Examination Wed. Oct. 30, 2019, 8:30–12:30

SSY121 Introduction to Communication Engineering

- Contact persons: Mohammad Nazari (phone: 031 772 17 71) will visit the exam after approximately 1 and 3 hours.
- Instructions:
  - Write in English.
  - Use a pencil and eraser.
  - There is no page limit. Extra sheets of paper are available.
  - Solve the problems in any order (they are not ordered by difficulty).
  - Before handing in, sort the pages in problem order. Label each page with problem number and running page number. Do not hand in drafts or unused paper.
  - If any data is missing, make reasonable assumptions.
  - Chalmers' examination rules apply.
  - MP3/Music players **are not** allowed during the exam
- Allowed aids:
  - Mathematics Handbook by Råde and Westergren (any edition, including Beta) or equivalent
  - Chalmers-approved calculator
- Grading principles:
  - Explain the line of reasoning clearly. A good solution with a minor error usually gives close to full points, even if the answer is incorrect.
  - An answer without a clear motivation usually gives 0 points, even if it is correct.
  - Whenever possible, check if your answer is reasonable. An obviously unreasonable answer usually gives 0 points, even if the solution is almost correct.
- Solutions and results:
  - Solutions will be posted on the course website no later than 7 days after the exam.
  - The grading can be reviewed on Wednesday November 20, 2019, at 12:00–13:00 in Landahlsrummet (7430) on floor 7 in the EDIT building.

1. **True or false questions:** Justify *ALL* your answers using short and concise explanations (maximum 30 words per item). (Total points: 10)

   (a) The random end-to-end transmission delay, if is not compensated for, causes a phase rotation of the received signal constellation. (1)
   True; since the delay is multiplied by the carrier frequency, and can be seen as a phase rotation of the received signal.

   (b) A digital communication system uses a QPSK modulation scheme and operates at a negligible BER, say $P_e = 0$. If the phase synchronization unit stops working, the system can still maintain $P_e = 0$, ***if and only if*** the random phase shift ϕ satisfies $|\phi| \leq \frac{\pi}{6}$. (1)
   False. $|\phi| \leq \frac{\pi}{6}$ is a sufficient but not necessary condition to have $P_e = 0$. The BER is still negligible if and only if $|\phi| < \frac{\pi}{4}$.

   (c) The main source of nonlinear distortion in wireless communication systems is the nonlinear filters used at the receiver. (1)
   False. The main source of nonlinear distortion is power amplifiers which are used at the transmitter.

   (d) The minimum number of matched filters in the receiver is equal to M, if an M-ary modulation scheme is used. (1)
   False. The minimum number of matched filters is the dimensionality of the signal space $N \leq M$.

   (e) In a digital communication system, if the prior knowledge about the data source is available, a *maximum likelihood* (ML) detector is preferred for making the decision. (1)
   False. The ML detector does not use the prior information, but if the priors are available, the *maximum a posteriori* (MAP) detector is used.

   (f) According to the theorem of irrelevance, only noise in the dimensions of the signals affects the detection. (1)
   True. The part of noise which cannot be projected in the dimensions of the signals does not affect the detection.

   (g) One drawback of the OFDM systems is the high amount of inter-symbol interference (ISI) caused as a result of high peak-to-average-power ratio (PAPR). (1)
   False. OFDM is less sensitive to ISI than single carrier communication, and ISI can be completely eliminated using a cyclic prefix of length greater than the maximum delay spread.

   (h) Error control coding aims to eliminate redundant information. (1)
   False. It creates redundancy and aims at detecting and correcting bit errors.

   (i) As the frequency is switched from one to another in FSK signaling, large spectral side lobs may appear. To prevent this, continuous-phase modulation techniques are exploited. (1)
   True, the abrupt change of frequency results in large spectral lobs outside of the main spectral band of the signal, due to the phase discontinuity.

   (j) The reason why RRC pulse-shaping is preferred in practice compared to $\mathrm{rect}(t/T)$, is that the pulse $\mathrm{rect}(t/T)$ fulfills Nyquist criterion, but it is not T-orthogonal for a symbol period T. (1)
   False. $\mathrm{rect}(t/T)$ is both Nyquist and T-orthogonal, but the reason it is not used is the significant power in its secondary lobes.

