Course: Discrete Mathematics
Course Code: CSE 125

Sheet: 01
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Topic: Logic Sentences, Logical Connectives

Proposition
It is a declarative sentence that is either true or false. A propositional consists of propositional variables and connectives. We denote the propositional variables by capital letters (A, B, etc). The connectives connect the propositional variables.

E1: Following declarative sentences are propositions

1. Dhaka is the capital of Bangladesh
2. Riyadh is the capital of Egypt
3. 1+1=2
4. 2+2=3

Propositions 1 and 3 are true, whereas 2 and 4 are false

E2: Following are not Propositions

1. How are you?
2. Read it carefully.
3. X+1=2
4. x+y=z

Propositional Variables
Variables that represent propositions. Just as letters are used to denote numerical variables.

Propositional Calculus or propositional logic
The area of logic that deals with propositions is called propositional calculus or logic

Compound Propositions
Which are formed from existing propositions using logical operators.

Connectives

In propositional logic generally we use five connectives which are –

OR (∨)

AND (∧)

Negation/ NOT (¬)

Implication / if-then (→)

If and only if (⇔).

OR (∨) (disjunction of two propositions)

The OR operation of two propositions A and B (written as A∨B) is true if at least any of the propositional variable A or B is true.

The truth table is as follows –

A	B	A ∨ B
True	True	True
True	False	True
False	True	True
False	False	False

AND (∧) (injunction of two proposition)

The AND operation of two propositions A and B (written as A ∧B) is true if both the propositional variable A and B is true.

The truth table is as follows –

A	B	A ∧ B
True	True	True
True	False	False
False	True	False
False	False	False

Negation (¬)

The negation of a proposition A (written as ¬A) is false when A is true and is true when A is false.

The truth table is as follows –

A	¬ A
True	False
False	True

Implication / if-then (→) /conditional statement

An implication A→B is the proposition "if A, then B". It is false if A is true and B is false. The rest cases are true.

The truth table is as follows –

A	B	A → B
True	True	True
True	False	False
False	True	True
False	False	True

If and only if (⇔)/biconditional

A⇔B is bi-conditional logical connective which is true when p and q are same, i.e. both are false or both are true.

The truth table is as follows –

A	B	A ⇔ B
True	True	True
True	False	False
False	True	False
False	False	True

Exclusive OR of Two Propositions

Let p and q be propositions. The exclusive or of p and q, denoted by $p \oplus q$, is the proposition that is true when exactly one of p and q is true and is false otherwise.

A	B	$A \oplus B$
True	True	False
True	False	True
False	True	True
False	False	False

- Read Converse, Contrapositive and inverse from page 29
- Example 9

Inverse, Converse, and Contra-positive

Implication / if-then (→) is also called a conditional statement. It has two parts –

- Hypothesis, p
- Conclusion, q

As mentioned earlier, it is denoted as p→q.

Example of **Conditional Statement** – "If you do your homework, you will not be punished." Here, "you do your homework" is the hypothesis, p, and "you will not be punished" is the conclusion, q.

Inverse – An inverse of the conditional statement is the negation of both the hypothesis and the conclusion. If the

statement is “If p, then q”, the inverse will be “If not p, then not q”. Thus the inverse of $p \rightarrow q$ is $\neg p \rightarrow \neg q$.

Example – The inverse of “If you do your homework, you will not be punished” is “If you do not do your homework, you will be punished.”

Converse – The converse of the conditional statement is computed by interchanging the hypothesis and the conclusion. If the statement is “If p, then q”, the converse will be “If q, then p”. The converse of $p \rightarrow q$ is $q \rightarrow p$.

Example – The converse of "If you do your homework, you will not be punished" is "If you will not be punished, you do your homework”.

Contra-positive – The contra–positive of the conditional is computed by interchanging the hypothesis and the conclusion of the inverse statement. If the statement is “If p, then q”, the contra–positive will be “If not q, then not p”. The contra–positive of $p \rightarrow q$ is $\neg q \rightarrow \neg p$.

Example – The Contra–positive of " If you do your homework, you will not be punished” is "If you are punished, you did not do your homework”.

FROM BOOK: