Examination Mon. Oct. 23, 2017, 14:00–18:00

SSY121 – Introduction to Communication Engineering

- Contact persons: Mohammad Nazari (031 - 772 1771) and Fredrik Brännström (031 - 772 1787) will visit the exam after approximately 1 and 3 hours.
- Instructions:
  - Write in English.
  - Use a pencil and eraser.
  - There is no page limit. Extra sheets of paper are available.
  - Solve the problems in any order (they are not ordered by difficulty).
  - Before handing in, sort the pages in problem order. Label each page with problem number and running page number. Do not hand in drafts or unused paper.
  - If any data is missing, make reasonable assumptions.
  - Chalmers' examination rules apply, available at student.portal.chalmers.se /en/chalmersstudies/Examinations/Pages/Examinationroominstructions.aspx
  - MP3/Music players **are not** allowed during the exam
- Allowed aids:
  - Mathematics Handbook by Råde and Westergren (any edition, including Beta) or equivalent
  - Chalmers-approved calculator
- Grading principles:
  - Explain the line of reasoning clearly. A good solution with a minor error usually gives close to full points, even if the answer is incorrect.
  - An answer without a clear motivation usually gives 0 points, even if it is correct.
  - Whenever possible, check if your answer is reasonable. An obviously unreasonable answer usually gives 0 points, even if the solution is almost correct.
- Exam review:
  - The grading of the exam can be reviewed on Thursday November 2, 2017, at 14:00–15:00 in Room BI Rummet 6414, floor 6, Department of Electrical Engineering, EDIT building.

1. **True or false questions:** Justify *ALL* your answers using short and concise explanations (maximum 30 words per item). (Total points: 10)

   (a) BPSK and QPSK use two orthogonal basis functions, but 8-PSK uses three orthogonal basis functions. (1)
   False. BPSK uses one orthogonal basis function, while QPSK and 8-PSK use two orthogonal basis function.

   (b) In M-PSK signaling over AWGN channels, if the phase drift is small enough, the receiver can correctly detect the transmitted signal, even without a synchronization unit. (1)
   False. In AWGN channels, it is not possible to tolerate the phase drift without any phase synchronization unit.

   (c) The Minimum Distance receiver is optimal (in terms of minimizing error probability) in the channels with the input-output relation $y(t) = x(t) \times n(t)$, where $y(t)$, $x(t)$, and $n(t)$ are the output, the input, and the noise of the channel, respectively. (1)
   False. The MD receiver is optimal if the channel is AWGN, and the symbols are equally likely.

   (d) For all M-ary modulation signaling schemes, the MAP receiver has the same conditional error probability for all symbols, if the symbols are equally likely. (1)
   False. M-PAM is a counter example. The first and the last symbols have different conditional error probabilities, since the noise can affect their correct reception only in one direction (either positive or negative noise).

   (e) For a given symbol rate $R_s = 1/T_s$, root raised-cosine pulses are T_s-orthogonal pulses, raised-cosine pulses are Nyquist pulses, and sinc pulses are both Nyquist and T_s-orthogonal pulses. (1)
   True.

   (f) If a zero-mean N-dimensional constellation is shifted in the space, so that the new constellation is not zero-mean anymore, for a given energy constraint, the performance of the new constellation will be identical to the first one. (1)
   False, Shifting the constellation does not change the minimum distance but increases the average symbol energy. Therefore, for a given energy constraint, the new constellation will be worse than the original one.

   (g) Systems using OFDM are seriously impaired by nonlinear amplifiers and nonlinear channel phenomena. (1)
   True, since the amplitude of an OFDM signal varies a lot.

   (h) In a digital communication system using BPSK modulation, phase errors happen due to some imperfections. If the phase error is 45°, 3dB more power is required to have the same performance as previous. (1)
   True. Because the power should be doubled.

   (i) The frequency domain interpretation of the matched filter is the complex conjugate of the transmitted signal spectrum multiplied by an exponential function representing the sampling delay. (1)
   True. $H(f) = S^*(f)e^{-j2\pi fT}$

(j) For a fixed bit rate, 16-QAM modulation is more bandwidth efficient compared to 8-PSK. (1)

True, since each symbol of 16-QAM represents 1 bit more than 8-PSK symbols.

