Multi-carrier Communications

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Outline

Multicarrier modulation

- Data transmission using multiple carriers
- Orthogonal Frequency Division Multiplexing (OFDM)
  - Multicarrier modulation with overlapping subchannels
  - Discrete implementation of multicarrier modulation
  - Matrix Representation of OFDM
  - Parameterisation of OFDM
- Exploitation of multi-carrier modulation
- Hardware impairments
- DFT-precoded OFDMA

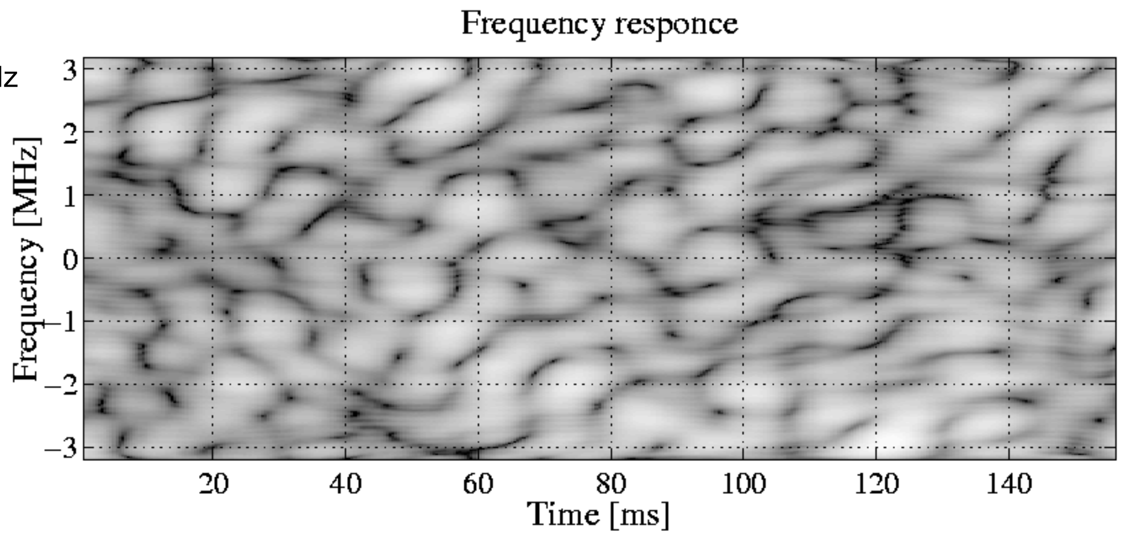
Recall: Frequency-Selective Channels

- Channels for which $T_b = 1/R_b$ is comparable with the root mean square (RMS) delay spread are said to be time dispersive or frequency-selective.
- Using binary modulation over such channels will introduce significant intersymbol interference (ISI).
- We can then
  - Use an equalizer to mitigate the ISI (actually this exploits the frequency diversity in the channel).
  - Use higher-order modulation to lengthen the pulses to $T_s = k\,T_b$, where k is the number of bits per symbol.
  - Use *multicarrier modulation* techniques to enable parallel transmission of narrowband, flat fading, long duration signals with relatively low-complexity equalization.

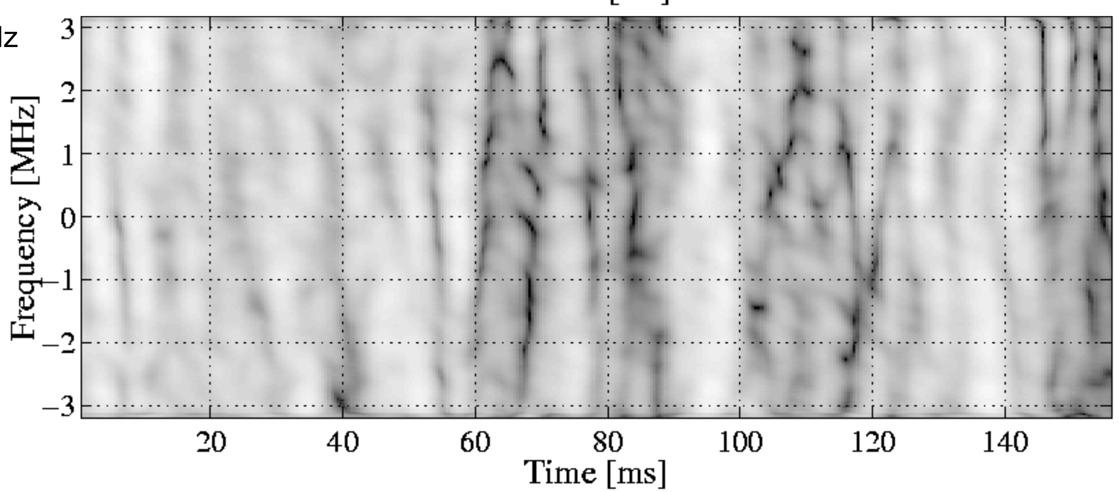
Typical Channel Behaviour at 2.4 GHz Carrier Frequency

(5 MHz channel bandwidth and user speed ~50 km/h)

Coherence bandwidth 0.6 MHz

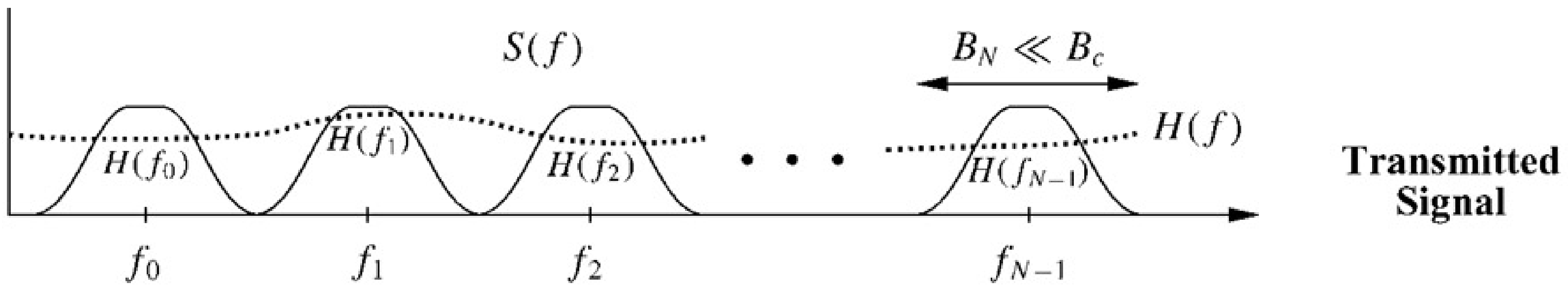


Coherence bandwidth 4.9 MHz



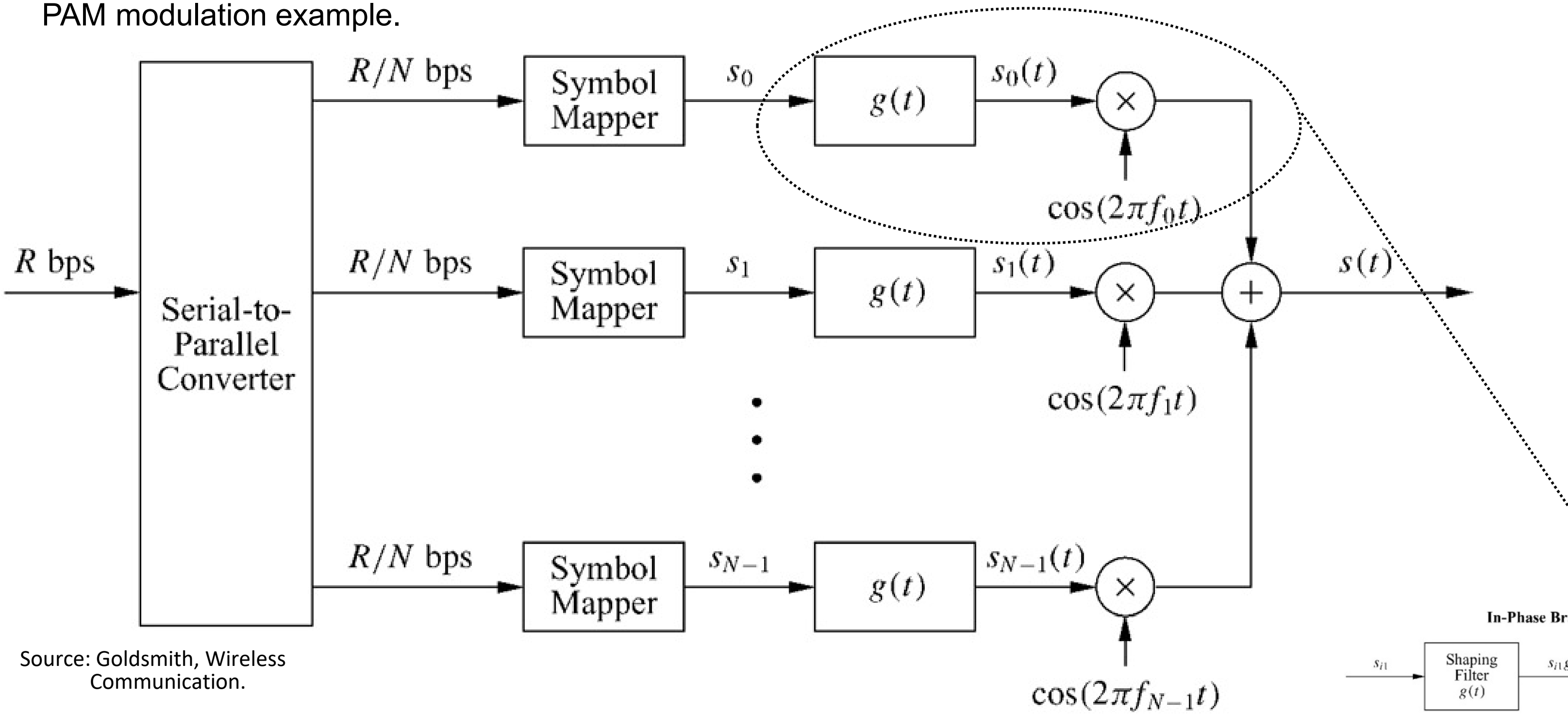
Why Multicarrier Modulation?

- Break up a large system bandwidth *B* into *N* subchannels each with bandwidth B_N:
  - $B_N = B/N << B_c$
  - $T_N \approx 1/B_N >> \tau_{\max}$
  - Almost flat fading on each subchannel
  - Almost no inter-symbol-interference (ISI)
  - Easy 1-tap frequency domain equalization on each subchannel



Multi-carrier Transmitter

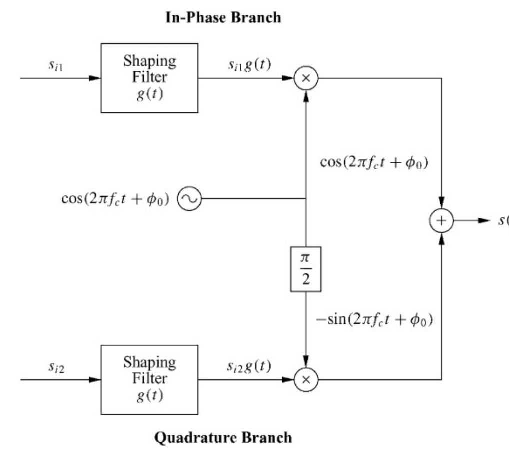
PAM modulation example.



