Examination Mon. Oct. 24, 2016, 14:00–18:00

SSY121 – Introduction to Communication Engineering

- Contact persons: Mohammad Nazari (031 - 772 1771) and Fredrik Brännström (031 - 772 1787) will visit the exam after approximately 1 and 3 hours.
- Instructions:
  - Write in English.
  - Use a pencil and eraser.
  - There is no page limit. Extra sheets of paper are available.
  - Solve the problems in any order (they are not ordered by difficulty).
  - Before handing in, sort the pages in problem order. Label each page with problem number and running page number. Do not hand in drafts or unused paper.
  - If any data is missing, make reasonable assumptions.
  - Chalmers' examination rules apply, available at student.portal.chalmers.se /en/chalmersstudies/Examinations/Pages/Examinationroominstructions.aspx
  - MP3/Music players **are not** allowed during the exam
- Allowed aids:
  - Mathematics Handbook by Råde and Westergren (any edition, including Beta) or equivalent
  - Chalmers-approved calculator
- Grading principles:
  - Explain the line of reasoning clearly. A good solution with a minor error usually gives close to full points, even if the answer is incorrect.
  - An answer without a clear motivation usually gives 0 points, even if it is correct.
  - Whenever possible, check if your answer is reasonable. An obviously unreasonable answer usually gives 0 points, even if the solution is almost correct.
- Solutions and results:
  - Solutions will be posted on the course website no later than 7 days after the exam.
  - The grading of the exam can be reviewed on Monday November 7, 2016, at 12:00–13:00 in Room Landahlsrummet (7430), floor 7, Department of Signals and Systems, EDIT building.

1. **True or false questions:** Justify *ALL* your answers using short and concise explanations (maximum 30 words per item). (Total points: 10)

(a) A digital communication system uses a phase-shift keying constellation with eight points (8-PSK) over the additive white Gaussian noise (AWGN) channel. The matched filter receiver implementation requires at least 8 filters matched to the signals $\{s_m(t), 1 \leq m \leq 8\}$. (1)

False. The minimum number of matched filters is the dimensionality of the signal space which is $N = 2$ for 8-PSK.

(b) The Symbol error rate for an envelope detector in an AWGN channel with equiprobable M-PSK signaling is $\frac{M-1}{M}$. (The envelope detector decides only based on the amplitude/magnitude of the received signal.) (1)

True. Because in M-PSK signaling, all the symbols have equal amplitude, and therefore, the detector cannot differentiate the different symbols based on the amplitude of the received signal. Hence, the probability that the detector is correct is $\frac{1}{M}$, i.e., the symbol error rate is $\frac{M-1}{M}$.

(c) In a binary pulse-amplitude modulation (PAM) modulation system using a rectangular pulse, the length of the pulse is reduced from 1 second to 0.25 seconds. To maintain the same probability of error, the amplitude of the pulse should be increased by a factor of four. (1)

False. The amplitude needs to increase by a factor of two, so that the symbol energy is constant, and the same performance is maintained.

(d) One of the challenges for the frequency-shift keying (FSK) signals is phase discontinuities resulting in large spectral lobes. To avoid this, continuous phase modulation (CPM) schemes are preferred. (1)

True. There are no phase discontinuities in CPM.

(e) In the class of orthogonal signal constellations, for a fixed average bit energy, E_b, the minimum distance in the constellation, $d_{\min}$, decreases with increasing M. (1)

False. In orthogonal signal constellations, $d_{\min} = \sqrt{2E_\mathrm{b}\log_2 M}$ which increases with increasing M.

(f) Minimum-shift keying (MSK) signaling is an orthogonal signaling scheme exploiting minimum frequency separation required for both orthogonality of transmitted signals, and providing minimum probability of error in the coherent receiver. (1)

False. MSK uses minimum frequency separation $\Delta f = \frac{1}{2T}$ for orthogonality. However, a larger frequency separation $\Delta f = \frac{0.715}{T} = \frac{1.43}{2T}$ is required to minimize the BER in the coherent receiver.

