AMAT 363 - Homework III
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PROBLEM #1:

For each of the following statements, determine whether it is true or false. Label "T" if it is true, otherwise label "F".

a. Confidence intervals are constructed as interval estimators for population mean, population proportion, population variance, or population standard deviation. T

b. When constructing confidence intervals for a population parameter, confidence levels should be specified. T

c. For a confidence interval constructed for a certain parameter, the higher the confidence level is, the wider the confidence interval will be. T

d. Suppose that we are given a random sample from a certain unknown distribution with unknown variance and we are interested in constructing a 90% confidence interval for the population mean. Then it is still possible as long as the sample size is large. F

e. One can interpret the width of a confidence interval as the precision of the interval estimator; one can interpret the associated confidence level as the reliability of the interval estimator. T

f. Given a 90% confidence interval for a certain population parameter, one can understand it as follows: with probability 0.9, the interval will cover this parameter and so it is essentially an interval estimator. T

g. We would expect about 90% of a sample to lie within the 95% confidence interval of the sample mean. F

h. A confidence interval may not be symmetric about the population parameter. T

i. Let T be a random variable that has a t-distribution and $t_{\alpha,v}$ be a t critical value. Then we have $\Pr(T \leq -t_{\alpha,v}) = \alpha$. T

j. The left endpoint and the right endpoint of a confidence interval are deterministic. More precisely, with a given random sample, then the two endpoints are then fixed. T

a) T
b) T
c) T
d) F
e) T

f) T
g) F
h) T
i) T
j) T

Based on definition 8.5

A) $f(x|\theta)=1 \quad \theta-.5<x<\theta+.5$

Definition $=\left(\bar{X}-Z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}},\ \bar{X}+Z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}}\right)$

thus $L(X): -\frac{1}{2}+\alpha/2$

$U(X): \frac{1}{2}-\alpha/2$

lower: $-\frac{1}{2}+\frac{\alpha}{2}$

upper: $\frac{1}{2}-\frac{\alpha}{2}$

$CI\left(-\frac{1}{2}+\frac{\alpha}{2},\ \frac{1}{2}-\frac{\alpha}{2}\right)$

B) $f(x|\theta)=\frac{2x}{\theta^2} \quad 0<x<\theta,\ \theta>0$

$=\int_{x_1}^{x_2}\frac{2x}{\theta^2}\,dx \longrightarrow \frac{x_2^2-x_1^2}{\theta^2}\geq 1-\alpha$

$\frac{(\theta-x)^2-(x)^2}{\theta^2}\geq 1-\alpha$

$(\theta-x)(\theta-x)$

$=\theta^2-\theta x-\theta x+x^2$

$=\frac{\theta^2-2\theta x+\cancel{x^2-(x)^2}}{\theta^2}\geq 1-\alpha$

$=\frac{\theta^2-2\theta x}{\theta^2}\geq 1-\alpha$

$=\frac{\cancel{\theta}(\theta-2x)}{\cancel{\theta^2}}\geq 1-\alpha$

$=\frac{\theta-2x}{\theta}\geq 1-\alpha$

$\theta-2x\geq(\theta)(1-\alpha)$

$\theta-2x\geq\theta-\theta\alpha$

$\frac{-2x}{-2}\geq\frac{-\theta\alpha}{-2}$

$\boxed{x\leq\frac{\theta\alpha}{2}}$

CI $\left(\frac{\theta\alpha}{2},\ \theta-\frac{\theta\alpha}{2}\right)$

(3)

Based on 8.2 textbook

A) $\bar{x} - 1.96 \frac{\sigma}{\sqrt{n}} \le \mu \le \bar{x} + 1.96 \frac{\sigma}{\sqrt{n}}$

$= 2(1.96) \frac{\sigma}{\sqrt{n}} < \frac{\sigma}{4}$

$= \frac{3.92}{\sqrt{n}} < \frac{1}{4}$

$= \sqrt{n} \ge 3.92(4)$

$= \sqrt{n} \ge 15.68$

$= \sqrt{n} \ge 245.9$

$\boxed{n = 246}$

B) $\bar{x} \pm 2_{1-\alpha/2} \frac{s}{\sqrt{n}}$

$= P\left(2 z_{1-\alpha/2} \frac{s}{\sqrt{n}} > \frac{\sigma}{4}\right) \ge .90$

Square + multiply by $(n-1)$

$= P\left(4t^2_{n-1, .025} \frac{s^2}{n} < \frac{\sigma^2}{16}\right) \ge .90$

$$\boxed{= P\left(\frac{(n-1)s^2}{\sigma^2} \le \frac{(n-1)n}{t^2_{n-1, .025} \cdot 64}\right) \ge .90}$$

This gives smallest n size

8.12 textbook

(4)

$$\text{mean} = \mu_{X_{n+1}} - \mu_{\bar{x}}$$
$$= \mu - \mu$$
$$= \mu$$

$$SD = \sigma^2_{X_{n+1}} + \sigma^2_{\bar{x}}$$
$$= \sigma^2 + \frac{\sigma^2}{n}$$
$$= \sigma^2\left(1 + \frac{1}{n}\right)$$

$$n\left(0, \sigma\sqrt{\left(1 + \frac{1}{n}\right)}\right)$$

$$P\left(\bar{x} - z_{\frac{\alpha}{2}}\sigma\sqrt{1+\frac{1}{n}} \le X_{n+1} \le \bar{x} + z_{\frac{\alpha}{2}}\sigma\sqrt{1+\frac{1}{n}}\right)$$
$$= P\left(|z| \le z_{\frac{\alpha}{2}}\right)$$
$$= 1 - \alpha$$

$$\rightarrow \boxed{\bar{x} \pm z_{\frac{\alpha}{2}}\sigma\sqrt{1+\frac{1}{n}}}$$

Based on 8.12

$$T = \frac{\bar{x} - X_{n+1}}{S\sqrt{1+\frac{1}{n}}}$$

use s^2 as estimate

follows t distribution, n-1 degree freedom

b)

$$\boxed{\bar{x} \pm t_{\frac{\alpha}{2}, n-1}\, S\sqrt{1+\frac{1}{n}}}$$