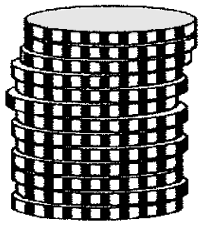
Lecture 4: Stacks

Stack

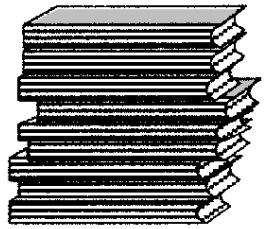
What is a Stack?

- A method of organizing data
- Defined structure and operations

- A stack is a data structure of ordered items such that items can be inserted and removed only at one end.
- Stacks typically used for temporary storage of data.
- A stack is a LIFO (Last-In/First-Out) data structure.
- A stack is sometimes also called a pushdown store.



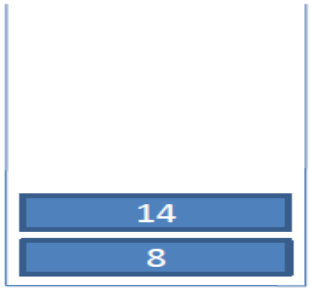
Stack of coins



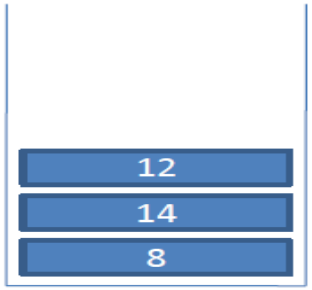
Stack of books

Example

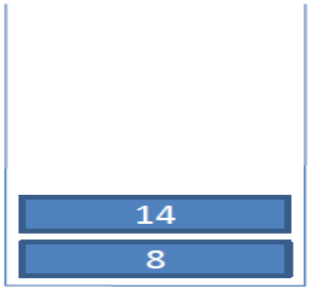
push 8



push 14

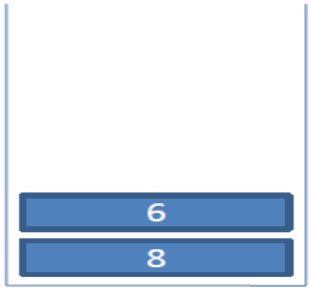


push 12



pop 12

pop 14



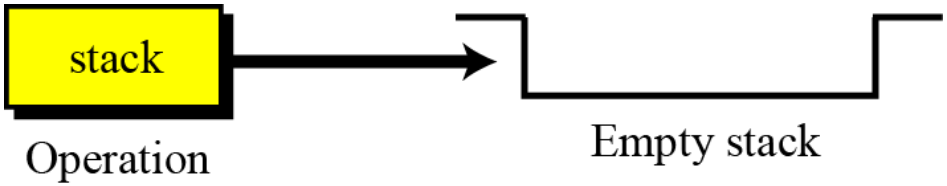
push 6

pop 6
pop 8

stack operation

- creates an empty stack

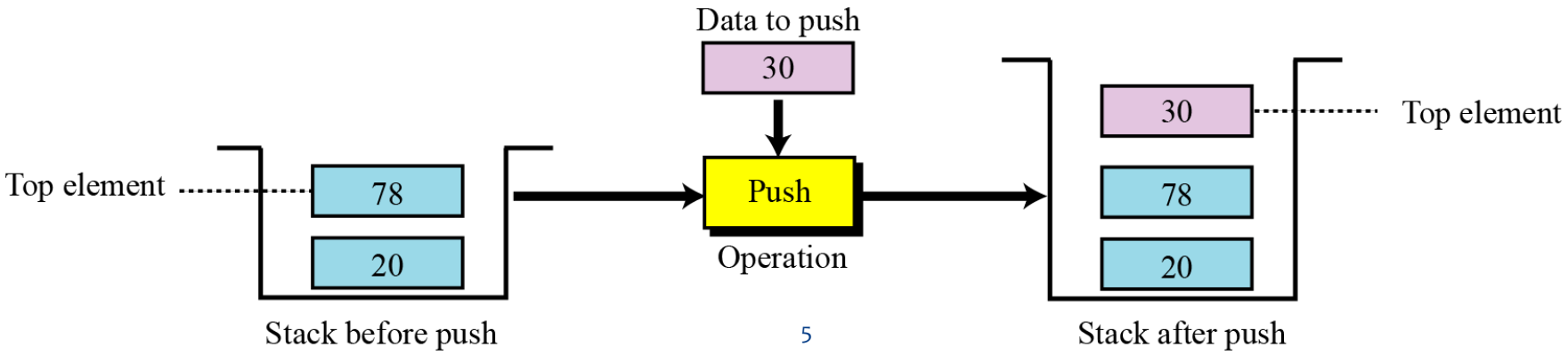
stack (stackName)



push operation

- Inserts an item at the top of the stack

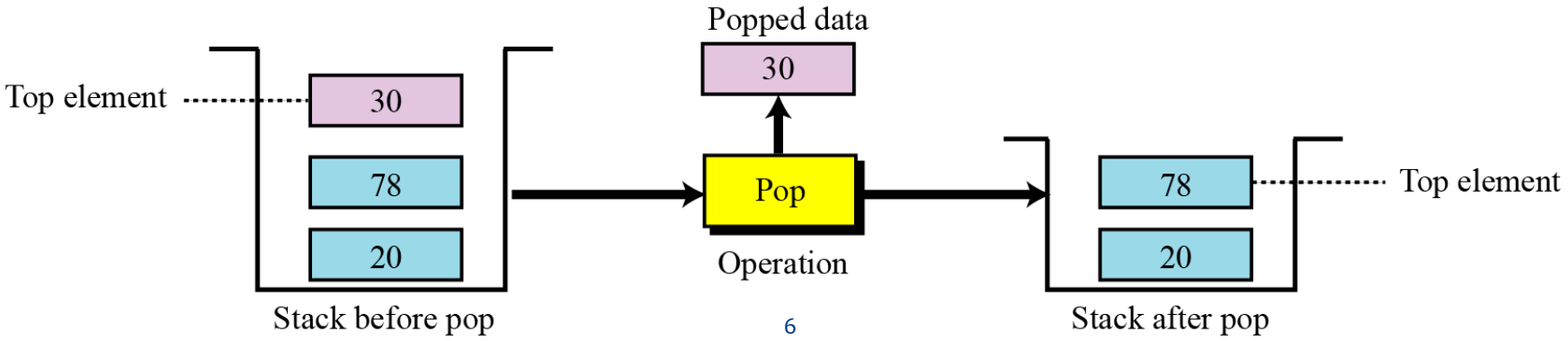
push (stackName, dataItem)



pop operation

- Deletes the item at the top of the stack.

pop (stackName, dataItem)

