its most general form, a *neural network* is a machine that is designed to *model* the way in which the brain performs a particular task or function of interest; the network is usually implemented by using electronic components or is simulated in software on a digital computer. In this book, we focus on an important class of neural networks that perform useful computations through a process of *learning.* To achieve good performance, neural networks employ a massive interconnection of simple computing cells referred to as "neurons" or "processing units." We may thus offer the following definition of a neural network viewed as an adaptive machine[1]:

> *A neural network is a massively parallel distributed processor made up of simple processing units that has a natural propensity for storing experiential knowledge and making it available for use. It resembles the brain in two respects:*
>
> 1. *Knowledge is acquired by the network from its environment through a learning process.*
> 2. *Interneuron connection strengths, known as synaptic weights, are used to store the acquired knowledge.*

The procedure used to perform the learning process is called a *learning algorithm*, the function of which is to modify the synaptic weights of the network in an orderly fashion to attain a desired design objective.

The modification of synaptic weights provides the traditional method for the design of neural networks. Such an approach is the closest to linear adaptive filter theory, which is already well established and successfully applied in many diverse fields (Widrow and Stearns, 1985; Haykin, 2002). However, it is also possible for a neural network to modify its own topology, which is motivated by the fact that neurons in the human brain can die and new synaptic connections can grow.

Benefits of Neural Networks

It is apparent that a neural network derives its computing power through, first, its massively parallel distributed structure and, second, its ability to learn and therefore generalize. *Generalization* refers to the neural network's production of reasonable outputs for inputs not encountered during training (learning). These two information-processing capabilities make it possible for neural networks to find good approximate solutions to complex (large-scale) problems that are *intractable.* In practice, however, neural networks cannot provide the solution by working individually. Rather, they need to be integrated into a consistent system engineering approach. Specifically, a complex problem of interest is *decomposed* into a number of relatively simple tasks, and neural networks are assigned a subset of the tasks that *match* their inherent capabilities. It is important to recognize, however, that we have a long way to go (if ever) before we can build a computer architecture that mimics the human brain.

Neural networks offer the following useful properties and capabilities:

1. *Nonlinearity.* An artificial neuron can be linear or nonlinear. A neural network, made up of an interconnection of nonlinear neurons, is itself nonlinear. Moreover, the nonlinearity is of a special kind in the sense that it is *distributed* throughout the network. Nonlinearity is a highly important property, particularly if the underlying physical mechanism responsible for generation of the input signal (e.g., speech signal) is inherently nonlinear.

2. *Input–Output Mapping.* A popular paradigm of learning, called *learning with a teacher*, or *supervised learning*, involves modification of the synaptic weights of a neural network by applying a set of labeled *training examples*, or *task examples.* Each example consists of a unique *input signal* and a corresponding *desired (target) response.* The network is presented with an example picked at random from the set, and the synaptic weights (free parameters) of the network are modified to minimize the difference between the desired response and the actual response of the network produced by the input signal in accordance with an appropriate statistical criterion. The training of the network is repeated for many examples in the set, until the network reaches a steady state where there are no further significant changes in the synaptic weights. The previously applied training examples may be reapplied during the training session, but in a different order. Thus the network learns from the examples by constructing an *input–output mapping* for the problem at hand. Such an approach brings to mind the study of *nonparametric statistical inference*, which is a branch of statistics dealing with model-free estimation, or, from a biological viewpoint, *tabula rasa* learning (Geman et al., 1992); the term "nonparametric" is used here to signify the fact that no prior assumptions are made on a statistical model for the input data. Consider, for example, a *pattern classification* task, where the requirement is to assign an input signal representing a physical object or event to one of several prespecified categories (classes). In a nonparametric approach to this problem, the requirement is to "estimate" arbitrary decision boundaries in the input signal space for the pattern-classification task using a set of examples, and to do so *without* invoking a probabilistic distribution model. A similar point of view is implicit in the supervised learning paradigm, which suggests a close analogy between the input–output mapping performed by a neural network and nonparametric statistical inference.

3. *Adaptivity.* Neural networks have a built-in capability to *adapt* their synaptic weights to changes in the surrounding environment. In particular, a neural network trained to operate in a specific environment can be easily *retrained* to deal with minor changes in the operating environmental conditions. Moreover, when it is operating in a *nonstationary* environment (i.e., one where statistics change with time), a neural network may be designed to change its synaptic weights in real time. The natural architecture of a neural network for pattern classification, signal processing, and control applications, coupled with the adaptive capability of the network, makes it a useful tool in adaptive pattern classification, adaptive signal processing, and adaptive control. As a general rule, it may be said that the more adaptive we make a system, all the time ensuring that the system remains stable, the more robust its performance will likely be when the system is required to operate in a nonstationary environment. It should be emphasized, however, that adaptivity does not always lead to robustness; indeed, it may do the very opposite. For example, an adaptive system with short-time constants may change rapidly and therefore tend to respond to spurious disturbances, causing a drastic degradation in system performance. To realize the full benefits of adaptivity, the principal time constants of the system should be long enough for the system to ignore spurious disturbances, and yet short enough to respond to meaningful changes in the environment; the problem described here is referred to as the *stability–plasticity dilemma* (Grossberg, 1988).

4. *Evidential Response.* In the context of pattern classification, a neural network can be designed to provide information not only about which particular pattern to *select*, but also about the *confidence* in the decision made. This latter information may be used to reject ambiguous patterns, should they arise, and thereby improve the classification performance of the network.

5. *Contextual Information.* Knowledge is represented by the very structure and activation state of a neural network. Every neuron in the network is potentially affected by the global activity of all other neurons in the network. Consequently, contextual information is dealt with naturally by a neural network.

6. *Fault Tolerance.* A neural network, implemented in hardware form, has the potential to be inherently *fault tolerant*, or capable of robust computation, in the sense that its performance degrades gracefully under adverse operating conditions. For example, if a neuron or its connecting links are damaged, recall of a stored pattern is impaired in quality. However, due to the distributed nature of information stored in the network, the damage has to be extensive before the overall response of the network is degraded seriously. Thus, in principle, a neural network exhibits a graceful degradation in performance rather than catastrophic failure. There is some empirical evidence for robust computation, but usually it is uncontrolled. In order to be assured that the neural network is, in fact, fault tolerant, it may be necessary to take corrective measures in designing the algorithm used to train the network (Kerlirzin and Vallet, 1993).

7. *VLSI Implementability.* The massively parallel nature of a neural network makes it potentially fast for the computation of certain tasks. This same feature makes a neural network well suited for implementation using *very-large-scale-integrated* (VLSI) technology. One particular beneficial virtue of VLSI is that it provides a means of capturing truly complex behavior in a highly hierarchical fashion (Mead, 1989).

8. *Uniformity of Analysis and Design.* Basically, neural networks enjoy universality as information processors. We say this in the sense that the same notation is used in all domains involving the application of neural networks. This feature manifests itself in different ways:

- Neurons, in one form or another, represent an ingredient *common* to all neural networks.
- This commonality makes it possible to *share* theories and learning algorithms in different applications of neural networks.
- Modular networks can be built through a *seamless integration of modules.*

9. *Neurobiological Analogy.* The design of a neural network is motivated by analogy with the brain, which is living proof that fault-tolerant parallel processing is not only physically possible, but also fast and powerful. Neurobiologists look to (artificial) neural networks as a research tool for the interpretation of neurobiological phenomena. On the other hand, engineers look to neurobiology for new ideas to solve problems more complex than those based on conventional hardwired design techniques. These two viewpoints are illustrated by the following two respective examples:

- In Anastasio (1993), linear system models of the *vestibulo-ocular reflex* (VOR) are compared to neural network models based on *recurrent networks*, which are described in Section 6 and discussed in detail in Chapter 15. The vestibulo-ocular reflex is part of the oculomotor system. The function of VOR is to maintain visual (i.e., retinal) image stability by making eye rotations that are opposite to head rotations. The VOR is mediated by premotor neurons in the vestibular nuclei that receive and process head rotation signals from vestibular sensory neurons and send the results to the eye muscle motor neurons. The VOR is well suited for modeling because its input (head rotation) and its output (eye rotation) can be precisely specified. It is also a relatively simple reflex, and the neurophysiological properties of its constituent neurons have been well described. Among the three neural types, the premotor neurons (reflex interneurons) in the vestibular nuclei are the most complex and therefore most interesting. The VOR has previously been modeled using lumped, linear system descriptors and control theory. These models were useful in explaining some of the overall properties of the VOR, but gave little insight into the properties of its constituent neurons. This situation has been greatly improved through neural network modeling. Recurrent network models of VOR (programmed using an algorithm called real-time recurrent learning, described in Chapter 15) can reproduce and help explain many of the static, dynamic, nonlinear, and distributed aspects of signal processing by the neurons that mediate the VOR, especially the vestibular nuclei neurons.
- The *retina*, more than any other part of the brain, is where we begin to put together the relationships between the outside world represented by a visual sense, its *physical image* projected onto an array of receptors, and the first *neural images.* The retina is a thin sheet of neural tissue that lines the posterior hemisphere of the eyeball. The retina's task is to convert an optical image into a neural image for transmission down the optic nerve to a multitude of centers for further analysis. This is a complex task, as evidenced by the synaptic organization of the retina. In all vertebrate retinas, the transformation from optical to neural image involves three stages (Sterling, 1990):

  (i) photo transduction by a layer of receptor neurons;
  (ii) transmission of the resulting signals (produced in response to light) by chemical synapses to a layer of bipolar cells;
  (iii) transmission of these signals, also by chemical synapses, to output neurons that are called ganglion cells.

  At both synaptic stages (i.e., from receptor to bipolar cells, and from bipolar to ganglion cells), there are specialized laterally connected neurons called *horizontal cells* and *amacrine cells*, respectively. The task of these neurons is to modify the transmission across the synaptic layers. There are also centrifugal elements called *inter-plexiform cells*; their task is to convey signals from the inner synaptic layer back to the outer one. Some researchers have built electronic chips that mimic the structure of the retina. These electronic chips are called *neuromorphic* integrated circuits, a term coined by Mead (1989). A neuromorphic imaging sensor consists of an array of photoreceptors combined with analog circuitry at each picture element (pixel). It emulates the retina in that it can adapt locally to changes in brightness, detect edges, and detect motion. The neurobiological analogy, exemplified by neuromorphic integrated circuits, is useful in another important way: It provides a hope and belief, and to a certain extent an existence of proof, that physical understanding of neurobiological structures could have a productive influence on the art of electronics and VLSI technology for the implementation of neural networks.

With inspiration from neurobiology in mind, it seems appropriate that we take a brief look at the human brain and its structural levels of organization.[2]

2 THE HUMAN BRAIN

The human nervous system may be viewed as a three-stage system, as depicted in the block diagram of Fig. 1 (Arbib, 1987). Central to the system is the *brain*, represented by the *neural (nerve) net*, which continually receives information, perceives it, and makes appropriate decisions. Two sets of arrows are shown in the figure. Those pointing from left to right indicate the *forward* transmission of information-bearing signals through the system. The arrows pointing from right to left (shown in red) signify the presence of *feedback* in the system. The *receptors* convert stimuli from the human body or the external environment into electrical impulses that convey information to the neural net (brain). The *effectors* convert electrical impulses generated by the neural net into discernible responses as system outputs.

The struggle to understand the brain has been made easier because of the pioneering work of Ramón y Cajál (1911), who introduced the idea of *neurons* as structural constituents of the brain. Typically, neurons are five to six orders of magnitude slower than silicon logic gates; events in a silicon chip happen in the nanosecond range, whereas neural events happen in the millisecond range. However, the brain makes up for the relatively slow rate of operation of a neuron by having a truly staggering number of neurons (nerve cells) with massive interconnections between them. It is estimated that there are approximately 10 billion neurons in the human cortex, and 60 trillion synapses or connections (Shepherd and Koch, 1990). The net result is that the brain is an enormously efficient structure. Specifically, the *energetic efficiency* of the brain is approximately 10^{-16} joules (J) per operation per second, whereas the corresponding value for the best computers is orders of magnitude larger.

