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367 Chapter 5 HW

Binomial

① i) $P_B(x) = \binom{n}{x} p^x q^{n-x}$ $\quad p = .80\ n = 15\ q = .20$

$P_B(x) = \binom{15}{.80} (.80)^x (.20)^{15-x}$

ii) $P_B(10) = C_{10}^{15} (.80)^{10} (.20)^5 =$ **.1032**

iii) $P(X < 10) = 1 - P_B(X \geq 10)$

$= 1 - C_{10}^{15} (.80)^{10} (.20)^5 + C_{11}^{15} (.80)^{11} (.20)^4$

$+ C_{12}^{15} (.80)^{12} (.20)^3 + \binom{15}{13} (.80)^{13} (.20)^2 + \binom{15}{14}^{14} (.80)(.20)^{1}$

$+ C_{15}^{15} (.80)^{15} = 1 - .938949$

$=$ **0.0610514**

iv) $P_B(X \geq 10) = 1 - P(X \leq 9) = 1 - 0.061 =$ **.9389**

v) $P_B(8 \leq X \leq 10) = \binom{15}{8} (.80)^8 (.20)^7 + \binom{15}{9} (.80)^9 (.20)^6$

$+ \binom{15}{10} (.80)^{10} (.20)^5$

$= .0138 + .04 + .103 =$ **.1568**

vi) $E(X) = (15)(.80) =$ **12**

vii) $V(X) = (15)(.80)(.20) =$ **2.4**

(2) Binomial

i) $P_B(x) = \binom{n}{x} p^x q^{n-x} \Rightarrow \binom{10}{x} (0.2)^x (.80)^{10-x}$

ii) $P_B(X=4) = \boxed{C_4^{10} (.20)^4 (.80)^6} = \boxed{.08}$

iii) $P_B(X<3) = C_2^{10}(.20)^2(.80)^8 + C_1^{10}(.20)^1(.80)^9 + C_0^{10}(.20)^0 (.80)^{10}$

iv) $E(X) = \boxed{(10)(.20)}$

v) $V(X) = \boxed{(10)(.2)(.80)}$

vi) $(X \leq 2)$ $C_2^{15}(.20)^2(1-.20)^{13} + C_1^{15}(.20)^1(.80)^{14} +$

$C_0^{15}(.20)^0(.80)^{15}$

$= \boxed{.398023}$

vii) $P_B(X \geq 3)$ $1 - P_B(X \leq 2)$

$1 - .398023 = \boxed{0.6020}$

(3) a) $(.85)^2(.15) = \boxed{.1084}$

★ b) $P(X<4)$ $\boxed{\sum_{t=1}^{3} (.85)^{t-1}(.15)} \Rightarrow \boxed{.38}$

c) $P(X>3) = 1 - P(X \leq 3) \rightarrow 1 - \left[\sum_{t=1}^{3} (.85)^{t-1}(.15)\right]$

$1 - \sum_{x=0}^{3} P(x=y) \Rightarrow 1 - .38 = \boxed{.62}$

d) $E(X) = \frac{1}{.15} = \boxed{6.66}$

e) $V(X) = \frac{(.85)}{(.15)^2} = \boxed{37.77}$

(4)

i) [illegible] $= \boxed{.249}$ $P_6(1) = \binom{1-1}{1-1}\left(\frac{1}{19}\right)^1\left(\frac{18}{19}\right)$

ii) [illegible] $P(X \leq 30) = \boxed{.25}$ $P_6(30) = \binom{30-1}{3-1}\left(\frac{1}{19}\right)^3\left(\frac{18}{19}\right)^{27}$

iii) [illegible] $\frac{r}{p}$ [illegible] $= \frac{3}{(1/19)} = \boxed{19.3}$

iv) $1/(1/19)^p = \boxed{19}$

v) $\frac{18/19}{(1/19)^2} = \boxed{19.18}$

vi)

