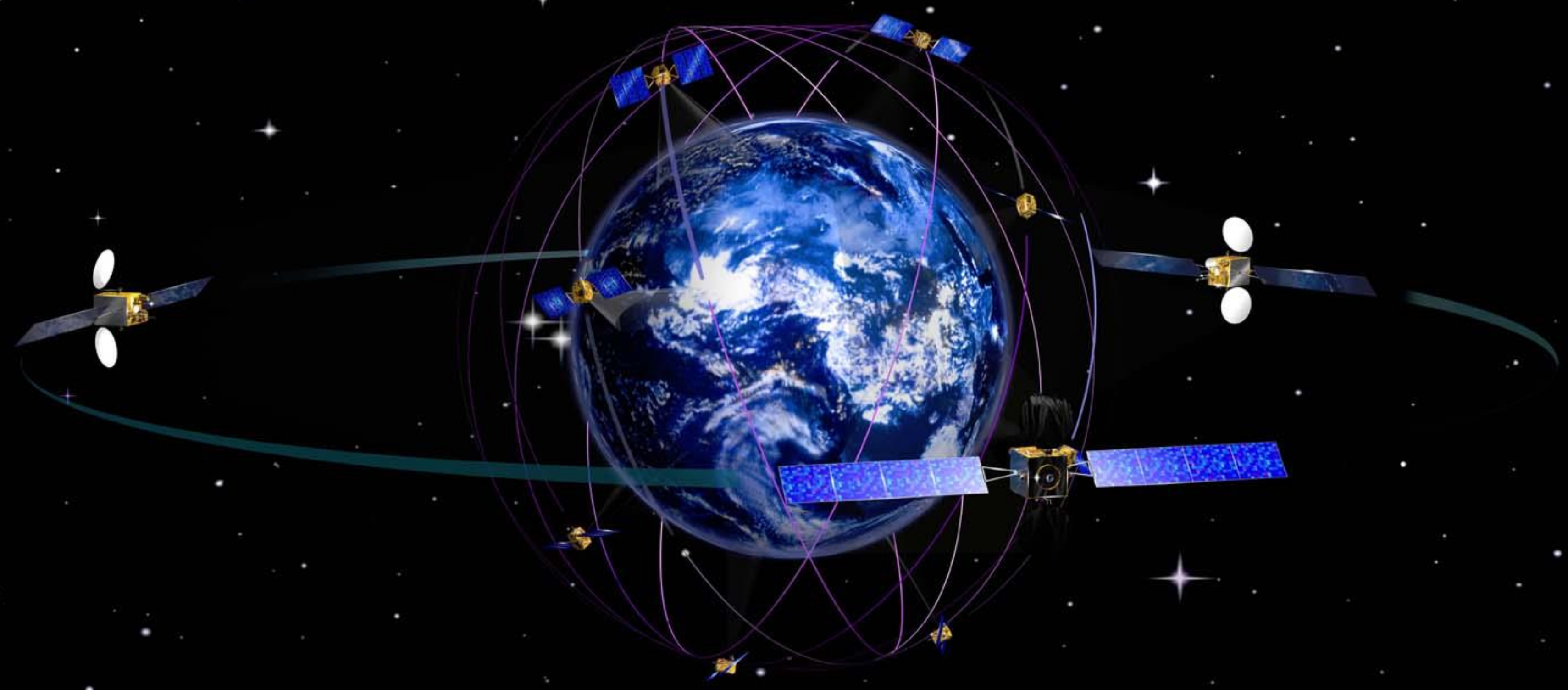
Satellite Communications
– RRY100 –

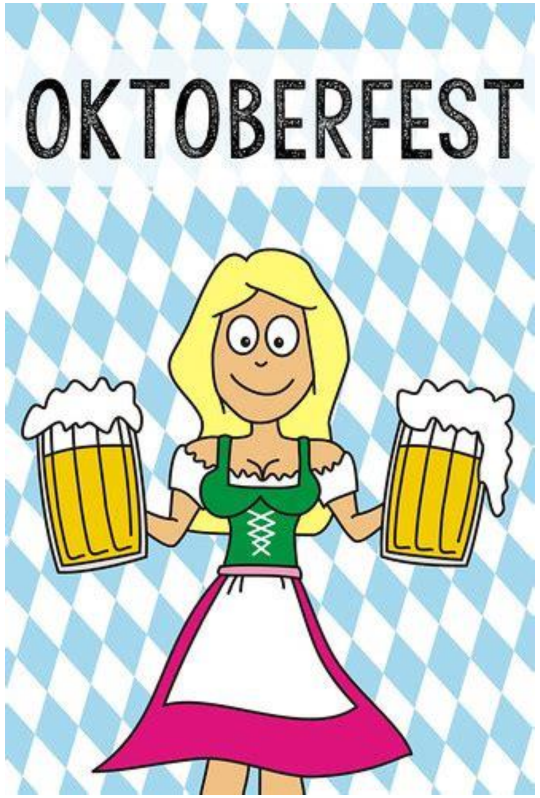


2024 Study Period 1
Lecturer: Rüdiger Haas

Lecture-09: Modulation and multiplex

What was special on the last weekend?

No, it is not
(only) Oktoberfest...



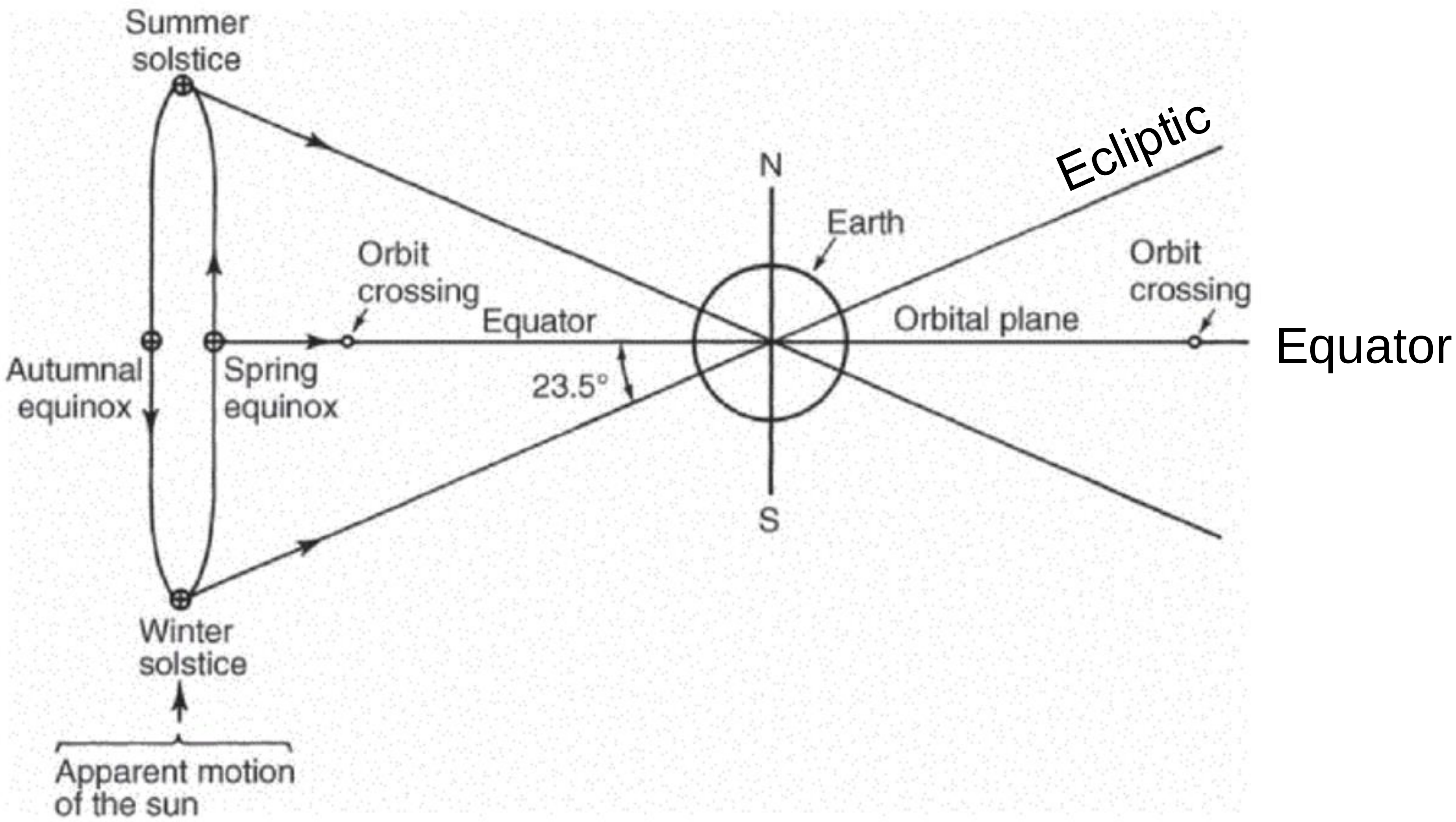


Figure 2.28 Apparent movement of the sun with respect to the orbit of geostationary satellites.

Reference: Maral, Gérard Michel, Bousquet Sun, Zhili. (2020). *Satellite Communications Systems - Systems, Techniques and Technology (6th Edition).* John Wiley & Sons. Retrieved from https://app.knovel.com/hotlink/toc/id:kpSCSSTT02/satellite-communications/satellite-communications

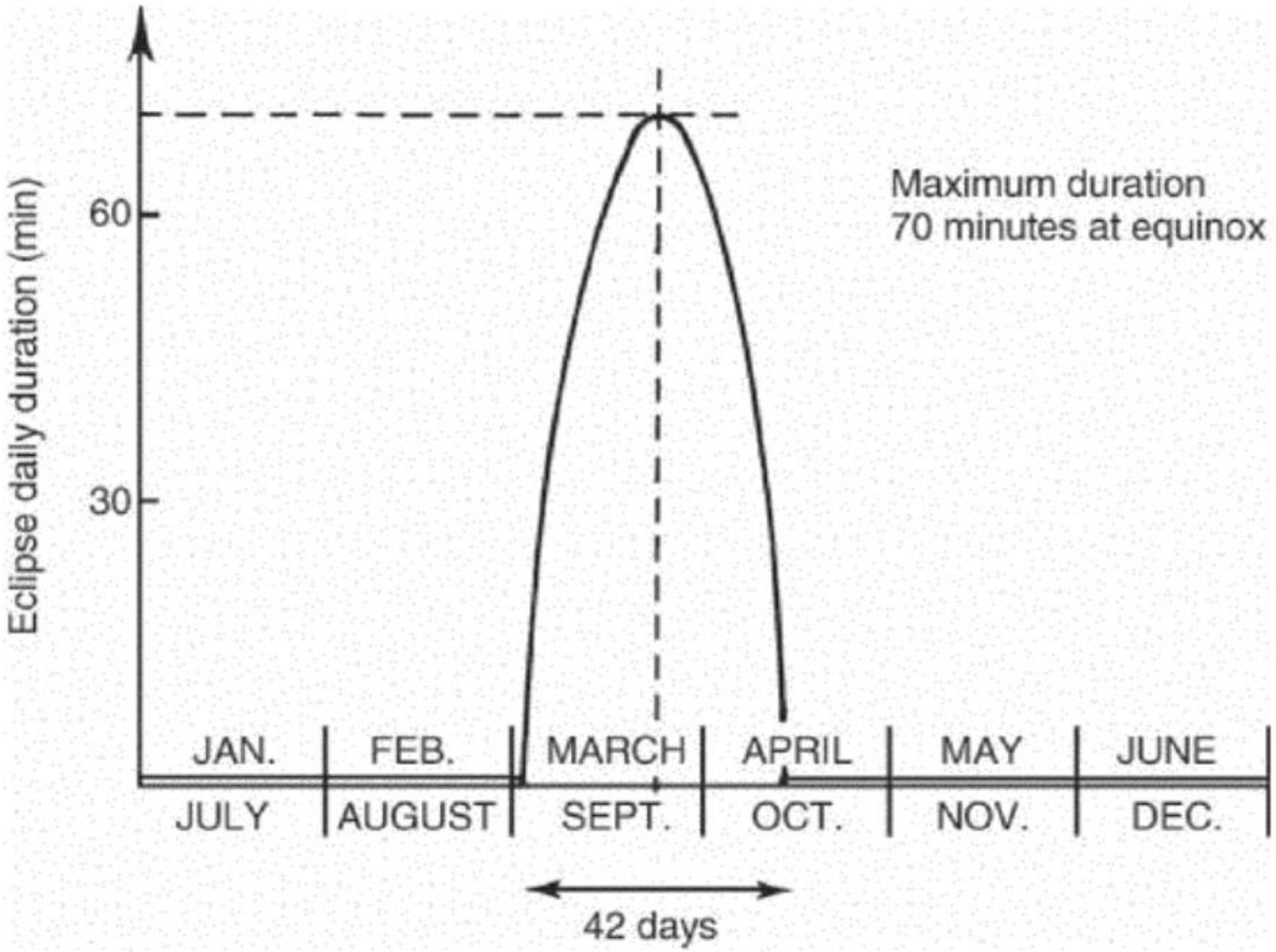
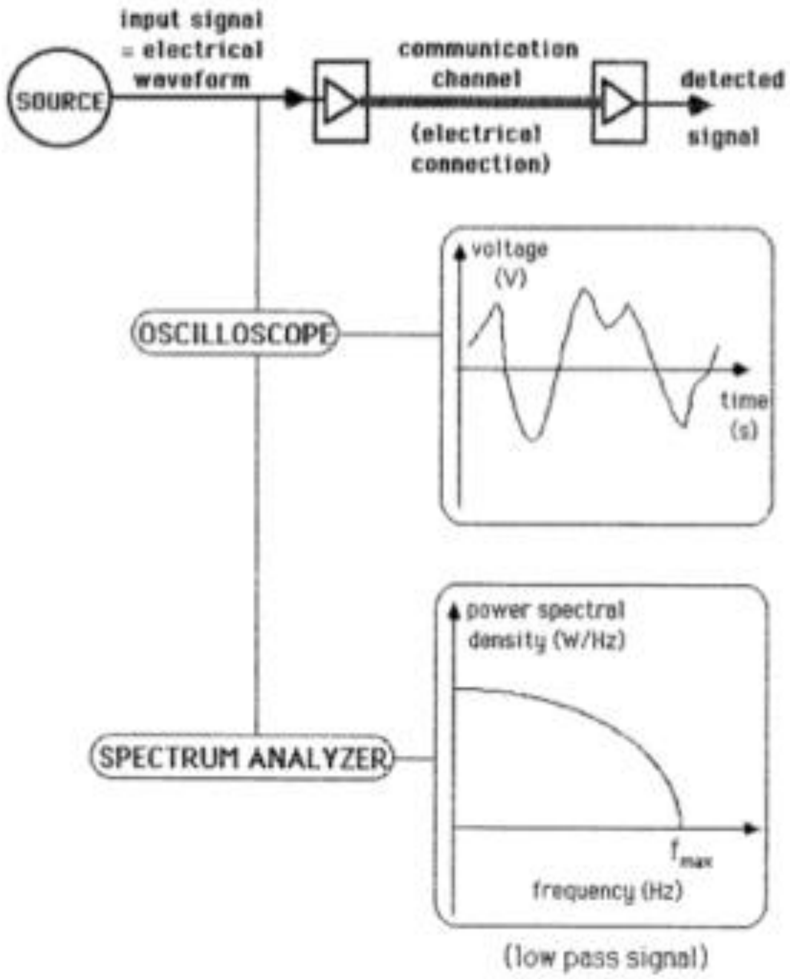
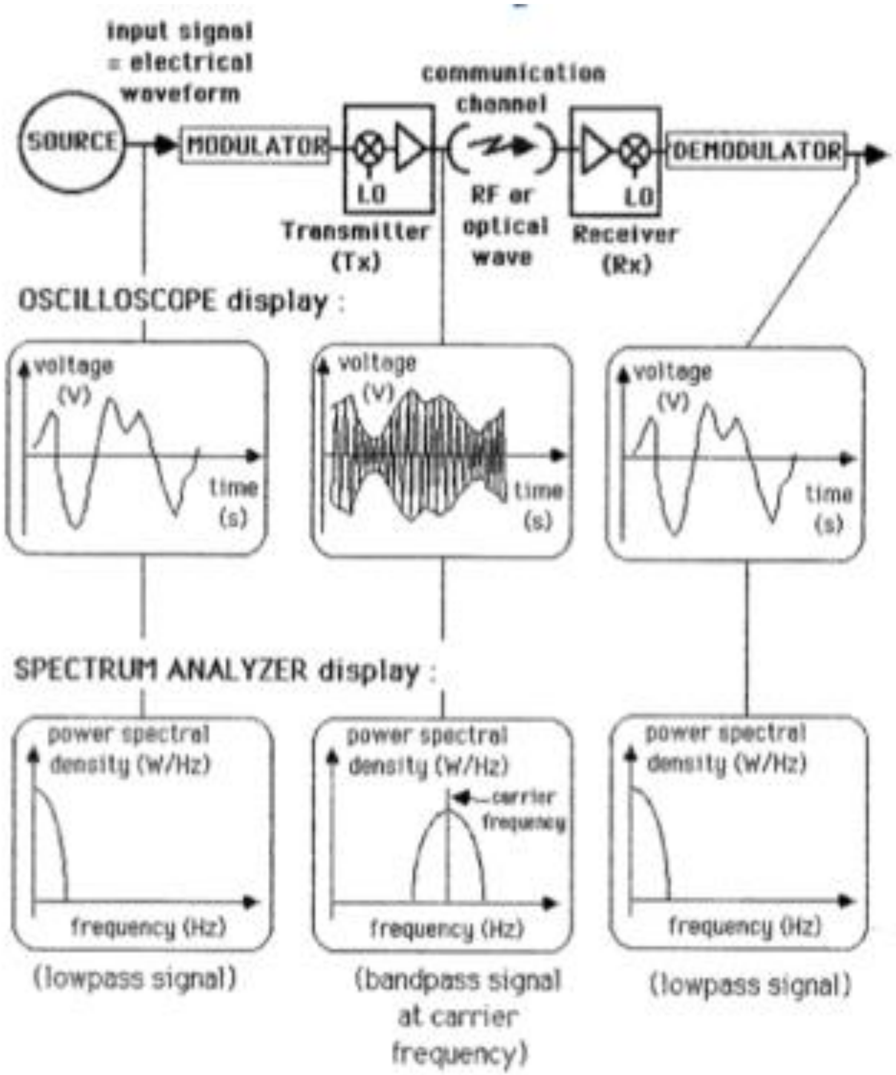


