasses all coded systems, aligning with expectations. The initial assumption of the $Viterbi$ algorithm suggests a low probability of bit errors.

Consequently, in a highly noisy environment (low signal-to-noise ratio), where the bit error probability is already substantial, the $Viterbi$ algorithm struggles to operate effectively. Despite this, in terms of bit error rate (BER), all three encoders outperform the uncoded system. Among them, ε_2 exhibits the poorest performance due to its minimal coding gain and higher complexity. It's performance is inferior to ε_3, attributed to the shorter distance between codewords corresponding to adjacent states in the trellis structure.

The optimal choice among the three encoders depends on the acceptable or required complexity. At high $Eb/N0$, ε_3 demonstrates a better BER compared to ε_1, although ε_1 has lower complexity than all other encoders. Thus, a trade-off between complexity and BER needs consideration before implementing either ε_1 or ε_3 in practical applications.

Encoder	**Coding gain**	**Asymptotic gain**
ε_1	3.3	3.6
ε_2	3.2	3.6
ε_2	4.1	4.4

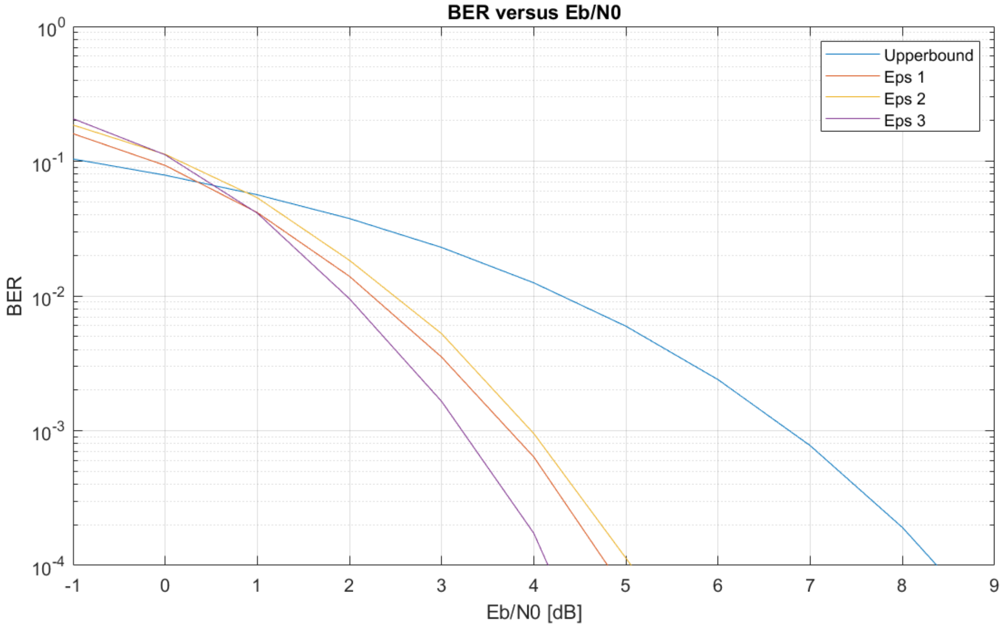

Figure 2: BER vs Eb/N0 for the three given encoders

3 Coding can Increase Efficiency

System	Spectral efficiency	Power efficiency
$1 : \varepsilon_3$ and χ_{BPSK}	0.5	4.1
$2 : \varepsilon_3$ and χ_{QPSK}	1	4
$3 : \varepsilon_4$ and χ_{AMPM}	2	6.1

No system is strictly superior to another. A lower coding rate increases redundancy, enhancing coding strength but also decreasing bandwidth efficiency. Further analysis reveals that the modulation scheme influences system complexity. $BPSK$, $QPSK$, and $AMPM$ modulation require 2, 4, and 8 constellation points respectively, leading to increased complexity with the transition from $BPSK$ to $AMPM$. Thus, a trade-off exists among complexity, spectral efficiency, and bit error rate (BER). For instance, while the BER of coded and uncoded $BPSK$ and $QPSK$ performs similarly, $BPSK$ exhibits lower complexity compared to $QPSK$.

In practical applications aiming for low cost and relatively low BER, System 1 might be preferred over System 2 due to its lower complexity and nearly equivalent performance. Additionally, if implementing a practical application with limited bandwidth, System 3 could be chosen despite its higher complexity and higher BER compared to other systems. Therefore, the purpose of the practical application needs analysis to determine if the BER is sufficiently low for the chosen field, especially for safety applications where a low BER is crucial, requiring consideration of alternative systems.

The uncoded system performs poorly, operating far from capacity, requiring a significantly higher E_b/N_0 than the theoretical limit. However, all three systems gradually approach capacity when encoded. System 2 is closest to the capacity curve, In all cases, encoding brings the curves closer to capacity compared to uncoded systems. This aligns with Shannon's theory, emphasizing the necessity of coding to achieve capacity.

In the uncoded scenario, Systems 1 and 2 perform similarly. However, in the coded case, System 2 demonstrates double the spectral efficiency of System 1, as it sends two bits per symbol using $QPSK$, while System 1 maps 1 bit per symbol with $BPSK$. System 2 also exhibits higher coding gain and requires less power for the same BER, making it more power-efficient. Overall, coding reduces bit error rate and enhances power efficiency. While all these systems get closer to capacity when encoded, there is a trade-off with spectral efficiency.

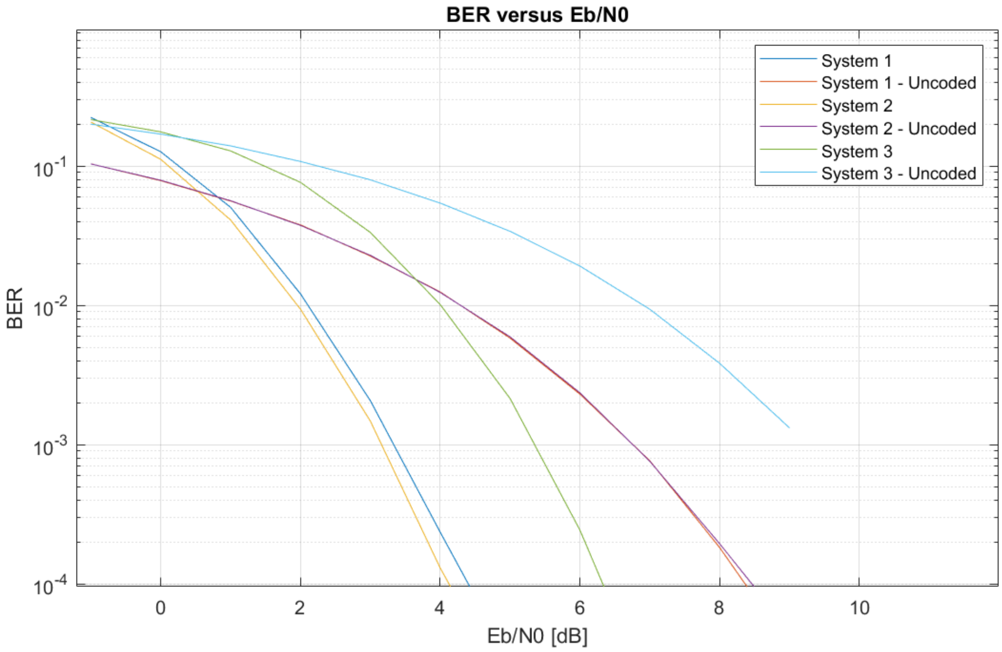

Figure 3: BER vs Eb/N0 for the three given systems