EE2703 Final Exam

EE19B049(Jahnavi Pragada)

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Abstract

This problem is about radiation from a loop antenna of length λ.

The main content of the assignment is:

- Finding the current in loop using vectors.
- Finding magnetic field due to the loop along z-axis.
- Fitting the field to a fit of the type B_z $\approx$cz^b.

Given data for solving the problem

A long wire carries a current

$I = \frac{4\pi}{\mu_0} cos(\phi)exp(jt)$

through a loop of wire. Here, ϕ is the angle in polar coordinates i.e., in (r , ϕ , z) coordinates. The wire is on the $x-y$ plane and centered at the origin. The radius of the loop is 10 cm and is also equal to $1/k = c/\omega$. (This means that the circumference is λ)

The problem is to compute and plot the magnetic field $\vec{B}$ along the z axis from 1cm to 1000 cm, plot it and then fit the data to $|\vec{B}| = cz^b$. of grid will give.

The computation involves the calculation of the vector potential

$\vec{A}(r, \phi, z) = \frac{\mu_0}{4\pi} \int \frac{(I\phi)\phi exp(jkR)ad\phi}{R})$

where $\vec{R} = |\vec{r} - \vec{r_0}|$and $k = \omega/c = 0.1$. $\vec{r}$ is the point where we want the field, and $\vec{r'} = a\vec{r'}$ is the point on the loop. Due to the This can be reduced to a sum:

$$\vec{A}_{ijk} = \sum_{l=0}^{N-1} \frac{cos(\phi_l')exp(-jkR_{ijkl})\vec{dl'}}{R_{ijkl}} .....(1)$$

where $\vec{r}$ is at r_i, ϕ_j, z_k and $\vec{r'}$ is at $acos\phi_l' x + asin\phi_l' \hat{y}$. Note that Eq. (1) is valid for any (x_i, y_j, z_k) ,and is summed over the current elements in the loop. You must implement this as a vector operation over both l and over a vector of (x_i, y_j, z_k) values. From $\vec{A}$, you can obtain $\vec{B}$ as

$$\vec{B} = \nabla X \vec{A}$$

Along the z axis this becomes ($\vec{A}$ is along and the curl gives only aB_z component along .

$$B_z(z) = \frac{A_y(\triangle x,0,z) - A_x(0,\triangle y,z) - A_y(-\triangle x,0,z) + A_x(0,\triangle y,z)}{4\triangle x \triangle y} .....(2)$$

Part 2

- Breaking the volume into a 3 by 3 by 1000 mesh, with mesh points separated by 1cm.(The 3 by 3 grid in x y is can be used to compute the curl using Eq.2).

The following code snippet do the process:

```
x=np.linspace(-1,1,num=3)
y=np.linspace(-1,1,num=3)
z=np.linspace(0,5000,num=1000)
X,Y,Z=np.meshgrid(x,y,z)
```

Part 3 and 4

- Obtained vectors $\vec{r_l}$ and $\vec{d_l}$ which will be used in further calculations.
- We found the Current in the loop with the equation given in the data and ploted it.

The python code snippet for finding vectors $\vec{r_l}$ and $\vec{d_l}$ is as follows:

```
r_l = rad*np.array([np.cos(phi), np.sin(phi), np.zeros(len(phi))]).T
d_l = 2*pi*rad*(1/secs)*np.array([-np.sin(phi), np.cos(phi), np.zeros(len(phi))]).T
```

The python code snippet for finding and ploting current is as follows:

```
I = 4*pi*(1/(4*pi*1e-7))*np.array([-np.cos(phi)*np.sin(phi), np.cos(phi)*np.cos(phi
plt.figure(0)
plt.quiver(r_l[:,0],r_l[:,1],I[:,0],I[:,1])
plt.xlabel("x $\longrightarrow$")
plt.ylabel("y $\longrightarrow$")
plt.title("Quiver Plot of Current in the Loop")
plt.grid()
plt.show()
```

The quiver plot of current in the loop obtained is as follows:

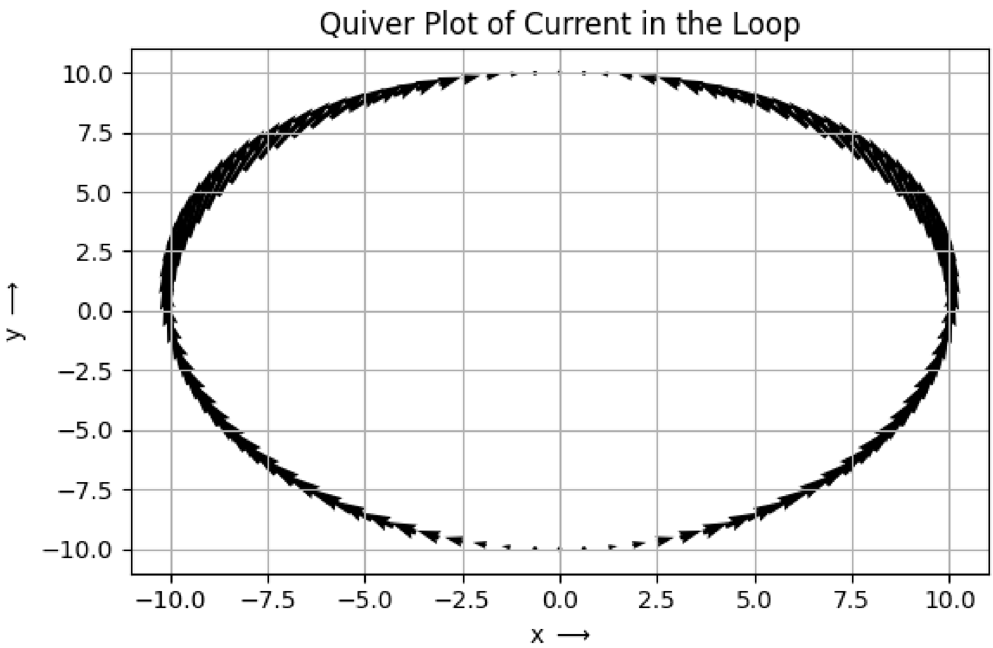

Figure 1: Current plot

Part 5 and 6

- Calculated magnitude of $\vec{R}$ which will be used to find magnetic potential.
- Clculated magnetic potential at a point in loop which will further be added to get Magnetic potential.

The python code snippet of function that can find these is as shown below:

```
def calc(l):
```

```
    k = 1/rad
    x_l, y_l, z_l = r_l[l]
    R_ijkl = np.sqrt((X-x_l)**2 + (Y - y_l)**2 + (Z - z_l)**2)
    A_ijkl = np.cos(l*2*pi/100)*np.exp(-1j*k*R_ijkl)/R_ijkl
    return A_ijkl
```

Part 7

- Adding up the potential at any point($\vec{A_{ijkl}}$) obtained from above function to obtain the potential($\vec{A_{ijk}}$)

The python code snippet is as follows:

```
A_x = np.zeros(X.shape)
A_y = np.zeros(Y.shape)
A_z = np.zeros(Z.shape)
for n in range(secs):
  A_ijkl = calc(n)
  A_x = A_x + A_ijkl*d_l[n,0]
  A_y = A_y + A_ijkl*d_l[n,1]
  A_z = A_z + A_ijkl*d_l[n,2]
```

Part 8 and 9

- Computing Magnetic Field $\vec{B}$ along z-axis.
- Plotting a loglog plot of $\vec{B_z}$ and z.

The python code snippet is as follows:

```
Bz = 0.25*(A_y[1,0,:]-A_x[0,1,:]-A_y[-1,0,:]+A_x[0,-1,:])

plt.figure(1)
plt.loglog(z, np.abs(Bz))
plt.xlabel("z $\longrightarrow$")
plt.ylabel("Bz $\longrightarrow$")
plt.title("loglog plot of Magnetic field")
plt.grid()
plt.show()
```

The loglogplot of Magnetic field alongz-axis is as follows :

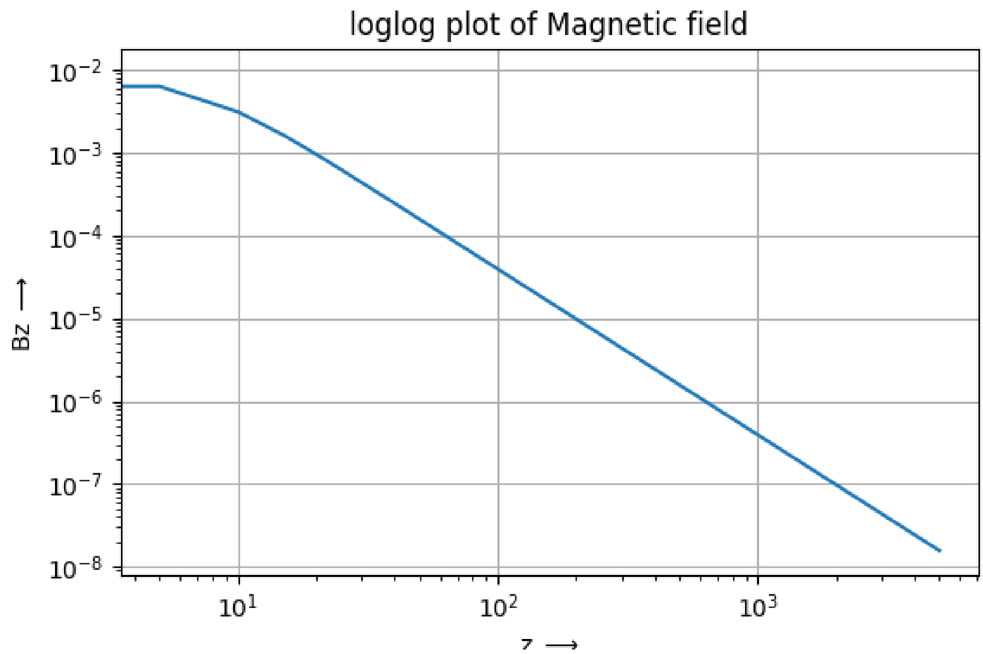

Figure 2: Magnetic field plot

Part 10

- Fitting the field obtained to a type given i.e.,B_z ≈cz^b

The following code snippet do the process:

```
B = np.hstack([np.ones(len(Bz[999:]))[:,np.newaxis],np.log(z[999:])[:,np.newaxis]])
log_c , b = np.linalg.lstsq(B,np.log(np.abs(Bz[999:]))) [0]
c=np.exp(log_c)
```