Examination Mon. Oct. 23, 2017, 14:00–18:00

SSY121 – Introduction to Communication Engineering

- Contact persons: Mohammad Nazari (031 - 772 1771) and Fredrik Brännström (031 - 772 1787) will visit the exam after approximately 1 and 3 hours.
- Instructions:
  - Write in English.
  - Use a pencil and eraser.
  - There is no page limit. Extra sheets of paper are available.
  - Solve the problems in any order (they are not ordered by difficulty).
  - Before handing in, sort the pages in problem order. Label each page with problem number and running page number. Do not hand in drafts or unused paper.
  - If any data is missing, make reasonable assumptions.
  - Chalmers' examination rules apply, available at student.portal.chalmers.se /en/chalmersstudies/Examinations/Pages/Examinationroominstructions.aspx
  - MP3/Music players **are not** allowed during the exam
- Allowed aids:
  - Mathematics Handbook by Råde and Westergren (any edition, including Beta) or equivalent
  - Chalmers-approved calculator
- Grading principles:
  - Explain the line of reasoning clearly. A good solution with a minor error usually gives close to full points, even if the answer is incorrect.
  - An answer without a clear motivation usually gives 0 points, even if it is correct.
  - Whenever possible, check if your answer is reasonable. An obviously unreasonable answer usually gives 0 points, even if the solution is almost correct.
- Exam review:
  - The grading of the exam can be reviewed on Thursday November 2, 2017, at 14:00–15:00 in Room BI Rummet 6414, floor 6, Department of Electrical Engineering, EDIT building.

1. **True or false questions:** Justify *ALL* your answers using short and concise explanations (maximum 30 words per item). (Total points: 10)

   (a) BPSK and QPSK use two orthogonal basis functions, but 8-PSK uses three orthogonal basis functions. (1)

   (b) In M-PSK signaling over AWGN channels, if the phase drift is small enough, the receiver can correctly detect the transmitted signal, even without a synchronization unit. (1)

   (c) The Minimum Distance receiver is optimal (in terms of minimizing error probability) in the channels with the input-output relation $y(t) = x(t) \times n(t)$, where $y(t)$, $x(t)$, and $n(t)$ are the output, the input, and the noise of the channel, respectively. (1)

   (d) For all M-ary modulation signaling schemes, the MAP receiver has the same conditional error probability for all symbols, if the symbols are equally likely. (1)

   (e) For a given symbol rate $R_\mathrm{s} = 1/T_\mathrm{s}$, root raised-cosine pulses are T_s-orthogonal pulses, raised-cosine pulses are Nyquist pulses, and sinc pulses are both Nyquist and T_s-orthogonal pulses. (1)

   (f) If a zero-mean N-dimensional constellation is shifted in the space, so that the new constellation is not zero-mean anymore, for a given energy constraint, the performance of the new constellation will be identical to the first one. (1)

   (g) Systems using OFDM are seriously impaired by nonlinear amplifiers and nonlinear channel phenomena. (1)

   (h) In a digital communication system using BPSK modulation, phase errors happen due to some imperfections. If the phase error is 45°, 3dB more power is required to have the same performance as previous. (1)

   (i) The frequency domain interpretation of the matched filter is the complex conjugate of the transmitted signal spectrum multiplied by an exponential function representing the sampling delay. (1)

   (j) For a fixed bit rate, 16-QAM modulation is more bandwidth efficient compared to 8-PSK. (1)

2. Consider a communication system in which the binary bits 0 and 1 are transmitted using $s_1(t) = s_1 I\{0 \leq t \leq 1\}$ and $s_2(t) = s_2 I\{0 \leq t \leq 1\}$ respectively, where $s_1 = 1$ and $s_2 = 2$. *A priori* probabilities are $p(s_1) = \frac{1}{3}$ and $p(s_2) = \frac{2}{3}$. Assume that the channel is AWGN, but the variance of the noise is $\sigma_1^2 = s_1^2 \cdot \sigma^2$, when 0 is transmitted, and $\sigma_2^2 = s_2^2 \cdot \sigma^2$, when 1 is transmitted, i.e., the noise is signal-dependent. (Total points: 7)

   (a) Find the decision rule for the optimal receiver (which minimizes the probability of error) in the simplest form. (4)
   Hint: You need to find two thresholds as the decision boundaries.