2. Consider a communication system in which the binary bits 0 and 1 are transmitted using $s_1(t) = s_1 I\{0 \leq t \leq 1\}$ and $s_2(t) = s_2 I\{0 \leq t \leq 1\}$ respectively, where $s_1 = 1$ and $s_2 = 2$. *A priori* probabilities are $p(s_1) = \frac{1}{3}$ and $p(s_2) = \frac{2}{3}$. Assume that the channel is AWGN, but the variance of the noise is $\sigma_1^2 = s_1^2 \cdot \sigma^2$, when 0 is transmitted, and $\sigma_2^2 = s_2^2 \cdot \sigma^2$, when 1 is transmitted, i.e., the noise is signal-dependent. (Total points: 7)

(a) Find the decision rule for the optimal receiver (which minimizes the probability of error) in the simplest form. (4)

Hint: You need to find two thresholds as the decision boundaries.

The optimal detection rule is that upon receiving the received vector r, decision is made in favor of bit 0 if $p(s_1) \cdot p(r|s_1) > p(s_2) \cdot p(r|s_2)$, and in favor of bit 1, otherwise, where $s_1 = 1$ and $s_2 = 2$ are vector representations of bits 0 and 1, respectively. The above decision rule is written as in the following:

$$\frac{1}{3} \cdot \frac{1}{\sqrt{2\pi \cdot \sigma^2}} \mathrm{e}^{-\frac{(r-1)^2}{2 \cdot \sigma^2}} > \frac{2}{3} \cdot \frac{1}{\sqrt{2\pi \cdot 4\sigma^2}} \mathrm{e}^{-\frac{(r-2)^2}{2 \cdot 4\sigma^2}}.$$

By simplifying the above equation and taking natural logarithm of both sides, the receiver decides in favor of 0 if

$$(r-2)^2 - 4(r-1)^2 > 0.$$

Accordingly,

$$\hat{B} = \begin{cases} 0 & \text{if } 0 < r < \frac{4}{3} \\ 1 & \text{otherwise} \end{cases}$$

where $\hat{B}$ is the detected bit.

(b) Based on the two thresholds found in (a), calculate the BER of the optimal receiver. (3)

Hint: If you have not found the thresholds, you may still solve this part by considering r_{th1} and r_{th2}.

To calculate the error probability, according to the MAP decision rule, one can write:

$$p_{e|s_1} = \Pr\left(r > \frac{4}{3} \text{ or } r < 0 | s_1 = 1\right) = Q\left(\frac{1}{\sigma}\right) + Q\left(\frac{1}{3\sigma}\right)$$
$$p_{e|s_2} = \Pr\left(0 < r < \frac{4}{3} | s_2 = 2\right) = Q\left(\frac{1}{3\sigma}\right) - Q\left(\frac{1}{\sigma}\right)$$
$$p_e = \frac{1}{3} p_{e|s_1} + \frac{2}{3} p_{e|s_2} = Q\left(\frac{1}{3\sigma}\right) - \frac{1}{3} Q\left(\frac{1}{\sigma}\right)$$

