Proposition :- A proposition is a declarative sentence that is either true or false but not both.

Connectives

And – $\wedge$

OR – $\vee$

Negation / Not – $\neg$

Implication – $\longrightarrow$

Biconditional – $\longleftrightarrow$

Negation of p

$\neg p$

p	$\neg p$
T	F
F	T

$p \wedge q$

└ Conjuction

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

$p \vee q$

└ OR

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Conditional statement

$$p \rightarrow q$$

The conditional statement $p \rightarrow q$ is false when p is true & q is false & true otherwise

Here,

p is hypothesis

q is Conclusion

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Biconditional

$p \leftrightarrow q$ → If and only if

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Exclusive or

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Inverse, Converse, Contrapositive

$p \rightarrow q$ — (If p, than q)

Hypothisis   Conclusion

$p \rightarrow$ You do your homework

$q \rightarrow$ You will not be punished

If you do your homework than you will not be punished.

Inverse :

Negation of both hypothisis & conclusion

$\neg p \rightarrow \neg q$

If not p than not q

* If you do your homework than you will not be punished.

Inverse : If you do not do your homework than you will be punished.

Converse: If q than p

$q \to p$

If ~~d~~ you $\underbrace{\text{do your homework}}_{p}$ you $\underbrace{\text{will not be}}_{q}$ punished.

⊛ If you will not be punished than you do your homework.

Contrapositive:

$\neg q \to \neg p$

If not q than not p

If you are punished than you did not do your homework.

⊛ If it is raining than the grass is wet.

Inverse: $\neg p \to \neg q$

If it is not raining than the grass is not wet

Converse :- $q \to p$

If the grass is wet than it is raining.

Contrapositive :-

$$\neg q \to \neg p$$

If the grass is not wet than it is not raining.

(*) Sentence এ whenever যখন থাকবে তখন আগে q হবে।

The home team wins whenever it is raining
$\underbrace{\text{The home team wins}}_{q}$ whenever $\underbrace{\text{it is raining}}_{p}$

Inverse :-

$$\neg p \to \neg q$$

If it is not raining than the home team does not wins.

Converse :

$$q \to p$$

If the home team wins than it is raining.

Constrapositive : $\neg q \to \neg p$

If the home team does not wins than it is not raining.

田 Let p and q be the proposition "The election is decited" and the votes have been counted "

a) $\neg p$

The election is not decited.

b) $p \vee q$

The election is not decided or the votes have been counted.

c) $\neg p \wedge q$

The election is not decided and but the votes have been counted.

d) $q \rightarrow p$

The votes have been counted whenever the election is decided.

e) $\neg q \rightarrow \neg p$

The votes haven't been counted whenever the electi is not decited

f) $\neg p \rightarrow q$

If the election is not decided than the votes haven't been counted.

g) $p \leftrightarrow q$ [If and only if, neccessary and sufficient

The election is not decited if and only if the votes have been counted.

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h) $\neg q \overset{(on)}{\vee} (\neg p \overset{and}{\wedge} q)$

Eithen the votes have not been counted on the election is not decided but the voted have been counted.

d) ~~$q \to p$~~

Truth table

a) $p \land \neg p$

p	$\neg p$	$p \land \neg p$
T	F	F
T	F	F
F	T	F
F	T	F

b) $(p \lor q) \to (p \land q)$

p	q	$p \lor q$	$p \land q$	$p \lor q \to p \land q$
T	T	T	T	T
T	F	T	F	F
F	T	T	F	F
F	F	F	F	T

c) $(p \lor \neg q) \to q$

p	q	$\neg q$	$p \lor \neg q$	$p \lor \neg q \to q$
T	T	F	T	T
T	F	T	T	F
F	T	F	F	T
F	F	T	T	F

d) $(p \to q) \leftrightarrow (\neg q \to \neg p)$

p	$\neg p$	q	$\neg q$	$p \to q$	$\neg q \to \neg p$	$(p\to q) \leftrightarrow (\neg q \to \neg p)$
T	F	T	F	T	T	T
T	F	F	T	F	F	T
F	T	T	F	T	T	T
F	T	F	T	T	T	T

Predicate

$x > 3$ — x is greater than 3

x — Variable (subject)

greater than 3 — (Predicate)

Eg: Let $P(x)$ denote the statement $x > 3$ what are the truth $P(4)$ & $P(2)$

$P(4) \Rightarrow x > 3$

$\Rightarrow 4 > 3$ T

$P(2) \Rightarrow x > 3$

$\Rightarrow 2 > 3$ F

Q: Let $Q(x,y)$ denote the statement $x = y + 3$ what are the truth valus of the proposition.

