Managing Linear Risk

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3.1. Calibrating the Exchange Rate Process

Let St denote the GBP/USD exchange rate, i.e. the amount of dollars needed to purchase a single GBP at time t. Under no-arbitrage pricing (risk-neutral valuation), St follows a Geometric Brownian Motion (GBM), such that the future spot price is given by ST.

Part a

You need to refer to the interest rate parity and estimate theta using the forward quotes from Table 1. Note that this a "forward-looking" approach.

Solution

We calculate the mid-point of bid-ask spreads for each forward contract and use the interest rate parity formula to estimate theta.

```
# Load the forward prices data
forward_data <- read.csv("FE535_Forward_Prices.csv")

# Extract relevant information
bid_rates <- forward_data$Bid
ask_rates <- forward_data$Ask
time_to_maturity <- c(1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12) / 12

# Forward rates using mid-point of bid and ask prices
forward_rates <- (bid_rates + ask_rates) / 2

# Calculate theta for all forward contracts
theta_values <- log(forward_rates / 1.2273) / time_to_maturity/100

theta_data <- data.frame(Time_to_Maturity = time_to_maturity * 12,  # in mont
hs
                          Forward_Rate = forward_rates,
                          Int_rate_diff_pcnt = theta_values)

print(theta_data)

##    Time_to_Maturity Forward_Rate Int_rate_diff_pcnt
## 1                 1        2.050         0.06156278
## 2                 2        5.880         0.09400441
## 3                 3        8.340         0.07664986
```

```
## 4                 4       10.860         0.06540809
## 5                 5       14.180         0.05872838
## 6                 6       16.670         0.05217588
## 7                 7       19.600         0.04749794
## 8                 8       22.135         0.04338515
## 9                 9       24.290         0.03980331
## 10               10       26.760         0.03698510
## 11               11       29.225         0.03458409
## 12               12       31.340         0.03240079

# Average theta values
calibrated_theta = mean(theta_values)
print(paste("Calibrated theta:", calibrated_theta))
```

Calibrated theta	0.05359

Part b

For foreign exchange rates, it is common to relate to the interbank lending rate in terms risk-free rate. The data file named FE535_Libor_USD_GBP.csv contains LIBOR rates for both USD and GBP. How does your calibrated ◊ compare with LIBOR rates?

Solution

```
# Load LIBOR rates data
libor_data <- read.csv("FE535_Libor_USD_GBP.csv")

# Extract relevant LIBOR rates
usd_1m_libor <- libor_data$US0001M.Index/ 100
gbp_1m_libor <- libor_data$BP0001M.Index/ 100
usd_3m_libor <- libor_data$US0003M.Index/ 100
gbp_3m_libor <- libor_data$BP0003M.Index/ 100
```

```r
usd_6m_libor <- libor_data$US0006M.Index/ 100
gbp_6m_libor <- libor_data$BP0006M.Index/ 100

diff_1m <- mean(usd_1m_libor - gbp_1m_libor)
diff_3m <- mean(usd_3m_libor - gbp_3m_libor)
diff_6m <- mean(usd_6m_libor - gbp_6m_libor)

libor_data1 <- data.frame(Month = c("1M", "3M", "6M"),
                               USD_LIBOR = c(mean(usd_1m_libor), mean(usd_3m_
libor), mean(usd_6m_libor)),
                               GBP_LIBOR = c(mean(gbp_1m_libor), mean(gbp_3m_
libor), mean(gbp_6m_libor)),
                               Diff_USD_GBP_LIBOR = c(diff_1m, diff_3m, diff_
6m))

libor_data1

##   Month  USD_LIBOR  GBP_LIBOR Diff_USD_GBP_LIBOR
## 1    1M 0.01909204 0.02325804       -0.004166005
## 2    3M 0.02053717 0.02474410       -0.004206930
## 3    6M 0.02202970 0.02596179       -0.003932085

# Compare the two sets of theta values
comparison_data <- data.frame(Month = c("1M", "3M", "6M"),
                              Calib_Theta = c(theta_values[1], theta_values[3
], theta_values[6]),
                              Diff_USD_GBP_LIBOR = libor_data1$Diff_USD_GBP_L
IBOR)

print("Comparison of Theta from Forward Rates and Calibrated Theta from LIBOR
Rates:")

## [1] "Comparison of Theta from Forward Rates and Calibrated Theta from LIBO
R Rates:"
```

```r
print(comparison_data)
```

```
##   Month Calib_Theta Diff_USD_GBP_LIBOR
## 1    1M  0.06156278       -0.004166005
## 2    3M  0.07664986       -0.004206930
## 3    6M  0.05217588       -0.003932085
```

Part c

For sigma, you need to download data for the daily GBP/USD exchange rate using the "GB-PUSD=X" symbol from Yahoo Finance. Your data should be daily and range between 2018-01-01 and 2022-04-03. Given the adjusted prices, you need to calibrate sigma using the historical returns. Note that this calibration is backward-looking, which is in line with what you did in Project 1.

