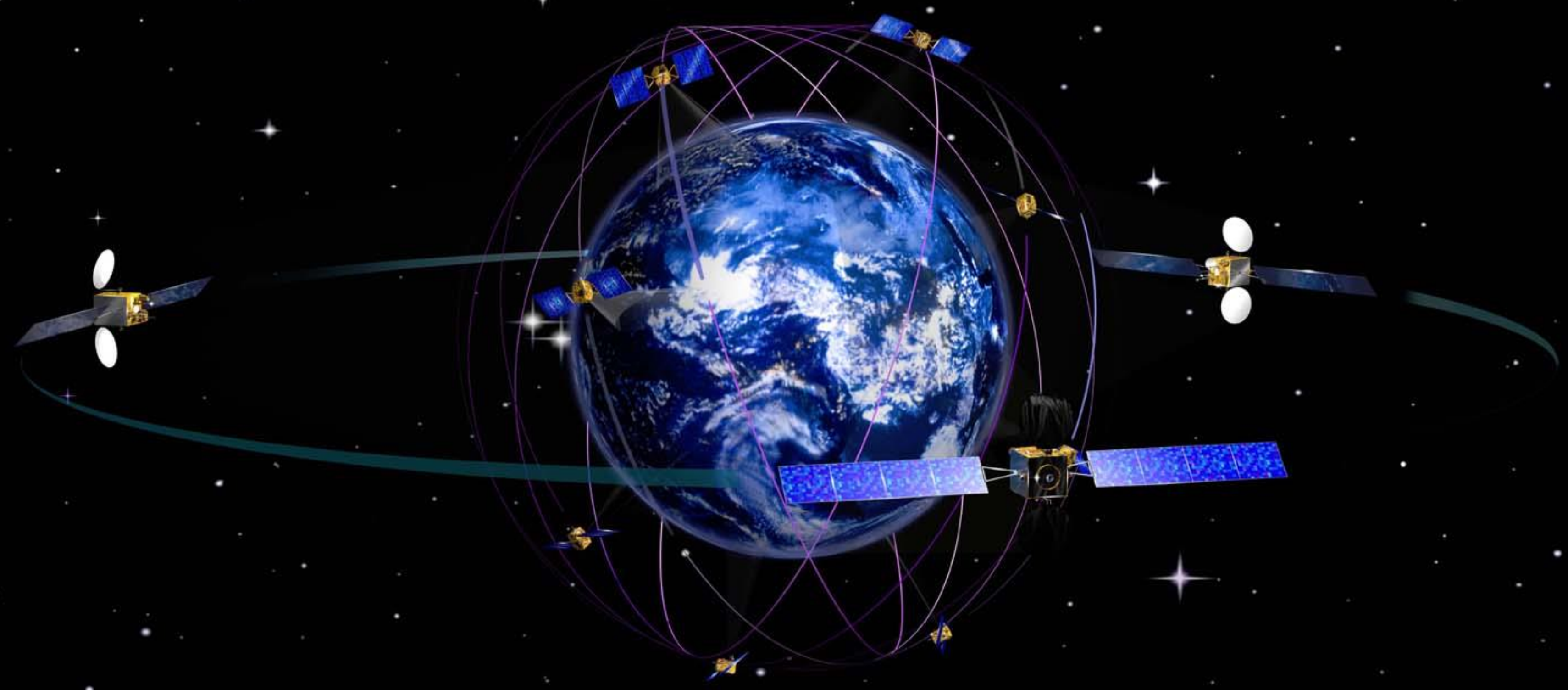
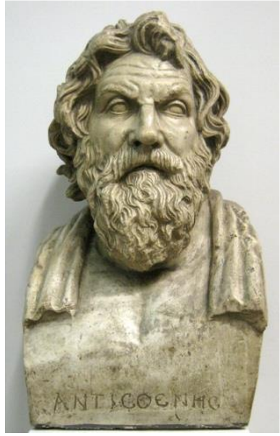
Satellite Communications
– RRY100 –



2024 Study Period 1
Lecturer: Rüdiger Haas

Lecture-4: Orbits and spacecrafts

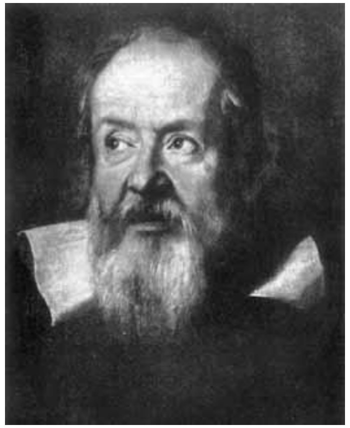
Some important persons related to satellite orbits:



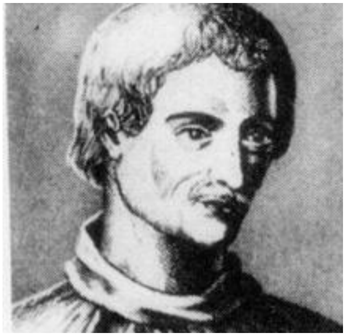
Aristarchus of S.
(310–230 BC)

Ptolomy
(ca 130 BC)

N. Copernicus
(1473–1543)



G. Galilei
(1564–1642)



B. Giordano
(1548–1600)

T. Brahe
(1546–1601)

J. Kepler
(1571–1630)

I. Newton
(1642–1727)

A. Einstein
(1879–1955)

- **Aristarchus of Samos** (310–230 BC): earth rotates around the sun (heliocentric)

- **Ptolomy** (ca. 130 BC): earth is centre of the universe (geocentric system, ”Ptolomaic” system), planetary orbits described by a complicated system of circles that move inside circles

- **Nicolaus Copernicus** (1473–1543): re-introduces the heliocentric system, Catholic church: ”This is an heretic idea!”

- **Galileo Galilei** (1564–1642): adopts the Copernican system, Catholic church’s Inquisition processes against him
  - Bruno Giordano (1548–1600): supports Galileo, Catholic church burns him
  - In 1993 the pope John Paul II officially admits that Galileo was right…

- **Tycho Brahe** (1546–1601): very good observed, provides the astronomical observations for Kepler; however a poor theoretician: planets move around the sun and the earth…

- **Johannes Kepler** (1571–1630): uses Brahe’s observations, adopts the Copernican system and develops the three Kepler’s laws, dynamic universe

- **Isaac Newton** (1642–1727): develops the laws of motion and law of universal gravitation

- **Albert Einstein** (1879–1955): precession of Mercury to prove his theories (SRT and GRT)

The three Kepler's laws:

1. The orbit of each planet is an ellipse and the sun is at one of its foci (1605, published 1609)
2. Lines joining the sun and the planets sweep equal areas in equal time ("law of areas", 1602, published 1609)
3. The square of the period of a planet is proportional to the cube of its mean distance from the sun (1618, published 1619)

The three Newton laws:

1. "law of inertia": a body stays in rest when no resulting force acts on it (==> reference systems)
2. Rate of change of momentum is proportional to force impressed on it and in same direction:

$$\sum_i^n \vec{F}_i = \frac{d\vec{p}}{dt}$$

3. "action is proportional to re-action": $\vec{F}_{ij} = -\vec{F}_{ji}$

The law of universal gravitation:

$$\vec{F} = -\frac{G \cdot m_1 \cdot m_2}{\left|\vec{r}\right|^2} \cdot \frac{\vec{r}}{\left|\vec{r}\right|}$$

$$G = (6672 \pm 4) \times 10^{-14} \quad \mathrm{m^3\ s^{-2}\ kg^{-1}}$$

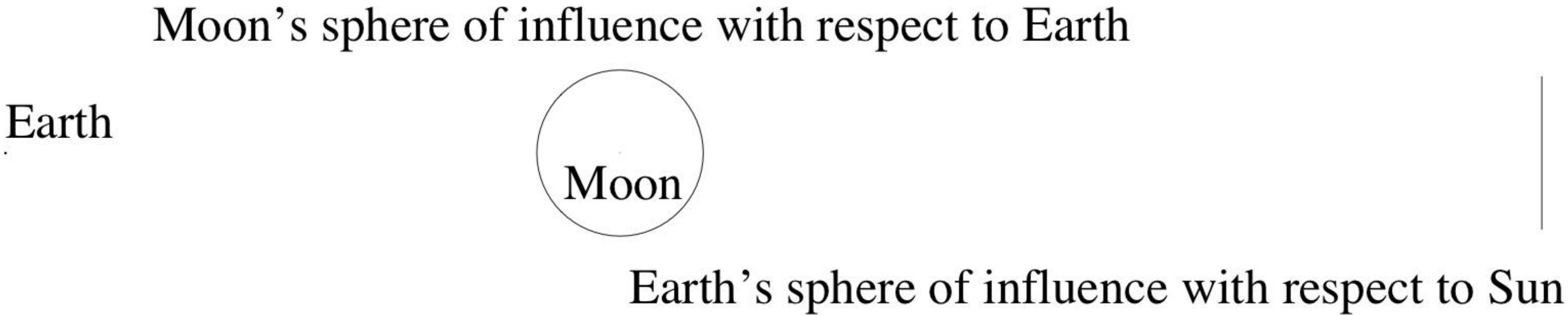
- **Most general ”The N-body problem”:**
  - Using Newton’s 2nd law and law of universal gravitation
  - System of 2nd order non-linear differential vector equations

$$m_i \cdot \frac{d^2 \vec{r}_i}{dt^2} + \frac{dm_i}{dt}\frac{d\vec{r}_i}{dt} = -G \cdot m_i \sum_{j \neq i}^{n} \frac{m_j}{\left|\vec{r}_{ij}\right|^2} \frac{\vec{r}_{ij}}{\left|\vec{r}_{ij}\right|} + \vec{F}_{other}$$

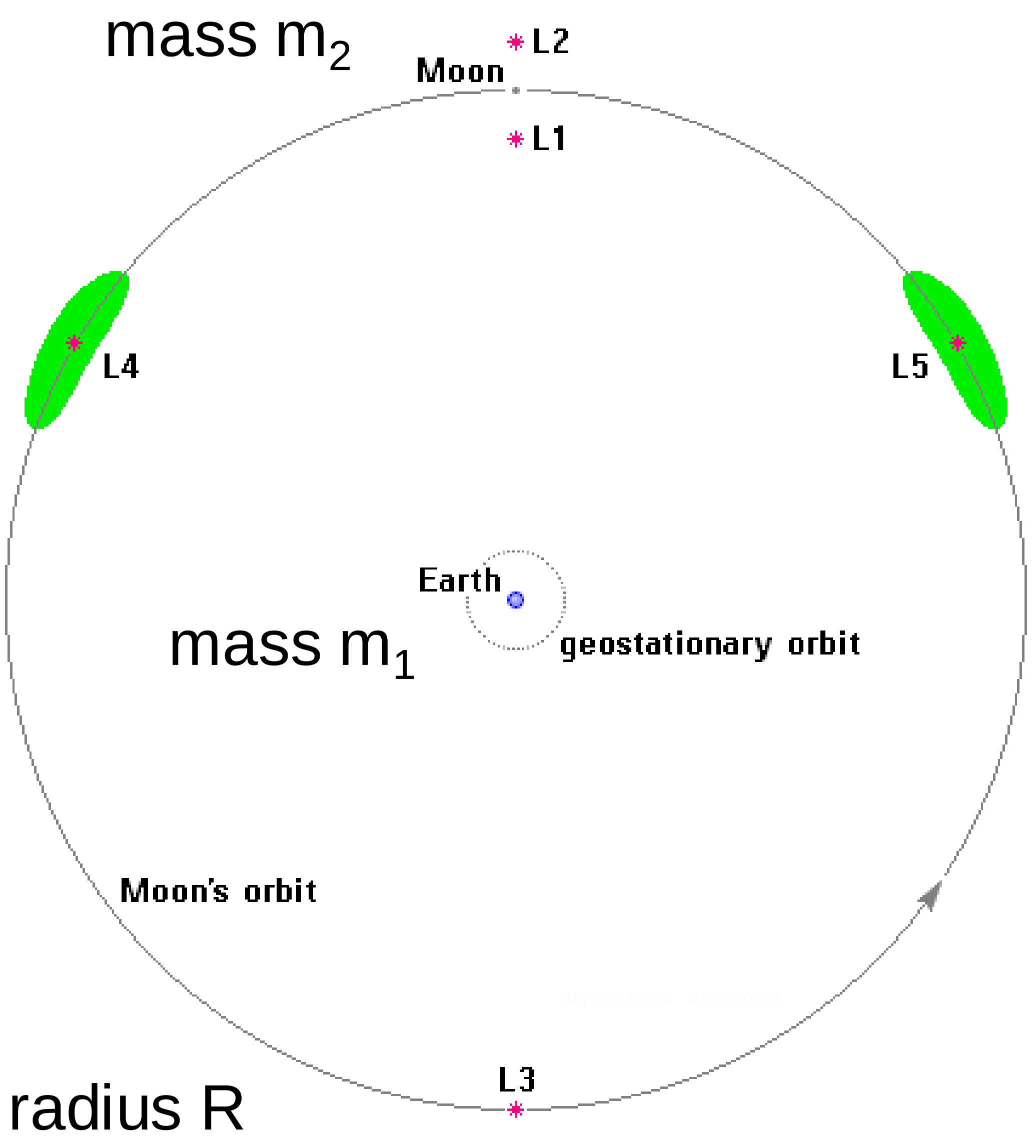
- **”Sphere of influence”:**
  - N-body problem is difficult to solve…
  - Often restriction to 3-body problem
  - When can we restrict to 2-body problem?
  - Yes, within the ”sphere of influence”. Approximation by Lagrange for one large (a_1, m_1) and one small (a_2, m_2) body:

$$R_{Influence} \approx a_2 \left(\frac{m_2}{m_1}\right)^{\frac{2}{5}} \qquad (m_2 < m_1)$$

Body	$R_{influence}$	
Mercury	0.1	Gm
Venus	0.6	Gm
Earth	1.0	Gm
Mars	0.6	Gm
Jupiter	50	Gm
Saturnus	55	Gm
Uranus	52	Gm
Neptunus	86	Gm
Moon	65	Mm



Stable points in a 3-body system: ”Lagrange points”:



$$L1 = \left[0, \quad R \cdot \left(1 - \left(\frac{\alpha}{3}\right)^{1/3}\right)\right]$$

$$L2 = \left[0, \quad R \cdot \left(1 + \left(\frac{\alpha}{3}\right)^{1/3}\right)\right]$$