(g) The nearest-neighbor (or minimum-distance) detector is the optimal detector provided that the signals are *both* equiprobable *and* have equal energy. (1)

False. Nearest-neighbor detector is optimal *in AWGN channel* provided that the messages are *equiprobable* a priori. Having equal energy is not necessary.

(h) In CDMA, different users are allotted small portions of the available bandwidth and each user transmits only in the allotted bandwidth without interfering in other user's bandwidth. (1)

False. In CDMA, all the users transmit simultaneously over the entire available spectrum.

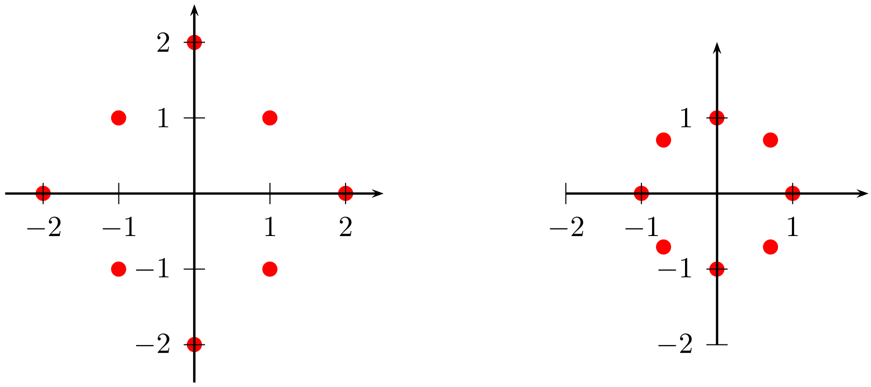
(i) For the one-sided bandwidth $B = 5$ MHz, ISI-free transmission with a symbol rate of $R_s = 1/T_s = 10$ Msymbol/s is *theoretically* possible using an RRC pulse with $\alpha > 0$, together with a matched filter receiver. (1)

Flase. ISI-free transmission is *theoretically* possible with the maximum symbol rate $R_s = 1/T_s = 10$ Msymbol/s, *but* using sinc pulse.

(j) In Shannon's communication model, the block modulator serves as the interface to the communication channel, and its primary purpose is to map the binary information sequence into signal waveforms. (1)

True. The process of mapping a digital/binary sequence to signals for transmission over the communication channel is done in the modulator block.

2. A transmitter and a receiver are communicating over the AWGN channel. The transmitter first transmits a header consisting of two symbols, equally likely, using BPSK whereafter the data is transmitted using either 8-QAM or 8-PSK, see Fig. 1. Based on the header, the receiver decides what modulation scheme to use for ML-decoding. (Total points: 11)



(a) Un-normalized 8-QAM (b) 8-PSK

Figure 1: Constellations in Problem 2.

(a) Give an expression for the probability that both header symbols are received erroneously in terms of the signal-to-noise ratio, E_s/N_0. (2)
The average symbol energy for BPSK is given as

$$E_\mathrm{s} = 2d^2/2 \Rightarrow d = \sqrt{E_\mathrm{s}},$$

where $2d$ is the distance between the points, and

$$\sigma = \sqrt{\frac{N_0}{2}}.$$

We have that the SER is given by

$$P_\mathrm{e} = Q\left(\frac{d}{\sigma}\right) = Q\left(\sqrt{\frac{2E_\mathrm{s}}{N_0}}\right).$$

Since the two receptions are independent, we have that the probability of having two consecutive errors given by

$$\Pr(\text{two errors}) = Q\left(\sqrt{\frac{2E_\mathrm{s}}{N_0}}\right)^2.$$

(b) Assume now that the transmitter is using 8-QAM for transmission after the header and that the header is received correctly, i.e., the receiver decode using 8-QAM as well. Find the normalizing constant to make the average symbol energy equal to one for the 8-QAM constellation in Fig. 1(a), give the high-SNR approximation for the SER and draw the decision regions for the normalized constellation. (3)
The normalizing constant is computed as

$$E_\mathrm{s} = \frac{4(2d^2) + 4(4d^2)}{8} = 3d^2 = 1 \Rightarrow d = \frac{1}{\sqrt{3}}.$$