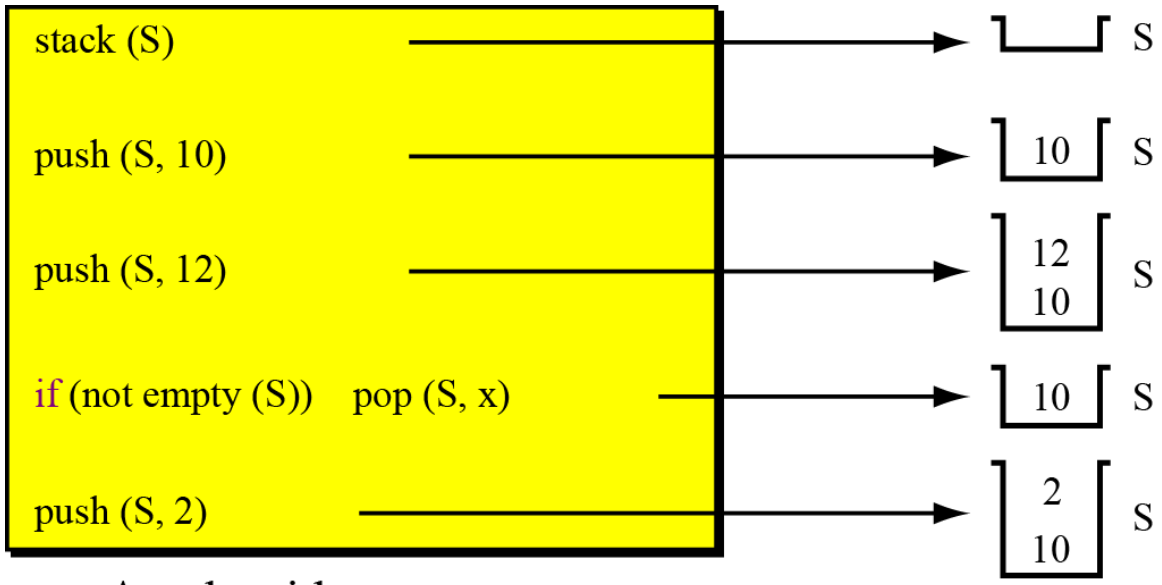


empty operation

- Checks the status of the stack.
- This operation returns true if the stack is empty and false if the stack is not empty.

```
empty (stackName)
```

Example



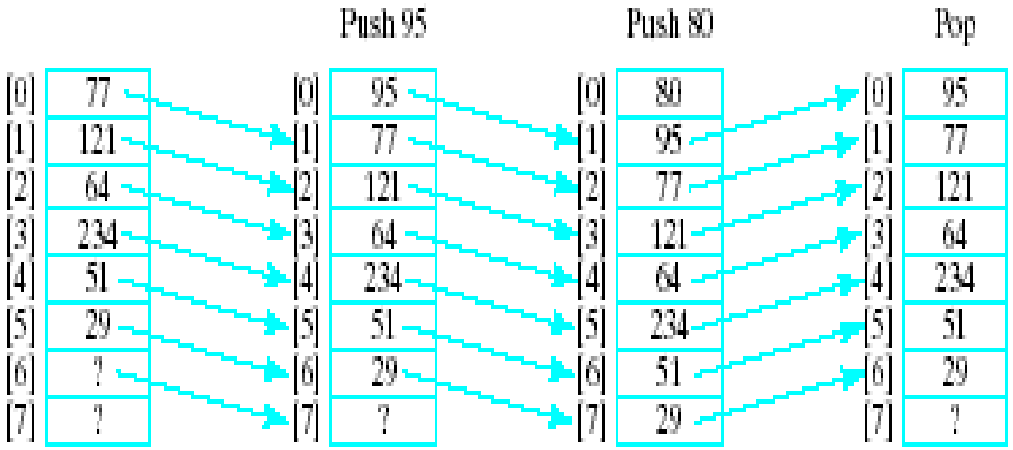
An algorithm segment

Storage structures

- Two choices
  - Select position 0 as top of the stack
  - Select position 0 as bottom of the stack

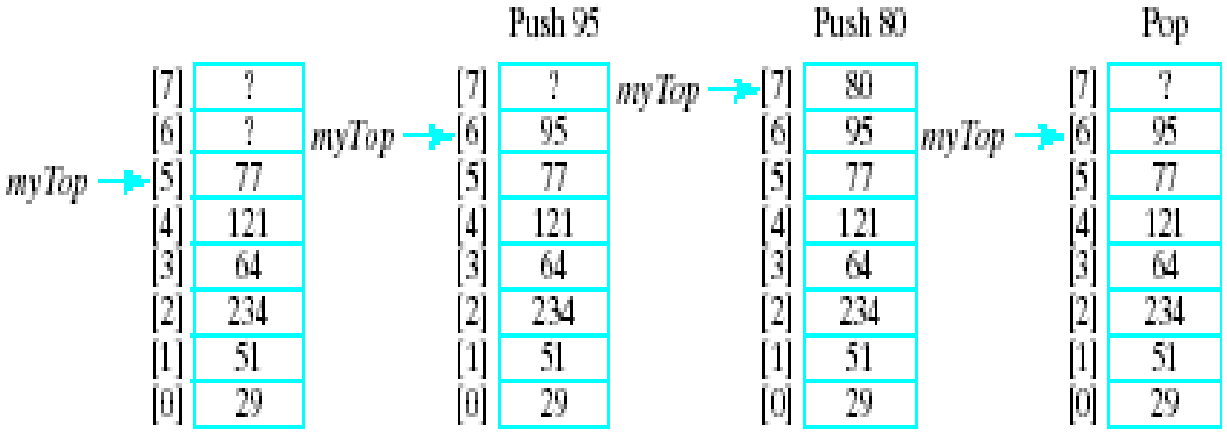
Select position 0 as top of the stack

- Problem … consider pushing and popping
  - Requires much shifting



Select position 0 as bottom of the stack

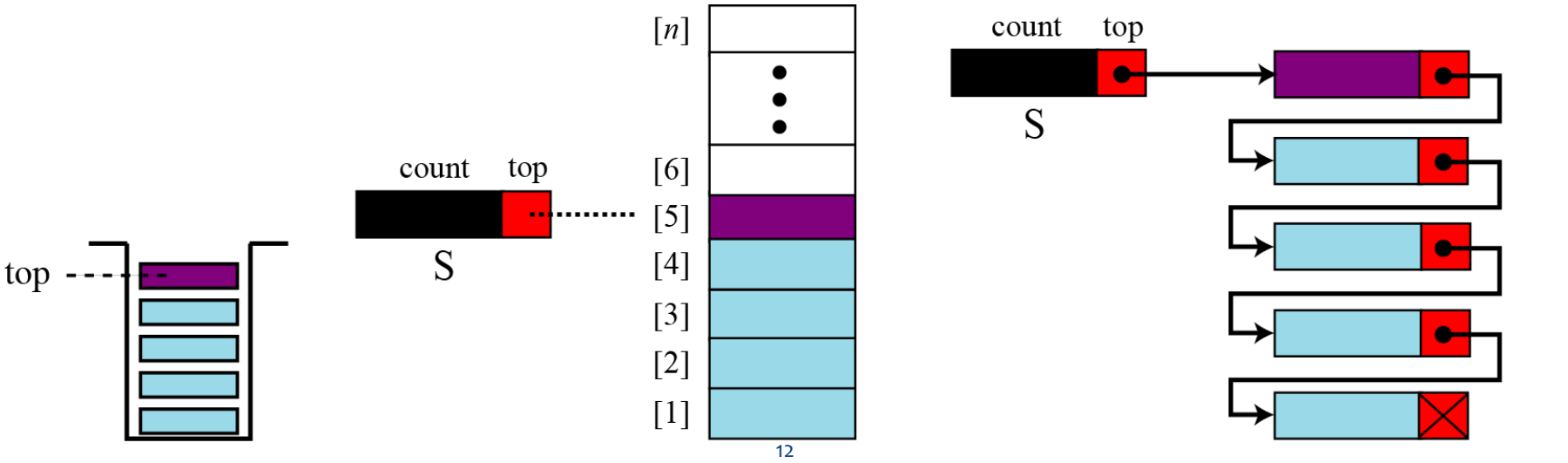
- A better approach is to let position 0 be the bottom of the stack



- Thus our design will include
  - An array to hold the stack elements
  - An integer to indicate the top of the stack

Implementation

- There are two ways we can implement a stack:
  - Static implementation (Using an array)
  - Dynamic implementation (Using a linked list)



Stack (Array Implementation)

- **Algorithm for push:**

Suppose STACK[SIZE] is a one dimensional array for implementing the stack, which will hold the data items. TOP is the pointer that points to the top most element of the stack. Let DATA is the data item to be pushed.