Figure 2.32 Daily duration of eclipses as a function of the date (simplified assumptions).

Reference: Maral, Gérard Michel, Bousquet Sun, Zhili. (2020). *Satellite Communications Systems - Systems, Techniques and Technology (6th Edition).* John Wiley & Sons. Retrieved from https://app.knovel.com/hotlink/toc/id:kpSCSSTT02/satellite-communications/satellite-communications

Baseband vs. RF transmission:





- **Modulation:**
  - Superimposing signals on a high frequency carrier
    - Amplitude modulation (AM)
      - Varying the amplitude of the carrier signal
      - Old technique
      - Suspect to interference, noise and non-linearity

$$\frac{S}{N} = m^2 \cdot \frac{C}{N}$$

m=modulation index [0 1]

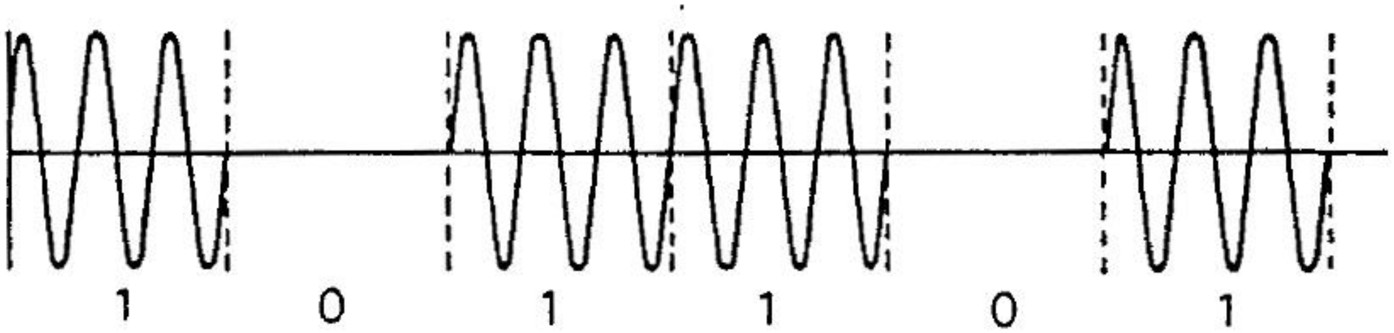
    - Frequency modulation (FM)
      - Varying the frequency of the carrier signal
      - Robust, less suspective to noise an interference than AM
      - Power versus bandwidth tradeoff possible

$$\frac{S}{N} = \frac{R_b}{R_t} \cdot \frac{C}{N}$$

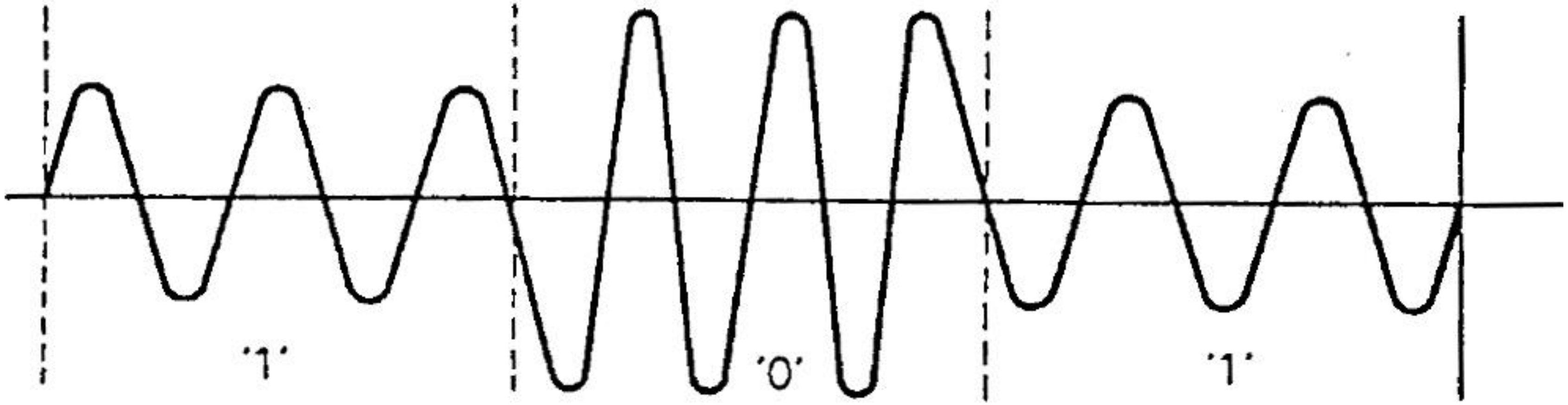
R_b= baseband bandwidth, R_t=transmission bandwidth

    - Phase modulation (PM)
      - Varying the phase of the carrier signal
      - Phase shift keying (PSK)

Example: Amplitude modulation (AM)

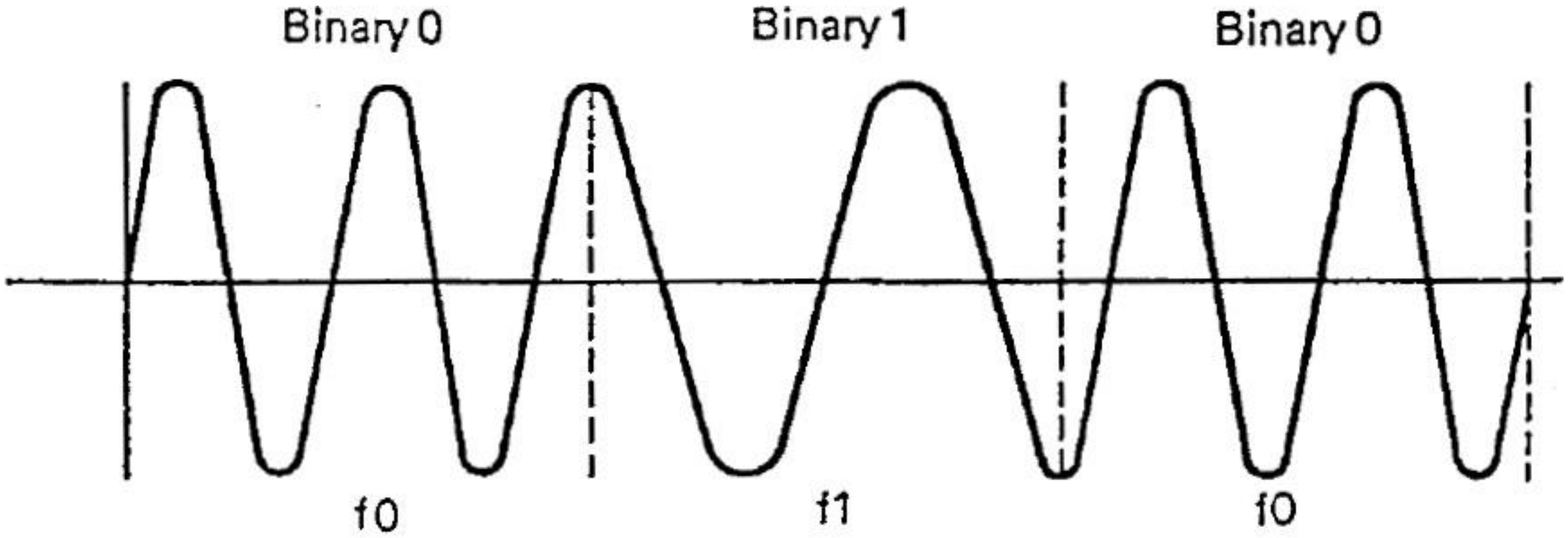


Waveform for ASK using on/off keying

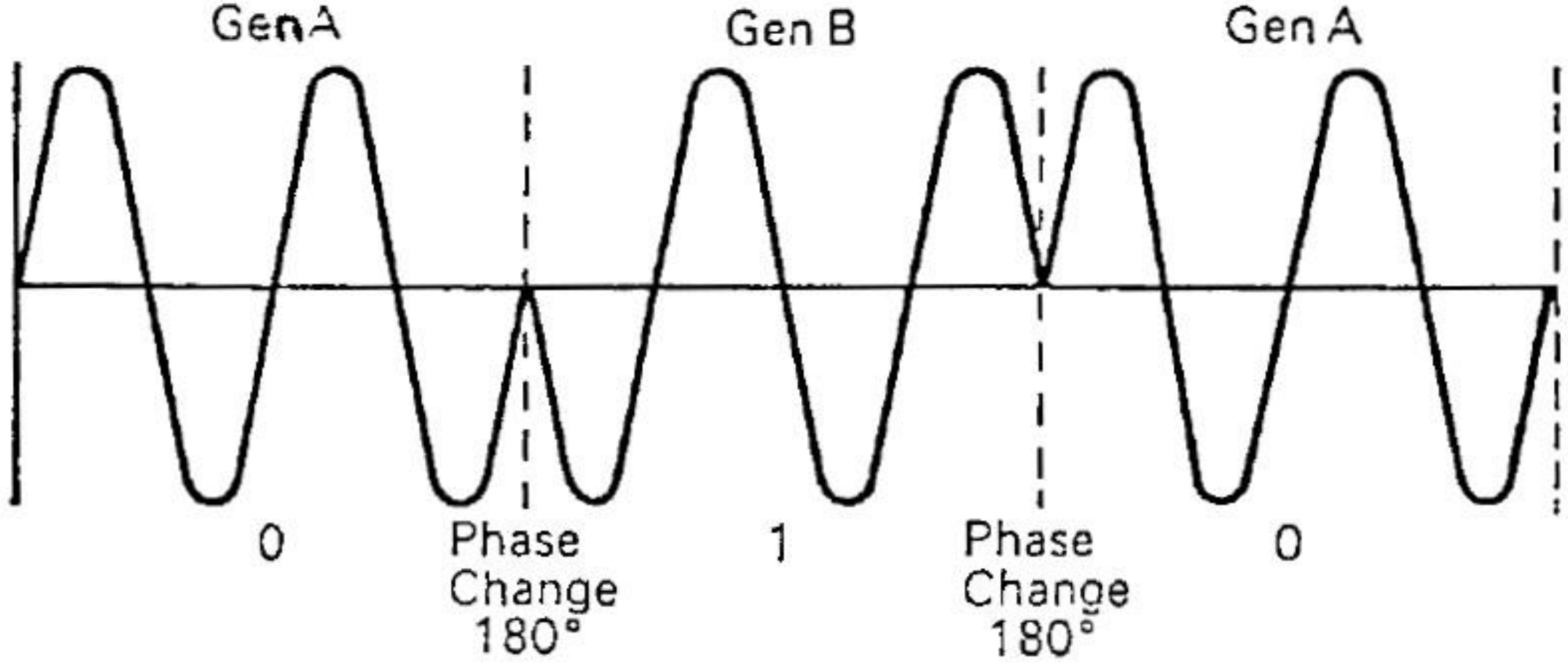


Waveform for ASK using different amplitude signals for the logic levels

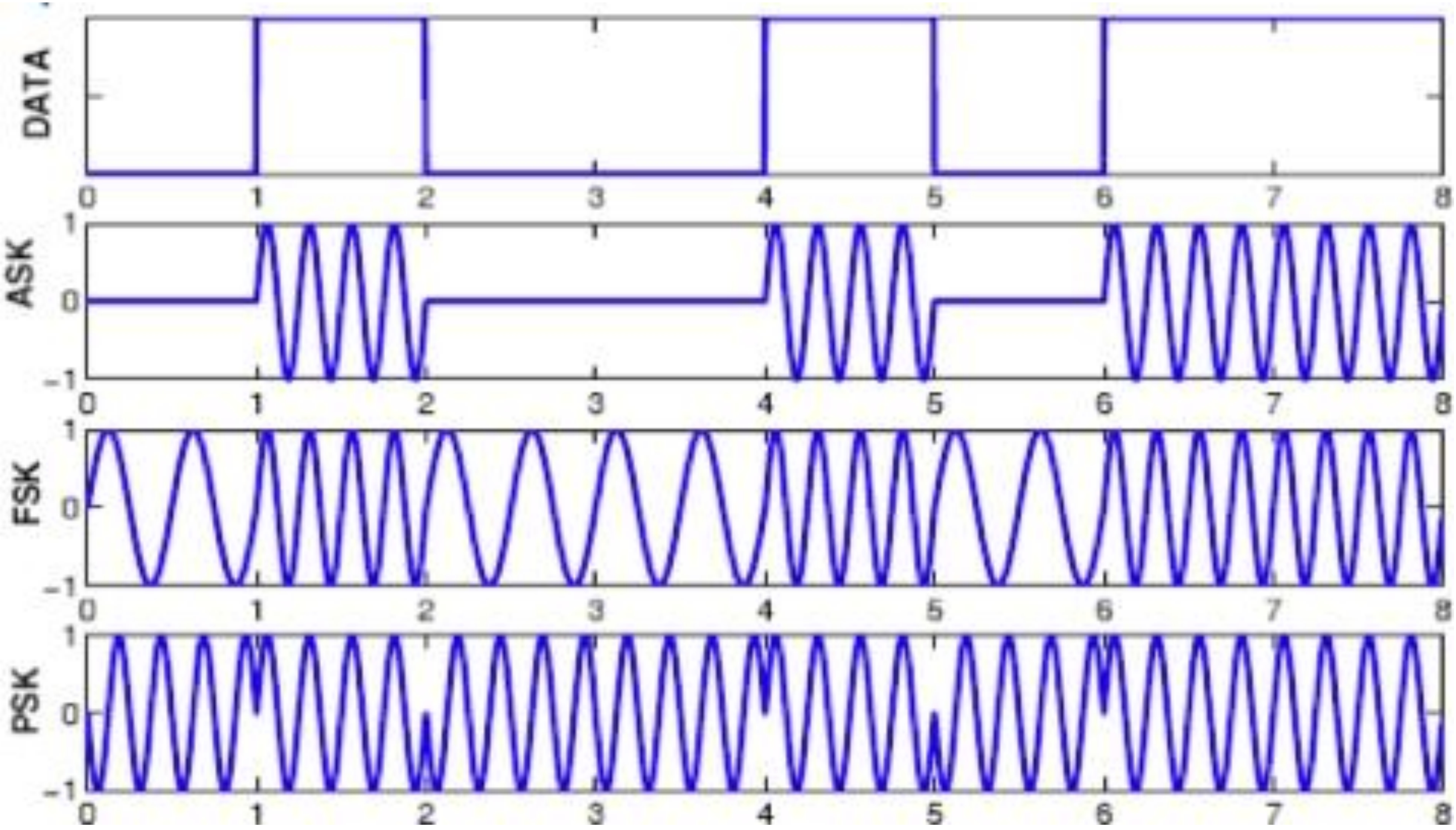
Example: Frequency modulation (FM)



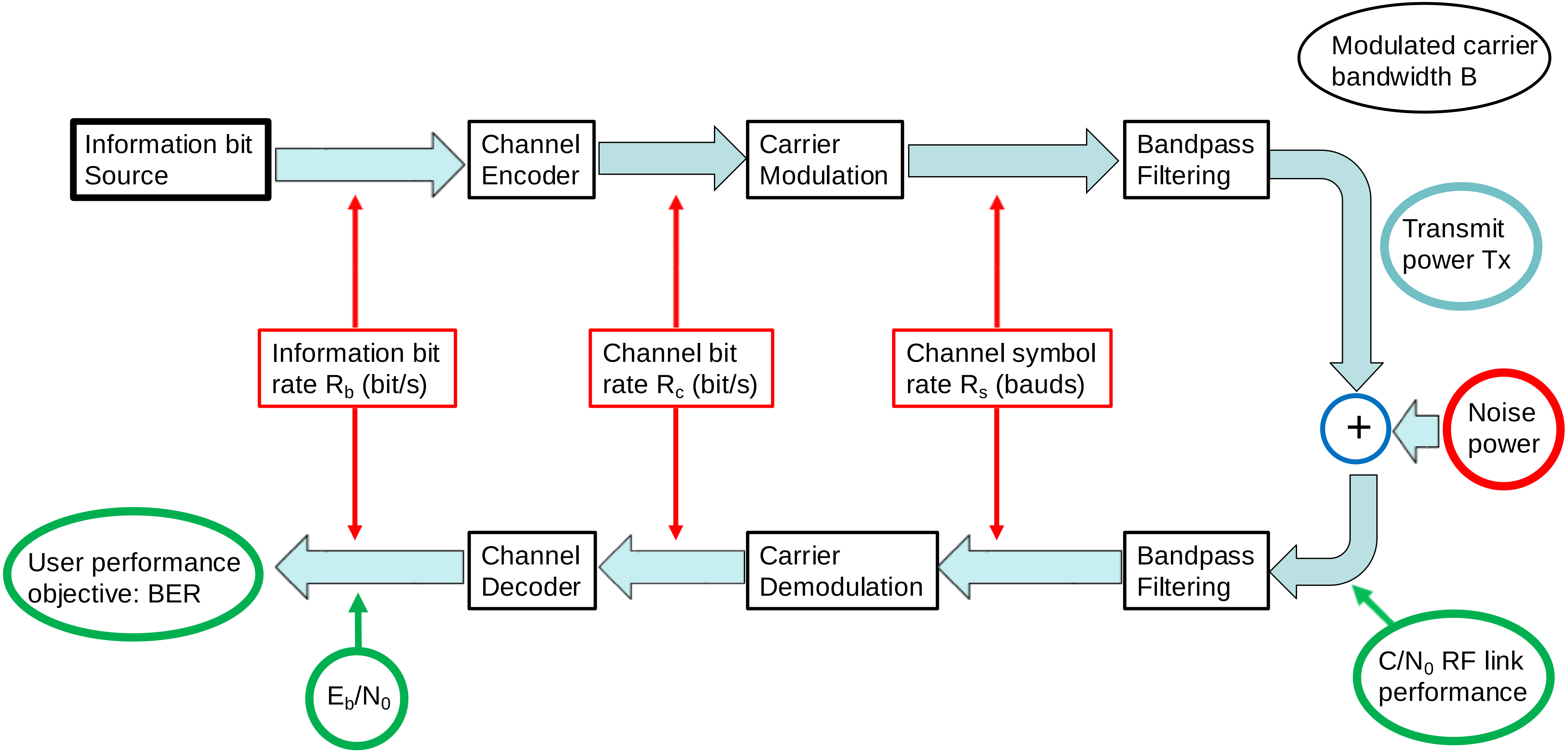
Example: Phase modulation (PM)



Example: AM – FM – PM



Digital communications channel model:



- **Information:**
  - Usually measured in units bits (binary digit)
  - The amount of information associated with the choice between two states with equal probability is equal to one bit
  - Information of the j^{th} symbol from a digital source is:

$$I(x_j) = \log_2\left(\frac{1}{p(x_j)}\right) = -\log_2\left(p(x_j)\right)$$

  - For p(x)=1/2 follows that information I=1
  - For p(x)=1 follows that information I=0

  - In general does the information in each symbol vary because of different probabilities p(x) for each symbol

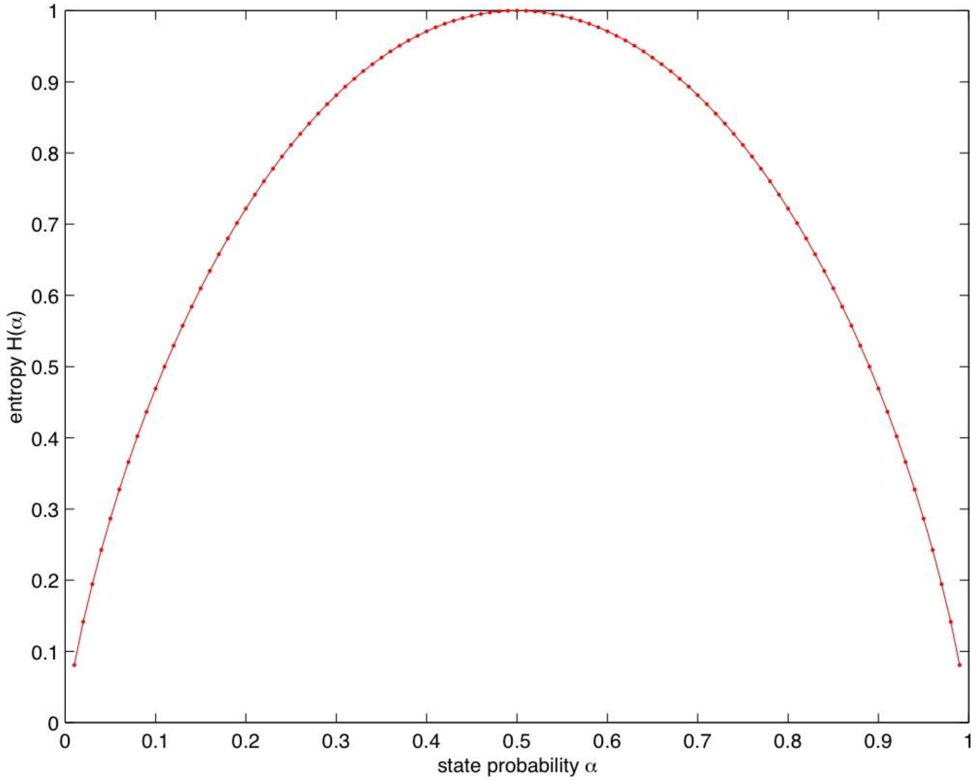
- **Example**
  - Assume a twelve digit word, e.g. ABCDABDCDCBA
  - Assume that each digit can take 4 possible stages (A, B, C, D) with equal probability and independent of the others
  - Thus, the number of possible combinations 4^{12}
  - Probability of one combination (word) $p(x)=4^{-12}$
  - Information of one combination $I(x)=\log_2(4^{12})=24$ bit

- **Entropy:**
  - Is the average information of L possible messages

$$H=\sum_{j=1}^{L} p(x_j)\cdot I(x_j) = -\sum_{j=1}^{L} p(x_j)\cdot \log_2\left(p(x_j)\right)$$

- A binary source transmitting 'ones' and 'zeros' with probabilities $p_1=\alpha$ and $p_2=(1-\alpha)$

$$H = -\sum_{j=1}^{2} p_j \cdot \log_2 p_j = -[\alpha \cdot \log_2 \alpha + (1-\alpha) \cdot \log_2 (1-\alpha)]$$
$$= -\alpha \cdot \log_2 \alpha - (1-\alpha) \cdot \log_2 (1-\alpha)$$



- Symmetric curve
- Entropy reaches maximum for $\alpha=0.5$
- In general: entropy is always highest when the probability is the same for all L states, i.e. $p(x_i)=1/L$

- Maximum entropy:

$$H_{max} = -\sum_{j=1}^{L} \left(\frac{1}{L}\right) \cdot \log_2 \left(\frac{1}{L}\right) = \log_2 L$$

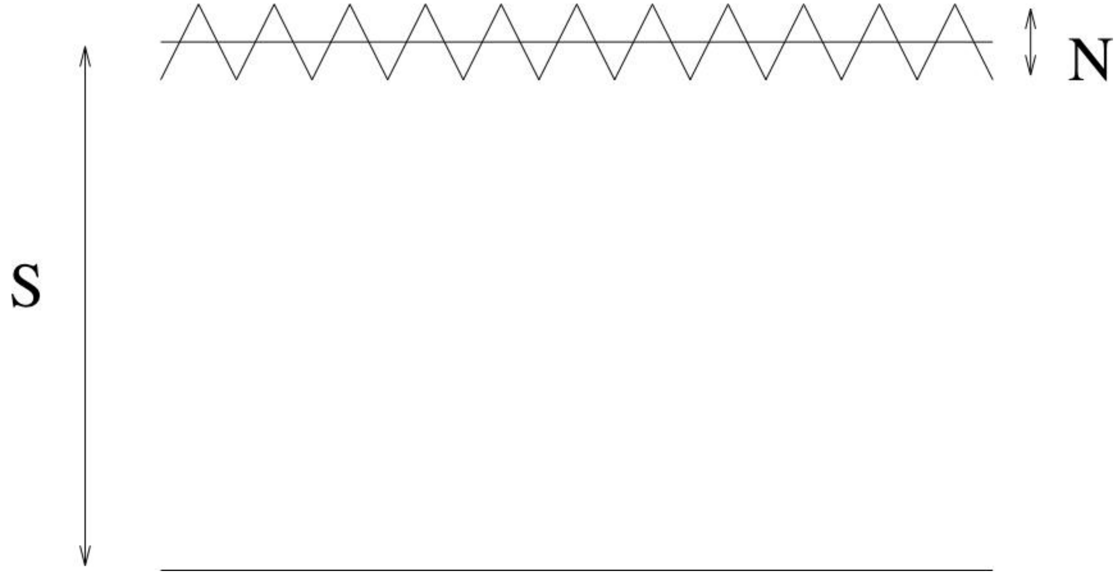
- **Information rate:**

- Information rate is entropy per time: $R = \frac{H}{T}$ [bit/s]

- In a completely noise-free environment (i.e. no disturbances) the necessary energy to transmit one bit can be as low as we wish.
- However, in practice there is of course always noise present.
- Thus, a minimum energy E_b [J/bit] or [Ws/bit] is needed to transmit one bit.
- We introduce γ , the ”system-to-noise-ratio per bit of information”.
- It is a useful parameter to compare different systems:

$$\gamma = \frac{E_b}{N_0} = \frac{S}{N_0 \cdot R} = \frac{S}{N} \cdot \frac{B}{R}$$

- Here, S is the minimum necessary signal power [W], N is the noise power [W] within the bandwidth B [Hz], N_0 the corresponding noise spectral density in [W/Hz], and R is the information rate in [bit/s]



- The number of significant different stages (levels) of a signal is at most (S/N+1)

- During time T we can transmit $B{\cdot}T$ pulses
- The maximum number of different messages that can be transmitted is thus

$$L = \left(1 + \frac{S}{N}\right)^{B \cdot T}$$

- L corresponds to the maximum amount of information that can be transmitted in the time interval T
- When all messages have identical probabilities we can calculate the maximum entropy H_{max} and relate it to the time interval T to express the information rate R_{max} or often called 'channel capacity' $_C$:

$$C = R_{\max} = \frac{H_{\max}}{T} = \left(\frac{1}{T}\right)\log_2(L)$$

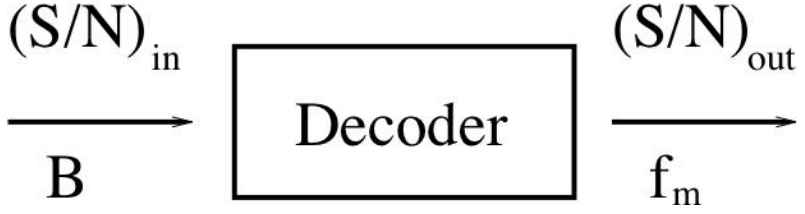
$$= \left(\frac{1}{T}\right)\log_2\left((1+\frac{S}{N})^{B\cdot T}\right) = B\cdot\log_2\left(1+\frac{S}{N}\right) \qquad \text{[bit/s]}$$

=> "Shannon's equation" or "Shannon-Hartley law"

- **Shannon-Hartley law:**
  - Used to improve the S/N ratio in the detection process
  - Information is coded (modulated) so that is occupies a larger bandwidth *B* in the transmission channel compared to the original signal bandwidth f_m
  - Information rate is constant $R_{in}=R_{out}$
  - Theoretically we can thus reach exponential improvement in signal to noise ratio *S/N*
  - In reality the S/N improvement depends on the type of modulation and is significantly lower than theory

$$R_{in} = B \cdot \log_2\left(1+\frac{S}{N}\right)_{in}$$

$$= f_m \cdot \log_2\left(1+\frac{S}{N}\right)_{out} = R_{out}$$



=> it follows: $$\left(\frac{S}{N}\right)_{out} = \left(\frac{S}{N}\right)_{in}^{\frac{B}{f_m}}$$

- Assumption: maximum channel capacity $C=R_{max}$ can be realized
- Reformulation of Shannon's equation:

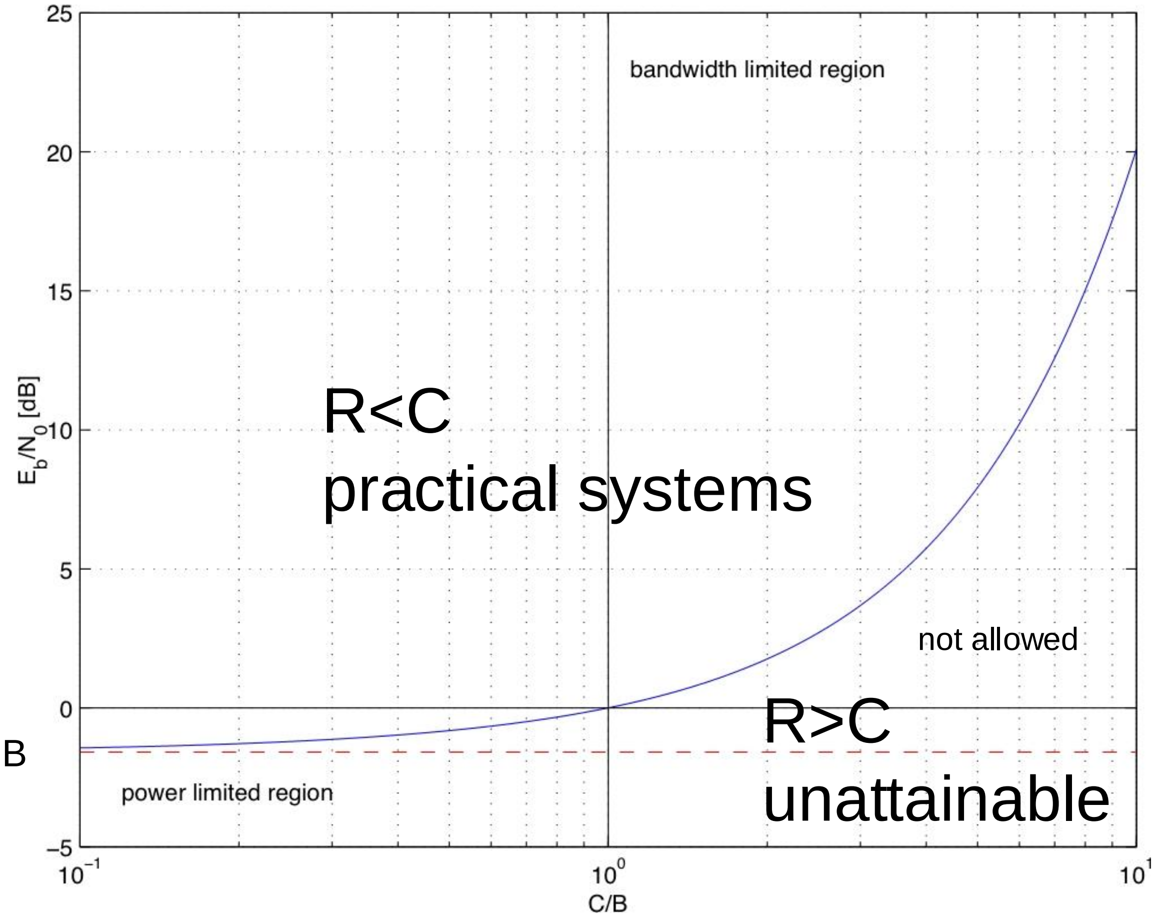
$$\frac{C}{B} = \log_2\left(1+\frac{S}{N}\right) = \log_2\left(1+\frac{S}{N_0 \cdot C}\cdot\frac{C}{B}\right) = \log_2\left(1+\frac{E_b}{N_0}\cdot\frac{C}{B}\right)$$

- Finally we get:

$$\frac{E_b}{N_0} = \frac{B}{C}\cdot\left(2^{\left(\frac{C}{B}\right)}-1\right)$$

- Limit:

$$\lim_{B\to\infty}\left(\frac{E_b}{N_0}\right) = \log_e 2$$



Limit expressed in dB: -1.59 dB

- **Analogue modulation:**
  - Amplitude, frequency or phase of the carrier is modulated continuously
  - Determined by incoming information
  - Quality factor for modulation is (S/N)out for given (S/N)in

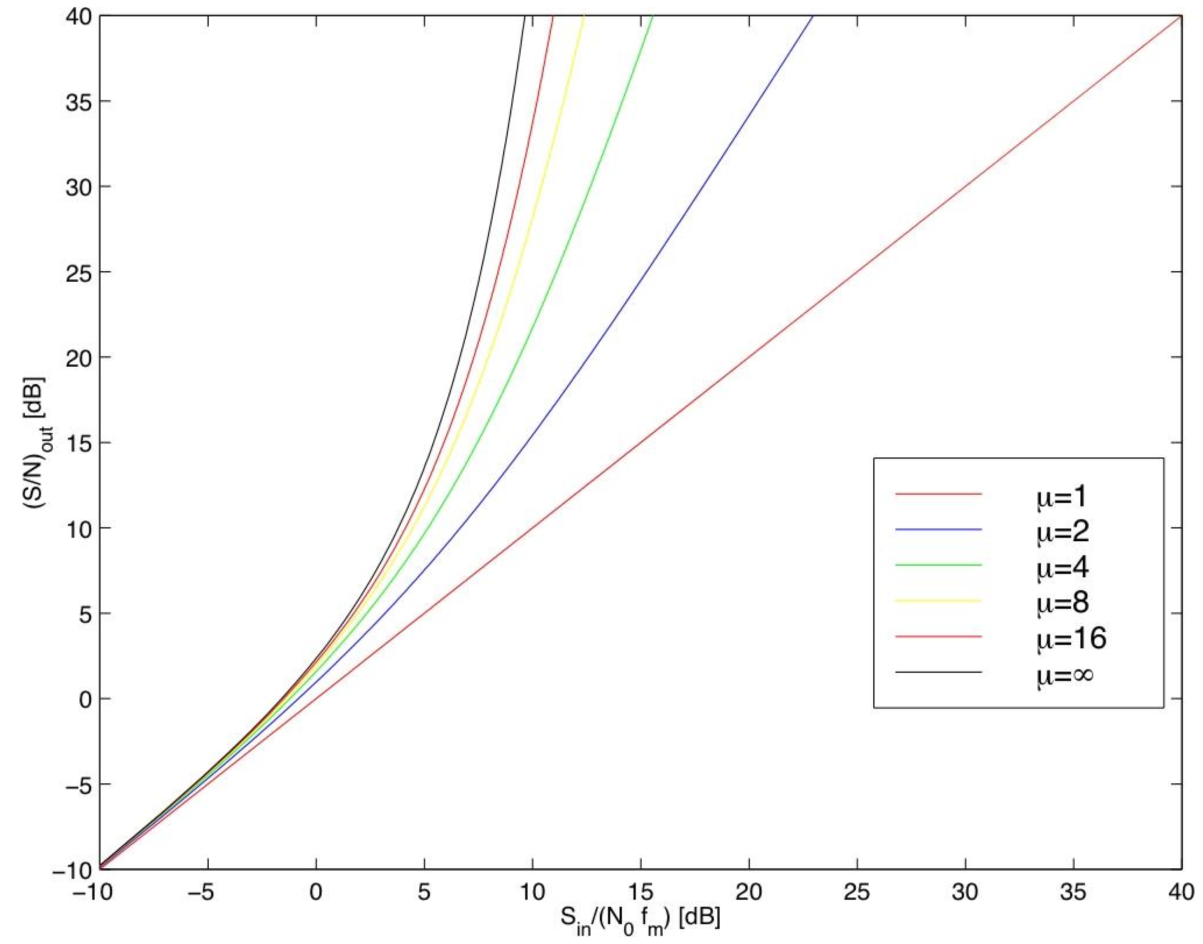
- **Digital modulation:**
  - Discontinuous change of amplitude, frequency or phase of the carrier
  - Controlled by digital flow of information
  - Quality factor is the probability of occurrence of an error

- **In both cases:**
  - quality factor depends on bandwidth expansion factor μ and the modulation process
- **Bandwidth expansion factor μ:**
  - ratio of transmitted RF bandwidth B and highest modulation frequency f_m (analogue)
  - ratio of data transmission rate (or information rate) to R (digital)

$$\mu = \frac{B}{f_m} \qquad \mu = \frac{B}{R}$$

- **Shannon's equation for analogue modulation:**

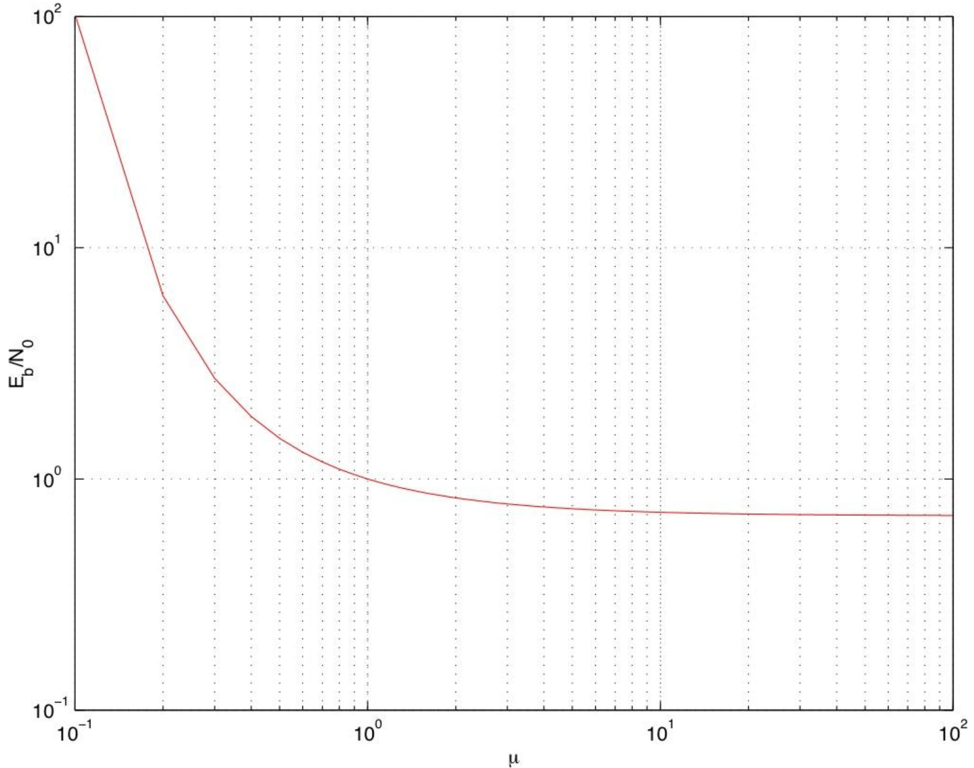
$$\left(\frac{S}{N}\right)_{out} \leq \left\lfloor 1+\left(\frac{S_m}{N_0 \cdot f_m} \cdot \frac{1}{\mu}\right)^{\mu} \right\rfloor - 1$$



- For digital modulation E_b/N_0 is directly related to μ:

$$\frac{E_b}{N_0} \geq \mu \cdot \left(2^{1/\mu} - 1\right)$$

- Gives a lower limit for required E_b/N_0
- Probability of an error is a function of E_b/N_0
- μ can be smaller or larger than 1
- If $\mu < 1$: more than one bit of information per symbol, bandwidth reduction is compensated by increasing transmitting power
- If $\mu > 1$: more than one symbol used for each bit of information, the bandwidth requirement increases but power requirement decreases



- Due to noise there is always a probability that a symbol is misinterpreted
- Error probabilities can be strictly derived
- We assume gaussian noise and absence of inter-symbol interference (ISI)
- It can be shown that the error probability is:

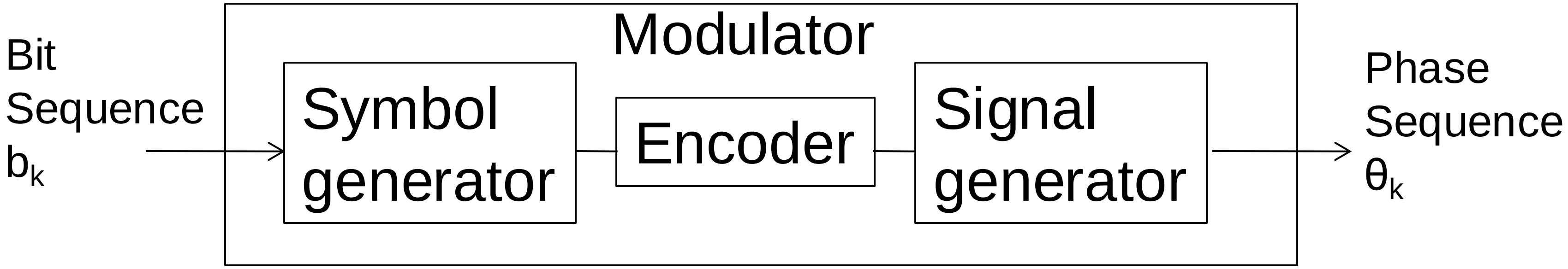
$$P(E) = \frac{1}{2} \cdot erfc\left(\frac{A}{\sigma \cdot \sqrt{2}}\right)$$

- Using the complementary error function:

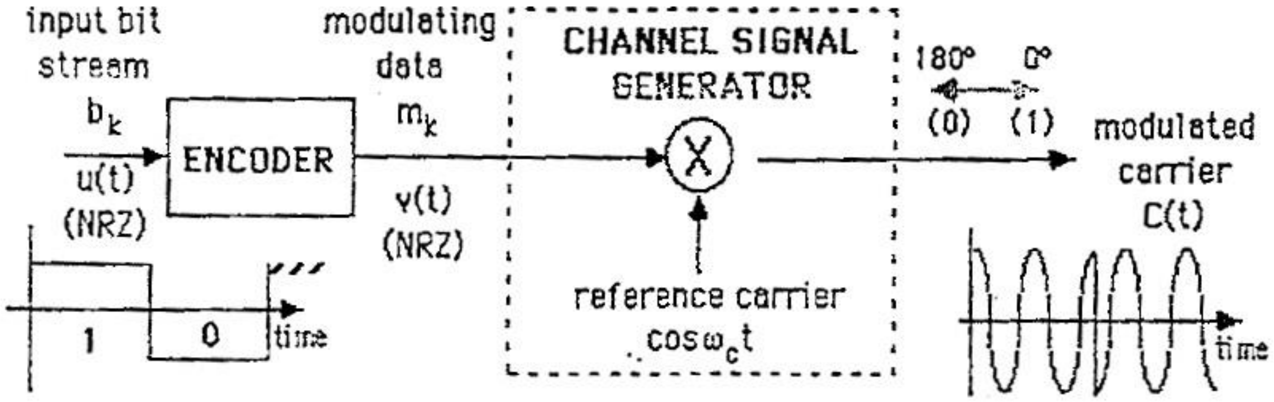
$$erfc(x) = \frac{2}{\sqrt{\pi}} \cdot \int_x^{\infty} e^{-u^2} du \qquad u = \frac{x}{\sigma \cdot \sqrt{2}}$$

Digital phase modulation PSK:

– well suited for satellite communications
– constant envelope, bandwidth efficient
– main types BPSK, QPSK, 8PSK
– direct or differential



Example: BPSK, direct encoding



DIRECT ENCODING :

<u>BPSK direct encoding:</u>

Input bit stream:	b_k	=	0 , 1 , 1 , 1 , 0 , 1 , 0 , 1
Modulating data:	m_k	=	0 , 1 , 1 , 1 , 0 , 1 , 0 , 1
Carrier phase:			**π** , 0 , 0 , 0 , **π** , 0 , **π** , 0

Example: BPSK, differential encoding

BPSK differential encoding, with initial phase = π, $m_0 = 0$:

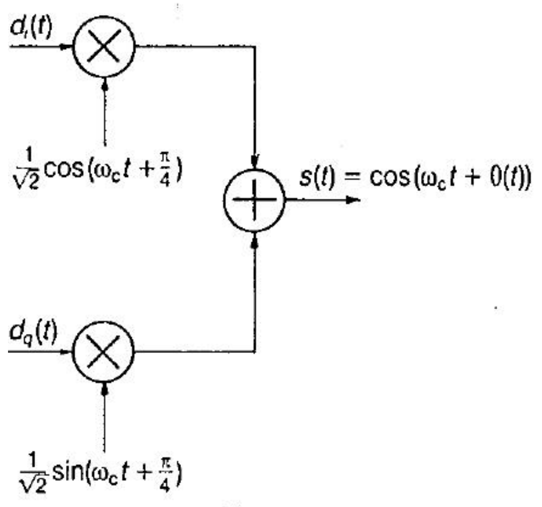
Input bit stream:	b_k	=		0	1	1	1	0	1	0	1
Modulating data:	m_k	=	0	0	1	0	1	1	0	0	1
Carrier phase:			π	π	0	π	0	0	π	π	0

BPSK differential encoding, with initial phase = 0, $m_0 = 1$:

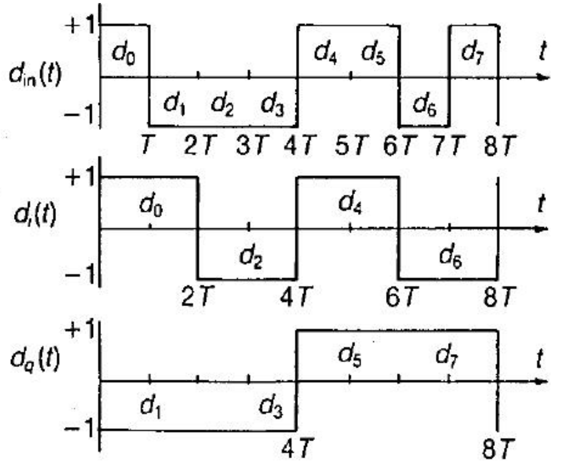
Input bit stream:	b_k	=		0	1	1	1	0	1	0	1
Modulating data:	m_k	=	1	1	0	1	0	0	1	1	0
Carrier phase:			0	0	π	0	π	π	0	0	π

- Bit 0 produces no phase shift
- Bit 1 produces phase shift by π
- Opposite phase sequences can be interpreted by demodulator as same bit sequences: => differential encoding oversteps any phase ambiguity at the demodulator side

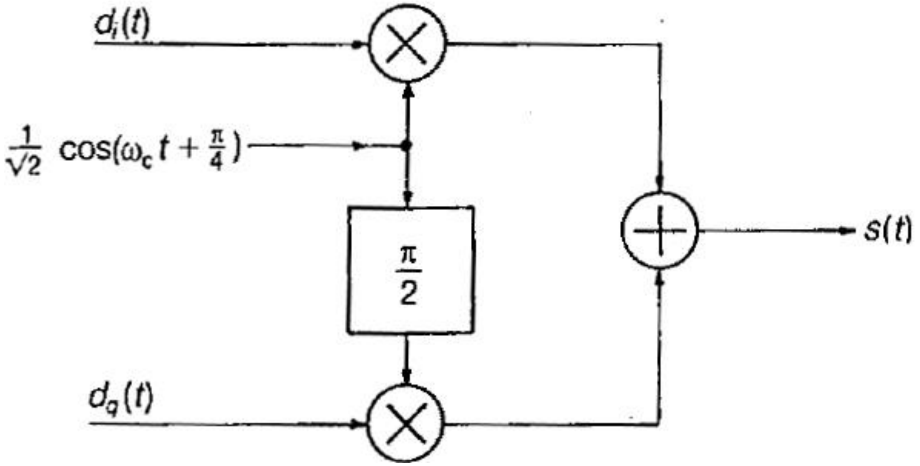
QPSK example:

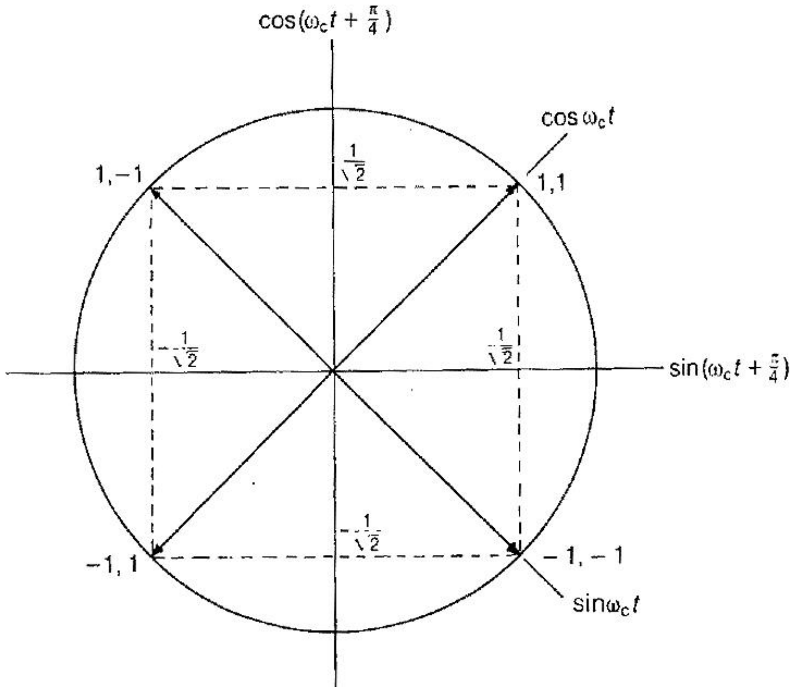


(a)

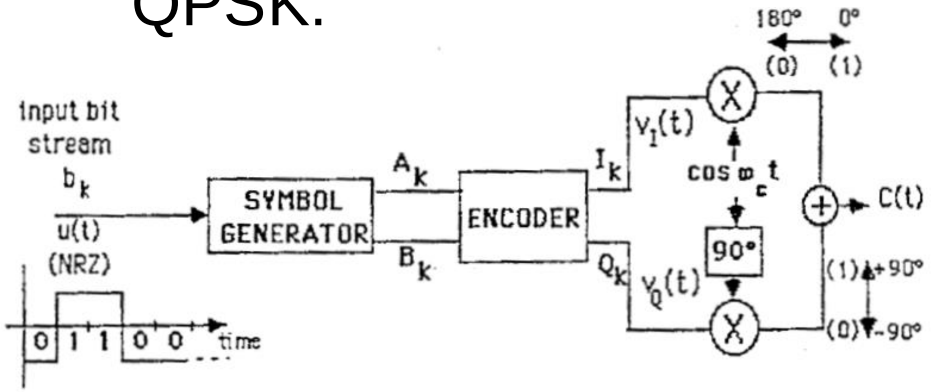


(b)





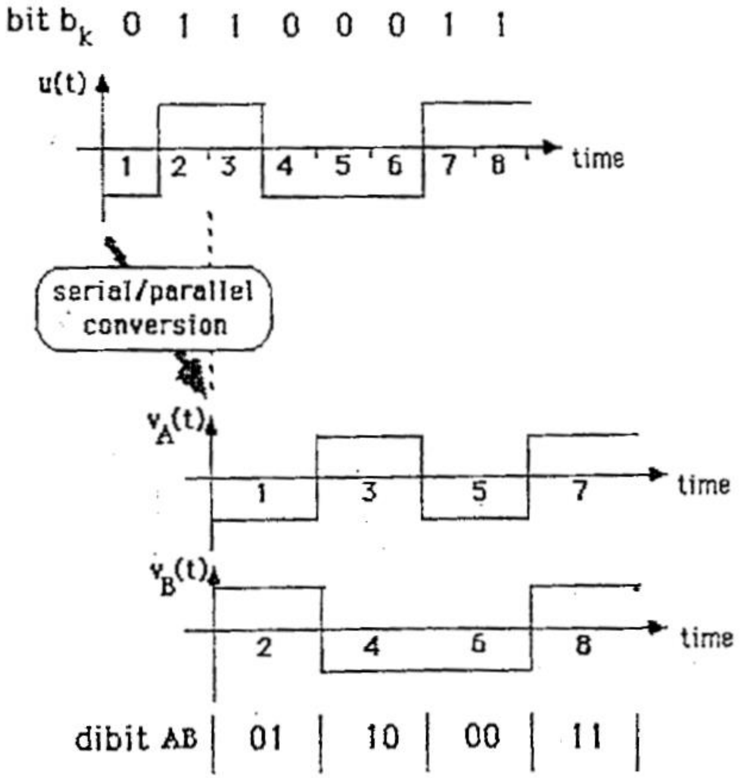
QPSK:



CHANNEL SYMBOL GENERATOR :

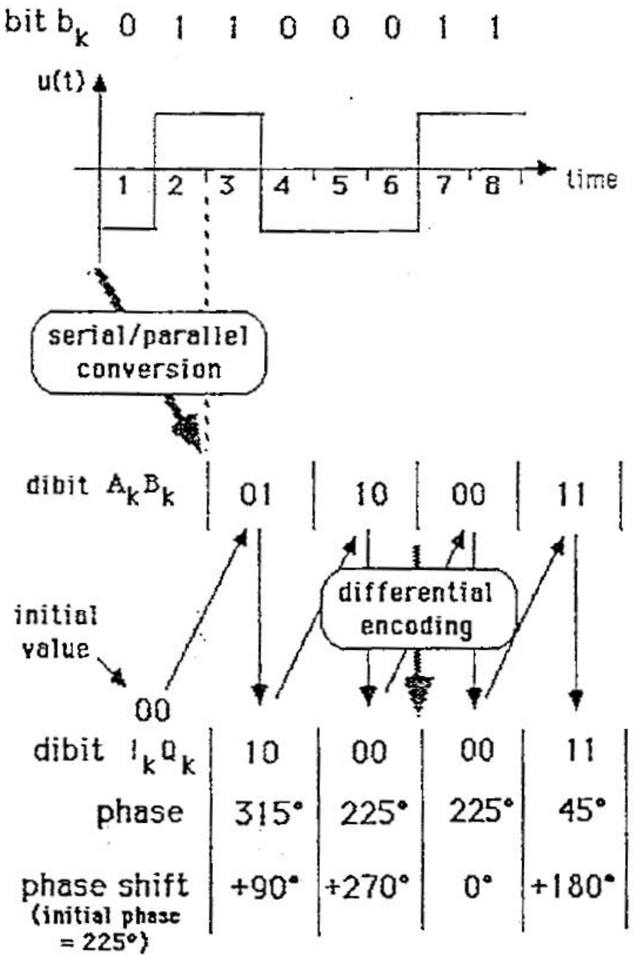
serial/parallel converter ➜ one symbol = 2 bits

($T_s = 2T_c$)



