Homework 2

Due Date: Sep. 20, 2023

Problem 1 (Matched Filter vs Correlator)

Let $x(t)$ and $y(t)$ be two complex functions. Let $\overleftarrow{x}(t)$ denote $x(-t)$.
Express the inner product $\langle x, y \rangle$ in terms of the convolution $x(t)$ and $y(t)$.

Problem 2 (Convolution)

Let a random variable X be uniformly distributed on the interval $[0, 2]$ and a random variable Y be exponential distributed, i.e., $f_Y(y) = A\mathrm{e}^{-2y}$, $y \geq 0$.

1. Find the constant A.
2. Find the distribution of $Z = X + Y$ if X and Y are independent.

Problem 3 (Centered Constellations)

Consider an arbitrary N-dimensional constellation with M points, i.e., $\mathcal{S} = \{\boldsymbol{s}_j \in \mathbb{R}^N,\ 1 \leq j \leq M\}$. Assume that all constellation points are equiprobable. Create a new constellation constellation $\tilde{\mathcal{S}} = \{\tilde{\boldsymbol{s}}_j \in \mathbb{R}^N,\ 1 \leq j \leq M,\ \ \tilde{\boldsymbol{s}}_j = \boldsymbol{s}_j - \boldsymbol{b}\}$, where $\boldsymbol{b} \in \mathbb{R}^N$.

1. Show that the minimum distance is the same for the two constellations.
2. Find $\boldsymbol{b}$ that minimizes the average energy for $\tilde{S}$.
3. Show that for $M = 2$ and a minimum distance d, the binary antipodal signaling (BPSK) has the least energy.