Homework 4

Due Date: October 16, 2023

Problem 1 (Timing Error)

Rectangular pulse $s(t) = \mathrm{I}\{0 \leq t \leq T\}$ is used for BPSK transmission over the AWGN channel, where T is the symbol interval. Information bits are independent and uniformly distributed. A matched filter receiver is used at the receiver side. Assume that a timing error of $\Delta t = T/4$ occurs at the receiver.

1. Sketch carefully the eye diagram for the matched filter output.
2. Calculate the probability of error. What is the probability of error when the SNR tends to infinity?
3. Assume that the timing error is $\Delta t = T/2$. Calculate the probability of error in this case. Find the probability of error when the SNR tends to infinity.

Problem 2 (Frame Synchronization)

A frame consist of a 5-bit preamble and 10 information bits. The information bits are independent and uniformly distributed. The preamble $[1, 1, 1, 0, 1]$ (binary version of a Barker code of length 5) is used to indicate the beginning of the frame. The receiver looks for the preamble and, once it is detected, switches to the data receiving mode. Suppose the channel inverts bits with probability $p = 0.05$.

1. What is the probability of missing the preamble.
2. What is the probability of declaring the detection of the preamble with a 2-bit offset (with respect to the correct preamble position)?
3. What is the probability of declaring the detection of the preamble with a 6-bit offset (with respect to the correct preamble position)?

Problem 3 (Link Budget)

A spacecraft located at 100000 km from the earth is sending data at 1 Mbit/s using BPSK modulation with a sync pulse. The carrier frequency is 2 MHz. The earth station uses a parabolic antenna, 50 m in diameter, and the spacecraft has an antenna with a gain of 10 dB. The noise temperature of the receiver front end is 300 K. Find the transmitted power in order to provide the BER of 10^{-5}.