Given input bit stream : $b_k = (0,1,1,0,0,0,1,1)$

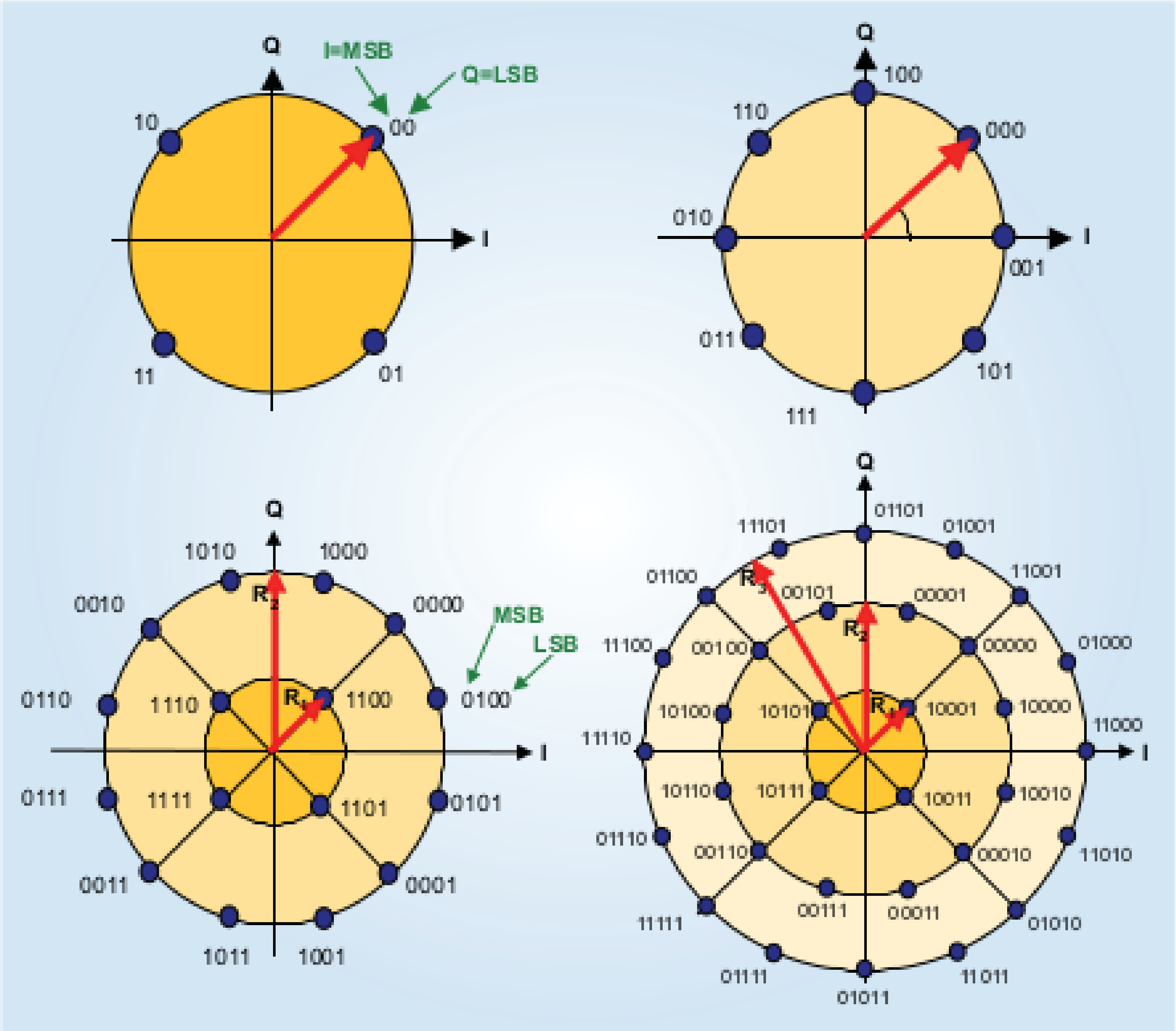
DIFFERENTIAL ENCODING :



dibit	phase shift
00	0
01	$\pi/2$
11	π
10	$3\pi/2$

QPSK

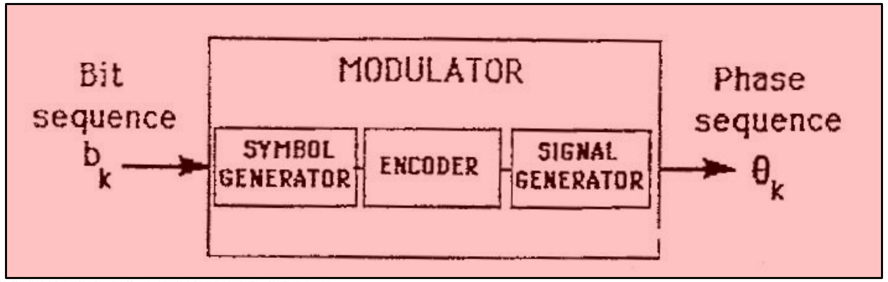
8PSK



16 APSK

32 APSK

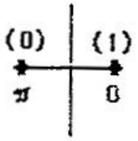
Summary PSK:



DIRECT ENCODING

BPSK

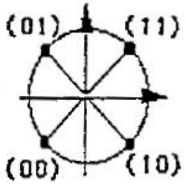
bit	phase status
0	π
1	0



b_k	0	1	1	0	0	0	1	1
θ_k	π	0	0	π	π	π	0	0

QPSK

dibit	phase status
00	$5\pi/4$
01	$3\pi/4$
11	$\pi/4$
10	$7\pi/4$



b_k	0 1	1 0	0 0	1 1
θ_k	$3\pi/4$	$7\pi/4$	$5\pi/4$	$\pi/4$

DIFFERENTIAL ENCODING

BPSK

bit	phase shift
0	0
1	π

b_k	0	1	1	0	0	0	1	1
θ_k	0	π	0	0	0	0	π	0

(0° phase start)

QPSK

dibit	phase shift
00	0
01	$\pi/2$
11	π
10	$3\pi/2$

b_k	0 1	1 0	0 0	1 1
θ_k	$3\pi/4$	$\pi/4$	$\pi/4$	$5\pi/4$

(45° phase start)

☞ With QPSK the four possible dibits (pairs of bits) are mapped in accordance with the Gray code : adjacent symbols differ by only one bit.

Direct modulation:
– BPSK, QPSK

Differential modulation:
– DE-BPSK, DE-QPSK

Coherent demodulation:

Differential demodulation:
– D-BPSK, D-QPSK

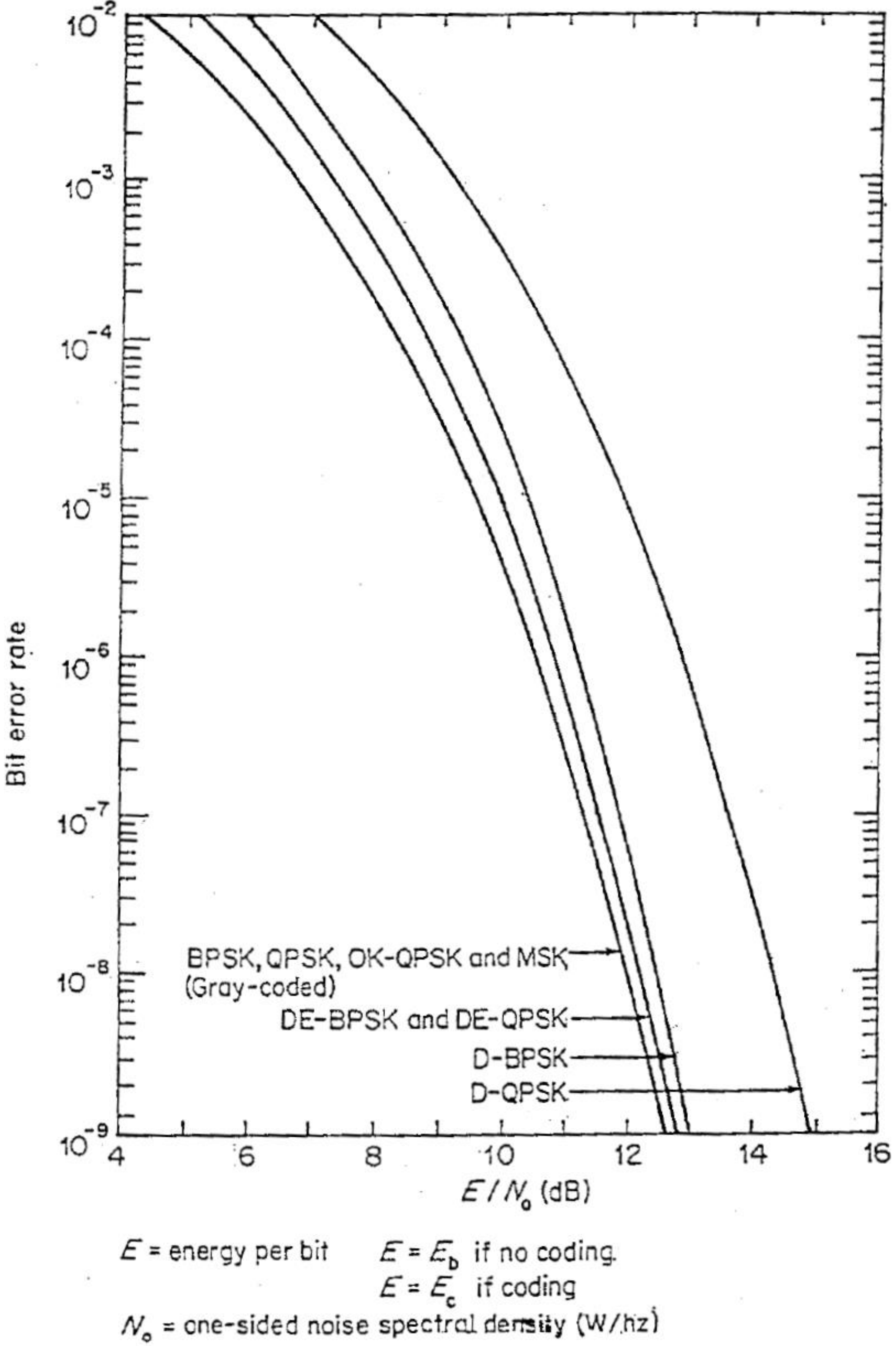
Bit Error Probability (BEP):

BPSK, QPSK

$$P(E) = \frac{1}{2} erfc\left(\frac{E_b}{N_0}\right)$$

DE-BPSK, DE-QPSK

$$P(E) = erfc\left(\frac{E_b}{N_0}\right)$$



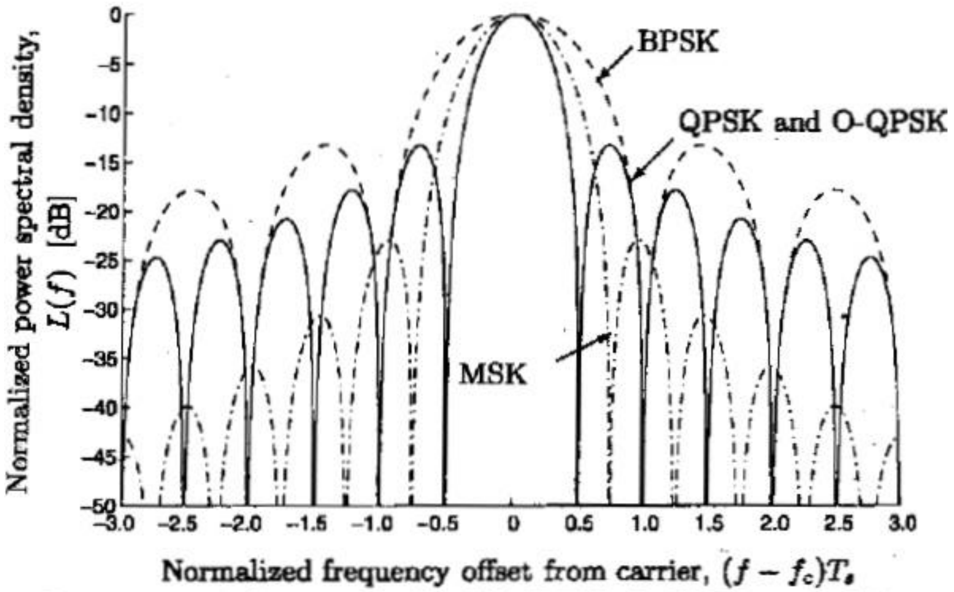
Theoretical E/N_0 needed to achieve a given error probabilities (ref. Maral & Bousquet, 1998)

	BPSK QPSK	DE-BPSK DE-QPSK
BEP	E/N_0 (dB)	E/N_0 (dB)
10E-3	6.8	7.4
10E-4	8.4	8.8
10E-5	9.6	9.9
10E-6	10.5	10.8
10E-7	11.3	11.5
10E-8	12.0	12.2
10E-9	12.6	12.8

Power spectral density:

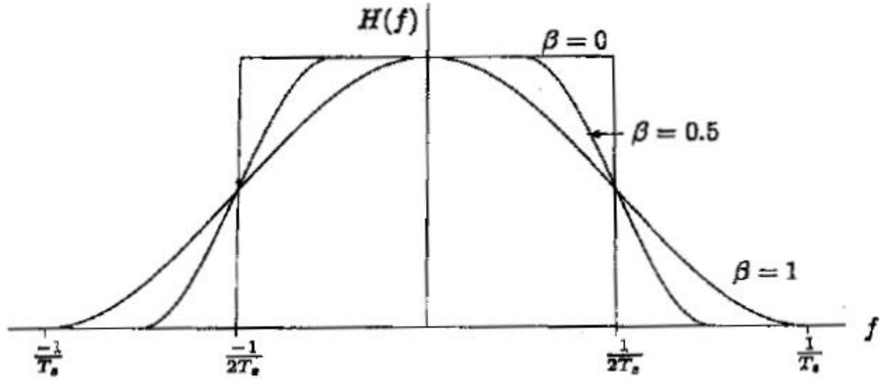
$$L(f) = T_s \cdot \left(\frac{\sin(\pi \cdot f \cdot T_s)}{\pi \cdot f \cdot T_s} \right)^2$$

$T_s = T_b$ for BPSK
$T_s = 2 \cdot T_b$ for QPSK

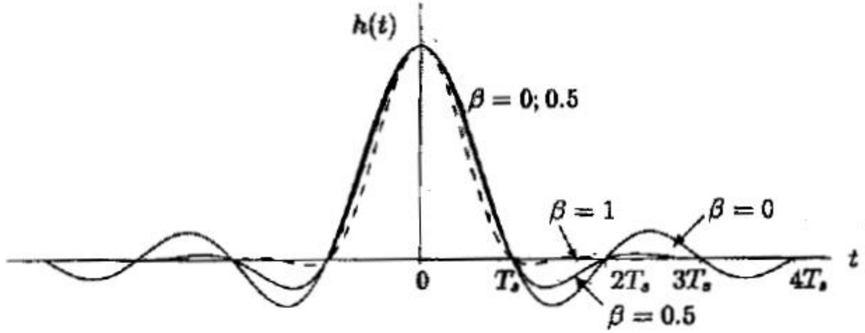


Power spectral densities for BPSK, QPSK, O-QPSK, and MSK

Filtering needed to avoid infinite bandwidth.
=> Nyquist filter with cosine roll-off



$$h(t) = \frac{\cos(\beta \cdot \pi \cdot t / T_S)}{1 - (2 \cdot \beta \cdot t / T_S)^2} \cdot \frac{\sin(\pi \cdot t / T_S)}{\pi \cdot t / T_S}$$



