

HOMEWORK 3

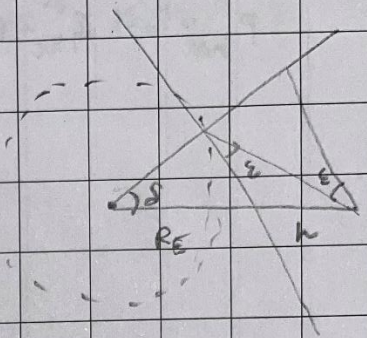
1.

Homework 3

1) inclination = 84°

$$\varepsilon = \arctan \left(\frac{\cos \delta - (R_E / r)}{\sin \delta} \right)$$

From figure: $\cos \delta = \frac{R_E}{r} \cos \varepsilon$

$$\delta = \arccos \left(\frac{R_E}{r} \cos \varepsilon \right)$$


Passage time $t = \frac{\text{length of arc}}{\text{velocity}}$ (for $\text{LEO: } v = \sqrt{\frac{\mu}{r}}$)

we know: $s = R \cdot \delta \Rightarrow t = \frac{(R_E + h) \left(\arccos \left(\frac{R_E}{r} \cos \varepsilon \right) \right)}{\sqrt{\frac{\mu}{r}}}$

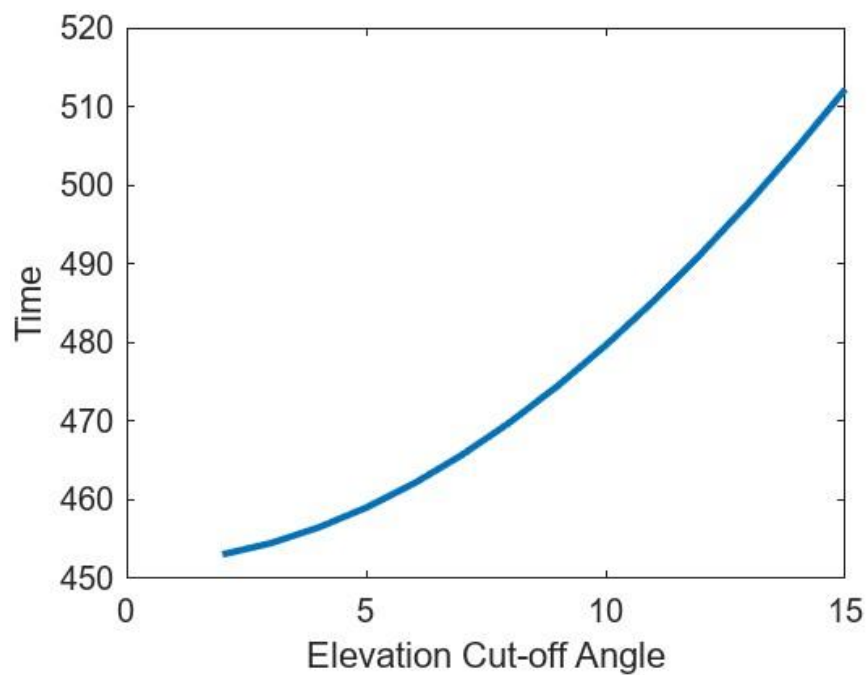
$$\Rightarrow t = \frac{7152 \left(\arccos \left(0.89 \cos \varepsilon \right) \right)}{2860.78}$$

2)

2. Simulation from matlab:

Code:

```
Re = 6371 * 1000;  
h = 781 * 1000;  
r = Re + h;  
mu = 3.986e14;  
v = sqrt(mu / r);  
theta_degrees = 2:15;  
theta_radians = deg2rad(theta_degrees);  
  
%formula  
cos_theta = cos(theta_radians);  
cos_delta = (Re * cos_theta) / r;  
delta = acos(cos_delta); % Central angle in radians  
s = r * delta; % Arc length  
t = s / v; % Passage time in seconds  
  
%plot  
figure;  
plot(theta_degrees, t, 'LineWidth', 2, 'MarkerSize', 5);  
xlabel('Elevation Cut-off Angle');  
ylabel('Time');
```



3.

3) Doppler shift of non moving Rx: $\Delta f = \frac{\gamma v \cos \beta}{c} f_0$

$\Rightarrow \beta = 90^\circ - \epsilon \Rightarrow \cos \beta = \cos(90^\circ - \epsilon) \Rightarrow \cos \beta = \sin \epsilon$

$\therefore \Delta f = \gamma \frac{\sin \epsilon}{c} \cdot f_0$

For low Earth Orbit: $\gamma = \sqrt{\frac{\mu}{R}}$

$= \sqrt{\frac{3.986 \times 10^{14}}{7152 \times 10^3}} = 7465.431 \text{ m/s}$

$\Rightarrow \Delta f = \frac{7465.431 (\text{m/s})}{3 \times 10^8} \times 1626.5 \times 10^6$

$\Delta f = 40452.682 (\text{Hz})$

4)

4. Matlab simulation

```
f0 = 1626.5e6;
c = 3e8;
v = 7465.431;

elevation_angles = linspace(0, 90, 90);
doppler_shifts = 40452.682*sind(elevation_angles)

figure;
plot(elevation_angles, doppler_shifts, 'LineWidth', 2);
xlabel('Elevation Angle ');
ylabel('Doppler Shift');
```