1. If TOP = SIZE – 1, then:
    (a) Display “The stack is in overflow condition”
    (b) Exit
2. TOP = TOP + 1
3. STACK [TOP] = ITEM
4. Exit

Stack (Array Implementation)

```
1.  //This function will add/insert an element to Top of the stack
2.  void push()
3.  {
4.  int item;
5.  if (Top == MAXSIZE–1)          //if the top pointer already reached the maximum allowed size then
6.  {                              //we can say that the stack is full or overflow

7.  printf("\nThe Stack Is Full");
8.  getch();
9.  }                               //Otherwise an element can be added or inserted by incrementing the
                                   //stack pointer Top as follows
10. else
11. {
12. printf("\nEnter The Element To Be Inserted = ");
13. scanf("%d",&item);
14. top++;
15.  stack[top] = item;
            }
            }
```

Stack (Array Implementation)

- **Algorithm for pop:**

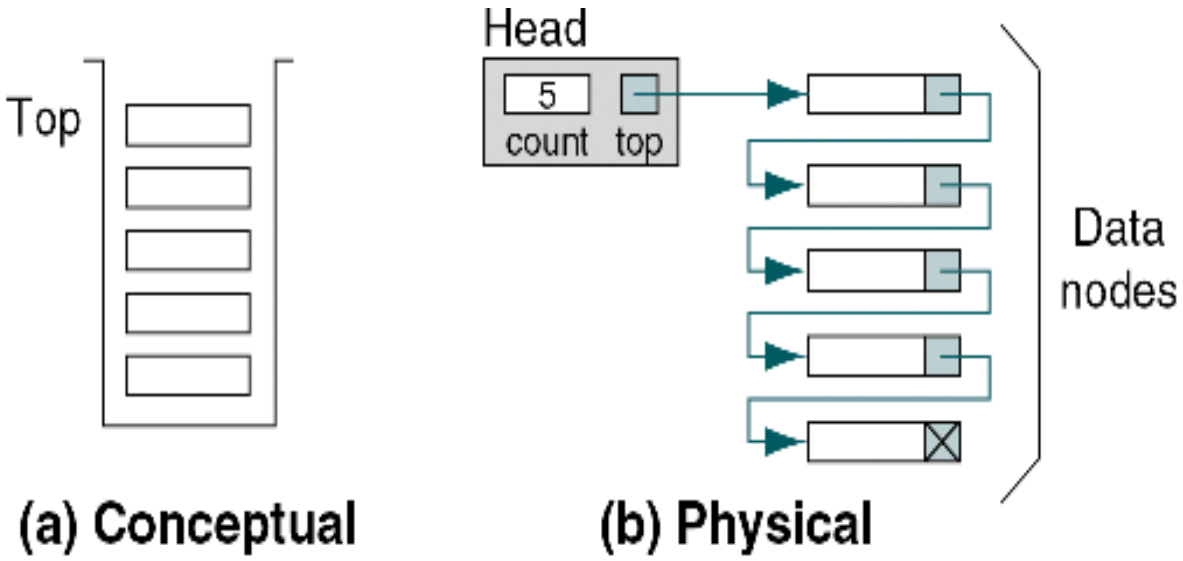
Suppose STACK[SIZE] is a one dimensional array for implementing the stack, which will hold the data items. TOP is the pointer that points to the top most element of the stack. DATA is the popped (or deleted) data item from the top of the stack.

1. If TOP < 0, then
   (a) Display “The Stack is empty”
   (b) Exit
2. Else remove the Top most element
3. DATA = STACK[TOP]
4. TOP = TOP – 1
5. Exit

Stack (Array Implementation)

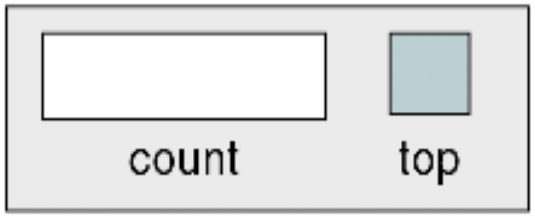
```
1.  //This function will delete an element from the Top of the stack
2.  void pop()
3.  {
4.  int item;                          //If the Top pointer points to NULL, then the stack is empty
5.  if (Top == -1)                     // that is NO element is there to delete or pop
6.  printf("\nThe Stack Is Empty");
7.  //Otherwise the top most element in the stack is popped or deleted by decrementing the Top
    pointer
8.  else
9.  {
10. item= stack[top];
11. Top=top-1;
12. printf("\nThe Deleted Element Is = %d",item);
13. }
14. }
```

Stack (Linked List Implementation)

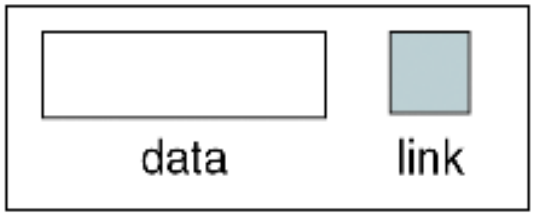


Conceptual and Physical Stack Implementations

Stack (Linked List Implementation)



Stack head structure



Stack node structure

```
stack
   count
   top
end stack


node
   data
   link
end node
```

Stack (Linked List Implementation)

ALGORITHM 3-1 Create Stack

```
Algorithm createStack
Creates and initializes metadata structure.
   Pre    Nothing
   Post   Structure created and initialized
   Return stack head
 1 allocate memory for stack head
 2 set count to 0
 3 set top to null
 4 return stack head
end createStack
```

Stack (Linked List Implementation)