   (b) Based on the two thresholds found in (a), calculate the BER of the optimal receiver. (3)
   Hint: If you have not found the thresholds, you may still solve this part by considering r_{th1} and r_{th2}.

3. Consider the three waveforms $f_n(t)$ shown in Figure 1. (Total points: 7)

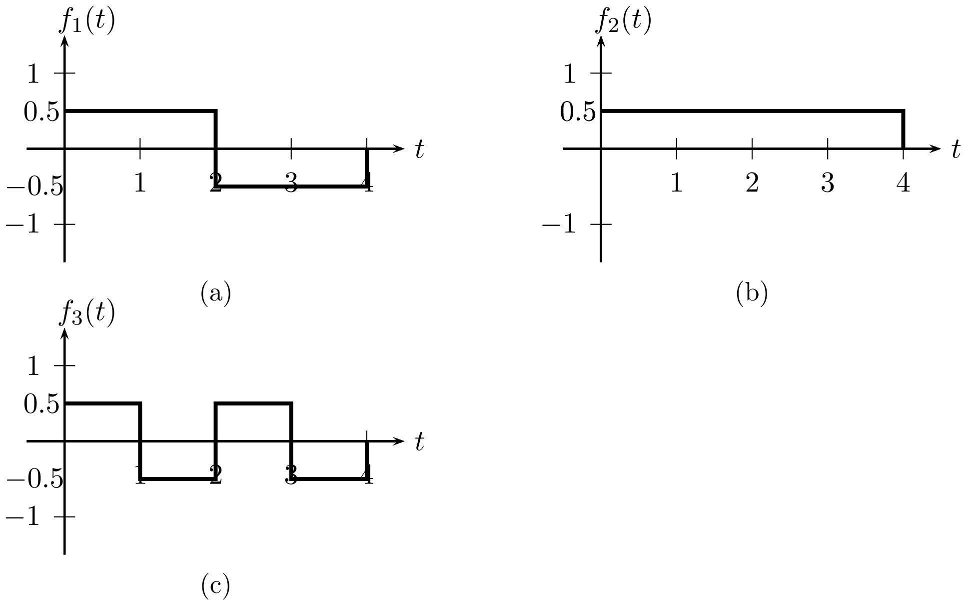


Figure 1

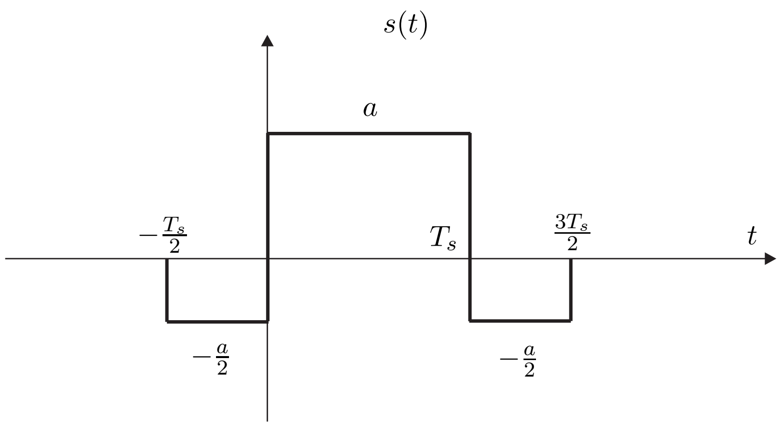
   (a) Show that these waveforms are orthonormal. (4)

   (b) Express the waveform $x(t)$ as a linear combination of $f_n(t),\ n = 1, 2, 3,$ if

$$x(t) = \begin{cases} 1 & 0 \leq t < 1 \\ 2 & 1 \leq t < 2 \\ -1 & 2 \leq t < 3 \end{cases},$$

   and determine the weighting coeeficients. (3)

4. Assume a BPSK data transmission system with the given pulse shape $s(t)$, symbol separation T_s, and signal vector representation of $\{-1, 1\}$. A matched filter receiver is implemented. (Total points: 8)



   (a) What are the maximum and minimum possible values at the output of the transmitter when a train of symbols is transmitted? (Motivate your answer.) (2)

   (b) Sketch the receiver's block diagram and find the causal impulse response of the receiver's filter? What is the optimum sampling instant? (3)

   (c) Sketch the matched filter output signal (noise-free case) for the transmitted symbol of $+1$. Values over the axis must be calculated and marked correctly. (3)

5. Answer the following questions (about working in projects) using concise explanations with a maximum of 50 words per item. (Total points: 4)

   (a) Describe the SCRUM process. (2)

   (b) Describe the execution phase in a project. (2)

6. The symbol error rate for the AWGN channel in the high-SNR regime is approximately given as

$$P_e \approx \alpha(M)\mathrm{Q}\left(\frac{d(M, E_s)}{\sigma}\right)$$

where $\alpha(M)$ is the average number of nearest neighbors in the symbol constellation. The function $d(M, E_\mathrm{s})$ is half the minimum distance between two symbols.