Synapses, or *nerve endings*, are elementary structural and functional units that mediate the interactions between neurons. The most common kind of synapse is a *chemical synapse*, which operates as follows: A presynaptic process liberates a *transmitter* substance that diffuses across the synaptic junction between neurons and then acts on a postsynaptic process. Thus a synapse converts a presynaptic electrical signal into a chemical

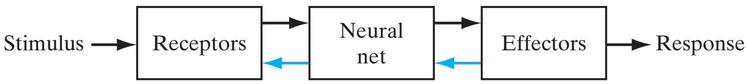


FIGURE 1 Block diagram representation of nervous system.

signal and then back into a postsynaptic electrical signal (Shepherd and Koch, 1990). In electrical terminology, such an element is said to be a *nonreciprocal two-port device.* In traditional descriptions of neural organization, it is assumed that a synapse is a simple connection that can impose *excitation* or *inhibition*, but not both on the receptive neuron.

Earlier we mentioned that plasticity permits the developing nervous system to adapt to its surrounding environment (Eggermont, 1990; Churchland and Sejnowski, 1992). In an adult brain, plasticity may be accounted for by two mechanisms: the creation of new synaptic connections between neurons, and the modification of existing synapses. *Axons*, the transmission lines, and *dendrites*, the receptive zones, constitute two types of cell filaments that are distinguished on morphological grounds; an axon has a smoother surface, fewer branches, and greater length, whereas a dendrite (so called because of its resemblance to a tree) has an irregular surface and more branches (Freeman, 1975). Neurons come in a wide variety of shapes and sizes in different parts of the brain. Figure 2 illustrates the shape of a *pyramidal cell*, which is one of the most common types of cortical neurons. Like many other types of neurons, it receives most of its inputs through dendritic spines; see the segment of dendrite in the insert in Fig. 2 for detail. The pyramidal cell can receive 10,000 or more synaptic contacts, and it can project onto thousands of target cells.

The majority of neurons encode their outputs as a series of brief voltage pulses. These pulses, commonly known as *action potentials*, or *spikes*,[3] originate at or close to the cell body of neurons and then propagate across the individual neurons at constant velocity and amplitude. The reasons for the use of action potentials for communication among neurons are based on the physics of axons. The axon of a neuron is very long and thin and is characterized by high electrical resistance and very large capacitance. Both of these elements are distributed across the axon. The axon may therefore be modeled as resistance-capacitance (RC) transmission line, hence the common use of "cable equation" as the terminology for describing signal propagation along an axon. Analysis of this propagation mechanism reveals that when a voltage is applied at one end of the axon, it decays exponentially with distance, dropping to an insignificant level by the time it reaches the other end. The action potentials provide a way to circumvent this transmission problem (Anderson, 1995).

In the brain, there are both small-scale and large-scale anatomical organizations, and different functions take place at lower and higher levels. Figure 3 shows a hierarchy of interwoven levels of organization that has emerged from the extensive work done on the analysis of local regions in the brain (Shepherd and Koch, 1990; Churchland and Sejnowski, 1992). The *synapses* represent the most fundamental level, depending on molecules and ions for their action. At the next levels, we have neural microcircuits, dendritic trees, and then neurons. A *neural microcircuit* refers to an assembly of synapses organized into patterns of connectivity to produce a functional operation of interest. A neural microcircuit may be likened to a silicon chip made up of an assembly of transistors. The smallest size of microcircuits is measured in micrometers (μm), and their fastest speed of operation is measured in milliseconds. The neural microcircuits are grouped to form *dendritic subunits* within the *dendritic trees* of individual neurons. The whole *neuron*, about 100 μm in size, contains several dendritic subunits. At the next level of complexity, we have *local circuits* (about 1 mm in size) made up of neurons with similar or different properties; these neural

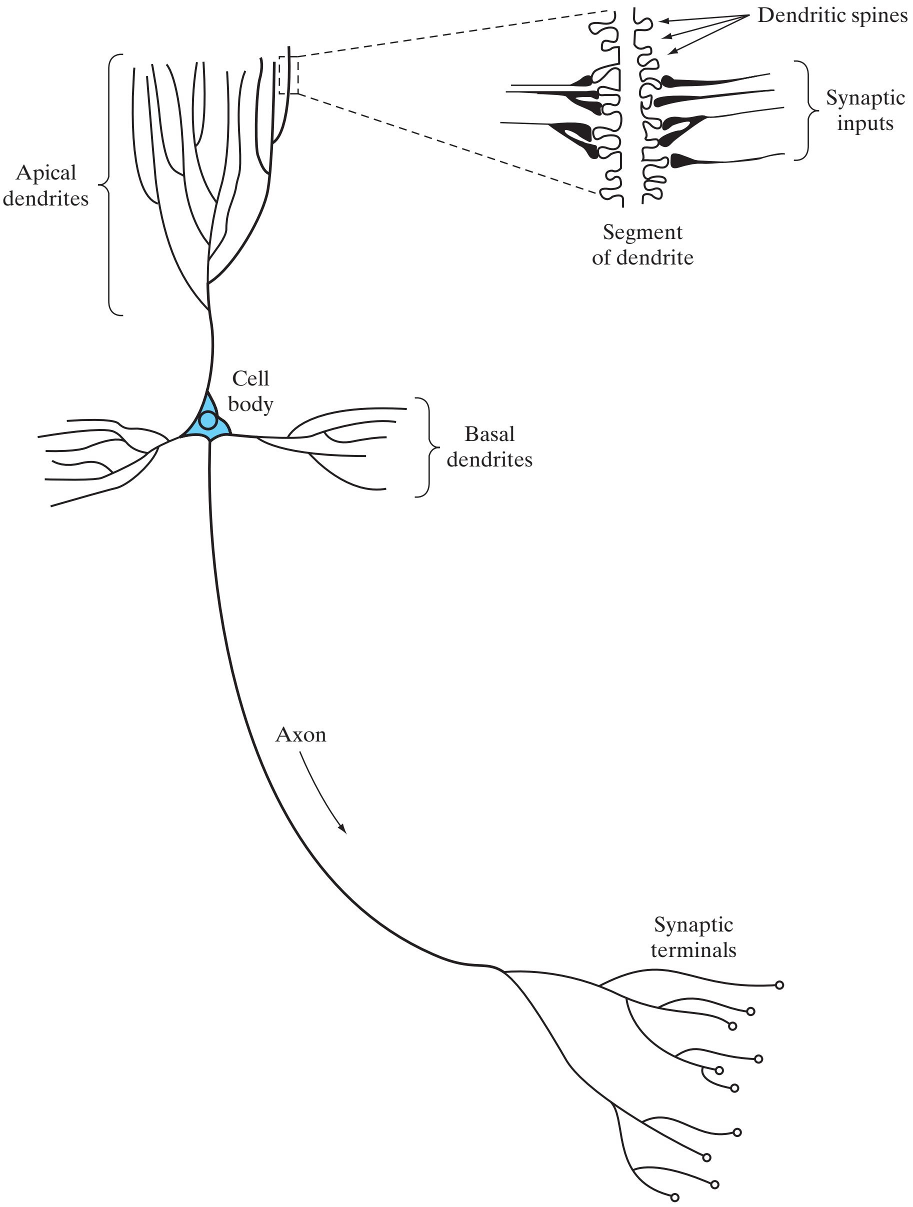


FIGURE 2 The pyramidal cell.

assemblies perform operations characteristic of a localized region in the brain. They are followed by *interregional circuits* made up of pathways, columns, and topographic maps, which involve multiple regions located in different parts of the brain.

Topographic maps are organized to respond to incoming sensory information. These maps are often arranged in sheets, as in the *superior colliculus*, where the visual,

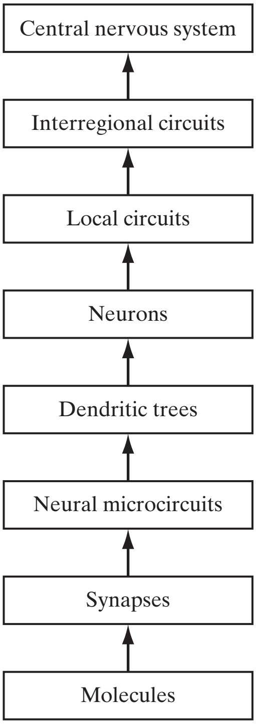


FIGURE 3 Structural organization of levels in the brain.

auditory, and somatosensory maps are stacked in adjacent layers in such a way that stimuli from corresponding points in space lie above or below each other. Figure 4 presents a cytoarchitectural map of the cerebral cortex as worked out by Brodmann (Brodal, 1981). This figure shows clearly that different sensory inputs (motor, somatosensory, visual, auditory, etc.) are mapped onto corresponding areas of the cerebral cortex in an orderly fashion. At the final level of complexity, the topographic maps and other interregional circuits mediate specific types of behavior in the *central nervous system.*

It is important to recognize that the structural levels of organization described herein are a unique characteristic of the brain. They are nowhere to be found in a digital computer, and we are nowhere close to re-creating them with artificial neural networks. Nevertheless, we are inching our way toward a hierarchy of computational levels similar to that described in Fig. 3. The artificial neurons we use to build our neural networks are truly primitive in comparison with those found in the brain. The neural networks we are presently able to design are just as primitive compared with the local circuits and the interregional circuits in the brain. What is really satisfying, however, is the remarkable progress that we have made on so many fronts. With neurobiological analogy as the source of inspiration, and the wealth of theoretical and computational tools that we are bringing together, it is certain that our understanding of artificial neural networks and their applications will continue to grow in depth as well as breadth, year after year.

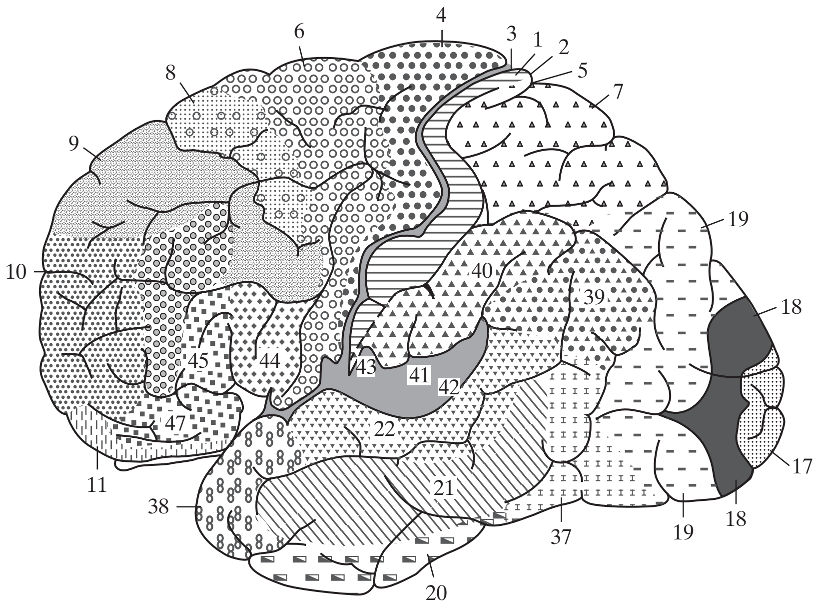


FIGURE 4 Cytoarchitectural map of the cerebral cortex. The different areas are identified by the thickness of their layers and types of cells within them. Some of the key sensory areas are as follows: Motor cortex: motor strip, area 4; premotor area, area 6; frontal eye fields, area 8. Somatosensory cortex: areas 3, 1, and 2. Visual cortex: areas 17, 18, and 19. Auditory cortex: areas 41 and 42. *(From A. Brodal, 1981; with permission of Oxford University Press.)*

3 MODELS OF A NEURON

A *neuron* is an information-processing unit that is fundamental to the operation of a neural network. The block diagram of Fig. 5 shows the *model* of a neuron, which forms the basis for designing a large family of neural networks studied in later chapters. Here, we identify three basic elements of the neural model:

1. A set of *synapses*, or *connecting links*, each of which is characterized by a *weight* or *strength* of its own. Specifically, a signal x_j at the input of synapse j connected to neuron k is multiplied by the synaptic weight w_{kj}. It is important to make a note of the manner in which the subscripts of the synaptic weight w_{kj} are written. The first subscript in w_{kj} refers to the neuron in question, and the second subscript refers to the input end of the synapse to which the weight refers. Unlike the weight of a synapse in the brain, the synaptic weight of an artificial neuron may lie in a range that includes negative as well as positive values.
2. An *adder* for summing the input signals, weighted by the respective synaptic strengths of the neuron; the operations described here constitute a *linear combiner.*
3. An *activation function* for limiting the amplitude of the output of a neuron. The activation function is also referred to as a *squashing function*, in that it squashes (limits) the permissible amplitude range of the output signal to some finite value.