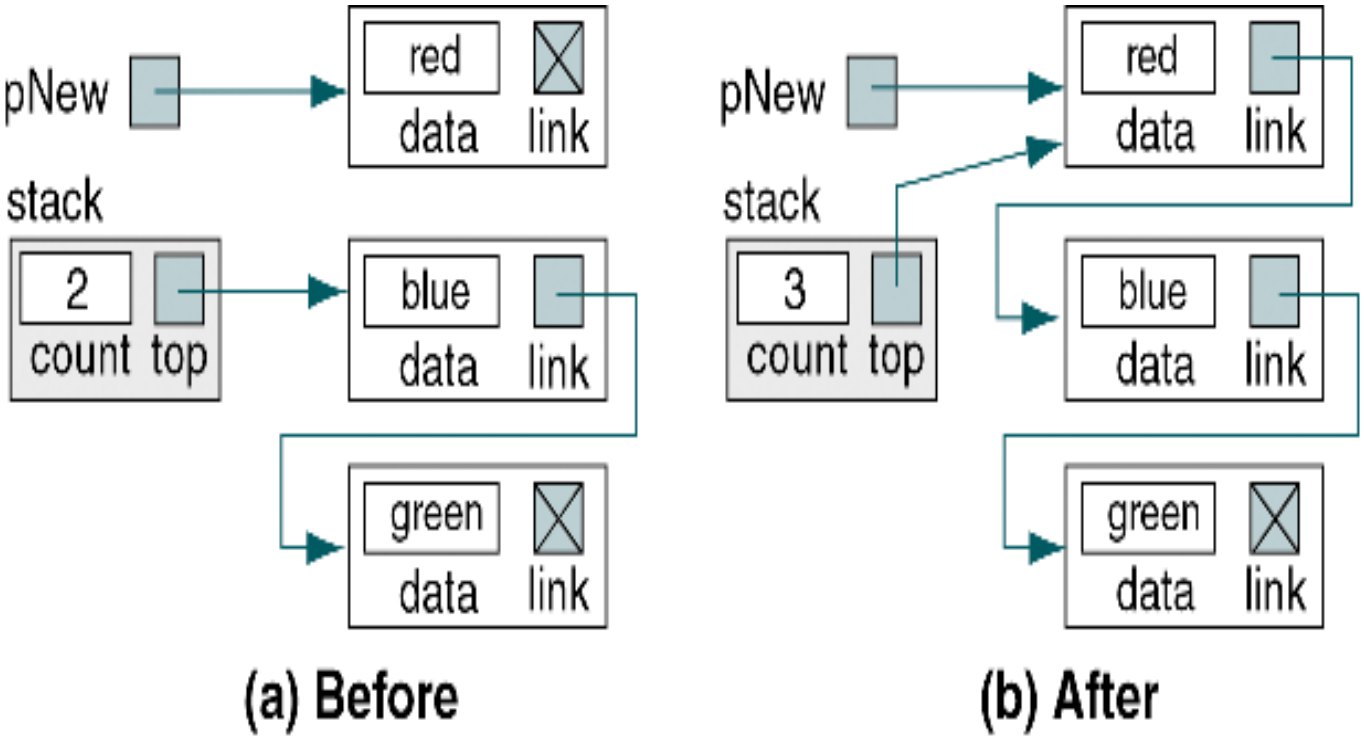


FIGURE 3-9 Push Stack Example

Stack (Linked List Implementation)

ALGORITHM 3-2 Push Stack Design

```
Algorithm pushStack (stack, data)
Insert (push) one item into the stack.
   Pre  stack passed by reference
        data contain data to be pushed into stack
   Post data have been pushed in stack
 1 allocate new node
 2 store data in new node
 3 make current top node the second node
 4 make new node the top
 5 increment stack count
end pushStack
```

Push()

```
1.  void push()
2.  {
3.  struct node *ptr;
4.  printf("\n\nEnter ITEM: ");
5.  scanf("%d", &item);
6.  if (top == NULL)
7.  {
8.  top = (struct node *)malloc(sizeof(struct node));
9.  top->info = item;
10. top->link = NULL;
11. }
12. else
13. {
14. ptr = top;
15. top = (struct node *)malloc(sizeof(struct node));
16. top->info = item;
17. top->link = ptr;
18. }
19. printf("\nItem inserted: %d\n", item);
20. }
```

Push()

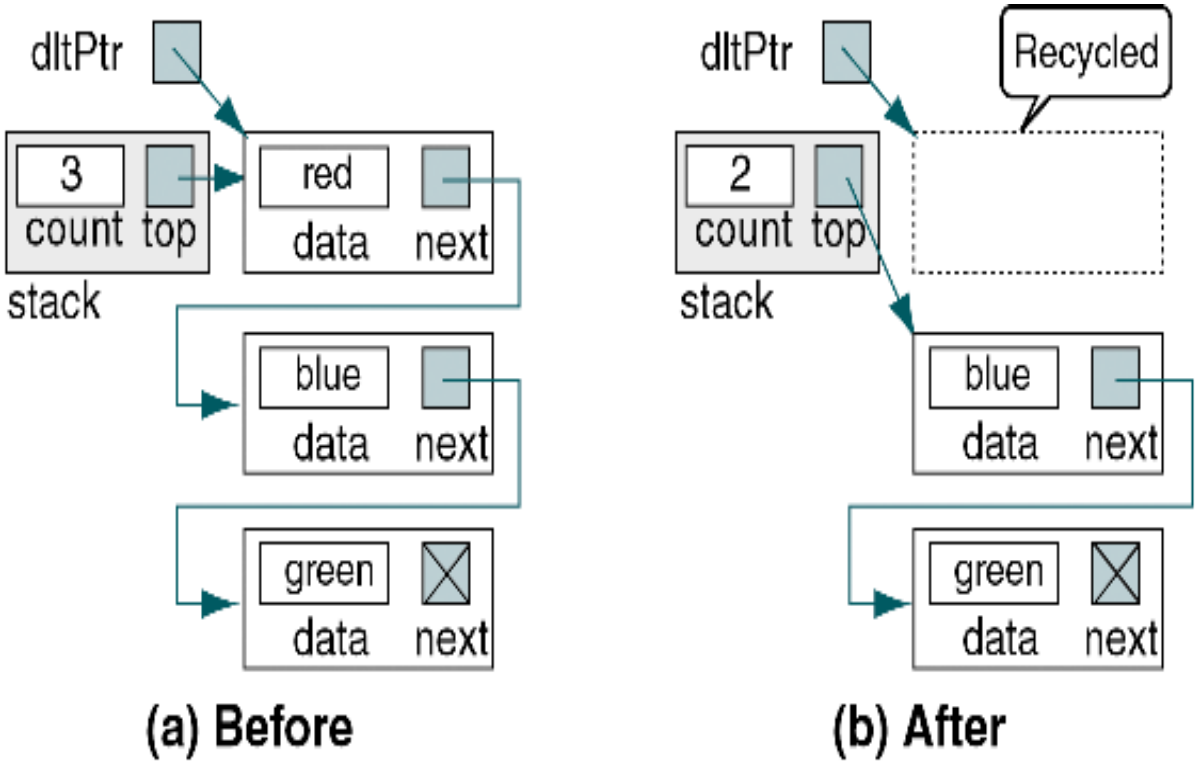


FIGURE 3-10 Pop Stack Example

Pop()

ALGORITHM 3-3 Pop Stack

```
Algorithm popStack (stack, dataOut)
This algorithm pops the item on the top of the stack and
returns it to the user.
   Pre    stack passed by reference
          dataOut is reference variable to receive data
   Post   Data have been returned to calling algorithm
   Return true if successful; false if underflow
1  if (stack empty)
   1   set success to false
2  else
   1   set dataOut to data in top node
   2   make second node the top node
   3   decrement stack count
   4   set success to true
3  end if
4  return success
end popStack
```

Pop()

```
1.  void pop()
2.  {
3.  struct node *ptr;
4.  if (top == NULL)
5.  printf("\n\nStack is empty\n");
6.  else
7.  {
8.  ptr = top;
9.  item = top->info;
10. top = top->link;
11. free(ptr);
12. printf("\n\nItem deleted: %d", item);
13. }
14. }
```

Traversing

```
1.  void display()
2.  {
3.  struct node *ptr;
4.  if (top == NULL)
5.  printf("\n\nStack is empty\n");
6.  else
7.  {
8.  ptr = top;
9.  while(ptr != NULL)
10. {
11. printf("\n\n%d",  ptr->info);
12. ptr = ptr->link;
13. }
```

Applications of Stacks

- Stack applications can be classified into four broad categories: *reversing data*, *pairing data*, *postponing data usage* and *backtracking steps*.
- Direct applications
  - Page-visited history in a Web browser
  - Undo sequence in a text editor
  - Chain of method calls in the Java Virtual Machine or C++ runtime environment
  - Program execution
  - Parsing
  - Evaluating postfix expressions
- Indirect applications
  - Auxiliary data structure for algorithms
  - Component of other data structures

Reversing data items

- Reversing data items requires a given set of data items be reordered so that the first and last items are exchanged, with all of the positions between the first and last also being relatively exchanged. For example, the list (2, 4, 7, 1, 6, 8) becomes (8, 6, 1, 7, 4, 2).

Pairing data items

- The following two expressions are evaluated differently because of the parentheses in the second expression:

$3 \times 6 + 2 = 20$ $3 \times (6 + 2) = 24$

- The compiler uses a stack to check that all opening parentheses are paired with a closing parentheses.

Example

- Algorithm to check if all opening parentheses are paired with a closing parenthesis.

```
Algorithm: CheckingParentheses (expression)
Purpose: Check the pairing of parentheses in an expression
Pre: Given the expression to be checked
Post: Error messages if unpaired parentheses are found
Return: None
{
    stack (S)
    while (more character in the expression)
    {
        Char ← next character
        if (Char = '(')          push (S, Char)
        else
        {
            if (Char = ')')
            {
                if (empty (S))   print (unmatched opening parenthesis)
                else             pop (S, x)
            }
        }
    }
    if (not empty (S))           print (a closing parenthesis not matched)
    return
}
```

Applications of Stacks

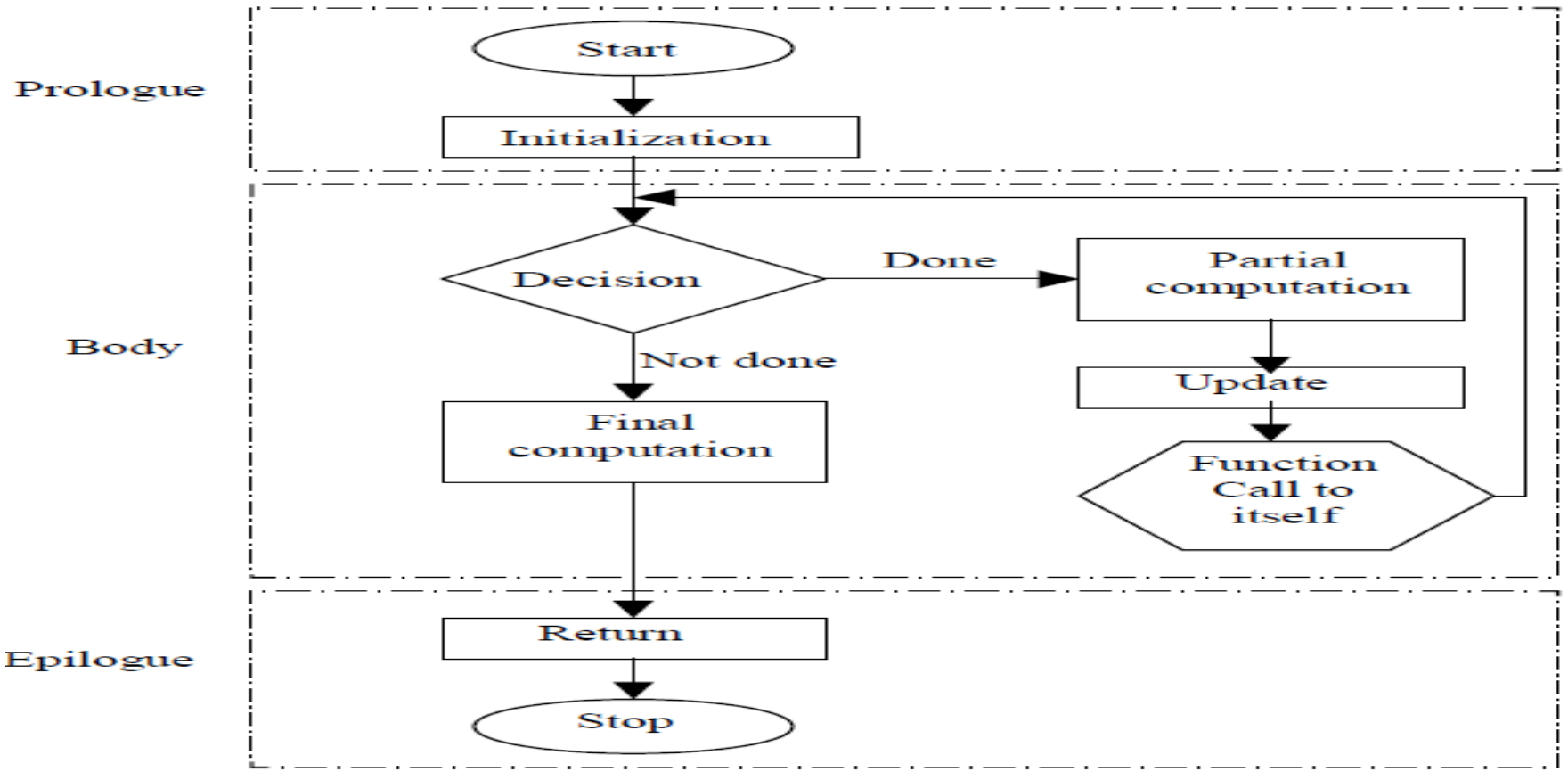
- **RECURSION**
- Recursion occurs when a function is called by itself repeatedly; the function is called recursive function. The general algorithm model for any recursive function contains the following steps:

1. ***Prologue:*** Save the parameters, local variables, and return address.

2. ***Body:*** If the base criterion has been reached, then perform the final computation and go to step 3; otherwise, perform the partial computation and go to step 1 (initiate a recursive call).

3. ***Epilogue:*** Restore the most recently saved parameters, local variables, and return address.

Recursion


Prologue
Start
Initialization
Body
Decision
Done
Partial computation
Update
Function Call to itself
Not done
Final computation
Epilogue
Return
Stop

TOWER OF HANOI

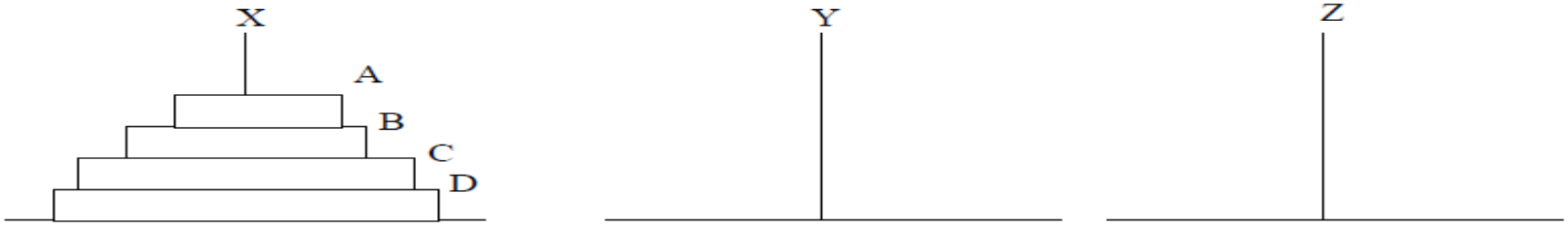


Fig. 3.4. Initial position of the Tower of Hanoi

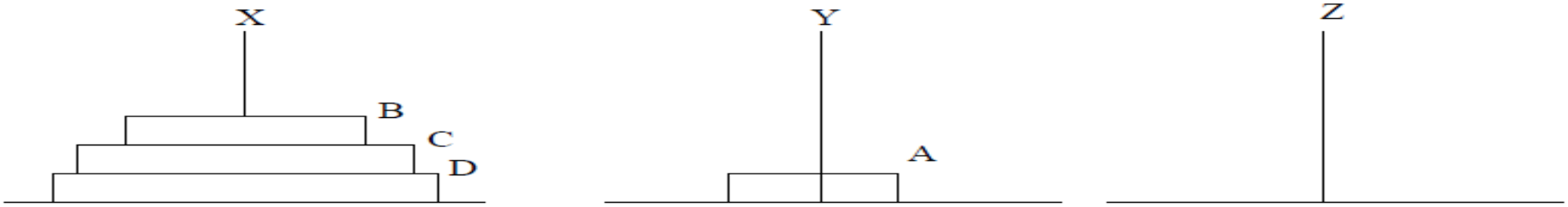


Fig. 3.5. Move disk A from the peg X to peg Y

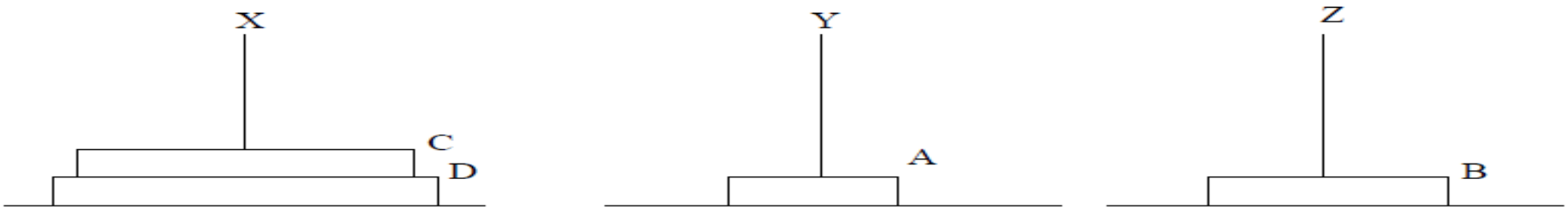


Fig. 3.6. Move disk B from the peg X to peg Z

TOWER OF HANOI

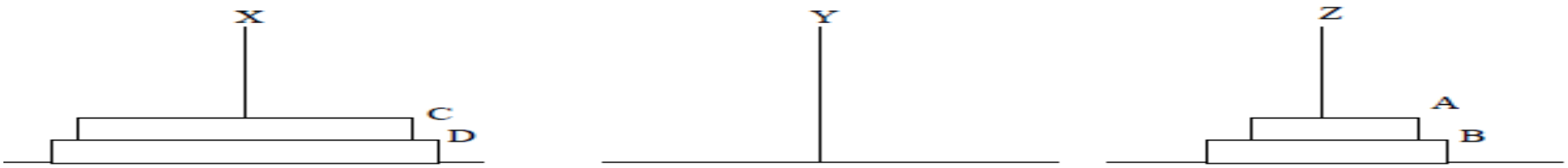


Fig. 3.7. Move disk A from the peg Y to peg Z

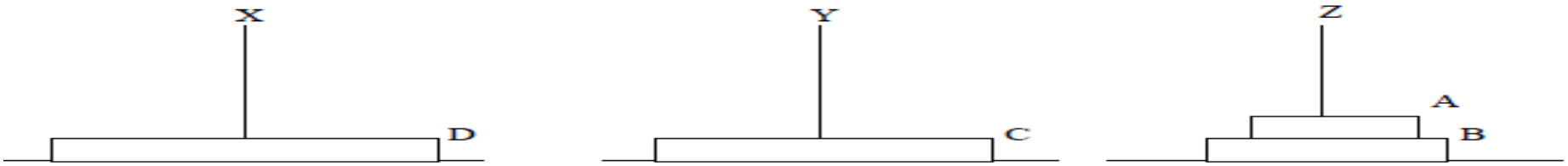


Fig. 3.8. Move disk C from the peg X to peg Y

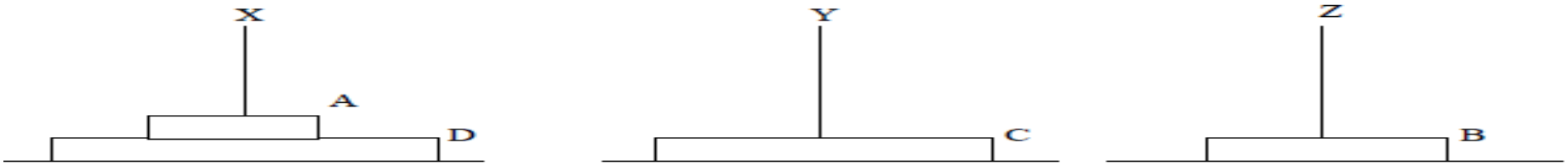


Fig. 3.9. Move disk A from the peg Z to peg X

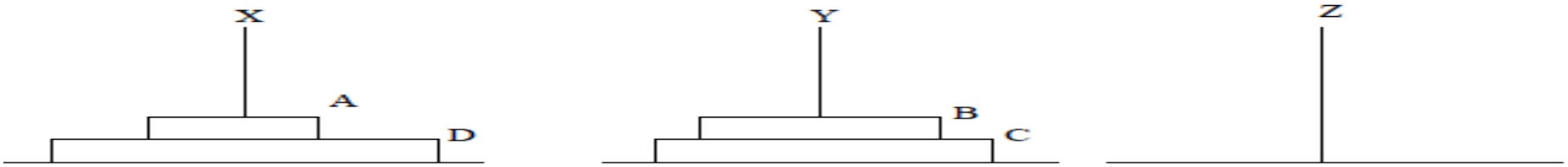


Fig. 3.10. Move disk B from the peg Z to peg Y

TOWER OF HANOI

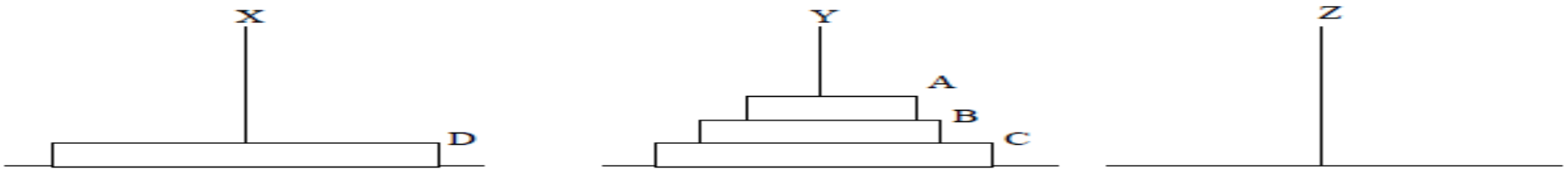


Fig. 3.11. Move disk A from the peg X to peg Y

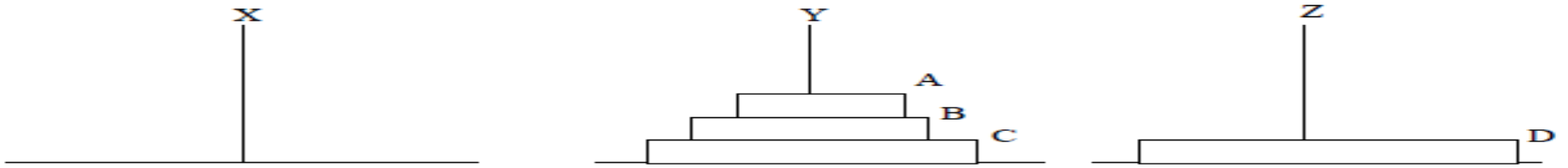


Fig. 3.12. Move disk D from the peg X to peg Z

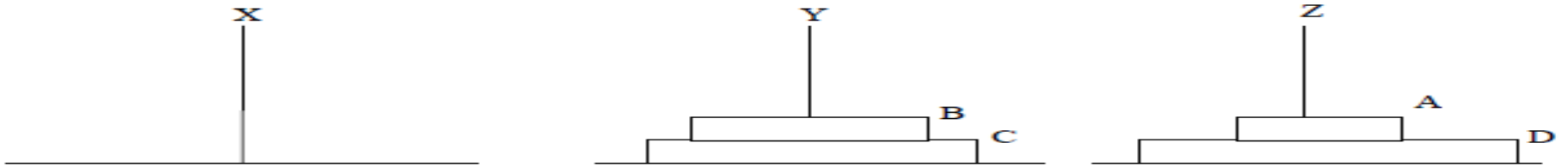


Fig. 3.13. Move disk A from the peg Y to peg Z

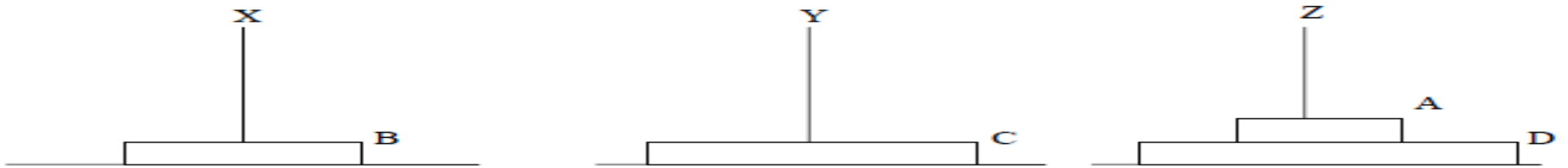


Fig. 3.14. Move disk B from the peg Y to peg X

TOWER OF HANOI

