AMAT 367 HW 2: Jahnavi Bonagiri

1a $S: \{FlUFF\}$ $F=3$ $L=1$ $U=1$

$\frac{5!}{3!1!1!} = \boxed{20} = \binom{5}{3,1,1}$

b. $S = \{ROTOR\}$

$R=2$
$O=2$

$\frac{4!}{2!2!} = \boxed{6}$

ROTOR
RRTOO
OOTRR
ORTRO
ORTOR

$\Big\}$ 6

2. $S = \{1,2,2,3,3,3,0\}$

1-1
2-2
3-3
0-1

7 total numbers.

$\binom{7}{1,2,3,1} = \frac{7!}{1!2!3!1!} = \boxed{420}$

ABCDE

3.
a. $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$

$\frac{120}{2} = \boxed{60}$

b. $4! = 4 \times 3 \times 2 \times 1 = \boxed{24}$

4. 6 - RB
3 - T
2 - PC
1 - FF

$6!\,3!\,2!\,1! = \boxed{8640}$

5. a. $8! = 8\times7\times6\times5\times4\times3\times2\times1 = \boxed{40320}$

ways to choose

b. $4! \cdot 2^4 = \boxed{384}$

c. $7! = 7\times6\times5\times4\times3\times2\times1 = \boxed{5040}$

6. $x_1 + x_2 + x_3 + x_4 = 30$

$$\binom{30+4-1}{30} = \binom{33}{30} = \frac{33!}{30!\,3!} = \boxed{5456}$$

7. a. $$\binom{n-1+r}{r} = \binom{8-1+30}{30} = \binom{37}{30} = \frac{37!}{30!\,7!}$$

$$= \boxed{10\ 295\ 472}$$

b. $r = 30 - 4 = 26.$

$$\binom{8-1+26}{26} = \binom{33}{26} = \frac{33!}{26!\,7!} = \boxed{4272048}$$

8. $\binom{n}{r+1} = \binom{n-r}{r+1}\binom{n}{r}$

$$= \frac{n!}{(r+1)!\,(n-(r+1))!} \qquad = \frac{n-r}{r+1}\binom{n}{r}$$

$$= \frac{n!}{(r+1)\,r!\,(n-r-1)!} \cdot \frac{(n-r)}{(n-r)}$$

$$= \frac{(n-r)}{r+1} \cdot \frac{n!}{r!\,\underbrace{(n-r)(n-r-1)!}_{(n-r)!}}$$

$$= \frac{(n-r)}{r+1} \cdot \frac{n!}{(n-r)!\,r!}$$

$$= \frac{n-r}{r+1} \cdot \binom{n}{r}$$

9a. $$\sum_{i=0}^{10} \binom{10}{i} 2^i = \binom{10}{0}2^0 + \binom{10}{1}2^1 + \binom{10}{2}2^2 + \binom{10}{3}2^3 +$$
$$\binom{10}{4}2^4 + \binom{10}{5}2^5 + \binom{10}{6}2^6 + \binom{10}{7}2^7 +$$
$$\binom{10}{8}2^8 + \binom{10}{9}2^9 + \binom{10}{10}2^{10}$$

$$\Rightarrow \left[\frac{10!}{0!\,10!}\cdot 1\right] + \left[\frac{10!}{1!(9)!}\,2\right] + \left[\frac{10!}{2!\,8!}\cdot 4\right] + \left[\frac{10!}{3!\,7!}\cdot 8\right] + \left[\frac{10!}{4!\,6!}\cdot 16\right] +$$
$$\left[\frac{10!}{5!\,5!}\cdot 32\right] + \left[\frac{10!}{6!\,4!}\cdot 64\right] + \left[\frac{10!}{7!\,3!}\cdot 128\right] + \left[\frac{10!}{8!\,2!}\cdot 256\right] + \left[\frac{10!}{9!\,1!}\cdot 512\right]$$
$$+ \left[\frac{10!}{10!\,0!}\,1024\right] = 1+20+180+960+3360+8064+13440+15360+11520+$$
$$5120+1024 = \boxed{59049} \leftarrow$$

9b $(1-x)^6$

$= (1-x)(1-x)(1-x)(1-x)(1-x)(1-x)$

$= (1)^6 + (6)1^5(-x)^1 + (15)1^4(-x)^2 + (20)(1)^3(-x)^3 + (15)(1)^2(-x)^4$

$+ (6)(1)^1(-x)^5 + (1)(1)(-x)^6$

simplifies to

$= \boxed{1-6x+15x^2-20x^3+15x^4-6x^5+x^6}$

$= \binom{6}{0}x^6 - \binom{6}{1}6x^5 + \binom{6}{2}15x^4 - \binom{6}{3}20x^3 + \binom{6}{4}15x^2$

$- \binom{6}{5}6x + \binom{6}{6}1 \Rightarrow \sum_{i=0}^{6}\binom{6}{i}\cdot 1^{(6-i)}(-x)^i$

9c $(x+x^{-1})^5$

$= \left(x+\frac{1}{x}\right)^5$

$\binom{5}{0}(x)^5\left(\frac{1}{x}\right)^0 + \binom{5}{1}(x)^4\left(\frac{1}{x}\right)^1 + \binom{5}{2}(x)^3\left(\frac{1}{x}\right)^2$

$+\binom{5}{3}(x)^2\left(\frac{1}{3}\right)^3 + \binom{5}{4}(x)^1\left(\frac{1}{x}\right)^4 + \binom{5}{5}x^0\left(\frac{1}{x}\right)^5$

$nC_r = \frac{n!}{r!(n-r)!}$

$= x^5 + 5x^4\left(\frac{1}{x}\right) + 10x^3\left(\frac{1}{x}\right)^2 + 10x^2\left(\frac{1}{x}\right)^3 + 5x\left(\frac{1}{x}\right)^4 + \frac{1}{x^5}$

$= \boxed{x^5 + 5x^3 + 10x + \frac{10}{x} + \frac{5}{x^3} + \frac{1}{x^5}} = \sum_{i=0}^{5}\binom{5}{i}x^{5-i}\left(\frac{1}{x}\right)^i$