

## Introduction

### High School Fellowship

This past summer I had the opportunity to work on the Ultra-Cold Neutron (UCN) project at TRIUMF, sharing the High School Fellowship with three other graduating high school students from BC: Micah Gay, Robert Lee, and Ray Liu. It was a pleasure getting to know them, other students working at the facility, and the UCN project members. I learned far more than I could have predicted and enjoyed my six weeks at TRIUMF immensely. I left more knowledgeable about science, particularly experimental physics, and hope to work in similar environments in the future. I'd like to especially thank my supervisor, Dr. Rüdiger Picker, as well as all others who helped coordinate the fellowship, in particular Dana Giasson and Marcello Pavan.

### EDM Experiment

#### EDM Cell

Once the UCN beamline is properly prepared, the first planned experiment is a measurement of the neutron's electric dipole moment (nEDM). This measurement will take place in the EDM cell, a vacuum chamber in which homogenous electric and magnetic fields are used to bring ultracold neutrons through the Ramsay cycle to make the measurement. The cell has two chambers separated by a high-voltage electrode. Two additional electrodes at the very top and bottom of the cell are grounded to create an electric field in each chamber. Neutrons are piped into an insulated sub-region of each chamber, surrounded by glass. Inside this insulated area, it is important that electric field is highly homogenous, although this could never be completely the case without infinitely long electrodes. The bulk of my work this summer revolved around modelling this EDM cell, focussing particularly on the electrodes that induce the electric field.

#### Simple Electrode Shapes

After spending time familiarizing myself with the UCN project and the nEDM experiment, I produced a Lua script that built a simplified version of possible cell arrangement, modelled on a similar script made for a previous generation of electrodes.

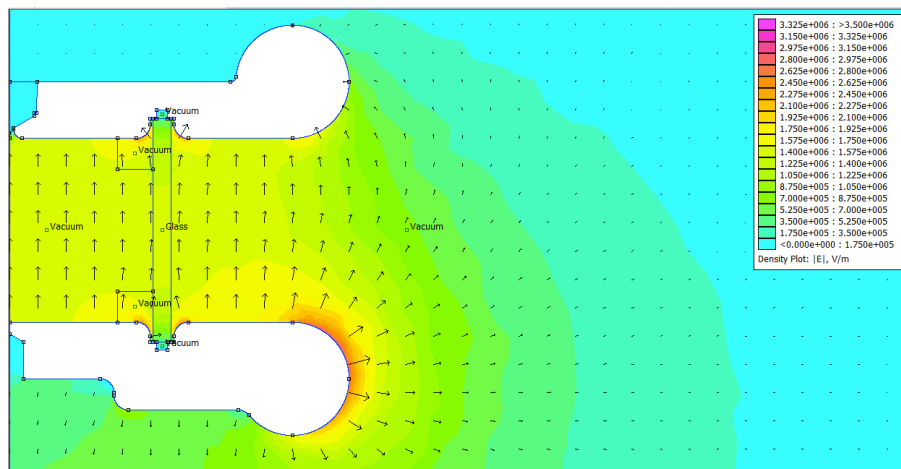


Figure 1 Previous generation of electrodes, constructed from "electrode FEA v2.0.lua"

I had two main goals in doing so: to ensure that the general electrode arrangement might have sufficient electric field homogeneity and to find appropriate parameters on which to base models. Picking appropriate parameters meant building the model around a set of variables such that a proper electrode could be produced for an set of values assigned to the variables – the insulator would be in the O-ring trench, the end of the electrode would be farther from the center than the insulator, and the electrode would curve the right way. The parameters I chose are illustrated in *Figure 2*: the dimensions of the vacuum chamber, the separation between the electrodes, the electrode radius up until the O-ring groove, the electrode half-thicknesses, and values constructing the insulator and O-ring trench.

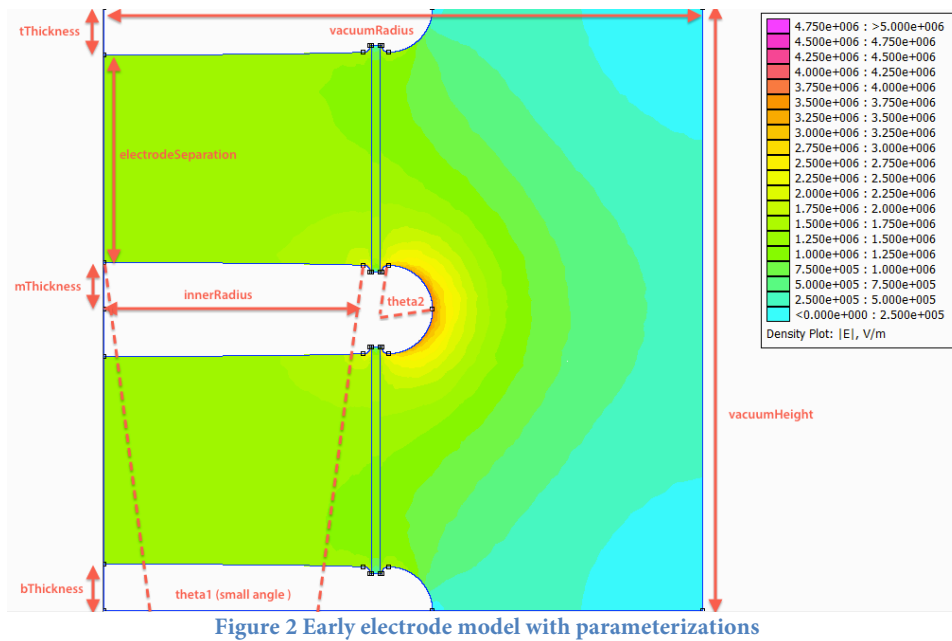


Figure 2 Early electrode model with parameterizations

### Complex Electrode Shapes

I first modelled flat electrodes, and then electrodes with slight curves produced by simple arcs. However, my next task was to model electrodes with particular curves, more complex than arcs, so I learned to create these in FEMM (the program I used to model electrodes) using many small arcs to approximate proper curvature.

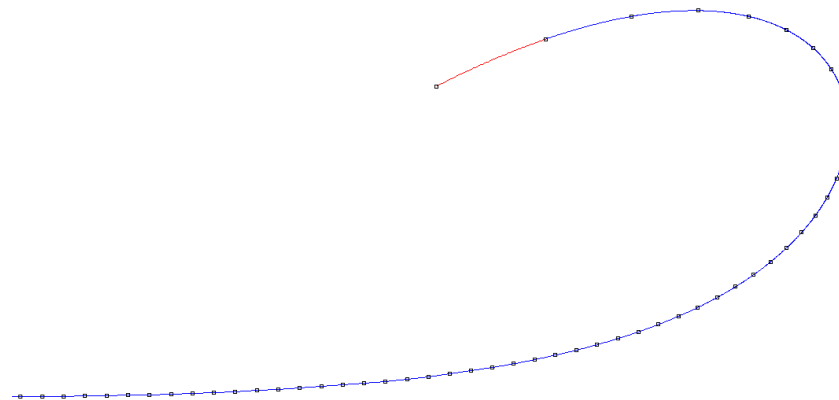


Figure 3 An example curve, created from Ernst-C equations

In order to construct electrodes from these curves, I ended the curve when it became vertical (in practice, the last point before it began to slope back towards  $r = 0$ ) and put two curves together.

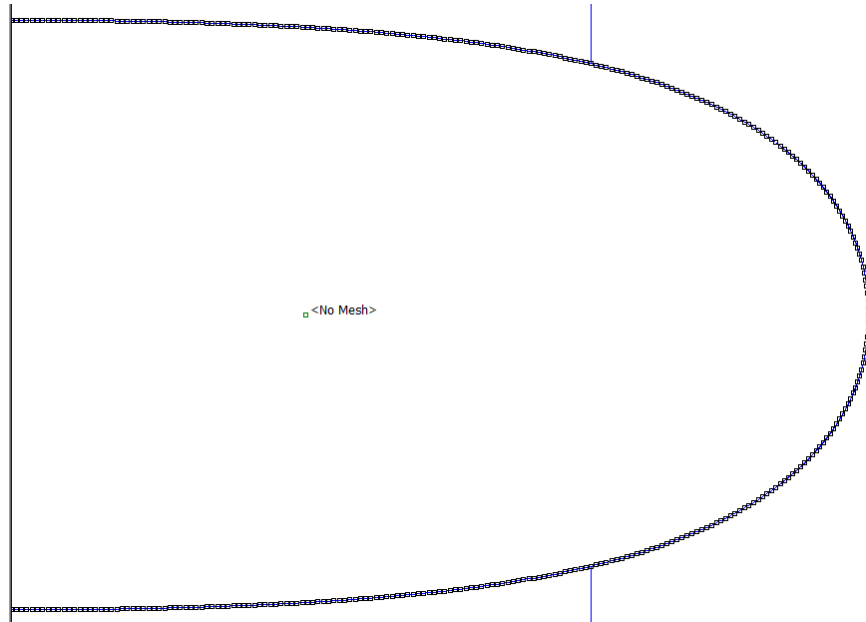


Figure 4 Ernst-C electrode constructed using the curve in Figure 3

### Results

At this point I was able to take proper measurements comparing the usefulness of different electrode shapes. I found that the simplest shape, a flat electrode, was the best across the measurements I took, but was unable to explain why this would be. Perhaps the complex electrode profiles I used were not well suited to the EDM cell or maybe my methods of measuring homogeneity were inaccurate.

Finally, I united a number of Lua documents into one master script that can produce several types of electrodes and makes it easy to incorporate more. At this time, the script does not allow electrode modifications that break their curvature (like pipes leading into the cell or an O-ring trench), but these could be implemented with a few small adaptations.

## Electrode Shapes

### Electric Field Homogeneity

Electric field homogeneity is one of the most important factors in comparing electrode shapes for the UCN project. It is crucial that the neutrons in the EDM cell experience a similar electric field throughout the cell. To my knowledge, no standardized technique exists for quantifying electric field homogeneity, but there are several measurements that can be used to compare homogeneity across experimental setups.

Electric field homogeneity is essentially the degree to which the electric field vectors at various points between the electrodes align in direction and magnitude. FEMM allows two main types of

measurements: line integrals and block integrals. Each has its advantages and disadvantages, line integrals allowing more particular measurements over certain sets of contours and block integrals allowing simple measurements over larger areas. Since FEMM models three-dimensional objects from two-dimensional drawings on the screen, it offers two ways to produce this third dimension: planar models have a predetermined depth into the screen and axisymmetric models are rotated about  $r = 0$ . Since my models were axisymmetric, the contour over which a line integral was taken represented an area of the cell and the area over which a block integral was taken represented a volume. When using line integrals, I quantified homogeneity by calculating:

$$\frac{|\vec{E}_{max}| - |\vec{E}_{min}|}{|\vec{E}_{max}|} \propto \frac{1}{homogeneity}$$

Where  $|\vec{E}_{max}|$  was the largest electric field strength and  $|\vec{E}_{min}|$  was the smallest measured along a given contour. The maximum value was most often at  $r = 0$  (the center of the electrodes) and the electric fields mostly decreased as  $r$  increased.

When using block integrals, I quantified homogeneity by calculating:

$$\frac{E_r}{E_z} \propto \frac{1}{homogeneity}$$

Where  $E_r$  was the average electric field along the  $r$ -axis (the  $x$ -axis in graphs of electrodes) and  $E_z$  was the average electric field along the  $z$ -axis (the  $y$ -axis in graphs of electrodes).

Both of these measurements were proportional to inverse homogeneity: smaller values meant more useful electrodes. Changes in these values reflect changes in homogeneity that occur with varying electrode shapes and separations, and with vacuum chamber dimensions. Physicists have designed electrode shapes that should produce high homogeneities and some of these shapes, Rogowski and Ernst profiles, are described below.

Of course, there should be other ways to quantify homogeneity, perhaps more accurately. However, these two methods proved relatively easy to implement and useful for overall homogeneity comparisons between various electrodes.

### Electrode Thickness

Electric field magnitudes have practical limits, over which dangerous breakdowns are likely to occur; it is therefore important to reduce electric fields, even where a homogenous field is not required. The maximum recommended DC design voltage is  $3 \text{ kV/mm}$  in a vacuum (but lower values are safer)<sup>1</sup> and the electric field in the EDM cell is naturally highest around the end of the middle electrode. In this area, the radial field is at its maximum in the cell and the electrodes

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<sup>1</sup> [https://www.nasa.gov/sites/default/files/files/684078main\\_12188C\\_RevisedforPosting\\_Battel\\_HVEngineering\\_Workshop\\_DAY1.pdf](https://www.nasa.gov/sites/default/files/files/684078main_12188C_RevisedforPosting_Battel_HVEngineering_Workshop_DAY1.pdf)

curvature is large, increasing the electric field. Therefore, this area is an important area to consider when discussing electrode shape and size. Although thicker electrode shapes allow more gradual curvatures that can reduce electric field, space is premium in experimental apparatuses like the UCN nEDM cell. Therefore, when two profiles provide similar electric field homogeneity, the thinner one is more practical. This is the case with the Ernst-B and Ernst-C profiles described below: the latter is designed to be thinner than the former while maintaining similar homogeneity.