(Total points: 10)

(a) Consider an M-PSK constellation with equiprobable symbols. Derive $\alpha(M)$ and $d(M, E_\mathrm{s})$.

(2)

(b) Now, consider an M-QAM constellation with equiprobable symbols. Derive $\alpha(M)$ and $d(M, E_\mathrm{s})$.
(*hint: for an M-PAM constellation with the constellation symbols* $\mathcal{C} = \{\pm A, \pm 3A, \ldots, \pm(M-1)A\}$, *we have* $E_\mathrm{s} = \frac{A^2(M^2-1)}{3}$)

(6)

(c) Sketch the asymptotic gain between the M-PSK and M-QAM considered in a) and b) as a function of the number of bits per symbol (ratio of the $d(M, E_\mathrm{s})$ in a) and b)). Interpret your result.

(2)

7. There are five updates to the physical layer in WiFi 802.11a compared to WiFi 802.11n that made the maximum throughput increase from 54 Mbit/s to 600 Mbit/s. Give example of two (and ONLY two) of them.

(Total points: 2)

Formula sheet, SSY121

Version 2.1, August 10, 2016

This sheet is an allowed aid at written exams in SSY121, Introduction to Communication Engineering, at Chalmers in 2016. It will be handed out with the exam problems. Students may not bring their own copy.

Decibels

$$\left(\frac{E_1}{E_2}\right)_{\mathrm{dB}} = 10\log_{10}\frac{E_1}{E_2}$$

Energies E_{s} and E_{b}

$$E_{\mathrm{s}} = \sum_{i=1}^{M} \mathbb{P}\left[\mathsf{S} = \mathsf{s}_i\right] ||\mathsf{s}_i||^2$$

$$E_{\mathrm{s}} = E_{\mathrm{b}} \log_2 M$$

Normalized minimum distance

$$d_{\min} = \frac{D_{\min}}{\sqrt{2E_{\mathrm{b}}}}$$

Nyquist criterion

- In time domain
$$v(nT_{\mathrm{s}}) = 0, \quad n = \pm 1, \pm 2, \dots$$
- In frequency domain
$$\sum_{n=-\infty}^{\infty} \Re\left\{V\left(f - \frac{n}{T_{\mathrm{s}}}\right)\right\} = T_{\mathrm{s}} v(0)$$
$$\sum_{n=-\infty}^{\infty} \Im\left\{V\left(f - \frac{n}{T_{\mathrm{s}}}\right)\right\} = 0,$$
where $T_{\mathrm{s}}v(0)$ is a real constant.
- If the $v(t)$ is symmetric respect to zero, the definition in frequency domain is
$$\sum_{n=-\infty}^{\infty} V\left(f - \frac{n}{T_{\mathrm{s}}}\right) = T_{\mathrm{s}} v(0)$$

T_{s}-orthogonality

- In time domain
$$\int_{-\infty}^{\infty} v(t)v(t - nT_{\mathrm{s}})dt = 0, \quad n = \pm 1, \pm 2, \dots$$
- In frequency domain
$$\sum_{n=-\infty}^{\infty} \left|V\left(f - \frac{n}{T_{\mathrm{s}}}\right)\right|^2 = T_{\mathrm{s}} E_v$$

