Examination Wed. Oct. 28, 2020, 14:00–18:00

SSY121 Introduction to Communication Engineering

- Contact person: Examiner Fredrik Brännström (070 – 872 1685)
- Instructions:
  - Write in English.
  - Use a pencil and eraser.
  - There is no page limit.
  - Solve the problems in any order (they are not ordered by difficulty).
  - If any data is missing, make reasonable assumptions.
  - Make sure that each paper is clearly marked with your name, exam problem number and running page number.
  - Scan or photograph your solutions. Make sure to have a good lightning and preferably use a document scanning app, e.g., CamScanner or Genius Scan.
  - Name your image files Problem_YY_Page_XX.
    Example: Problem_01_Page_02.jpg
  - If you want, you can combine images for the same problem into a single document (e.g. PDF). *Example*: Problem_YY.pdf
  - You can also combine all problems into one single file.
    Example: Exam_SSY121_Fredrik.pdf
- Allowed aids:
  - All aids are allowed. However, **it is not permitted to cooperate with or take help from another person**.
- Grading principles:
  - Explain the line of reasoning clearly. A good solution with a minor error usually gives close to full points, even if the answer is incorrect.
  - An answer without a clear motivation usually gives 0 points, even if it is correct.
  - Whenever possible, check if your answer is reasonable. An obviously unreasonable answer usually gives 0 points, even if the solution is almost correct.
- Solutions and results:
  - After the exams are graded, a date and time for an online grading review will be posted on the course website in Canvas.

1. Consider a communication system using the following four signals for transmission,

$$\begin{aligned}
s_1(t) &= +2\mathrm{I}\{0 \le t \le 1\} - 2\mathrm{I}\{1 \le t \le 2\} + 2\mathrm{I}\{2 \le t \le 3\},\\
s_2(t) &= -2\mathrm{I}\{0 \le t \le 1\} + 2\mathrm{I}\{4 \le t \le 5\},\\
s_3(t) &= +2\mathrm{I}\{2 \le t \le 3\} + 2\mathrm{I}\{4 \le t \le 5\},\\
s_4(t) &= +2\mathrm{I}\{0 \le t \le 1\} + 2\mathrm{I}\{3 \le t \le 5\},
\end{aligned}$$

where the indicator function $\mathrm{I}\{\cdot\}$ is equal to 1 when the condition is fulfilled.
(Total points: 7)

   (a) Draw the signals $s_1(t), s_2(t), s_3(t)$, and $s_4(t)$. (1)

   (b) Find a **minimal set** of orthonormal basis for $\{s_1(t), s_2(t), s_3(t), s_4(t)\}$. (4)

   (c) Represent the set of signals $\{s_1(t), s_2(t), s_3(t), s_4(t)\}$ in terms of the basis functions. (2)

2. Assume a pulse

$$v(t) = \begin{cases} 1 & 0 \le t < 0.1 \text{ and } 0.3 \le t < 0.4 \\ 0, & \text{otherwise} \end{cases}$$

with a duration of 1 second is used to modulate the BPSK symbol $a \in \{-1, +1\}$ to create the transmitted signal $s(t) = av(t)$. A matched filter is used at the receiver. (Total points: 10)

   (a) Sketch the output of the matched filter when $s(t)$ is received without noise for $a = 1$ and write down its expression. (2)

   (b) Sketch the eye-diagram for the case $a \in \{-1, +1\}$ assuming a noise-free channel. Find in what range the timing error could be in order to have a zero symbol error rate. (2)

   (c) Suppose the transmitted signal $s(t)$ when $a = 1$ is passed through a channel with a frequency response $H(f) = 1 + e^{-j2\pi f t_0} + e^{-j4\pi f t_0}$. Sketch the channel output $y(t)$ when $t_0 = 0.1$ and write down its expression. (3)

   (d) The channel output $y(t)$ from (c) is now passed through the filter matched to $v(t)$. Sketch the matched filter output when $t_0 = 0.1$ assuming no noise in the receiver and write down its expression. (3)

3. A spacecraft located $50,000$ km from earth is sending data at a rate of R bits/sec using a BPSK modulation scheme. The antenna gain at the transmitter side and the transmitted power are 0 dB and 10 W, respectively. The earth station receiver uses a parabolic antenna, 200 m in diameter, and the noise temperature of the receiver front end is $T = 300$ K. (Total points: 4)

   (a) Determine the received power level. (2)

   (b) If an average SNR of $\frac{E_b}{N_0} = 10$ dB is required, determine the maximum bit rate that the spacecraft can transmit with. (2)

4. Consider a communication system using a pulse $v(t)$ having a spectrum $V(f)$ shown in Figure 1, where $a > 0$ and $0 < b < 1$. (Total points: 3)

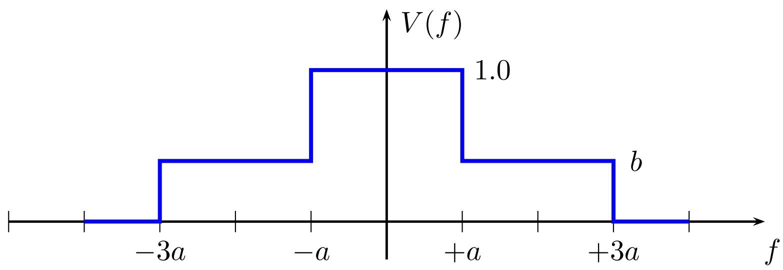

Figure 1: Spectrum of the pulse.