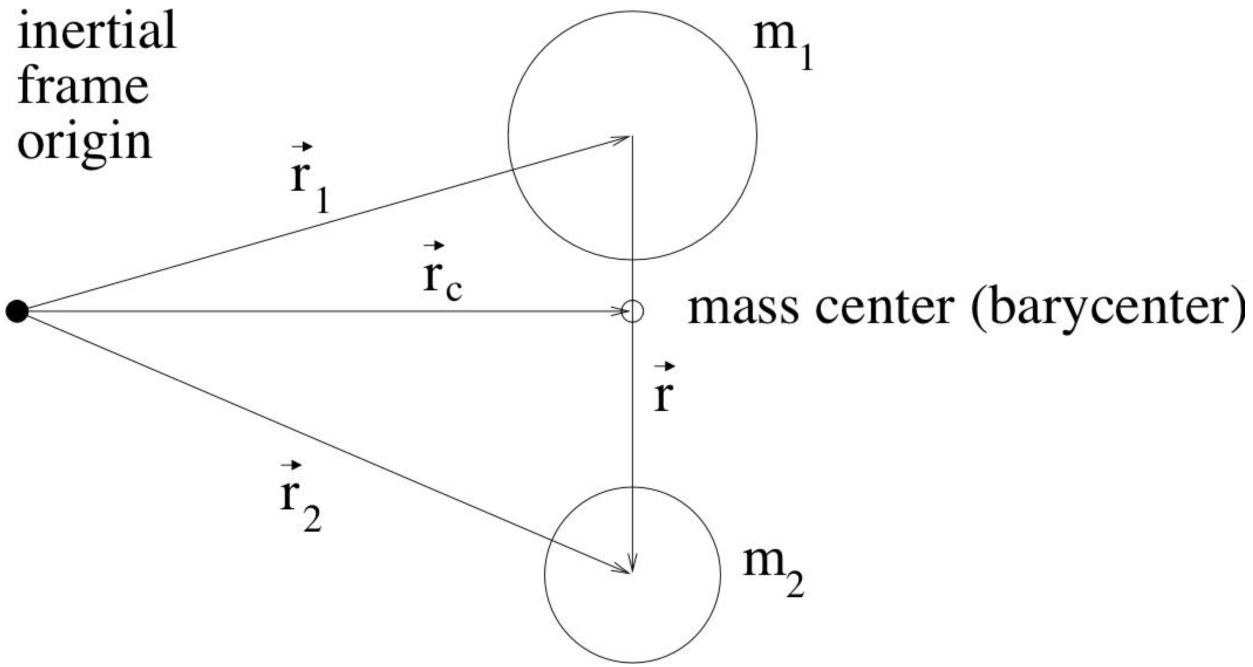
$$L3 = \left[0, \quad -R \cdot \left(1 + \frac{5}{12} \cdot \alpha\right)\right]$$

$$L4 = \left[-\frac{\sqrt{3}}{2} \cdot R, \quad \frac{R}{2}\left(\frac{m_1 - m_2}{m_1 + m_2}\right)\right]$$

$$L5 = \left[\frac{\sqrt{3}}{2} \cdot R, \quad \frac{R}{2}\left(\frac{m_1 - m_2}{m_1 + m_2}\right)\right]$$

$$a = \frac{m_1}{m_1 + m_2}$$

- **The 2-body problem:**



  - Law of motion for the 2 bodies
  - Definition of a common mass centre (”barycentre”)
  - Working with the difference vector
  - Simplification for one large (m_1) and one small mass (m_2)

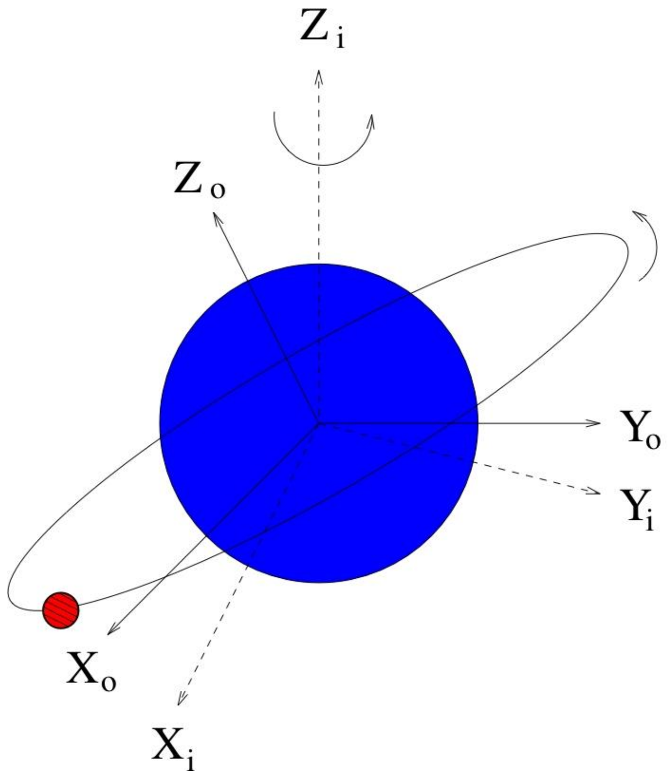
- We get finally:

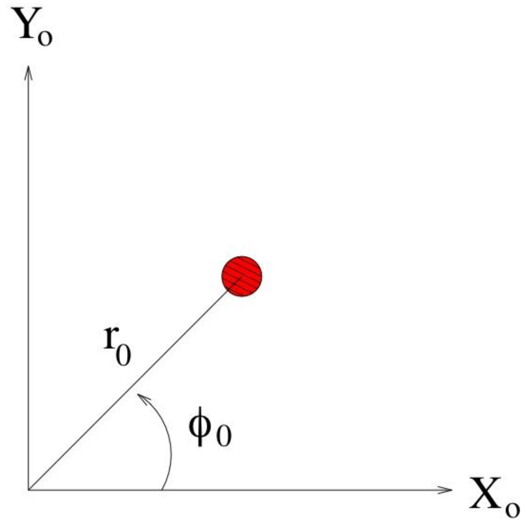
$$\ddot{\vec{r}} = -\frac{G \cdot m_1}{|\vec{r}|^3}\vec{r}$$

  - This is an approximation only…
  - Useful for artificial satellites, astronautical problems
  - Motion of a small body (e.g. spacecraft) in the gravitational field of a massive body (e.g. planet)
  - The inertial system is centred in the mass centre of the massive body
  - System of 2nd order linear differential equations
  - 6 underdetermined constraints for the solution
  - These are the 6 orbital elements

- Orbital angular momentum of the small body: perpendicular to position and velocity vector
- Its time derivative is zero, i.e. the orbital angular momentum is constant
- The small body moves in an orbital plane, i.e. the 3D-motion reduces to a 2D-motion in the orbital plane

$$\vec{r} = \begin{pmatrix} r_0 \cdot \cos\theta_0 \\ r_0 \cdot \sin\theta_0 \\ 0 \end{pmatrix}$$





$$\dot{\vec{r}} = \begin{pmatrix} \frac{dr_0}{dt}\cos\theta_0 - r_0 \cdot \frac{d\theta_0}{dt}\sin\theta_0 \\ \frac{dr_0}{dt}\sin\theta_0 + r_0 \cdot \frac{d\theta_0}{dt}\cos\theta_0 \\ 0 \end{pmatrix}$$

- Introduce an orbital plane coordinate system
- Express the position vector with distance and angle
- Write the equation of motion as

$$\ddot{\vec{r}} = -\frac{G \cdot m}{\left|\vec{r}\right|^3}\vec{r} = -\frac{G \cdot m}{{r_0}^2}\begin{pmatrix}\cos\theta_0 \\ \sin\theta_0 \\ 0\end{pmatrix}$$

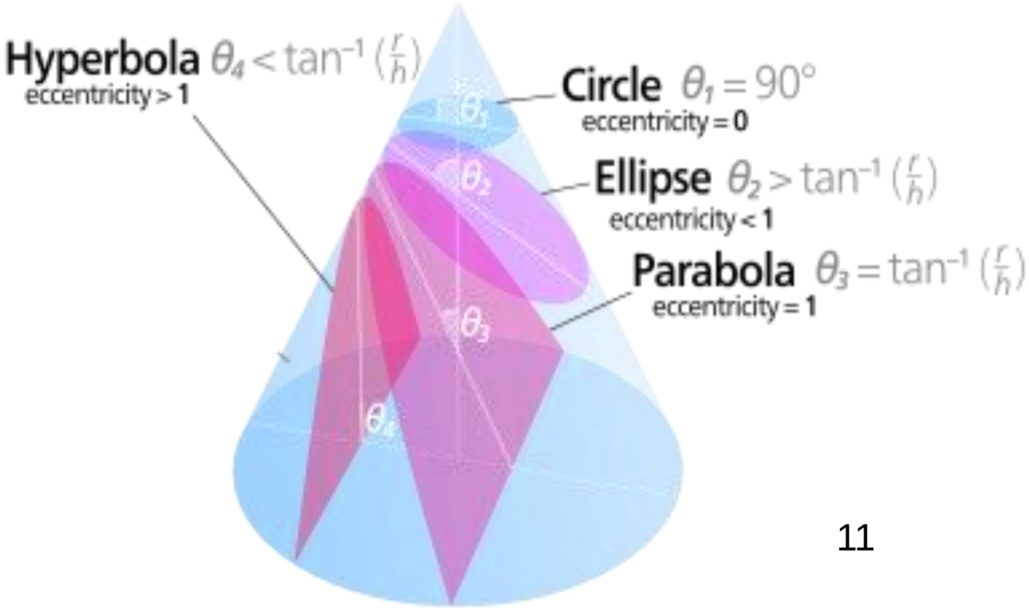
- Integrate the equation of motion
- Finally we get the equation of an ellipse

$$r_0 = \frac{p}{1 + e \times \cos(q_0 - f_0)}$$

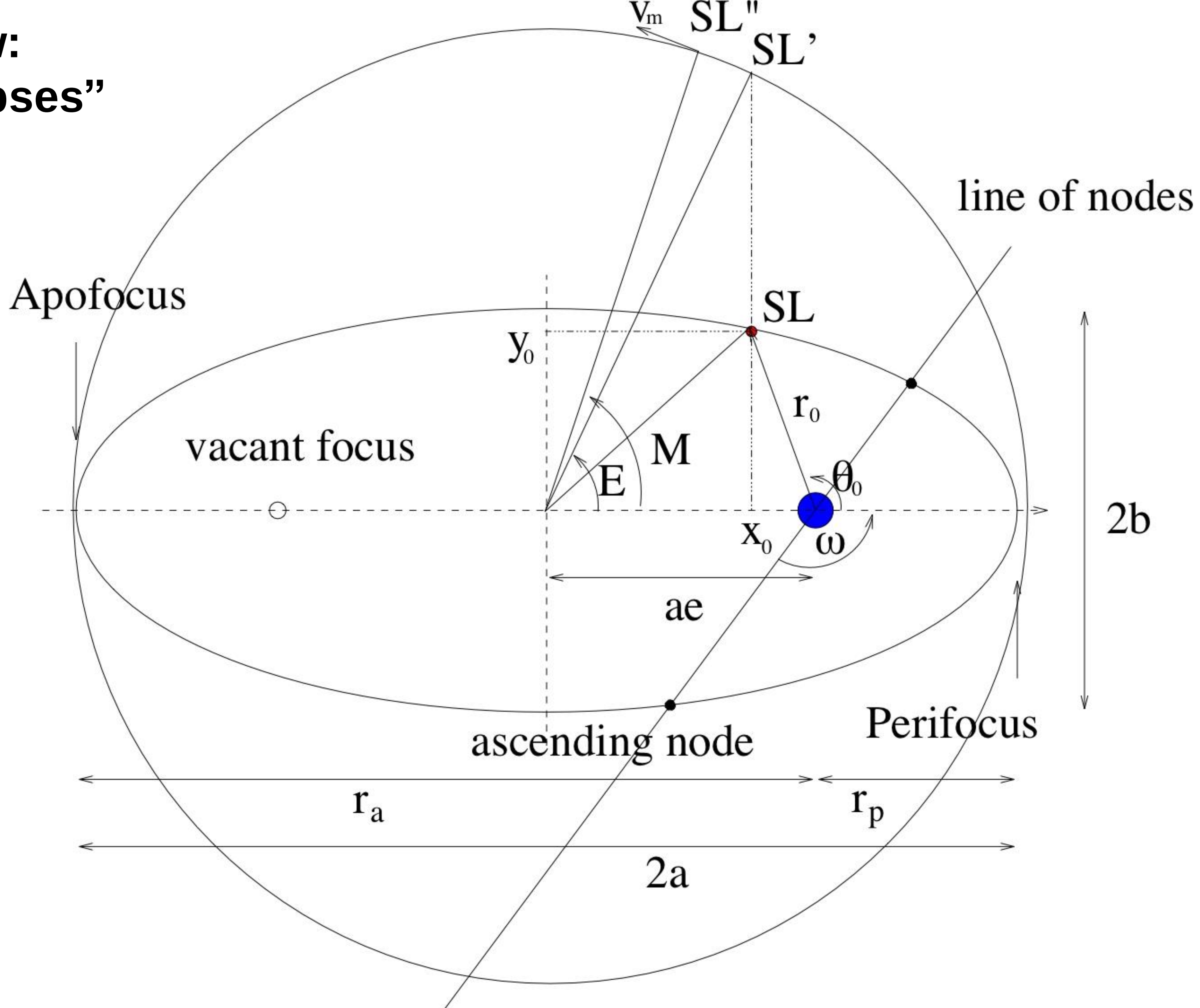
- "*e*" is the eccentricity:

  Circle *e*=0; Ellipse *e*<1; Hyperbola *e*>1, Parabola *e*=1

- Conic sections



Keplers 1st law:
”orbits are ellipses”



- a = semi-major, b = semi-minor axis
- e = eccentricity
- v_m = mean velocity, η_m= mean angular velocity
- M = mean anomaly, E = eccentric anomaly
- r_0 = distance to satellite, θ_0 = true anomaly
- x_0 and y_0 = coordinates in the orbital plane

- Some mathematical relations:

$$e^2 = \frac{a^2 - b^2}{a^2}$$

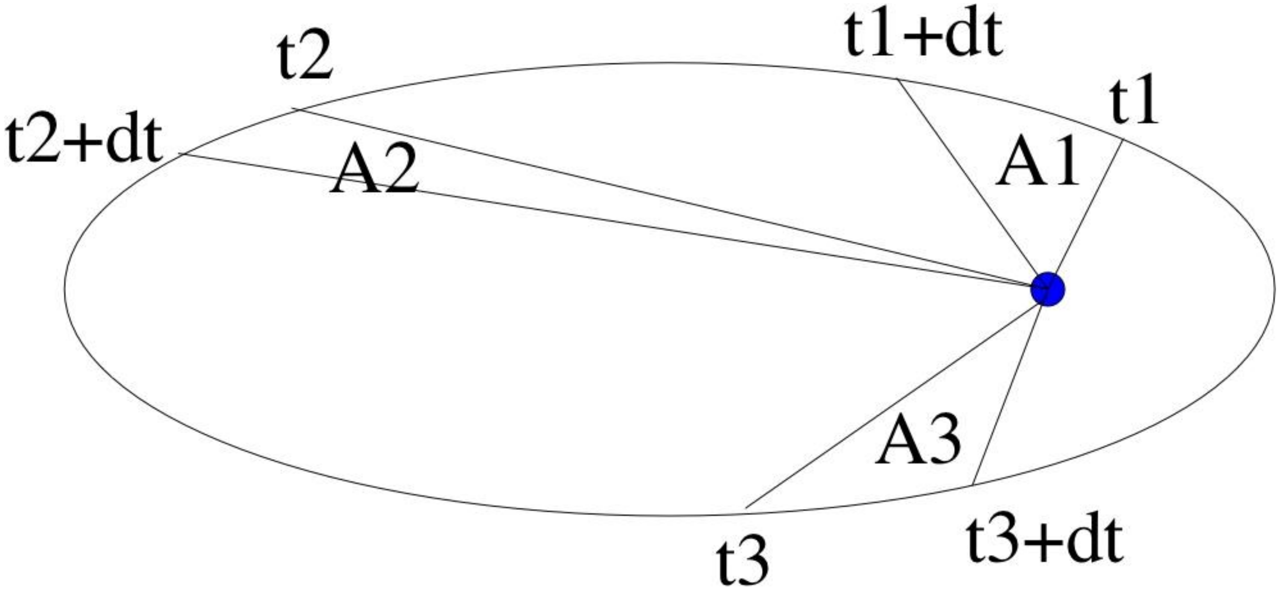
$$M = h_m \times \left(t - t_p\right)$$

$$M = E - e \times \sin E$$

$$E_{n+1} = E_n - \frac{E_n - M - e \times \sin E_n}{1 - \cos E_n}$$

Iterative equation in unit radian, starting with E_0=M

Kepler’s 2nd law ”the law of areas”:



$$A1 = A2 = A3$$

Kepler’s 3rd law:

$$T = \frac{2 \times p \times a^{3/2}}{\left(G \times M_E\right)^{1/2}}$$

==> The orbital period is independent of eccentricity

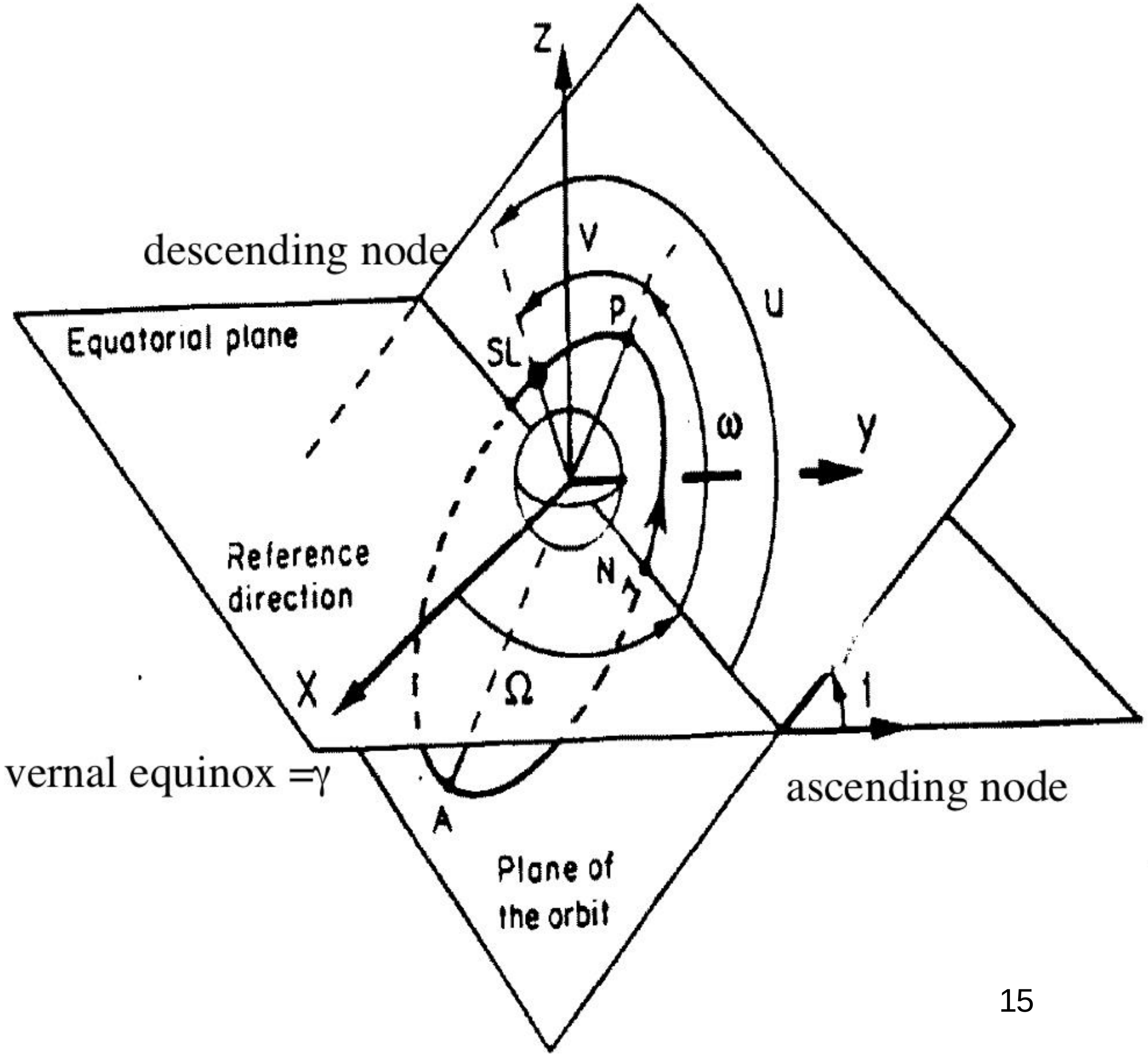
Orientation of the orbital plane in 3D space:

ω = angle between ascending node and orbital perifocus

i = inclination of the orbital plane

Ω = rectascension of the ascending node

γ = reference direction, ”vernal equinox”, ascending node of the equatorial plane w.r.t. ecliptic plane



- **We need 6 Kepler elements to describe an orbit:**
  - Semi-major axis a
  - Eccentricity e
  - Epoch of perifocus pass t_p
  - Inclination of the orbital plane i
  - Argument of perifocus ω
  - Rectascension of the ascending node Ω

- Given e.g. in two-line notation (NASA), two line elements (TLE):

```
----------------------------------------------------------------------
1   code  code     tp       decay    code     code     #        #
2   code  i        Ω        e        ω        M        rev/day  rev
----------------------------------------------------------------------


IRIDIUM 8
1 24792U 97020A   01024.21407331 -.00000100  00000-0 -42661-4 0   4474
2 24792  86.3981 300.9143 0002947  73.9660 286.1872 14.34215711 194933
INTELSAT 805
1 25371U 98037A   01022.26446194 -.00000287  00000-0  00000+0 0   2884
2 25371   0.1088 313.6221 0063248 287.0220 281.4067  1.00269924   9559
```

Equations for orbital calculations in the orbital plane

$$T^2 = \frac{4 \cdot \pi^2 \cdot a^3}{G \cdot M_E}$$

$$\eta_m = (2 \cdot \pi)/T$$

$$M = \eta_m \cdot (t - t_p)$$

$$E_{n+1} = E_n - \frac{E_n - M - e \cdot \sin E_n}{1 - \cos E_n}$$

$$r_0 = a \cdot (1 - e \cdot \cos E)$$

$$\theta_0 = \arccos\left(\frac{\cos E - E}{1 - e \cdot \cos E}\right)$$

$$x_0 = r_0 \cdot \cos \theta_0$$

$$y_0 = r_0 \cdot \sin \theta_0$$

- Transformation into inertial coordinate system:

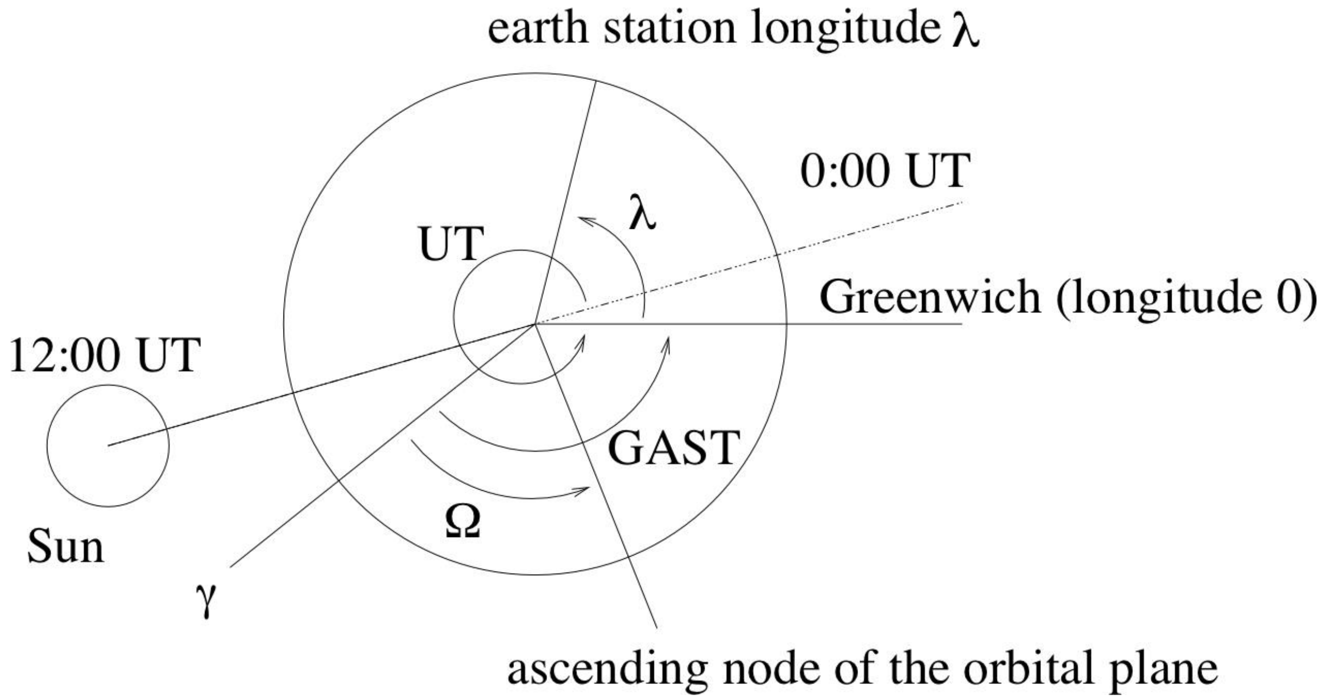
$$X_i^I = R_{ij.3}(\Omega) \cdot R_{jk.1}(i) \cdot R_{kl.3}(\omega) \cdot X_l^O$$

$$= r_0 \cdot \begin{bmatrix} \cos\Omega \cdot \cos(\omega + \theta_0) - \sin\Omega \cdot \cos i \cdot \sin(\omega + \theta_0) \\ \sin\Omega \cdot \cos(\omega + \theta_0) + \cos\Omega \cdot \cos i \cdot \sin(\omega + \theta_0) \\ \sin i \cdot \sin(\omega + \theta_0) \end{bmatrix}$$

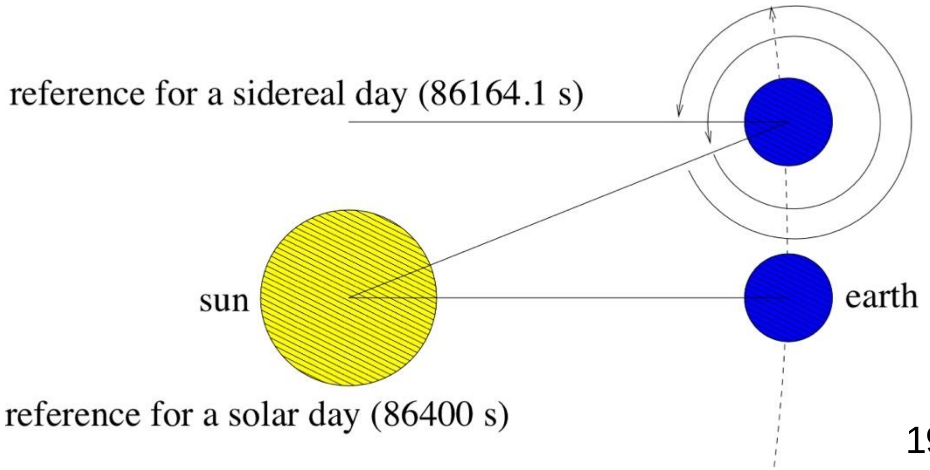
- Transformation from the inertial system to the rotating earth system:

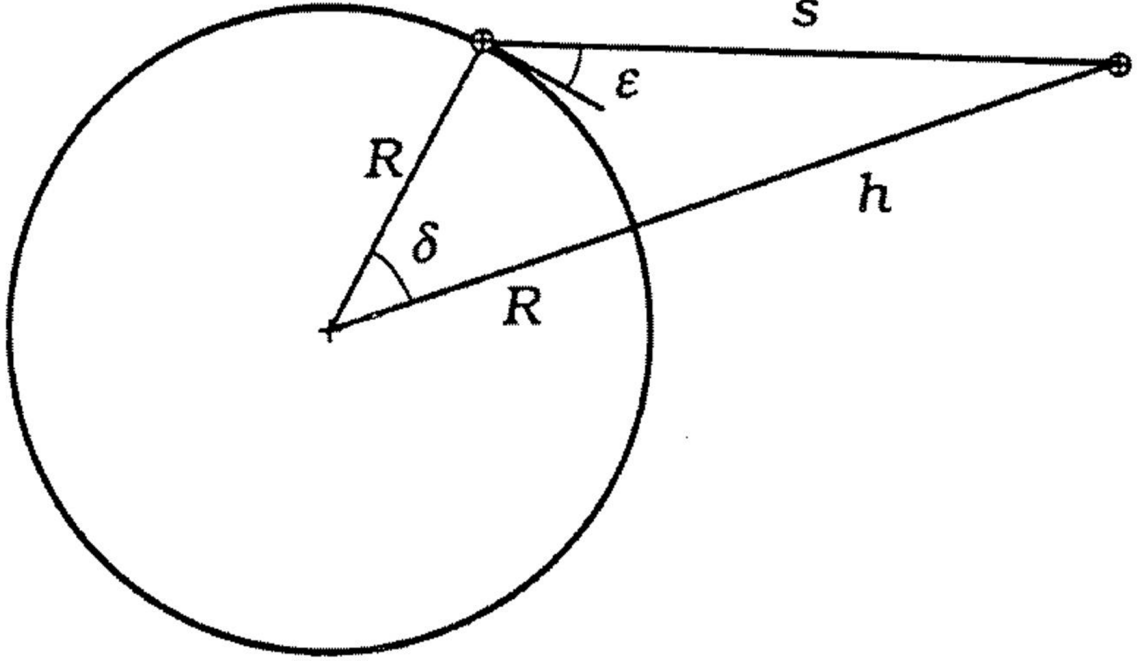
$$X_i^E = R_{ij.3}(GAST) \times X_j^I$$

$$= -\begin{bmatrix} \cos GAST \cdot X_1^I + \sin GAST \cdot X_2^I \\ \sin GAST \cdot X_1^I + \cos GAST \cdot X_2^I \\ X_3^I \end{bmatrix}$$



$$GAST = 24110.54841 + 8640184.812886 \times Cent + 0.093104 \times Cent^2 - 6.2 \times 10^{-6} \times Cent^3 + \frac{sid}{sol} \times UT$$

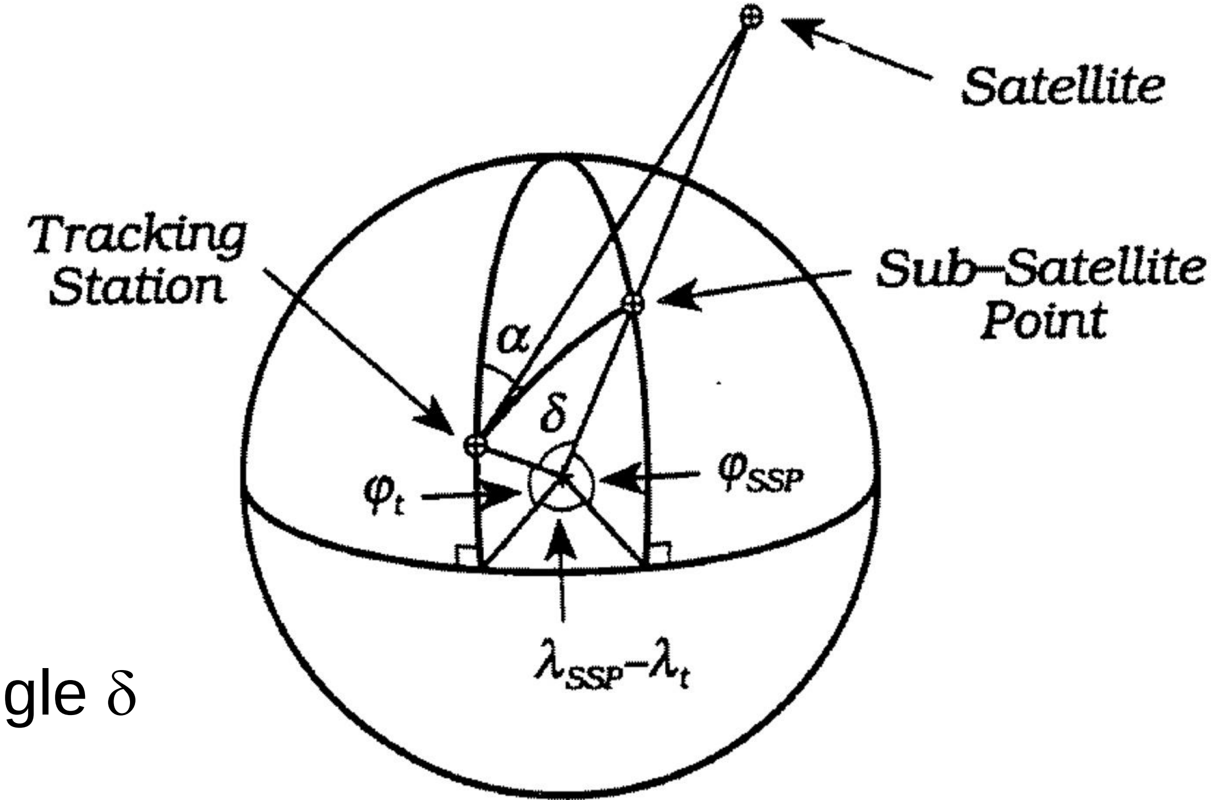




The vector from an earth station to a satellite can be described by two angles ε, α and distance s.