## EDM Cell

The environment surrounding the electrodes is also a factor in determining electric fields, and therefore the construction of electrodes. Because the outside of the vacuum chamber is grounded, it interacts with the charged middle electrode. Consequently, both the vacuum chamber's height and radius are important considerations, determining how close these grounded areas are to the electrodes. In order to reduce the vacuum chamber's impact in many simulations, its dimensions were often set to ten times normal sizes. I also conducted simulations to analyze how a varying vacuum chamber size impacted electric field homogeneity. However, I never explored the effect of vacuum chambers with shapes other than simple cylinders.

In a static system,  $\vec{E} = -\nabla\phi$  where  $\phi$  is a function for the electric potential and  $\nabla$  determines how this function varies in each direction. In the simpler case of ideal, infinitely long parallel plates,  $|\vec{E}| = -\Delta\phi/s$  where  $\Delta\phi$  is the potential difference between the plates and  $s$  is the distance between them. These two equations, the second more simply but the first more formally, show the importance of the separation distance between electrodes in determining electric fields surrounding them. The mathematical impact of the electrode separation is clear in the equations for the curved electrode profiles described below. However, simple simulations showed that if the voltages are varied proportional to the electrode separations, characteristics of the system change very little (as predicted by the second equation).

## Profiles

### Rogowski Profile

The Rogowski profile, a particular class of electrode shapes designed to produce a more uniform electric field,<sup>2</sup> is constructed from the following formula:<sup>3</sup>

$$f(x) = \frac{s}{\pi} \left( \frac{\pi}{2} + e^{x\frac{\pi}{s}} \right)$$

Where  $s$  is the separation between a pair of electrodes. For negative values of  $x$ , the profile is essentially

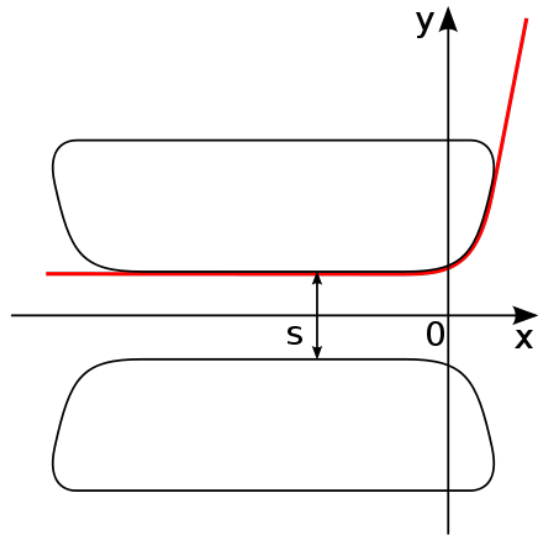


Figure 5 Example electrodes constructed using the Rogowski profile with separation  $s$

<sup>2</sup> U. Nundy, D.V.S. Satyanaraya, and N.S.N. Bannerjee, "Uniform Field Electrodes," *Centre for Advanced Technology*

<sup>3</sup> <https://de.wikipedia.org/wiki/Rogowski-Profil> (source for picture as well)

horizontal; as  $x$  grows larger, the profile slopes increasingly away from the  $x$ -axis. However, the curve never becomes vertical, so an arc beginning at an arbitrary point along the curve is necessary in order to model an electrode from this function. Since the arc must begin with the correct slope in the right location, this rounded end makes modelling more difficult. However, it is also useful, giving more control over the radius of the full Rogowski electrode.

I did not conduct any simulations using the Rogowski profile. The Ernst profiles were designed as improvements on the Rogowski profile, so I focussed on these instead. Moreover, my original code was designed for profiles that become vertical at some point, unlike the Rogowski profile. Although I added functionality for this near the end of the summer, I did not have time to test it extensively.

### Ernst Profiles

Noting the approximations necessary in order to actually build a Rogowski electrode and the large width and ineffective compacting methods of other physicists, Gerard Ernst produced more complex electrode equations that he claimed better compact electrodes while maintaining surface electric field strength uniformity.<sup>4</sup> This suggests that these profiles are the best for producing electric field homogeneity, but that did not seem to be the case in my simulations. Ernst's equations allow for a variety of profiles depending on constants raised to certain powers. We looked at two of these profiles: Ernst-B and Ernst-C. The primary difference between the two is thickness, where Ernst-B electrodes are thicker than Ernst-C electrodes.

$$\text{Ernst-B: } x = \frac{s}{\pi+2k_0} \left( t - \frac{1}{4} \left( 1 - \sqrt{1 - k_0^2} \right) \times \sinh 2t \right), y = \frac{s}{\pi+2k_0} \left( \frac{\pi}{2} + k_0 \times \cosh t \right)$$

$$\text{Ernst-C: } x = \frac{s}{\pi+2k_0} (t - k_1 \times \sinh 2t), y = \frac{s}{\pi+2k_0} \left( \frac{\pi}{2} + k_0 \times \cosh t - k_2 \times \cosh 3t \right)$$

Where  $s$  is the separation between a pair of electrodes,  $k_0$  is the primary constant associated with the profiles, changing to produce different versions of the electrode,  $k_1 = \frac{1}{8} k_0^2$ , and  $k_2 = \frac{1}{90} k_0^3$ .

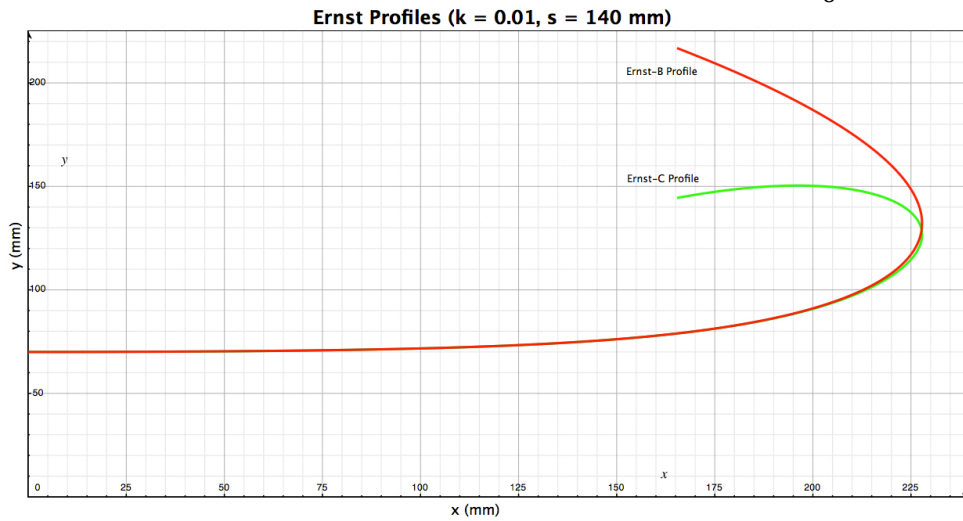


Figure 6 Profiles generated using the Ernst-B and Ernst-C equations

<sup>4</sup> Gerard Ernst, "Uniform-Field Electrodes with Minimum Width," *Twente Institute of Technology*, 1984

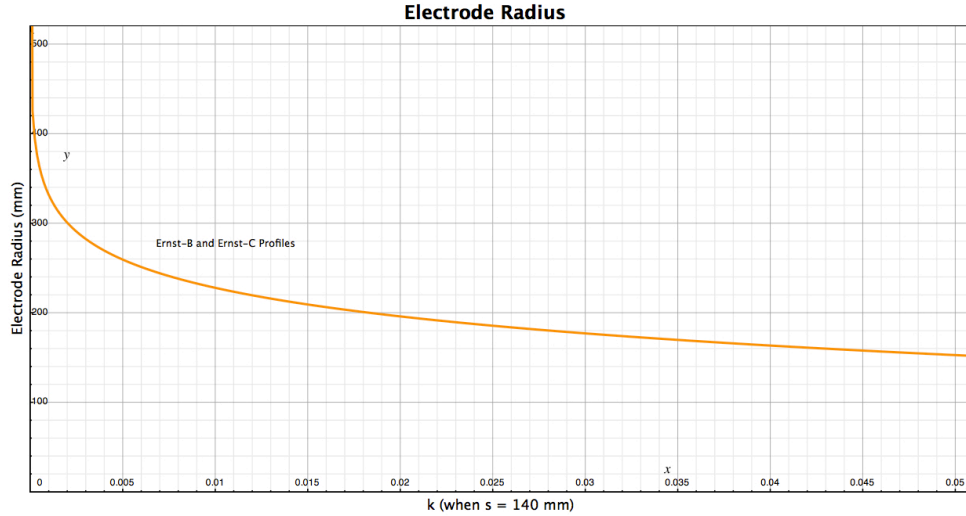


Figure 7 The radii of the two profiles differ by less than a millimetre until  $k_0 = 0.45$  (not shown on graph), so the radii are essentially equal in the ranges I simulated

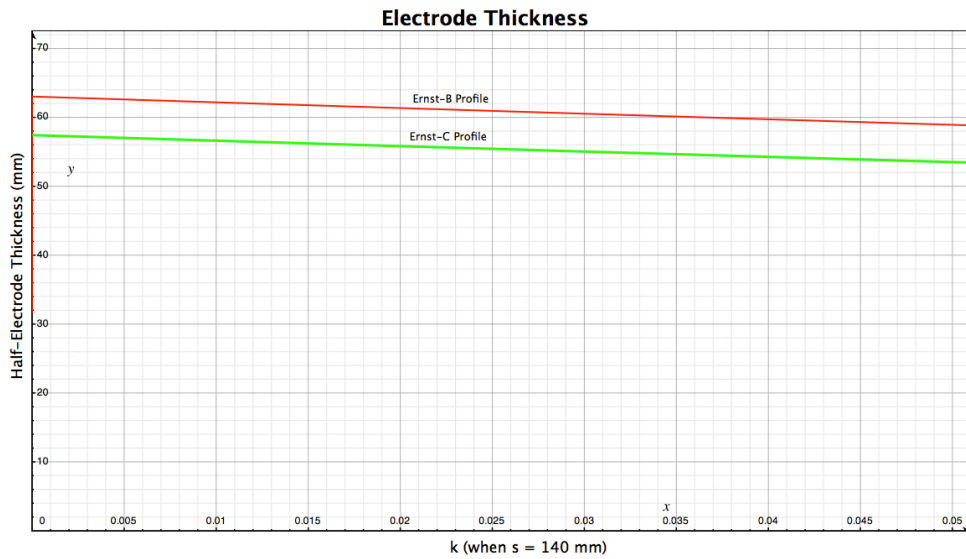


Figure 8 The half-electrode thicknesses appear linear and approximately 5.5 mm apart, meaning that an Ernst-C electrode would be about 11 mm thinner than an Ernst-B electrode

### Flat Electrode

The last and simplest electrode profile has a flat surface. As with the Rogowski profile, the flat electrode never becomes vertical (it is always horizontal) so an arc at an arbitrary point along the surface is required to produce an electrode. In theory, since the other two shapes are meant to improve on the imperfections of flat electrodes, flat electrodes should yield the least electric field uniformity. Not only does this not seem to be the case, flat electrodes are also the easiest to model and build, especially when including surface complications like O-ring trenches. To make comparable electrodes in my tests, I modelled flat electrodes whose radii and thicknesses (at  $r = 0$ ) were equal to those of whichever Ernst-C profiles I was comparing against. Flat electrodes clearly offer greater customizability, and this would be a good area for future testing.

## Modelling

### FEMM/Lua Overview

FEMM is a free program designed for modelling magnetics, electrostatics, heat flow, and current flow simulations in two dimensions. As described above, it allows axisymmetric and planar models to make calculations in three dimensions. Its two primary features are its relative simplicity for great functionality and its compatibility with Lua for building models. These two features work well in conjunction: it is easy to take measurements on models defined by nodes, segments and arcs, and areas, each element with its own predefined characteristics, and it is easy to automate the building and measuring processes using Lua.

My use of Lua to build FEMM models evolved significantly over my six weeks at TRIUMF. In my first couple days, I spent hours working through tutorials and building models by hand. Next I learned to build simple elements with Lua and how to build models with variable dimensions like that in *Figure 2*. Finally, I learned how to use functions to determine these variables, allowing me to create curved profiles in FEMM and model complex electrodes like that in *Figure 4*. Using Lua and FEMM together can be finicky at times – sometimes only one of two seemingly equivalent algorithms would work properly – but everything I needed to do proved possible in one way or another.

Although some tutorials and threads on the Internet exist to help solve FEMM problems, I did not find very much useful support. This is unsurprising, because the program is relatively old and useful for niche purposes. The resource I found most useful was the Version 4.2 User's Manual found at <http://www.femm.info/Archives/doc/manual42.pdf>.

### Basic Electrodes

FEMM models are made up of nodes, segments, and arcs. Nodes are defined by their  $x$  and  $y$ -coordinates and have no other properties. Segments and arcs are both defined by the two nodes they connect, and arcs also by the angle of the imaginary circle segment they span. In electrostatics problems, segments and arcs can be assigned “conductors”, a type of object property determining voltage. Segments and arcs bound areas in models whose properties are defined by block labels. Block labels can be added at any point in an area and apply to the entire contiguous area, usually determining the mesh precision used for calculations (FEMM's solvers divide models into a bunch of triangles to take measurements, smaller triangles allowing greater precision).