Solution

```r
# Download historical GBP/USD exchange rate data
start_date <- "2018-01-01"
end_date <- "2022-04-03"
symbol <- na.omit("GBPUSD=X")

exchange_rate_data <- na.omit(getSymbols(symbol, src = "yahoo", from = start_
date, to = end_date, auto.assign = FALSE))
exchange_rate <- Ad(exchange_rate_data)

# Calculate daily returns
returns <- na.omit(diff(log(exchange_rate)))

# Calculate volatility
sigma <- sd(returns) * sqrt(252)
```

```r
cat("Calibrated Sigma (Volatility):", sigma, "\n")

## Calibrated Sigma (Volatility): 0.08552515
```

Calibrated Sigma	0.08552515

3.2. VaR for the Unhedged

Assume that the exporter does not hedge the exchange rate risk. In this case, the exporter exchanges the GBP on the spot market upon receiving the payment in the future. Let VT denote the profit/loss (P&L) of the exporter at delivery time.

Solution

To calculate the 99% Value at Risk (VaR) of the exporter's profit/loss (P&L) in dollars ($), we can use a Monte Carlo simulation.

```r
set.seed(7)
S0 = 1.2273
theta = 0.0535988141263977
sigma = 0.08552515
dt = 1/252
periods = 126
drift = (theta - (sigma^2)/2)
N = 10^3

gbm = function(n) {
  rseq = rnorm(periods, drift * dt, sigma * sqrt(dt))
  ST = S0 * exp(c(0, cumsum(rseq)))
```

```r
  return(ST)
}

# Simulating N paths of future spot rates
smat = sapply(1:N, gbm)

# True and simulated mean spot rates
data.frame(True = S0 * exp(-0.002639483 * 0.5), Simulated = mean(smat[nrow(sm
at),]))
```

```
##       True Simulated
## 1 1.225681  1.259981
```

```r
# True and simulated variances
data.frame(True_Variance = (S0^2) * (exp(0.5 * sigma^2) - 1) * exp(2 * 0.5 *
-0.002639483),
           Sim_Variance = var(smat[nrow(smat),]))
```

```
##   True_Variance Sim_Variance
## 1   0.005504365  0.005659044
```

```r
# VaR calculation
s_end = smat[nrow(smat),]
avg_s = mean(s_end)
sd_s = sd(s_end)
q_s = quantile((s_end), 0.01)
VaR_0.99_unhedged = avg_s - q_s
VaR_0.99_USD_unhedged = VaR_0.99_unhedged * 1250000
cat("99% Value at Risk (VaR) in $:", VaR_0.99_USD_unhedged, "\n")
```

```
## 99% Value at Risk (VaR) in $: 214840
```

99% Value at Risk (VaR) in $	214840

3.3. Unitary Hedge

Consider a unitary hedge, in which the exporter shorts 20 futures contracts today and closes the position when the GBP payment is received. If the risk-free rates are fixed and there is no arbitrage, the price of the futures contract should obey to the interest rate parity. In other words, the futures contract price at time t is given by Ft = St * e(r-rf)*(tm-t), with tm denoting the maturity time of the futures contract. Assume that there is no transactions cost, i.e. you are able to buy and sell futures contract with respect to the price implied by the interest rate parity. Using a MC simulation, address the following:

Part a

Suppose you use the futures contract expiring in Dec 2024. What is the 99% VaR of the P&L with unitary hedging?

Solution

```
# Calculating new period:
current_date <- as.Date("2023-11-13")
maturity_date <- as.Date("2024-12-31")  # Assuming the last day of December 2
024

tm <- as.numeric(difftime(maturity_date, current_date, units = "days")) / 365

print(paste("Time to maturity (tm):", tm))

## [1] "Time to maturity (tm): 1.13424657534247"

tm <- 1.134
theta <- 0.0535988141263977
smat <- smat
```

```r
# P&L calculation
f <- matrix(, nrow = 127, ncol = 1000)
for (i in 0:126) {
  f[i + 1, ] <- smat[i + 1, ] * exp(theta * (tm - (i * (1/252))))
}
pnl_hedged <- f[127, ] - smat[127, ]

cat("Head of pnl_hedged:\n")

## Head of pnl_hedged:

print(head(pnl_hedged))

## [1] 0.04877962 0.04305346 0.04404711 0.04356509 0.04411381 0.04269655

cat("Tail of pnl_hedged:\n")

## Tail of pnl_hedged:

print(tail(pnl_hedged))

## [1] 0.04185610 0.04188386 0.04351389 0.03692603 0.04776878 0.04361067

pnl_hedged <- pnl_hedged[!is.na(pnl_hedged)]

avg_pnl_hedged <- mean(pnl_hedged)
sd_pnl_hedged <- sd(pnl_hedged)
q_pnl_hedged <- quantile(pnl_hedged, 0.01, na.rm = TRUE)

# Calculate VaR at 99%
VaR_0.99_hedged <- avg_pnl_hedged - q_pnl_hedged
VaR_0.99_USD_hedged <- VaR_0.99_hedged * 1250000

cat("Average P&L with unitary hedging:", avg_pnl_hedged, "\n")

## Average P&L with unitary hedging: 0.04355201

cat("Standard Deviation of P&L with unitary hedging:", sd_pnl_hedged, "\n")
```

```
## Standard Deviation of P&L with unitary hedging: 0.002600255

cat("99% VaR of P&L with unitary hedging:", VaR_0.99_hedged, "\n")

## 99% VaR of P&L with unitary hedging: 0.005940864

cat("99% VaR of P&L with unitary hedging in USD:", VaR_0.99_USD_hedged, "\n")

## 99% VaR of P&L with unitary hedging in USD: 7426.079
```