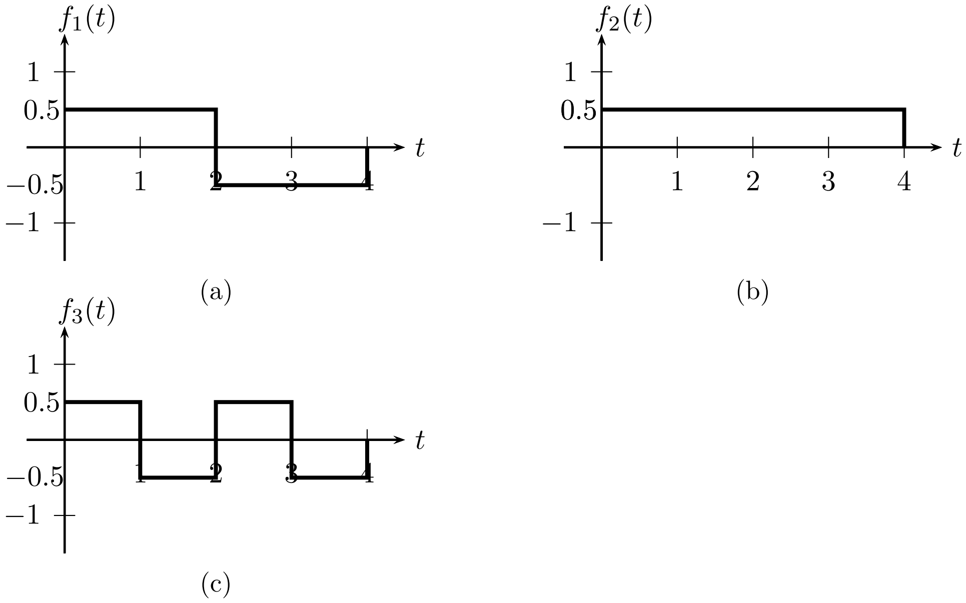


Figure 1

3. Consider the three waveforms $f_n(t)$ shown in Figure 1. (Total points: 7)

   (a) Show that these waveforms are orthonormal. (4)

   We need to show that

$$\int_{-\infty}^{\infty} f_m(t) f_n(t) \mathrm{d}t = \begin{cases} 0, & m \neq n \\ 1, & m = n \end{cases}.$$

$$\begin{aligned}
\langle f_1(t), f_2(t) \rangle &= \int_{-\infty}^{\infty} f_1(t) f_2(t) \mathrm{d}t = \int_0^4 f_1(t) f_2(t) \mathrm{d}t \\
&= \frac{1}{4}\int_0^2 \mathrm{d}t - \frac{1}{4}\int_2^4 \mathrm{d}t = 0, \\
\langle f_1(t), f_3(t) \rangle &= \int_{-\infty}^{\infty} f_1(t) f_3(t) \mathrm{d}t = \int_0^4 f_1(t) f_3(t) \mathrm{d}t \\
&= \frac{1}{4}\int_0^1 \mathrm{d}t - \frac{1}{4}\int_1^2 \mathrm{d}t - \frac{1}{4}\int_2^3 \mathrm{d}t + \frac{1}{4}\int_3^4 \mathrm{d}t = 0, \\
\langle f_2(t), f_3(t) \rangle &= \int_{-\infty}^{\infty} f_2(t) f_3(t) \mathrm{d}t = \int_0^4 f_2(t) f_3(t) \mathrm{d}t \\
&= \frac{1}{4}\int_0^1 \mathrm{d}t - \frac{1}{4}\int_1^2 \mathrm{d}t + \frac{1}{4}\int_2^3 \mathrm{d}t - \frac{1}{4}\int_3^4 \mathrm{d}t = 0.
\end{aligned}$$

   Additionally, we have

$$\langle f_1(t), f_1(t) \rangle = \langle f_2(t), f_2(t) \rangle = \langle f_3(t), f_3(t) \rangle = \int_0^4 \frac{1}{4} \mathrm{d}t = 1.$$

   (b) Express the waveform $x(t)$ as a linear combination of $f_n(t)$, $n = 1, 2, 3$, if

$$x(t) = \begin{cases} 1 & 0 \leq t < 1 \\ 2 & 1 \leq t < 2 \\ -1 & 2 \leq t < 3 \end{cases},$$

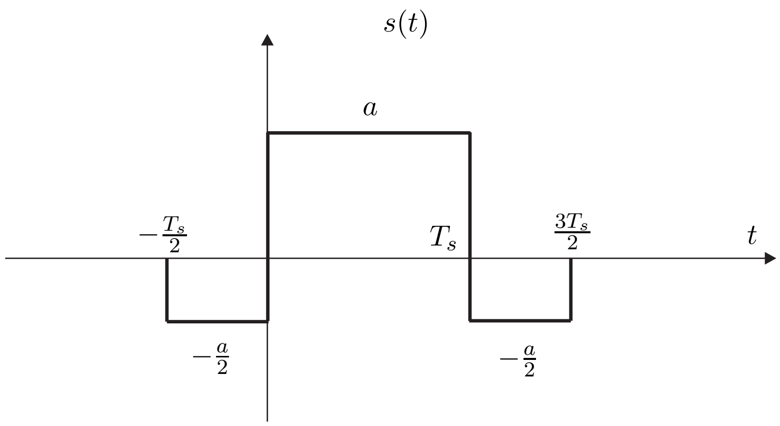
and determine the weighting coeeficients. (3)