There are several steps to actually implementing this using Lua. Specific instances of these elements do not have identifiers, so they are instead selected using their coordinates. FEMM's select function chooses the node or segment closest to the supplied coordinates, so it is best to select nodes using exact coordinates and segments using the coordinates of a node at one end with the  $x$  or  $y$ -coordinate altered by  $\pm 0.1$  to ensure the right segment is selected. Block labels should be selected in the same way as nodes. Segments and block labels must first be selected before adding properties, although code can be simplified by selecting several segments or block labels before adding properties to all of them with one line of code.



## Curved Electrodes

FEMM's simple tools make it easy to model electrodes built from only nodes, segments, and arcs of circles; however, it is more difficult to build the curves necessary for Rogowski and Ernst profiles. FEMM does not support curves along mathematical functions, so I had to approximate these profiles using two main functions. The first generates a 2D array composed of  $(x, y, dy/dx)$  values corresponding to points along the curve, where  $x$  and  $y$  are found from parametric functions of  $t$  and  $dy/dx$  is approximated by calculating the slope of the line between the points on either side of the point in question. The approximation is better for smaller increments of  $t$  and functions with small values of  $d^2y/dx^2$ . The second function loops through this array, first adding the next node in FEMM, and then finding the angle between the "derivatives" of this new point and the previous point and constructing the appropriate arc between the two points. I found it easiest to add characteristics to these arcs one at a time, as they were added, to avoid having to loop back through my 2D array to find their coordinates again.

Here, the limitations of arcs become more apparent. Unfortunately, FEMM's arcs support only angles between 1 and 90 degrees. When a negative value (or an equivalent value larger than 180 degrees) is required, the order of the arc's points must instead be flipped in `ei_addarc()` and when an angle less than 1 degree is required, a segment must be used instead. Moreover, the arc's angles are defined in degrees while Lua trigonometric functions use radians, necessitating conversions.

This electrode construction method makes it difficult to add additional elements to the electrodes like O-ring trenches. I did not implement a solution but I did think about the problem at length. I think the best solution would be to pass arguments to the electrode building functions detailing start and stop  $x$ -values between which the program does not add nodes along the curve, and having the program return the coordinates of the electrode at these  $x$ -values. Then, with this information it would be possible to build custom shapes at the surface of the electrode with proper gaps in the profile.

## Closed Shape Generator (Python)

The code required to build FEMM models using Lua is structured such that the coordinates for a specific point are often repeated many times, first to add the node, then to add any segments and arcs originating at that node, then to select those segments and arcs, and even to place a block label within the bounds of a shape. It can be tedious to edit all these statements if the position of one point is changed somehow, especially since each point value can be a long string of variables. Therefore, I wrote a short Python script that takes a text file listing each point of a shape in order, and generates all the necessary node and segment code in Lua. The script also allows you to provide more information about the model in the text file in order to add segment and block properties, draw arcs instead of lines between certain points, and omit certain segments (omissions are helpful for using a separate Lua function to draw complex curves).

This script is most useful for closed shapes with consistent properties throughout. It can still be useful if some segment or block has different properties from the rest, but that change would have to be made manually each time the code is regenerated. I primarily used this script in my

early electrode models, before adding complex curves, in order for the entire electrode to adjust when I changed the variables that generate a point, without having to find every instance of that point in the electrode code. Later, I used it for the simple shapes in my more complex models, like the vacuum chamber itself.

## Measurements

### Electric Field Homogeneity by $k_0$

The most fundamental variable for determining the shape of electrodes built from Ernst profiles is  $k_0$ . It is therefore logical to examine how the value of  $k_0$  impacts the electric field homogeneity of Ernst-B and Ernst-C electrodes, and flat electrodes modelled with the same radius and thickness as the Ernst-C electrodes. FEMM's postprocessor allows colour gradients illustrating electric field strength, where warmer colours imply greater electric field, and three of these are included below. For each trial, the electrode separation is 140 mm, the vacuum chamber's height is the sum of the electrodes' thicknesses and the two separations, and the vacuum chamber's radius is twice the full radius of an electrode.

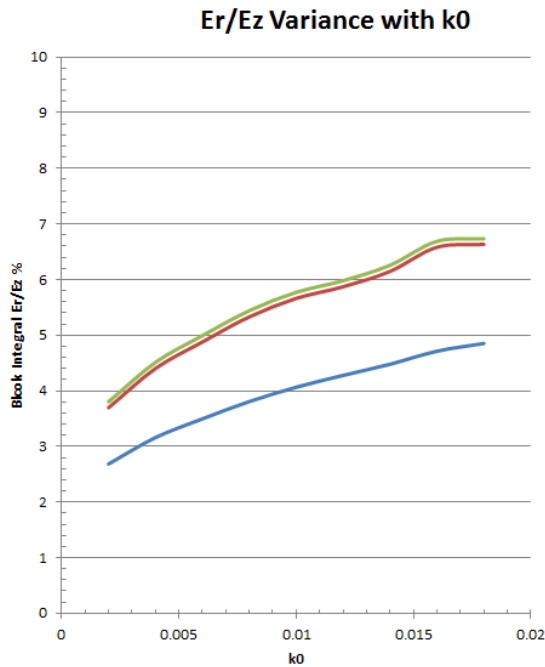


Figure 9 Smaller values of  $k_0$  correspond to greater homogeneity

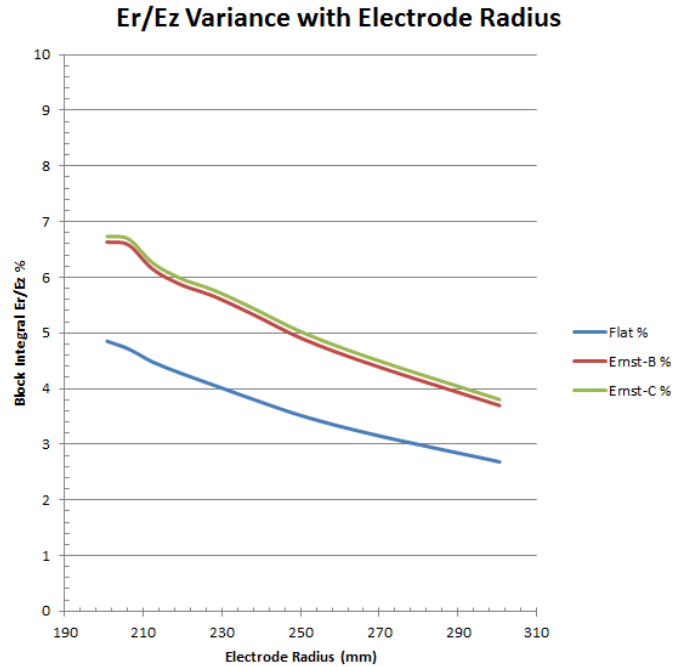


Figure 10 Because smaller values of  $k_0$  lead to greater electrode radii, greater electrode radii in turn correspond to greater homogeneity

