

While much of the literature on the screening of nuclear reactions has focused on the screening of fusion reactions, this same physics is important when reaction pairs have equilibrated, be it local quasi-equilibrium (QSE) or global Nuclear Statistical Equilibrium (NSE) (Hix & Thielemann 1996; Bravo & García-Senz 1999). If equilibrium abundances calculated from thermonuclear reaction networks using these fusion rates are to match the abundances calculated from equilibrium expressions, the relevant screening factors must be calculated consistently. Using the velocity integrated cross section  $\langle\sigma v\rangle$ , the screened cross section for the reaction of nuclei  $i$  and  $j$  is defined in terms of the unscreened cross section by

$$\langle\sigma v\rangle_{\text{screened nuclei}} = \exp(H_{ij})\langle\sigma v\rangle_{\text{bare nuclei}},$$

where  $\exp(H_{ij})$  is termed the screening enhancement factor, sometimes denoted  $f_{scr}$ .

Screening enters the equilibrium expressions for NSE or QSE as a modification of the nuclear binding energy,  $H(Z)$ , that is the difference in the Helmholtz free energy due to screening in assembling the nucleus  ${}^AZ$ . Some screening laws, like Salpeter's original weak and strong screening laws (Salpeter 1954), are derived analytically from this same change in the free energy (see also Dewitt et al. 1973). Given an expression for the change in free energy  $H(Z)$ , the screening factor for reaction rates can be computed from

$$H_{ij} = (H(Z_i + Z_j) - H(Z_i) - H(Z_j))/k_B T.$$

Other screening laws are the product of fits to Monte Carlo plasma simulations (see, for example, ?). Within statistical uncertainties, these alternative methods produce consistent results (see, for example, ??).

Previously, Hix & Thielemann (1996) had constructed the NSE screening term sequentially from  $H_{p,\gamma}$  terms to maintain consistency with the nuclear reaction network results, whereas Bravo & García-Senz (1999) computed NSE screening terms from global Coulomb corrections using various moments in powers of  $Z$  over the NSE abundance distribution. Where  $H_{p,\gamma} = (H(Z + 1) - H(Z) - H(1))/k_B T$ , these results should be consistent. (Note  $H(1) = 0$ , there is no reaction that builds a proton.)

The necessity of this functional form for screening comes from the requirement that the change in the difference in the binding energy due to screening between two species must be independent of the reaction pathway. For example, the abundances of  ${}^AZ$  and  ${}^{(A+4)}(Z+2)$  can be kept in equilibrium by  $(\alpha, \gamma)$  and its reverse or by the series  $(p, \gamma)(p, \gamma)(n, \gamma)(n, \gamma)$  and their reverses. When constructed from  $H(Z)$ , this requirement is manifestly met,  $\Delta BE = H(Z + 2) - H(Z)$ . Without this, you can not define a unique relation for the equilibrium screening correction that covers all of the reaction paths that are simultaneously in equilibrium between  ${}^AZ$  and  ${}^{(A+4)}(Z + 2)$ . Further note that because the reaction pathways  $(p, \gamma)(p, \gamma)(n, \gamma)(n, \gamma)$  and  $(p, \gamma)(n, \gamma)(p, \gamma)(n, \gamma)$  are simultaneously in equilibrium,  $H(Z)$  can not depend on  $A$ .

For three reactant reactions, the screening factor for reaction rates is conventionally calculated as  $H_{ijk} = H_{ij} + H_{(i+j)k}$  because three particle reactions of importance in astrophysics are generally sequential pairs of reactions where a short-lived intermediate nucleus  $(i + j)$

is formed by the first pair of reactants and captures the third reactant before it decays. To remain consistent with the requirements of equilibrium screening, we must construct  $H_{ijk}$  from  $H$ . In general, this approach should work as well for astrophysical 3 reactant reactions as for 2.

$$H(Z_i + Z_j + Z_k) - H(Z_i) - H(Z_j) - H(Z_k) \quad (1)$$

$$= (H(Z_i + Z_j + Z_k) - H(Z_i + Z_j) - H(Z_k)) \quad (2)$$

$$+ (H(Z_i + Z_j) - H(Z_i) - H(Z_j)) \quad (3)$$

$$= H_{(i+j)k} + H_{ij} \quad (4)$$

$$= H_{ijk} . \quad (5)$$

Thus the screening factors of 3 reactant reactions can be adequately calculated from the change in free energy.

