Bayesian HW 4W

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① $P(Pam) = \frac{15}{30} = .50$ $P(F|Pam) = .04$

$P(Pia) = \frac{5}{30} = 1/6$ $P(F|Pia) = .06$

$P(Pablo) = \frac{10}{30} = 1/3$ $P(F|Pablo) = .03$

$$P(Pam|F) = \frac{P(Pam)\,P(F|Pam)}{P(Pam)P(F|Pam) + P(Pia)P(F|Pia) + P(Pablo)P(F|Pablo)}$$

$$= \frac{(.50)(.04)}{(.50)(.04) + (1/6)(.06) + (1/3)(.03)} = \boxed{.50} = \boxed{50\%}$$

② $P(12 \text{ sided die}) = .50$ $P(2,9|12) = \left(\frac{1}{12}\right)\left(\frac{1}{12}\right) = 1/144$

$P(20 \text{ sided die}) = .50$ $(1/144)(2) = 1/72$

$P(2,9|20) = \left(\frac{1}{20}\right)\left(\frac{1}{20}\right) = 1/400\,(2)$

$\Downarrow$

$1/200$

$$P(12|2,9) = \frac{P(12)\,P(2,9|12)}{P(12)P(2,9|12) + P(20)P(2,9|20)}$$

$$= \frac{(.50)(1/72)}{(.50)(1/72) + (.50)(1/200)} = \boxed{.735 = 73\%}$$

(3)

$P(S) = 50\%$ $\quad P(G|S) = 80\%$

$P(W) = 20\%$ $\quad P(G|W) = 5\%$

$P(M) = 30\%$ $\quad P(G|M) = 20\%$

$$P(S|G) = \frac{P(S)\,P(G|S)}{P(S)P(G|S) + P(W)\,P(G|W) + P(M)\,P(G|M)}$$

$$= \frac{(.50)(.80)}{(.50)(.80) + (.20)(.05) + (.30)(.20)} = \boxed{.851} = \boxed{85.1\ \%}$$

(4)

A 327.1 ± 1.2

B 332.5 ± 3.2

C 321.2 ± 7.5

D 318.0 ± 2.2

E 325.3 ± 3.5

optimal solution $= K\exp\left[\sum_{i=1}^{K} \frac{-1}{2\sigma^2}(d_i - z)^2\right]$

$$\log P = \log K + \sum_{i=1}^{n} \frac{-1}{2\sigma^2}(d_i - z)^2$$

(A)

$$\frac{d\log P}{dz} = 0 + \sum_{i=1}^{n} \frac{1}{\sigma^2}(d_i - \hat{z}) \rightarrow \sum_{i=1}^{n} \frac{d_i}{\sigma_{z_i}^2}$$

$$\hat{z} = \frac{\sum_{i=1}^{n} \frac{d_i}{\sigma_{z_i}^2}}{\sum_{i=1}^{n} \frac{1}{\sigma_{z_i}^2}}$$

$$d_i = \frac{\frac{327.1}{1.2^2} + \frac{332.5}{3.2^2} + \frac{321.2}{7.5^2} + \frac{318.0}{2.2^2} + \frac{325.3}{3.5^2}}{\left(\frac{1}{1.2^2} + \frac{1}{3.2^2} + \frac{1}{7.5^2} + \frac{1}{2.2^2} + \frac{1}{3.5^2}\right)}$$

$\boxed{d_i = 325.64}$

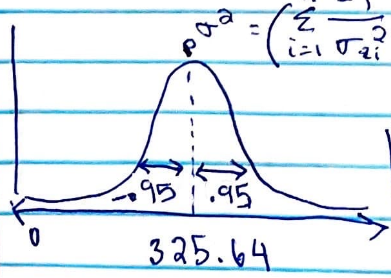
$$\sigma = \left(\frac{1}{1.2^2} + \frac{1}{3.2^2} + \frac{1}{7.5^2} + \frac{1}{2.2^2} + \frac{1}{3.5^2}\right)^{-.5} = \boxed{.95}$$

$$\sigma^2 = -\left(\frac{d^2\log P}{dz^2}\right)^{-1}$$

(B)

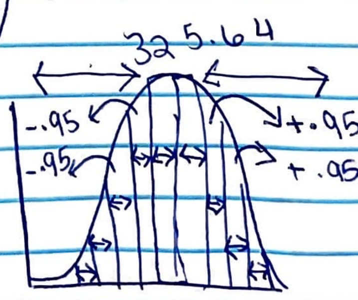
$$\frac{d^2\log P}{dz^2} = \sum_{i=1}^{n} \frac{d}{dz}\left(\frac{1}{\sigma_{z_i}^2}(d_i - \hat{z})\right) = \sum_{i=1} \frac{-1}{\sigma_{z_i}}$$

$$\sigma^2 = \left(\sum_{i=1}^{n} \frac{-1}{\sigma_{z_i}^2}\right)^{-1} \longrightarrow \sigma = \left(\sum_{i=1}^{n} \frac{1}{\sigma_{z_i}^2}\right)^{-1/2} =$$



$\boxed{325.64 \pm .95}$

Plotting error bars



(5) $P(r,\theta|I) = C$

(A) $\int_0^R \int_0^{2\pi} \underbrace{P(r,\theta|I)}_{C} \, r\,dr\,d\theta = 1$

$= C \int_0^{2\pi} \frac{r^2}{2} \Big|_0^R d\theta$

$= \cancel{2}\pi \cdot \frac{CR^2}{\cancel{2}} = C\pi R^2 = 1$

solve for C

$\boxed{C = \frac{1}{\pi R^2}}$

(B) $\int_0^R \int_0^{2\pi} \frac{1}{\pi R^2} r\,dr\,d\theta$

$2\cancel{\pi} \int_0^R \frac{1}{\cancel{\pi} R^2} r\,dr$

$= \boxed{\int_{R_1}^{R_2} \frac{2r}{R^2} dr}$

(C) $\int_2^3 \frac{2r}{R^2} r\,dr$

$R = 6m \Rightarrow \frac{2r}{R^2} = \frac{2r}{6^2} = \frac{1}{18}$

$\frac{1}{18} \cdot \frac{r^2}{2} \Big|_2^3$

$= \frac{3^2}{36} - \frac{2^2}{36} = \boxed{\frac{5}{36}}$

(6) $P(t|I) = \frac{t^{k-1}}{\Gamma(k)} \theta^{-k} e^{-t/\theta}$

(A) $\log P = \cancel{\log \frac{1}{\Gamma(k)}}^{0} + \log(t^{k-1}) + \cancel{\log \theta^{-k}}^{0} + \cancel{\log e^{-t/\theta}}$

$$\underbrace{\frac{d \log P}{dt} = \left(\frac{1}{t^{k-1}}\right)(k-1)t^{k-2} - \frac{1}{\theta}}_{} = 0$$

$$\frac{(k-1)t^{-2}}{t^{-1}} = \frac{(k-1)}{t}$$

$$\cancel{\frac{k-1}{t} = \frac{1}{\theta}} \Rightarrow \boxed{\hat{t} = \theta(k-1)}$$

(B) second derivative

$$\frac{d^2 \log P}{dt^2_p} = \frac{d}{dt}\left(\frac{k-1}{t} - \frac{1}{\theta}\right) = -\frac{(k-1)}{t^2}$$

$$\sqrt{\frac{\hat{t}^2}{k-1}} = \frac{\sqrt{\theta(k-1)^2}}{\sqrt{k-1}}$$

Plug in

$$\boxed{\sigma = \frac{\theta(k-1)}{\sqrt{k-1}}}$$