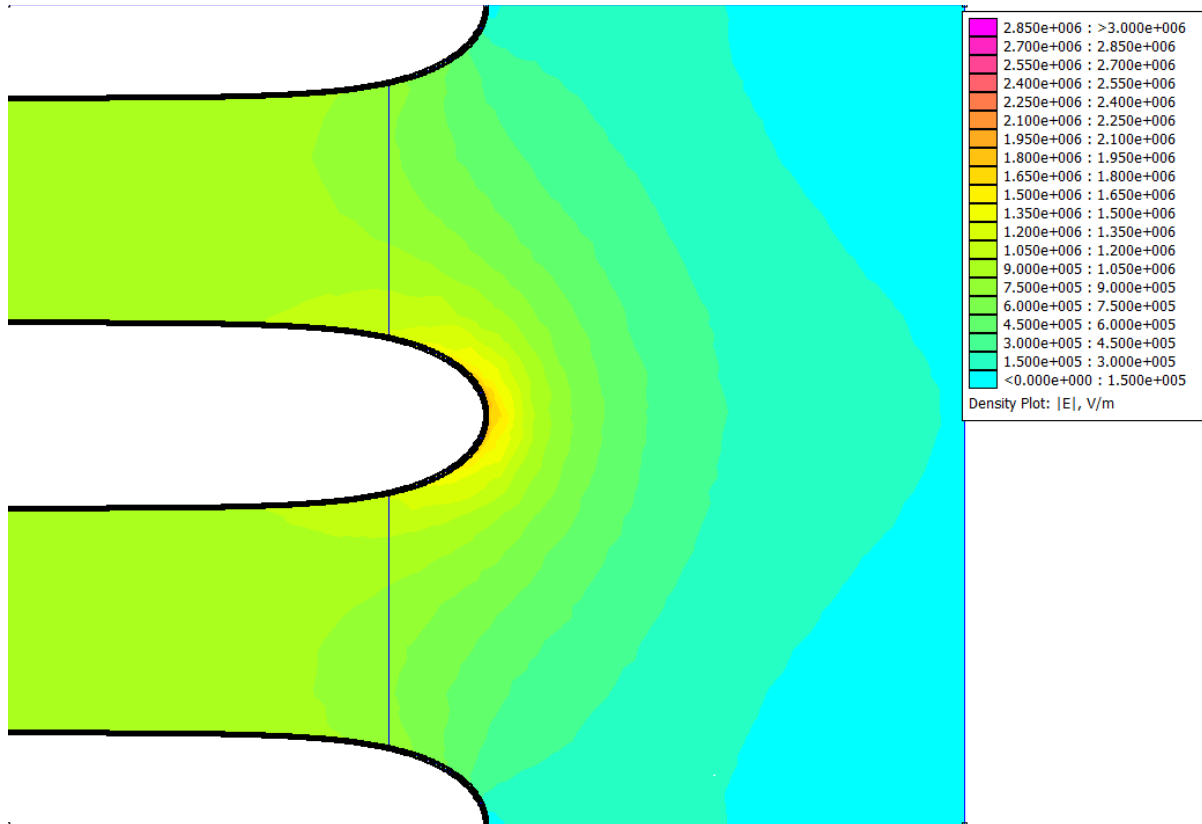


Figure 11 Ernst-C when  $k_0 = 0.002$ , corresponding to radius of 300.6 mm

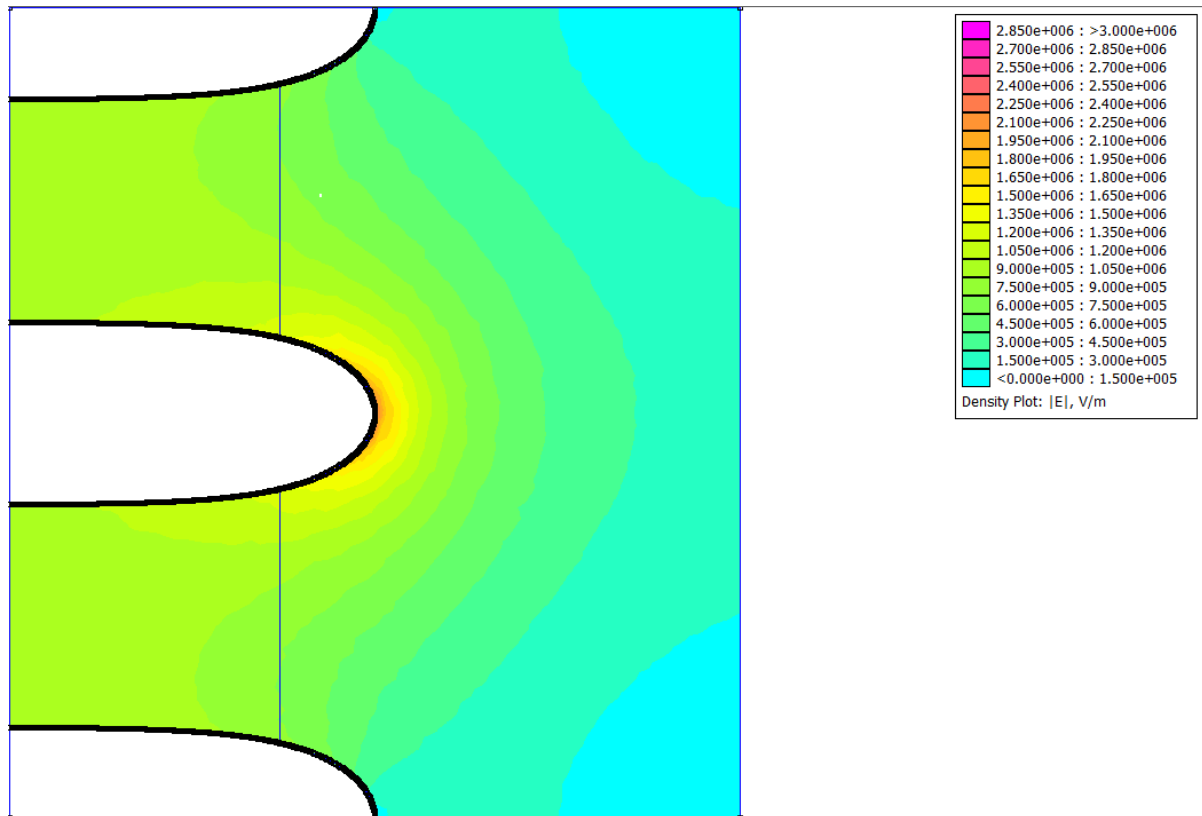


Figure 12 Ernst-C when  $k_0 = 0.01$ , corresponding to radius of 227.8 mm

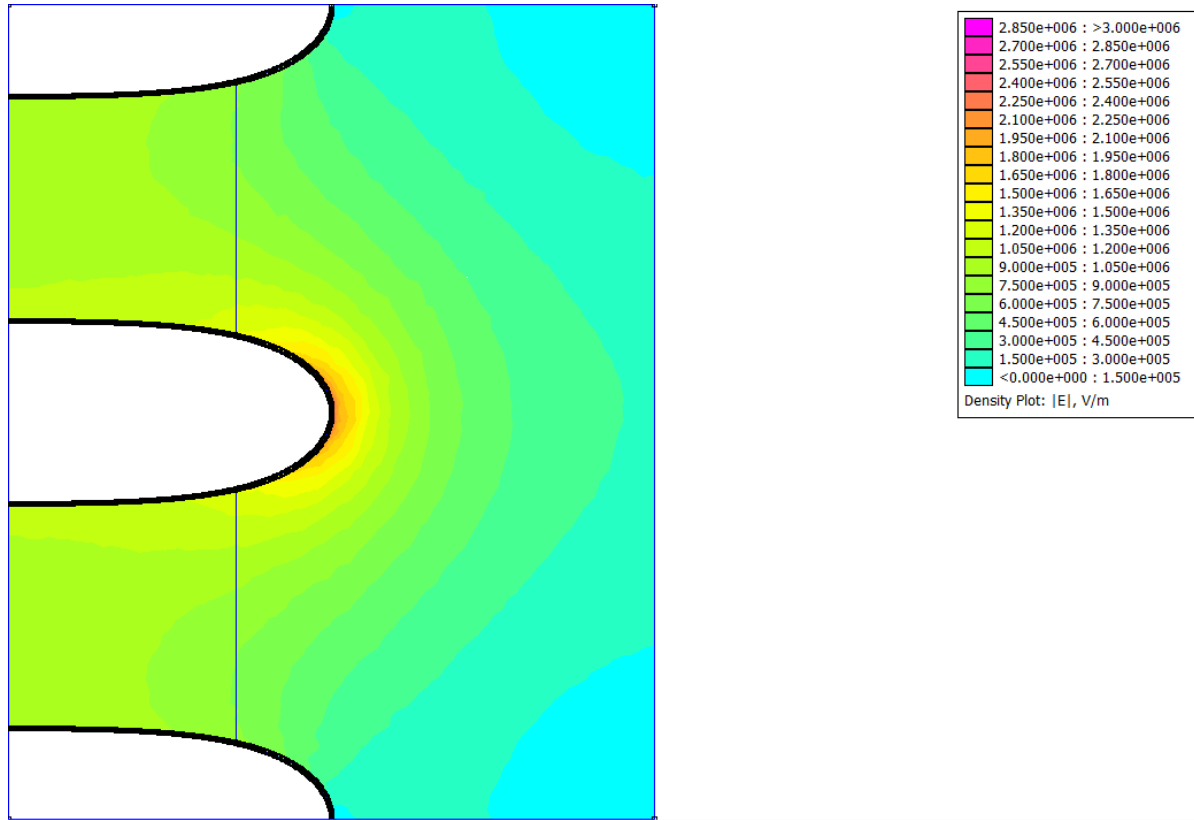


Figure 13 Ernst-C when  $k_0 = 0.018$ , corresponding to radius of 200.8 mm

I gathered data for values of  $k_0$  between 0.002 and 0.018 (inclusive), measuring the ratio of the average electric field along the  $r$ -axis to the average electric field along the  $z$ -axis in an area between the electrodes whose farthest boundary was defined by the “inner radius”, a variable created to mimic the position of the insulator in the EDM cell. Electric field information around the whole cell was certainly not important for homogeneity because the neutrons would not be able to travel through the insulator to the very end of the electrodes. The inner radius was calculated by subtracting half the electrode thickness from its radius, matching the clear ending point of a flat electrode with a half-circle tip.

### Homogeneity by Vacuum Chamber Size

To better understand how electrode shape alone impacts electric field homogeneity, I ran several simulations with a very large vacuum chamber so that the grounded sides would interact less with the charged electrode. However, in order to understand what impact this could have on tests, I also measured how homogeneity varied with vacuum chamber dimensions for multipliers one to ten on normal base dimensions.

Each test had an electrode separation of 140 mm, but all tests other than that with multiplier one had three full electrodes instead of one full electrode in the middle and two half electrodes on the top and bottom. I was looking for large-scale changes, so I did no tests using multipliers between one and two; however, examining this range could be useful in the future to help determine the size of the vacuum chamber in the actual EDM cell.

I found that the electric field homogeneity increased with the multiplier, although very gradually after a major jump from multiplier one to two.

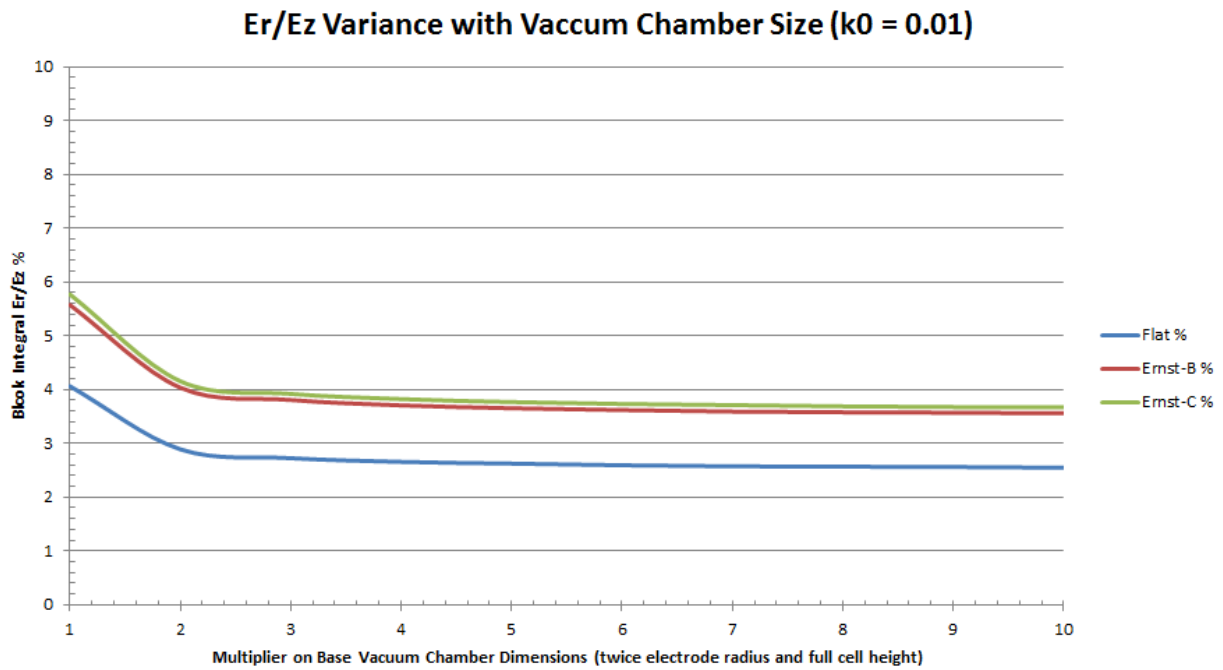


Figure 14 Larger multipliers correspond to greater homogeneity, but there is a large jump between multipliers one and two