### 0.1. Weak Screening

For weak screening, Graboske, Dewitt, Grossman & Cooper (1973; ApJ 181 457), give  $H_{12}$  in terms of charge independent parameter  $\Lambda_0$ , an average charge parameter  $\tilde{z}$  which depends on the abundances of all nuclear species, and the ion charges  $Z_i$  and  $Z_j$  as

$$H_{ij} = Z_i Z_j \tilde{z} \Lambda_0 .$$

$\Lambda_0$  and  $\tilde{z}$  are defined in Dewitt et al. (1973). This expression approaches the classical Debye-Hückel case as the electrons become non-degenerate. Choosing the corresponding

$$H(Z) = \frac{1}{2} Z^2 \tilde{z} \Lambda_0 ,$$

then

$$H(Z_i + Z_j) - H(Z_i) - H(Z_j) = \frac{1}{2} ((Z_i + Z_j)^2 - Z_i^2 - Z_j^2) \tilde{z} \Lambda_0 = Z_i Z_j \tilde{z} \Lambda_0 = H_{ij}$$

Thus the equivalence of the approaches does indeed hold for two-reactant reactions. For three-reactant reactions,

$$H_{ijk} = H_{ij} + H_{(i+j)k} = Z_i Z_j \tilde{z} \Lambda_0 + (Z_i + Z_j) Z_k \tilde{z} \Lambda_0 = (Z_i Z_j + Z_i Z_k + Z_j Z_k) \tilde{z} \Lambda_0 .$$

In terms of  $H(Z)$

$$\begin{aligned} H(Z_i + Z_j + Z_k) - H(Z_i) - H(Z_j) - H(Z_k) &= \frac{1}{2} ((Z_i + Z_j + Z_k)^2 - Z_i^2 - Z_j^2 - Z_k^2) \tilde{z} \Lambda_0 \\ &= (Z_i Z_j + Z_i Z_k + Z_j Z_k) \tilde{z} \Lambda_0 = H_{ijk} , \end{aligned} \quad (7)$$

thus the equivalence of the approaches also holds for three-reactant reactions.

## 0.2. Intermediate Screening

For intermediate screening, Dewitt et al. (1973) give  $H_{12}$  in terms of charge independent parameter  $\Lambda_0$ , and several average charge parameters  $\tilde{z}$ ,  $\bar{z}$ , and  $\langle z^{1.58} \rangle$ , which depends on the abundances of all nuclear species, and the ion charges  $Z_i$  and  $Z_j$  as

$$H_{ij} = 0.380 \frac{\langle z^{1.58} \rangle}{\tilde{z}^{0.58} \bar{z}^{0.28}} \Lambda_0^{0.860} [(Z_i + Z_j)^{1.860} - Z_i^{1.860} - Z_j^{1.860}] .$$

Choosing the corresponding

$$H(Z) = 0.380 \frac{\langle z^{1.58} \rangle}{\tilde{z}^{0.58} \bar{z}^{0.28}} \Lambda_0^{0.860} Z^{1.860} ,$$

then

$$H(Z_i + Z_j) - H(Z_i) - H(Z_j) = 0.380 \frac{\langle z^{1.58} \rangle}{\tilde{z}^{0.58} \bar{z}^{0.28}} \Lambda_0^{0.860} [(Z_i + Z_j)^{1.860} - Z_i^{1.860} - Z_j^{1.860}] = H_{ij}$$

Thus the equivalence of the approaches does indeed hold for two-reactant reactions. For three-reactant reactions,

$$H_{ijk} = H_{ij} + H_{(i+j)k} \quad (8)$$

$$= 0.380 \frac{\langle z^{1.58} \rangle}{\tilde{z}^{0.58} \bar{z}^{0.28}} \Lambda_0^{0.860} [(Z_i + Z_j)^{1.860} - Z_i^{1.860} - Z_j^{1.860}] \quad (9)$$

$$+ 0.380 \frac{\langle z^{1.58} \rangle}{\tilde{z}^{0.58} \bar{z}^{0.28}} \Lambda_0^{0.860} [(Z_i + Z_j + Z_k)^{1.860} - (Z_i + Z_j)^{1.860} - Z_k^{1.860}] \quad (10)$$

$$= 0.380 \frac{\langle z^{1.58} \rangle}{\tilde{z}^{0.58} \bar{z}^{0.28}} \Lambda_0^{0.860} [(Z_i + Z_j + Z_k)^{1.860} - Z_i^{1.860} - Z_j^{1.860} - Z_k^{1.860}] . \quad (11)$$

In terms of  $H(Z)$

$$H(Z_i + Z_j + Z_k) - H(Z_i) - H(Z_j) - H(Z_k) \quad (12)$$

$$= 0.380 \frac{\langle z^{1.58} \rangle}{\tilde{z}^{0.58} \bar{z}^{0.28}} \Lambda_0^{0.860} [(Z_i + Z_j + Z_k)^{1.860} - Z_i^{1.860} - Z_j^{1.860} - Z_k^{1.860}] \quad (13)$$

$$= H_{ijk} , \quad (14)$$

thus the equivalence of the approaches also holds for three-reactant reactions.

## REFERENCES

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