Bayesian Data Analysis (PHY/CSI/INF 451/551)
HW#2w

1. Independent Pair of Fair 6-Sided Dice.
Consider an independent pair of fair 6-sided dice with sides indexed by i and j.

a. Since they are independent, how does p(i | j, I) relate to p(i | I) ?

Since they are Independent, they both do not affect each other

i is given prior and is independent

b. What is the probability of rolling i=2 on the first 6-sided die? That is, what is p(i = 2 | I) ?

1/6 is the probability since there are 6 sides

c. What quantity does p(i = 2, j = 4 | I) represent? And what is its value?

$P(i=2) = 1/6$. 36 is the sample space of the 2 die

$P(j=4) = 1/6$. $(1/6)(1/6) = 1/36$

d. What is the average value (also called the expected value) of i ?

$$\frac{(1+2+3+4+5+6)}{6} = 3.5$$

e. Is it possible to ever observe this expected value? Why or why not?

No, it isn't possible because dice can only have integer values. You can't have 3.5 as a value on a die.

f. What is the expected value of i+j ?

$i+j\ (3.5+3.5) = 7$

g. What is the most probable value of i+j ?

$P(10) = 3/36$
$P(11) = 2/36$
$P(12) = 1/36$

The most possible value of $\boxed{i+j=7}$

Sum of 1 = impossible

It is because the most outcomes when rolling 2 die add up to 7

Sum of 2 = (1,1) = 1/36
Sum of 3 = (1,2)(2,1) = 2/36.
Sum of 4 = (2,2)(3,1)(1,3) = 3/36
Sum of 5 = (2,3)(3,2)(4,1)(1,4) = 4/36
Sum of 6 = (4,2)(2,4)(5,1)(1,5)(3,3) = 5/36

(3,4)(4,3)(1,6)(6,1)(5,2)(2,5)

These are all the different combinations that are possible to add up to 7. It has the highest probability of 6/36.

Sum of 8 = (4,4)(5,3)(2,6)(6,2)(3,5) = 5/36
Sum of 9 = (5,4)(4,5)(6,3)(3,6) = 4/36

2. Imagine that we have a pair of six-sided dice that are attached to one another with a string. Below is a partially-filled in table of the probabilities for rolling different values indexed by i and j.

$P(i,j\|I)$	$j = 1$	$j = 2$	$j = 3$	$j = 4$	$j = 5$	$j = 6$
$i = 1$	0	0	0	0	0	0
$i = 2$	0	4/36	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{1}{36}$	0
$i = 3$	0	$\frac{2}{36}$	$\frac{4}{36}$	$\frac{1}{36}$	$\frac{2}{36}$	0
$i = 4$	0	$\frac{2}{36}$	$\frac{1}{36}$	$\frac{4}{36}$	$\frac{2}{36}$	0
$i = 5$	0	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{4}{36}$	0
$i = 6$	0	0	0	0	0	0

a. Fill in the missing entry in the table above.

missing entry is 4/36

b. What is the expected value of i+j ?

$i+j = 7$

$(3.5)+(3.5) = 7$ The expected value is 7

c. What is the probability p(i | I) for all values of i ?

$p(i = 1 \mid I) = 0$

$p(i = 2 \mid I) = 9/36 = 1/4$

$p(i = 3 \mid I) = 9/36 = 1/4$

$p(i = 4 \mid I) = 9/36 = 1/4$

$p(i = 5 \mid I) = 9/36 = 1/4$

$p(i = 6 \mid I) = 0$

d. What is the expected value of i ? $\frac{2+3+4+5}{4} = \boxed{3.5}$

The expected value of $i = 3.5$

3. In the problem #1, we found that $p(i \mid j, I) = p(i \mid I)$ when i and j are independent.

Prove (algebraically using the sum and/or the product rules) that if $p(x, y \mid I) = p(x \mid I)\, p(y \mid I)$ then $p(x \mid y, I) = p(x \mid I)$ meaning that if x and y are independent, then you can just drop any conditioning of x on y.

$$P(x, y \mid I) = P(y \mid I)\, P(x \mid y, I)$$
$$P(x \mid I)\, P(y \mid x, I)$$

$$\frac{P(y \mid I)\, P(x \mid y, I)}{P(x \mid I)} = \frac{\cancel{P(x \mid I)}\, P(y \mid x, I)}{P(x \mid I)}$$

$$\frac{P(y \mid I)\, P(x \mid y, I)}{P(x, I)} = P(y \mid x, I) \implies \boxed{P(y \mid x, I) = P(y \mid I)}$$

$$\frac{\cancel{P(y \mid I)}\, P(x \mid y, I)}{\cancel{P(y \mid I)}} = \frac{P(x \mid I)\, P(y \mid x, I)}{\cancel{P(y \mid I)}}$$

$$\boxed{P(x \mid y, I) = P(x \mid I)}$$

↳ Proved using product rule that any conditioning can be dropped

4. Rolling 6's.

a. Imagine that you have **one set of 6** six-sided dice. Calculate the probability that you will **roll at least one 6** if you roll all six dice.

$1 - 5/6$ = gives you prob of a six

Probability of no six = $5/6$

$$\boxed{1 - (5/6)^6}$$ ← raise to 6 to represent 6 rolls.

$= 0.665$

b. Consider that you have **two sets of 6** six-sided dice (12 dice total). Calculate the probability that you will **roll at least two 6's** if you roll all 12 dice.

$$1 - \left(\frac{5}{6}\right)^{12} - \binom{12}{1}\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^{11}$$

$$\boxed{= 0.6187}$$

c. Consider that you have **three sets of 6** six-sided dice (18 dice total). Calculate the probability that you will **roll at least three 6's** if you roll all 18 dice.

$$1 - \left(\frac{5}{6}\right)^{18} - \binom{18}{1}\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^{17} - \binom{18}{2}\left(\frac{1}{6}\right)^{2}\left(\frac{5}{6}\right)^{16}$$

$$\boxed{= 0.597}$$

d. Are the probabilities in parts a-c equal? Why or why not?

They are not equal. There are many ways of rolling the dice. These equations use "at least" meaning there can be multiple outcomes. Probability will not give a set result. There are more ways of rolling 12 dice than 6 and so on, thus will not give the same answer