2. Two equiprobable messages m_1 and m_2 are to be transmitted through a channel with input X and output Y related by $Y = \rho X + N$, where N is zero-mean Gaussian noise with variance σ^2 and ρ is a random variable independent of the noise. In any of the following cases, obtain the optimal decision rule *mathematically*. Also, find the resulting error probability for the first two cases, i.e, (a) and (b).

   Hint: To obtain the decision rule, find the conditional distribution of the received symbol, i.e.,

$$p(Y|X) = \Pr(\rho = a_1)p(Y|X, \rho = a_1) + \Pr(\rho = a_2)p(Y|X, \rho = a_2)$$

   for the different cases. (Total points: 8)

   (a) Antipodal signaling ($X = \pm A$) is used, and ρ is ± 1 with equal probability. (3)

   The receiver makes a decision in favor of A if $p(Y|A) > p(Y|-A)$, or if

$$1/2p(Y|A, \rho = 1)+1/2p(Y|A, \rho = -1) > \\ 1/2p(Y|-A, \rho = 1) + 1/2p(Y|-A, \rho = -1).$$

   From $Y = \rho X + N$, the above relation simplifies to

$$\exp(-\frac{(Y-A)^2}{2\sigma^2}) + \exp(-\frac{(Y+A)^2}{2\sigma^2}) > \exp(-\frac{(Y+A)^2}{2\sigma^2}) + \exp(-\frac{(Y-A)^2}{2\sigma^2}).$$

   Since both sides of inequality are equal, any received Y can be equally detected as A or $-A$, and the error probability will be $1/2$.

   (b) Antipodal signaling ($X = \pm A$) is used, and ρ is 0 and 1 with equal probability. (3)

   The receiver makes a decision in favor of A if $p(Y|A) > p(Y|-A)$, or if

$$1/2p(Y|A, \rho = 1)+1/2p(Y|A, \rho = 0) > \\ 1/2p(Y|-A, \rho = 1) + 1/2p(Y|-A, \rho = 0).$$

   From $Y = \rho X + N$, the above relation simplifies to

$$\exp(-\frac{(Y-A)^2}{2\sigma^2}) + \exp(-\frac{Y^2}{2\sigma^2}) > \exp(-\frac{(Y+A)^2}{2\sigma^2}) + \exp(-\frac{Y^2}{2\sigma^2}).$$

   After simplifying the above inequality, we find that the threshold is zero, i.e., the receiver makes a decision in favor of A, if $Y > 0$, and in favor of $-A$, if $Y < 0$. To obtain the error probability, we have

$$P_e = 1/2P_{e|\rho=0} + 1/2P_{e|\rho=1} = 1/4 + 1/2\mathrm{Q}(A/\sigma).$$

   (c) On-off signaling ($X = 0$ or A) is used, and ρ is ± 1 with equal probability. (2)

   The receiver makes a decision in favor of A if $p(Y|A) > p(Y|0)$, or if

$$1/2p(Y|A, \rho = 1)+1/2p(Y|A, \rho = -1) > \\ 1/2p(Y|0, \rho = 1) + 1/2p(Y|0, \rho = -1).$$

   From $Y = \rho X + N$, the above relation simplifies to

$$\exp(-\frac{(Y-A)^2}{2\sigma^2}) + \exp(-\frac{(Y+A)^2}{2\sigma^2}) > \exp(-\frac{Y^2}{2\sigma^2}) + \exp(-\frac{Y^2}{2\sigma^2}),$$

   which simplifies to the following decision rule for decision in favor of A

$$\cosh(-\frac{YA}{\sigma^2}) > \exp(-\frac{A^2}{2\sigma^2}).$$

3. Summarize the purpose of the Gram-Schmidt method and motivate, in words, the steps that have to be carried out. (Total points: 4)

The Gram-Schmidt method is used provide a minimal orthogonal basis, not necessarily unique, based on some functions $f_1, f_2, \ldots, f_n$ to begin with.

Step 1: The method starts by picking one of the starting functions, say f_1 and normalizes it to yield the first basis function.

Step 2: Next, a function f_i that has not yet been selected is chosen. The inner product between f_i and all the obtained basis functions is calculated. Next, the inner products are subtracted from f_i and if this quantity is non-zero, it is normalized and added to the set of basis functions.