Source: Goldsmith, Wireless Communication.

$$s(t) = \sum_{i=0}^{N-1} s_i g(t) \cos\left(2\pi f_i t + \varphi_i\right),$$

or more general QAM on each subchannel:

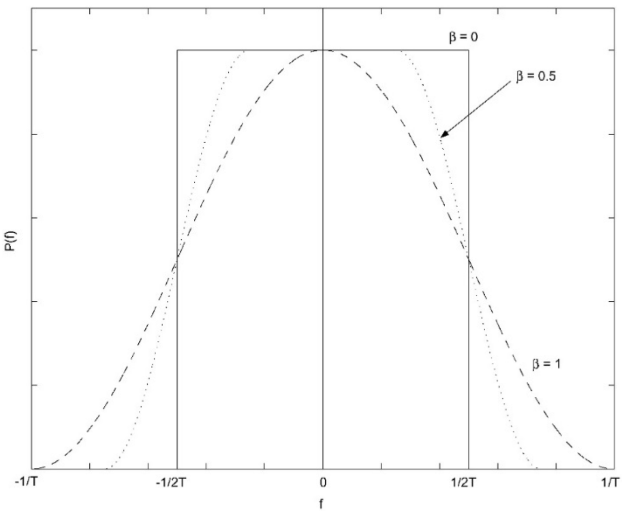


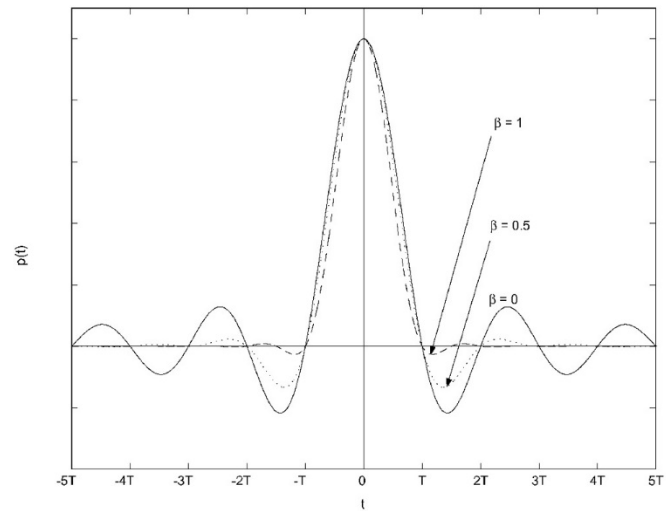
High complexity/cost:

- *N* transmit filters
- *N* modulators

Penalty with Non-overlapping Subchannels

- Pulse shaping loss with realistic pulses (e.g. raised cosine):



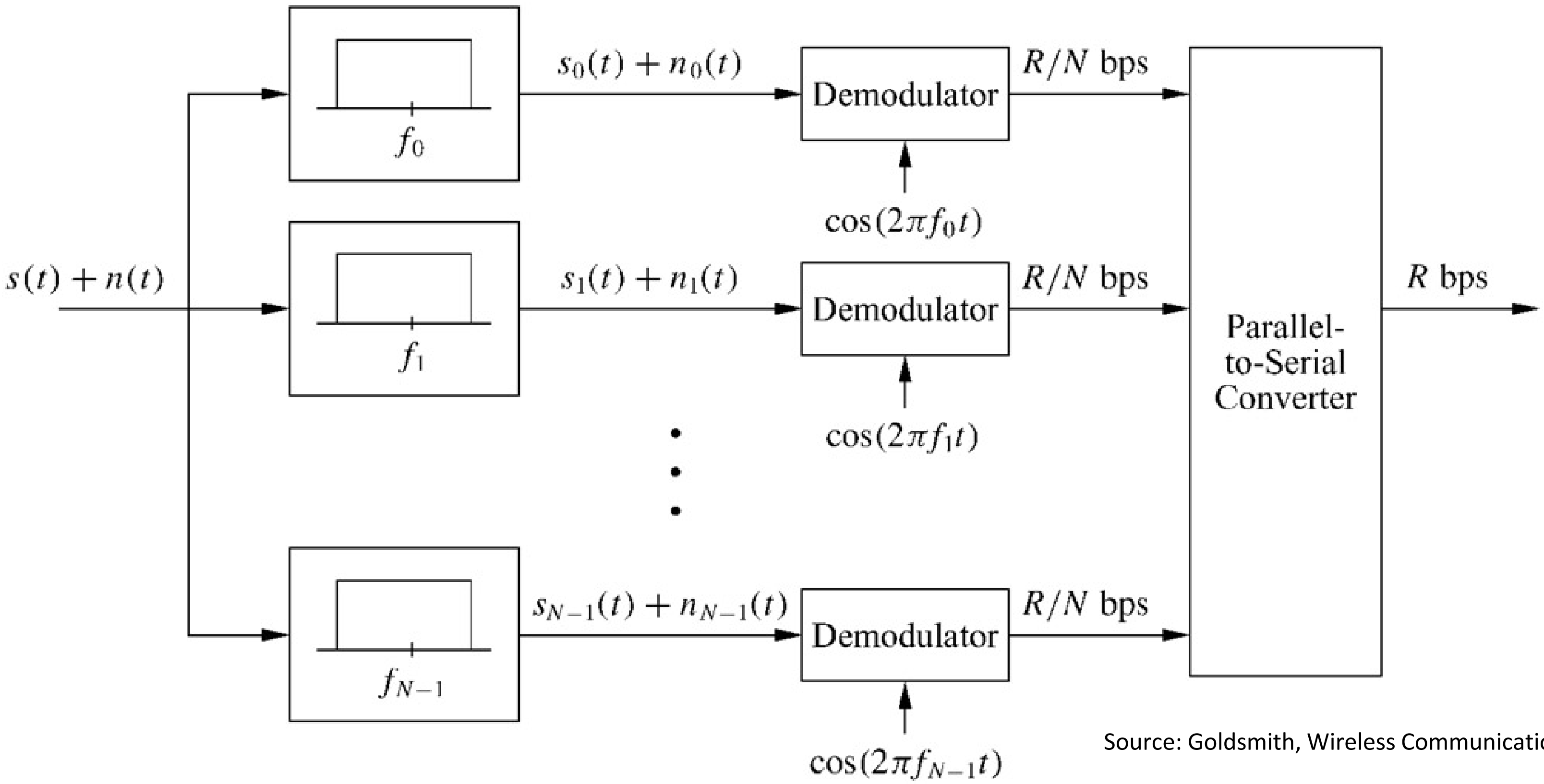


$B_N = (1+\beta)/T_N$ => system bandwidth $B = NB_N = N(1+\beta)/T_N$

- To compensate for $\beta > 1$, either
  - Increase system bandwidth B (if possible),
  - Lower number of subchannels N, or
  - Increase symbol time T_N

In all cases the spectral efficiency is decreased by the factor $(1+\beta)$, or more generally $(1+\beta+\varepsilon)$ considering practical time-limited impulse responses.

Multicarrier Receiver in White Noise



Source: Goldsmith, Wireless Communication.

High complexity/cost:

- N receiver filters
- N demodulators

OFDM

Orthogonal Frequency Division Multiplexing

Multicarrier with Overlapping Subcarriers

Use the fact that two subcarriers separated with a multiple of $1/T_N$ are orthogonal over a finite period T_N:

$$\frac{1}{T_N}\int_0^{T_N}\cos\left(2\pi\left(f_0+\frac{i}{T_N}\right)t+\varphi_i\right)\cos\left(2\pi\left(f_0+\frac{j}{T_N}\right)t+\varphi_j\right)dt$$

$$=\frac{1}{T_N}\int_0^{T_N}.5\cos\left(2\pi\frac{(i-j)t}{T_N}+\varphi_i-\varphi_j\right)dt$$

$$+\frac{1}{T_N}\int_0^{T_N}.5\cos\left(2\pi\left(2f_0+\frac{i+j}{T_N}\right)t+\varphi_i+\varphi_j\right)dt$$

Narrowband assumption

$$\approx\frac{1}{T_N}\int_0^{T_N}.5\cos\left(2\pi\frac{(i-j)t}{T_N}+\varphi_i-\varphi_j\right)dt$$

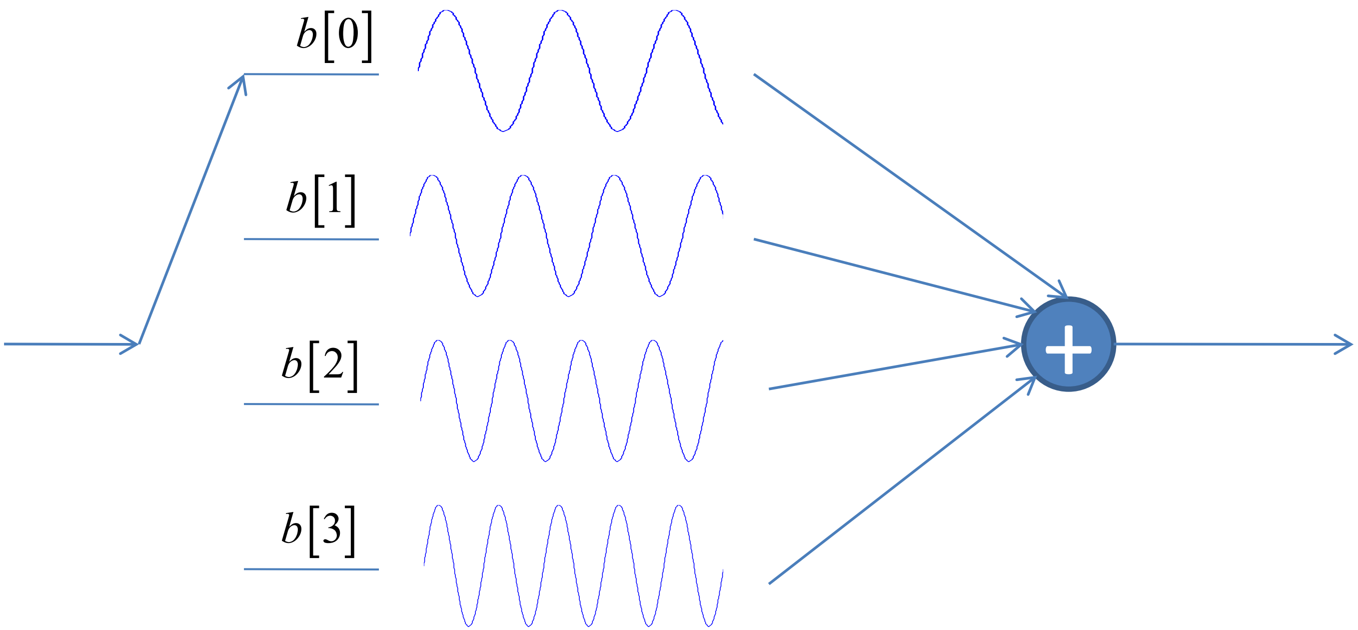
$$=.5\delta(i-j),$$

Subcarriers are orthogonal also on the receiver side:

- Provided correct frequency offset is preserved

Orthogonal Frequency-Division Multiplexing (OFDM)

Uses a large number of minimum spaced orthogonal *sub-carriers* to carry the information.

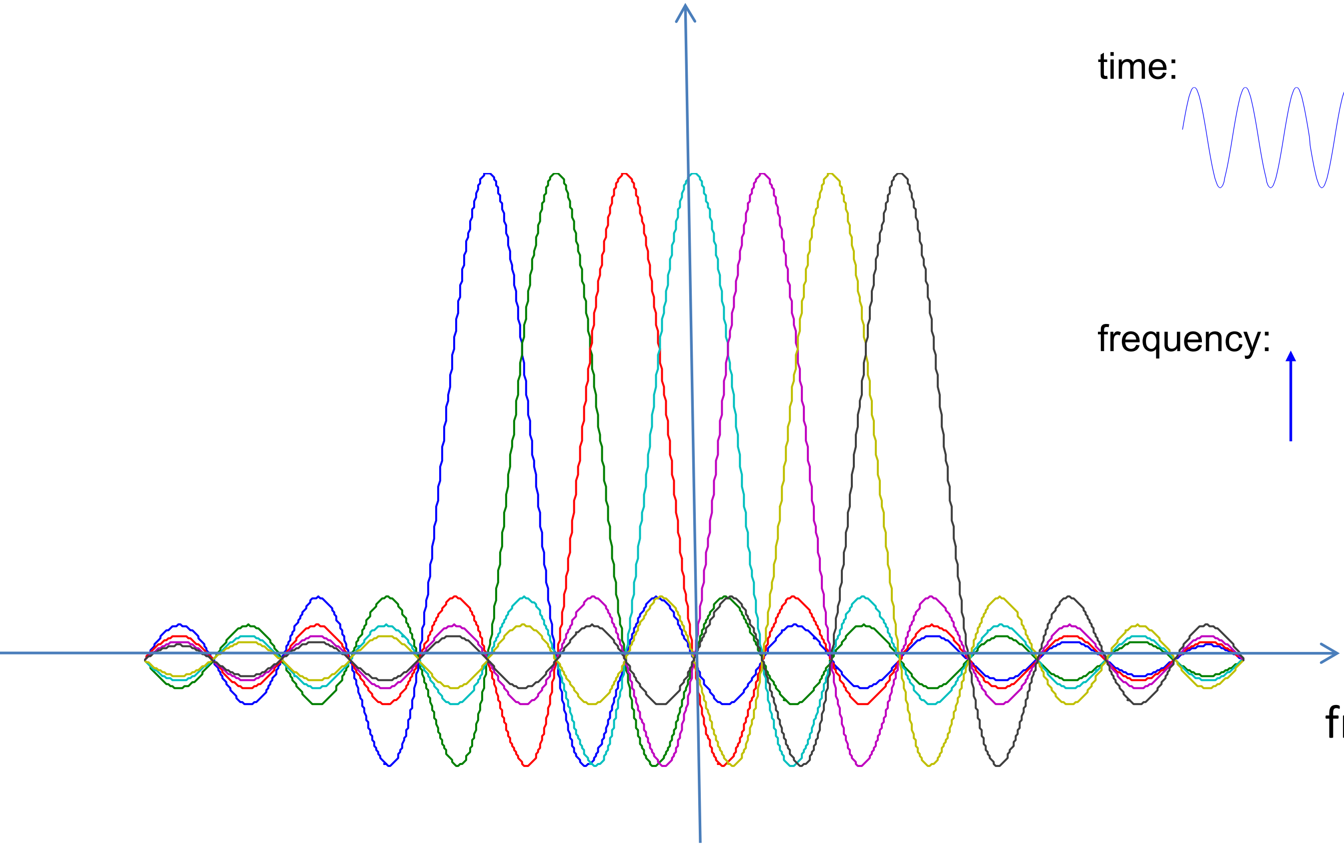


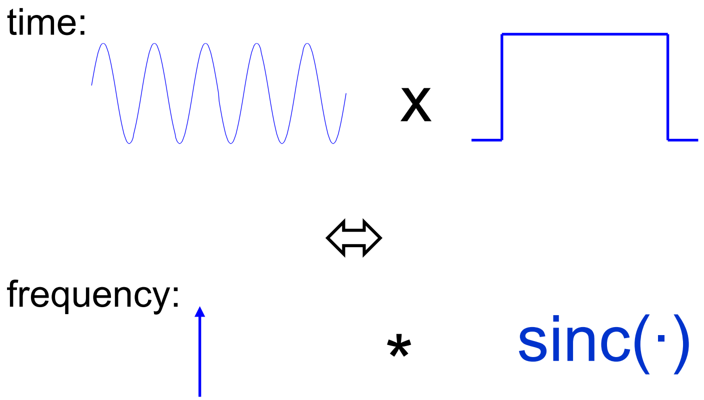
Baseband:

$$\sum_{k=0}^{N-1} b[k]e^{j2\pi f_k t}$$

OFDM in Frequency Domain

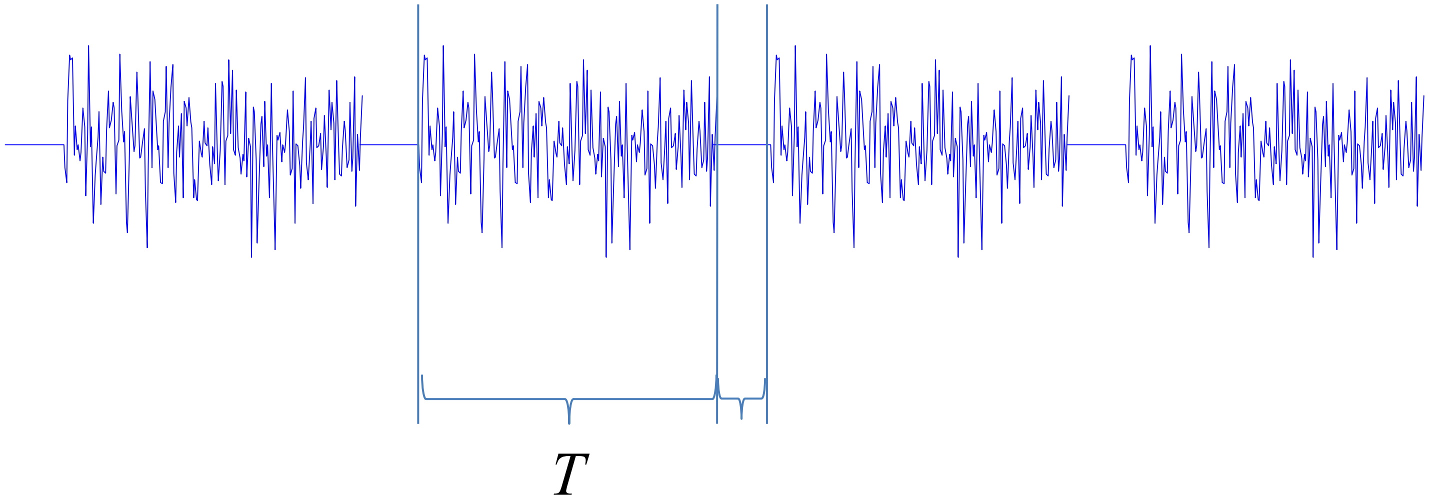
Due to the block-based transmission, no distinct subcarriers are used in OFDM. The carriers are partly overlapping in frequency, but overall they are ortogonal.





Guard Time

By inserting a guard time (at least as long as the delay spread) after each OFDM symbol of length *T*, the intersymbol interference (ISI) is almost completely eliminated.

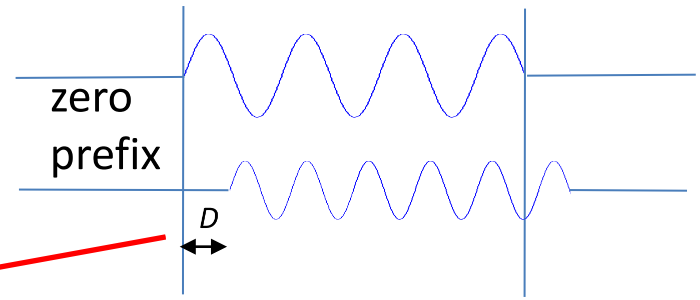


The guard time should be choosen at least as long as the delay spread. Then, the overall OFDM symbol length *T* should be choosen so that the guard time do not dominate (maybe ~10 times the guard time).

Intercarrier Interference

The guard time protects against ISI, but we still have inter-carrier-interference (ICI) due to the finite length subcarriers and the delay spread.

Let us study an example where we add two carriers of different frequencies, whereof one is slightly delayed.

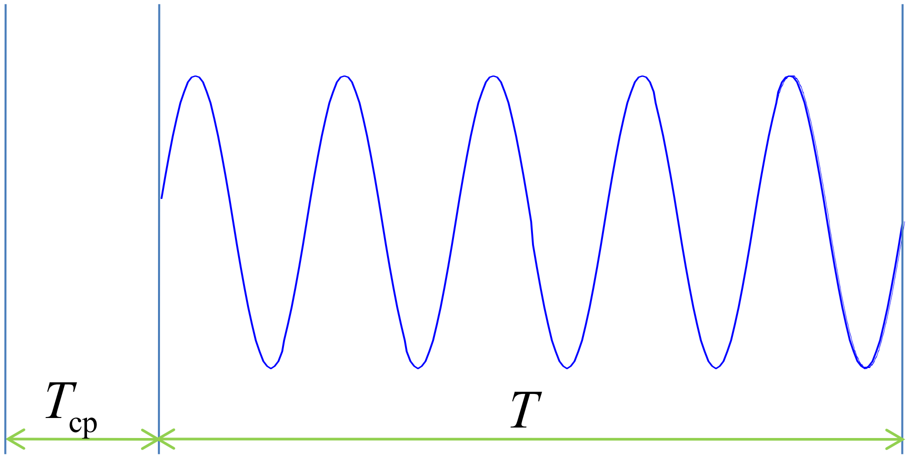


$$\int_D^T e^{j2\pi\frac{k}{T}t}\, a e^{-j2\pi\frac{m}{T}(t-D)}\, dt = a e^{j2\pi\frac{m}{T}D}\int_D^T e^{j2\pi\frac{k-m}{T}t}\, dt = a e^{j2\pi\frac{m}{T}D}\left[\frac{e^{j2\pi\frac{k-m}{T}t}}{j2\pi\frac{k-m}{T}}\right]_D^T$$

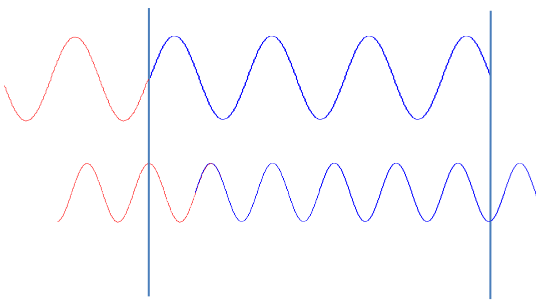
Typically non-zero also when $k \neq m$.

We get some intercarrier interference, particularly from neighbour carriers (k-m is small).

Cyclic Prefix



We add a cyclic prefix to each subcarrier



$$\int_0^T e^{j2\pi\frac{k}{T}t} a e^{-j2\pi\frac{m}{T}(t-D)} dt = a e^{j2\pi\frac{m}{T}D} \int_0^T e^{j2\pi\frac{k-m}{T}t} dt = a e^{j2\pi\frac{m}{T}D} \left[\frac{e^{j2\pi\frac{k-m}{T}t}}{j2\pi\frac{k-m}{T}} \right]_0^T = 0$$

Regardless of the delay *D*, orthogonality is preserved

$$\int_0^T e^{j2\pi\frac{k}{T}t} a e^{-j2\pi\frac{k}{T}(t-D)} dt = a e^{j2\pi\frac{k}{T}D} \int_0^T \underbrace{e^{j2\pi\frac{k-k}{T}}}_{1} dt = aTe^{j2\pi\frac{k}{T}D}$$

By adding a cyclic prefix to the original signal, we get rid of both ICI and ISI.
The cost is both a lower spectral efficiency and a power loss,
since the cyclic prefix cannot carry any information.

Discrete Implementation of OFDM

Discrete time signal $x[n]$:

$$x[n] = \sum_{k=0}^{N-1} b[k] e^{j2\pi \frac{kn}{N}}, \quad 0 \le n \le N-1.$$

- Discrete carrier frequencies $f_k = k/N$, which after D/A conversion and upconvertion by a carrier becomes the subchannel center frequencies.
- The $\{b[k]\}$ sequence is the output from a QAM symbol mapper.
- With N being a power of 2, the Inverse Fast Fourier Transform (IFFT) can be used at the transmitter.

OFDM Reception (Flat Channel)

Using:

$$x[n] = \sum_{k=0}^{N-1} b[k] e^{j2\pi \frac{kn}{N}}$$

$$\begin{aligned}
\tilde{b}[l] &= \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-j2\pi \frac{ln}{N}} \\
&= \frac{1}{N} \sum_{n=0}^{N-1} \sum_{k=0}^{N-1} b[k] e^{j2\pi \frac{kn}{N}} e^{-j2\pi \frac{ln}{N}} \\
&= \frac{1}{N} \sum_{n=0}^{N-1} \sum_{k=0}^{N-1} b[k] e^{j2\pi \frac{(k-l)n}{N}} \\
&= \frac{1}{N} \sum_{k=0}^{N-1} b[k] \sum_{n=0}^{N-1} e^{j2\pi \frac{(k-l)n}{N}} \\
&= \frac{1}{N} \sum_{k=0}^{N-1} b[k] N \qquad \text{if } k = l, \text{ 0 otherwise} \\
&= b[l]
\end{aligned}$$

- Likewise the transmitter, with *N* being a power of 2, the Fast Fourier Transform (FFT) can be used at the receiver.

How to Recover the Data After a Frequency-Selective Channel?

We would like to treat also the channel as a discrete time FIR filter $h[0]$, $h[1]$, …$h[\mu]$.

$$y[n] = h[n] * x[n] = x[n] * h[n] = \sum_k h[k]\, x[n-k].$$

With the cyclic prefix:

$$\tilde{x}[n] = \overbrace{\boxed{x[N-\mu], x[N-\mu+1], \ldots, x[N-1]}}^{\text{Cyclic Prefix}} \; \overbrace{\boxed{x[0], x[1], \ldots, x[N-\mu-1]} \; \boxed{x[N-\mu], x[N-\mu+1], \ldots, x[N-1]}}^{\text{Original Length-}N\text{ Sequence}}$$

n=0

Append Last μ Symbols to Beginning

We get:

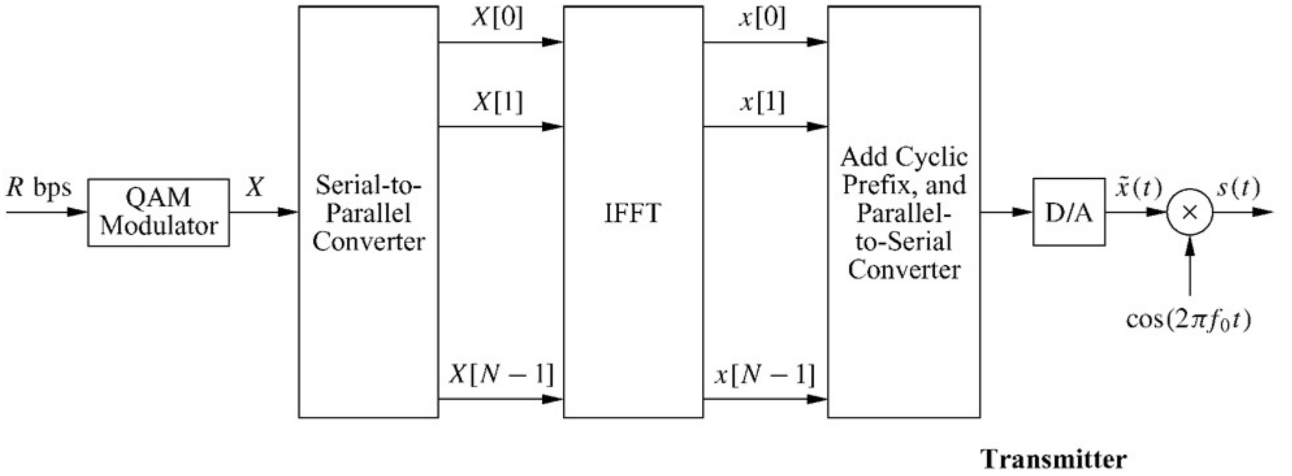
$$\begin{aligned} y[n] &= \tilde{x}[n] * h[n] \\ &= \sum_{k=0}^{\mu} h[k]\, \tilde{x}[n-k] \\ &= \sum_{k=0}^{\mu} h[k]\, x[n-k]_N \\ &= x[n] \circledast h[n], \end{aligned}$$

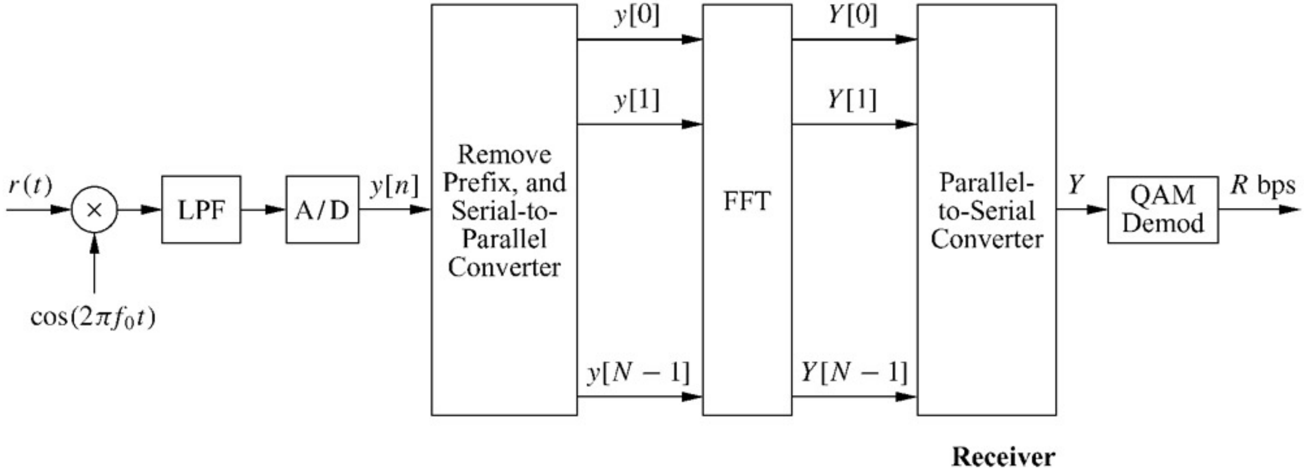
Cyclic prefix size: μ

Thus, without noise:

$$\boxed{Y[i] = \mathrm{DFT}\{y[n] = x[n] \circledast h[n]\} = X[i]\, H[i], \quad 0 \le i \le N-1,}$$

OFDM Transceiver





Matrix Representation of OFDM

Recall:

$$\tilde{x}[n] = \overbrace{x[N-\mu], x[N-\mu+1], \ldots, x[N-1]}^{\text{Cyclic Prefix}}\ \overbrace{x[0], x[1], \ldots, x[N-\mu-1]\ \ x[N-\mu], x[N-\mu+1], \ldots, x[N-1]}^{\text{Original Length-}N\text{ Sequence}}$$

Append Last μ Symbols to Beginning

and *h[n]={h[0], h[1], …h[μ]}*

=> $$y[n] = \tilde{x}[n] * h[n] = \sum_{k=0}^{\mu} h[k]\,\tilde{x}[n-k] = \sum_{k=0}^{\mu} h[k]\,x[n-k]_N = x[n] \circledast h[n],$$

In matrix notation (with noise):

$$\begin{bmatrix} y_{N-1} \\ y_{N-1} \\ \vdots \\ \vdots \\ \vdots \\ y_0 \end{bmatrix} = \begin{bmatrix} h_0 & h_1 & \cdots & h_\mu & 0 & \cdots & 0 \\ 0 & h_0 & \cdots & h_{\mu-1} & h_\mu & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & h_0 & \cdots & h_{\mu-1} & h_\mu \\ \vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ h_2 & h_3 & \cdots & h_{\mu-2} & \cdots & h_0 & h_1 \\ h_1 & h_2 & \cdots & h_{\mu-1} & \cdots & 0 & h_0 \end{bmatrix} \begin{bmatrix} x_{N-1} \\ x_{N-2} \\ \vdots \\ \vdots \\ x_0 \end{bmatrix} + \begin{bmatrix} \nu_{N-1} \\ \nu_{N-2} \\ \vdots \\ \vdots \\ \vdots \\ \nu_0 \end{bmatrix},$$

<=> $$\boxed{\mathbf{y} = \widetilde{\mathbf{H}}\mathbf{x} + \mathbf{v}}$$

Parameterization of OFDM

Semi-static situations, delay spread is the limitation:

- Start with maximum delay spread T_m
- Assign OFDM symbol duration $T_N=T_m/x_{CPoh}$, and CP=$T_m=x_{CPoh}T_N$
- Subcarrier bandwidth becomes $B_N=1/T_N$
- Number of subcarriers becomes $N=B/B_N$

Thus, the smaller required CP overhead, the larger T_N, smaller B_N and larger number of subcarriers N with given system bandwidth B.

- If N is not a power of 2, adjust upwards to closest value $N=2^k$, k integer to maintain maximum x_{CPoh}.

Mobile situations: Doppler spread, carrier frequency offsets and phase noise can be the limitation, i.e. a minimum requirement on B_N in order to have a slowly varying channel during the OFDM sybmol duration:

- $(1+x_{CPoh})T_N<<1/(f_D+\Delta f_{CFO})$ <=> $B_N=1/T_N>>(1+x_{CPoh})(f_D+\Delta f_{CFO})$

Complexity requirements (memory, FFT size) can put a limit on N, i.e. also a minimum requirement on B_N with given B.

Examples

- WLAN example IEEE 802.11a (Goldsmith ch. 12):

B=20 MHz, T_m=0,8 µs (longest path 240 m)
With N=64 (48 for data) => B_N=312.5 kHz, T_N=4 µs and x_{CPoh}=20%

- WINNER N=2048:

	Base Coverage Urban	Microcellular	Indoor
Subcarrier distance Δf	39062.5 Hz	48828.125 Hz	
Useful symbol duration T_N	25.6 µs	20.48 µs	
Guard interval T_G	3.2 µs	**2.00** µs	
Total symbol duration	28.8 µs	**22.48** µs	
used subcarriers	[-576:576] subcarrier 0 unused	[-920:920] subcarrier 0 unused	
Signal bandwidth	**2 x 45 MHz**	**89.84 MHz**	
System bandwidth	2 x 50 MHz	100.0 MHz	
FFT bandwidth, sampling rate	80.0 MHz	100.0 MHz	

Taking Advantage of the Multiple Carriers in Frequency Selectivity Channels

Per subcarrier techniques:

- Equalization (1-tap):
  - Invert the channel on the receiver side: $1/H(f_i)$
  - MMSE equalization in case of noisy channel estimates: $H(f_i)/(|H(f_i)|^2+SNR^{-1})$

- Channel inversion based precoding (tx must know the channel):
  - Invert the channel on the transmitter side: assign power $P_i = 1/|H(f_i)|^2$ on subcarrier *i*.

- Link adaptation (tx must know the channel):
  - Power loading
  - Bit loading (adaptive modulation and/or rate adaptation)

Joint subcarrier processing:

- Coding+interleaving

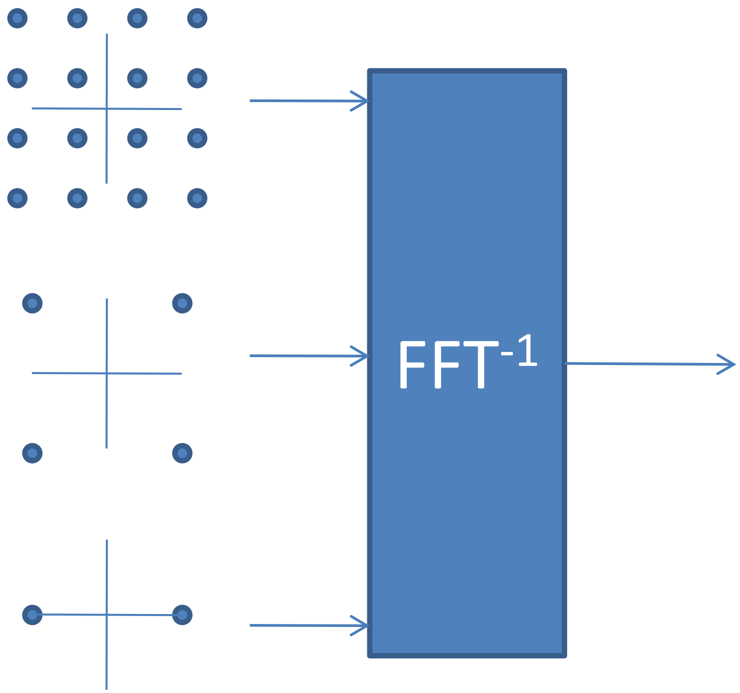
- Spreading

- DFT precoding

Multi-user resource allocation

Adaptive Modulation with OFDM

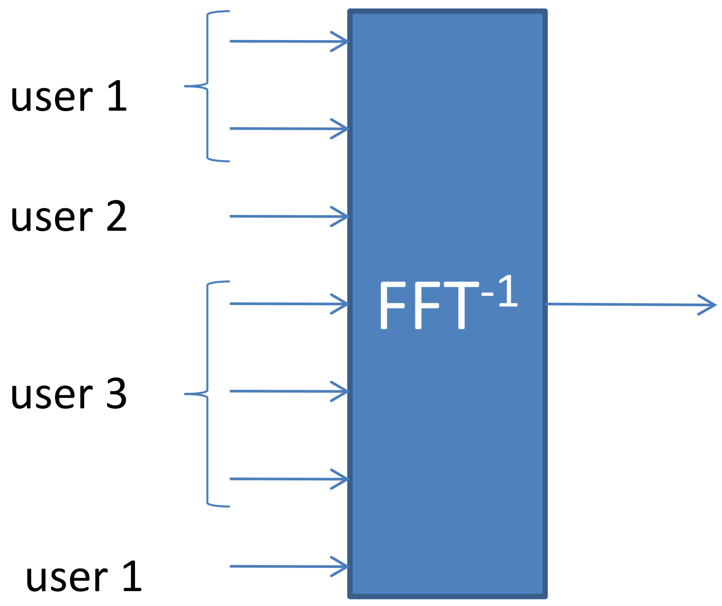
On a frequency-selective channel OFDM can be used in connection with *Adaptive Modulation*, which basically means that high-quality subcarriers can use high-rate modulation (e.g. 64-QAM or 256-QAM), while subcarriers with low quality uses low-rate modulation with better noise robustness (e.g. BPSK or QPSK).



This is a unique advantage of OFDM, with its narrow subcarrier bandwidths where frequency selectivity can be maximally exploited. Many systems exploit this property.

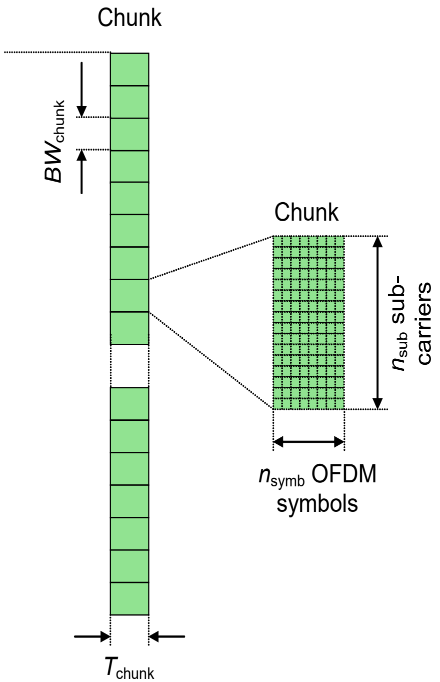
OFDMA

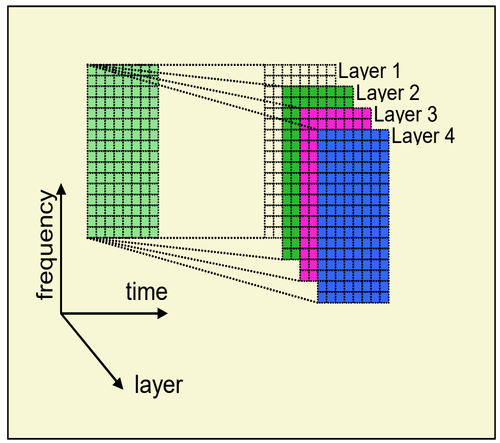
Orthogonal Frequency Domain Multiple Access (OFDMA) is a multiple-access technique, where the subcarriers of an OFDM system is subdivided among several users.



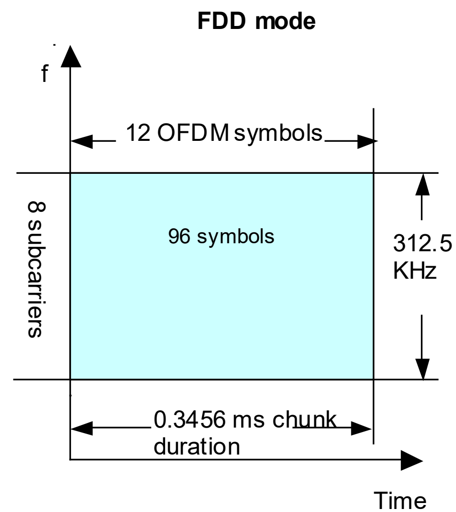
Based on the channel conditions, adaptive user-to-subcarrier assignment can be achieved, leading to **a large improvement in total spectral efficiency of the system.**

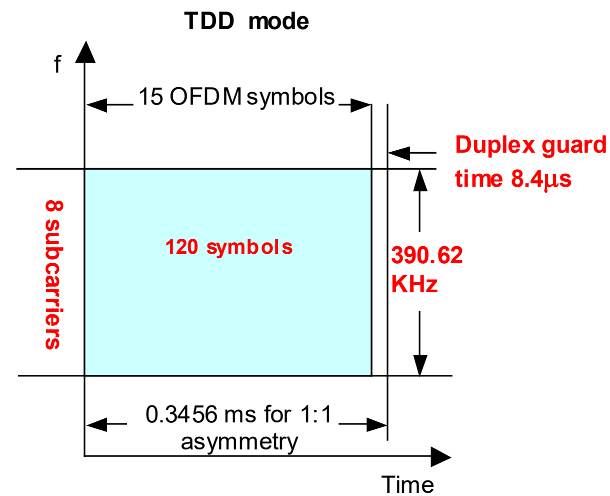
Time-Frequency-Space Partitioning





The channel is essentially flat within a resource blocks (chunk).