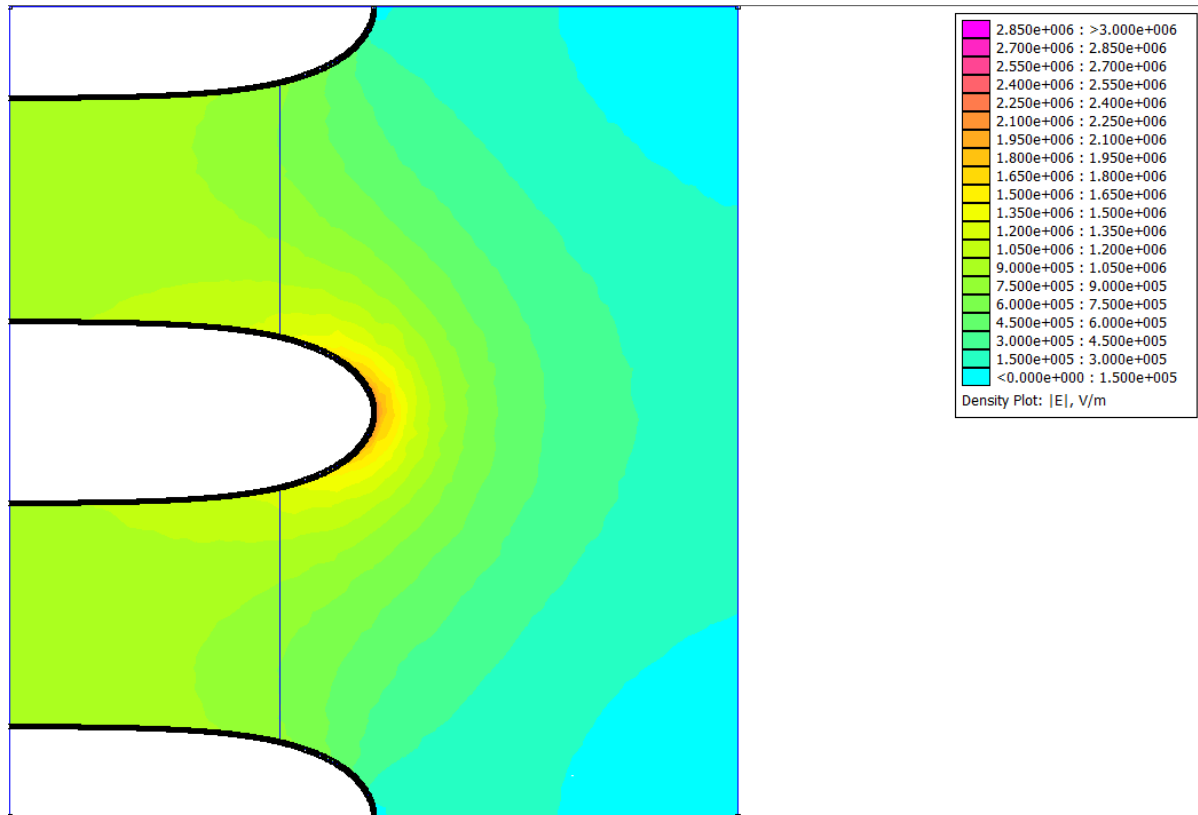


Figure 15 Ernst-C profile when multiplier = 1

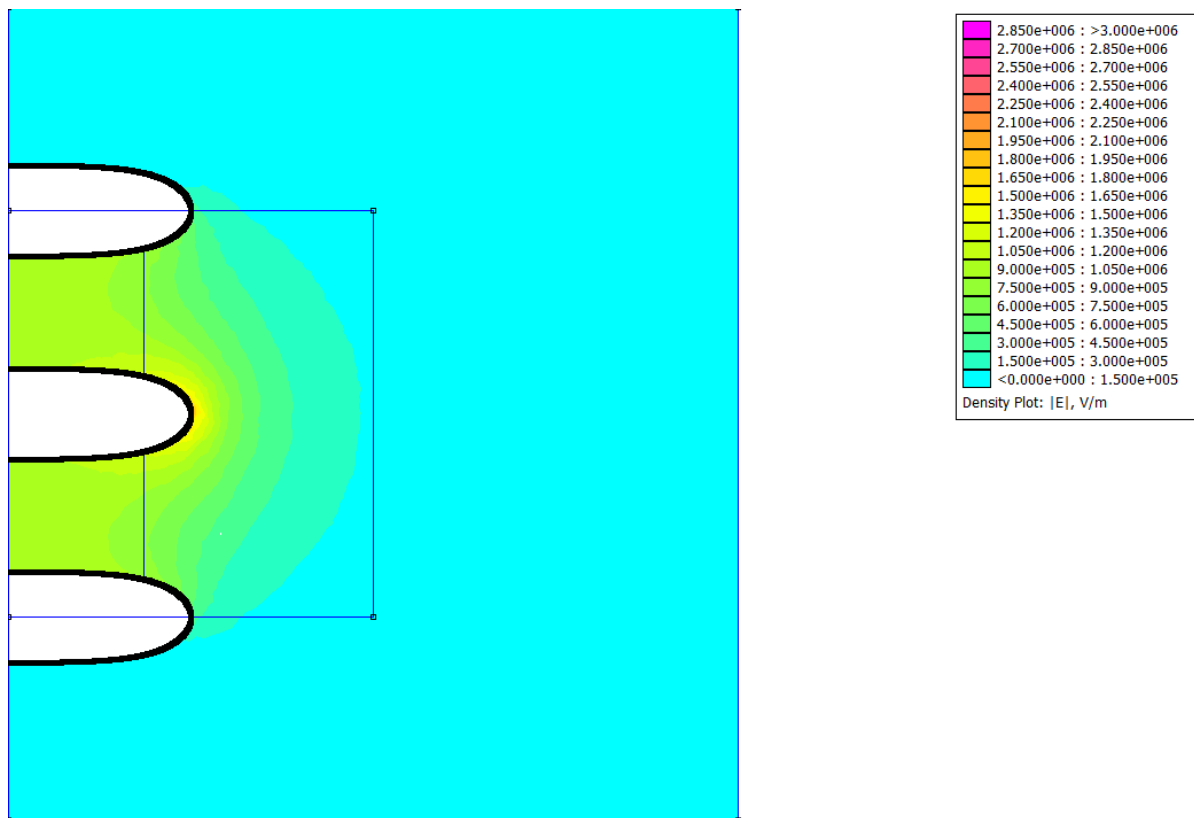


Figure 16 Ernst-C profile when multiplier = 2

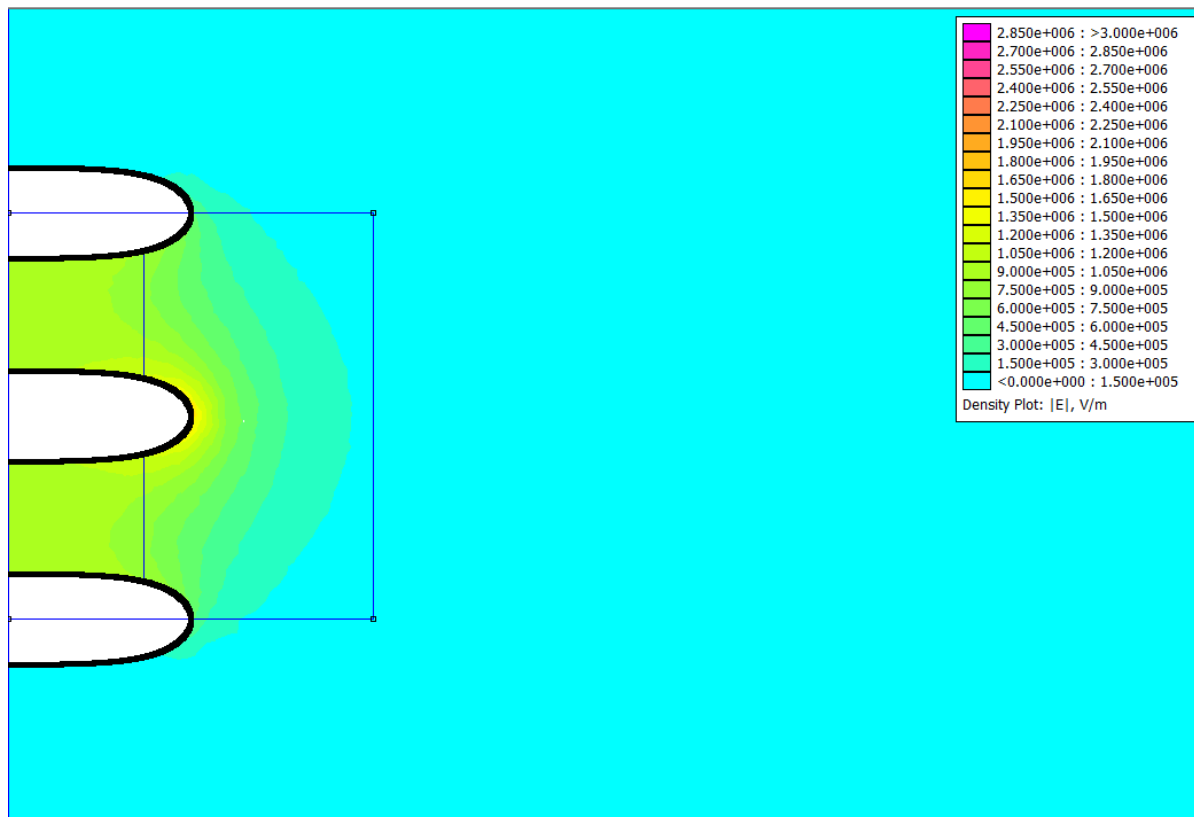


Figure 17 Ernst-C profile when multiplier = 10

## Electric Field Homogeneity by Electrode Separation

Electrode separation is another key variable in modelling electrodes. However, the voltage value assigned to the middle electrode was defined in terms of the electrode separation (high voltage =  $1000 \text{ V} \times \text{electrode separation}$ ), ensuring that the electric field between the electrodes was always  $1 \text{ kV/mm}$ . Since the middle electrode's voltage scales with electrode separation, the electric field should change very little. My measurements appeared to confirm this. Note also that the electrodes themselves scale with electrode separation, so the pictures appear similar.