The smallest distance between the points is $D_{\text{min}} = \sqrt{2}d = \sqrt{\frac{2E_s}{3}}$. The SER at high SNR is given as

$$P_e = \frac{2 \cdot K_{\text{min}}}{M} Q\left(\sqrt{\frac{D_{\text{min}}^2}{2N_0}}\right) = \frac{2 \cdot 8}{8} Q\left(\sqrt{\frac{E_s}{3N_0}},\right)$$

since $K_{\text{min}} = M = 8$, and the decision regions are given below

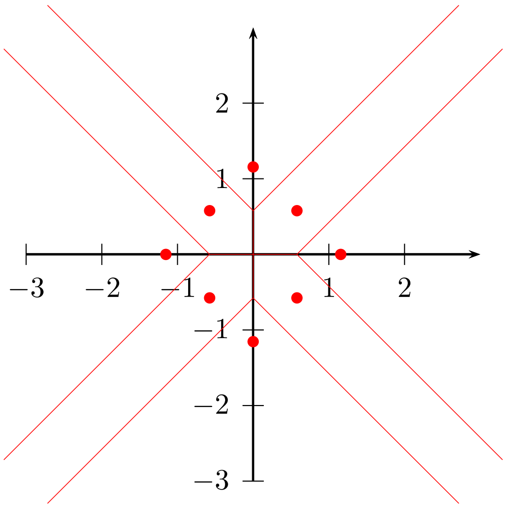


Figure 2: Decision regions in Problem 2(b).

(c) Now, assume the header is received erroneously and that the receiver is decoding with the 8-PSK constellation when the transmitter is transmitting using the (normalized) 8-QAM constellation. Find the high-SNR approximation for the SER. Compare and interpret the result with what you obtained in (b). (6)

We need to compute the pairwise error probability which translates to computing the length of the two green lines in the figure below. They are given as

$$D_1 = \sqrt{2}d \sin(\pi/8) \approx 0.541d$$

and

$$D_2 = 2d \sin(\pi/8) \approx 0.765d.$$

Note that $D_1 < D_2$ and that we have 8 different scenarios where the noise being larger than D_1 causes an error. We also know from (b) that $d = \sqrt{E_s/3}$, hence, the high-SNR approximation is given by

$$\begin{aligned} P_e &= \frac{8Q\left(\frac{D_1}{\sqrt{N_0/2}}\right)}{4} \\ &= 2Q\left(2\sin(\pi/8)\sqrt{\frac{E_s}{3N_0}}\right) \end{aligned}$$

Comparing with the result from (b), one note that at high SNR, the mismatched case perform worse than the matched case since $2\sin(\pi/8) \approx 0.765$ is less than one.

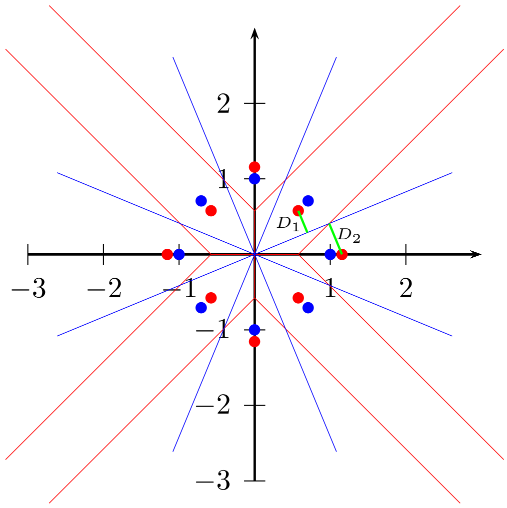


Figure 3: Decision regions for Problem 2(c).

3. Some questions regarding NFC and WiFi 802.11. (Total points: 4)

   (a) Bluetooth low energy (BLE) beacons are important because they address a number of challenges that marketers have been trying to solve for many years. Briefly describe two (and only two) of these challenges. (2)

   Two of the following:

   - **Secure, proximity-based communication** – Giving two devices the ability to securely communicate with each other when they are in close proximity is the business challenge that NFC technology has been attempting to solve. BLE beacons can also solve this challenge, but with the added benefit that the device does not need to be physically held against a sensor as it can stay in your pocket or purse the whole time.
   - **Indoor geo-location** – GPS technology is great for outdoor use, but satellite signals are significantly less effective inside a building. BLE beacons offer a cost-effective solution to in-building location services, with the added benefit of being cheap to deploy and being significantly less of a drain on a smartphone?s battery than GPS technology.
   - **Wide-reaching distribution** – The vast majority of smartphones produced in the past 2 years support BLE technology, meaning that the critical mass of users that is required to make a success of a new technology is already in place.