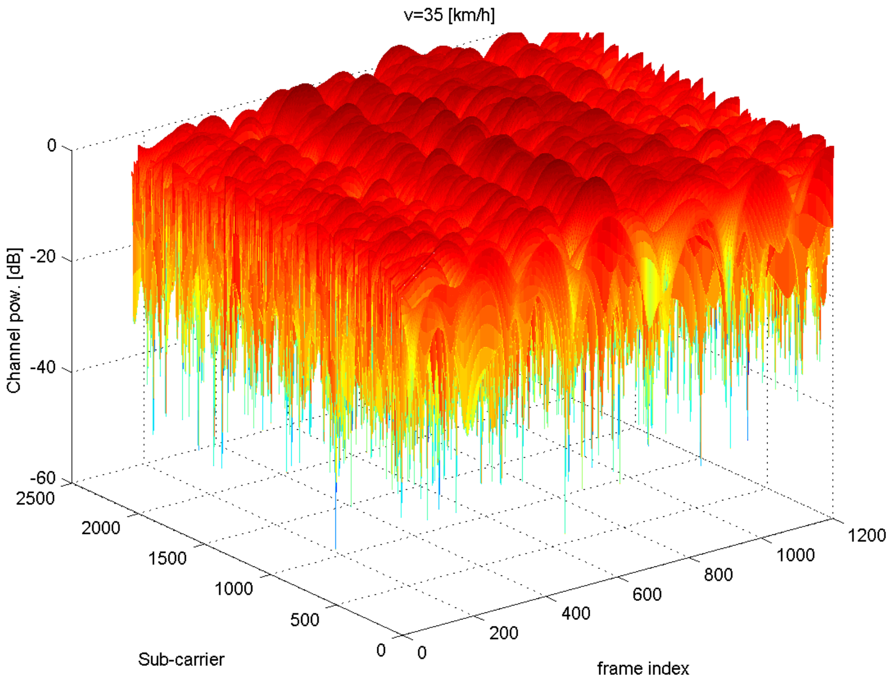


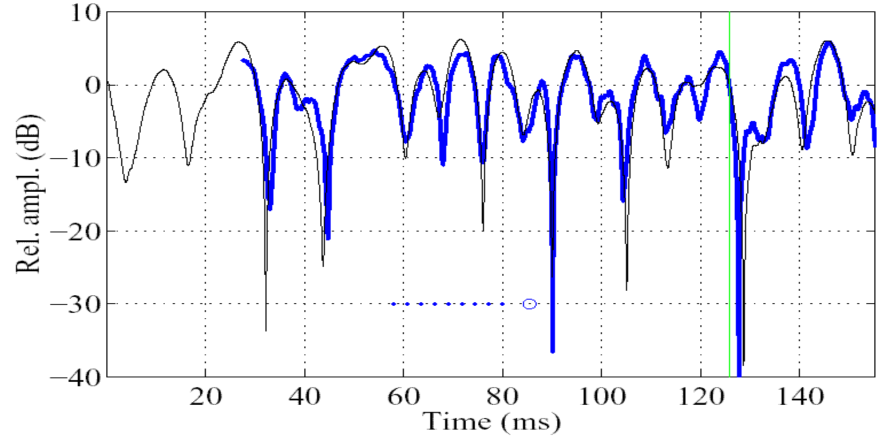


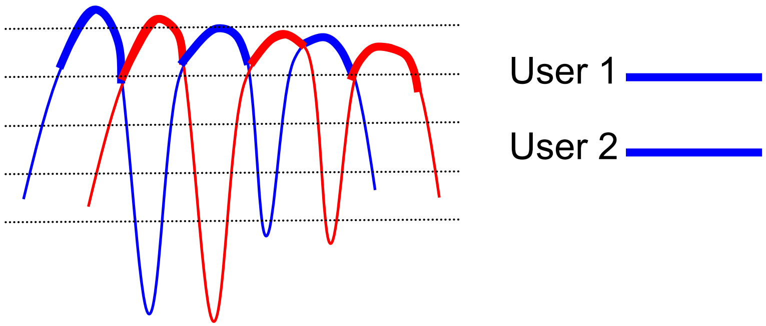
a) Physical channel structure and resource blocks (chunks)
b) Chunk layers obtained by spatial re-use.

Multi-user Scheduling and Link Adaptation Gains

A *time-frequency* selective channel

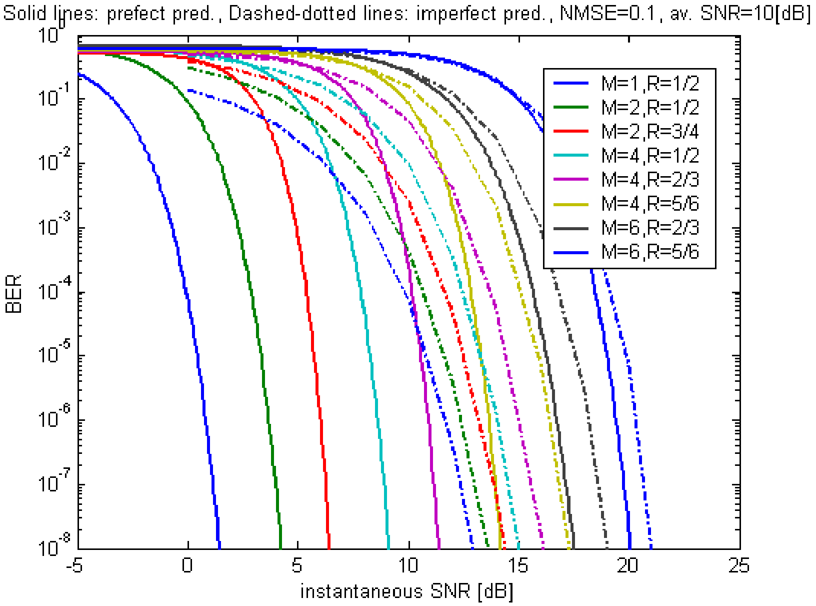


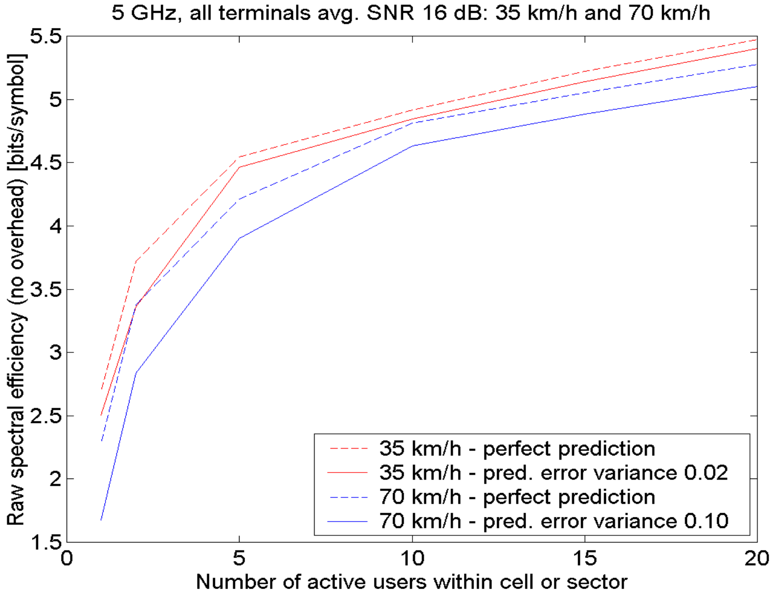




- Collect channel quality information to a multi-user Resource Scheduler and Link Adaptation units by using channel prediction (SINR prediction) and send this information over a feedback channel (FDD)

Multi-user Scheduling and Link Adaptation Gains cont.

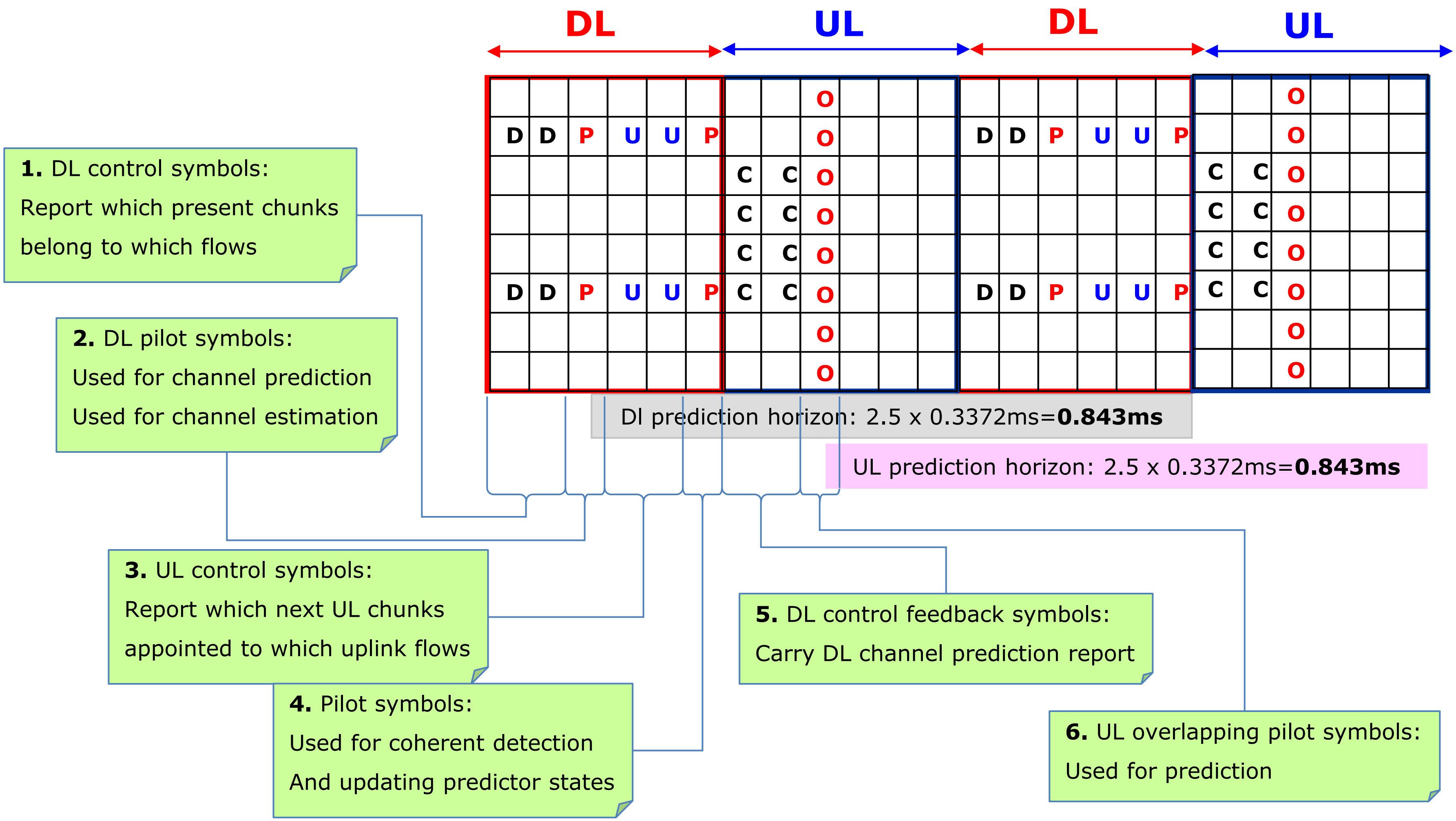




- Utilize the information on channel variability in order to achieve statistical multiplexing gain on the time-frequency (and space) resources
- Use link adaptation and more or less opportunistic scheduling (under QoS and certain fairness criteria)
- Spectral efficiency increases with number of users (multi-user diversity gain).

