

Pendulum.

Frictional force
Damping motion

Topic :

Definition of a simple
harmonic motion.

$$F \propto -x$$
$$F = -kx$$

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Simple harmonic motion is such a motion where the displacement is proportional to the applied force and is negatively directed.

$$\underline{F} = -k \underline{x}$$

where, k is a constant, it is called spring constant.



Sine wave

Equation of motion for a SHO

We know that the motion of a harmonic oscillator can be written as

$$\boxed{\bar{F} = -k\bar{x}}$$

$$m a = - k x \quad (\text{using Newton's law})$$

$$m \frac{dv}{dt} = - k x$$

$$\begin{aligned} a &= \frac{dv}{dt} \\ &= \end{aligned}$$

$$\omega m \frac{d}{dt} \left(\frac{dx}{dt} \right) = - k x$$

$$\begin{aligned} v &= \frac{dx}{dt} \\ &= \end{aligned}$$

$$\frac{m}{\cancel{m}} \frac{d^2x}{dt^2} + \frac{k}{\cancel{m}} x = \cancel{\omega_m}$$

$$\frac{d^2x}{dt^2} + \frac{k}{m} x = 0$$

Let us suppose $\frac{k}{m} = \omega^2$

$$\frac{d^2x}{dt^2} + \omega^2 x = 0 \quad \text{--- } ①$$

So eq. ① is a second order differential
equation of a simple harmonic motion.

Find a solution of the eq. of motion of a SHO

We know that the equation of motion of a simple harmonic oscillator can be written as,

$$\frac{d^2x}{dt^2} + \omega^2 x = 0 \quad \text{--- (1)}$$

Let us consider a sinusoidal solution for equation (1)

$$x = A \sin(\omega t - \phi)$$

$$x \Rightarrow \sin(kx - \omega t)$$

$$y = \underline{\sin \theta}$$

$$y = \underline{\cos \theta}$$



$$\theta = 0^\circ \Rightarrow \sin \theta = 0$$

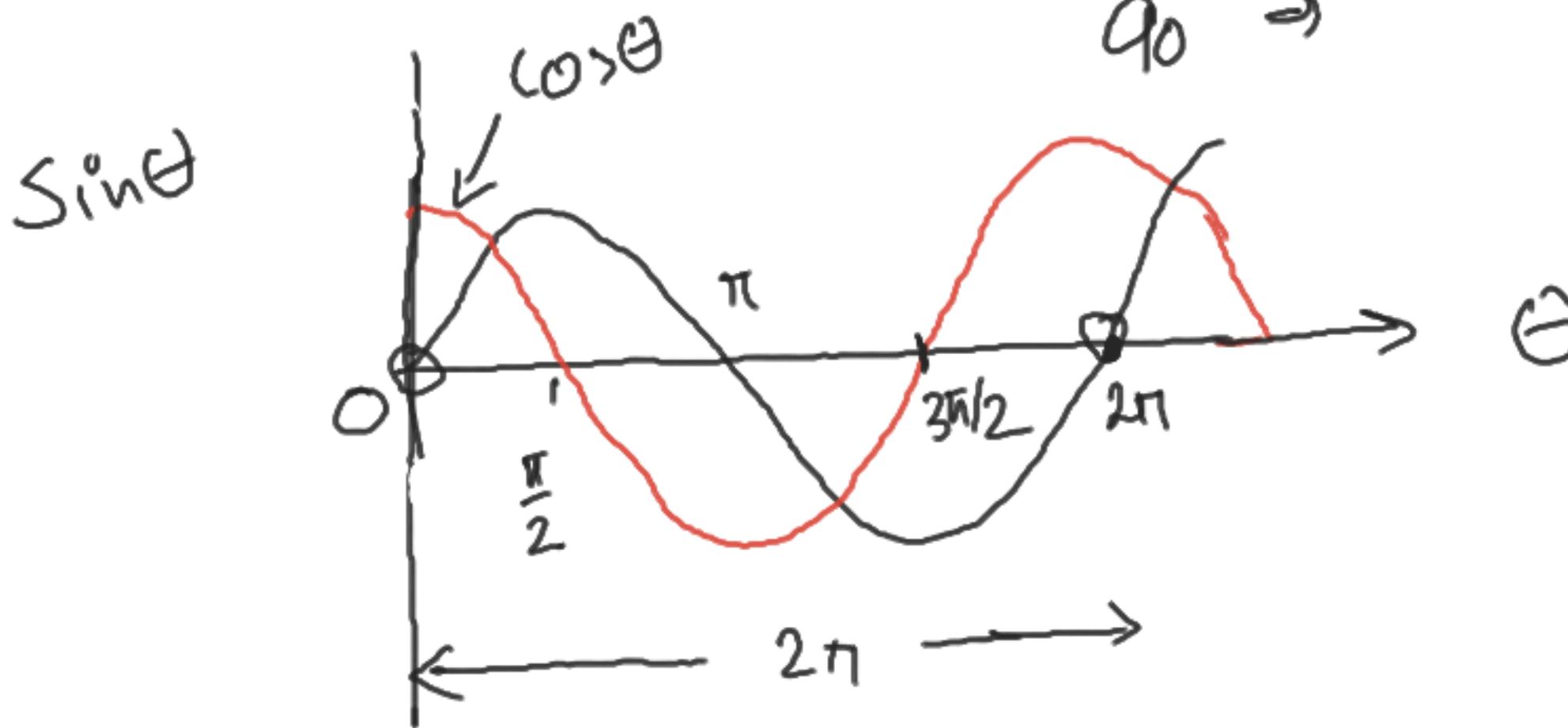


FIG. 4.

$$\sin \theta = \frac{\text{ଲକ୍ଷ୍ୟ}}{\text{ହାତ୍ୟକ}}$$



0, 2π, 4π, 6π

FIG. 5

We assume that the solution will be

$$x = A \sin(kx - \omega t) \longrightarrow ②$$

where, A = amplitude

k = wave number

ω = angular frequency

L.H.S of eq. ① $\frac{d^2}{dt^2} [A \sin(kx - \omega t)] + \omega^2 A \sin(kx - \omega t)$

$$\text{LHS} = \frac{d^2}{dt^2} \left[A \sin(kx - \omega t) + \frac{\omega^2 A \sin(kx - \omega t)}{2} \right]$$

$$= -\omega^2 A \frac{d}{dt} \left[\cos(kx - \underline{\omega t}) \right] + \cancel{\text{II}}$$

$$= +\omega^2 A (-) \sin(kx - \omega t) + \cancel{\text{II}}$$

$$= -\underline{\omega^2 A \sin(kx - \omega t)} + \underline{\omega^2 A \sin(kx - \omega t)}$$

$$= 0$$

LHS = RHS

$$\frac{d^2x}{dt^2}$$

$$x = A \sin \omega t$$

$$\frac{d}{dt} (A \sin \omega t)$$

$$\frac{d}{dt} \left[\frac{d}{dt} (A \sin \omega t) \right]$$

Find the

$$\omega A \frac{d}{dt} [\cos \omega t]) \\ - \omega^2 \underline{A \sin \omega t} = -\omega^2 x$$

What is this

definition of SHO?

Find the

equation of a SHO.

solution of a SHO