   (b) Describe the difference between SU-MIMO and MU-MIMO in WiFi 802.11ac. (2)

   i. Single-user multiple-input multiple-output (SU-MIMO) exploits the presence of multiple transmit and receive antennas to improve both the capacity and the reliability of a transmission.
   ii. Multi-user multiple-input multiple-output (MU-MIMO) is a technique where multiple stations, each with potentially multiple antennas, transmit and/or receive independent data streams simultaneously. It allows stations having multiple antennas to transmit several data streams to multiple users at the same time over the same frequency channel.

4. In a binary antipodal signaling scheme, $s_1(t) = s(t)$ and $s_2(t) = -s(t)$, and the energy of $s(t)$ is E_s. The prior probabilities of messages 1 and 2 are p and $1-p$, respectively, and the variance of the noise is $\sigma^2 = \frac{N_0}{2}$.
(Total points: 6)

   (a) Find the decision boundary for the optimal receiver. (3)
   According to the MAP decision rule, on the decision boundary we have:

$$p(s_1) \cdot p(r_\mathrm{th}|s_1) = p(s_2) \cdot p(r_\mathrm{th}|s_2)$$

   where r_th is the threshold or decision boundary, and $s_1 = \sqrt{E_\mathrm{s}}$ and $s_2 = -\sqrt{E_\mathrm{s}}$ are vector representation of the signals $s_1(t)$ and $s_2(t)$, respectively.

$$p \cdot \frac{1}{\sqrt{2\pi\sigma^2}} \mathrm{e}^{-\frac{(r_\mathrm{th}-\sqrt{E_\mathrm{s}})^2}{2\sigma^2}} = (1-p) \cdot \frac{1}{\sqrt{2\pi\sigma^2}} \mathrm{e}^{-\frac{(r_\mathrm{th}+\sqrt{E_\mathrm{s}})^2}{2\sigma^2}}$$

   By simplifying the above equation and taking the natural logarithm of both sides, we have

$$r_\mathrm{th} = \frac{2\sigma^2}{4\sqrt{E_\mathrm{s}}} \ln \frac{1-p}{p} = \frac{N_0}{4\sqrt{E_\mathrm{s}}} \ln \frac{1-p}{p}$$

   (b) Find the probability of error for the optimal receiver. (3)
   According to the decision boundary obtained in the previous part, and figure ??:

$$\begin{aligned} P_\mathrm{e} &= p \cdot P_1 + (1-p) \cdot P_2 \\ &= p \cdot \Pr\left(\sqrt{E_\mathrm{s}} + n < r_\mathrm{th}\right) + (1-p) \cdot \Pr\left(-\sqrt{E_\mathrm{s}} + n > r_\mathrm{th}\right) \\ &= pQ\left(\frac{\sqrt{E_\mathrm{s}} - r_\mathrm{th}}{\sqrt{\frac{N_0}{2}}}\right) + (1-p)Q\left(\frac{\sqrt{E_\mathrm{s}} + r_\mathrm{th}}{\sqrt{\frac{N_0}{2}}}\right) \end{aligned}$$

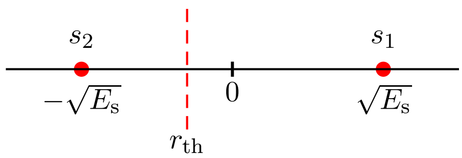


Figure 4: Decision regions for binary antipodal signaling.