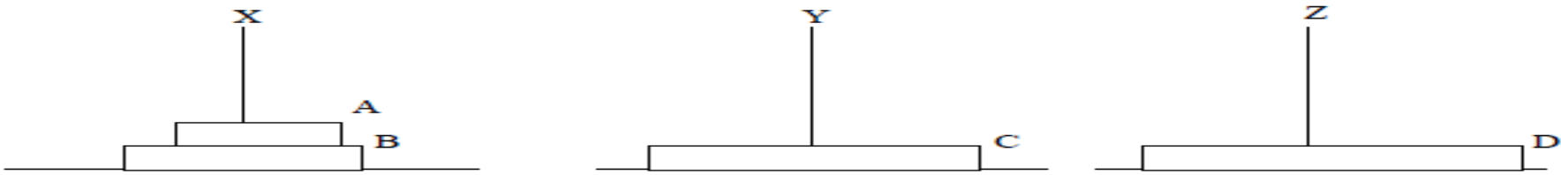


Fig. 3.15. Move disk A from the peg Z to peg X

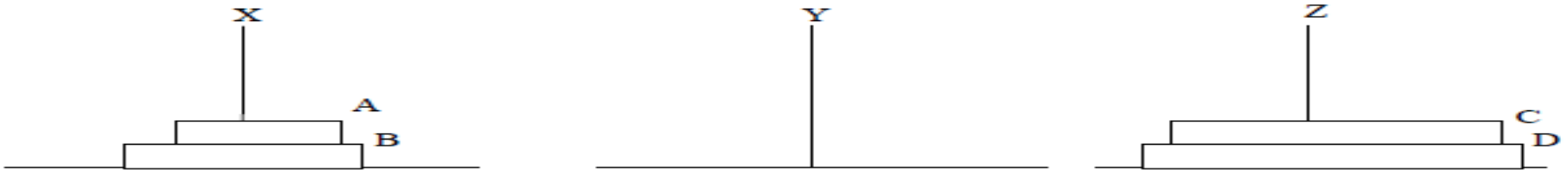


Fig. 3.16. Move disk C from the peg Y to peg Z

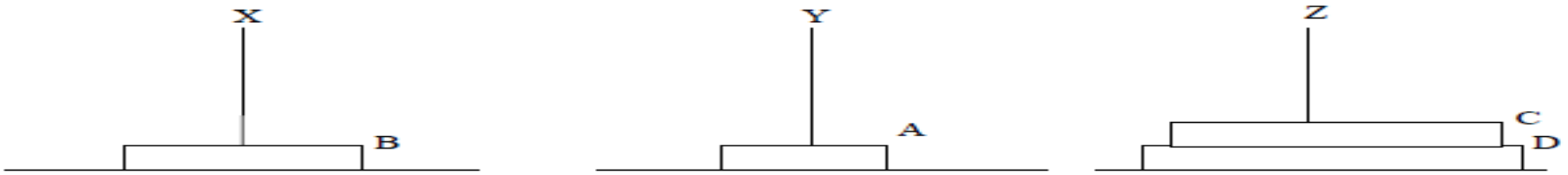


Fig. 3.17. Move disk A from the peg X to peg Y

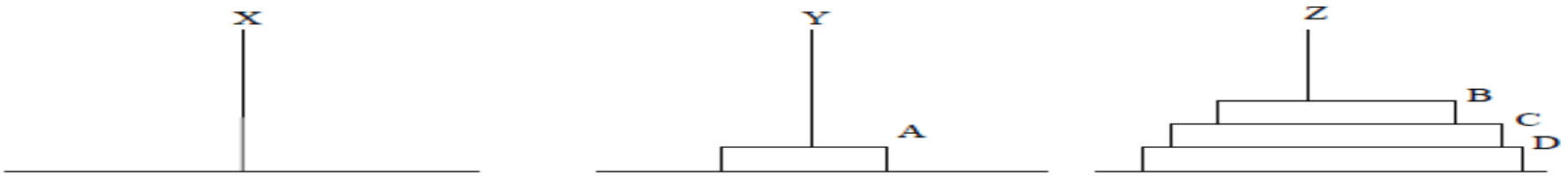


Fig. 3.18. Move disk B from the peg X to peg Z

TOWER OF HANOI

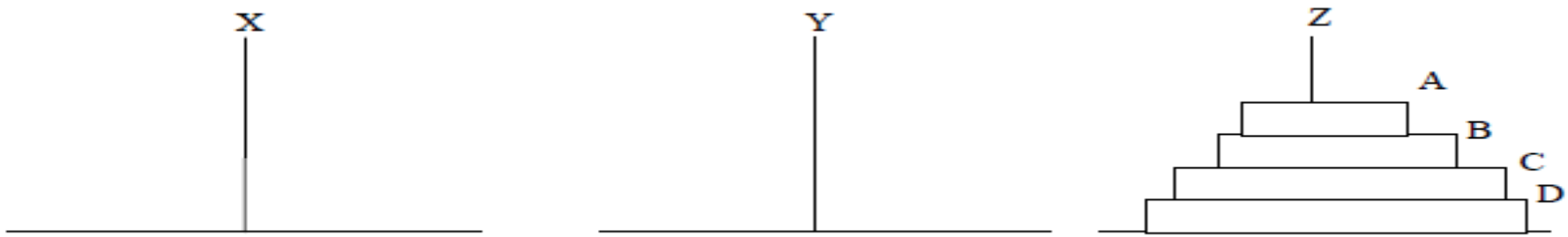


Fig. 3.19. Move disk A from the tower Y to tower Z

TOWER OF HANOI

- Recursive solution for the Tower of Hanoi with algorithm
  - Let's call the three pegs Src(Source), Aux(Auxiliary) and st(Destination).

1) Move the top N – 1 disks from the Source to Auxiliary tower
2) Move the Nth disk from Source to Destination tower
3) Move the N – 1 disks from Auxiliary tower to Destination tower. Transferring the top N – 1 disks from Source to Auxiliary tower can again be thought of as a fresh problem and can be solved in the same manner. So once you master solving Tower of Hanoi with three disks, you can solve it with any number of disks with the above algorithm.