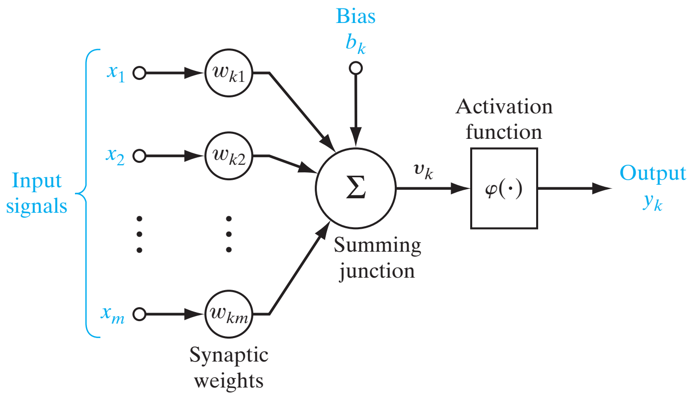


FIGURE 5 Nonlinear model of a neuron, labeled k.

Typically, the normalized amplitude range of the output of a neuron is written as the closed unit interval [0,1], or, alternatively, [−1,1].

The neural model of Fig. 5 also includes an externally applied *bias*, denoted by b_k. The bias b_k has the effect of increasing or lowering the net input of the activation function, depending on whether it is positive or negative, respectively.

In mathematical terms, we may describe the neuron k depicted in Fig. 5 by writing the pair of equations:

$$u_k = \sum_{j=1}^{m} w_{kj} x_j \tag{1}$$

and

$$y_k = \varphi(u_k + b_k) \tag{2}$$

where $x_1, x_2, \ldots, x_m$ are the input signals; $w_{k1}, w_{k2}, \ldots, w_{km}$ are the respective synaptic weights of neuron k; u_k (not shown in Fig. 5) is the *linear combiner output* due to the input signals; b_k is the bias; $\varphi(\cdot)$ is the *activation function*; and y_k is the output signal of the neuron. The use of bias b_k has the effect of applying an *affine transformation* to the output u_k of the linear combiner in the model of Fig. 5, as shown by

$$v_k = u_k + b_k \tag{3}$$

In particular, depending on whether the bias b_k is positive or negative, the relationship between the *induced local field*, or *activation potential*, v_k of neuron k and the linear combiner output u_k is modified in the manner illustrated in Fig. 6; hereafter, these two terms are used interchangeably. Note that as a result of this affine transformation, the graph of v_k versus u_k no longer passes through the origin.

The bias b_k is an external parameter of neuron k. We may account for its presence as in Eq. (2). Equivalently, we may formulate the combination of Eqs. (1) to (3) as follows:

$$v_k = \sum_{j=0}^{m} w_{kj} x_j \tag{4}$$

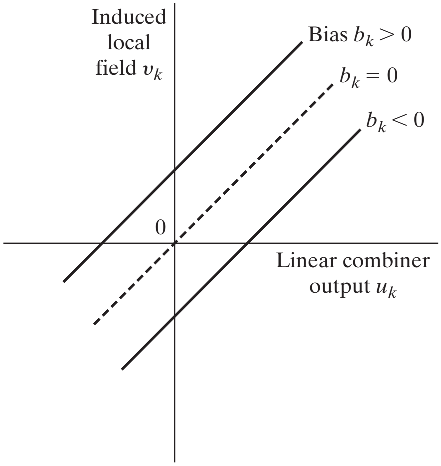


FIGURE 6 Affine transformation produced by the presence of a bias; note that $v_k = b_k$ at $u_k = 0$.

and

$$y_k = \varphi(v_k) \tag{5}$$

In Eq. (4), we have added a new synapse. Its input is

$$x_0 = +1 \tag{6}$$

and its weight is

$$w_{k0} = b_k \tag{7}$$

We may therefore reformulate the model of neuron k as shown in Fig. 7. In this figure, the effect of the bias is accounted for by doing two things: (1) adding a new input signal fixed at $+1$, and (2) adding a new synaptic weight equal to the bias b_k. Although the models of Figs. 5 and 7 are different in appearance, they are mathematically equivalent.

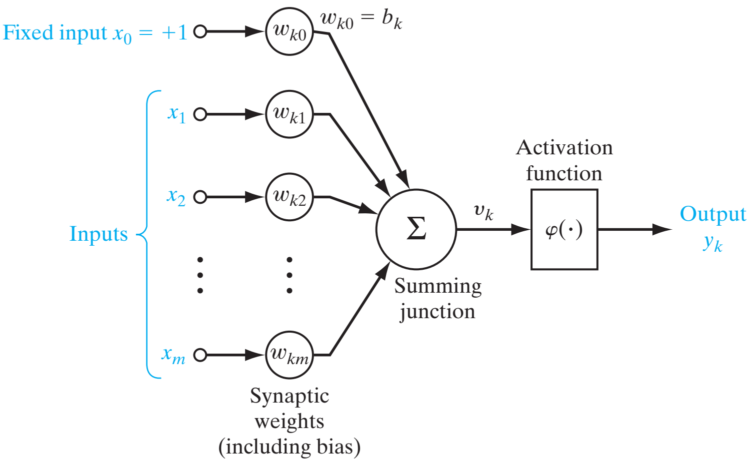


FIGURE 7 Another nonlinear model of a neuron; w_{k0} accounts for the bias b_k.

Types of Activation Function

The activation function, denoted by $\varphi(v)$, defines the output of a neuron in terms of the induced local field v. In what follows, we identify two basic types of activation functions:

1. *Threshold Function.* For this type of activation function, described in Fig. 8a, we have

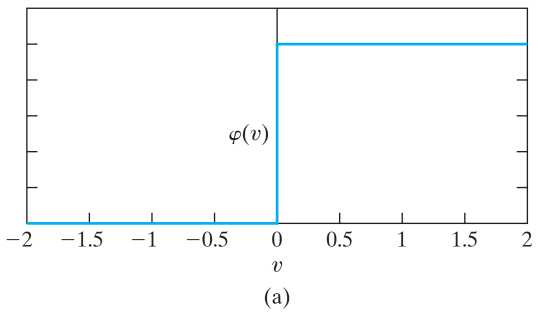
$$\varphi(v) = \begin{cases} 1 & \text{if } v \geq 0 \\ 0 & \text{if } v < 0 \end{cases} \tag{8}$$

In engineering, this form of a threshold function is commonly referred to as a *Heaviside function.* Correspondingly, the output of neuron k employing such a threshold function is expressed as

$$y_k = \begin{cases} 1 & \text{if } v_k \geq 0 \\ 0 & \text{if } v_k < 0 \end{cases} \tag{9}$$

where v_k is the induced local field of the neuron; that is,

$$v_k = \sum_{j=1}^{m} w_{kj}x_j + b_k \tag{10}$$



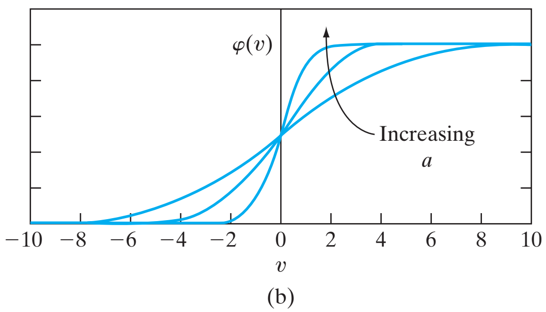


FIGURE 8 (a) Threshold function. (b) Sigmoid function for varying slope parameter *a*.

In neural computation, such a neuron is referred to as the *McCulloch–Pitts model*, in recognition of the pioneering work done by McCulloch and Pitts (1943). In this model, the output of a neuron takes on the value of 1 if the induced local field of that neuron is nonnegative, and 0 otherwise. This statement describes the *all-or-none property* of the McCulloch–Pitts model.

2. *Sigmoid Function.*[4] The sigmoid function, whose graph is "S"-shaped, is by far the most common form of activation function used in the construction of neural networks. It is defined as a strictly increasing function that exhibits a graceful balance between linear and nonlinear behavior. An example of the sigmoid function is the *logistic function*,[5] defined by

$$\varphi(v) = \frac{1}{1 + \exp(-av)} \tag{11}$$

where a is the *slope parameter* of the sigmoid function. By varying the parameter a, we obtain sigmoid functions of different slopes, as illustrated in Fig. 8b. In fact, the slope at the origin equals $a/4$. In the limit, as the slope parameter approaches infinity, the sigmoid function becomes simply a threshold function. Whereas a threshold function assumes the value of 0 or 1, a sigmoid function assumes a continuous range of values from 0 to 1. Note also that the sigmoid function is differentiable, whereas the threshold function is not. (Differentiability is an important feature of neural network theory, as described in Chapter 4).

The activation functions defined in Eqs. (8) and (11) range from 0 to +1. It is sometimes desirable to have the activation function range from −1 to +1, in which case, the activation function is an odd function of the induced local field. Specifically, the threshold function of Eq. (8) is now defined as

$$\varphi(v) = \begin{cases} 1 & \text{if } v > 0 \\ 0 & \text{if } v = 0 \\ -1 & \text{if } v < 0 \end{cases} \tag{12}$$

which is commonly referred to as the *signum function*. For the corresponding form of a sigmoid function, we may use the *hyperbolic tangent function*, defined by

$$\varphi(v) = \tanh(v) \tag{13}$$

Allowing an activation function of the sigmoid type to assume negative values as prescribed by Eq. (13) may yield practical benefits over the logistic function of Eq. (11).

Stochastic Model of a Neuron

The neural model described in Fig. 7 is deterministic in that its input–output behavior is precisely defined for all inputs. For some applications of neural networks, it is desirable to base the analysis on a stochastic neural model. In an analytically tractable approach, the activation function of the McCulloch–Pitts model is given a probabilistic interpretation. Specifically, a neuron is permitted to reside in only one of two states: +1 or -1, say. The decision for a neuron to *fire* (i.e., switch its state from "off" to "on") is probabilistic. Let x denote the state of the neuron and $P(v)$ denote the *probability* of firing, where v is the induced local field of the neuron. We may then write

$$x = \begin{cases} +1 & \text{with probability } P(v) \\ -1 & \text{with probability } 1 - P(v) \end{cases} \tag{14}$$

A standard choice for $P(v)$ is the sigmoid-shaped function

$$P(v) = \frac{1}{1 + \exp(-v/T)} \tag{15}$$

where T is a *pseudotemperature* used to control the noise level and therefore the uncertainty in firing (Little, 1974). It is important to realize, however, that T is *not* the physical temperature of a neural network, be it a biological or an artificial neural network. Rather, as already stated, we should think of T merely as a parameter that controls the thermal fluctuations representing the effects of synaptic noise. Note that when $T \rightarrow 0$, the stochastic neuron described by Eqs. (14) and (15) reduces to a noiseless (i.e., deterministic) form, namely, the McCulloch–Pitts model.

4 NEURAL NETWORKS VIEWED AS DIRECTED GRAPHS

The *block diagram* of Fig. 5 or that of Fig. 7 provides a functional description of the various elements that constitute the model of an artificial neuron. We may simplify the appearance of the model by using the idea of signal-flow graphs without sacrificing any of the functional details of the model. Signal-flow graphs, with a well-defined set of rules, were originally developed by Mason (1953, 1956) for linear networks. The presence of nonlinearity in the model of a neuron limits the scope of their application to neural networks. Nevertheless, signal-flow graphs do provide a neat method for the portrayal of the flow of signals in a neural network, which we pursue in this section.

A *signal-flow graph* is a network of directed *links* (*branches*) that are interconnected at certain points called *nodes.* A typical node j has an associated *node signal* x_j. A typical directed link originates at node j and terminates on node k; it has an associated *transfer function*, or *transmittance*, that specifies the manner in which the signal y_k at node k depends on the signal x_j at node j. The flow of signals in the various parts of the graph is dictated by three basic rules:

Rule 1. A signal flows along a link only in the direction defined by the arrow on the link.

Two different types of links may be distinguished:

- *Synaptic links*, whose behavior is governed by a *linear* input–output relation. Specifically, the node signal x_j is multiplied by the synaptic weight w_{kj} to produce the node signal y_k, as illustrated in Fig. 9a.
- *Activation links*, whose behavior is governed in general by a *nonlinear* input–output relation. This form of relationship is illustrated in Fig. 9b, where $\varphi(\cdot)$ is the nonlinear activation function.