Step 2 is now repeated until all functions in the original set have been exhausted.

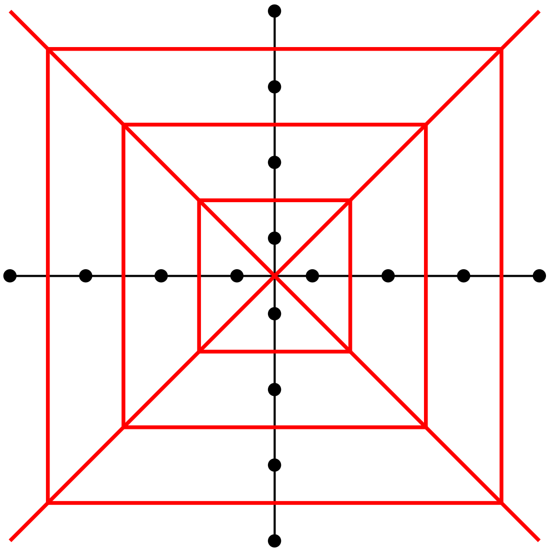

Figure 2: Decision Boundaries

4. For the 16-ary signal constellation shown in Fig. 1, assume that the SNR is sufficiently high that errors occur only between adjacent points.

(Total points: 6)

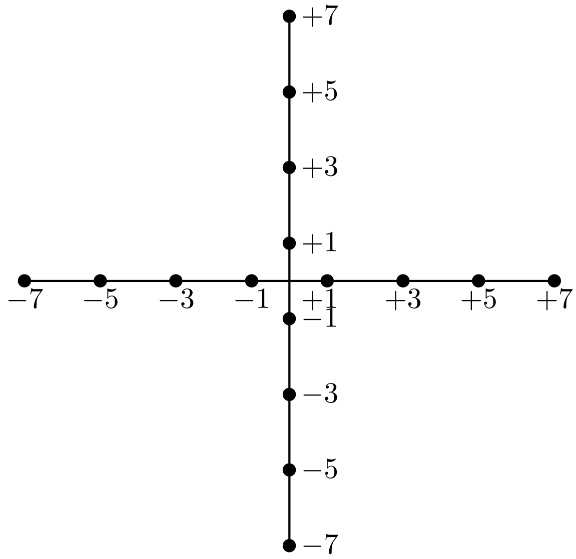

Figure 1: 16-ary Constellation

(a) Determine the decision boundaries for the optimum nearest neighbor detector. (2)

Please see Fig. 2.

(b) Give an expression for the SER in the following form

$$\mathrm{SER} = a_1 \mathrm{Q}\left(\sqrt{\frac{k_1 E_s}{N_0}}\right),$$

i.e., find the constants a_1 and k_1. (2)

The average signal energy

$$E_s = \frac{4 \cdot (d^2 + 9d^2 + 25d^2 + 49d^2)}{16} = 21d^2,$$

where the squared minimum Euclidean distance is $D_{\min}^2 = 2d^2$. The number of signal pairs with distance $D_{\min}$ is $K = 4$ and $M = 16$.

$$\mathrm{SER} = \frac{2K}{M}\mathrm{Q}\left(\sqrt{\frac{D_{\min}^2}{2N_0}}\right) = \frac{2K}{M}\mathrm{Q}\left(\sqrt{\frac{d^2}{N_0}}\right) = \frac{1}{2}\mathrm{Q}\left(\sqrt{\frac{E_s}{21N_0}}\right).$$

Therefore, $a_1 = \frac{1}{2}$ and $k_1 = \frac{1}{21}$.