TOWER OF HANOI

- **Explicit Pattern**

Number of Disks	Number of Moves
1	1
2	3
3	7
4	15
5	31

- *Powers of two help reveal the pattern:*

Number of Disks (n)	Number of Moves
1	2^1 - 1 = 2 - 1 = 1
2	2^2 - 1 = 4 - 1 = 3
3	2^3 - 1 = 8 - 1 = 7
4	2^4 - 1 = 16 - 1 = 15
5	2^5 - 1 = 32 - 1 = 31

Evaluation of Expressions

- The representation and evaluation of expressions is of great interest to computer scientists.
  - *x = a/b - c+d*e - a*c*
- If we examine expression, we notice that they contains:
  - operators: ==, +, -, ||, &&, !
  - operands: a,b,c…
  - parentheses: ( )
- How to generate the machine instructions corresponding to a given expression?

  precedence rule + associative rule

Precedence hierarchy and associativity

Token	Operator	Precedence[1]	Associativity
() [] –> .	function call array element struct or union member	17	left-to-right
–– ++	increment, decrement[2]	16	left-to-right
–– ++ ! ~ – + & * sizeof	decrement, increment[3] logical not one's complement unary minus or plus address or indirection size (in bytes)	15	right-to-left
(type)	type cast	14	right-to-left
* / %	multiplicative	13	left-to-right
+ –	binary add or subtract	12	left-to-right
<< >>	shift	11	left-to-right
> >= < <=	relational	10	left-to-right
== !=	equality	9	left-to-right
&	bitwise and	8	left-to-right
^	bitwise exclusive or	7	left-to-right
|	bitwise or	6	left-to-right
&&	logical and	5	left-to-right
||	logical or	4	left-to-right
?:	conditional	3	right-to-left
= += –= /= *= %= <<= >>= &= ^= |=	assignment	2	right-to-left
,	comma	1	left-to-right

1. The precedence column is taken from Harbison and Steele.
2. Postfix form
3. Prefix form

Figure 3.12: Precedence hierarchy for C

Expression

➢ 1. **Infix notation**

- The *infix notation* is what we come across in our general mathematics, where the operator is written in-between the operands. For example : The expression to add two numbers A and B is written in infix notation as:

A + B

2. **Prefix notation**

- The *prefix notation* is a notation in which the operator(s) is written before the operands, it is also called polish notation in the honor of the polish mathematician Jan Lukasiewicz who developed this notation. The same expression when written in prefix notation looks like:

+ A B

Expression

- 3. **Postfix notation**

- In the *postfix notation* the operator(s) are written after the operands, so it is called the postfix notation (post means after), it is also known as *suffix notation* or *reverse polish notation*. The above expression if written in postfix expression looks like:

A B +

Expression

- The method of converting infix expression A + B * C to postfix form is:

A + B * C	Infix Form
A + (B * C)	Parenthesized expression
A + (B C *)	Convert the multiplication
A (B C *) +	Convert the addition
A B C * +	Postfix form

The rules to be remembered during infix to postfix conversion are:

1. Parenthesize the expression starting from left to light.
2. During parenthesizing the expression, the operands associated with operator having higher precedence are first parenthesized. For example in the above expression B * C is parenthesized first before A + B.
3. The sub-expression (part of expression), which has been converted into postfix, is to be treated as single operand.

4. Once the expression is converted to postfix form, remove the parenthesis.

Example 1

Give postfix form for A + [(B + C) + (D + E) * F] / G

Solution. Evaluation order is

A + { [(BC +) + (DE +) * F] / G}

A + { [(BC +) + (DE + F *] / G}

A + { [(BC + (DE + F * +] / G} .

A + [BC + DE + F *+ G /]

ABC + DE + F * + G / + Postfix Form

Example 2

Give postfix form for (A + B) * C / D + E ^ A / B

Solution. Evaluation order is

[(AB +) * C / D] + [(EA ^) / B]

[(AB +) * C / D] + [(EA ^) B /]

[(AB +) C * D /] + [(EA ^) B /]

(AB +) C * D / (EA ^) B / +

AB + C * D / EA ^ B / + Postfix Form

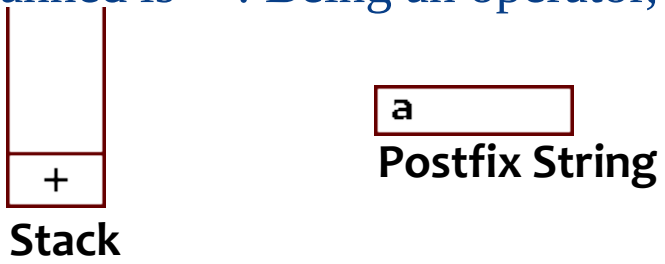
Algorithm

1. Push “(” onto stack, and add“)” to the end of P.
2. Scan P from left to right and repeat Steps 3 to 6 for each element of P until the stack is empty.
3. If an operand is encountered, add it to P.
4. If a left parenthesis is encountered, push it onto stack.
5. If an operator $\otimes$ is encountered, then:
   (*a*) Repeatedly pop from stack and add P each operator (on the top of stack), which has the same precedence as, or higher precedence than $\otimes$.
   (*b*) Add $\otimes$ to stack.
6. If a right parenthesis is encountered, then:
   (*a*) Repeatedly pop from stack and add to P (on the top of stack until a left parenthesis is encountered.
   (*b*) Remove the left parenthesis. [Do not add the left parenthesis to P.]
7. Exit.

Example 3

- **Infix String : a+b*c-d**

- Initially the Stack is empty and our Postfix string has no characters. Now, the first character scanned is 'a'. 'a' is added to the Postfix string. The next character scanned is '+'. Being an operator, it is pushed to the stack.



- Next character scanned is 'b' which will be placed in the Postfix string. Next character is '*' which is an operator. Now, the top element of the stack is '+' which has lower precedence than '*', so '*' will be pushed to the stack.

Example 3

- The next character is 'c' which is placed in the Postfix string. Next character scanned is '-'. The topmost character in the stack is '*' which has a higher precedence than '-'. Thus '*' will be popped out from the stack and added to the Postfix string. Even now the stack is not empty. Now the topmost element of the stack is '+' which has equal priority to '-'. So pop the '+' from the stack and add it to the Postfix string. The '-' will be pushed to the stack.

Stack

abc*+

Postfix String

- Next character is 'd' which is added to Postfix string. Now all characters have been scanned so we must pop the remaining elements from the stack and add it to the Postfix string. At this stage we have only a '-' in the stack. It is popped out and added to the Postfix string. So, after all characters are scanned, this is how the stack and Postfix string will be :

Stack

abc*+d-

Postfix String

Applications of Stacks

- Consider the following arithmetic infix expression P

P = A + (B / C - (D * E ^ F) + G) * H

Character scanned	*Stack*	*Postfix Expression (Q)*
A	(	A
+	(+	A
(	(+ (	A
B	(+ (	A B
/	(+ (/	A B
C	(+ (/	A B C
–	(+ (-	A B C /
(	(+ (- (	A B C /
D	(+ (- (	A B C / D
*	(+ (- (*	A B C / D
E	(+ (- (*	A B C / D E
^	(+ (- (* ^	A B C / D E
F	(+ (- (* ^	A B C / D E F
)	(+ (-	A B C / D E F ^ *
+	(+ (+	A B C / D E F ^ * -
G	(+ (+	A B C / D E F ^ * - G
)	(+	A B C / D E F ^ * - G +
*	(+ *	A B C / D E F ^ * - G +
H	(+ *	A B C / D E F ^ * - G + H
)		A B C / D E F ^ * - G + H * +