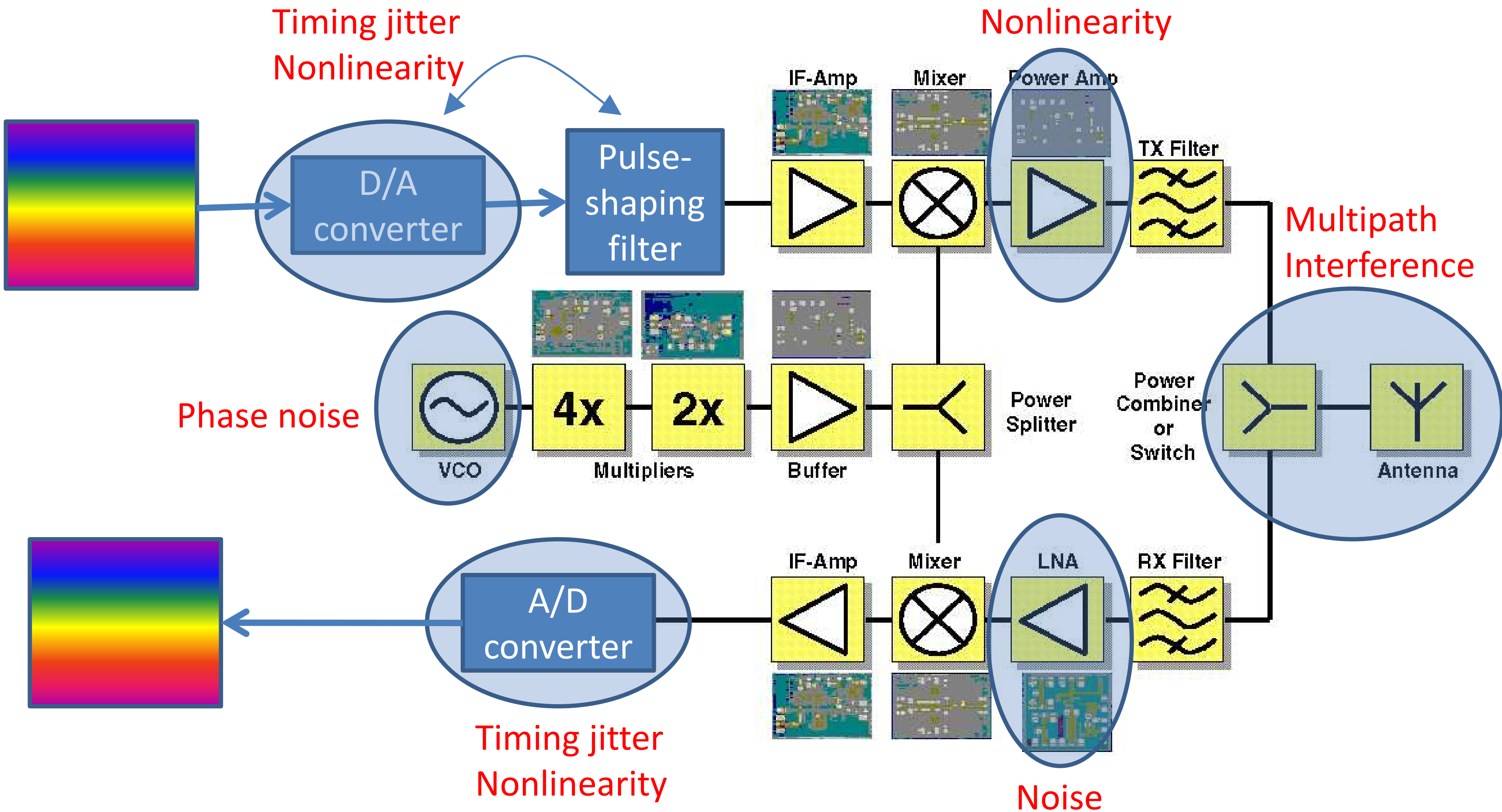
Ref: M. Sternad, T. Svensson, T. Ottosson, A. Ahl´en, A. Svensson, and A. Brunstrom, “Towards systems beyond 3G based on adaptive OFDMA transmission,” Proceding of the IEEE, Special Issue on Adaptive Transmission, vol. 95, no. 12, pp. 2432–2455, Dec 2007.

Frame Structure of FDD DL and UL
Enabling Frequency-Adaptive Transmission at Vehicular Speeds



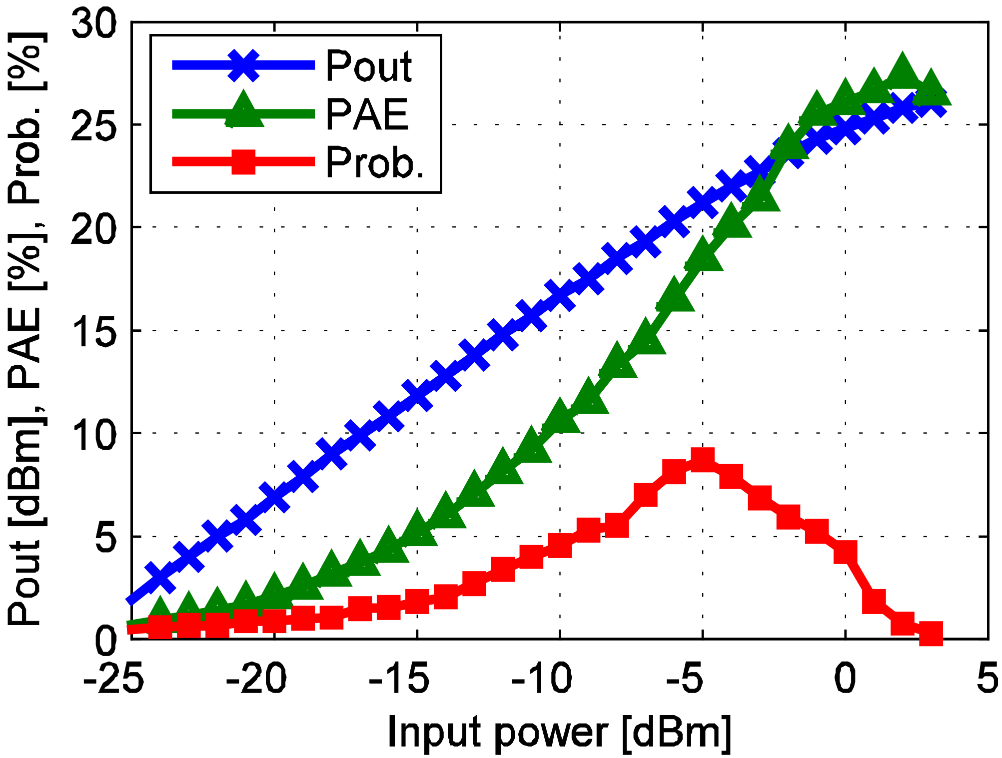
- *WINNER II Reference Design deliverable D6.13.14*

A communication system



Slides from Thomas Eriksson, Chalmers

Nonlinear amplifiers



The efficiency of an amplifier (the output power compared to the consumed power) is highest when the input/output power is high. A problem is that the amplifier is usually quite nonlinear for high input powers.

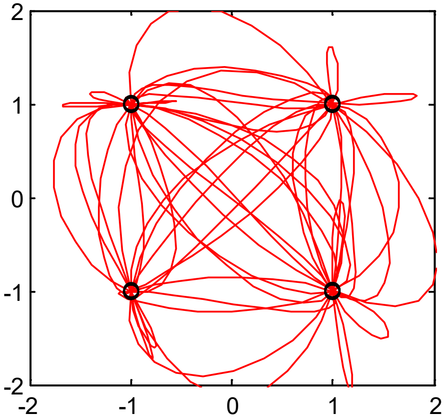
Many communication systems cannot handle nonlinearities particularly well. To overcome the difficulties, the power of the input signal is often reduced (Input BackOff, IBO) to run the amplifier in a more linear mode. Unfortunately, this leads to a low efficiency of the amplifier.

Slides from Thomas Eriksson, Chalmers

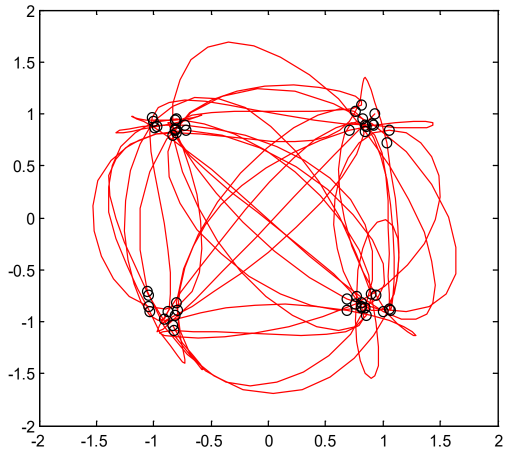
The effect of a nonlinear amplifier

Single-carrier modulation

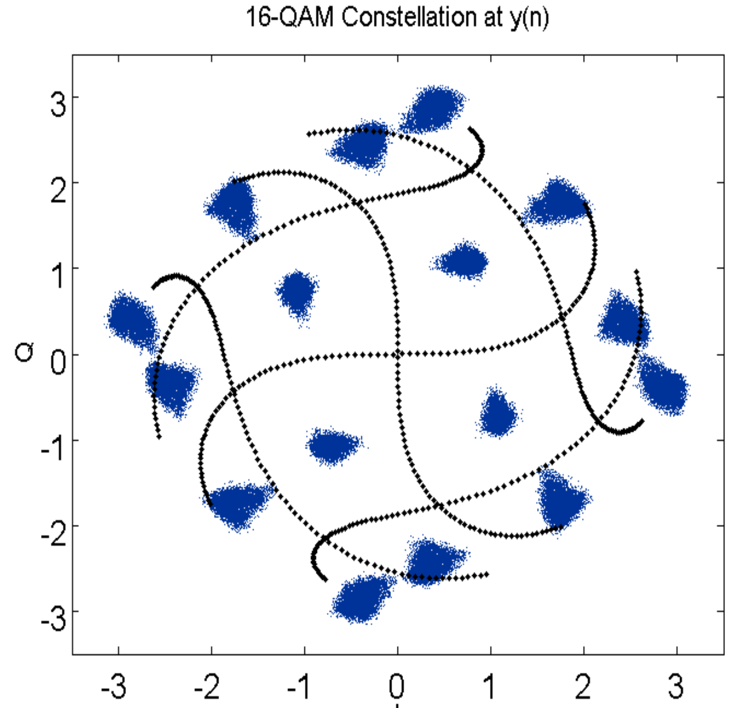
QPSK, linear amplifier



QPSK, nonlinear amplifier (clipping)



16-QAM, Saleh nonlinearity



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We see:

- Deformation of the signal space
- Intersymbol Interference (ISI)

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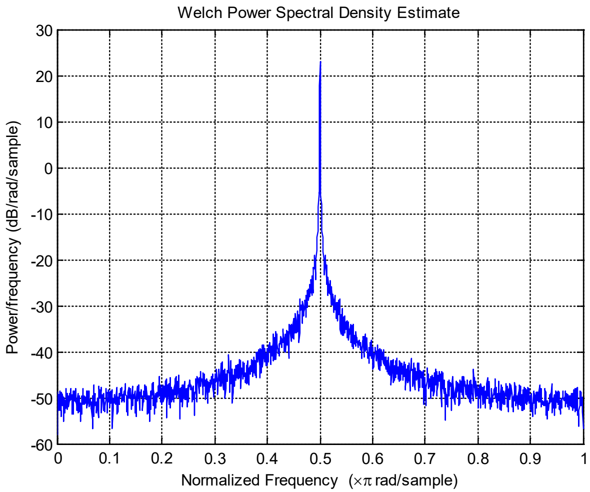
Phase noise in the oscillator

All oscillators suffers to some extent from *phase noise*:

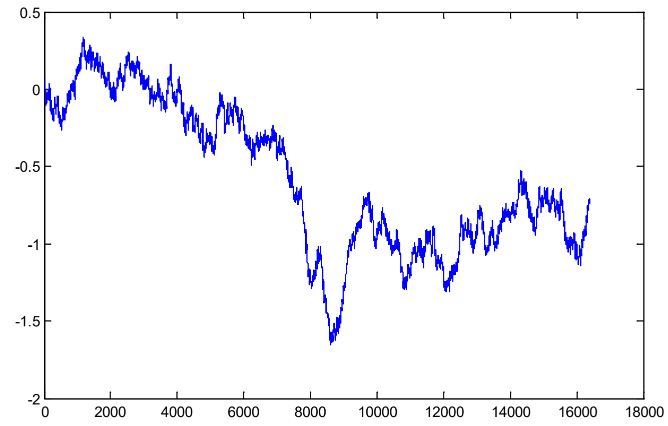
$$X(t) = A\cos\left(2\pi f_c t + \phi(t)\right)$$

The phase noise is visible as a widening of the spectral peak of the oscillator.