(5) $P_x(X=x) = (.4)(.6)^{x-1}$

i) $(.4)(.6)^{x-1}$

ii) $P(x=1) = (.4)(.6)^0 = .4$

iii) $P(X \le 3)$

$1 - (.60)^3 = .784$

$\sum_{x=1}^{3} (.60)^{x-1}(.40)$

iv) $P(x > 5) = 1 - P(x \le 4)$

$= 1 - P(x \le 4) = 1 - [1 - (.60)^4] = .1296$

$\Rightarrow 1 - \left[\sum_{x=1}^{4} (.60)^{x-1}(.40)\right]$

v) $E(x) = \frac{1}{p} = \frac{1}{.4} = 2.5$

vi) $V(x) = \frac{q}{(p)^2} = \frac{(.60)}{p^2} = \frac{(.60)}{(.40)^2} = 3.75$

(6) i) $P(x=3) = (.80)^2(.20) = .1280$

ii) $P(x=7) \Rightarrow (.80)^{7-1}(.20) = .16$

iii) $\frac{r}{p} = \frac{3}{.20} = 15$

iv) variance $= \frac{r(1-p)}{p^2} = \frac{3(.80)}{(.20)^2} = .60$

(7) $\frac{\binom{6}{0}\binom{19}{10}}{\binom{25}{16}} = .0283$

(8) i) $P_H = \frac{\binom{m}{x}\binom{n-m}{k-x}}{\binom{n}{k}} = \frac{\binom{13}{3}\binom{23-13}{6-3}}{\binom{23}{6}} = \boxed{\frac{\binom{13}{3}\binom{10}{3}}{\binom{23}{6}}}$

ii) $E(X) = \frac{A(n)}{N} = \frac{13(6)}{23} = \boxed{3.39}$

iii) $V(X) = \left(\frac{6 \cdot 13}{23}\right)\left(\frac{(23-13)(23-6)}{(23(23-1)}\right) = \boxed{1.7}$

(9) i) $P_B = \binom{n}{x} p^x q^{n-x}$

$p = .90 \quad q = .10$

Binomial random variable

ii) $P(X=x) = \binom{n}{x} p^x (1-p)^{n-x}$

$P_B(x) = \binom{10}{x} (.90)^x (.10)^{10-x}$

iii) $P(X<9) = 1 - P(X \geq 9)$

$\binom{10}{9}(.90)^9(.10)^1 + \binom{10}{10}(.9)^{10}(.10)^0$

$1 - .7361 = \boxed{.2639}$

iv) $E(X) = (10)(.9) = \boxed{9}$

v) $\text{var}(X) = n \cdot p \cdot q = (10)(.90)(.10) = \boxed{.90}$

(10) i) $\sum_{x=1}^{\infty} \frac{(10.2)^x}{x!} e^{-10.2}$ $\quad 3(3.4) = \boxed{10.2}$ Poisson random variable

ii) $P(X=x) = \frac{n^x}{x!} e^{-x}$ $\quad \boxed{\sum_{n=1}^{\infty} \frac{10.2^x}{x!} e^{-10.2}}$

$$\sum_{x=0}^{2} \frac{(10.2)^x}{x!} e^{-10.2}$$

iii) $P(x<3) = P(X=2) = \frac{10.2^2}{2!} e^{-10.2}$

$P(x=1) = \frac{10.2}{1!} e^{-10.2}$

$P(x=0) = \frac{10.2^0}{0!} e^{-10.2}$ $= .0024$

★ iv) $E(x) = 10.2 = \lambda$

v) $V(x) = \lambda = 10.2$

(11) i) Hypergeometric distribution

$n = 50 \quad N = 100 \quad K = 30$

ii) $P(X=x) = \frac{\binom{30}{x}\binom{100-30}{50-x}}{\binom{100}{50}}$

iii) $P(X=15) = \frac{\binom{30}{15}\binom{70}{35}}{\binom{100}{50}}$

iv) $E(x) = \frac{K(m)}{N} = \frac{30(50)}{100} = 15$

v) $V(x) = \frac{30(50)}{100}\left(\frac{50}{100}\frac{(70)}{(100-1)}\right) \Rightarrow \frac{Km}{n}\left(\frac{(n-m)(n-K)}{n(n-1)}\right)$

(12) i) Geometric random variable

P = .30 q = .70

ii) $P(x = y) = (p)(q)^{x-1} = \boxed{(.70)^{x-1}(.30)}$

iii) $P(x = 5) = \boxed{(.70)^{5-1}(.30)} = \boxed{.07203}$

(13)

i) $p = .005$ $n = 600$ Poisson distribution

ii) $P(x = y) = \frac{x^x}{x!} e^{-x} = \boxed{\sum_{x=0}^{\infty} \frac{(3)^y e^{-3}}{x!}}$

Answer ⟶ iii) $1 - P(x < 3)$: $\boxed{\left(1 - \left[\sum_{x=0}^{3} \frac{(3)^3 e^{-3}}{3!}\right]\right)}$

(14) i) $P_B(x) = \binom{n}{x} p^x q^{n-x}$ $p = .70$ $q = .30$

$\boxed{P_B = \binom{7}{3}(.70)^3(.30)^4}$

ii) $E(y) = 7(.70) = \boxed{4.9}$

iii) $V(x) = n \cdot p \cdot q = (7)(.70)(.30) = \boxed{1.47}$