$$\begin{aligned}
x_1 = \langle x(t), f_1(t)\rangle &= \int_0^4 x(t) f_1(t)\mathrm{d}t \\
&= \int_0^1 \frac{1}{2} \times 1\mathrm{d}t + \int_1^2 \frac{1}{2} \times 2\mathrm{d}t + \int_2^3 \frac{-1}{2} \times (-1)\mathrm{d}t = 2, \\
x_2 = \langle x(t), f_2(t)\rangle &= \int_0^4 x(t) f_2(t)\mathrm{d}t \\
&= \int_0^1 \frac{1}{2} \times 1\mathrm{d}t + \int_1^2 \frac{1}{2} \times 2\mathrm{d}t + \int_2^3 \frac{1}{2} \times (-1)\mathrm{d}t = 1, \\
x_3 = \langle x(t), f_3(t)\rangle &= \int_0^4 x(t) f_3(t)\mathrm{d}t \\
&= \int_0^1 \frac{1}{2} \times 1\mathrm{d}t + \int_1^2 \frac{-1}{2} \times 2\mathrm{d}t + \int_2^3 \frac{1}{2} \times (-1)\mathrm{d}t = -1.
\end{aligned}$$

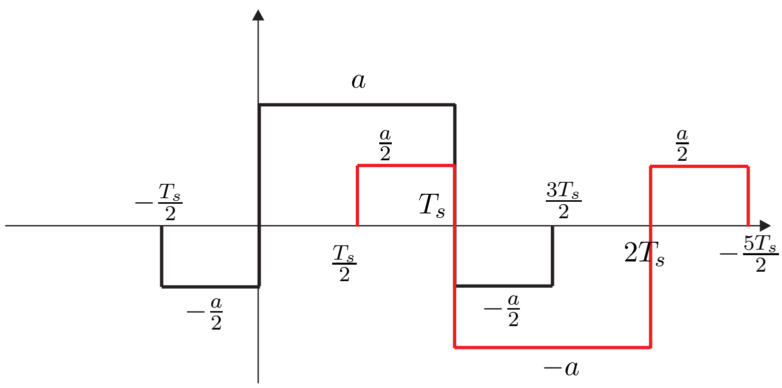
Thus,

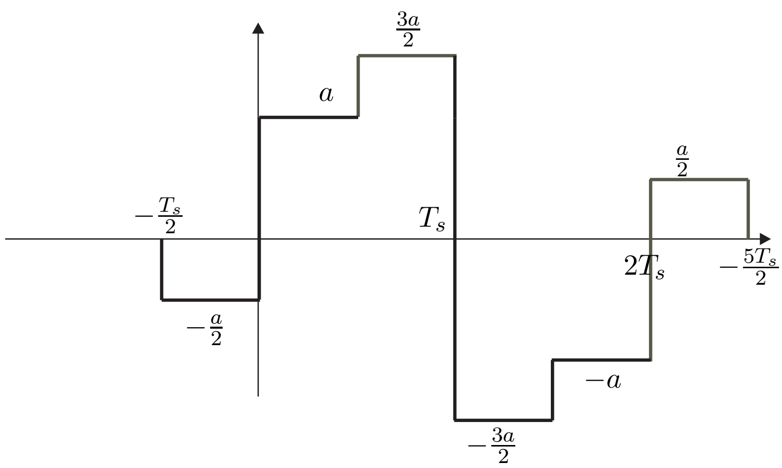
$$x(t) = 2f_1(t) + f_2(t) - f_3(t).$$

4. Assume a BPSK data transmission system with the given pulse shape $s(t)$, symbol separation T_s, and signal vector representation of $\{-1, 1\}$. A matched filter receiver is implemented. (Total points: 8)

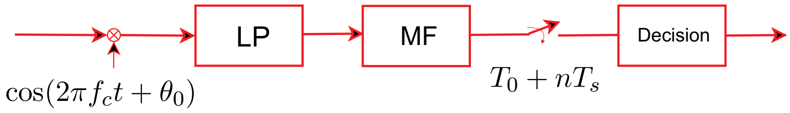


(a) What are the maximum and minimum possible values at the output of the transmitter when a train of symbols is transmitted? (Motivate your answer.) (2)
The maximum and minimum values are $\frac{3a}{2}$, and $-\frac{3a}{2}$, respectively. (Answer must be motivated e.g., sketching a graph)