Sinc, Raised-cosine, and Root raised-cosine pulses

$$v_{\mathrm{sinc}}(t) = \mathrm{sinc}(t/T_{\mathrm{p}}) = \frac{\sin(\pi t/T_{\mathrm{p}})}{\pi t/T_{\mathrm{p}}}$$

$$V_{\mathrm{sinc}}(f) = \begin{cases} T_{\mathrm{p}}, & |f| < \frac{1}{2T_{\mathrm{p}}} \\ 0, & |f| \geq \frac{1}{2T_{\mathrm{p}}} \end{cases}$$

$$v_{\mathrm{RC}}(t) = \mathrm{sinc}\left(\frac{t}{T_{\mathrm{p}}}\right) \frac{\cos\left(\frac{\pi\alpha t}{T_{\mathrm{p}}}\right)}{1 - \left(\frac{2\alpha t}{T_{\mathrm{p}}}\right)^2}$$

$$V_{\mathrm{RC}}(f) = \begin{cases} T_{\mathrm{p}}, & |f| < f_1 \\ \frac{T_{\mathrm{p}}}{2}\left(1 + \cos\left[\frac{\pi T_{\mathrm{p}}}{\alpha}\left(|f| - \frac{1-\alpha}{2T_{\mathrm{p}}}\right)\right]\right), & f_1 \leq |f| < f_2 \\ 0, & |f| \geq f_2, \end{cases}$$

where $f_1 = \dfrac{1-\alpha}{2T_{\mathrm{p}}}$ and $f_2 = \dfrac{1+\alpha}{2T_{\mathrm{p}}}$

$$v_{\mathrm{RRC}}(t) = \sqrt{T_{\mathrm{p}}} \frac{\sin\left(\frac{(1-\alpha)\pi t}{T_{\mathrm{p}}}\right) + \frac{4\alpha t}{T_{\mathrm{p}}}\cos\left(\frac{(1+\alpha)\pi t}{T_{\mathrm{p}}}\right)}{\pi t\left(1 - \left(\frac{4\alpha t}{T_{\mathrm{p}}}\right)^2\right)}$$

$$V_{\mathrm{RRC}}(f) = \sqrt{V_{\mathrm{RC}}(f)}$$

Correlation receiver

$$\min_i \left\{ \int_{-\infty}^{\infty} [y(t) - s_i(t)]^2 \, dt \right\}$$
$$\max_i \left\{ \int_{-\infty}^{\infty} y(t)s_i(t) \, dt - \frac{E_{s_i}}{2} \right\},$$

where $E_{s_i} = \int_{-\infty}^{\infty} s_i^2(t)\, dt$.

PAM (baseband)

$$s(t) = \sum_{k=0}^{\infty} a_k v(t - kT_{\mathrm{s}}),$$

where $a_k \in \{\pm(M-1), \pm(M-3), \dots, \pm 1\}$.

PAM (passband)

$$s(t) = \sum_{k=1}^{\infty} a_k v(t - kT_{\mathrm{s}})\sqrt{2}\cos w_c t,$$

where $a_k \in \{\pm(M-1), \pm(M-3), \dots, \pm 1\}$

2D Modulations

$$s(t) = \sum_{k=0}^{\infty} a_k v(t - kT_{\mathrm{s}})\sqrt{2}\cos(w_c t) - \sum_{k=0}^{\infty} b_k v(t - kT_{\mathrm{s}})\sqrt{2}\sin(w_c t)$$

M-PSK

$$s(t) = \sum_{k=0}^{\infty} v(t - kT_\mathrm{s})\sqrt{2}\cos\left(w_c t + \frac{2i\pi}{M}\right),$$

where $i = 0, 1, \ldots, M-1$.

M-FSK

$$s_i(t) = \cos\left(2\pi\left[f_c + \frac{h}{2T_\mathrm{s}} i\right] t\right),$$

where $i = \pm(M-1), \pm(M-3), \ldots, \pm 1$.

Link budget

$$P_\mathrm{R} = P_\mathrm{T} G_\mathrm{T} G_\mathrm{R} \left(\frac{\lambda}{4\pi d}\right)^2,$$

where $c = \lambda f = 3 \cdot 10^8 \mathrm{m/s}$.

Parabolic dish antenna

$$G_\mathrm{Par} = \frac{4\pi A}{\lambda^2}$$

1D Gaussian PDF (i.i.d.)

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

ND Gaussian PDF with variance σ^2 (i.i.d.)

$$f_\mathsf{Z}(\mathsf{z}) = \frac{1}{(2\pi\sigma^2)^{N/2}} \exp\left(-\frac{||\mathsf{z} - \boldsymbol{\mu}||^2}{2\sigma^2}\right)$$

Bayes' rule

$$f_{\mathsf{X}|\mathsf{Y}}(\mathsf{x}|\mathsf{y}) = \frac{f_\mathsf{X}(\mathsf{x})}{f_\mathsf{Y}(\mathsf{y})} f_{\mathsf{Y}|\mathsf{X}}(\mathsf{y}|\mathsf{x})$$

Additive white Gaussian noise (AWGN)

The following formulas are for the AWGN channel $\mathsf{Y} = \mathsf{S} + \mathsf{Z}$, where S is the transmitted symbol and Z is Gaussian noise.