$Q(1,2)$ & $Q(3,0)$

$Q(1,2) \Rightarrow x = y + 3$

$\Rightarrow 1 = 2 + 3$

$\Rightarrow 1 \stackrel{?}{=} 5$

$\therefore 1 \neq 5$ F

$Q(3,0) \Rightarrow x \stackrel{?}{=} y + 3$

$\Rightarrow 3 \stackrel{?}{=} 0 + 3$

$\Rightarrow 3 = 3$

T

Q: Let $R(x, y, z)$ denote the statement $x+y = z$.
what are the truth values of the proposition
$R(1, 2, 3)$ & $R(0, 0, 1)$

$R(1, 2, 3) \Rightarrow x+y = z$

$\Rightarrow 1+2 = 3$

$\Rightarrow 3 = 3$ T

$R(0, 0, 1) \Rightarrow x+y = z$

$\Rightarrow 0+0 = 1$

$\Rightarrow 0 \neq 1$ F

State the value of x after the statement if $P(x)$ then $x := 1$ is execute where $P(x)$ is the statement "$x > 1$". Thus if $x > 1$ then $x := 1$

a) $x = 0$

$\Rightarrow x > 1$

$\Rightarrow 0 > 1$ is false & will not execute.

b) $n = 1$

$n > 1$

$1 > 1$ F

c) $n = 2$

$2 > 1$ - True, the assignment statement will execute & $n = 1$.

Eg: Let $P(n)$ denote the statement $n \leq 4$ what are the truth valus —

a) $P(0)$

$P(n) = n \leq 4$

$P(0) = 0 \leq 4$

T

b) ~~$P(4) = 6$~~ ≤ 4

b) $P(4) = 4 \leq 4$ T

c) $P(6) = 6 \leq 4$ F

Logical equivalences.

Tautology — Always T

Contradiction - Always F

$p \vee \neg p$ & $p \wedge \neg p$

p	$\neg p$	$p \vee \neg p$	$p \wedge \neg p$
T	F	T	F
T	F	T	F
F	T	T	F
F	T	T	F

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Show that the conditional statement is a tautology by using truth table.

$(p \wedge q) \rightarrow p$

p	q	$p \wedge q$	$(p \wedge q) \rightarrow p$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

Law

$$\left.\begin{aligned} p \wedge T &\equiv p \\ p \vee F &\equiv p \end{aligned}\right\}$$ Identity Law

p	T	F	$p \wedge T$	$p \vee F$
T	T	F	T	T
F	T	F	F	F

$$\left.\begin{aligned} p \vee T &\equiv T \\ p \wedge F &\equiv F \end{aligned}\right\}$$ Domination Law

p	T	F	$p \vee T$	$p \wedge F$
T	T	F	T	F
F	T	F	T	F

$\neg(\neg p) \equiv p$ [Double nagation Law]

$$\left.\begin{aligned} (p \vee q) \vee r &\equiv p \vee (q \vee r) \\ (p \wedge q) \wedge r &\equiv p \wedge (q \wedge r) \end{aligned}\right\}$$ Associative Laws.

$$\left.\begin{aligned} p \vee q &\equiv q \vee p \\ p \wedge q &\equiv q \wedge p \end{aligned}\right\}$$ Commutative Law.

$$p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$$
$$p \land (q \lor r) \equiv (p \land q) \lor (p \land r)$$
Distributive La[ws]

$$p \lor \neg p \equiv T$$
$$p \land \neg p \equiv F$$
Negation Laws.

$$\neg(p \land q) \equiv \neg p \lor \neg q$$
$$\neg(p \lor q) \equiv \neg p \land \neg q$$
De Morgan's Laws

$$p \lor (p \land q) \equiv p$$
$$p \land (p \lor q) \equiv p$$
Absorption Laws.