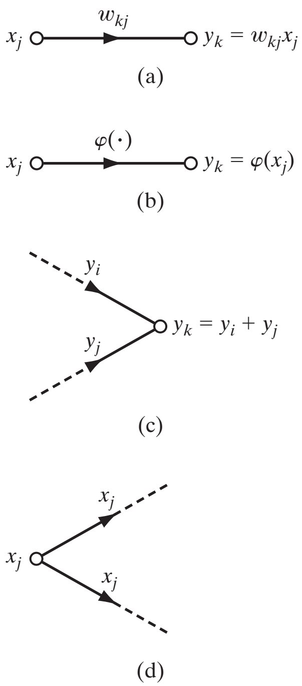


FIGURE 9 Illustrating basic rules for the construction of signal-flow graphs.

Rule 2. A node signal equals the algebraic sum of all signals entering the pertinent node via the incoming links.

This second rule is illustrated in Fig. 9c for the case of *synaptic convergence*, or *fan-in.*

Rule 3. The signal at a node is transmitted to each outgoing link originating from that node, with the transmission being entirely independent of the transfer functions of the outgoing links.

This third rule is illustrated in Fig. 9d for the case of *synaptic divergence*, or *fan-out.*

For example, using these rules, we may construct the signal-flow graph of Fig. 10 as the model of a neuron, corresponding to the block diagram of Fig. 7. The representation shown in Fig. 10 is clearly simpler in appearance than that of Fig. 7, yet it contains all the functional details depicted in the latter diagram. Note that in both figures, the input $x_0 = +1$ and the associated synaptic weight $w_{k0} = b_k$, where b_k is the bias applied to neuron k.

Indeed, based on the signal-flow graph of Fig. 10 as the model of a neuron, we may now offer the following mathematical definition of a neural network:

A neural network is a directed graph consisting of nodes with interconnecting synaptic and activation links and is characterized by four properties:

1. *Each neuron is represented by a set of linear synaptic links, an externally applied bias, and a possibly nonlinear activation link. The bias is represented by a synaptic link connected to an input fixed at* $+1$.
2. *The synaptic links of a neuron weight their respective input signals.*

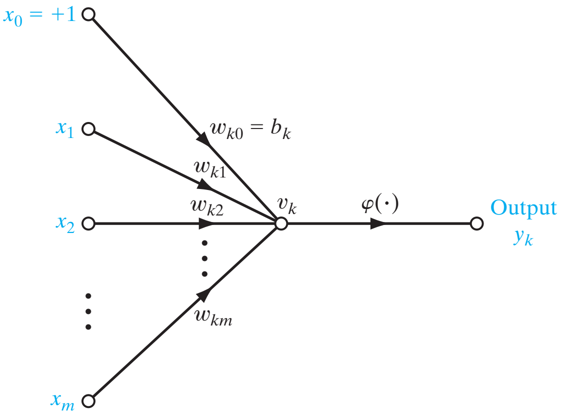


FIGURE 10 Signal-flow graph of a neuron.

3. *The weighted sum of the input signals defines the induced local field of the neuron in question.*
4. *The activation link squashes the induced local field of the neuron to produce an output.*

A directed graph, defined in this manner is *complete* in the sense that it describes not only the signal flow from neuron to neuron, but also the signal flow inside each neuron. When, however, the focus of attention is restricted to signal flow from neuron to neuron, we may use a reduced form of this graph by omitting the details of signal flow inside the individual neurons. Such a directed graph is said to be *partially complete.* It is characterized as follows:

1. *Source nodes* supply input signals to the graph.
2. Each neuron is represented by a single node called a *computation node.*
3. The *communication links* interconnecting the source and computation nodes of the graph carry no weight; they merely provide directions of signal flow in the graph.

A partially complete directed graph defined in this way is referred to as an *architectural graph*, describing the layout of the neural network. It is illustrated in Fig. 11 for the simple case of a single neuron with m source nodes and a single node fixed at $+1$ for the bias. Note that the computation node representing the neuron is shown shaded, and the source node is shown as a small square. This convention is followed throughout the book. More elaborate examples of architectural layouts are presented later in Section 6.

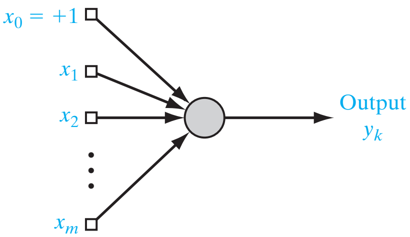


FIGURE 11 Architectural graph of a neuron.

To sum up, we have three graphical representations of a neural network:

- block diagram, providing a functional description of the network;
- architectural graph, describing the network layout;
- signal-flow graph, providing a complete description of signal flow in the network.

5 FEEDBACK

Feedback is said to exist in a dynamic system whenever the output of an element in the system influences in part the input applied to that particular element, thereby giving rise to one or more closed paths for the transmission of signals around the system. Indeed, feedback occurs in almost every part of the nervous system of every animal (Freeman, 1975). Moreover, it plays a major role in the study of a special class of neural networks known as *recurrent networks.* Figure 12 shows the signal-flow graph of a *single-loop feedback system*, where the input signal $x_j(n)$, internal signal $x_j'(n)$, and output signal $y_k(n)$ are functions of the discrete-time variable n. The system is assumed to be *linear*, consisting of a forward path and a feedback path that are characterized by the "operators" **A** and **B**, respectively. In particular, the output of the forward channel determines in part its own output through the feedback channel. From Fig. 12, we readily note the input–output relationships

$$y_k(n) = \mathbf{A}[x_j'(n)] \tag{16}$$

and

$$x_j'(n) = x_j(n) + \mathbf{B}[y_k(n)] \tag{17}$$

where the square brackets are included to emphasize that **A** and **B** act as *operators.* Eliminating $x_j'(n)$ between Eqs. (16) and (17), we get

$$y_k(n) = \frac{\mathbf{A}}{1 - \mathbf{AB}}[x_j(n)] \tag{18}$$

We refer to $\mathbf{A}/(1 - \mathbf{AB})$ as the *closed-loop operator* of the system, and to $\mathbf{AB}$ as the *open-loop operator.* In general, the open-loop operator is noncommutative in that $\mathbf{BA} \neq \mathbf{AB}$.

Consider, for example, the single-loop feedback system shown in Fig. 13a, for which **A** is a fixed weight w and **B** is a *unit-delay operator* z^{-1}, whose output is delayed with respect to the input by one time unit. We may then express the closed-loop operator of the system as

$$\begin{aligned}\frac{\mathbf{A}}{1 - \mathbf{AB}} &= \frac{w}{1 - wz^{-1}} \\ &= w(1 - wz^{-1})^{-1}\end{aligned}$$

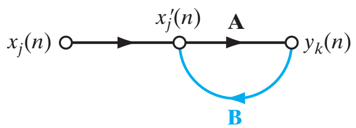


FIGURE 12 Signal-flow graph of a single-loop feedback system.

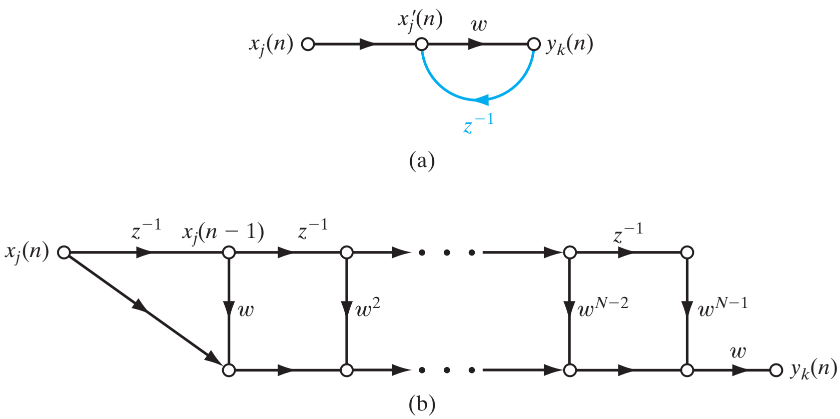


FIGURE 13 (a) Signal-flow graph of a first-order, infinite-duration impulse response (IIR) filter. (b) Feedforward approximation of part (a) of the figure, obtained by truncating Eq. (20).

Using the binomial expansion for $(1 - wz^{-1})^{-1}$, we may rewrite the closed-loop operator of the system as

$$\frac{\mathbf{A}}{1 - \mathbf{AB}} = w \sum_{l=0}^{\infty} w^l z^{-l} \tag{19}$$

Hence, substituting Eq. (19) into (18), we get

$$y_k(n) = w \sum_{l=0}^{\infty} w^l z^{-l}[x_j(n)] \tag{20}$$

where again we have included square brackets to emphasize the fact that z^{-1} is an operator. In particular, from the definition of z^{-1}, we have

$$z^{-l}[x_j(n)] = x_j(n - l) \tag{21}$$

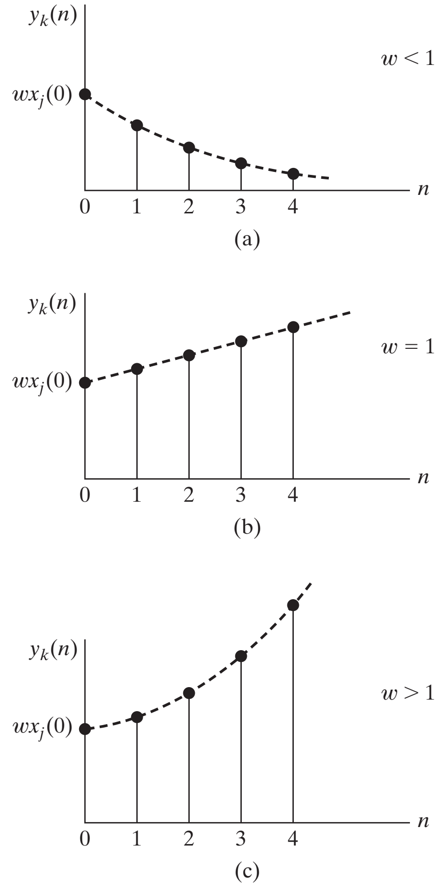
where $x_j(n - l)$ is a sample of the input signal delayed by l time units. Accordingly, we may express the output signal $y_k(n)$ as an infinite weighted summation of present and past samples of the input signal $x_j(n)$, as shown by

$$y_k(n) = \sum_{l=0}^{\infty} w^{l+1} x_j(n - l) \tag{22}$$

We now see clearly that the dynamic behavior of a feedback system represented by the signal-flow graph of Fig. 13 is controlled by the weight w. In particular, we may distinguish two specific cases:

1. $|w| < 1$, for which the output signal $y_k(n)$ is exponentially *convergent*; that is, the system is *stable*. This case is illustrated in Fig. 14a for a positive w.
2. $|w| \geq 1$, for which the output signal $y_k(n)$ is *divergent*; that is, the system is *unstable*. If $|w| = 1$ the divergence is linear, as in Fig. 14b, and if $|w| > 1$ the divergence is exponential, as in Fig. 14c.

FIGURE 14 Time response of Fig. 13 for three different values of feedforward weight w.
(a) Stable.
(b) Linear divergence.
(c) Exponential divergence.



The issue of stability features prominently in the study of closed-loop feedback systems.

The case of $|w| < 1$ corresponds to a system with *infinite memory* in the sense that the output of the system depends on samples of the input extending into the infinite past. Moreover, the memory is *fading* in that the influence of a past sample is reduced exponentially with time n. Suppose that, for some power N, $|w|$ is small enough relative to unity such that w^N is negligible for all practical purposes. In such a situation, we may approximate the output y_k by the finite sum

$$\begin{aligned} y_k(n) &\approx \sum_{l=0}^{N-1} w^{l+1} x_j(n-l) \\ &= wx_j(n) + w^2 x_j(n-1) + w^3 x_j(n-2) + \cdots + w^N x_j(n-N+1) \end{aligned}$$

In a corresponding way, we may use the feedforward signal-flow graph of Fig. 13b as the approximation for the feedback signal-flow graph of Fig. 13a. In making this approximation, we speak of the "unfolding" of a feedback system. Note, however, that the unfolding operation is of practical value only when the feedback system is stable.