- Elevation ε
- Azimuth α

r_0=R+h



Central angle δ

Latitude and longitude of the sub-satellite point:

$$\phi_{SSP} = \arcsin\left(\sin i \cdot \sin(\omega + \theta_0)\right)$$

$$\lambda_{SSP} = \Omega - GAST + \arctan\left(\cos i \cdot \tan(\omega + \theta_0)\right) + A$$

$$A = 0 \qquad \text{if: } \cos(\omega + \theta_0) \geq 0$$

$$A = \pi \cdot sng\left(\cos i \cdot \sin(\omega + \theta_0)\right) \qquad \text{if: } \cos(\omega + \theta_0) \leq 0$$

Satellite altitude and range:

$$h = r_0 - R \qquad s = \sqrt{h^2 + 4 \cdot R_e \cdot r_0 \cdot \sin^2(\delta/2)}$$

Central angle:

$$\delta = \arccos\left[\sin \phi_{SSP} \cdot \sin \phi_t + \cos \phi_{SSP} \cdot \cos \phi_t \cdot \cos(\lambda_{SSP} - \lambda_t)\right]$$

Elevation and azimuth:

$$\varepsilon = \arctan\left(\frac{\cos\delta - (R/r_0)}{\sin\delta}\right)$$

$$\alpha = \operatorname{sgn}\left(\sin(\lambda_{SSP} - \lambda_t)\right) \cdot \arccos\left(\frac{\sin\phi_{SSP} - \sin\phi_t \cdot \cos\delta}{\cos\phi_t \cdot \sin\delta}\right)$$

Spherical trigonometry:

- ”great circles”, include centre of the sphere e.g. meridians, equator
- Spherical distances measured along ”great circles” (a,b,c)
- Great circles intersect at intersection nodes
- Spherical angles between two planes at intersection nodes
- Spherical triangles are bound by great circles (A,B,C)

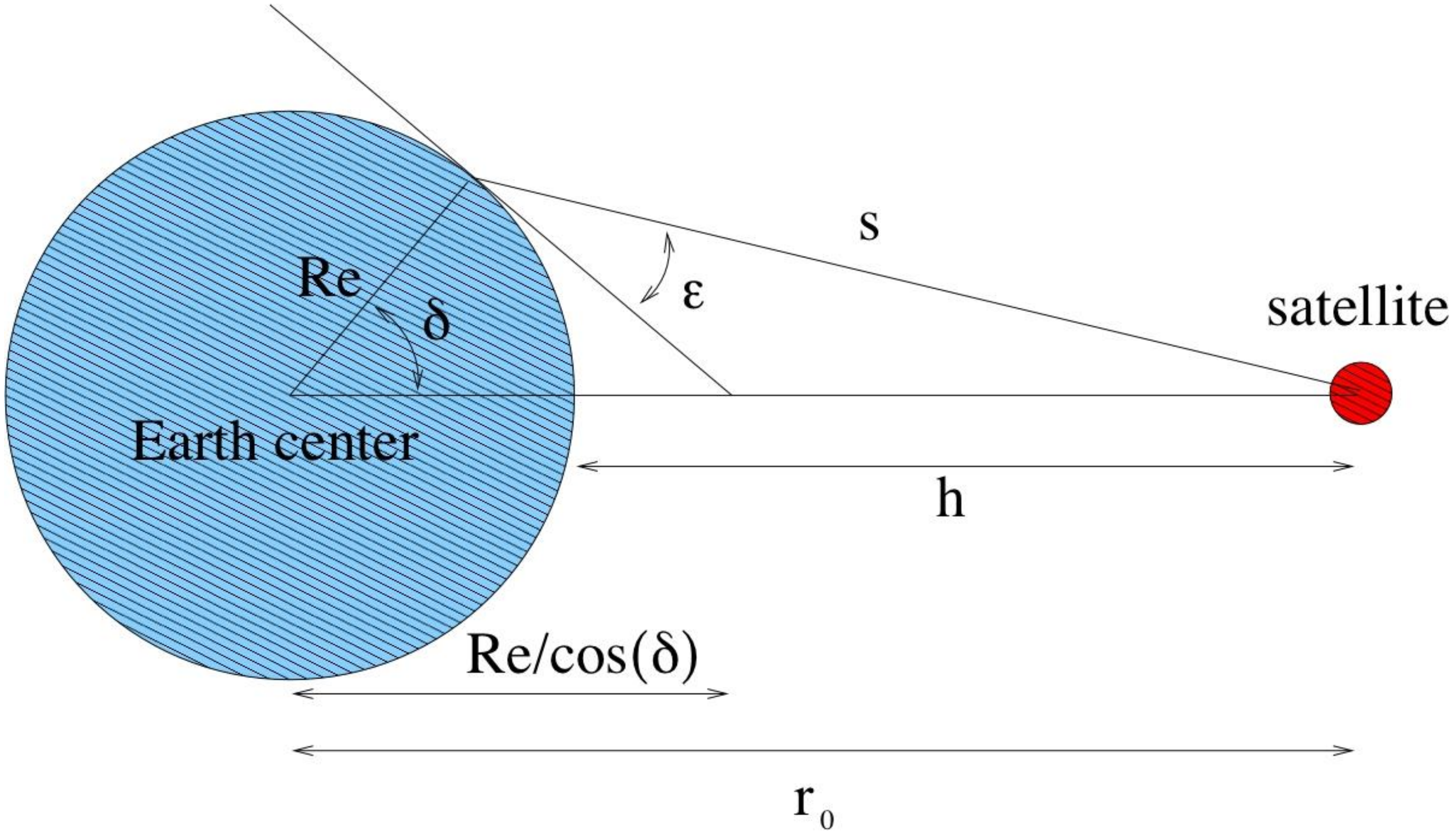
- Spherical sine-law: $$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$$

- Spherical cosine-law: $$\cos a = \cos b \times \cos c + \sin b \times \sin c \times \cos A$$

- Spherical ”third theorem”: $$\sin a \times \sin B = -\sin b \times \sin c \times \cos A + \cos b \times \sin c$$

Satellite visibility:

- The satellite can only be visible when it is above the local horizon, i.e. the elevation must be >0
- Visible if $r_0 > R_e/\cos\delta$



Coverage aspects:

D_A = coverage arc length of one satellite

A = area of spherical cap covered by one satellite

A/Ae= ratio w.r.t. total earth surface

N = number of satellites needed for global coverage,
assuming 21% overlap

P = minimum number of orbital planes needed

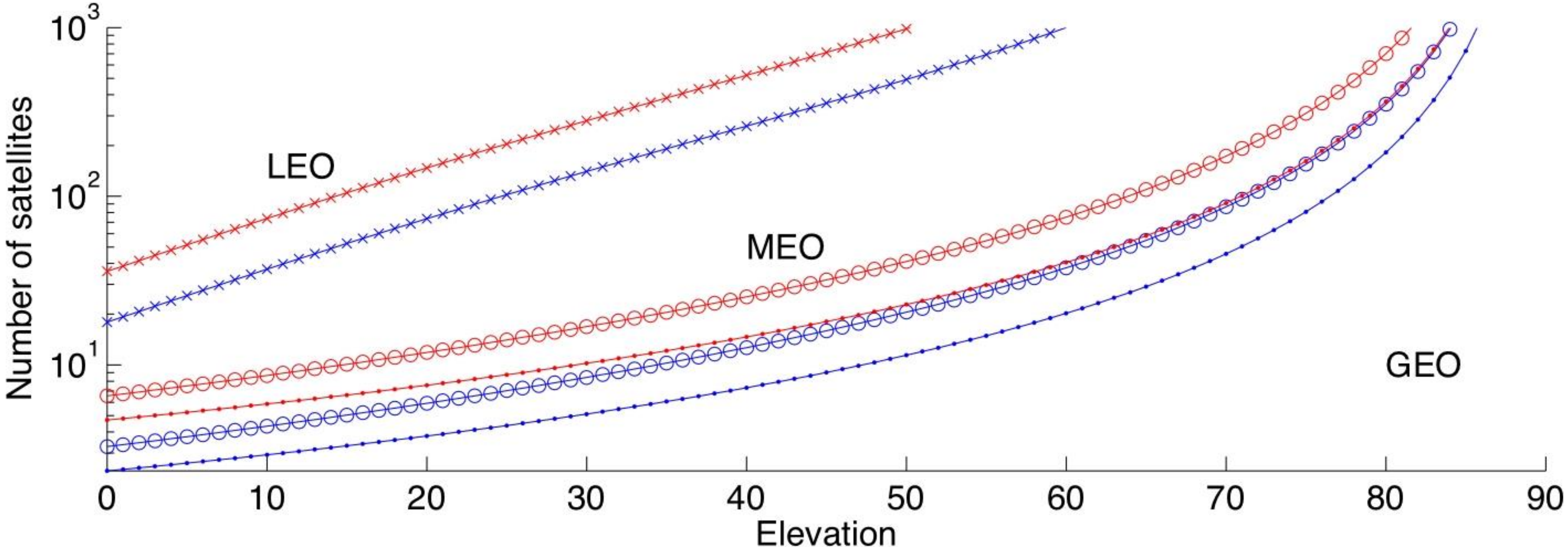
$$D_A = 2 \times \delta \times R_e$$

$$A = 2 \times \pi \times R_e^2 \times (1 - \cos\delta)$$

$$\frac{A}{A_e} = \frac{1}{2}(1 - \cos\delta)$$

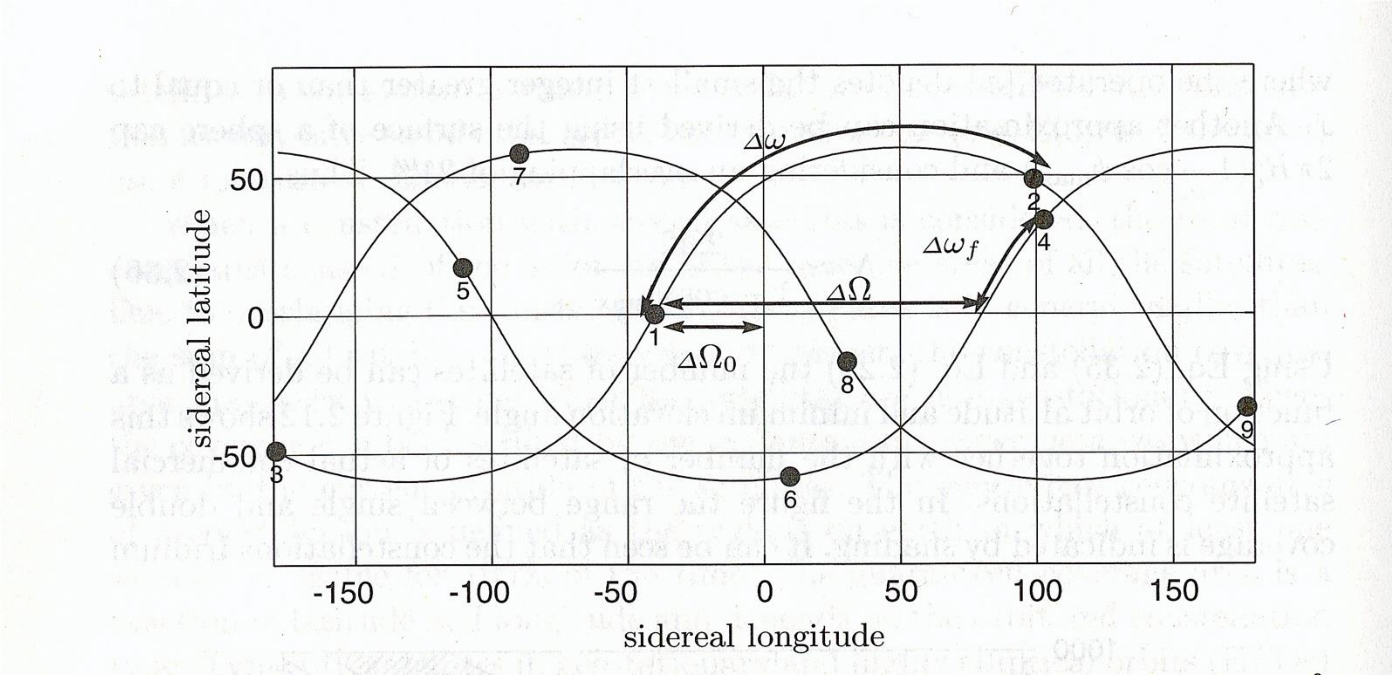
$$N \approx \frac{2.42}{1 - \cos\delta}$$

$$P = \frac{2 \times \pi}{3 \times \delta}$$



- Inclined Walker constellation
  - Walker notation N/P/F
  - Number of satellites N, number of orbital planes P, phasing factor F (determines angular offset between satellites in adjacent orbit planes)

$$\Delta\omega_f = 2 \cdot \pi \cdot \frac{F}{N}$$



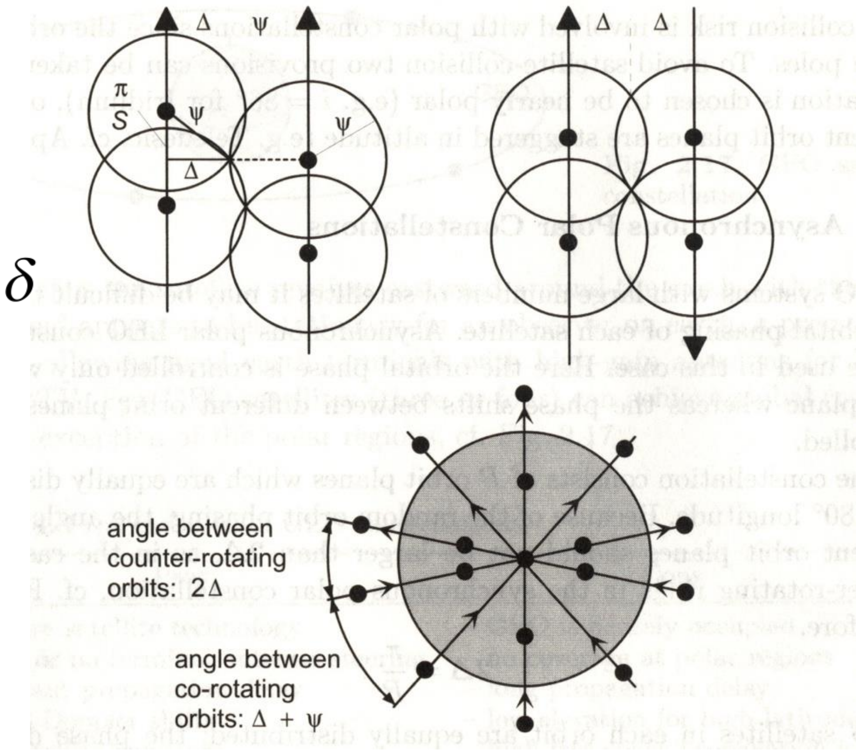
Walker constellation 9/3/1, Ref. Lutz-Werner-Jahn, 2000.