(c) Consider the labeling as shown in Fig 3, and give an expression for the BER in the following form

$$\mathrm{BER} = a_2\mathrm{Q}\left(\sqrt{\frac{k_2 E_b}{N_0}}\right),$$

i.e., find the constants a_2 and k_2. (2)

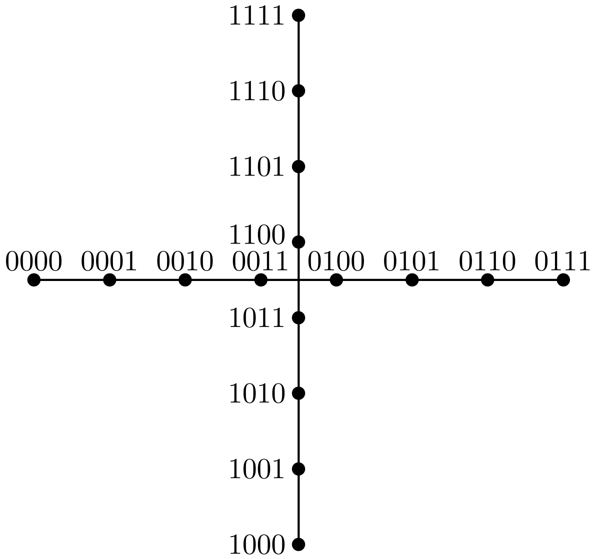

Figure 3: Constellation with Labeling

The number of different bits in signal pairs with distance $D_{\min}$ is $H_{\min} = 10$ and $m = 4$.

$$\mathrm{BER} = \frac{2H_{\min}}{M \cdot m}\mathrm{Q}\left(\sqrt{\frac{D_{\min}^2}{2N_0}}\right) = \frac{2H_{\min}}{M \cdot m}\mathrm{Q}\left(\sqrt{\frac{E_s}{21N_0}}\right)$$
$$= \frac{2H_{\min}}{M \cdot m}\mathrm{Q}\left(\sqrt{\frac{m \cdot E_b}{21N_0}}\right) = \frac{5}{16}\mathrm{Q}\left(\sqrt{\frac{4E_b}{21N_0}}\right).$$

Therefore, $a_2 = \frac{5}{16}$ and $k_2 = \frac{4}{21}$.

5. We are given a standard BPSK constellation to be used together with a pulse of duration 1s. The pulse is constant for the first 0.25s whereafter it is zero for the remaining 0.75s. At the receiver, a matched filter is used. Derive the output from the matched filter and draw the eye-diagram under ideal sampling. (Total points: 7)

   Let $p(t)$ denote the pulse. We have the output from the matched filter as

$$\begin{aligned} p_{\mathrm{mf}}(t) &= \int_{-\infty}^{\infty} p(\tau)p(t-\tau)d\tau \\ &= 4\int_{0}^{t} \mathbf{1}(0 \le t < 0.25)d\tau + 4\int_{t-0.25}^{0.25} \mathbf{1}(0.25 \le t < 0.5)d\tau \\ &= 4t\mathbf{1}(0 \le t < 0.25) + 4(0.5-t)\mathbf{1}(0.25 \le t < 0.5) \end{aligned}$$

   which corresponds to a triangular pulse of duration equal to 0.5 seconds. Hence, the output of the matched filter is a pulse where the first 0.5s corresponds to a triangular pulse with peak at 1 and the last 0.5s is zero. The eye-diagram is shown in Fig. 4.

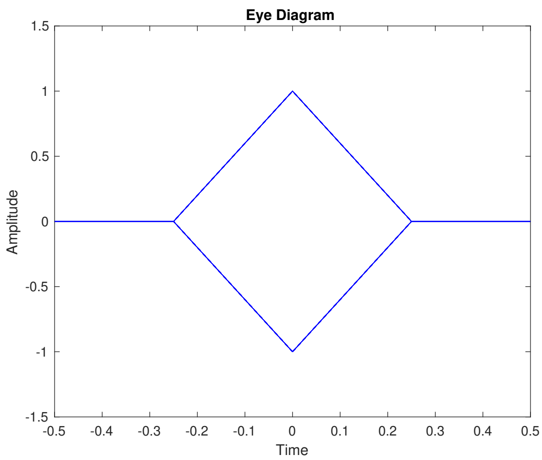

Figure 4: Eye diagram of the MF output.