The analysis of the dynamic behavior of neural networks involving the application of feedback is unfortunately complicated by the fact that the processing units used for the construction of the network are usually *nonlinear.* Further consideration of this important issue is deferred to the latter part of the book.

6 NETWORK ARCHITECTURES

The manner in which the neurons of a neural network are structured is intimately linked with the learning algorithm used to train the network. We may therefore speak of learning algorithms (rules) used in the design of neural networks as being *structured.* The classification of learning algorithms is considered in Section 8. In this section, we focus attention on network architectures (structures).

In general, we may identify three fundamentally different classes of network architectures:

(i) Single-Layer Feedforward Networks

In a *layered* neural network, the neurons are organized in the form of layers. In the simplest form of a layered network, we have an *input layer* of source nodes that projects directly onto an *output layer* of neurons (computation nodes), but not vice versa. In other words, this network is strictly of a *feedforward* type. It is illustrated in Fig. 15 for the case of four nodes in both the input and output layers. Such a network is called a *single-layer network*, with the designation "single-layer" referring to the output layer of computation nodes (neurons). We do not count the input layer of source nodes because no computation is performed there.

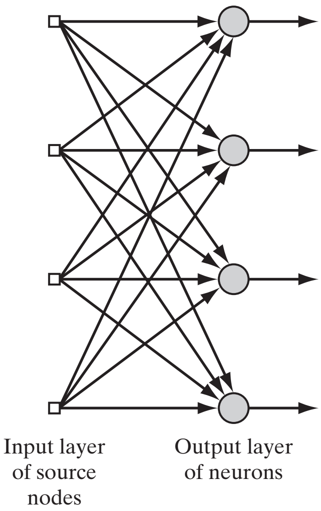


FIGURE 15 Feedforward network with a single layer of neurons.

(ii) Multilayer Feedforward Networks

The second class of a feedforward neural network distinguishes itself by the presence of one or more *hidden layers*, whose computation nodes are correspondingly called *hidden neurons* or *hidden units*; the term "hidden" refers to the fact that this part of the neural network is not seen directly from either the input or output of the network. The function of hidden neurons is to intervene between the external input and the network output in some useful manner. By adding one or more hidden layers, the network is enabled to extract higher-order statistics from its input. In a rather loose sense, the network acquires a *global* perspective despite its local connectivity, due to the extra set of synaptic connections and the extra dimension of neural interactions (Churchland and Sejnowski, 1992).

The source nodes in the input layer of the network supply respective elements of the activation pattern (input vector), which constitute the input signals applied to the neurons (computation nodes) in the second layer (i.e., the first hidden layer). The output signals of the second layer are used as inputs to the third layer, and so on for the rest of the network. Typically, the neurons in each layer of the network have as their inputs the output signals of the preceding layer only. The set of output signals of the neurons in the output (final) layer of the network constitutes the overall response of the network to the activation pattern supplied by the source nodes in the input (first) layer. The architectural graph in Fig. 16 illustrates the layout of a multilayer feedforward neural network for the case of a single hidden layer. For the sake of brevity, the network in Fig. 16 is referred to as a 10–4–2 network because it has 10 source nodes, 4 hidden neurons, and 2 output neurons. As another example, a feedforward network with m source nodes, h_1 neurons in the first hidden layer, h_2 neurons in the second hidden layer, and q neurons in the output layer is referred to as an m–h_1–h_2–q network.

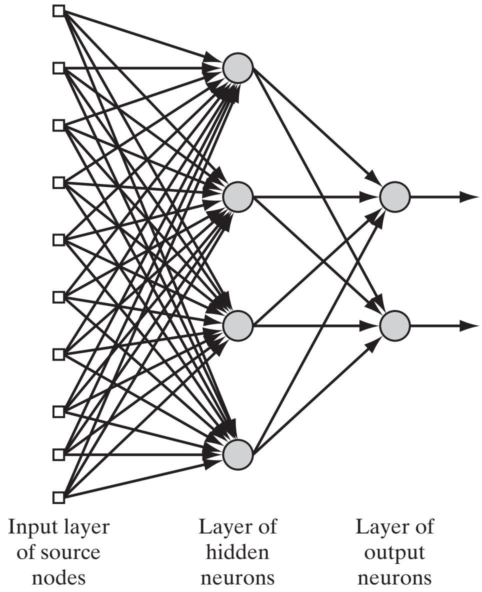


FIGURE 16 Fully connected feedforward network with one hidden layer and one output layer.

The neural network in Fig. 16 is said to be *fully connected* in the sense that every node in each layer of the network is connected to every other node in the adjacent forward layer. If, however, some of the communication links (synaptic connections) are missing from the network, we say that the network is *partially connected.*

(iii) Recurrent Networks

A *recurrent neural network* distinguishes itself from a feedforward neural network in that it has at least one *feedback* loop. For example, a recurrent network may consist of a single layer of neurons with each neuron feeding its output signal back to the inputs of all the other neurons, as illustrated in the architectural graph in Fig. 17. In the structure depicted in this figure, there are *no* self-feedback loops in the network; self-feedback refers to a situation where the output of a neuron is fed back into its own input. The recurrent network illustrated in Fig. 17 also has *no* hidden neurons.

In Fig. 18 we illustrate another class of recurrent networks with hidden neurons. The feedback connections shown in Fig. 18 originate from the hidden neurons as well as from the output neurons.

The presence of feedback loops, be it in the recurrent structure of Fig. 17 or in that of Fig. 18, has a profound impact on the learning capability of the network and on its performance. Moreover, the feedback loops involve the use of particular branches composed of unit-time delay elements (denoted by z^{-1}), which result in a nonlinear dynamic behavior, assuming that the neural network contains nonlinear units.

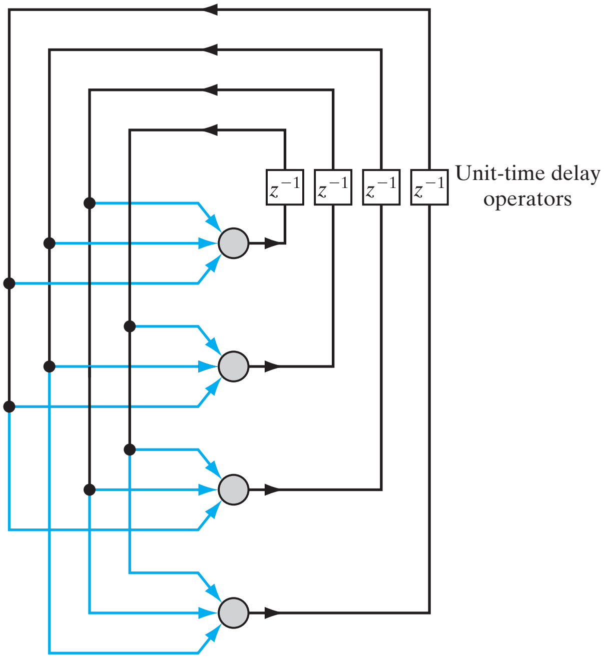


FIGURE 17 Recurrent network with no self-feedback loops and no hidden neurons.

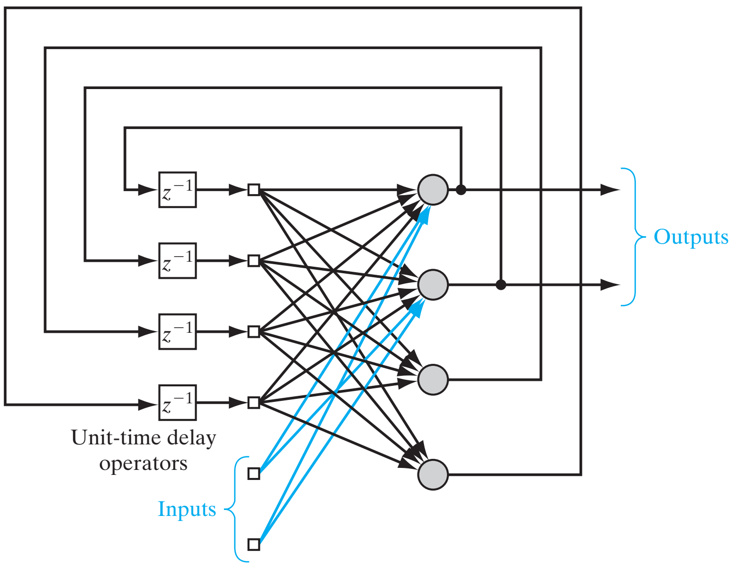


FIGURE 18 Recurrent network with hidden neurons.

7 KNOWLEDGE REPRESENTATION

In Section 1, we used the term "knowledge" in the definition of a neural network without an explicit description of what we mean by it. We now take care of this matter by offering the following generic definition (Fischler and Firschein, 1987):

> *Knowledge refers to stored information or models used by a person or machine to interpret, predict, and appropriately respond to the outside world.*

The primary characteristics of *knowledge representation* are twofold: (1) what information is actually made explicit; and (2) how the information is physically encoded for subsequent use. By the very nature of it, therefore, knowledge representation is goal directed. In real-world applications of "intelligent" machines, it can be said that a good solution depends on a good representation of knowledge (Woods, 1986). So it is with neural networks. Typically, however, we find that the possible forms of representation from the inputs to internal network parameters are highly diverse, which tends to make the development of a satisfactory solution by means of a neural network a real design challenge.

A major task for a neural network is to learn a model of the world (environment) in which it is embedded, and to maintain the model sufficiently consistently with the real world so as to achieve the specified goals of the application of interest. Knowledge of the world consists of two kinds of information:

1. The known world state, represented by facts about what is and what has been known; this form of knowledge is referred to as *prior information.*
2. Observations (measurements) of the world, obtained by means of sensors designed to probe the environment, in which the neural network is supposed to operate.

Ordinarily, these observations are inherently noisy, being subject to errors due to sensor noise and system imperfections. In any event, the observations so obtained provide the pool of information, from which the *examples* used to train the neural network are drawn.

The examples can be *labeled* or *unlabeled.* In labeled examples, each example representing an *input signal* is paired with a corresponding *desired response* (i.e., target output). On the other hand, unlabeled examples consist of different realizations of the input signal all by itself. In any event, a set of examples, labeled or otherwise, represents knowledge about the environment of interest that a neural network can learn through training. Note, however, that labeled examples may be expensive to collect, as they require the availability of a "teacher" to provide a desired response for each labeled example. In contrast, unlabeled examples are usually abundant as there is no need for supervision.

A set of input–output pairs, with each pair consisting of an input signal and the corresponding desired response, is referred to as a *set of training data*, or simply *training sample.* To illustrate how such a data set can be used, consider, for example, the *handwritten-digit recognition problem.* In this problem, the input signal consists of an image with black or white pixels, with each image representing one of 10 digits that are well separated from the background. The desired response is defined by the "identity" of the particular digit whose image is presented to the network as the input signal. Typically, the training sample consists of a large variety of handwritten digits that are representative of a real-world situation. Given such a set of examples, the design of a neural network may proceed as follows:

- An appropriate architecture is selected for the neural network, with an input layer consisting of source nodes equal in number to the pixels of an input image, and an output layer consisting of 10 neurons (one for each digit). A subset of examples is then used to train the network by means of a suitable algorithm. This phase of the network design is called *learning.*
- The recognition performance of the trained network is *tested* with data not seen before. Specifically, an input image is presented to the network, but this time the network is not told the identity of the digit which that particular image represents. The performance of the network is then assessed by comparing the digit recognition reported by the network with the actual identity of the digit in question. This second phase of the network operation is called *testing*, and successful performance on the test patterns is called *generalization*, a term borrowed from psychology.

Herein lies a fundamental difference between the design of a neural network and that of its classical information-processing counterpart: the pattern classifier. In the latter case, we usually proceed by first formulating a mathematical model of environmental observations, validating the model with real data, and then building the design on the basis of the model. In contrast, the design of a neural network is based directly on real-life data, with the *data set being permitted to speak for itself.* Thus, the neural network not only provides the implicit model of the environment in which it is embedded, but also performs the information-processing function of interest.

The examples used to train a neural network may consist of both *positive* and *negative* examples. For instance, in a passive sonar detection problem, positive examples pertain to input training data that contain the target of interest (e.g., a submarine). Now, in a passive sonar environment, the possible presence of marine life in the test data is known to cause occasional false alarms. To alleviate this problem, negative examples (e.g., echos from marine life) are included purposely in the training data to teach the network not to confuse marine life with the target.