- Polar constellations:
  - Number of required satellites N
  - Number of orbital planes P
  - Number of satellites per plane S=N/P
  - Satellite shift in co-rotating orbits by π/S
  - Continuous coverage requires overlap

$$N \approx \frac{4}{1-\cos\alpha}$$

$$\cos\Delta = \frac{\cos\delta}{\cos(\pi/S)}$$

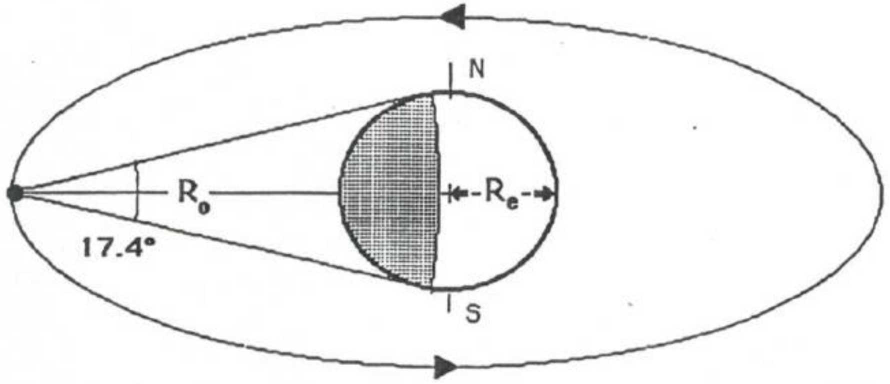
$$\psi = \delta$$



Co-rotating and counter-rotating polar orbits.
Ref. Lutz-Werner-Jahn, 2000

GEOSTATIONARY SATELLITE ORBIT

The satellite orbits in the equatorial plane (inclination i = 0°) on a circular orbit at an angular velocity equal to that of the Earth.



R_e = Earth radius = 6 378 km

R_o = satellite altitude = 35 786 km

The Earth is viewed from the satellite with an angle equal to 17.4°.

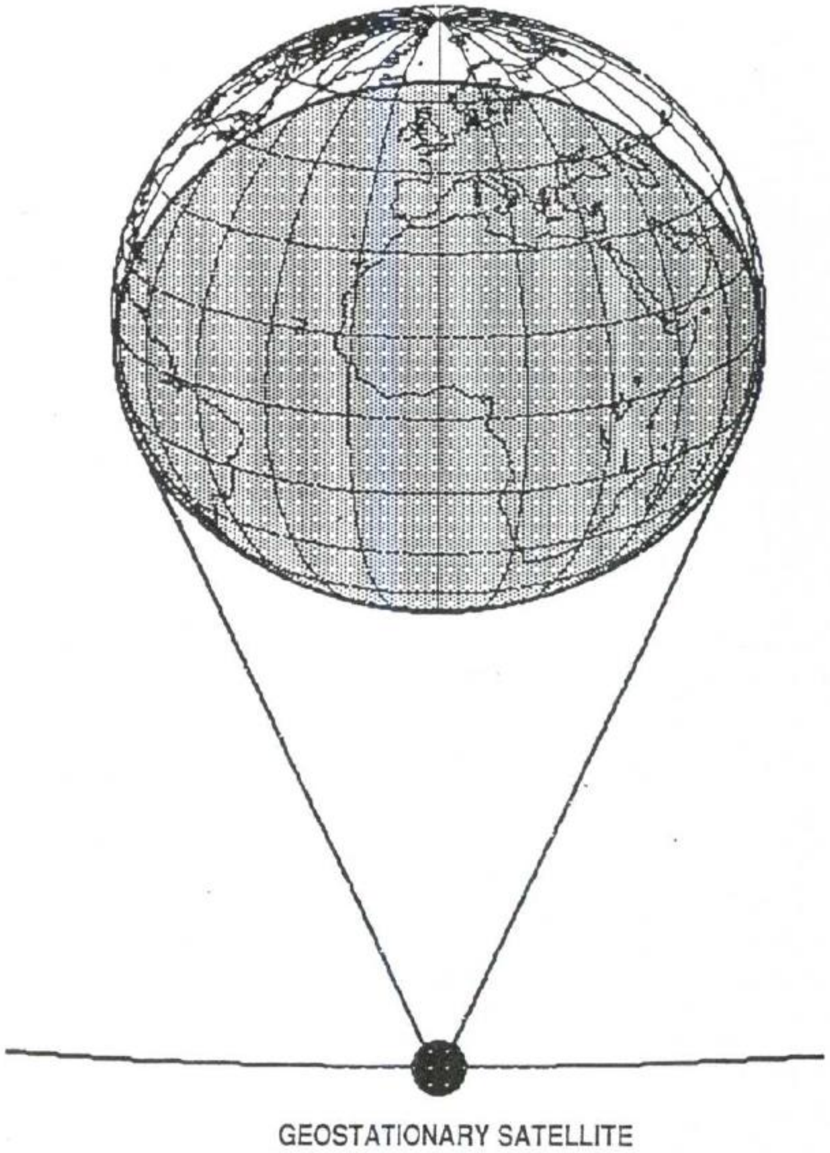
PROPAGATION TIME DELAY on a earth-satellite-earth link is comprised between :

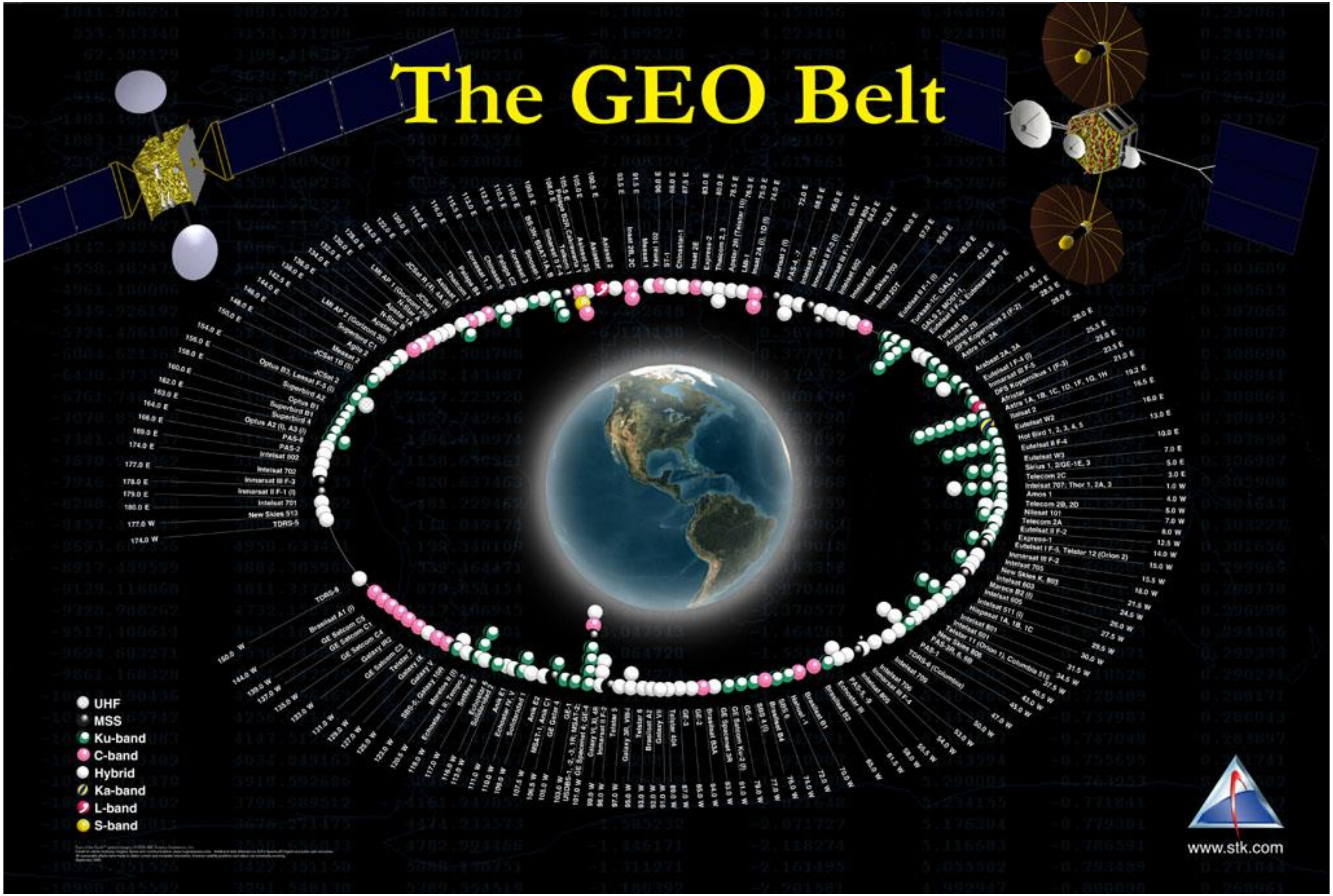
min = $2\,R_o/c$ = 238 ms

max = $2\,((R_o + R_e)/c)\cos(17.4°/2)$ = 278 ms

where c = speed of light = 3×10^8 m/s

GEOSTATIONARY SATELLITE COVERAGE

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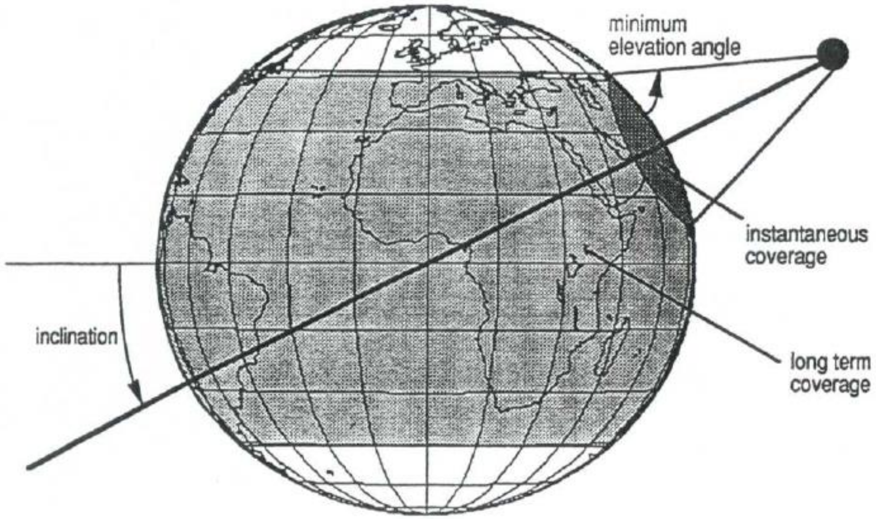
CIRCULAR NON POLAR LOW EARTH ORBIT (LEO)



The LONG TERM COVERAGE consists of an annular zone about the equator extending to latitudes depending on :

- the inclination of the orbit,

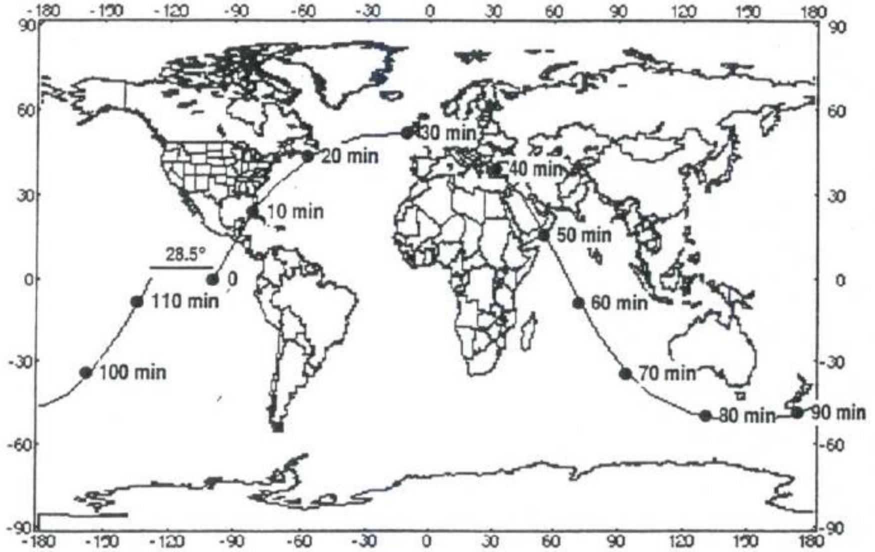
- the minimum elevation angle,

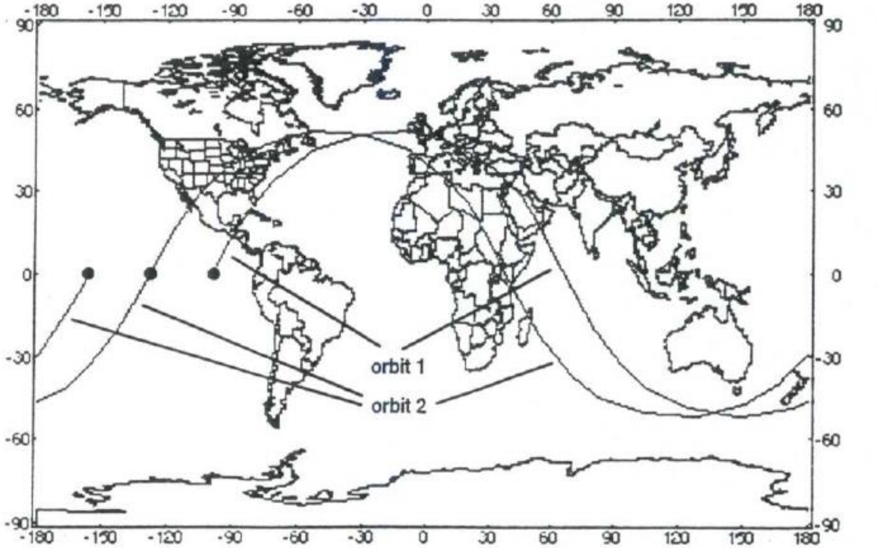
- the altitude of the satellite.

CIRCULAR NON POLAR LOW EARTH ORBIT (LEO)

Example : GLOBALSTAR orbit

Semimajor axis : a = 7792km , Period : 1hr 54 min 05 s = 114 min 05 s = 6845 s
Altitude : 1414 km
Inclination : i = 52°





CIRCULAR POLAR LOW EARTH ORBIT (LEO)

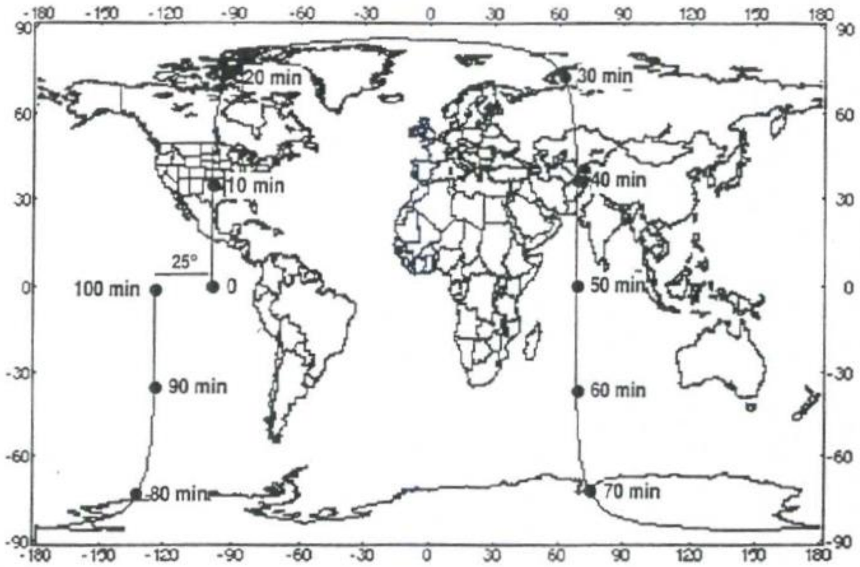
Inclination i = 90° (or near 90°)

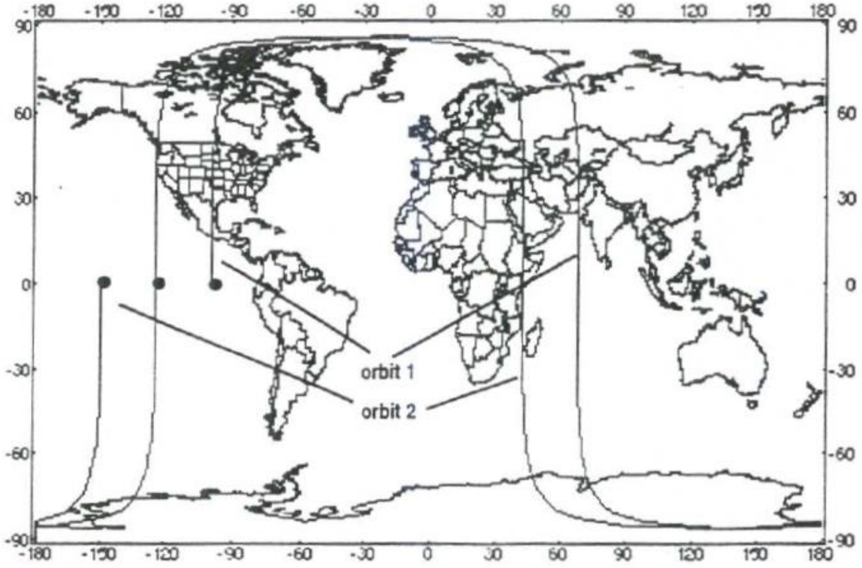
The LONG TERM COVERAGE is a world-wide one.