   (a) What is the maximum symbol rate expressed in a to achieve ISI-free communication using a matched filter receiver when $b = 0.4$? (1)

   (b) What value of b, $0 < b < 1$, maximizes the symbol rate to achieve ISI-free communication using a matched filter receiver, and what is the maximum rate expressed in a for this case? (2)

5. In a binary communication system, the symbols $s_1 = +A$ and $s_2 = -A$ with the *a priori* probabilities $p(s_1) = p_1$ and $p(s_2) = p_2$, where $p_1 + p_2 = 1$, are sent over a fading communication channel with additive noise. The received symbol r can be written as

$$r = \rho s + n,$$

where s is the transmitted symbol and n is the noise variable characterized by the probability density function

$$p(n) = \frac{1}{\sqrt{2\sigma^2}} \exp\left(\frac{-|n|}{\sigma/\sqrt{2}}\right).$$

The communication channel undergoes a random fading process described by the random variable ρ taking the values 0 and 1, with equal probability.
(Total points: 12)

(a) Determine the optimal *decision rule* in this communication scheme in the general case. (3)

(b) Determine and plot the optimal *decision regions* in case $p_1 = p_2$. (3)

(c) Find the error probability of the optimal receiver as a function of A and σ when $p_1 = p_2$. *If you have not found the decision boundary in (b), you can assume it is equal to* r_{th}*, and still solve this part.* (4)

(d) Assume now that the additive noise is normally distributed with the probability density function (2)

$$p(n) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-|n|^2}{2\sigma^2}},$$

and the decision rule for the optimal receiver is given by

$$\hat{s} = \begin{cases} s_1, & r > r_{\text{th}} \\ s_2, & r < r_{\text{th}} \end{cases}.$$

Find the error probability as a function of A and σ when $p_1 = p_2$.

6. Consider the 16-point QAM constellation shown in Figure 2, with the minimum distance of $D_{\min} = 2d$. This constellation is used for communication over an AWGN channel with noise variance σ^2. Consider the following relations between the *a priori* probabilities:

$$p(s_i) = p(s_j)\ \forall i, j \in \{1, 4, 13, 16\},$$
$$p(s_k) = p(s_l)\ \forall k, l \in \{2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 14, 15\},$$
$$p(s_1) = 2p(s_2).$$

(Total points: 12)

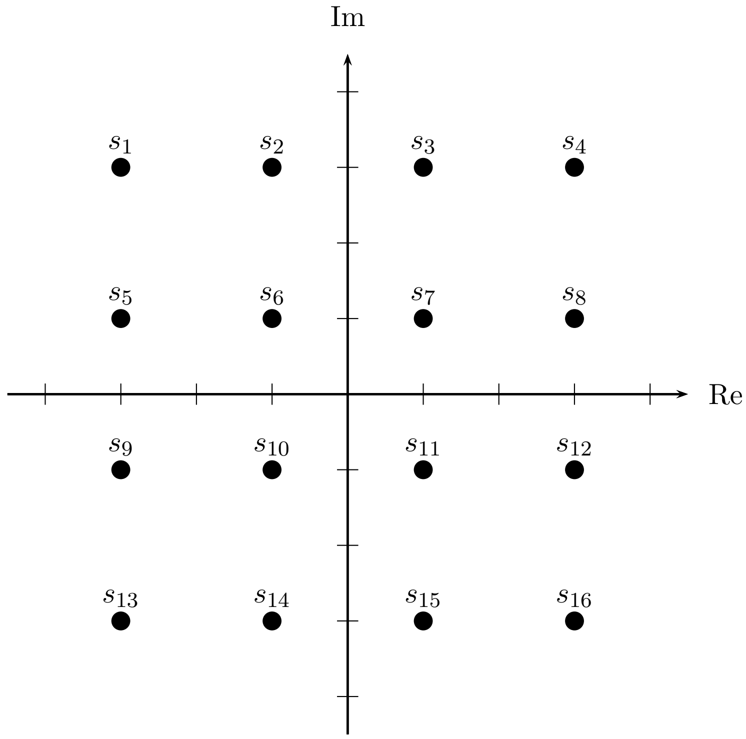

Figure 2: 16-point QAM constellation

(a) Find the *decision boundaries*, i.e., thresholds, as functions of d and σ^2 and specify the *decision regions* by drawing the boundaries for the MAP receiver. (4)

(b) Compute the average energy of the constellation in terms of d, both for the case above with unequal probabilities and in the case when all signal alternatives are equally likely. (3)

(c) If the transmitted symbol is denoted by s and the detected symbol by $\hat{s}$, find the exact conditional probability $\text{Prob}(\hat{s} = s_3 | s = s_1)$ as a function of d and σ. Assume that the noise components are independent and identically distributed. (3)

(d) Assume that the exact SER for the non-equiprobable case is given by $P_e(d, \sigma^2) = P$. Obtain the exact SER $\tilde{P}_e(d, \sigma^2)$ as a function of P for the new constellation with the signals $\tilde{s}_i = s_i - \mathbf{b}$, $i = 1, \cdots, 16$, where $\mathbf{b} = [-1, 1]$. Assume that $p(\tilde{s}_i) = p(s_i)$, $i = 1, \cdots, 16$. (2)