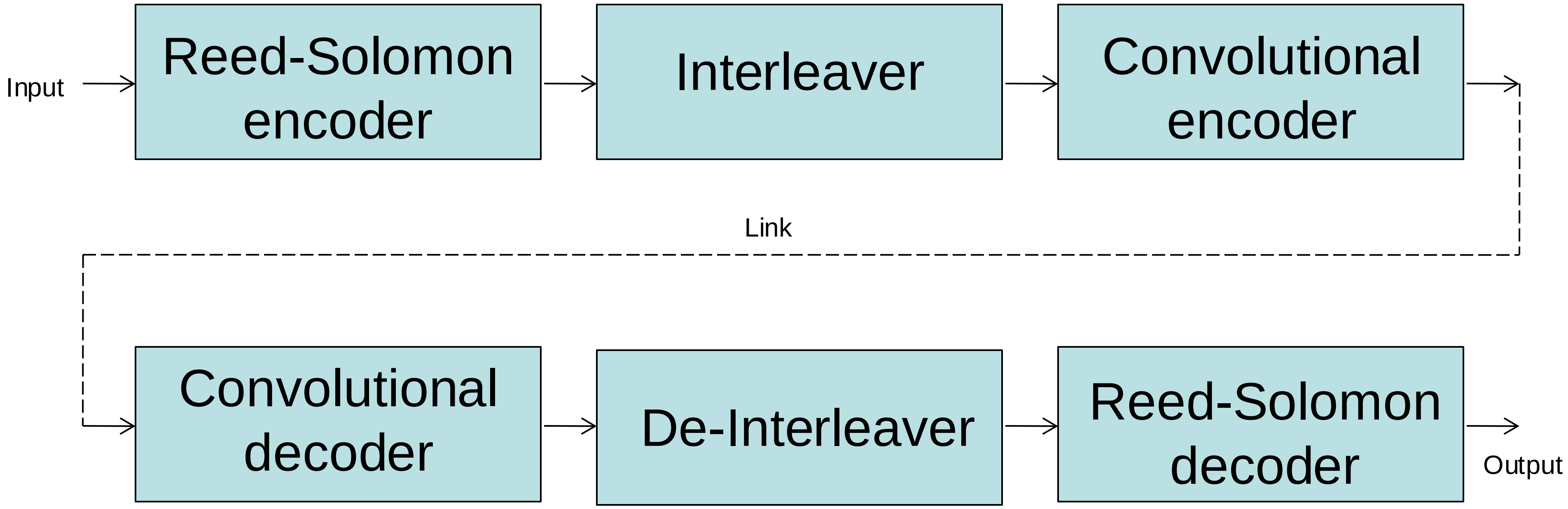
Required bandwidth B:

$$B = \frac{1 + \beta}{T_s} = \frac{(1 + \beta) \cdot R_b}{\log_2 M}$$

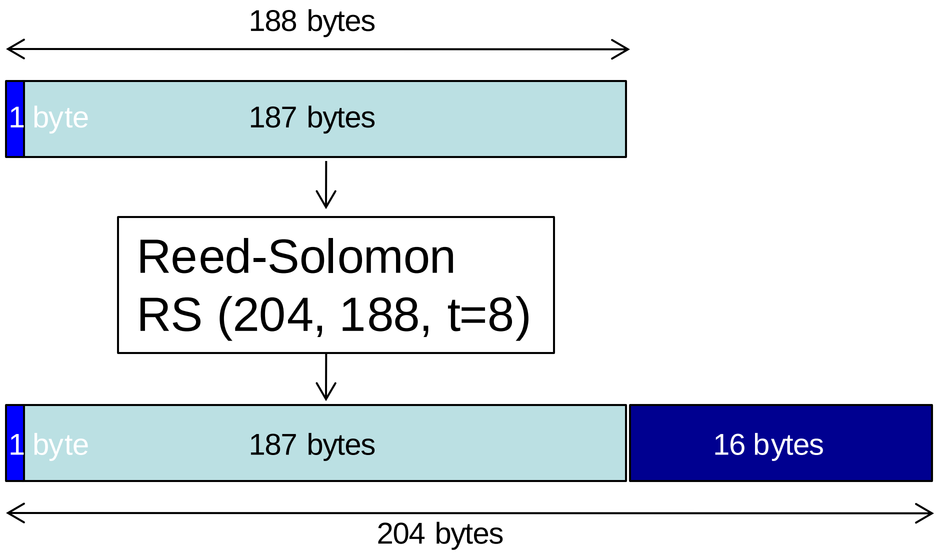
Additional channel coding:

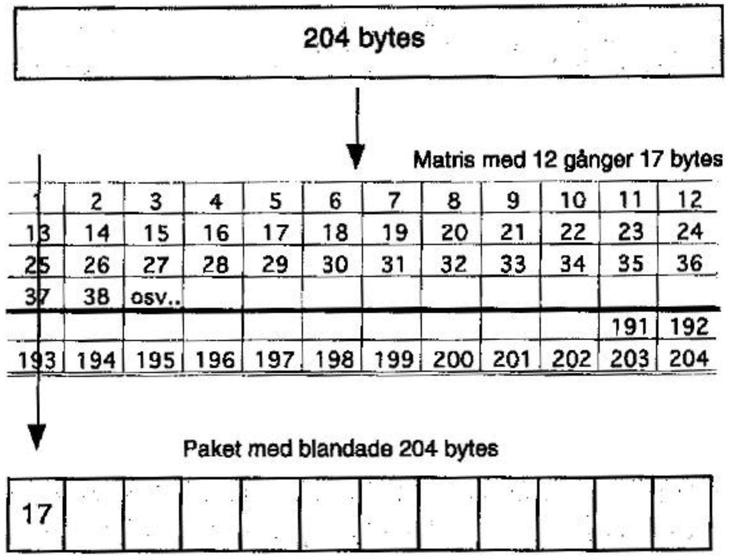
– trades power efficiency against spectral efficiency

– Reed-Solomon coding
– Interleaving
– Convolutional coding



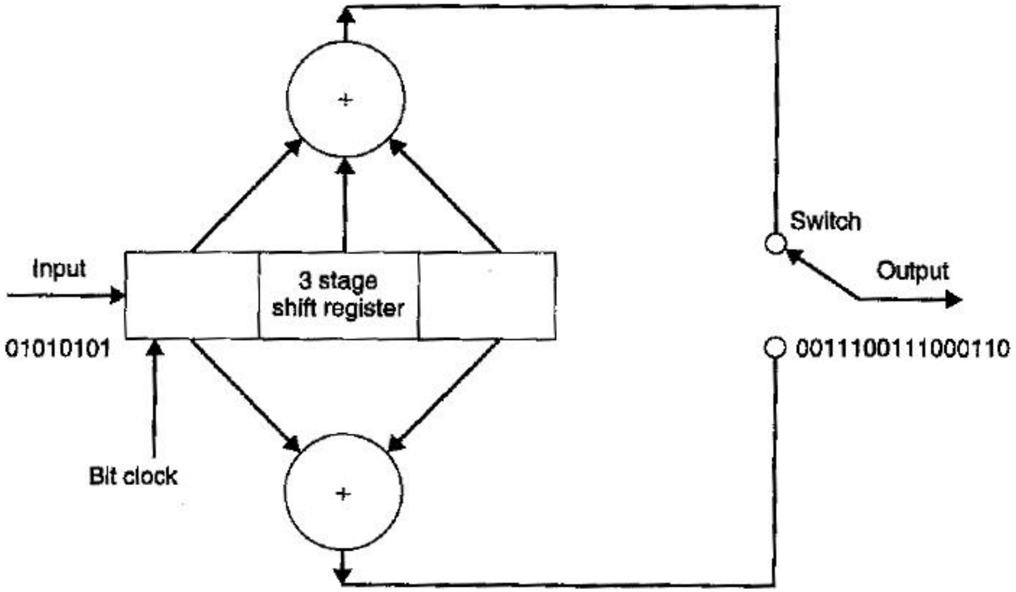
e.g. Reed-Solomon coding





Interleaving

Interleaving coding and decoding.



Convolutional 1/2 coding.

Example for interleaving:

Original message:

THE CAT SAT ON THE TABLE

T	*	**E**		**C**
A	*		**S**	**A**
*		**O**	**N**	
T	**H**	*		**T**
*	**B**	**L**	*	

Rain and no interleaving:

T*E CA* SA* ON TH* T*BL*

Reconstructed message:

T*E CA* SA* ON TH* T*BL*

Original message:

THE CAT SAT ON THE TABLE

T	**H**	*		*
*	*		**S**	**A**
T		**O**	*	
T	**H**	**E**		*
A	**B**	**L**	**E**	

Rain and interleaving:

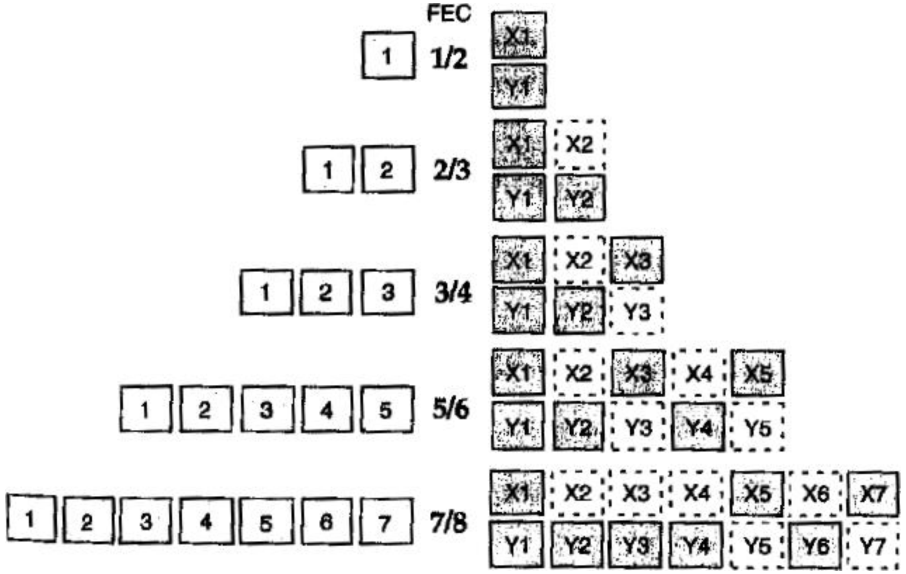
T*TTAH* HB* OEL S* E*A *

Reconstructed message:

THE *** SAT O* THE *ABLE

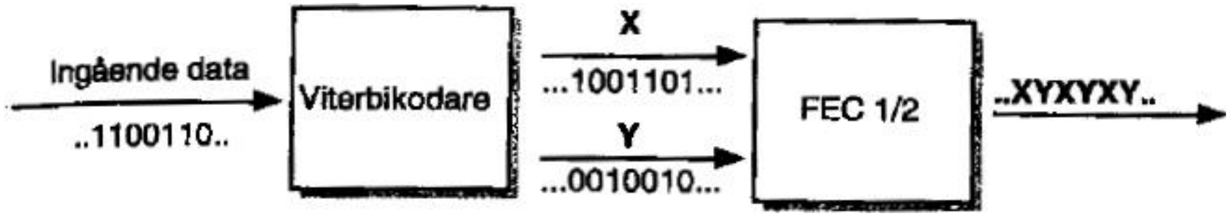
FEC – forward error correction

1/2 corrects 99 %



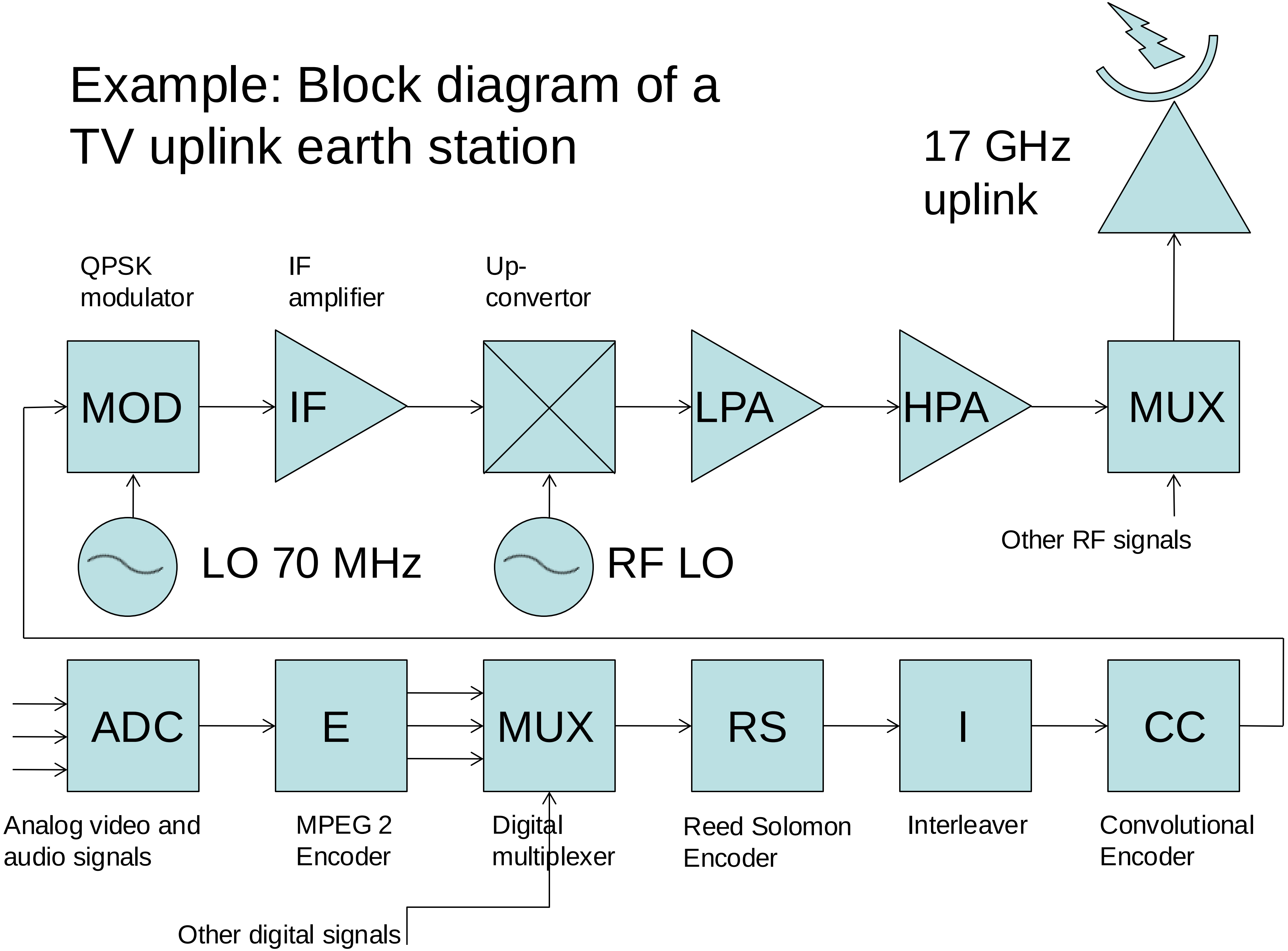
7/8 corrects 80 %

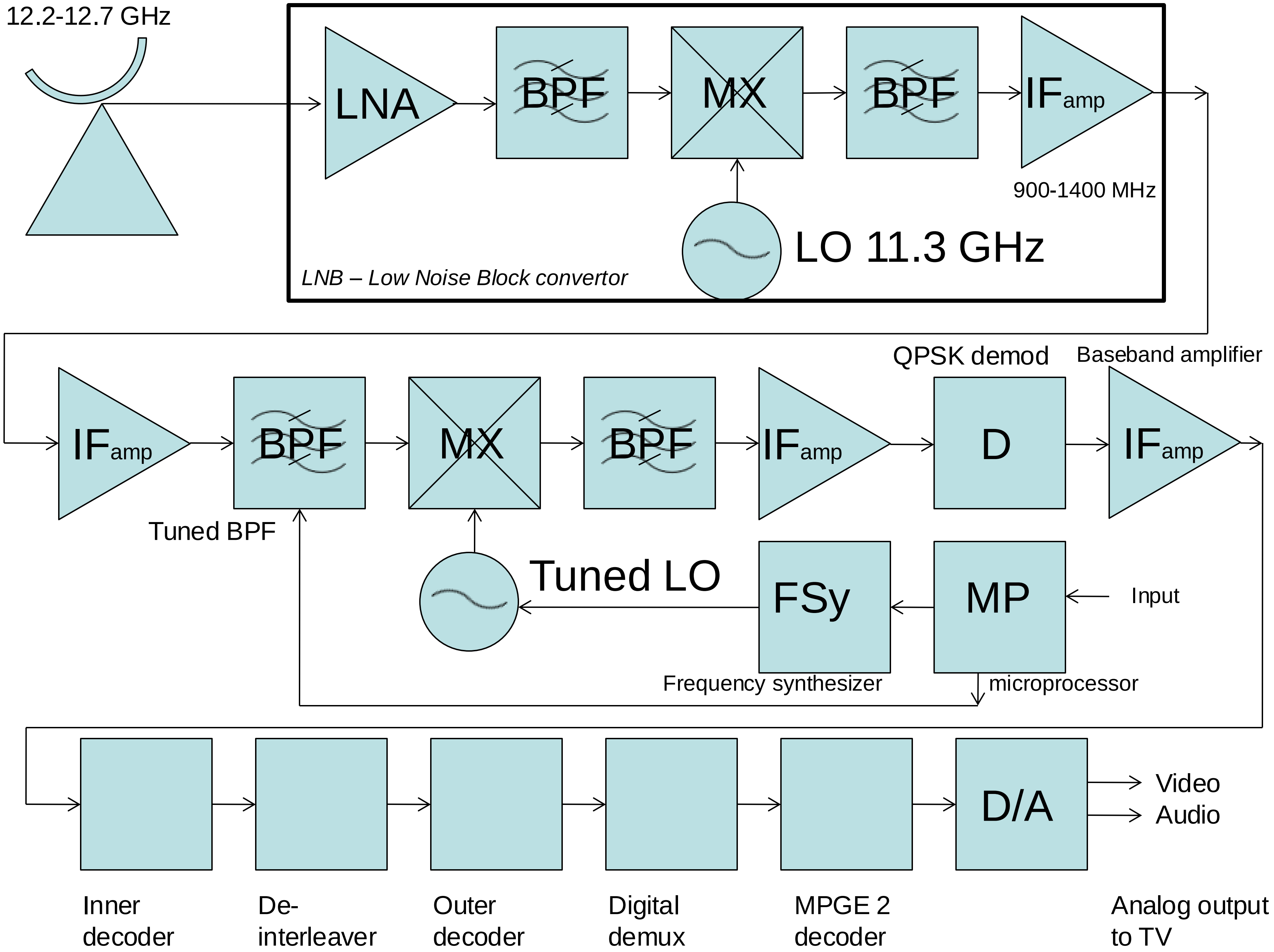
Viterbi-coding.
(convolutional coding)



– Several dB can be gained by appropriate coding.
– However, larger bandwidth is needed.

Example: Block diagram of a TV uplink earth station




12.2-12.7 GHz
LNA
BPF
MX
BPF
IFamp
900-1400 MHz
LO 11.3 GHz
LNB – Low Noise Block convertor
QPSK demod
Baseband amplifier
IFamp
BPF
MX
BPF
IFamp
D
IFamp
Tuned BPF
Tuned LO
FSy
MP
Input
Frequency synthesizer
microprocessor
D/A
Video
Audio
Inner decoder
De-interleaver
Outer decoder
Digital demux
MPGE 2 decoder
Analog output to TV

Various ” rates”:

R_b [bit/s] => bit rate (i.e. the “real information”)

R_c [bit/s] => channel rate (after coding, i.e. added code bits)

$R_c=R_b/\rho$ with $\rho=n/(n+r)$; n = information bits, r = coding bits

R_s [bit/s] => symbol rate (after modulation)

$R_s = R_c/(\log_2 M)$ (e.g. with M=2 for BPSK, M=4 for QPSK)

E_b [Ws/bit] => energy per bit

C [W] => carrier power

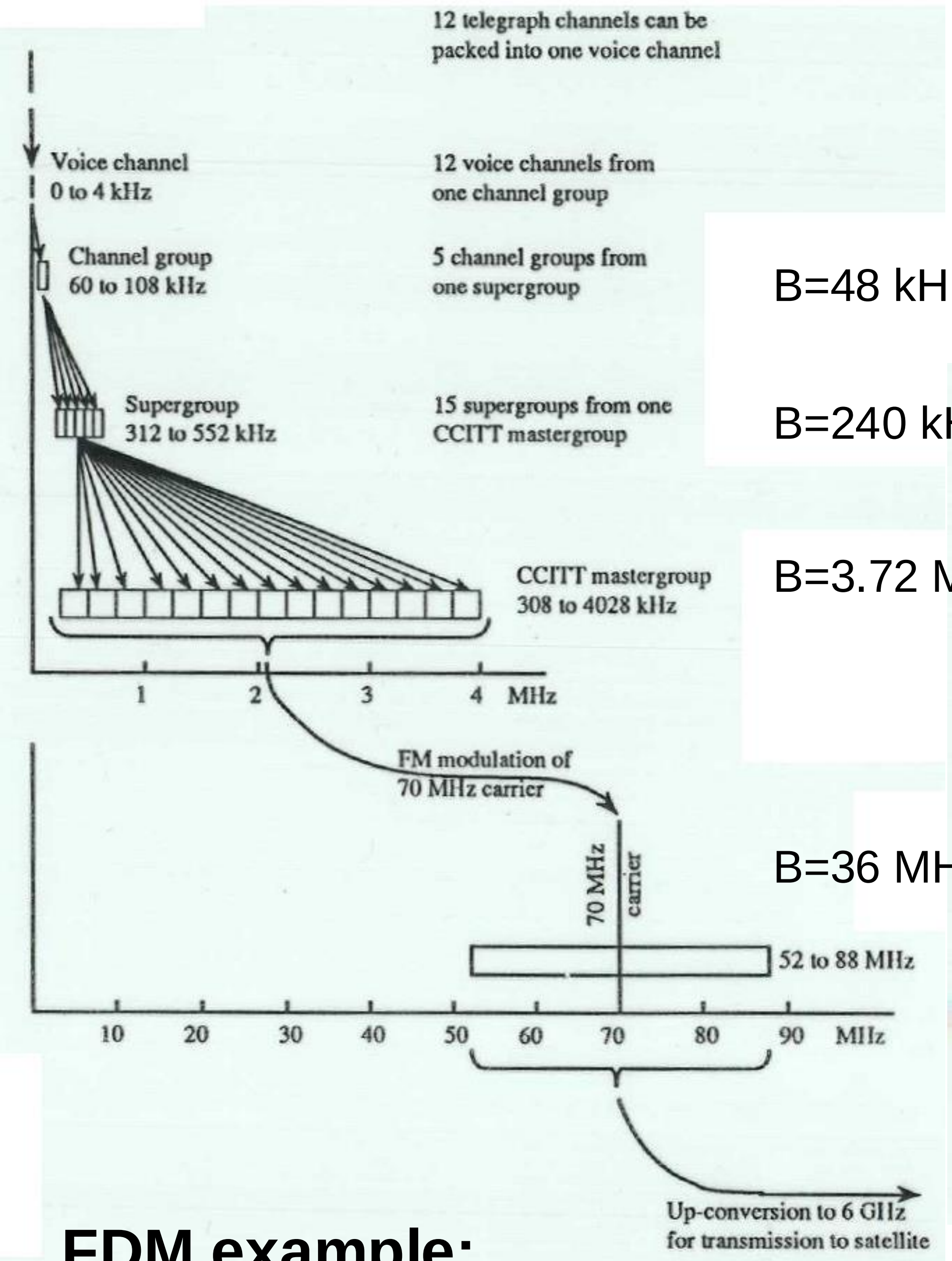
$C = E_b \bullet R_b$

N_0 => noise spectral density [W/Hz]

$$\frac{C}{N_0} = \frac{E_b}{N_0} R_b$$

- **Multiplexing:**
  - Sharing a channel when the users are co-located
  - Point-to-point or point-to-multipoint
  - e.g. several users at a ground station
  - e.g. satellite to several ground stations
  - Keeping the signals seperate and avoid interference
    - Frequency division multiplex (FDM)
      - Composing several user frequencies
      - Using microwave filters, resonance cavities
      - Analogue signals usually use FDM
    - Time division multiplex (TDM)
      - Composing several user signals into time frames
      - Digital signals usually use TDM

FDM example:

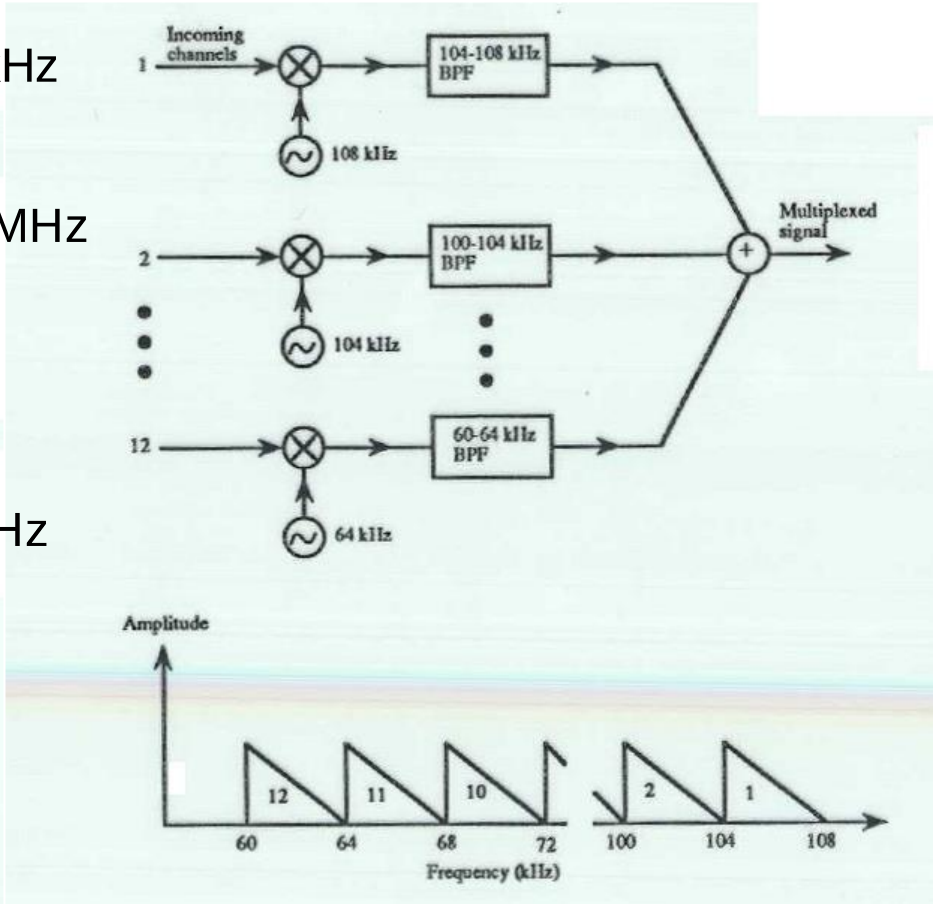


B=48 kHz

B=240 kHz

B=3.72 MHz

B=36 MHz



TDM example:

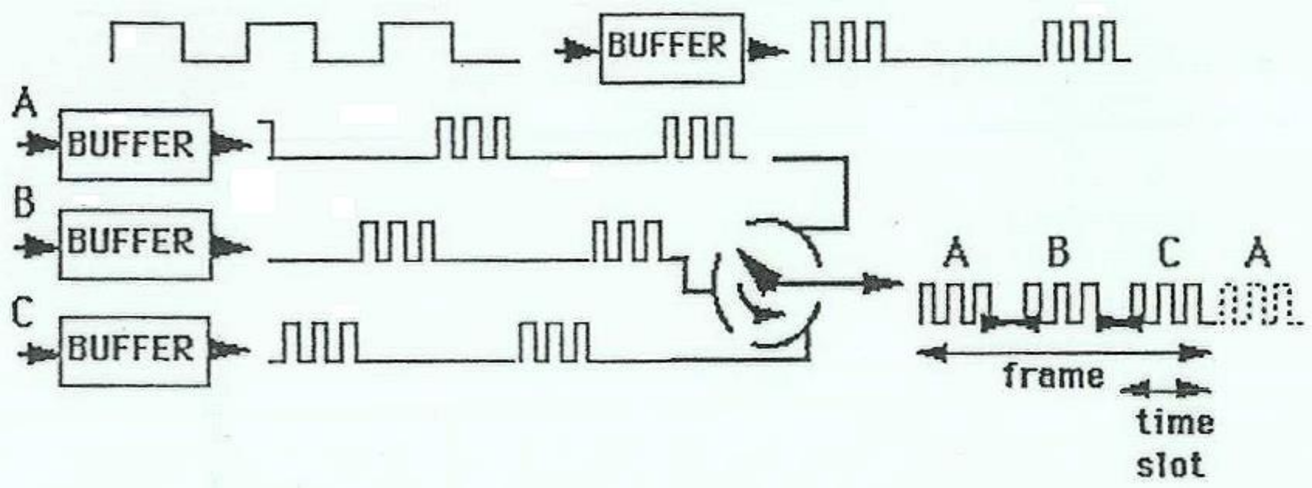


Table 3.4 Characteristics of CEPT and T-carrier multiplexes

	CEPT		USA/Canada		Japan	
Hierarchy level	Throughput (Mbit/s)	Capacity (channels)	Throughput (Mbit/s)	Capacity (channels)	Throughput (Mbit/s)	Capacity (channels)
1	2 048	30	1 544	24	1 544	24
2	8 448	120	6 312	96	6 312	96
3	34 368	480	44 736	672	32 064	480
4	139 264	1 920	274 176	4032	97 728	1 440
5	557 056	7 680			400 352	5 760

Short summary of today's topics

- Modulation: AM, FM, PM
- Information rate, Shannon-Hartley law
- BPSK, QPSK, other variants
- Necessary E_b/N_0
- Filtering, rolloff
- Channel coding, Reed-Solomon, Vitterbi
- Multiplexing in frequency and time domain