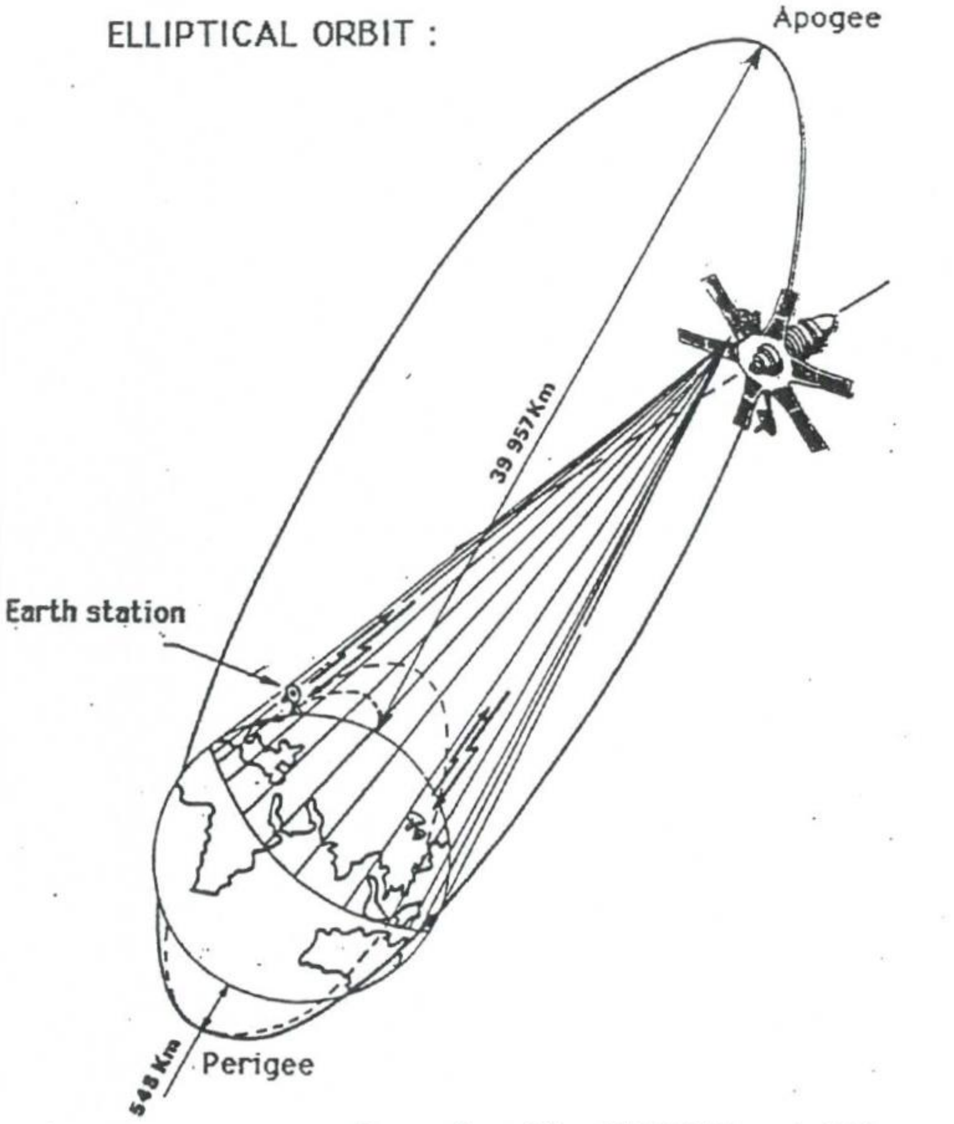
CIRCULAR POLAR LOW EARTH ORBIT (LEO)

Example : IRIDIUM orbit

Semimajor axis : a = 7158.8km , Period : 1hr 40 min 28 s =100 min 28s = 6028 s
Altitude : 780 km
Inclination : i = 86.4°

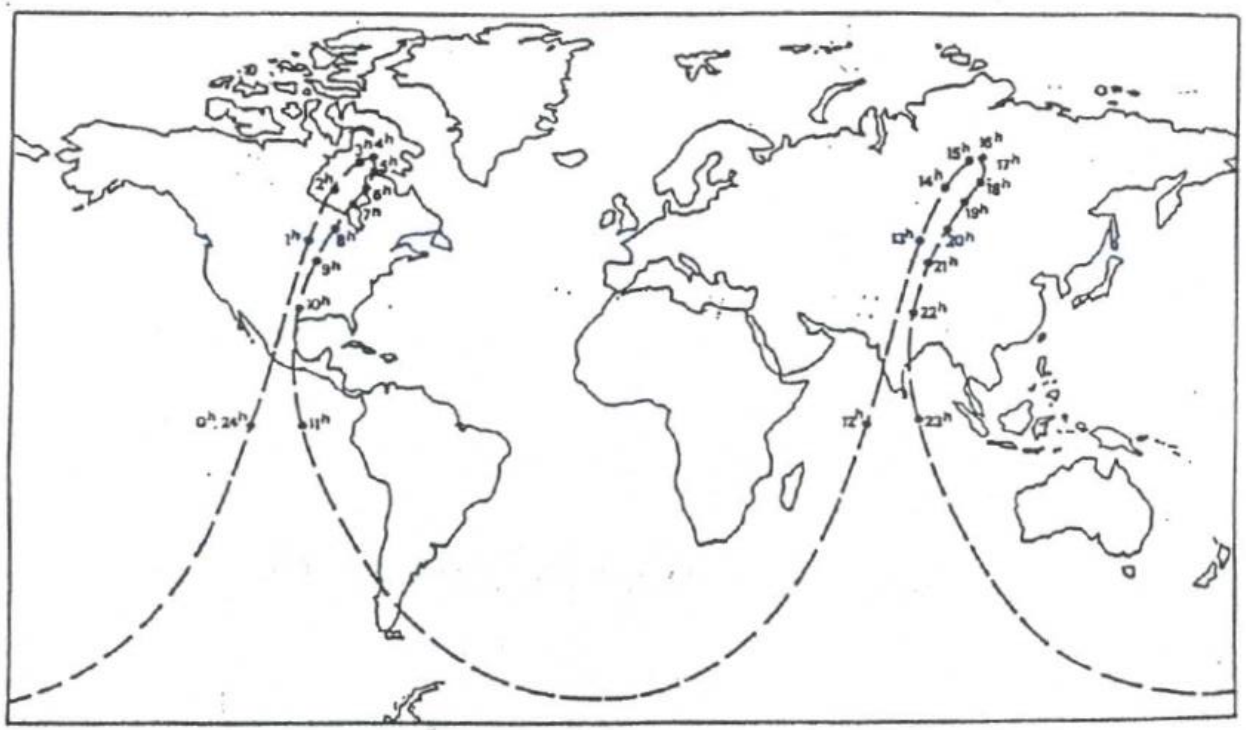






Example : The MOLNYA satellite system

Molnya satellite with i=64 deg, P=12 h



- **Orbital perturbations:**
  - Earth is not perfectly spherical, but more like a ”potatoe”
  - Gravitational attraction varies with position

  - ”Nodal regression”: change in right ascension

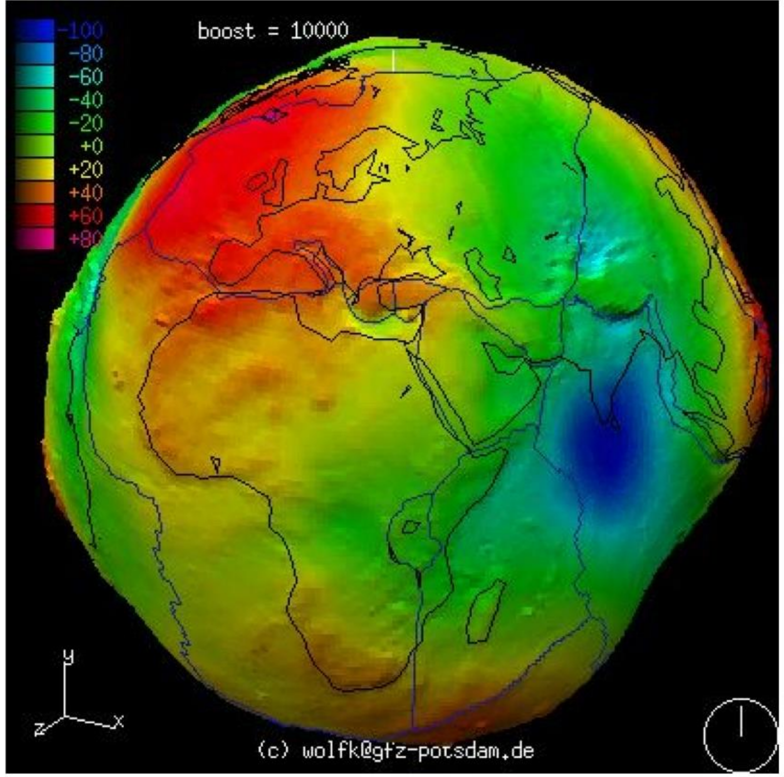
$$\frac{d\Omega}{dt} = -\frac{3}{2}\eta_m \frac{R_e^{\ 2}}{a^2(1-e^2)^2} J_2 \cos i$$

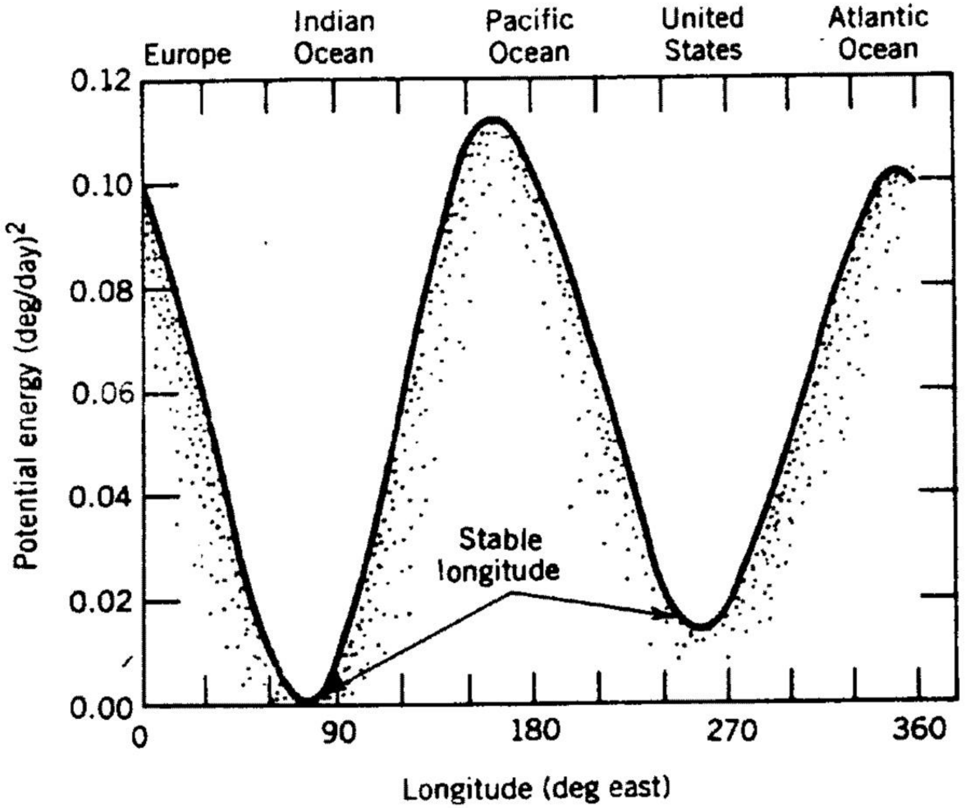
  - ”Apsidal rotation”: change in argument of perigee

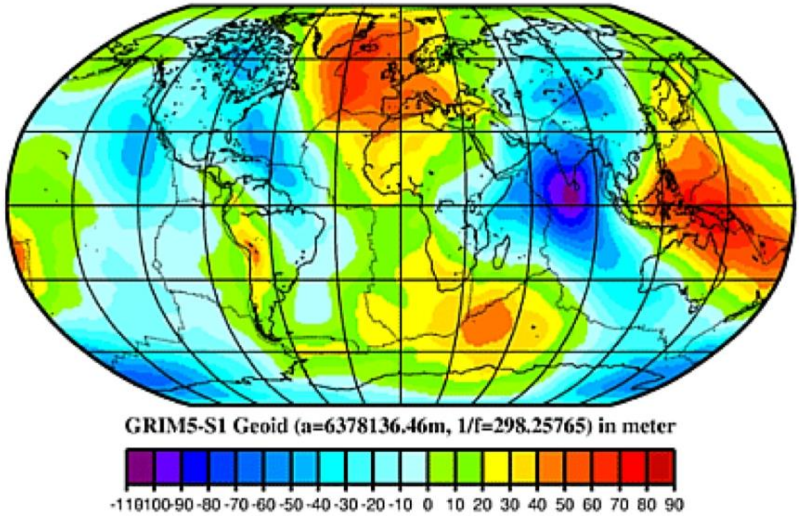
$$\frac{dw}{dt} = \frac{3}{4} h_m \frac{R_e^{\ 2}}{a^2(1-\ e^2)^2} J_2 \left(5 \times \cos^2 i - 1\right)$$

  - Change in mean anomaly:

$$\frac{dM}{dt} = \eta_m \left(1 + \frac{3}{4}\frac{R_e^{\ 2}}{a^2(1-e^2)^2}\sqrt{1-e} \cdot J_2\left(3 \cdot \cos^2 i - 1\right)\right)$$