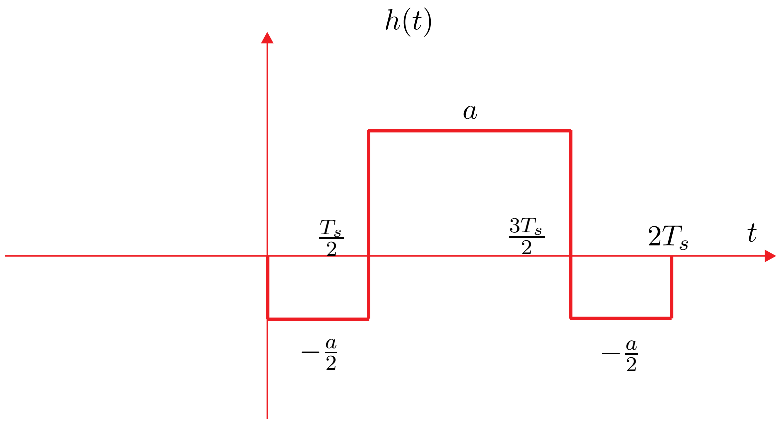


(b) Sketch the receiver's block diagram and find the causal impulse response of the receiver's filter? What is the optimum sampling instant? (3)



(1)

The MF response can be found by flipping and shifting the shaping signal, $s(t)$. Shifting to right is done to make the filter response causal.

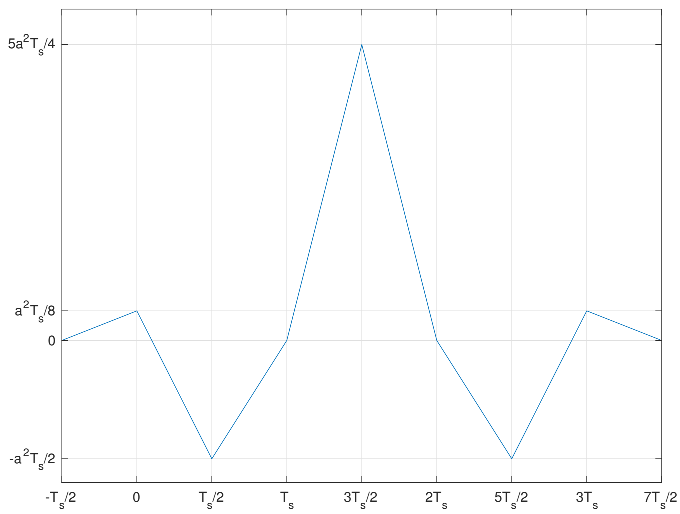


(2)

The optimum sampling time instant is $T_0 = \frac{3T_s}{2}$, which is the amount of time shifting to make the filter causal. (1)

(c) Sketch the matched filter output signal (noise-free case) for the transmitted symbol of $+1$. Values over the axis must be calculated and marked correctly. (3)

Calculating the convolution of the MF response and the received signal gives:



5. Answer the following questions (about working in projects) using concise explanations with a maximum of 50 words per item. (Total points: 4)

   (a) Describe the SCRUM process. (2)

   In the SCRUM methodology a sprint is the basic unit of development. Each sprint starts with a planning meeting, where the tasks for the sprint are identified and an estimated commitment for the sprint goal is made. A Sprint ends with a review or retrospective meeting where the progress is reviewed and lessons for the next sprint are identified. During each sprint, the team creates finished portions of a product

   (b) Describe the execution phase in a project. (2)

      i. Specify sub-units and interfaces in sufficient detail
      ii. Find a step-wise plan for testing the solution
      iii. Decide on a coordination practice
      iv. Get the job done

6. The symbol error rate for the AWGN channel in the high-SNR regime is approximately given as

$$P_e \approx \alpha(M)\mathrm{Q}\left(\frac{d(M,E_s)}{\sigma}\right)$$

where $\alpha(M)$ is the average number of nearest neighbors in the symbol constellation. The function $d(M, E_\mathrm{s})$ is half the minimum distance between two symbols.

(Total points: 10)

(a) Consider an M-PSK constellation with equiprobable symbols. Derive $\alpha(M)$ and $d(M, E_\mathrm{s})$.