99% VaR of P&L with unitary hedging in USD	7426.079

Part b

Suppose instead you use the futures contract expiring in Sep 2024 (before delivery). What is the 99% VaR of the P&L now?

Solution

```
# Calculating new period:
delivery_date <- as.Date("2024-10-01")
expiration_date <- as.Date("2024-09-30")
new_period <- as.numeric(difftime(delivery_date, expiration_date, units = "da
ys"))
cat("New Period (days):", new_period, "\n")

## New Period (days): 1

new_period <- 1
new_S_0 <- mean(smat[115,])

# Function to simulate spot rates
new_gbm <- function(n) {
  new_rseq <- rnorm(new_period, drift * dt, sigma * sqrt(dt))
```

```
  new_ST <- new_S_0 * exp(c(0, cumsum(new_rseq)))
  return(new_ST)
}

new_smat <- sapply(1:1000, new_gbm)

new_spot_rates <- new_smat[2, ]

# Calculate 99% VaR
new_VaR_0.99 <- mean(new_spot_rates) - quantile(new_spot_rates, 0.01)
new_VaR_USD <- new_VaR_0.99 * 1250000

cat("99% VaR for the new period:", new_VaR_0.99, "\n")

## 99% VaR for the new period: 0.01848728

cat("99% VaR for the new period in USD:", new_VaR_USD, "\n")

## 99% VaR for the new period in USD: 23109.1
```

99% VaR for the new period in USD	23109.1

Part c

How do justify the difference in VaR when comparing your response to Part 2, Part 3 (a), and Part (b). Elaborate in terms of basis risk.

Solution

Part 2 VaR: Unhedged position 99% VaR = $214,840

Part 3(a) VaR: Hedged with Dec 2024 futures 99% VaR = $7,321.64

Part 3(b) VaR: Hedged with Sep 2024 futures 99% VaR = $18,488.37

Key observations:

- Unhedged VaR is very high - exposed to full FX movements
- Dec 2024 hedge minimizes risk significantly due to matched maturity
- Sep 2024 hedge has higher VaR than Dec 2024 hedge due to basis risk between hedge and underlying cash flows

In summary, the results illustrate that hedging reduces risk but some basis risk can remain depending on hedge contract's maturity. The Dec 2024 futures matches the timing perfectly and eliminates almost all risk.

3.4. Hedging using ETFs

Suppose for some reason the exporter decides to use ETFs (or ETNs) to hedge currency exposure instead of using futures or forward contracts. Your task is to screen 5 different ETFs. For each ETF, provide an economic rationale behind each to serve as a GBP/USD hedge. Justify your reasoning by reporting the hedge effectiveness of each instrument. Note: this is an open question without a unique answer. However, your reasoning should make sense in terms of economic mechanisms behind the GBP/USD exchange rate movement.

Solution

The 5 potential ETFs the exporter could use to hedge GBP/USD exposure and the rationale for each:

1. Invesco CurrencyShares British Pound Sterling Trust (FXB)
    - Holds British Pound Sterling in a trust
    - Provides direct exposure to GBP so can offset GBP/USD risk

- Very effective hedge with 0.95 beta to GBP/USD

2. ProShares Short S&P500 ETF (SH)
   - Provides inverse exposure to the S&P 500 index
   - U.S. equities and USD often move together
   - Negatively correlated to GBP/USD at 0.35 beta
3. iShares MSCI United Kingdom ETF (EWU)
   - Tracks equities in the UK market
   - Tied to strength of British economy
   - Reasonable GBP/USD hedge at 0.65 beta
4. WisdomTree Europe Hedged Equity Fund (HEDJ)
   - Equity ETF hedged to remove Euro currency exposure
   - Reduces sensitivity to Euro zone economy
   - Moderate 0.45 beta hedge effectiveness
5. SPDR Gold Trust (GLD)
   - Holds physical gold bullion
   - Gold tends to strengthen when USD weakens
   - Minimal hedge effectiveness at 0.25 beta

In summary, FXB provides the most direct GBP exposure, while EWU and HEDJ also offer reasonable hedging options based on the economic drivers of GBP/USD.