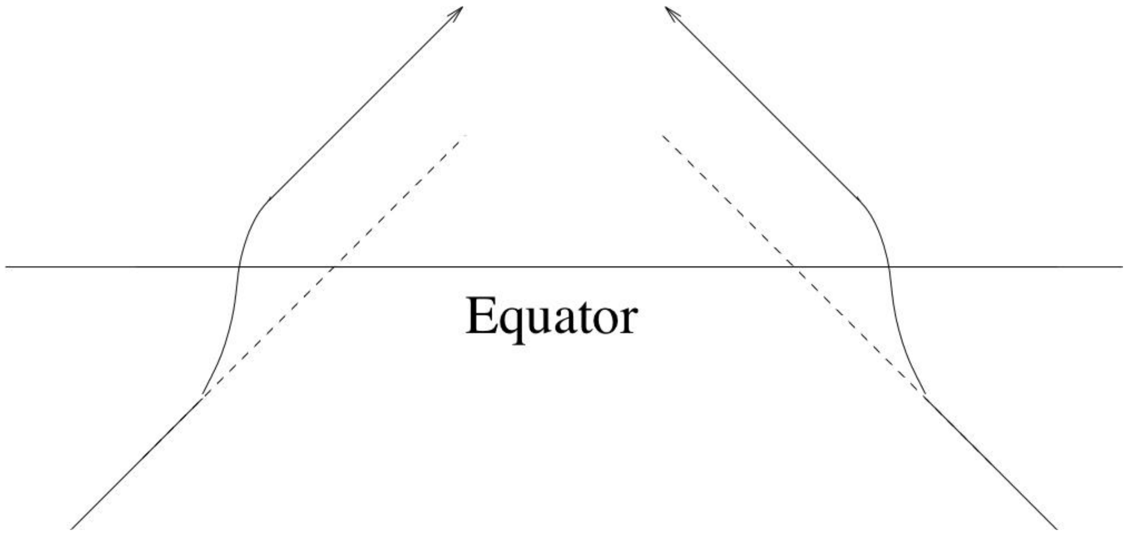


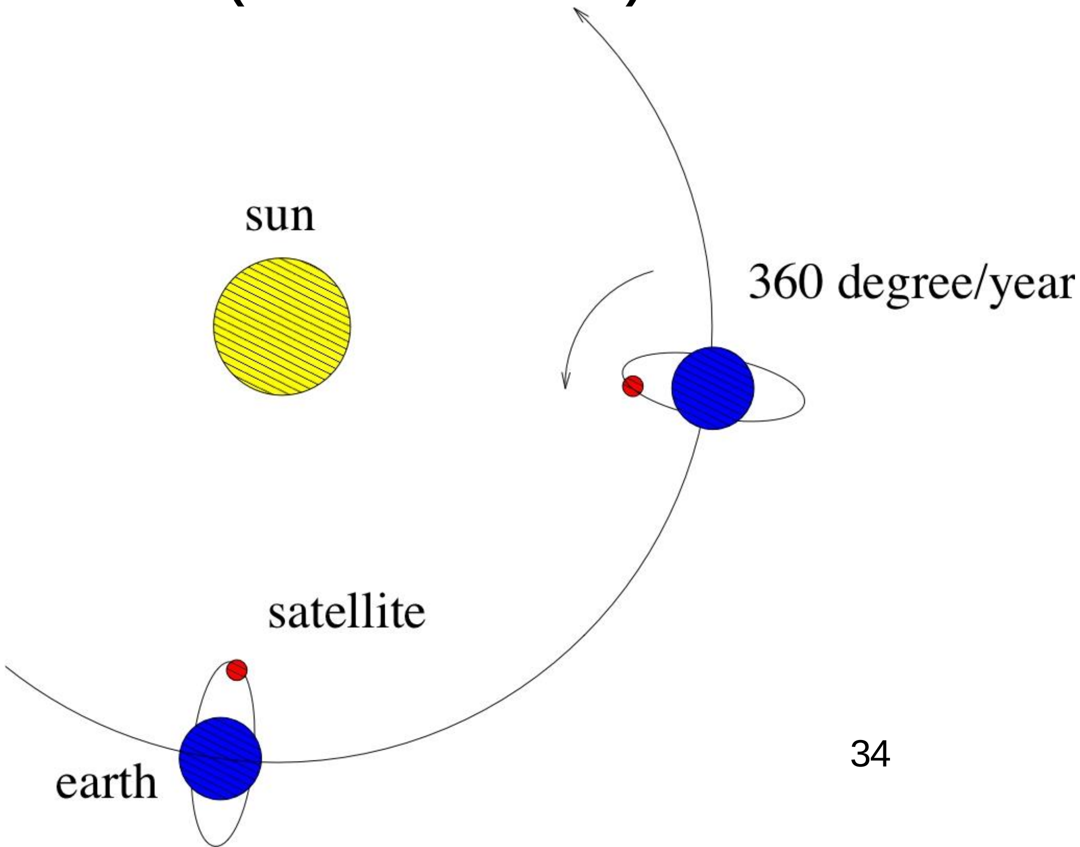


Pictures taken from GeoForschungsZentrum Potsdam

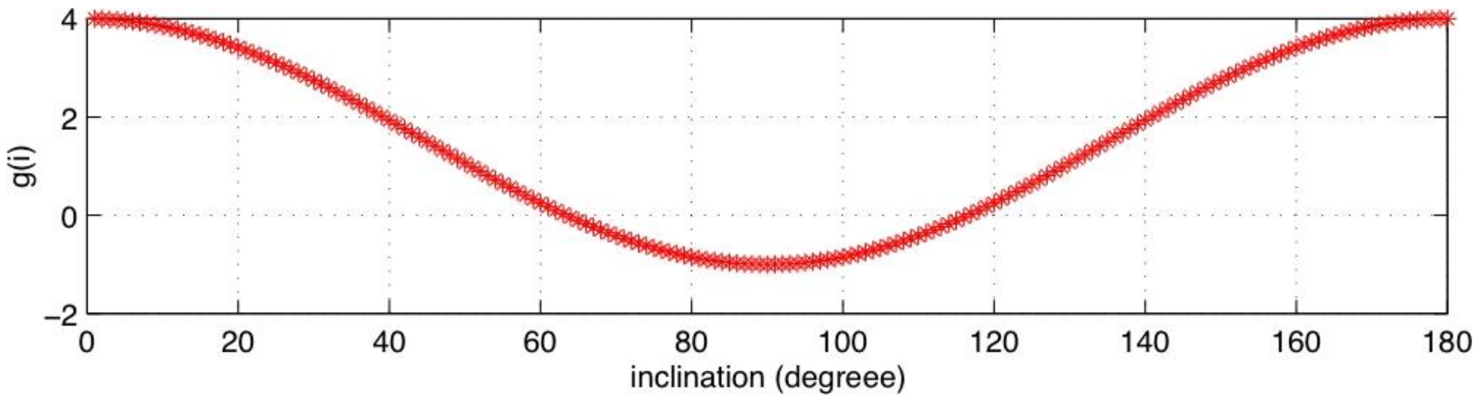
- The geopotential field causes longitudal drift of geostationary satellites.
- Satellites drift to ”graveyards”
- Continuous manoeuvers necessary to keep the geostationary satellites in their station keeping boxes.

- **Nodal regression:**
  - Satellites get deflected due to equatorial bulge
  - Ascending node moves westward for prograde orbits (0<i<90) and eastwards for retrograde orbits (90<i<180)
  - Makes sun-synchronous orbits possible (h<6000 km)



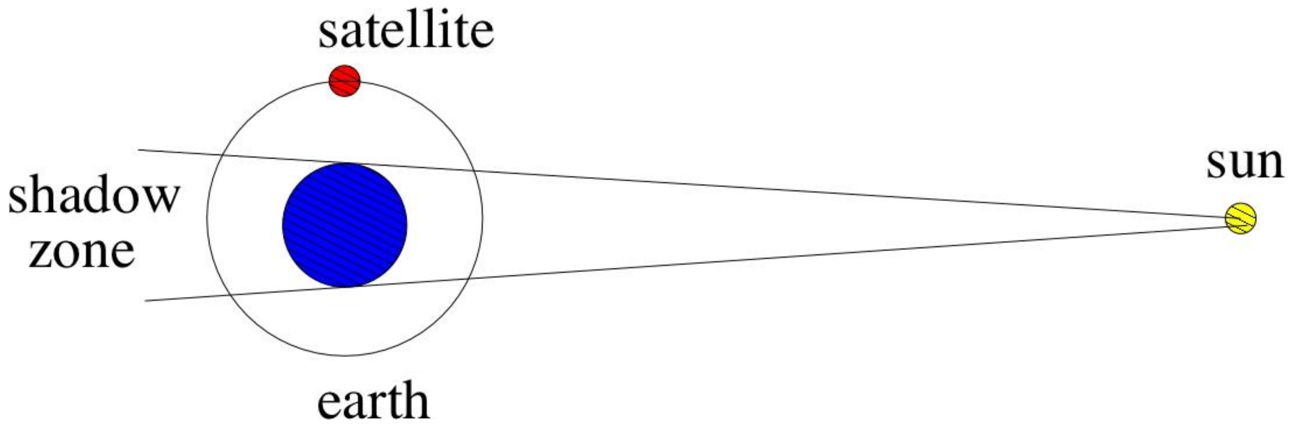


- **Apsidal rotation:**
  - Perigee moves in the orbital plane
  - In principle apogee and perigee can change position
  - Depends on inclination
  - Two inclinations (i=63.435°, i=116.565°) where the effect is zero
  - Molniya orbits have these inclinations



- **Third body effects and tides:**
  - Direct attration due to moon, sun, planets
  - Indirect effects due solid earth tides, ocean tides, etc.

- **Solar radiation pressure:**
  - Depends on amount of solar radiation on the satellite
  - Depends on mass and exposed surface area
  - Depends on path in the earth's shadow



- **Atmospheric drag:**
  - Depends on density of the atmosphere
  - Depends on mass and dimensions
  - Depends on satellite speed

Near earth environment:

- Charged particles in Van Allen belts
- Dangerous environment for satellite electronics
- Require satellite protection

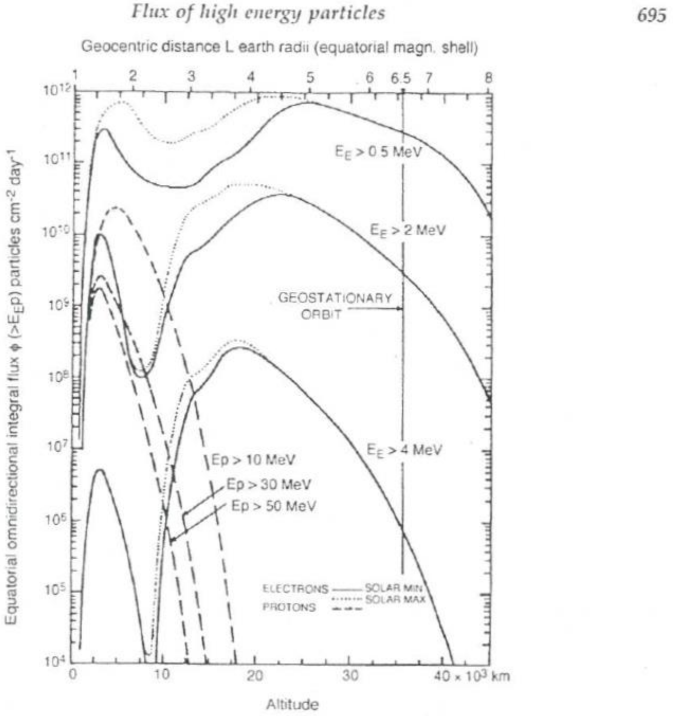
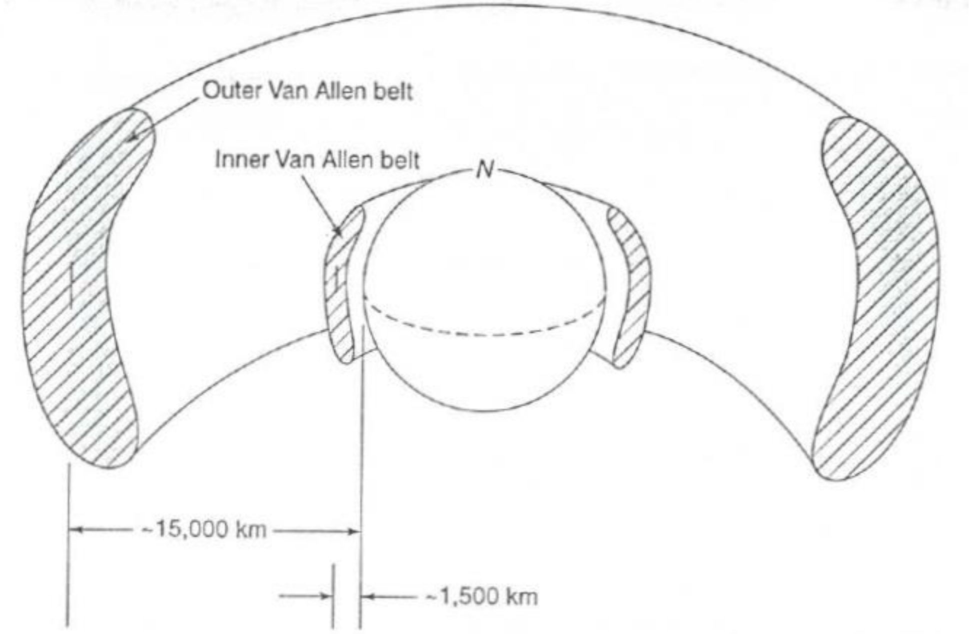
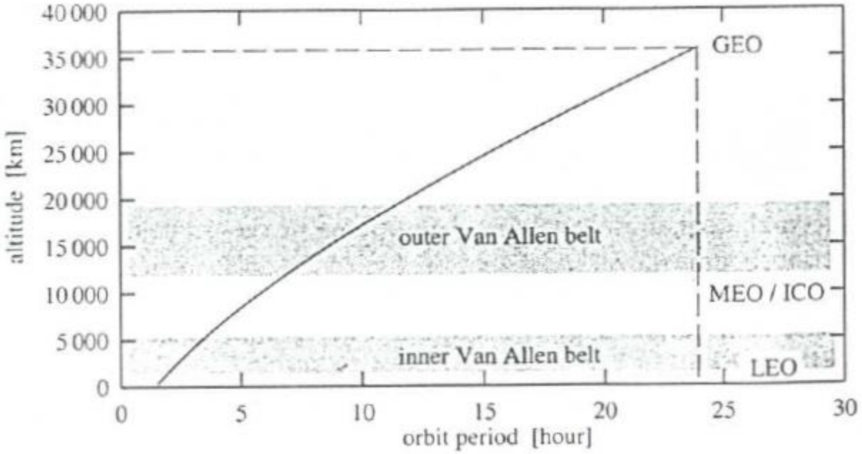
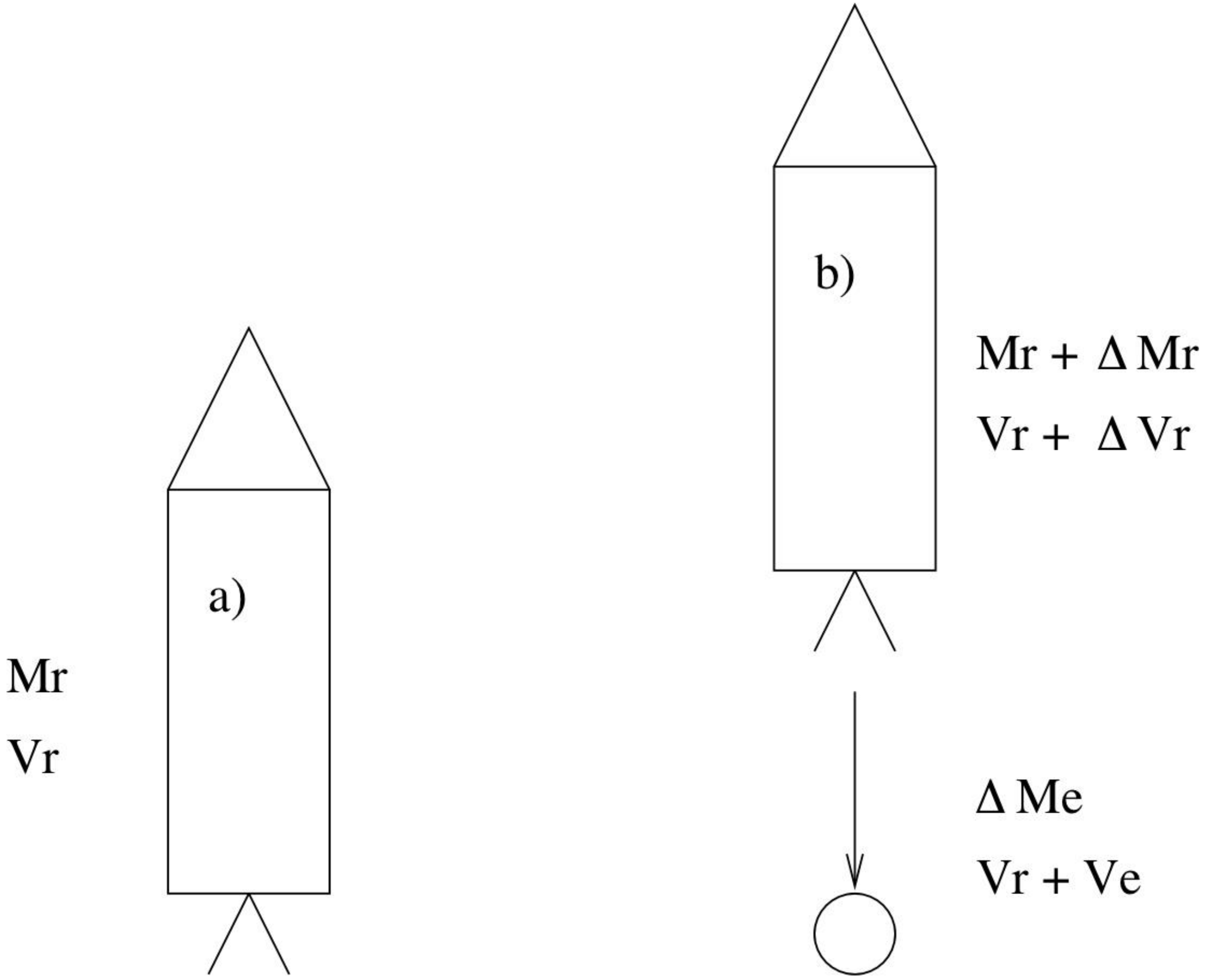


Figure 12.4 Earth's trapped electron and proton radial flux profiles [CRA-94]. (Reproduced with permission; © 1994 Elsevier.)