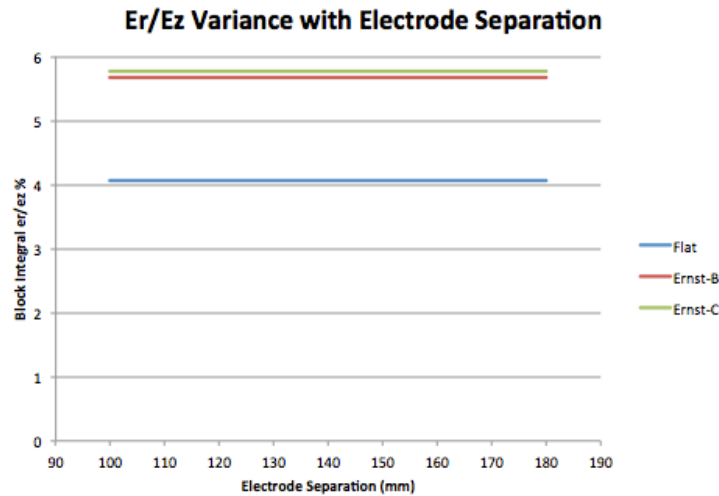


Figure 18 Electrode separation has little impact on homogeneity

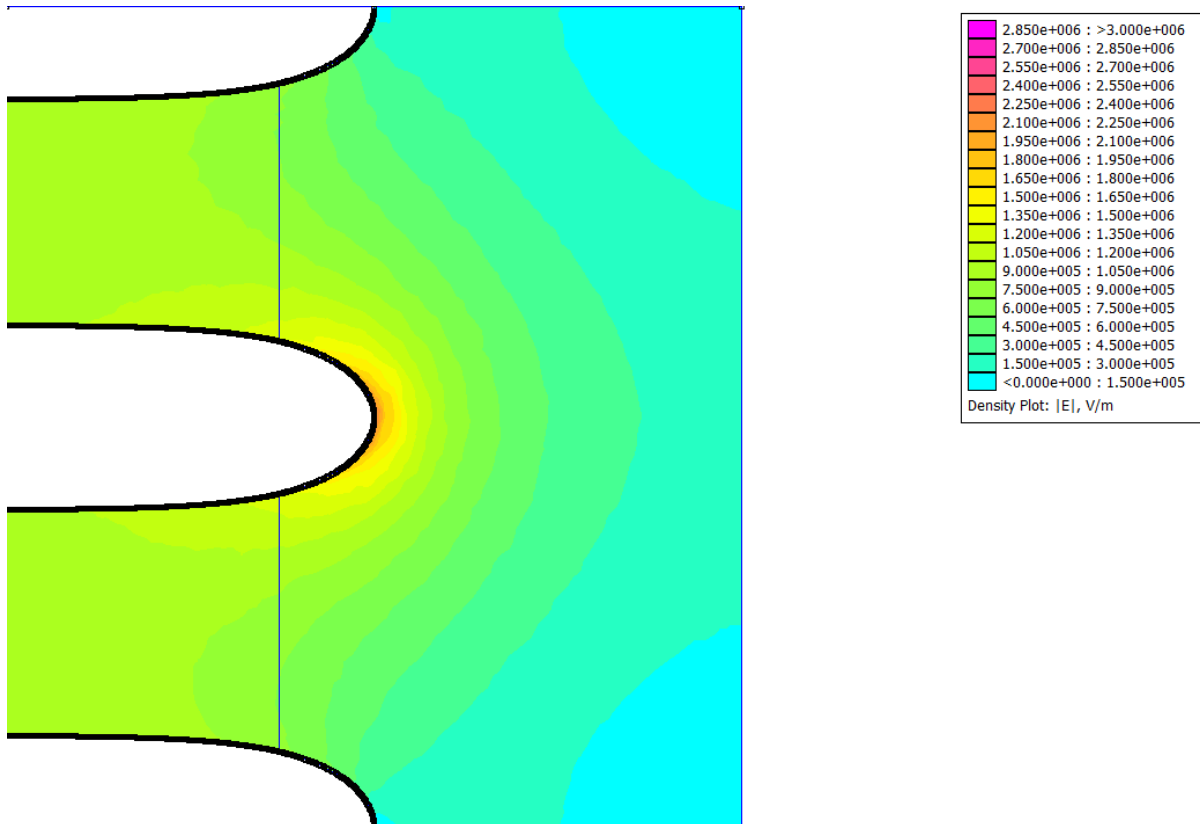


Figure 19 Ernst-C profile with 100 mm electrode separation

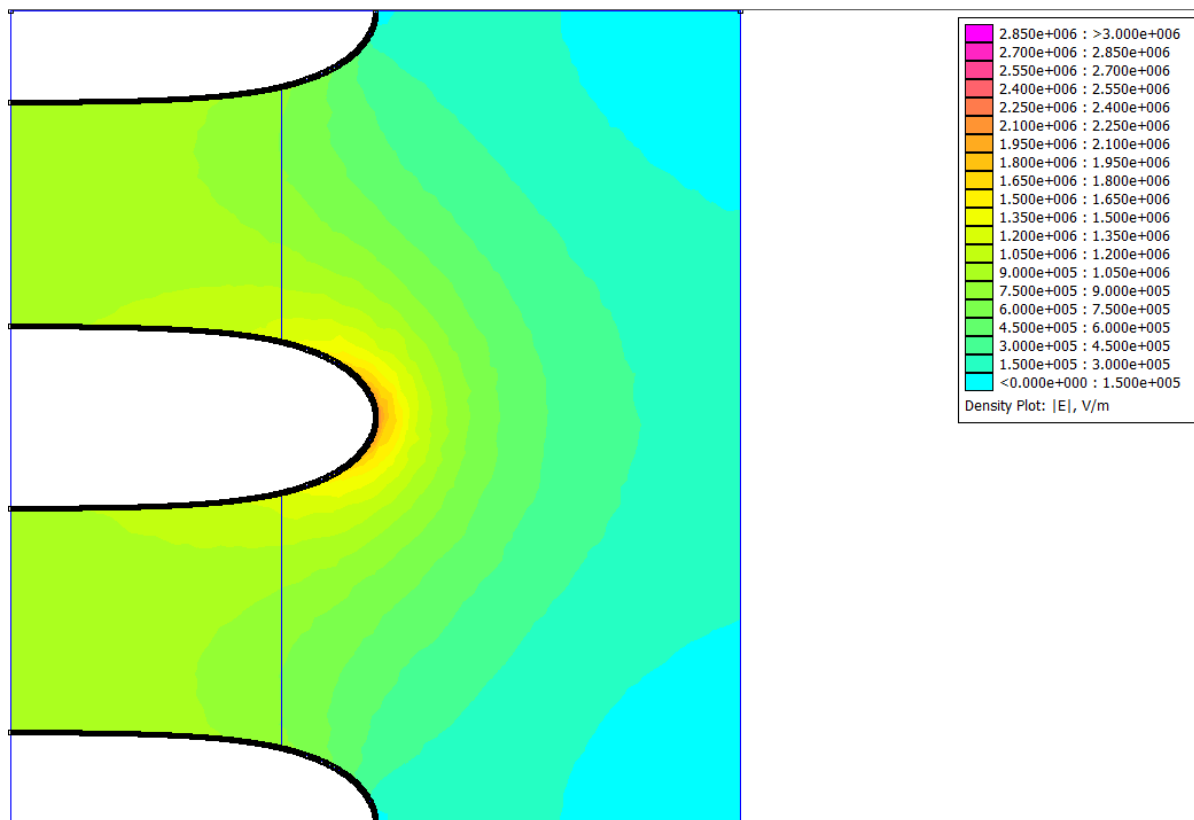


Figure 20 Ernst-C profile with 140 mm electrode separation

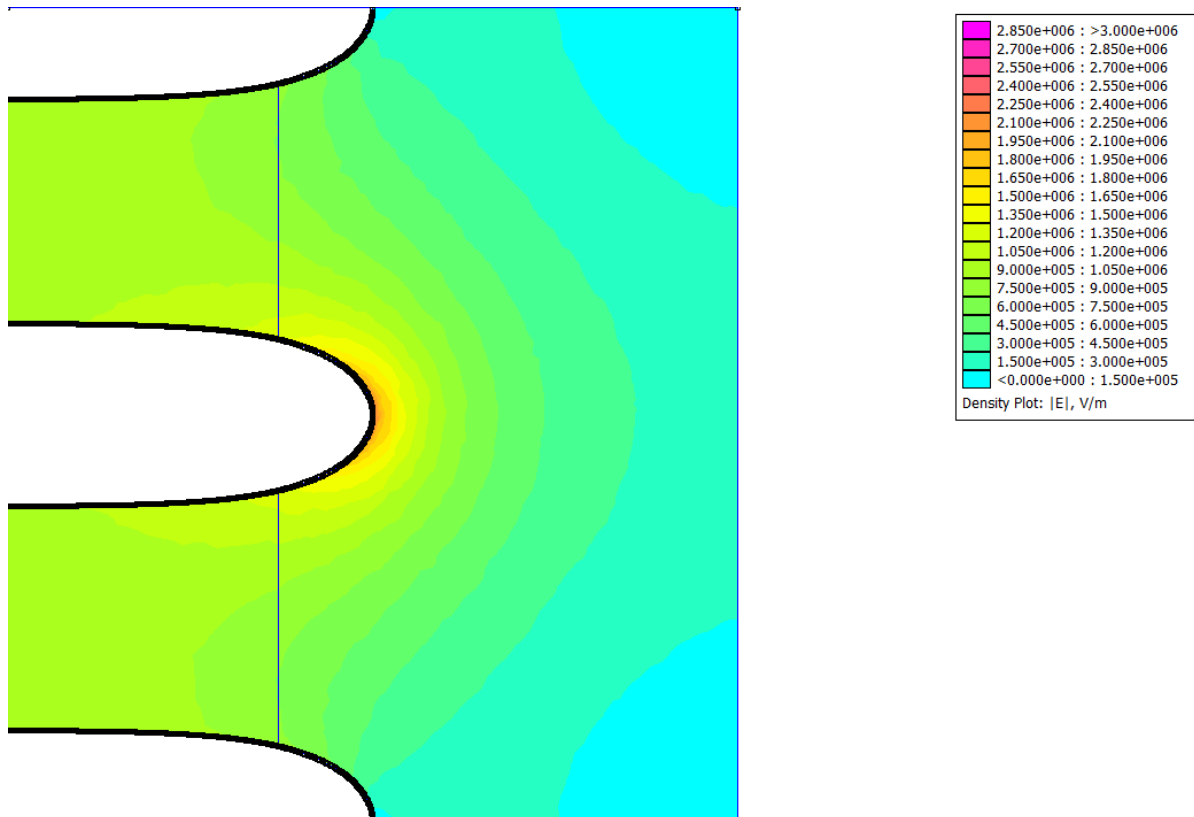


Figure 21 Ernst-C profile with 180 mm electrode separation



## Horizontal Contours

Measurements taken along contours trade completeness for detail: although an entire graph describes only one line between the electrodes, FEMM can give excellent information about how the electric field changes as  $r$  increases along that line. I chose to gather information for each electrode I tested along three contour lines spaced between the top two electrodes (my models were symmetric over  $z = 0$ , so this made no difference). For each of these contours, the locations of which are shown in *Figure 22*, I measured the electric field magnitudes and the electric field percent deviation given by

$$\frac{|\vec{E}_{max}| - |\vec{E}_{min}|}{|\vec{E}_{max}|} \times 100\%.$$

For each of my tests,  $k_0$  was set to 0.01, the electrode separation to 140 mm, and the multiplier on the vacuum chamber size to one. The resulting models are good illustrations of normal flat, Ernst-B, and Ernst-C electrodes, so I've attached pictures below. The Ernst-B and Ernst-C profiles have similar performance along each contour; the flat electrode performed best along the top two contours, more noticeably along the top, but performed worst along the bottom contour. The dotted line in the contour graphs denotes the inner radius described above.