(2)

- $\alpha(M)$

  In an M-PSK constellation, all symbols have 2 nearest neighbours. Therefore,

$$\alpha(M) = 2.$$

- $d(M, E_\mathrm{s})$

  We start by noting that each symbol in the constellation is given as

$$s_k = \{Ae^{j\frac{2\pi}{M}k}\}, \quad k = 0, \ldots, M-1. \tag{1}$$

  Hence, the average symbol energy is given as

$$E_\mathrm{s} = A^2. \tag{2}$$

  The phase between two neighboring points is $\frac{2\pi}{M}$. The ML decision region is a line from the origin drawn between the two symbols. By using some trigonometry, we find that

$$d(M, E_\mathrm{s}) = A\sin\left(\frac{\pi}{M}\right) = \sqrt{E_\mathrm{s}}\sin\left(\frac{\pi}{M}\right). \tag{3}$$

(b) Now, consider an M-QAM constellation with equiprobable symbols. Derive $\alpha(M)$ and $d(M, E_\mathrm{s})$.
(*hint: for an M-PAM constellation with the constellation symbols* $\mathcal{C} = \{\pm A, \pm 3A, \ldots, \pm(M-1)A\}$, *we have* $E_\mathrm{s} = \frac{A^2(M^2-1)}{3}$)

(6)

- $\alpha(M)$

  Consider Fig. 2. The rectangles are colored by how many neighbors each symbol have at minimum distance. The symbols in the green rectangle have four neighbors and the number of symbols grows like $(\sqrt{M}-2)^2$. The symbols in the blue rectangles have three neighbors and the number of symbols grows like $4*(\sqrt{M}-2)$. The number of symbols in the red squares is always 4 and the symbols within have 2 neighbors. Therefore, we get

$$
\begin{aligned}
\alpha(M) &= \frac{4*2+4(\sqrt{M}-2)*3+(\sqrt{M}-2)^2*4}{M} \\
&= \frac{4}{M}(2+3\sqrt{M}-6+M-4\sqrt{M}+4) \\
&= 4\left(1-\frac{1}{\sqrt{M}}\right)
\end{aligned}
\tag{4}
$$

- $d(M, E_s)$

  The M-QAM-constellation is nothing else than two $\sqrt{M}$-PAM constellations stacked on each other. Therefore, we can find the average energy of the constellation by adding the average energy of two $\sqrt{M}$-PAM constellations. Using the expression for E_s in a), we get

$$
E_s = E_{s,\text{real}} + E_{s,\text{imag}} = \frac{2A^2(M-1)}{3}. \tag{5}
$$

  Finally, noticing that $d(M, E_s) = A$, we get

$$
d(M, E_s) = \sqrt{\frac{3E_s}{2(M-1)}}. \tag{6}
$$

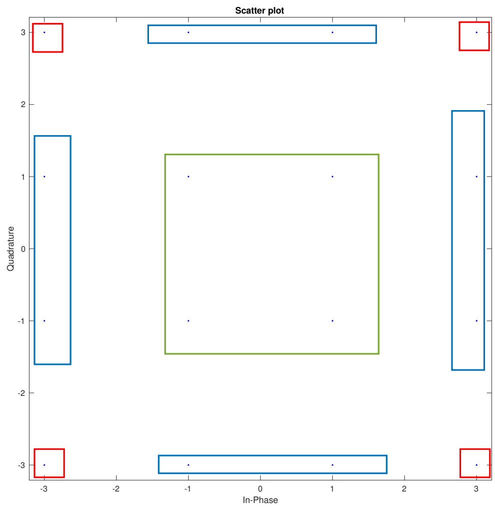
(c) Sketch the asymptotic gain between the M-PSK and M-QAM considered in a) and b) as a function of the number of bits per symbol (ratio of the $d(M, E_s)$ in a) and b)). Interpret your result.

(2)

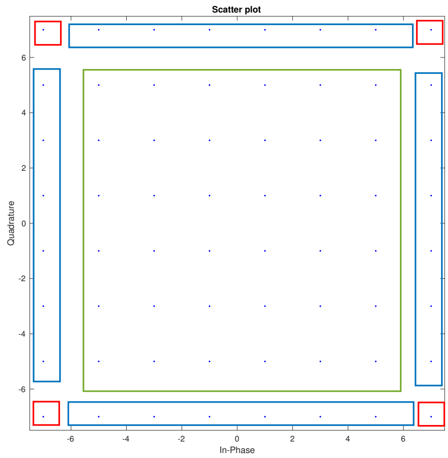
The asymptotic gain is given by the ratio of the argument of the Q-function, i.e.,

$$
\begin{aligned}
\frac{d(M, E_s)_{M\text{-PSK}}}{d(M, E_s)_{M\text{-QAM}}} &= \sqrt{\frac{E_s \sin^2\left(\frac{\pi}{M}\right)}{\frac{3E_s}{2(M-1)}}} \\
&= \sqrt{\frac{2}{3}(M-1)}\sin\left(\frac{\pi}{M}\right)
\end{aligned}
\tag{7}
$$