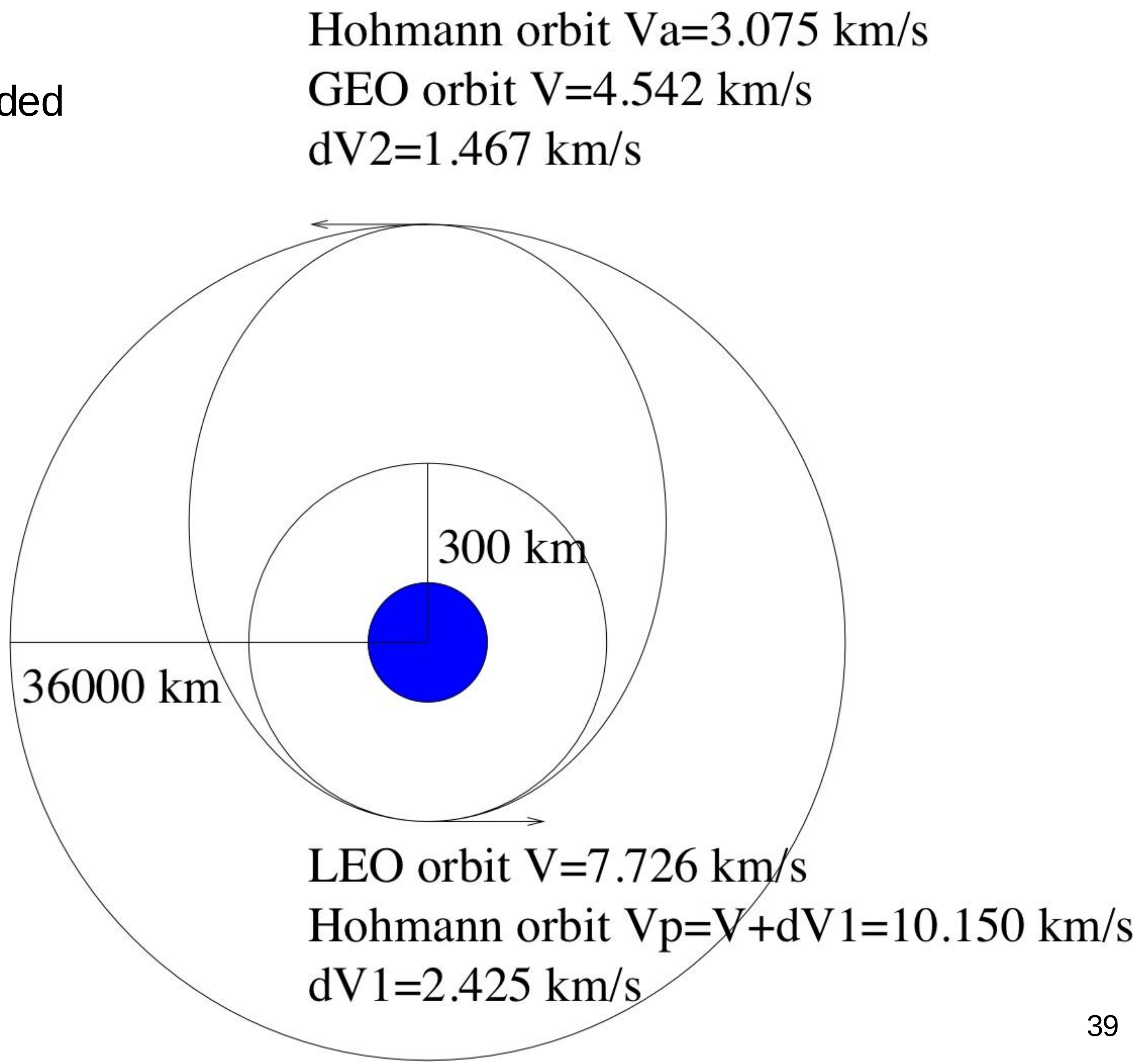


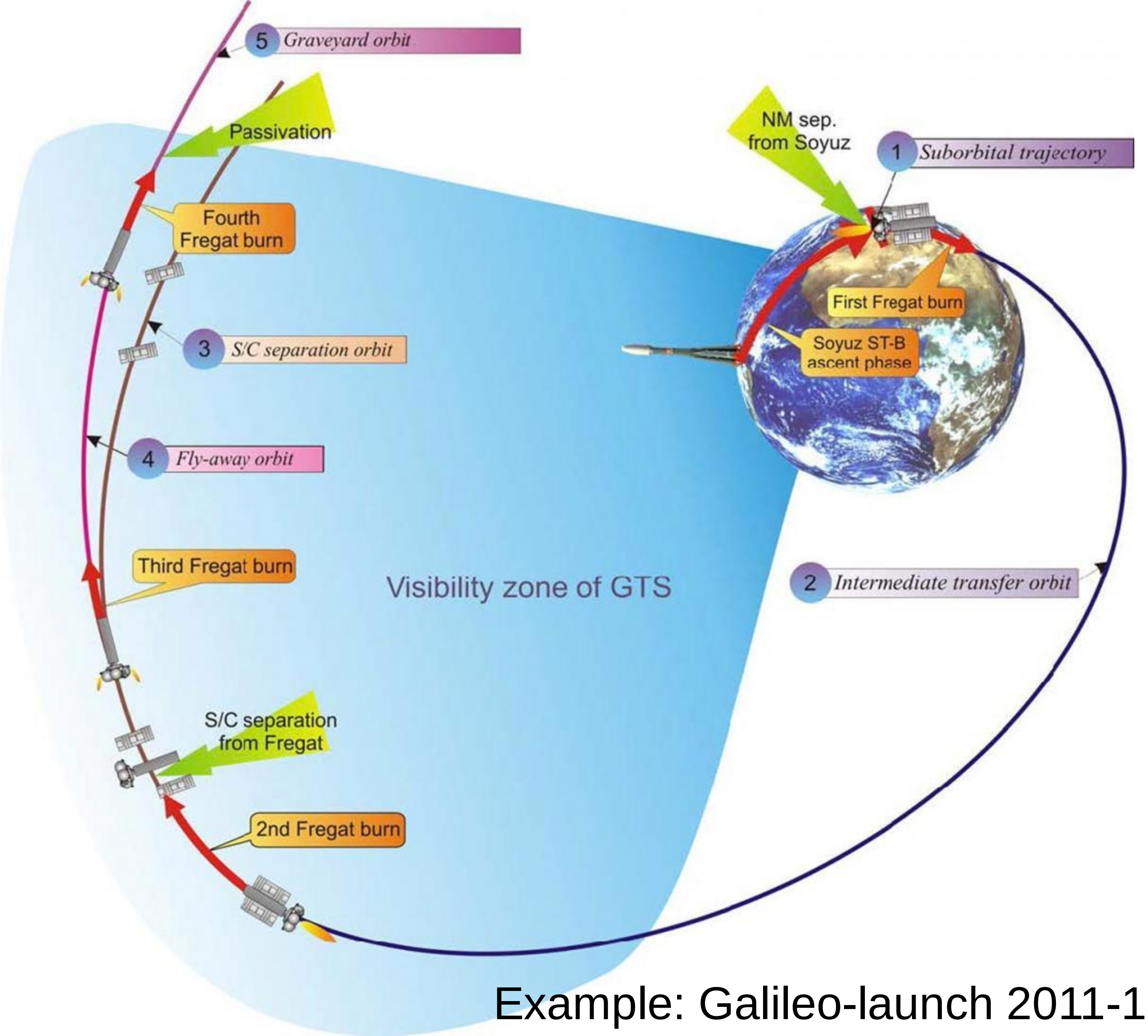




Principle of a rocket: Ejecting mass accelerates the rocket.

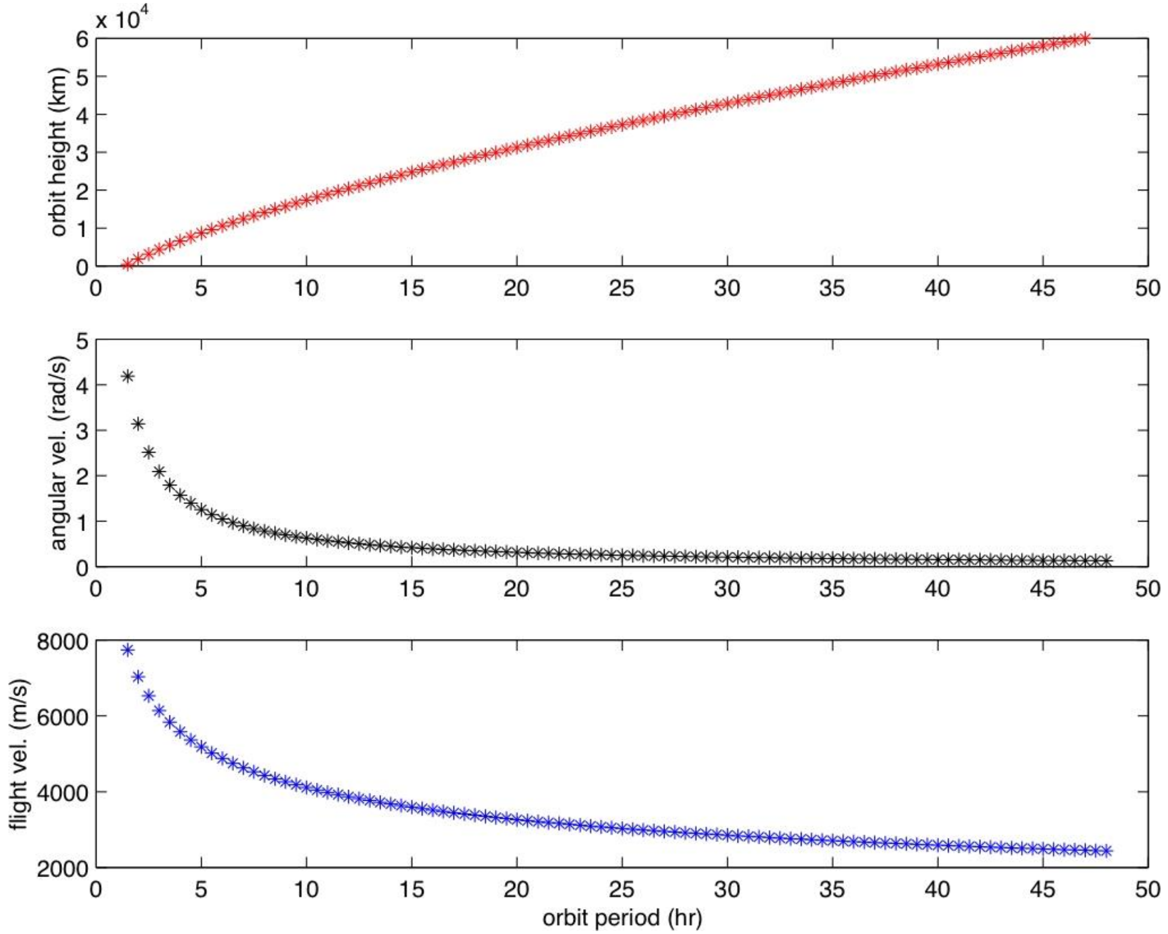
Transfer orbits needed to reach the geostationary orbit.





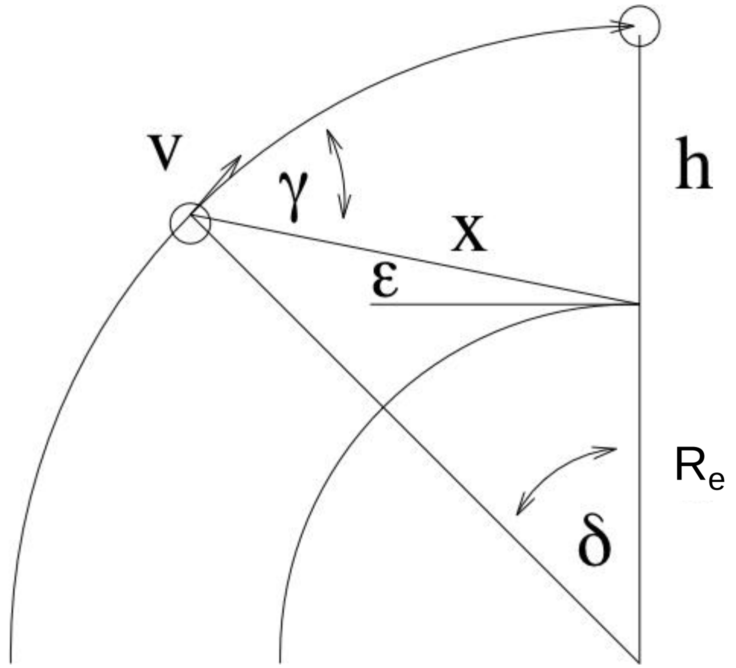
Example: Galileo-launch 2011-10-21

height (km)	radius (km)	period (s)	velocity (m/s)	type
200	6578	5309	7784	LEO
290	6668	5419	7732	LEO
800	7178	6052	7450	LEO
20000	26378	42636	3887	MEO
35786	42164	86164	3075	GEO



Doppler shift:

- Frequency of the radio signal
- Changes when distance between satellite and earth station changes
- Function of velocity
- Function of elevation
- Receivers need to handle it



$$df = \frac{f_{SL}}{c} \cdot \left(\frac{R_e}{R_e + h} \right)^2 \cdot \sqrt{g_0 \cdot (R_e + h)} \cdot \cos\varepsilon$$

	LEO	MEO
	800 km	20000 km
f(GHz)	df(Hz)	df(Hz)
1	698	98
5	3489	495
10	6978	989
15	10467	1484

Short summary of today's topics

- Short history of science of orbits
- Kepler's laws, Newton's laws, law of gravitation
- Two-body problem
- Kepler elements, transformations
- Visibility and coverage
- Examples of useful orbits
- Orbit perturbations
- Doppler effects for LEO satellites