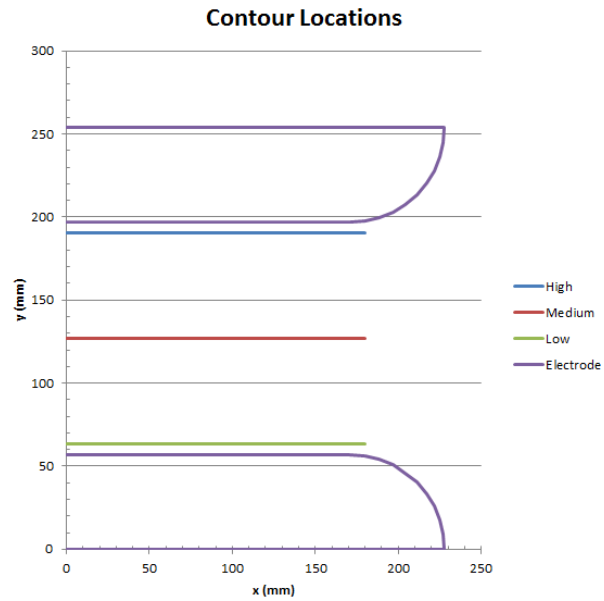


Figure 22 Positions of contours compared to simplified electrode model

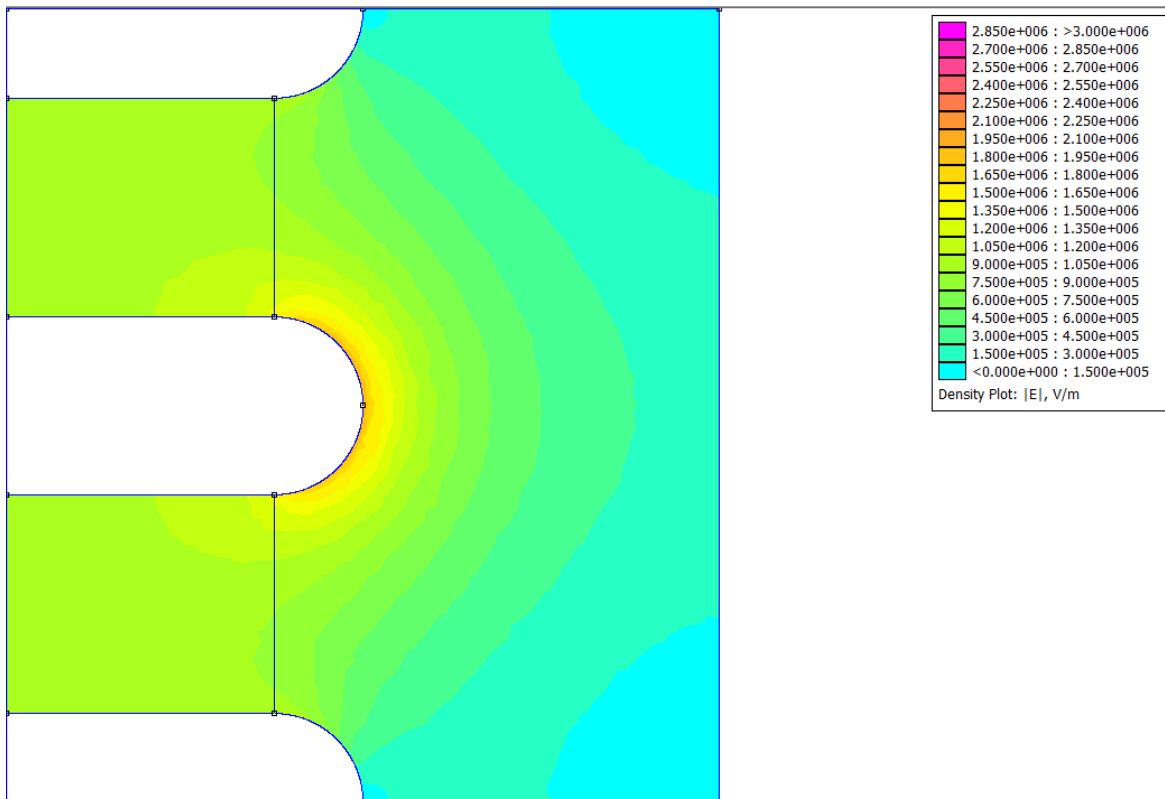


Figure 23 Simple flat electrode model

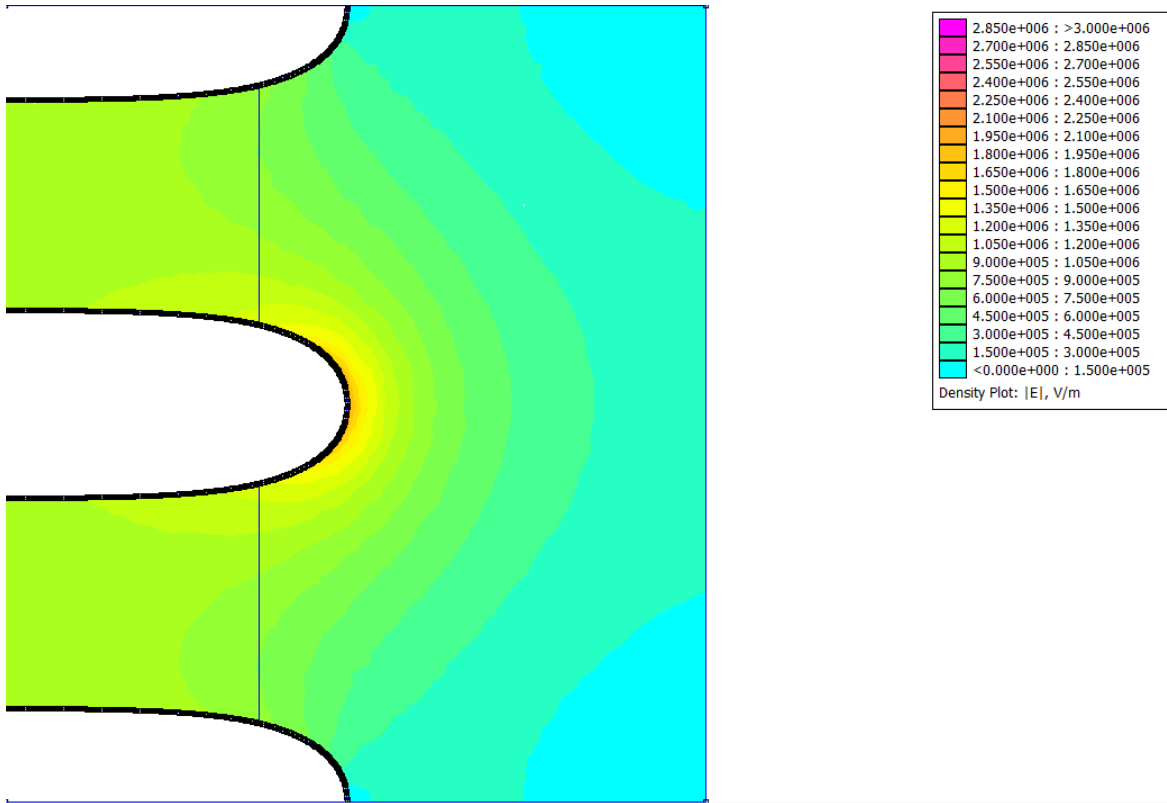


Figure 24 Simple Ernst-B model

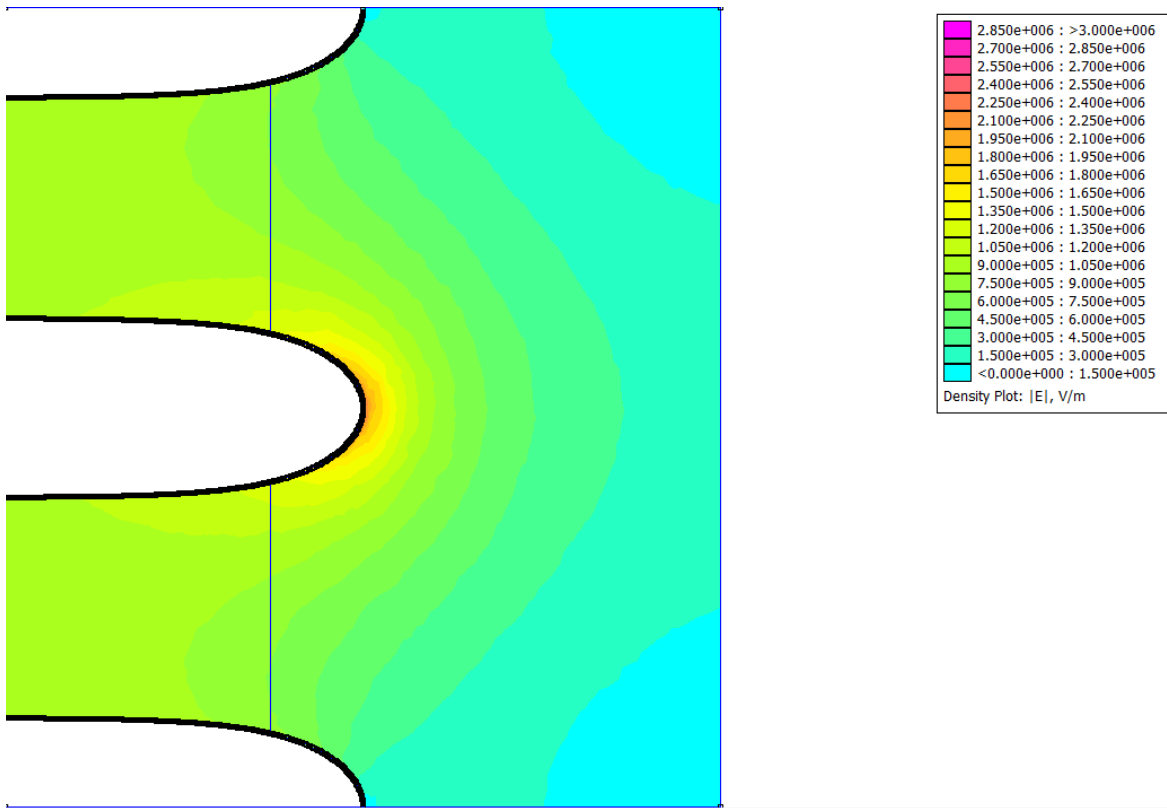


Figure 25 Simple Ernst-C model

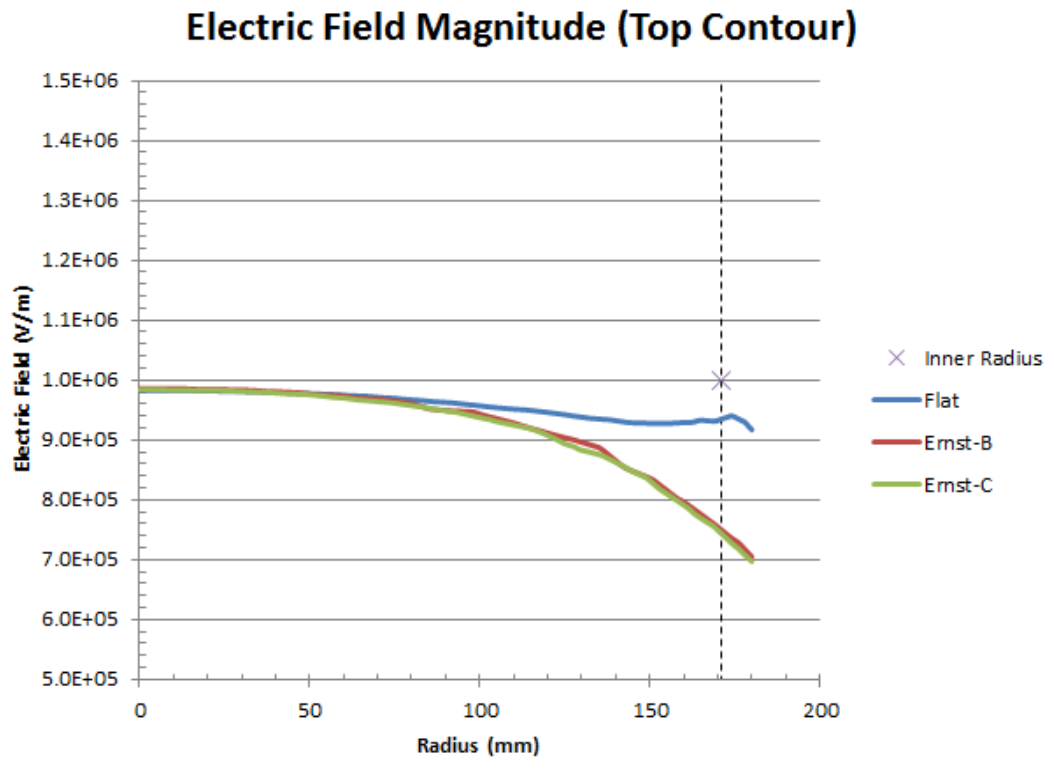


Figure 26 All electrodes' electric field magnitudes decrease along the top contour, but the flat electrode's decreases least

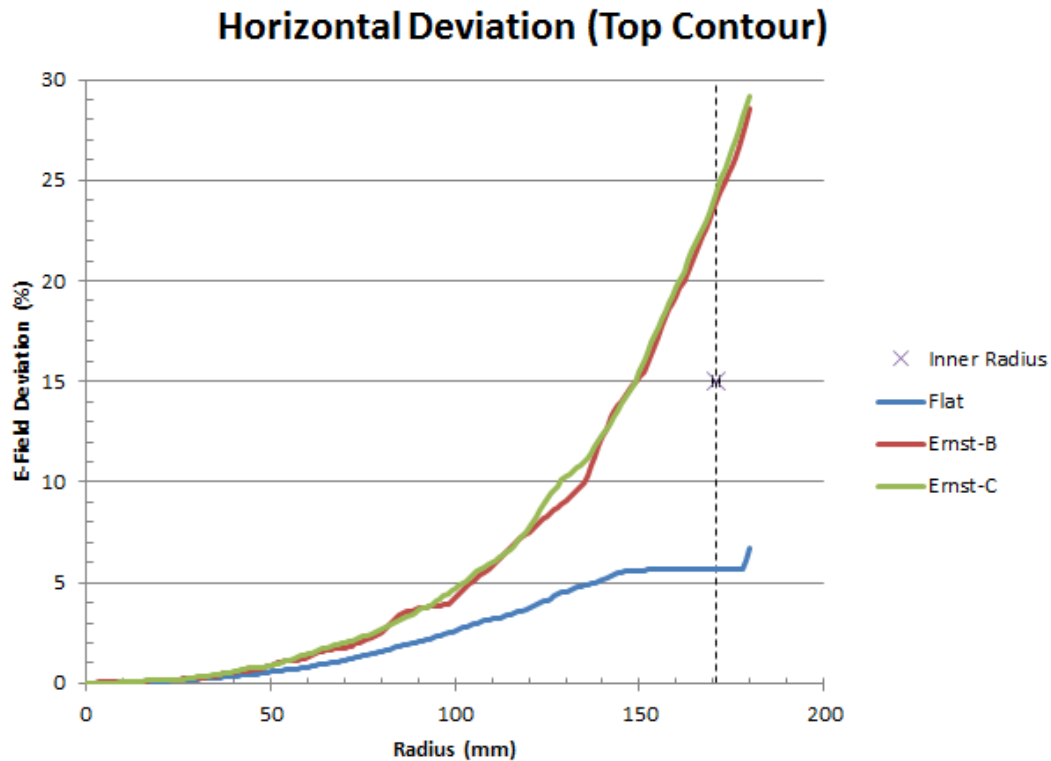


Figure 27 The flat electrode has the smallest percent deviation along the top contour

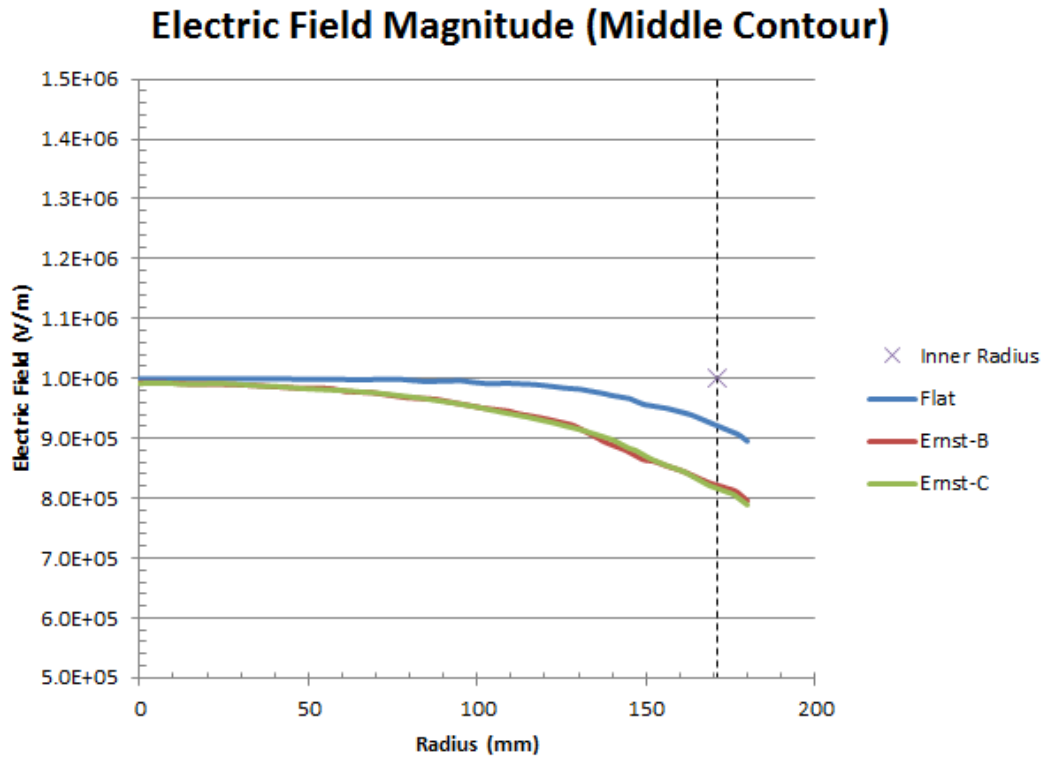


Figure 28 All electrodes' electric field magnitudes decrease along the top contour, but the flat electrode's decreases least

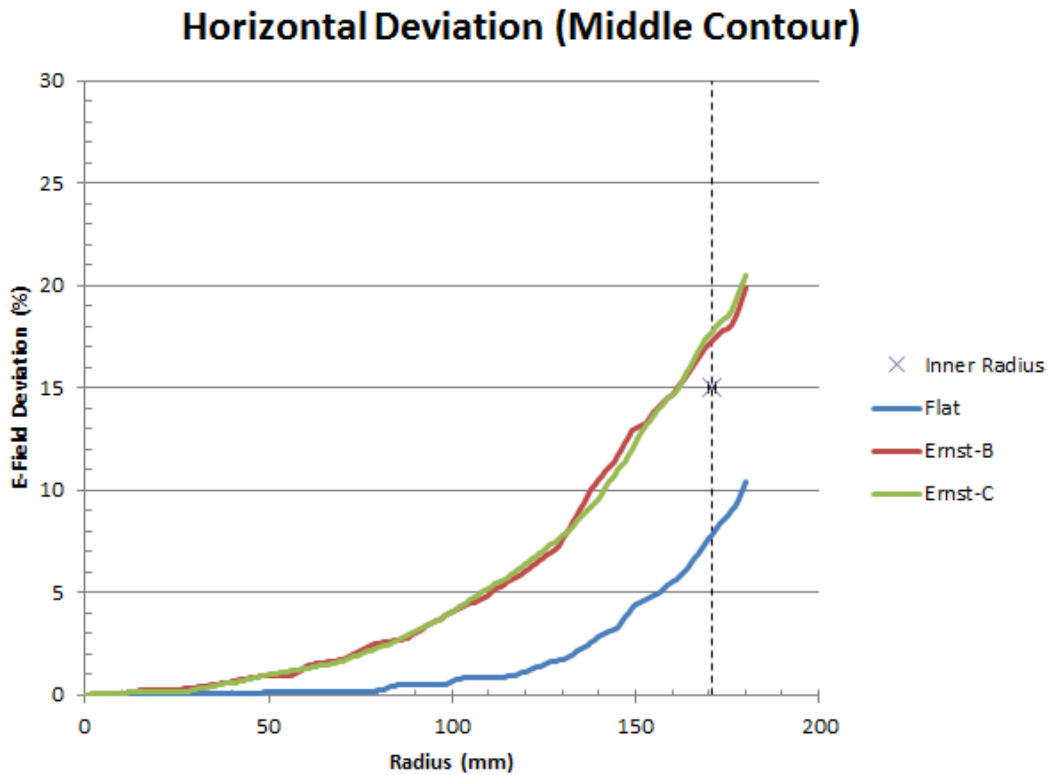


Figure 29 The flat electrode has the smallest percent deviation along the middle contour

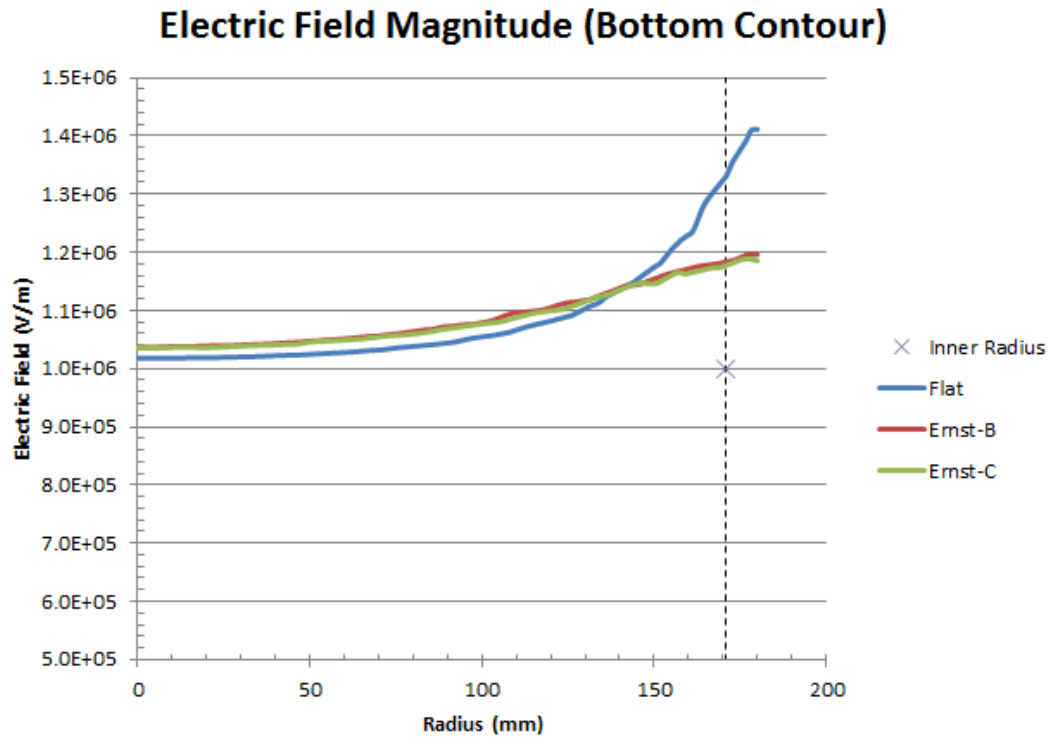


Figure 30 All electrodes' electric field magnitudes increase along the top contour, and the flat electrode's increase most