5. Suppose a digital communication system employs Gaussian shaped pulses of the form
$$p(t) = e^{-\pi a^2 t^2}.$$
To reduce ISI, we impose $p(T_\mathrm{p}) = 0.01$ where T_p is the time between transmitted symbols. The one-sided bandwidth of $p(t)$ is defined as the value of W for which $\frac{P(W)}{P(0)} = 0.01$, where $P(f)$ is the Fourier transform of $p(t)$. Determine W and compare it to the one-sided bandwidth of the Root raised cosine-pulse with roll-off factor $\alpha = 1$, where the one-sided bandwidth is defined as the lowest frequency, f, for which $P_\mathrm{RRC}(f) = 0$.
(Hint: Use the fact that $e^{-j\omega t} = \cos(\omega t) - j\sin(\omega t)$ and the identity $\int_{-\infty}^{\infty} e^{-cx^2}\cos(2\pi k x)dx = \sqrt{\frac{\pi}{c}}e^{-\frac{\pi^2 k^2}{c}}$, $c \in (0,\infty)$.) (Total points: 6)

The fourier transform of $p(t)$ may be found as

$$\begin{aligned}
P(f) &= \int_{-\infty}^{\infty} e^{-\pi a^2 t^2} e^{-j2\pi f t} dt \\
&= \int_{-\infty}^{\infty} e^{-\pi a^2 t^2} (\cos(2\pi f t) - i\sin(2\pi f t))dt \\
&= \int_{-\infty}^{\infty} e^{-\pi a^2 t^2} \cos(2\pi f t)dt - i\int_{-\infty}^{\infty} e^{-\pi a^2 t^2} \sin(2\pi f t)dt \\
&= \sqrt{\frac{\pi}{\pi a^2}} e^{-\frac{\pi^2 f^2}{\pi a^2}} + 0i \\
&= \frac{1}{a} e^{-\frac{\pi f^2}{a^2}}
\end{aligned}$$

where the second equality is due to Eulers formula and the second last is due to the hint and the sine function being odd.

Now we can find the expression for W from

$$\frac{P(W)}{P(0)} = \frac{\frac{1}{a}e^{-\pi W^2/a^2}}{\frac{1}{a}} = 0.01 \Rightarrow W^2 = \frac{-a^2\ln(0.01)}{\pi}.$$

From the ISI constraint, we also have

$$p(T_\mathrm{p}) = e^{-\pi a^2 T_\mathrm{p}^2} = 0.01 \Rightarrow T_\mathrm{p}^2 = \frac{-\ln(0.01)}{\pi a^2}.$$

Hence

$$(WT_\mathrm{p}) = \pm\sqrt{\frac{-\ln(0.01)}{\pi a^2}\frac{-a^2\ln(0.01)}{\pi}} = \pm\frac{\ln(0.01)}{\pi}$$

where a negative solution does not make sense. We get

$$WT_\mathrm{p} = \frac{-\ln(0.01)}{\pi} \Rightarrow W = \frac{1.466}{T_\mathrm{p}}.$$

An RRC pulse with $\alpha = 1$ have one-sided bandwidth equal to $W = 1/T_\mathrm{p}$. Hence, the Gaussian pulse requires a larger bandwidth than the RRC-pulse.

6. A communication system uses the four signals shown in Fig. 2 for transmission.

(Total points: 6)