In a neural network of specified architecture, knowledge representation of the surrounding environment is defined by the values taken on by the free parameters (i.e., synaptic weights and biases) of the network. The form of this knowledge representation constitutes the very design of the neural network, and therefore holds the key to its performance.

Roles of Knowledge Representation

The subject of how knowledge is actually represented inside an artificial network is, however, very complicated. Nevertheless, there are four rules for knowledge representation that are of a general commonsense nature, as described next.

Rule 1. Similar inputs (i.e., patterns drawn) from similar classes should usually produce similar representations inside the network, and should therefore be classified as belonging to the same class.

There is a plethora of measures for determining the similarity between inputs. A commonly used *measure of similarity* is based on the concept of Euclidian distance. To be specific, let $\mathbf{x}_i$ denote an m-by-1 vector

$$\mathbf{x}_i = [x_{i1}, x_{i2}, ..., x_{im}]^T$$

all of whose elements are real; the superscript T denotes matrix *transposition*. The vector $\mathbf{x}_i$ defines a point in an m-dimensional space called *Euclidean space* and denoted by $\mathbb{R}^m$. As illustrated in Fig. 19, the *Euclidean distance* between a pair of m-by-1 vectors $\mathbf{x}_i$ and $\mathbf{x}_j$ is defined by

$$\begin{aligned} d(\mathbf{x}_i, \mathbf{x}_j) &= \|\mathbf{x}_i - \mathbf{x}_j\| \\ &= \left[\sum_{k=1}^{m} (x_{ik} - x_{jk})^2\right]^{1/2} \end{aligned} \tag{23}$$

where x_{ik} and x_{jk} are the kth elements of the input vectors $\mathbf{x}_i$ and $\mathbf{x}_j$, respectively. Correspondingly, the similarity between the inputs represented by the vectors $\mathbf{x}_i$ and $\mathbf{x}_j$ is defined as the Euclidean distance $d(\mathbf{x}_i, \mathbf{x}_j)$. The closer the individual elements of the input vectors $\mathbf{x}_i$ and $\mathbf{x}_j$ are to each other, the smaller the Euclidean distance $d(\mathbf{x}_i, \mathbf{x}_j)$ is and therefore the greater the similarity between the vectors $\mathbf{x}_i$ and $\mathbf{x}_j$ will be. Rule 1 states that if the vectors $\mathbf{x}_i$ and $\mathbf{x}_j$ are similar, they should be assigned to the same class.

Another measure of similarity is based on the idea of a *dot product*, or *inner product*, which is also borrowed from matrix algebra. Given a pair of vectors $\mathbf{x}_i$ and $\mathbf{x}_j$ of the same dimension, their inner product is $\mathbf{x}_i^T\mathbf{x}_j$, defined as the *projection* of the vector $\mathbf{x}_i$ onto the vector $\mathbf{x}_j$, as illustrated in Fig. 19. We thus write

$$\begin{aligned} (\mathbf{x}_i, \mathbf{x}_j) &= \mathbf{x}_i^T\mathbf{x}_j \\ &= \sum_{k=1}^{m} x_{ik}x_{jk} \end{aligned} \tag{24}$$

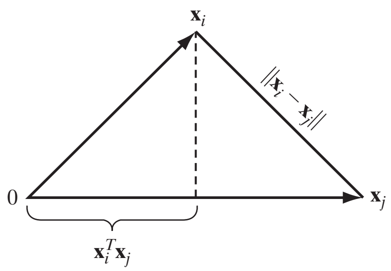


FIGURE 19 Illustrating the relationship between inner product and Euclidean distance as measures of similiarity between patterns.

The inner product $(\mathbf{x}_i, \mathbf{x}_j)$ divided by the product $\|\mathbf{x}_i\| \, \|\mathbf{x}_j\|$ is the *cosine of the angle* subtended between the vectors $\mathbf{x}_i$ and $\mathbf{x}_j$.

The two measures of similarity defined here are indeed intimately related to each other, as illustrated in Fig. 19. This figure shows clearly that the smaller the Euclidean distance $\|\mathbf{x}_i - \mathbf{x}_j\|$, and therefore the more similar the vectors $\mathbf{x}_i$ and $\mathbf{x}_j$ are, the larger the inner product $\mathbf{x}_i^T\mathbf{x}_j$ will be.

To put this relationship on a formal basis, we first normalize the vectors $\mathbf{x}_i$ and $\mathbf{x}_j$ to have unit length, that is,

$$\|\mathbf{x}_i\| = \|\mathbf{x}_j\| = 1$$

We may then use Eq. (23) to write

$$\begin{aligned} d^2(\mathbf{x}_i, \mathbf{x}_j) &= (\mathbf{x}_i - \mathbf{x}_j)^T(\mathbf{x}_i - \mathbf{x}_j) \\ &= 2 - 2\mathbf{x}_i^T\mathbf{x}_j \end{aligned} \tag{25}$$

Equation (25) shows that minimization of the Euclidean distance $d(\mathbf{x}_i, \mathbf{x}_i)$ corresponds to maximization of the inner product $(\mathbf{x}_i, \mathbf{x}_j)$ and, therefore, the similarity between the vectors $\mathbf{x}_i$, and $\mathbf{x}_j$.

The Euclidean distance and inner product described here are defined in deterministic terms. What if the vectors $\mathbf{x}_i$ and $\mathbf{x}_j$ are *stochastic*, drawn from two different populations, or ensembles, of data? To be specific, suppose that the difference between these two populations lies solely in their mean vectors. Let $\boldsymbol{\mu}_i$ and $\boldsymbol{\mu}_j$ denote the mean values of the vectors $\mathbf{x}_i$ and $\mathbf{x}_j$, respectively. That is,

$$\boldsymbol{\mu}_i = \mathbb{E}[\mathbf{x}_i] \tag{26}$$

where $\mathbb{E}$ is the *statistical expectation operator* over the *ensemble* of data vectors $\mathbf{x}_i$. The mean vector $\boldsymbol{\mu}_j$ is similarly defined. For a measure of the distance between these two populations, we may use the *Mahalanobis distance*, denoted by d_{ij}. The squared value of this distance from $\mathbf{x}_i$ to $\mathbf{x}_j$ is defined by

$$d_{ij}^2 = (\mathbf{x}_i - \boldsymbol{\mu}_i)^T\mathbf{C}^{-1}(\mathbf{x}_j - \boldsymbol{\mu}_j) \tag{27}$$

where $\mathbf{C}^{-1}$ is the *inverse* of the covariance matrix $\mathbf{C}$. It is assumed that the *covariance matrix* is the same for both populations, as shown by

$$\begin{aligned} \mathbf{C} &= \mathbb{E}[(\mathbf{x}_i - \boldsymbol{\mu}_i)(\mathbf{x}_i - \boldsymbol{\mu}_i)^T] \\ &= \mathbb{E}[(\mathbf{x}_j - \boldsymbol{\mu}_j)(\mathbf{x}_j - \boldsymbol{\mu}_j)^T] \end{aligned} \tag{28}$$

Then, for a prescribed $\mathbf{C}$, the smaller the distance d_{ij} is, the more similar the vectors $\mathbf{x}_i$ and $\mathbf{x}_j$ will be.

For the special case when $\mathbf{x}_j = \mathbf{x}_i$, $\boldsymbol{\mu}_i = \boldsymbol{\mu}_j = \boldsymbol{\mu}$, and $\mathbf{C} = \mathbf{I}$, where $\mathbf{I}$ is the identity matrix, the Mahalanobis distance reduces to the Euclidean distance between the sample vector $\mathbf{x}_i$ and the mean vector $\boldsymbol{\mu}$.

Regardless of whether the data vectors $\mathbf{x}_i$ and $\mathbf{x}_j$ are deterministic or stochastic, Rule 1 addresses the issue of how these two vectors are *correlated* to each other. *Correlation* plays a key role not only in the human brain, but also in signal processing of various kinds (Chen et al., 2007).

Rule 2. Items to be categorized as separate classes should be given widely different representations in the network.

According to Rule 1, patterns drawn from a particular class have an algebraic measure (e.g., Euclidean distance) that is small. On the other hand, patterns drawn from different classes have a large algebraic measure. We may therefore say that Rule 2 is the dual of Rule 1.

Rule 3. If a particular feature is important, then there should be a large number of neurons involved in the representation of that item in the network.

Consider, for example, a radar application involving the detection of a target (e.g., aircraft) in the presence of clutter (i.e., radar reflections from undesirable targets such as buildings, trees, and weather formations). The detection performance of such a radar system is measured in terms of two probabilities:

- *probability of detection*, defined as the probability that the system decides that a target is present when it is;
- *probability of false alarm*, defined as the probability that the system decides that a target is present when it is not.

According to the *Neyman–Pearson criterion*, the probability of detection is maximized, subject to the constraint that the probability of false alarm does not exceed a prescribed value (Van Trees, 1968). In such an application, the actual presence of a target in the received signal represents an important feature of the input. Rule 3, in effect, states that there should be a large number of neurons involved in making the decision that a target is present when it actually is. By the same token, there should be a very large number of neurons involved in making the decision that the input consists of clutter only when it actually does. In both situations, the large number of neurons assures a high degree of accuracy in decision making and tolerance with respect to faulty neurons.

Rule 4. Prior information and invariances should be built into the design of a neural network whenever they are available, so as to simplify the network design by its not having to learn them.

Rule 4 is particularly important because proper adherence to it results in a neural network with a *specialized structure*. This is highly desirable for several reasons:

1. Biological visual and auditory networks are known to be very specialized.

2. A neural network with specialized structure usually has a smaller number of free parameters available for adjustment than a fully connected network. Consequently, the specialized network requires a smaller data set for training, learns faster, and often generalizes better.
3. The rate of information transmission through a specialized network (i.e., the network throughput) is accelerated.
4. The cost of building a specialized network is reduced because of its smaller size, relative to that of its fully connected counterpart.

Note, however, that the incorporation of prior knowledge into the design of a neural network *restricts* application of the network to the particular problem being addressed by the knowledge of interest.

How to Build Prior Information into Neural Network Design

An important issue that has to be addressed, of course, is how to develop a specialized structure by building prior information into its design. Unfortunately, there are currently no well-defined rules for doing this; rather, we have some *ad hoc* procedures that are known to yield useful results. In particular, we may use a combination of two techniques:

1. *restricting the network architecture*, which is achieved through the use of local connections known as *receptive fields*[6];
2. *constraining the choice of synaptic weights*, which is implemented through the use of *weight-sharing.*[7]

These two techniques, particularly the latter one, have a profitable side benefit: The number of free parameters in the network could be reduced significantly.

To be specific, consider the partially connected feedforward network of Fig. 20. This network has a restricted architecture by construction. The top six source nodes constitute

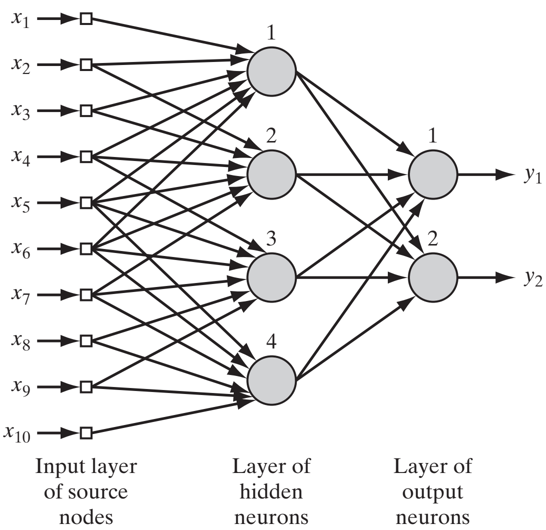


FIGURE 20 Illustrating the combined use of a receptive field and weight sharing. All four hidden neurons share the same set of weights exactly for their six synaptic connections.

the receptive field for hidden neuron 1, and so on for the other hidden neurons in the network. The *receptive* field of a neuron is defined as that region of the input field over which the incoming stimuli can influence the output signal produced by the neuron. The mapping of the receptive field is a powerful and shorthand description of the neuron's behavior, and therefore its output.