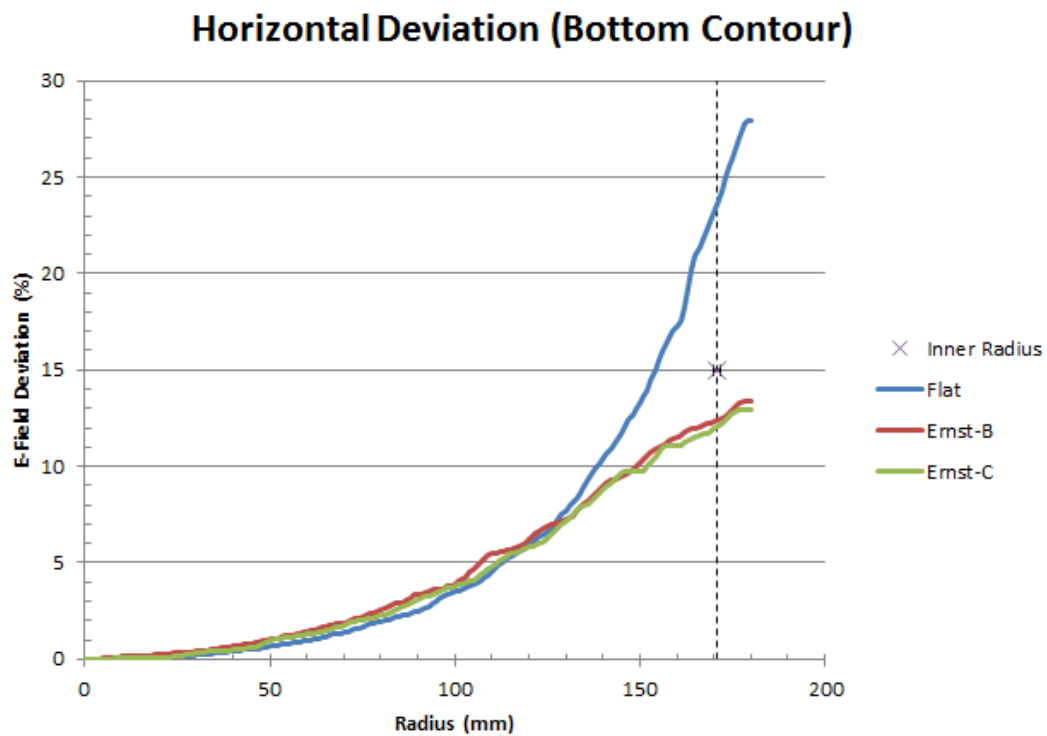


Figure 40 The flat electrode has the largest percent deviation along the middle contour

## Deviation Contours

The flat electrode gives consistently more homogenous numbers than the Ernst profiles, which is certainly counter to my expectations. Trying to understand these results, I looked for a way to see if, perhaps, the Ernst profiles were more homogenous up to a certain radius, after which the flat one was better. To examine this possibility, I made several graphs of “Deviation Contours”. To build these graphs, I set a maximum permitted percent deviation and, for each z-coordinate between the top two electrodes, searched horizontally until the percent deviation, given by

$\frac{|\vec{E}_{max}| - |\vec{E}_{min}|}{|\vec{E}_{max}|} \times 100\%$  was first greater than my maximum permitted value. Recording the x-coordinate at which this took place allowed me to visually compare the three electrode types. As the horizontal contours suggest, the Ernst profiles are best close to the middle electrode, but the flat electrode is best elsewhere.

If the nEDM experiment determines a value for the maximum allowed electric field deviation from centre, this method could be used to illustrate how much space between the electrodes can be used.

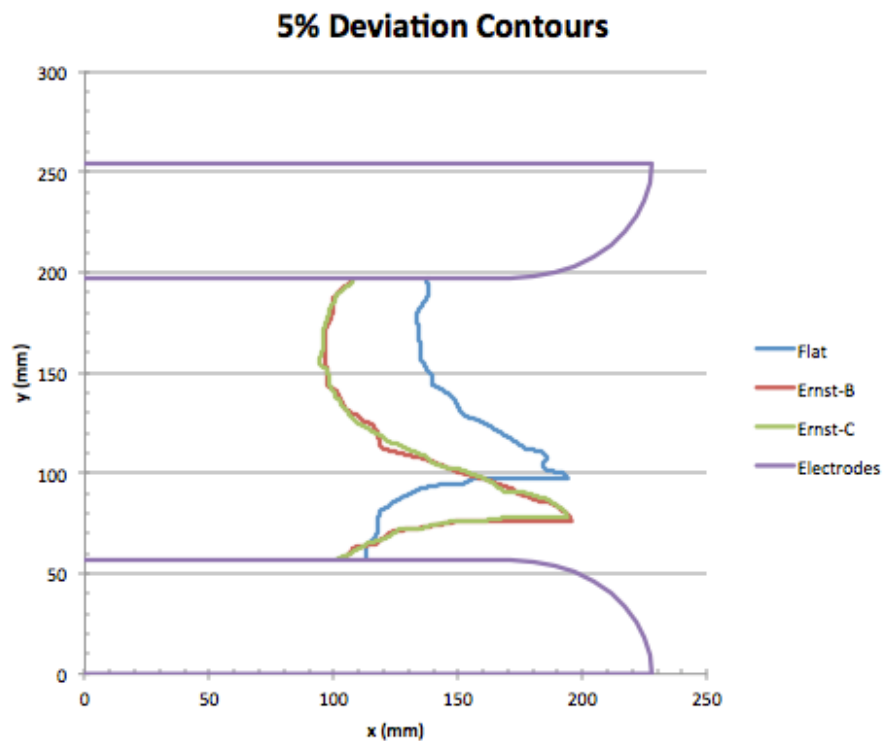


Figure 41 The flat electrode's electric field magnitude does not deviate by greater than 5% until a greater radius than the that of the other electrodes above y = 100 mm; below this, the flat electrode deviates at a smaller radius than the other electrodes except very near the electrode surface

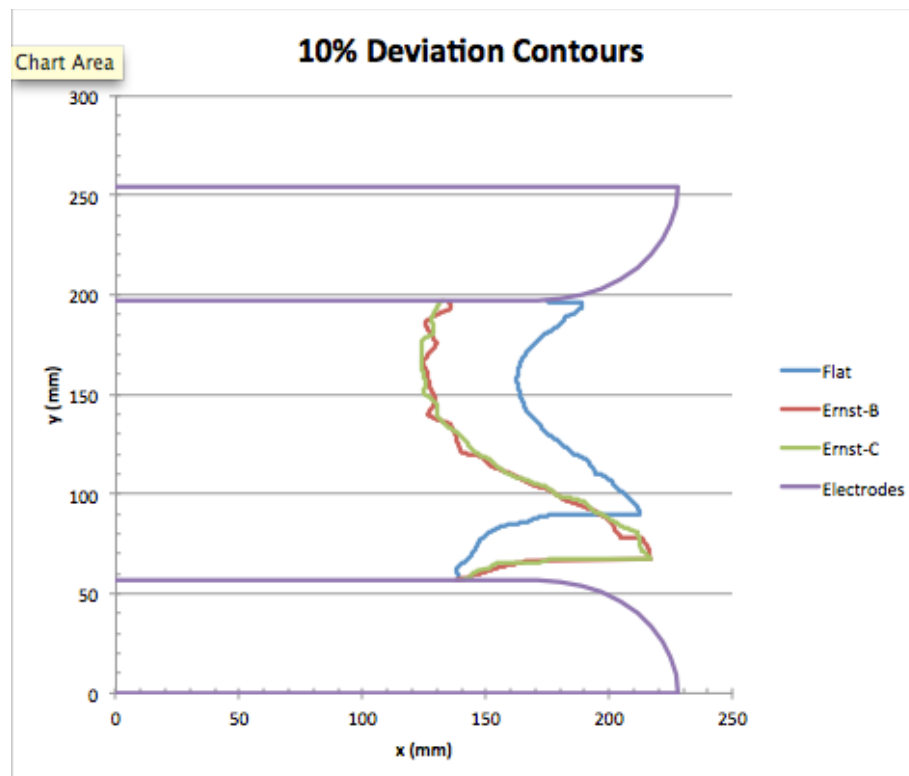


Figure 42 The 10% Deviation Contours exhibit a similar pattern to the 5% Deviation Contours

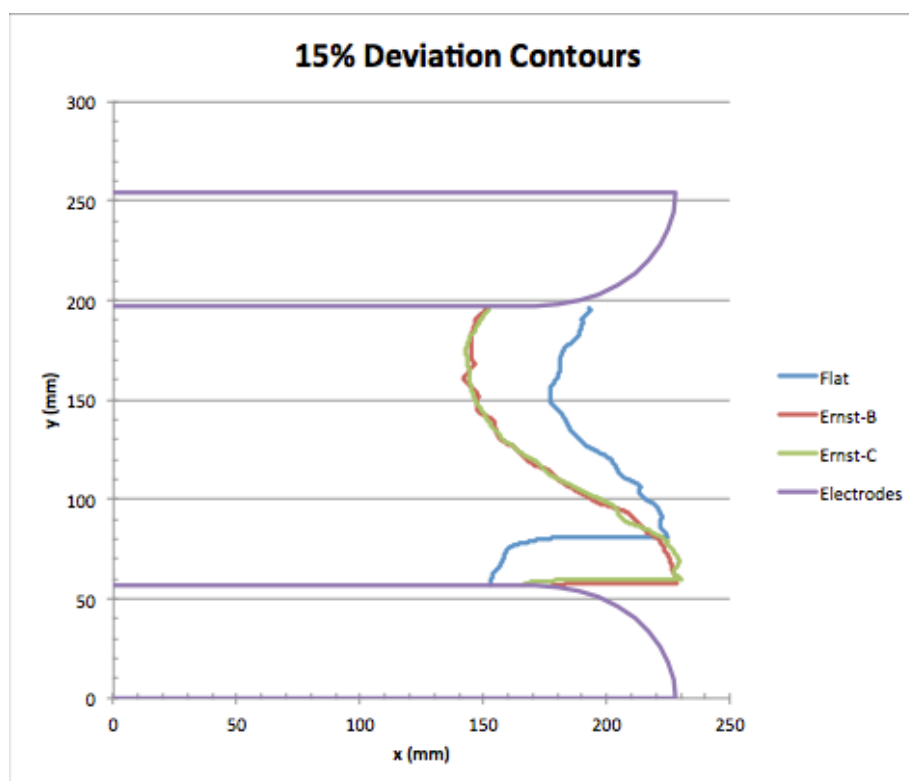


Figure 43 The 15% Deviation Contours exhibit a similar pattern to the 5% Deviation Contours

## Conclusion

Based on my measurements, it appears that the flat electrode is superior to the Ernst profiles in all metrics except contours close to the middle electrode. This, combined with more customizable dimensions and the relative ease of construction, make the flat electrodes more appealing for the EDM cell. However, this is not the expected result; Ernst profiles were designed to create electrodes with more uniform fields than simpler electrodes, but this does not seem to be the case. Gerard Ernst constructed his profile while searching for compact electrodes that produce highly homogenous electric fields “to obtain high output powers from transversely excited pulsed lasers” – I do not know why his homogeneity would be different from ours, but perhaps the distinction originates in the different intended use.<sup>5</sup> Supposedly, such profiles should create electric fields “such that the field is maximum at the center, and gradually diminish[es] towards the edges.”<sup>6</sup> Maybe this intentional diminishing creates fields less homogenous than flat electrodes.

Otherwise, it is possible that other definitions of homogeneity or different measurements would yield results in favour of the Ernst profiles. Or perhaps at the scale we were measuring, the Ernst profiles would normally be only marginally better than flat electrodes, and some other complications make the flat electrodes superior. Whatever the case, the flat electrodes do seem to create more homogenous fields.

Six weeks was not a very long time, and there are several potentially useful continuations of this project that I did not complete. Measurements of the electric field magnitude at the tip of the electrodes would give important information relating to breakdowns, because the electric field is highest in these areas. Exploring vacuum chamber dimensions with a multiplier between one and two could be useful for determining the ideal experimental setups. I did not conduct any tests with the Rogowski profile because the Ernst profile was supposed to yield more homogenous fields; however, since my expectations for the Ernst profiles were not met, perhaps it would be smart to do tests with the Rogowski profile as well. If the flat electrodes are indeed superior, further measurements could be used to optimize the radius and thickness, because these were always matched to the Ernst-C profile in my tests. Finally it could be helpful to add functionality to my Lua code allowing the implementation of O-ring trenches and intake pipes.

Once again, thanks to all those at TRIUMF who helped me gather the information for this report. I really enjoyed my time working on the UCN project and hope that I was of some use! I look forward to doing more physics as interesting as this in the future.

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<sup>5</sup> Gerard Ernst, “Uniform-Field Electrodes with Minimum Width,” *Twente Institute of Technology*, 1984

<sup>6</sup> U. Nundy, D.V.S. Satyanaraya, and N.S.N. Bannerjee, “Uniform Field Electrodes,” *Centre for Advanced Technology*