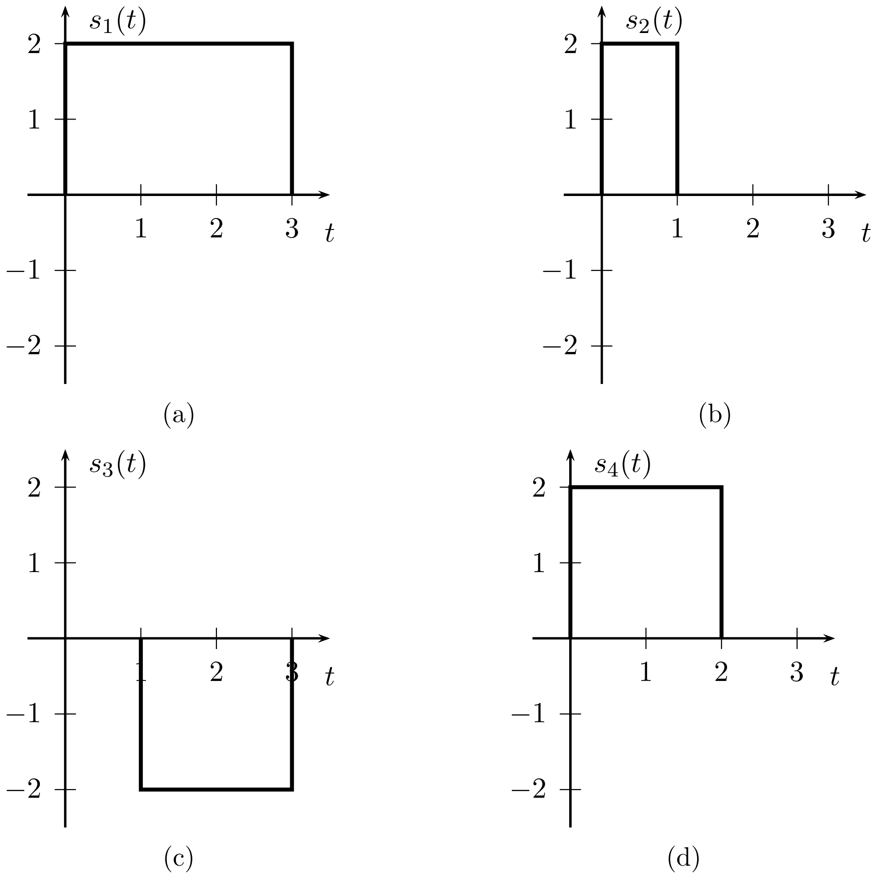


Figure 5: The signal alternatives in Problem 6.

(a) Find a *minimal* set of orthonormal basis for the space spanned by these signals. (4)
We use the Gram Schmidt procedure to find the orthonormal basis. The smart way of doing that is to start from $s_2(t)$. After calculations, the orthonormal functions are is in the Fig. ??.

(b) Express the signals as a superposition of the basis functions, and write their vector representation. (2)

$$\begin{aligned} s_1(t) &= 2\phi_1(t) + 2\phi_2(t) + 2\phi_3(t), \\ s_2(t) &= 2\phi_1(t), \\ s_3(t) &= -2\phi_2(t) - 2\phi_3(t), \\ s_4(t) &= 2\phi_1(t) + 2\phi_2(t). \end{aligned}$$

and the vector representation is as in the following:

$$\begin{aligned} s_1 &= [2, 2, 2], \\ s_2 &= [2, 0, 0], \\ s_3 &= [0, -2, -2], \\ s_4 &= [2, 2, 0]. \end{aligned}$$