6. There are four updates to the physical layer in WiFi 802.11a compared to WiFi 802.11n that made the maximum throughput increase from 54 Mbit/s to 150 Mbit/s. How much increase in throughput (in percentage) gave each of these four updates. (Total points: 4)

   (a) 52 data subcarriers instead of 48 subcarriers in 20 MHz: (1)
   $52/48 = 1.0833$, i.e., approximately 8%

   (b) The highest code rate for the channel code was updated from 3/4 to 5/6: (1)
   $(5/6)/(3/4) = 1.1111$, i.e., approximately 11%

   (c) The symbol time is 3.2 μs, where the added guard interval was shortened from 0.8 μs to 0.4 μs: (1)
   $(3.2 + 0.8)/(3.2 + 0.4) = 1.1111$, i.e., approximately 11%

   (d) The channel width was increased from 20 MHz to 40 MHz, i.e., 108 data subcarriers in 40 MHz instead of 52 data subcarriers in 20 MHz: (1)
   $108/52 = 2.077$, i.e., approximately 108%

7. Suppose a digital communication system employs exponential pulses of the form

$$p(t) = \alpha e^{\beta|t|}$$

where α and β are constants.

(Total points: 9)

(a) Give the conditions on α and β for $p(t)$ to have unit energy. (2)

The energy of the pulse is given as

$$\begin{aligned}
\|p(t)\|^2 &= \int_{-\infty}^{\infty} p(t)^2 dt \\
&= 2\alpha^2 \int_0^{\infty} e^{2\beta t} dt \\
&= 2\alpha^2 \left[\frac{1}{2\beta} e^{2\beta t}\right]_{t=0}^{t=\infty}.
\end{aligned}$$

Hence, we find that $\beta < 0$ for the term above not to explode. If we let $\beta < 0$, we get

$$\|p(t)\|^2 = -\frac{\alpha^2}{\beta} = \frac{\alpha^2}{|\beta|} = 1 \Rightarrow \alpha = \sqrt{|\beta|}.$$

(b) Show that the Fourier transform of $p(t)$ for the case of $\beta = -b$, where $b > 0$, is

$$P(f) = \frac{2\alpha b}{b^2 + (2\pi f)^2}.$$

Hint: the following integral identity may be useful

$$\int_0^{\infty} e^{-xA} \cos(xB) dx = \frac{A}{A^2 + B^2}.$$

(3)

The Fourier transform is given as

$$\begin{aligned}
P(f) &= \int_{-\infty}^{\infty} p(t) e^{-j2\pi f t} dt \\
&= \alpha \int_{-\infty}^{\infty} e^{-b|t| - j2\pi f t} dt \\
&= \alpha \int_{-\infty}^{0} e^{tb} e^{-j2\pi f t} dt + \alpha \int_0^{\infty} e^{-tb} e^{-j2\pi f t} dt \\
&= \alpha \int_{-\infty}^{0} e^{tb} (\cos(2\pi f t) - j\sin(2\pi f t)) dt + \alpha \int_0^{\infty} e^{-tb} (\cos(2\pi f t) - j\sin(2\pi f t)) dt \\
&= \alpha \int_0^{\infty} e^{-ub} (\cos(2\pi f u) + j\sin(2\pi f u)) du + \alpha \int_0^{\infty} e^{-tb} (\cos(2\pi f t) - j\sin(2\pi f t)) dt \\
&= 2\alpha \int_0^{\infty} e^{-bt} \cos(2\pi f t) dt
\end{aligned}$$

where in the last step, we simply made the substitution $t = -u$. By using the hint, we then find

$$P(f) = \frac{2\alpha b}{b^2 + (2\pi f)^2}.$$

(c) Use the Nyquist criterion in time to argue why $p(t)$ with $\alpha = 1$ and $\beta = -1$ is not a Nyquist pulse.

(1)

The pulse is clearly not Nyquist for any symbol time since the pulse is not equal to zero for any $t \neq 0$.

(d) You are now given an altered version of $p(t)$ as

$$p(t) = \alpha e^{\beta|x|}, |t| < T_\mathrm{s}$$

where T_s is the time between two symbols. It is argued that the pulse is now a Nyquist pulse if the symbol rate is $1/T_\mathrm{s}$. Argue about the consequences of the this approach and whether the argument holds or not.

(3)

The new suggestion will indeed make the pulse into a Nyquist pulse since samples taken at every symbol time does not have ISI.

The suggestion is to window the signal with a rectangular pulse which, in frequency domain, means that the signal is convolved with a sinc funtion. Therefore, the excess bandwidth of the windowed signal will increase and this can cause significant ICI if other signals are present nearby in the spectrum.