Conditional statement

$$p \to q \equiv \neg p \vee q$$

$$p \to q \equiv \neg q \to \neg p$$

$$p \vee q \equiv \neg p \to q$$

$$p \wedge q \equiv \neg (p \to \neg q)$$

$$(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$$

$$(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$$

$$(p \to q) \vee (p \to r) \equiv p \to (q \vee r)$$

$$(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$$

Bio-Conditional statement

$$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$$

$$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$$

$$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$$

$$\neg (p \leftrightarrow q) \equiv p \leftrightarrow \neg q$$

Q: Show that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology

$\equiv T$

$\overset{p}{(p \wedge q)} \rightarrow \overset{q}{(p \vee q)}$

$\equiv \neg(p \wedge q) \vee (p \vee q)$ [conditional law]

$\equiv (\neg p \vee \neg q) \vee (p \vee q)$ [De morgan's]

$\equiv (\neg p \vee p) \vee (\neg q \vee q)$ [associative & commutative Law]

$\equiv T \vee T$

$\equiv T$ Proved

Q: Show that $\neg(p \rightarrow q)$ is equivalent to $p \wedge \neg q$

$\neg(p \rightarrow q)$

$\equiv \neg(\neg p \vee q)$ [Conditional Law]

$\equiv \neg(\neg p) \wedge \neg q$ [De morgan's Law]

$\equiv p \wedge \neg q$ [Double negation]

Proved

Q: Show that $\neg(p \vee (\neg p \wedge q))$ & $(\neg p \wedge \neg q)$ are logically equivalent.

$\neg p \vee (\neg p \wedge q)$

$\equiv \neg p \wedge \neg(\neg p \wedge q)$ [De morgan's Law]

$\equiv \neg p \wedge [\neg(\neg p) \vee \neg q]$

$\equiv \cancel{(\neg p \wedge p)}$

$\equiv \neg p \wedge (p \vee \neg q)$

$\equiv (\neg p \wedge p) \vee (\neg p \wedge \neg q)$

$\equiv F \vee (\neg p \wedge \neg q)$

$\equiv (\neg p \wedge \neg q) \vee F$ [commulative Laws]

$T \vee F = T$ $[\neg p \wedge \neg q]$

$\equiv \neg p \wedge \neg q$

— Proved

Quantifier

All, Some, many, none, few.

Universal Quantifier $\longrightarrow$ For all

Existencial $\longrightarrow$ There exist

Let $P(n)$ be a statement

$x+1 > x$

$P(1) = 1+1 > 1$

$= 2 > 1$ T

$P(2) = 2+1 > 2$

$= 3 > 2$ T

For all x $P(n)$ is true.

$\forall x\, P(n)$ is true for all.

Eg: Let $Q(n)$ be the statement

$x < 2$

$Q(1) = 1 < 2$ — T

$Q(2) = 2 < 2$ — F

$Q(3) = 3 < 2$ — F

$\exists x$ $P(n)$ is true.

$\forall x\, P(x)$ — $P(x)$ is true for every x, for all x

$P(x)$ is true for every x.

F
F $\longrightarrow$ $\exists x\, P(x)$ is false
F

F
F $\longrightarrow$ $\exists x\, P(x)$ is true
T

Universal Quantifier

$P(x)$ for all values of x in the domain.

$$\forall x\, P(x)$$

Eg: Let $P(x)$ is the statement $x+1>x$ & Let us assume that domain set of all +ve integer. What is the truth value of $\forall x\, P(x)$

Ans: $P(x) = x+1>x$

$P(1) = 1+1>1$
$= 2>1$ — T

$P(2) = 2+1>2$
$= 3>2$ T

$P(3) = 3+1>3$
$= 4>3$ — T

$\forall x\, P(x)$ is true for all $P(x)$

Eg: Let $Q(n)$ to be statement "$x<2$" what is the truth value of the quantification.

$\forall x\, Q(n)$ where the domain consists of all real numbers?

$Q(n) = x<2$

$Q(1) = 1<2$ — T

$Q(3) = 3<2$ — F

$Q(1.5) = 1.5<2$ — T

$\forall n\, P(n)$ is false.

Eg: Let $P(n)$ is the statement $n^2>0$ & let us assume that domain set of all what is the true value of $\forall n\, P(n)$

$P(n) = n^2>0$

$P(0) = 0$

$P(2) = 2^2>0$

$= 4>0$ — T

$P(3) = 3^2>0$

$= 9>0$

$P(5) = 5^2>0$

$= 25>0$

Eg: Let $P(x)$ is the statement $x^2 < 10$ & the domain consists of positive integers not exceding what is the truth value of $\forall x\, P(x)$.

$P(x) = x^2 < 10$

$P(2) = 2^2 < 10 = 4 \not< 10$ F

$P(3) = 3^2 < 10 = 9 < 10$ F

$P(5) = 5^2 < 10 = 25 < 10$ T

$\forall x$ is a false.

Existencial quantification

There exists an elements x is domain

$\exists x\, P(x) \longrightarrow$ সবগুলো F হলে F But 1 টা যদি T হয় তাহলে T

Eg: Let $P(x)$ denote the statement $x > 3$ what the truth valus of the quantification $\exists x\, P(x)$ where the domain consist of all reals number

$P(x) = x > 3$

$P(-1) = -1 > 3$ — F

$P(2) = 2 > 3$ — F

$P(4) = 4 > 3$ — T

$\exists x\, P(x)$ is true:

Eg: $P(x) = x^2 > 10$ — Positive, integer, not exceeding 4.

$P(1) = 1^2 > 10 = 1 > 10$ — F

$P(4) = 16 > 10$ — T

$P(3) = 3^2 > 10 = 9 > 10$ — F

$\exists x \; P(x)$ is true.

Eg: $P(x) = x > 3$ — real number.
 L True.

(11) $x = x^2$

$P(0) \Rightarrow 0 = 0^2 = 0 = 0$ — T

$P(1) \Rightarrow 1 = 1^2 \Rightarrow 1 = 1$ — T

$P(3) \Rightarrow 3 = 3^2 \Rightarrow 3 = 9$ — F

$P(-1) \Rightarrow -1 = (-1)^2 \Rightarrow -1 = 1$ = F

(e) $\exists x \; P(x) \Longrightarrow$ T

(f) $\forall x \; P(x) \Longrightarrow$ F

Sets

A set is a unordered collection of district objects, called elements or numbers of the set.

$a \in A$ → a is element of set A

$a \notin A$ → a is not element of set A

set → denote by uppercase latter.

elements → " " lowercase latter.

$Q = \{1, 3, 5, 7, 9\}$

↳ set of odd positive integer.

Representation of set.

→ Tabular form.

→ Description form.

→ Set buider notation.

Listing of the elements of a set, separated by comma & exclused withis braket.

$A = \{1, 2, 3, 4, 5\}$

↳ set of five natural number.

$B = \{2, 4, 6, 8, 10 \ldots 50\}$

↳ set of even numbers upto 50

Set of even numbers less than or equal to 5

$C = \{1, 3, 5, 7 \ldots\}$

↳ set of odd positive numbers.

$O = \{1, 2, 3, 5, 7, 9\}$

↳ set of odd positive numbers less than 10.

Set builder notation:

$O = \{x \mid x$ is odd positive integer less than $10\}$

$\{x \in Z^+ \mid x$ is odd and $x < 10\}$

$\{\ \}$ → Represent that

$\mid$ → ~~st~~ such that

$N = \{0, 1, 2, 3 \ldots\} \to$ set of natutral numb:

$Z = \{\ldots -2, -1, 0, 1, 2 \ldots\} \to$ set of Integers "

$Z^+ = \{1, 2, 3 \ldots\} \to$ set of positive integers

R = Set ~~is~~ of real numbers

R^+ = set of positive real numbers.

C = set of complex number.

write the tabular form & set buiderc notation of " The set of positive integers less than 100"

$Z^+ = \{1, 2, 3, 4, 5, 6, 7 \ldots 90\}$ /

$\{x \in Z^+ \mid x$ is positive integer a $x < 100\}$

$\{x \mid x$ is positive integer less than $100\}$

$= \{N, Z, Q, R\}$

$\hookrightarrow Q$, the set of rational numbers.

Equal set :

Two sets are equal ~~two~~ if and only if they have same element.

$A = \{3, 5, 1\}$

$B = \{1, 3, 5\}$

└ set of equal set.

$\forall x (x \in A \leftrightarrow x \in B)$

Empty set / null set :

Set which has no element.

$\{\}$, Φ

Singleton set :

A set with one element is called a singleton set.

$\{\Phi\}$.