In the communication system, the phase noise appears as a random walk process on the phase

$$\phi_n = \phi_{n-1} + \Delta_n \qquad \tilde{X}_n = X_n e^{j\phi_n} = X_n e^{j\sum_{k=0}^{n}\Delta_n}$$



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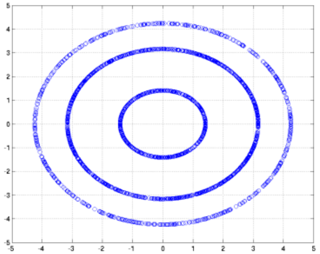
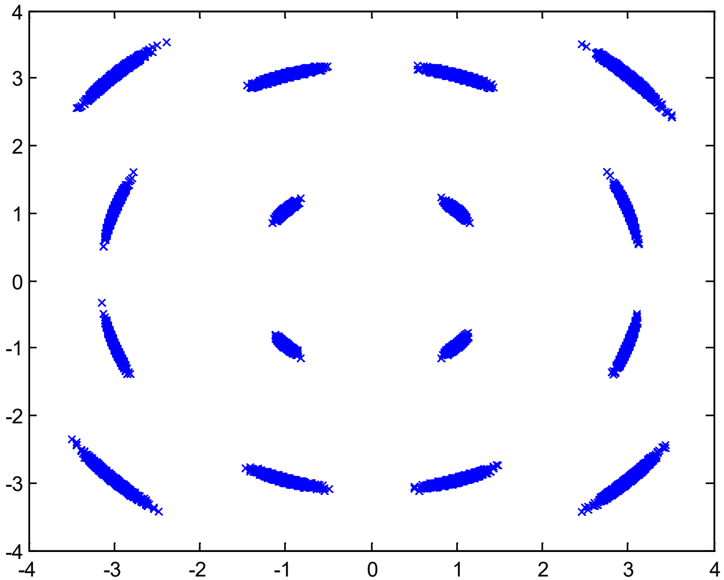
The effect of phase noise

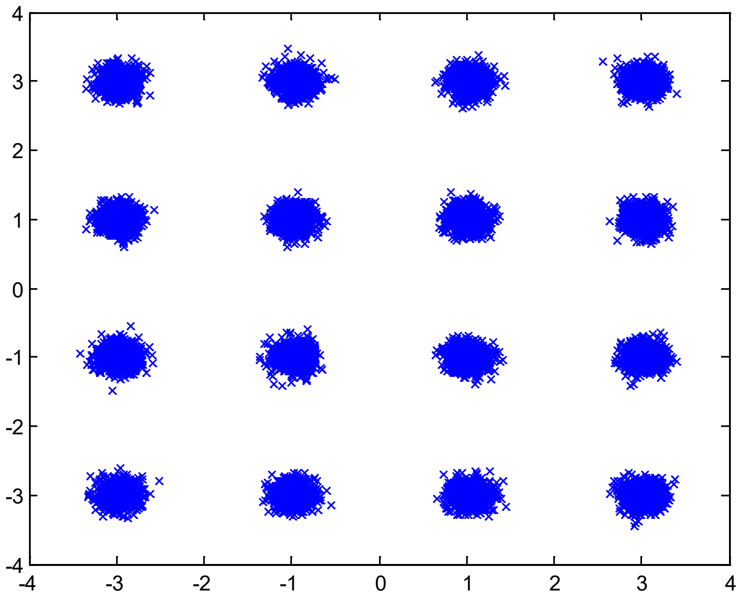
If not compensated at all, the effect of phase noise is a randomly rotated signal space. Therefore, some phase synchronization must be done in all communication systems, but there is always remaining noise.



16-QAM



OFDM 16-QAM



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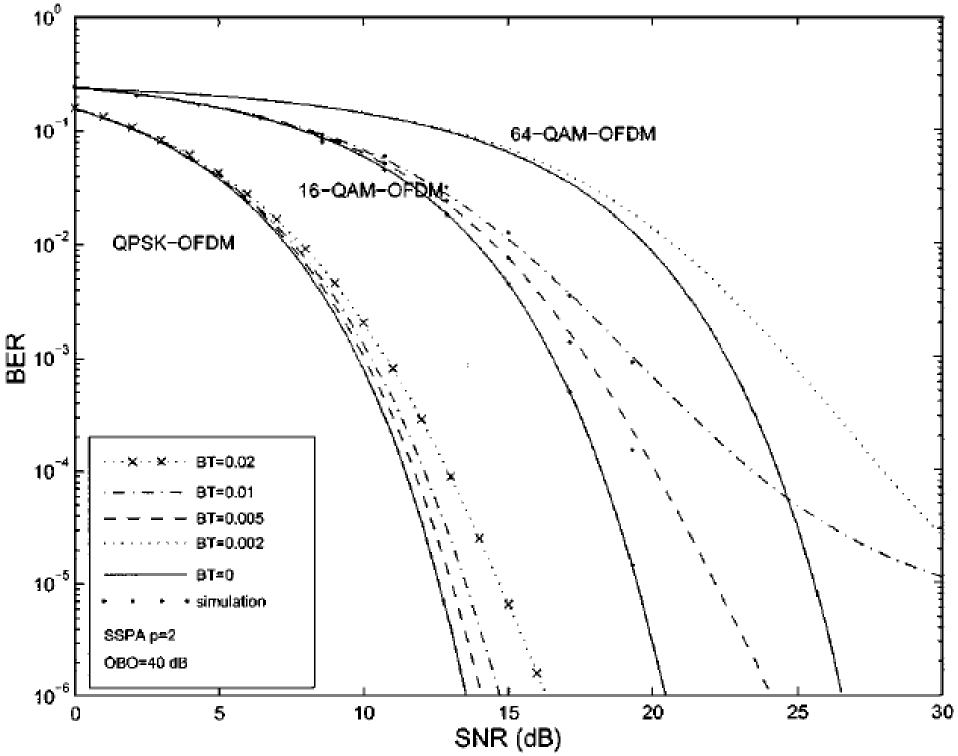
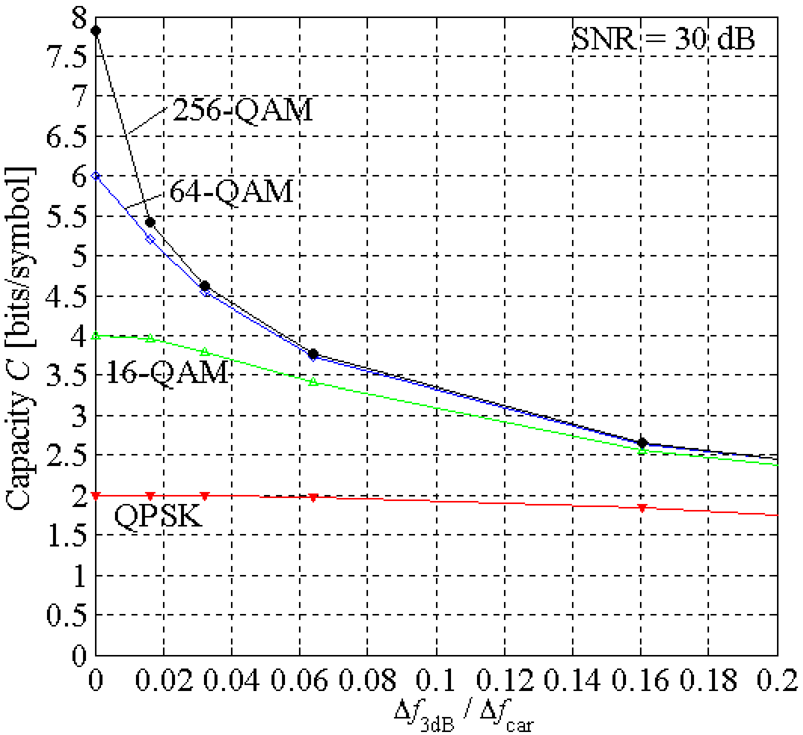
Performance with phase noise





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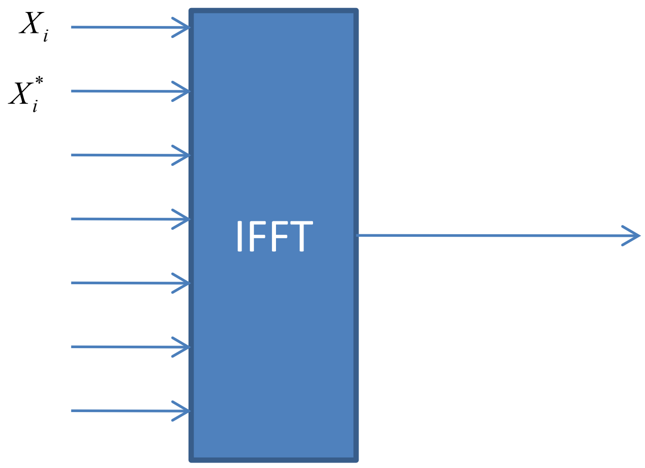
Compensation of phase noise

- Differential encoding (e.g. DPSK)
- Common Phase Error (CPE) removal
- Inter-Carrier Interference (ICI) removal
- Error control coding
- Coding for phase noise compensation



The majority of methods have been designed for OFDM systems, since it has been shown that phase noise is an order of magnitude more severe for an OFDM system.

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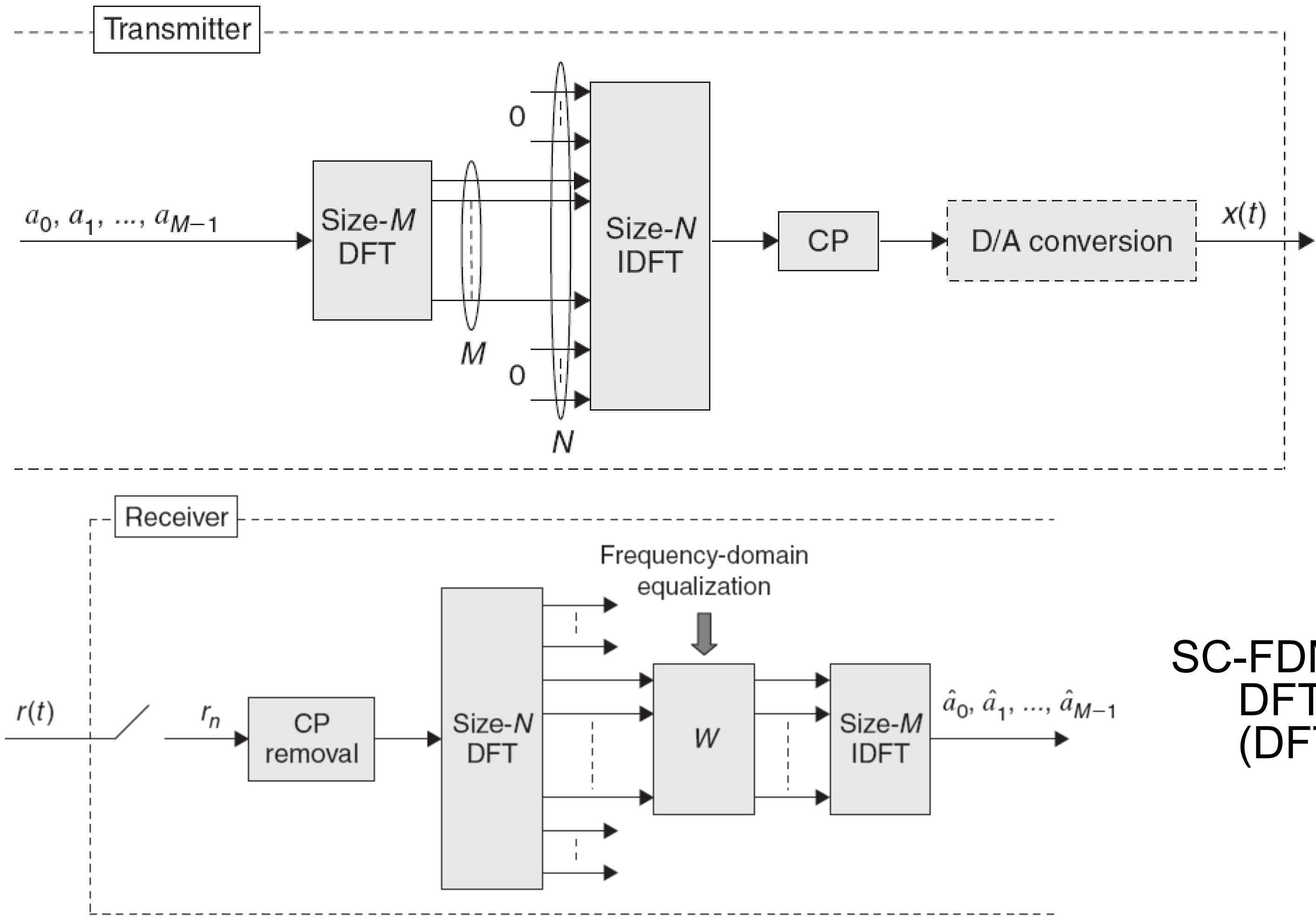
Single-Carrier FDMA, SC-FDMA (3GPP LTE Uplink)

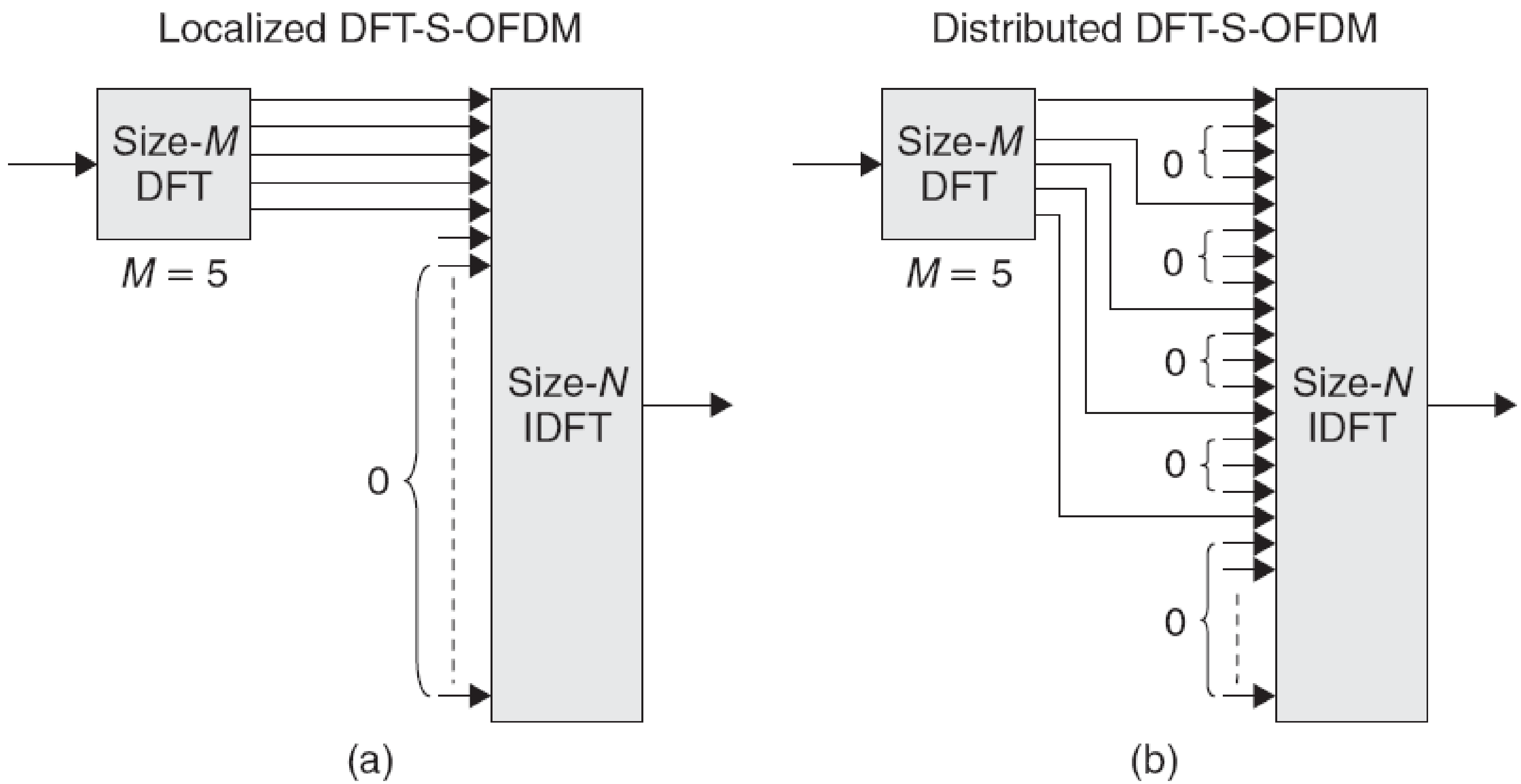
Less sensitive to HW impairments



SC-FDMA is also called DFT spread OFDM (DFT-S-OFDM).

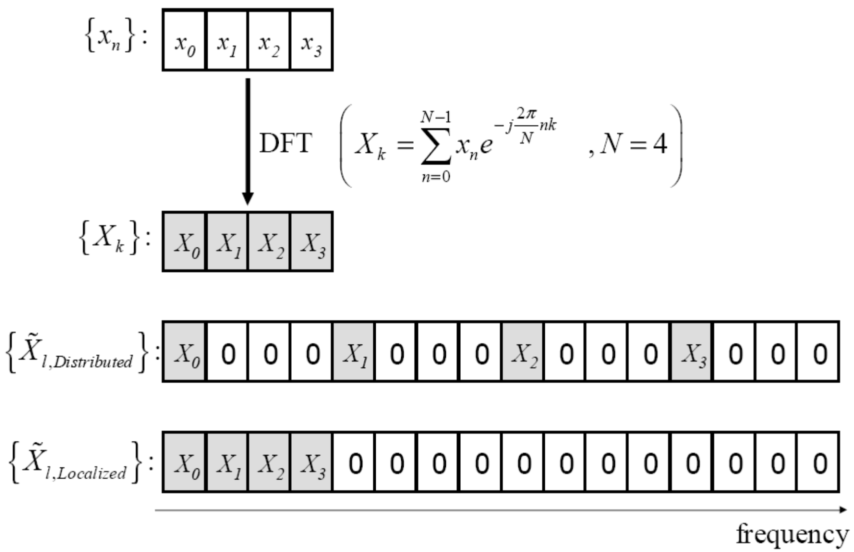
Source: E. Dahlman, S. Parkvall, J. Sköld, and P. Beming, 3G Evolution: HSPA and LTE for Mobile Broadband. Academic Press, 2007

Localized and Distributed Subcarrier Mapping

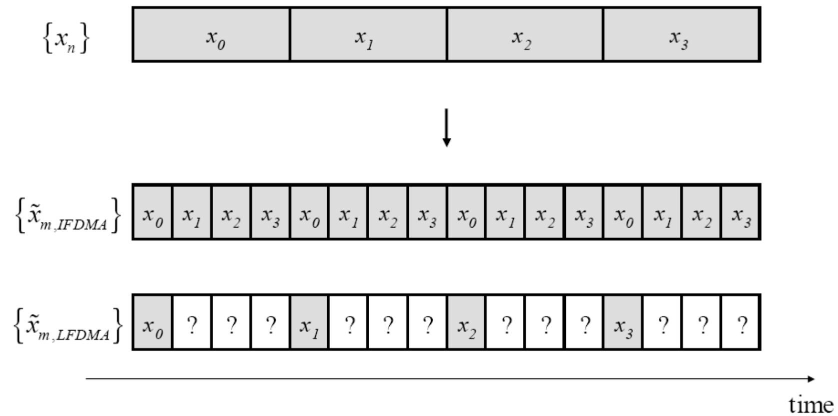


- Distributed mapping has better frequency diversity than localized mapping and better PAPR (before pulse shaping)
- Localized mapping + frequency-hopping (adds delay) easier to combine with adaptive scheduling and link adaptation. Channel estimation is easier. Adopted for LTE Release 8 standard.

Time Domain DFT-S-OFDM Signals



An example of SC-FDMA transmit symbols in the frequency domain for $N = 4$, $Q = 4$ and $M = 16$.

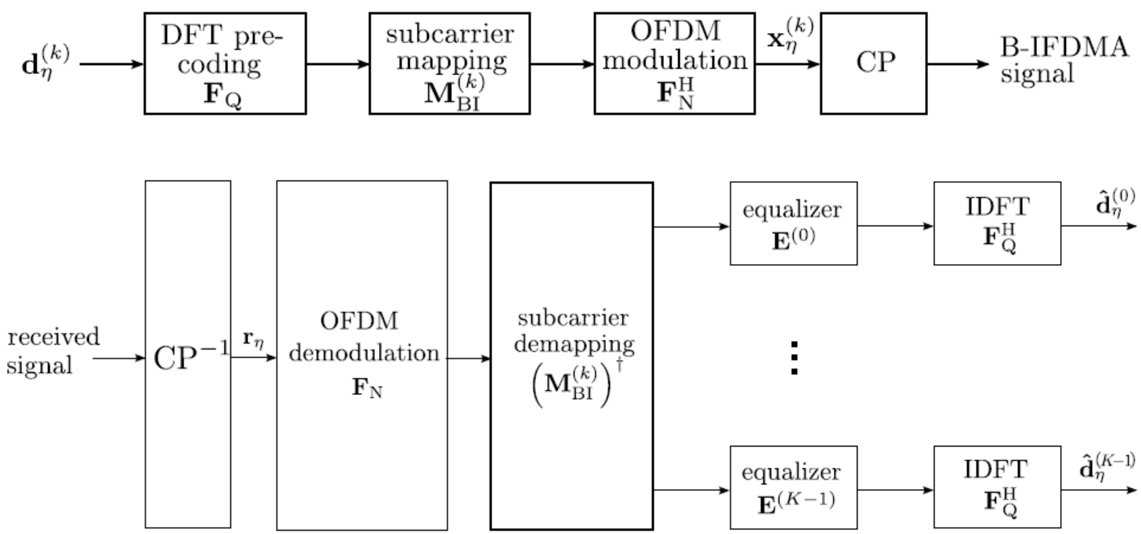


An example of SC-FDMA transmit symbols in the time domain for $N = 4$, $Q = 4$ and $M = 16$.

- Interleaved subcarrier allocation is the same as block-repeated serial modulation, with frequency-domain equalization made possible due to the cyclic prefix.
- Each data symbol is spread on all used subcarriers, so DFT-S-OFDM can be seen as a spread spectrum techniques as well, hence the name DFT-S-OFDM.

Source: H. G. Myung, J. Lim, and D. J. Goodman, “Peak-to-average power ratio of single carrier FDMA signals with pulse shaping,” in Proc. PIMRC06, Helsinki, Finland, Sept. 2006.

Generalized SC-FDMA



B-IFDMA transceiver, transmitter (top) and receiver (bottom).

$$\left[\mathbf{M}_\mathrm{BI}^{(k)}\right]_{n,q} = \begin{cases} 1 & n = l \cdot \frac{N}{L} + m + kM \\ 0 & \text{else} \end{cases}$$

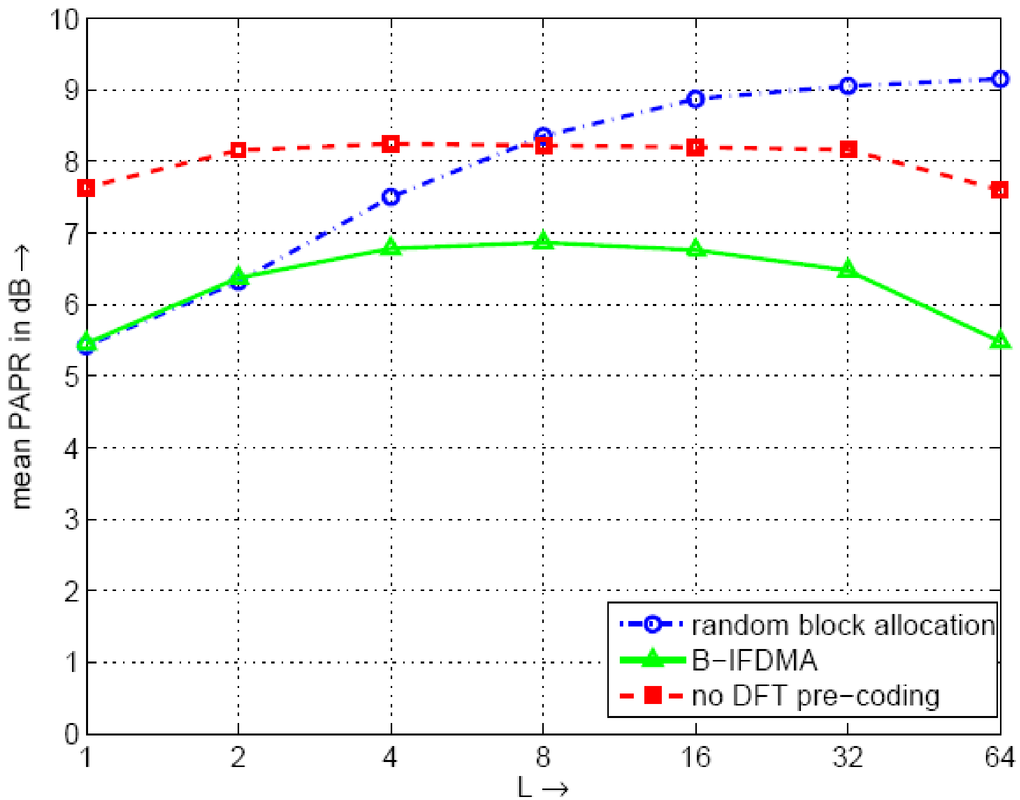
where $l = 0, \ldots, L-1$, $m = 0, \ldots, M-1$, and $q = m + l \cdot M$.

Block-Interleaved FDMA (B-IFDMA), adopted in WINNER:

- Assign *L* blocks of subcarriers to each user, each block containing *M* subcarriers, in total *Q* subcarriers.
- Provides a trade-off between the strengths and weaknesses for DFT-S-OFDM with distributed and localized mapping at somewhat worse envelope properties.
- DFT-S-OFDM with distributed and localized mapping are special cases of B-IFDMA

References:
• T. Svensson et al., “B-IFDMA - a power efficient multiple access scheme for non-frequency-adaptive transmission,” in 16th Mobile & Wireless Communications Summit, Budapest, Hungary, July 2007.
• T. Svensson, T. Frank, T. Eriksson, M. Sternad and A. Klein, “Block Interleaved Frequency Division Multiple Access for Power Efficiency, Robustness, Flexibility and Scalability,“ Jan 2009, Submitted.

Envelope Properties of the DFT-precoded Schemes with Pulse Shaping



Mean PAPR of B-IFDMA transmit signals with $Q = 64$ as a function of number of blocks L compared to the corresponding schemes without DFT pre-coding and schemes with random allocation of the subcarrier blocks.

Mean PAPR: Peak-to-average-power ratio relates to the amplifier efficiency (the lower value the better).

- L=1 corresponds to DFT-S-OFDM with localized mapping
- L=64 corresponds to DFT-S-OFDM with distributed mapping

Source: T. Svensson, T. Frank, T. Eriksson, M. Sternad and A. Klein, “Block Interleaved Frequency Division Multiple Access for Power Efficiency, Robustness, Flexibility and Scalability,“ Jan 2009, Submitted.

Summary on Multicarrier Modulation

- Data transmission using multiple carriers enables easy equalization and link adaptation, but is costly.
- Orthogonal Frequency Division Multiplexing (OFDM) implements multi-carrier modulation with virtual carriers using IDFT/DFT
  - Spectrally efficient
  - Low cost
  - Easy to equalize, implement link adaptation and frequency-domain multi-user scheduling (OFDMA)
  - Parameterization should be optimized for
- Hardware impairments
  - OFDM is more sensitive to non-linear amplifiers and phase noise than single-carrier.
- DFT-precoded OFDMA
  - A method to implement single-carrier like modulation using an OFDM transceiver.