Homework 3

Due Date: Oct. 4, 2023

Problem 1 (Closest Neighbour Approximation for the SER and the BER)

Consider a square 16-QAM constellation labelled by the so-called natural binary code shown in Figure 1. All points are equiprobable. Assume the high-snr regime, i.e., only the points at minimum distance can result in error.

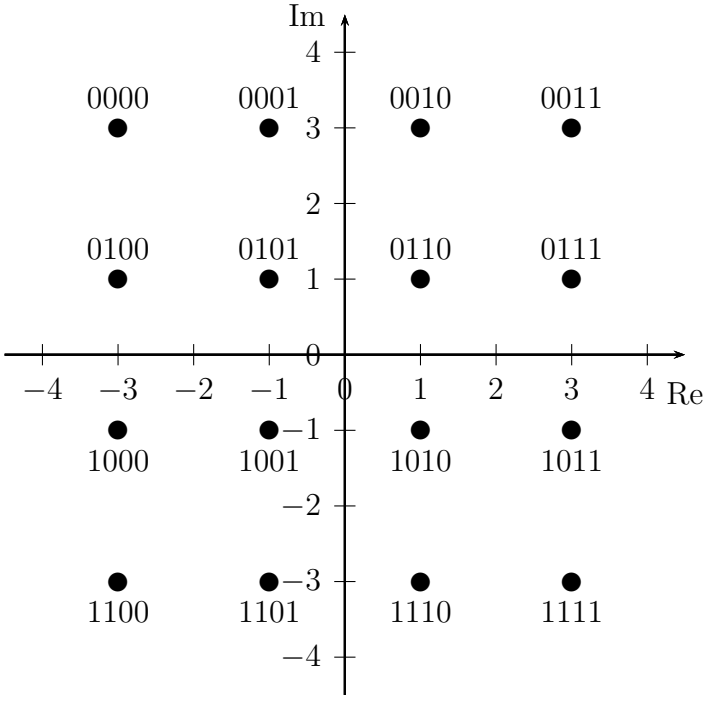

Figure 1: Problem 1.

1. Give an expression for the SER in the following form

$$\text{SER} = a_1 \text{Q}\left(\sqrt{\frac{k_1 E_s}{N_0}}\right),$$

   i.e., find the constants a_1 and k_1.

2. Give an expression for the BER in the following form

$$\text{BER} = a_2 \text{Q}\left(\sqrt{\frac{k_2 E_b}{N_0}}\right),$$

   i.e., find the constants a_2 and k_2.

Problem 2 (4-PAM)

Consider an equally spaces 4-PAM constellation shown in Fig. 2. The constellation is denoted as $\mathcal{S} = \{s_j, 1 \leq j \leq 4\}$. Let X denote the transmitted point and $\hat{X}$ the detected point. The symbols are equally likely.

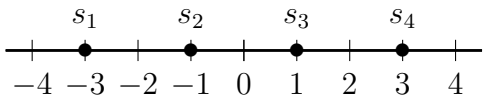

Figure 2: Problem 2.

1. Find the probability of misdetecting symbol s_1, i.e., $\Pr[\hat{X} \neq s_1 | X = s_1]$. Repeat for $\Pr[\hat{X} \neq s_2 | X = s_2]$. Express the probability as a function of E_s/N_0.
2. Calculate the probability of detecting symbol s_2 instead of symbol s_1, i.e., $\Pr[\hat{X} = s_2 | X = s_1]$. Repeat for $\Pr[\hat{X} = s_1 | X = s_2]$. Express the probability as a function of E_s/N_0.
3. Find a union bound on the symbol error probability and the high SNR approximation.

Problem 3 (Anti-Gray Code)

Assume an 8-PSK constellation with constellation points $s_i,\ i = 1, \ldots, 8$, where $s_i = \sqrt{E}\mathrm{e}^{j(i-1)\pi/4}$. Assume transmission over the AWGN channel in the high SNR regime.

1. Suggest the worst labeling in terms of the BER. Argue that there is no other labeling worse than the one you suggested.
2. Give an expression for the BER for the found labeling.