Maximum likelihood (ML) detection

$$\max_i \{\mathbb{P}[\mathsf{Y} = \mathsf{y} | \mathsf{S} = \mathsf{s}_i]\}$$

Maximum a posteriori (MAP) detection

$$\max_i \{\mathbb{P}[\mathsf{S} = \mathsf{s}_i | \mathsf{Y} = \mathsf{y}]\} \equiv \max_i \{\mathbb{P}[\mathsf{S} = \mathsf{s}_i]\, \mathbb{P}[\mathsf{Y} = \mathsf{y} | \mathsf{S} = \mathsf{s}_i]\}$$

Pairwise error probability (PEP)

$$\mathrm{PEP}^{(i,j)} = \mathrm{Q}\left(\sqrt{\frac{D_{i,j}^2}{2N_0}}\right),$$

where $D_{i,j}^2 = \|\mathsf{s}_j - \mathsf{s}_i\|^2$.

Symbol error probability (SEP) (exact)

$$P_\mathrm{e} = \sum_{i=1}^{M} \mathbb{P}[\mathsf{S} = \mathsf{s}_i] \sum_{j \neq i} \mathbb{P}[\mathsf{Y} \in \mathcal{R}_j | \mathsf{S} = \mathsf{s}_i]$$

SEP (union bound)

$$\begin{aligned} P_\mathrm{e} &\leq \sum_{i=1}^{M} \mathbb{P}[\mathsf{S} = \mathsf{s}_i] \sum_{j \neq i} \mathrm{PEP}^{(i,j)} \\ &= \sum_{i=1}^{M} \mathbb{P}[\mathsf{S} = \mathsf{s}_i] \sum_{j \neq i} \mathrm{Q}\left(\sqrt{\frac{D_{i,j}^2}{2N_0}}\right) \end{aligned}$$

SEP (High-SNR approximation for equally likely symbols)

$$P_\mathrm{e} \approx \frac{2K}{M} \cdot \mathrm{Q}\left(\sqrt{\frac{D_\mathrm{min}^2}{2N_0}}\right),$$

where K is the number of distinct signal pairs with distance $D_\mathrm{min} = \min_{i \neq j}\{D_{i,j}\}$ and M is the constellation size.

Bit error probability (BEP) (exact)

$$B_\mathrm{e} = \sum_{i=1}^{M} \mathbb{P}[\mathsf{S} = \mathsf{s}_i] \sum_{j \neq i} \frac{H_{i,j}}{m} \mathbb{P}[\mathsf{Y} \in \mathcal{R}_j | \mathsf{S} = \mathsf{s}_i],$$

where $H_{i,j}$ is the Hamming distance (the number of different bits) between the labels of symbols s_i and s_j.

BEP (union bound)

$$B_\mathrm{e} \leq \sum_{i=1}^{M} \mathbb{P}[\mathsf{S} = \mathsf{s}_i] \sum_{j \neq i} \frac{H_{i,j}}{m} \mathrm{Q}\left(\sqrt{\frac{D_{i,j}^2}{2N_0}}\right)$$

BEP (High-SNR approximation for equally likely symbols)

$$B_\mathrm{e} \approx \frac{2H_\mathrm{min}}{Mm} \cdot \mathrm{Q}\left(\sqrt{\frac{D_\mathrm{min}^2}{2N_0}}\right),$$

where H_min is the *total* number of bits differing between signal pairs at minimum distance and $m = \log_2(M)$ is the number of bits per symbol.

Q-function

See also tables in Mathematics Handbook, where $\mathrm{Q}(x) = 1 - \Phi(x)$

$\mathrm{Q}(x)$	x
10^{-1}	1.2816
10^{-2}	2.3263
10^{-3}	3.0902
10^{-4}	3.7190
10^{-5}	4.2649
10^{-6}	4.7534
10^{-7}	5.1993
10^{-8}	5.6120
10^{-9}	5.9978
10^{-10}	6.3613
10^{-11}	6.7060
10^{-12}	7.0345