HOMEWORK 3

Problem 1

a) We know $SER \simeq \frac{2K}{M} Q\sqrt{\frac{D_{min}}{2N_0}}$

From given figure: $K = 24$, $M = 16$] $D_{min} = 2$

$$Energy = \frac{1}{16}[4\times 18 + 8\times 10 + 4\times 2] \Rightarrow Energy = 10$$

$$SER \simeq \frac{2\times 24}{16} \cdot Q\sqrt{\frac{D_{min}}{2N_0}} \Rightarrow SER \simeq 3Q\sqrt{\frac{E_S}{5N_0}}$$

So we can say that $\boxed{a_1 = 3}$ and $\boxed{k_1 = \frac{1}{5}}$

b) We know $BER \simeq \frac{2H_{min}}{M} \cdot Q\sqrt{\frac{D_{min}^2}{2N_0}}$

$H_{min} = 2\times 4\times 4 \Rightarrow H_{min} = 32$; We know $M = 16$.

$$\therefore BER = \frac{2\times 12}{16} Q\sqrt{\frac{4}{2N_0}} \Rightarrow BER \simeq 4Q\sqrt{\frac{E_S}{5N_0}}$$

We can say $\boxed{a_2 = 4}$ and $\boxed{k_2 = \frac{1}{5}}$

PROBLEM 2

a) $E_S = \frac{1}{4}[9+1+1+9] \Rightarrow \boxed{E_S = 5}$ [Assuming all have equal probability]

$$P[\hat{x} \neq s_1 | x = s_1] = P[\hat{x} = s_2, s_3, s_4 | x = s_1]$$

We can write in terms of Q function :

$$= Q\left(\sqrt{\frac{D_{12}^2}{2N_0}}\right) + Q\left(\sqrt{\frac{D_{13}^2}{2N_0}}\right) + Q\left(\sqrt{\frac{D_{14}^2}{2N_0}}\right)$$

$$= Q\left(\sqrt{\frac{4}{2N_0}}\right) + Q\left(\sqrt{\frac{16}{2N_0}}\right) + Q\left(\sqrt{\frac{36}{2N_0}}\right)$$

$$= Q\left(\sqrt{\frac{E_S}{9 \cdot 5 N_0}}\right) + Q\left(\sqrt{\frac{E_S}{0.625 N_0}}\right) + Q\left(\sqrt{\frac{18}{5}\frac{E_S}{N_0}}\right)$$

b) We can do the same processas above;

$$P[\hat{x} \neq s_2 | x = s_2] = P[\hat{x} = s_1, s_3, s_4 | x = s_2]$$

$$= Q\left(\sqrt{\frac{4}{2N_0}}\right) + Q\left(\sqrt{\frac{4}{2N_0}}\right) + Q\left(\sqrt{\frac{16}{2N_0}}\right)$$

$$= Q\left(\sqrt{\frac{E_S}{2 \cdot 5 N_0}}\right) + Q\left(\sqrt{\frac{E_S}{2 \cdot 5 N_0}}\right) + Q\left(\sqrt{\frac{8}{5}\frac{E_S}{N_0}}\right) + Q\left(\sqrt{\frac{E_S}{0.625 N_0}}\right)$$

c) $P_e = \frac{1}{4} \sum_{i=1}^{M} \sum_{j \neq m} Q\left(\sqrt{\frac{D_{ij}^2}{2N_0}}\right)$

$$= 10\, Q\left(\sqrt{\frac{1}{2N_0}}\right)$$

$$\boxed{P_E = 10\, Q\left(\sqrt{\frac{E_S}{10 N_0}}\right)}$$

High SNR $\propto \frac{2k}{M} Q\left(\sqrt{\frac{D_{min}^2}{2N_0}}\right) = \frac{3 \times 2}{4} Q\left(\sqrt{\frac{4}{2N_0}}\right)$

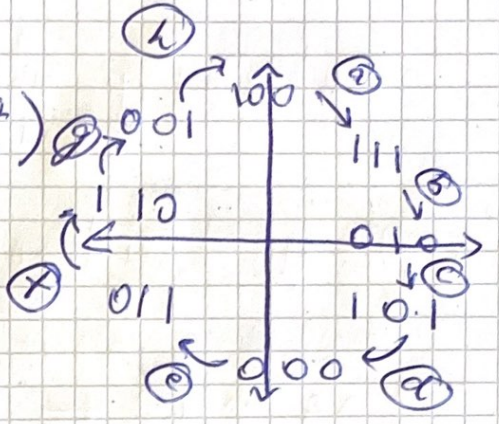
$$\Rightarrow \text{High SNR} = \frac{3Q\left(\sqrt{\frac{2 E_S}{N_0}}\right)}{2} = \boxed{\frac{3}{2} Q\left(\sqrt{\frac{E_S}{N_0 \cdot 5}}\right)}$$

PROBLEM 3

1) Worst labling means not grayycoded. That means more than 1 bit difference between 2 ~~symbols~~ points close to eachothers.

At Hmin is ~~was~~ increased

2) BER $= \frac{2 H_{min}}{M} Q\left(\sqrt{\frac{D_{min}^2}{2N_0}}\right)$



$M = 8$

$H_{min} = 2 + 2 + 2 + 2 + 2 + 3 + 3 + 2$

$= 18$

$ES = \frac{1}{8}\left(8 \cdot (\sqrt{E})^2\right)$

$= E$

$D_{min} = 0.586\sqrt{E}$

$BER = \frac{2 \times 18}{8} Q\left(\sqrt{\frac{(0.586)^2 E}{2N_0}}\right)$

$\boxed{BER = \frac{9}{4} Q\left(\sqrt{\frac{0.3434E}{2N_0}}\right)}$