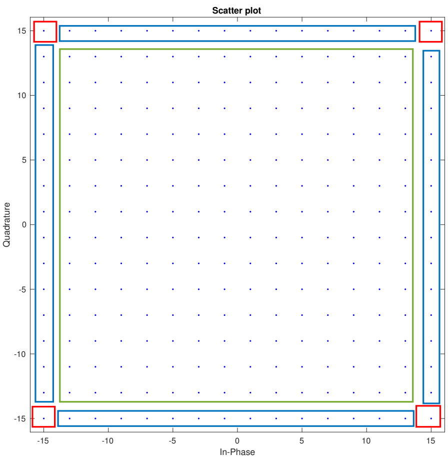
From Fig. 3, we see that as we increase $\log_2(M)$, i.e., the number of symbols in the constellation, the gain of using M-QAM increases drastically. We also see that for $M = 2$, we have the same performance, which is to be expected since 4-PSK is the same thing as 4-QPSK.



(a) 16-QAM



(b) 64-QAM



(c) 256-QAM

Figure 2

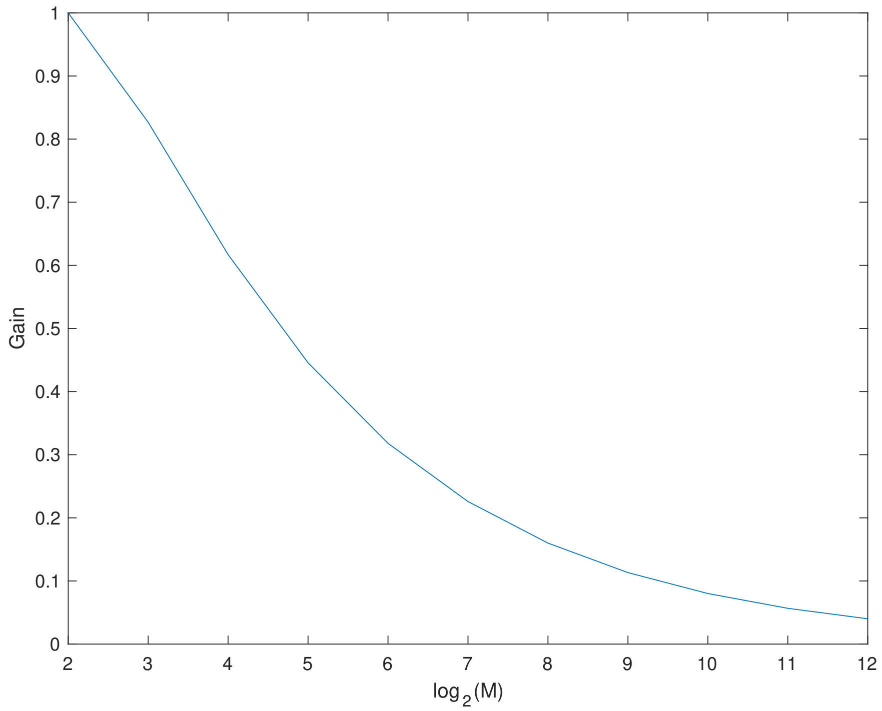


Figure 3: Asymptotic gain.

7. There are five updates to the physical layer in WiFi 802.11a compared to WiFi 802.11n that made the maximum throughput increase from 54 Mbit/s to 600 Mbit/s. Give example of two (and ONLY two) of them.

(Total points: 2)

Any two of the following updates:

(a) 52 data subcarriers instead of 48 subcarriers in 20 MHz.

(b) The highest code rate for the channel code was updated from 3/4 to 5/6

(c) The guard interval was shortened from 0.8 μs to 0.4 μs

(d) The channel width was increased from 20 MHz to 40 MHz, i.e., 108 data subcarriers in 40 MHz

(e) MIMO with up to 4 streams was introduced