To satisfy the weight-sharing constraint, we merely have to use the same set of synaptic weights for each one of the neurons in the hidden layer of the network. Then, for the example shown in Fig. 20 with six local connections per hidden neuron and a total of four hidden neurons, we may express the induced local field of hidden neuron j as

$$v_j = \sum_{i=1}^{6} w_i x_{i+j-1}, \qquad j = 1, 2, 3, 4 \tag{29}$$

where $\{w_i\}_{i=1}^{6}$ constitutes the same set of weights shared by all four hidden neurons, and x_k is the signal picked up from source node $k = i + j - 1$. Equation (29) is in the form of a *convolution sum*. It is for this reason that a feedforward network using local connections and weight sharing in the manner described herein is referred to as a *convolutional network* (LeCun and Bengio, 2003).

The issue of building prior information into the design of a neural network pertains to one part of Rule 4; the remaining part of the rule involves the issue of invariances, which is discussed next.

How to Build Invariances into Neural Network Design

Consider the following physical phenomena:

- When an object of interest rotates, the image of the object as perceived by an observer usually changes in a corresponding way.
- In a coherent radar that provides amplitude as well as phase information about its surrounding environment, the echo from a moving target is shifted in frequency, due to the Doppler effect that arises from the radial motion of the target in relation to the radar.
- The utterance from a person may be spoken in a soft or loud voice, and in a slow or quick manner.

In order to build an object-recognition system, a radar target-recognition system, and a speech-recognition system for dealing with these phenomena, respectively, the system must be capable of coping with a range of *transformations* of the observed signal. Accordingly, a primary requirement of pattern recognition is to design a classifier that is *invariant* to such transformations. In other words, a class estimate represented by an output of the classifier must not be affected by transformations of the observed signal applied to the classifier input.

There are at least three techniques for rendering classifier-type neural networks invariant to transformations (Barnard and Casasent, 1991):

1. *Invariance by Structure.* Invariance may be imposed on a neutral network by structuring its design appropriately. Specifically, synaptic connections between the neurons of the network are created so that transformed versions of the same input are forced to produce the same output. Consider, for example, the classification of an input image by a neural network that is required to be independent of in-plane rotations of the image about its center. We may impose rotational invariance on the network structure as follows: Let w_{ji} be the synaptic weight of neuron j connected to pixel i in the input image. If the condition $w_{ji} = w_{jk}$ is enforced for all pixels i and k that lie at equal distances from the center of the image, then the neural network is invariant to in-plane rotations. However, in order to maintain rotational invariance, the synaptic weight w_{ji} has to be duplicated for every pixel of the input image at the same radial distance from the origin. This points to a shortcoming of invariance by structure: The number of synaptic connections in the neural network becomes prohibitively large even for images of moderate size.

2. *Invariance by Training.* A neural network has a natural ability for pattern classification. This ability may be exploited directly to obtain transformation invariance as follows: The network is trained by presenting it with a number of different examples of the same object, with the examples being chosen to correspond to different transformations (i.e., different aspect views) of the object. Provided that the number of examples is sufficiently large, and if the the network is trained to learn to discriminate between the different aspect views of the object, we may then expect the network to generalize correctly to transformations other than those shown to it. However, from an engineering perspective, invariance by training has two disadvantages. First, when a neural network has been trained to recognize an object in an invariant fashion with respect to known transformations, it is not obvious that this training will also enable the network to recognize other objects of different classes invariantly. Second, the computational demand imposed on the network may be too severe to cope with, especially if the dimensionality of the feature space is high.

3. *Invariant Feature Space.* The third technique of creating an invariant classifier-type neural network is illustrated in Fig. 21. It rests on the premise that it may be possible to extract *features* that characterize the essential information content of an input data set and that are invariant to transformations of the input. If such features are used, then the network as a classifier is relieved of the burden of having to delineate the range of transformations of an object with complicated decision boundaries. Indeed, the only differences that may arise between different instances of the same object are due to unavoidable factors such as noise and occlusion. The use of an invariant-feature space offers three distinct advantages. First, the number of features applied to the network may be reduced to realistic levels. Second, the requirements imposed on network design are relaxed. Third, invariance for all objects with respect to known transformations is assured.

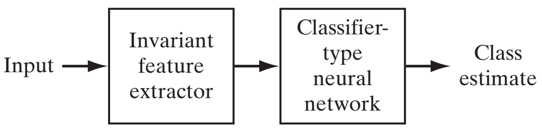


FIGURE 21 Block diagram of an invariant-feature-space type of system.

EXAMPLE 1: Autoregressive Models

To illustrate the idea of invariant-feature space, consider the example of a coherent radar system used for air surveillance, where the targets of interest include aircraft, weather systems, flocks of migrating birds, and ground objects. The radar echoes from these targets possess different spectral characteristics. Moreover, experimental studies have shown that such radar signals can be modeled fairly closely as an *autoregressive* (AR) *process* of moderate order (Haykin and Deng, 1991). An AR model is a special form of regressive model defined for complex-valued data by

$$x(n) = \sum_{i=1}^{M} a_i^* x(n-i) + e(n) \tag{30}$$

where $\{a_i\}_{i=1}^{M}$ are the *AR coefficients*, M is the *model order*, $x(n)$ is the *input*, and $e(n)$ is the *error* described as white noise. Basically, the AR model of Eq. (30) is represented by a *tapped-delay-line filter* as illustrated in Fig. 22a for $M = 2$. Equivalently, it may be represented by a *lattice filter* as shown in Fig. 22b, the coefficients of which are called *reflection coefficients.* There is a one-to-one correspondence between the AR coefficients of the model in Fig. 22a and the reflection coefficients of the model in Fig. 22b. The two models depicted here assume that the input $x(n)$ is complex valued, as in the case of a coherent radar, in which case the AR coefficients and the reflection coefficients are all complex valued. The asterisk in Eq. (30) and Fig. 22 signifies *complex conjugation.* For now, it suffices to say that the coherent radar data may be described by a set of *autoregressive coefficients*, or by a corresponding set of *reflection coefficients.* The latter set of coefficients has

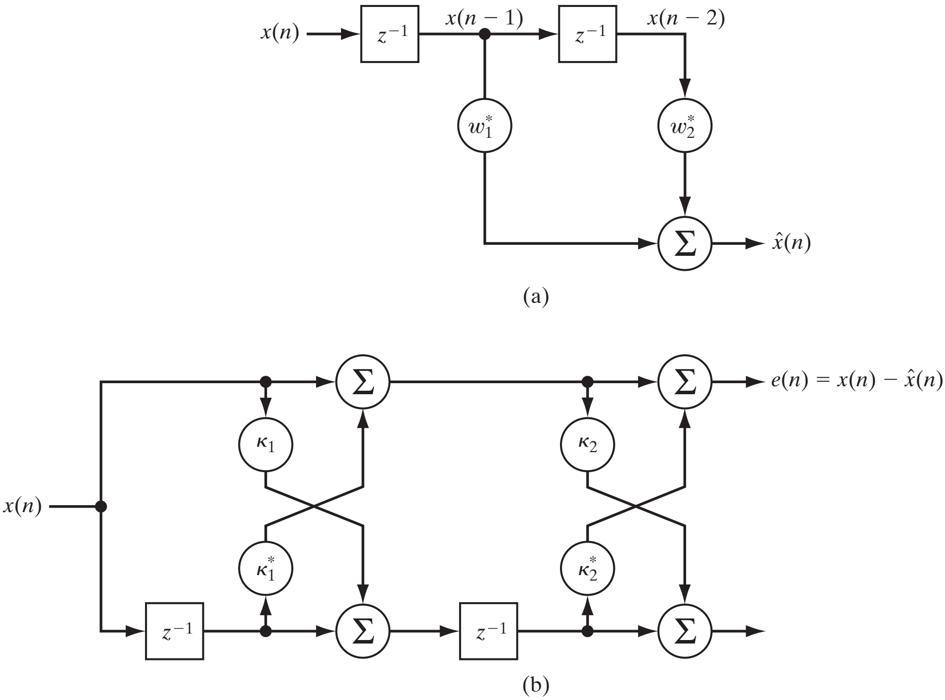


FIGURE 22 Autoregressive model of order 2: (a) tapped-delay-line model; (b) lattice-filter model. (The asterisk denotes complex conjugation.)

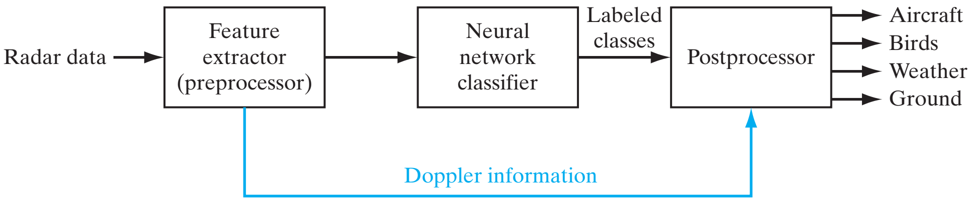


FIGURE 23 Doppler-shift-invariant classifier of radar signals.

a computational advantage in that efficient algorithms exist for their computation directly from the input data. The feature extraction problem, however, is complicated by the fact that moving objects produce varying Doppler frequencies that depend on their radial velocities measured with respect to the radar, and that tend to obscure the spectral content of the reflection coefficients as feature discriminants. To overcome this difficulty, we must build *Doppler invariance* into the computation of the reflection coefficients. The phase angle of the first reflection coefficient turns out to be equal to the Doppler frequency of the radar signal. Accordingly, Doppler frequency *normalization* is applied to all coefficients so as to remove the mean Doppler shift. This is done by defining a new set of reflection coefficients $\{\kappa'_m\}$ related to the set of ordinary reflection coefficients $\{\kappa_m\}$ computed from the input data as:

$$\kappa'_m = \kappa_m e^{-jm\theta} \qquad \text{for } m = 1, 2, ..., M \tag{31}$$

where θ is the phase angle of the first reflection coefficient. The operation described in Eq. (31) is referred to as *heterodyning*. A set of *Doppler-invariant radar features* is thus represented by the normalized reflection coefficients $\kappa'_1, \kappa'_2, ..., \kappa'_M$, with κ'_1 being the only real-valued coefficient in the set. As mentioned previously, the major categories of radar targets of interest in air surveillance are weather, birds, aircraft, and ground. The first three targets are moving, whereas the last one is not. The *heterodyned* spectral parameters of radar echoes from ground have echoes similar in characteristic to those from aircraft. A ground echo can be discriminated from an aircraft echo because of its small Doppler shift. Accordingly, the radar classifier includes a postprocessor as shown in Fig. 23, which operates on the classified results (encoded labels) for the purpose of identifying the ground class (Haykin and Deng, 1991). Thus, the *preprocessor* in Fig. 23 takes care of Doppler-shift-invariant feature extraction at the classifier input, whereas the *postprocessor* uses the stored Doppler signature to distinguish between aircraft and ground returns. ■

EXAMPLE 2: Echolocating Bat

A much more fascinating example of knowledge representation in a neural network is found in the biological sonar system of echolocating bats. Most bats use *frequency-modulated* (FM, or "chirp") signals for the purpose of acoustic imaging; in an FM signal, the instantaneous frequency of the signal varies with time. Specifically, the bat uses its mouth to broadcast short-duration FM sonar signals and uses its auditory system as the sonar receiver. Echoes from targets of interest are represented in the auditory system by the activity of neurons that are selective to different combinations of acoustic parameters. There are three principal neural dimensions of the bat's auditory representation (Simmons et al., 1992):

- *Echo frequency*, which is encoded by "place" originating in the frequency map of the cochlea; it is preserved throughout the entire auditory pathway as an orderly arrangement across certain neurons tuned to different frequencies.

- *Echo amplitude*, which is encoded by other neurons with different dynamic ranges; it is manifested both as amplitude tuning and as the number of discharges per stimulus.
- *Echo delay*, which is encoded through neural computations (based on cross-correlation) that produce delay-selective responses; it is manifested as target-range tuning.