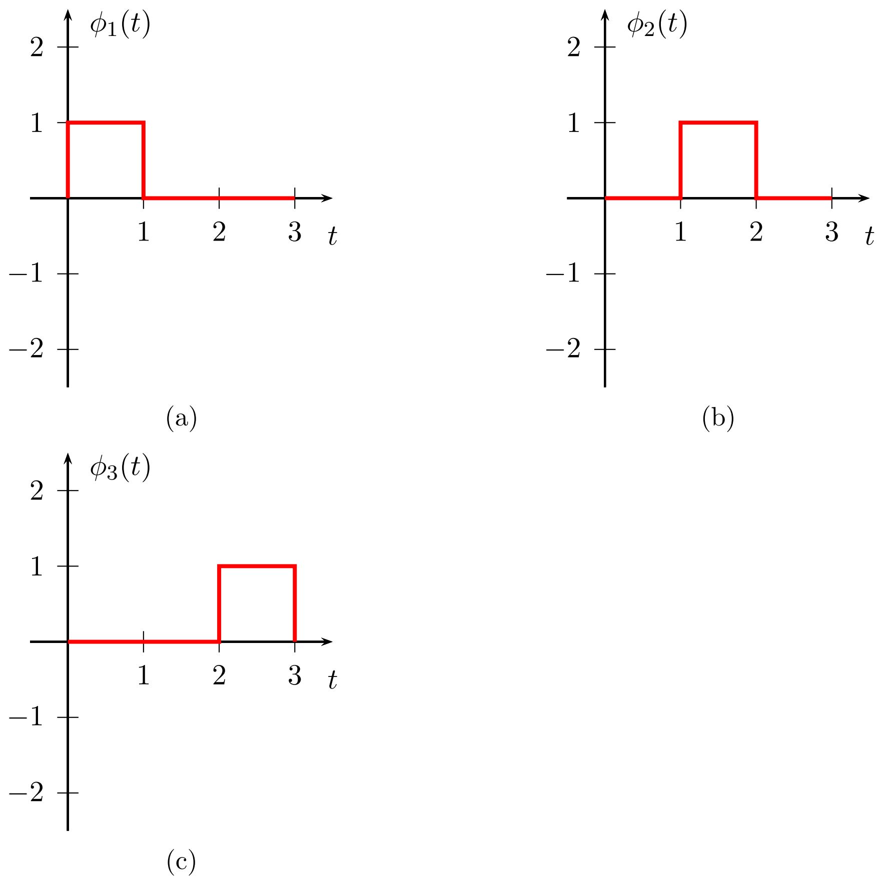


Figure 6: The orthonormal basis for Problem 6.

7. A binary digital communication system transmits data over a point-to-point microwave link using BPSK modulation. The distance between the transmitter (TX) and the receiver (RX) is 100 km. To offset the effect of channel attenuation, repeaters (R) are used every 10 km. Each repeater, receives the signal, amplifies it by its gain G_i, and forwards it. The communication channel follows the free-space path loss model. Assume that $G_\mathrm{T} = G_\mathrm{R} = 0$ dB, $G_i = G = 80$ dB for all repeaters, and that the carrier frequency is chosen so that $\frac{\lambda}{4\pi} = 1$. In the receiver side, a minimum received power of 0.15 μW is required to guarantee the quality of service. (Total points: 3)

   (a) Find the transmission power required to meet the quality of service. (2)

   Assuming $D = 100$ km and that the repeaters are located every $d = 10$ km, the number of required repeaters is $N = 9$ and there are $N + 1 = 10$ links from TX to RX. According to the free-space path loss model, we have:

$$P_\mathrm{R} = P_\mathrm{T}\left(\frac{\lambda}{4\pi d}\right)^2 \prod_{i=1}^{N} G_i \left(\frac{\lambda}{4\pi d}\right)^2 = P_\mathrm{T}\left(\frac{1}{10000^2}\right) = P_\mathrm{T}10^{-8},$$

   since $G_i(\lambda/(4\pi d))^2 = (10^{80/10})/10000^2 = 1$ for all i. By solving the above equation, we have $P_\mathrm{T} = P_\mathrm{R}10^8 = 0.15 \cdot 10^{-6} \cdot 10^8 = 15$ W.

   (b) If for some reason, a phase error $\phi = 45^\circ$ occurs at the final receiver (destination), how much power is required in the transmitter side to still meet the quality of service? (1)

We already know that the probability of error for BPSK is $Q\left(\sqrt{\frac{2E_s}{N_0}}\right)$. However, when the phase error happens, the probability of error is equal to $Q\left(\sqrt{\frac{2E_s\cos^2\phi}{N_0}}\right)$. Since $\cos^2\phi = 1/2$, the signal to noise ratio needs to be increased by a factor of 2 to guarantee the same performance as previous. Thus, the required power at the TX is $P_T = 15 \cdot 2 = 30$ W.

8. We want to transmit a data stream with bit rate $R_b = 8$ kbit/s using a raised cosine pulse with roll-off factor $\alpha = 0.4$ and a 128-ary modulation scheme. How much one-sided channel bandwidth W is required to have ISI-free transmission assuming that a sampling receiver is used?

(Total points: 2)

$$R_b = 8 \text{ kbit/s}$$
$$R_s = \frac{R_b}{\log_2 M} = \frac{R_b}{7}$$

For ISI-free transmission, $W \geq \frac{\alpha+1}{2T_s} = \frac{\alpha+1}{2}R_s$ must hold. Thus,

$$W_{\min} = \frac{\alpha+1}{2}R_s = \frac{(0.4+1)}{2}\frac{8000}{7} = 800 \text{ Hz}$$