The two principal characteristics of a target echo for image-forming purposes are *spectrum* for target shape and *delay* for target range. The bat perceives "shape" in terms of the arrival time of echoes from different reflecting surfaces (glints) within the target. For this to occur, *frequency* information in the echo spectrum is converted into estimates of the *time* structure of the target. Experiments conducted by Simmons and coworkers on the big brown bat, *Eptesicus fuscus,* critically identify this conversion process as consisting of parallel time-domain and frequency-to-time-domain transforms whose converging outputs create the common delay of range axis of a perceived image of the target. It appears that the unity of the bat's perception is due to certain properties of the transforms themselves, despite the separate ways in which the auditory time representation of the echo delay and frequency representation of the echo spectrum are initially performed. Moreover, feature invariances are built into the sonar image-forming process so as to make it essentially independent of the target's motion and the bat's own motion. ■

Some Final Remarks

The issue of knowledge representation in a neural network is directly related to that of network architecture. Unfortunately, there is no well-developed theory for optimizing the architecture of a neural network required to interact with an environment of interest, or for evaluating the way in which changes in the network architecture affect the representation of knowledge inside the network. Indeed, satisfactory answers to these issues are usually found through an exhaustive experimental study for a specific application of interest, with the designer of the neural network becoming an essential part of the structural learning loop.

8 LEARNING PROCESSES

Just as there are different ways in which we ourselves learn from our own surrounding environments, so it is with neural networks. In a broad sense, we may categorize the learning processes through which neural networks function as follows: learning with a teacher and learning without a teacher. By the same token, the latter form of learning may be subcategorized into unsupervised learning and reinforcement learning. These different forms of learning as performed on neural networks parallel those of human learning.

Learning with a Teacher

Learning with a teacher is also referred to as *supervised learning.* Figure 24 shows a block diagram that illustrates this form of learning. In conceptual terms, we may think of the teacher as having knowledge of the environment, with that knowledge being represented by a set of *input–output examples.* The environment is, however, *unknown* to the neural network. Suppose now that the teacher and the neural network are both exposed to a training vector (i.e., example) drawn from the same environment. By virtue of built-in

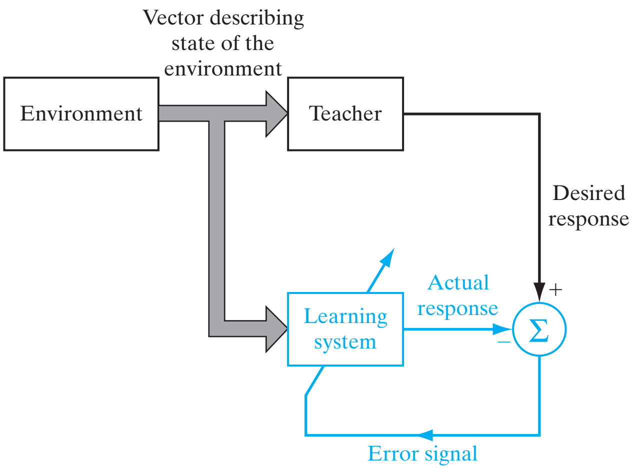


FIGURE 24 Block diagram of learning with a teacher; the part of the figure printed in red constitutes a feedback loop.

knowledge, the teacher is able to provide the neural network with a desired response for that training vector. Indeed, the desired response represents the "optimum" action to be performed by the neural network. The network parameters are adjusted under the combined influence of the training vector and the error signal. The *error signal* is defined as the difference between the desired response and the actual response of the network. This adjustment is carried out iteratively in a step-by-step fashion with the aim of eventually making the neural network *emulate* the teacher; the emulation is presumed to be optimum in some statistical sense. In this way, knowledge of the environment available to the teacher is transferred to the neural network through training and stored in the form of "fixed" synaptic weights, representing *long-term memory*. When this condition is reached, we may then dispense with the teacher and let the neural network deal with the environment completely by itself.

The form of supervised learning we have just described is the basis of *error-correction learning*. From Fig. 24, we see that the supervised-learning process constitutes a closed-loop feedback system, but the unknown environment is outside the loop. As a performance measure for the system, we may think in terms of the *mean-square error*, or the *sum of squared errors* over the training sample, defined as a function of the free parameters (i.e., synaptic weights) of the system. This function may be visualized as a multidimensional *error-performance surface*, or simply *error surface*, with the free parameters as coordinates. The true error surface is *averaged* over all possible input–output examples. Any given operation of the system under the teacher's supervision is represented as a point on the error surface. For the system to improve performance over time and therefore learn from the teacher, the operating point has to move down successively toward a minimum point of the error surface; the minimum point may be a local minimum or a global minimum. A supervised learning system is able to do this with the useful information it has about the *gradient* of the error surface corresponding to the current behavior of the system. The gradient of the error surface at any point is a vector that points in the direction of *steepest descent.* In fact, in the case of supervised learning from examples, the system may use an *instantaneous estimate* of the gradient vector, with the example indices presumed to be those of time. The use of such an estimate results in a motion of the operating point on the error surface that is typically in the form of a "random walk." Nevertheless, given an algorithm designed to minimize the cost function, an adequate set of input–output examples, and enough time in which to do the training, a supervised learning system is usually able to approximate an unknown input–output mapping reasonably well.

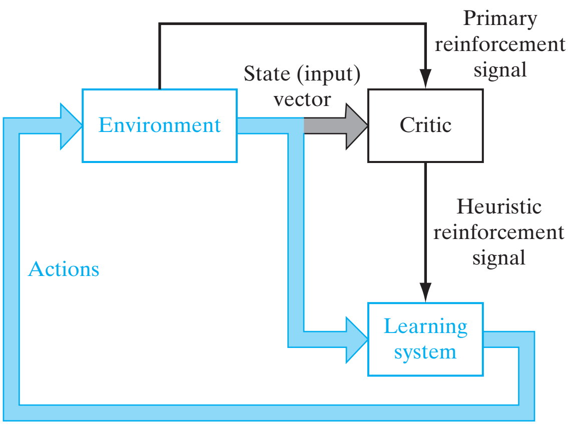
Learning without a Teacher

In supervised learning, the learning process takes place under the tutelage of a teacher. However, in the paradigm known as *learning without a teacher*, as the name implies, there is *no teacher* to oversee the learning process. That is to say, there are no labeled examples of the function to be learned by the network. Under this second paradigm, two subcategories are identified:

1. Reinforcement Learning

In *reinforcement learning*, the learning of an input–output mapping is performed through continued interaction with the environment in order to minimize a scalar index of performance. Figure 25 shows the block diagram of one form of a reinforcement-learning system built around a *critic* that converts a *primary reinforcement signal* received from the environment into a higher quality reinforcement signal called the *heuristic reinforcement signal*, both of which are scalar inputs (Barto et al., 1983). The system is designed to learn under *delayed reinforcement*, which means that the system observes a temporal sequence of stimuli also received from the environment, which eventually result in the generation of the heuristic reinforcement signal.

FIGURE 25 Block diagram of reinforcement learning; the learning system and the environment are both inside the feedback loop.



The goal of reinforcement learning is to minimize a *cost-to-go function*, defined as the expectation of the cumulative cost of *actions* taken over a sequence of steps instead of simply the immediate cost. It may turn out that certain actions taken earlier in that sequence of time steps are in fact the best determinants of overall system behavior. The function of the *learning system* is to *discover* these actions and feed them back to the environment.

Delayed-reinforcement learning is difficult to perform for two basic reasons:

- There is no teacher to provide a desired response at each step of the learning process.
- The delay incurred in the generation of the primary reinforcement signal implies that the learning machine must solve a *temporal credit assignment problem.* By this we mean that the learning machine must be able to assign credit and blame individually to each action in the sequence of time steps that led to the final outcome, while the primary reinforcement may only evaluate the outcome.

Notwithstanding these difficulties, delayed-reinforcement learning is appealing. It provides the basis for the learning system to interact with its environment, thereby developing the ability to learn to perform a prescribed task solely on the basis of the outcomes of its experience that result from the interaction.

2. Unsupervised Learning

In *unsupervised*, or *self-organized*, *learning*, there is no external teacher or critic to oversee the learning process, as indicated in Fig. 26. Rather, provision is made for a *task-independent measure* of the quality of representation that the network is required to learn, and the free parameters of the network are optimized with respect to that measure. For a specific task-independent measure, once the network has become tuned to the statistical regularities of the input data, the network develops the ability to form internal representations for encoding features of the input and thereby to create new classes automatically (Becker, 1991).

To perform unsupervised learning, we may use a competitive-learning rule. For example, we may use a neural network that consists of two layers—an input layer and a competitive layer. The input layer receives the available data. The competitive layer consists of neurons that compete with each other (in accordance with a learning rule) for the "opportunity" to respond to features contained in the input data. In its simplest form, the network operates in accordance with a "winner-takes-all" strategy. In such a strategy, the neuron with the greatest total input "wins" the competition and turns on; all the other neurons in the network then switch off.

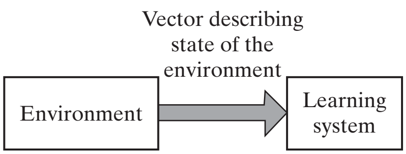


FIGURE 26 Block diagram of unsupervised learning.

9 LEARNING TASKS

In the previous section, we discussed different learning paradigms. In this section, we describe some basic learning tasks. The choice of a particular learning rule, is of course, influenced by the learning task, the diverse nature of which is testimony to the universality of neural networks.

Pattern Association

An *associative memory* is a brainlike distributed memory that learns by *association*. Association has been known to be a prominent feature of human memory since the time of Aristotle, and all models of cognition use association in one form or another as the basic operation (Anderson, 1995).

Association takes one of two forms: *autoassociation* and *heteroassociation*. In autoassociation, a neural network is required to *store* a set of patterns (vectors) by repeatedly presenting them to the network. The network is subsequently presented with a partial description or distorted (noisy) version of an original pattern stored in it, and the task is to *retrieve* (*recall*) that particular pattern. Heteroassociation differs from autoassociation in that an arbitrary set of input patterns is *paired* with another arbitrary set of output patterns. Autoassociation involves the use of unsupervised learning, whereas the type of learning involved in heteroassociation is supervised.

Let $\mathbf{x}_k$ denote a *key pattern* (vector) applied to an associative memory and $\mathbf{y}_k$ denote a *memorized pattern* (vector). The pattern association performed by the network is described by

$$\mathbf{x}_k \rightarrow \mathbf{y}_k, \qquad k = 1, 2, ..., q \tag{32}$$

where q is the number of patterns stored in the network. The key pattern $\mathbf{x}_k$ acts as a stimulus that not only determines the storage location of memorized pattern $\mathbf{y}_k$, but also holds the key for its retrieval.

In an autoassociative memory, $\mathbf{y}_k = \mathbf{x}_k$, so the input and output (data) spaces of the network have the same dimensionality. In a heteroassociative memory, $\mathbf{y}_k \neq \mathbf{x}_k$; hence, the dimensionality of the output space in this second case may or may not equal the dimensionality of the input space.

There are two phases involved in the operation of an associative memory:

- *storage phase*, which refers to the training of the network in accordance with Eq. (32);
- *recall phase*, which involves the retrieval of a memorized pattern in response to the presentation of a noisy or distorted version of a key pattern to the network.

Let the stimulus (input) $\mathbf{x}$ represent a noisy or distorted version of a key pattern $\mathbf{x}_j$. This stimulus produces a response (output) $\mathbf{y}$, as indicated in Fig. 27. For perfect recall, we should find that $\mathbf{y} = \mathbf{y}_j$, where $\mathbf{y}_j$ is the memorized pattern associated with the key pattern $\mathbf{x}_j$. When $\mathbf{y} \neq \mathbf{y}_j$ for $\mathbf{x} = \mathbf{x}_j$, the associative memory is said to have made an *error in recall*.

FIGURE 27 Input–output relation of pattern associator.

The number of patterns q stored in an associative memory provides a direct measure of the *storage capacity* of the network. In designing an associative memory, the challenge is to make the storage capacity q (expressed as a percentage of the total number N of neurons used to construct the network) as large as possible, yet insist that a large fraction of the memorized patterns is recalled correctly.

Pattern Recognition

Humans are good at pattern recognition. We receive data from the world around us via our senses and are able to recognize the source of the data. We are often able to do so almost immediately and with practically no effort. For example, we can recognize the familiar face of a person even though that person has aged since our last encounter, identify a familiar person by his or her voice on the telephone despite a bad connection, and distinguish a boiled egg that is good from a bad one by smelling it. Humans perform pattern recognition through a learning process; so it is with neural networks.

Pattern recognition is formally defined as *the process whereby a received pattern/signal is assigned to one of a prescribed number of classes.* A neural network performs pattern recognition by first undergoing a training session during which the network is repeatedly presented with a set of input patterns along with the category to which each particular pattern belongs. Later, the network is presented with a new pattern that has not been seen before, but which belongs to the same population of patterns used to train the network. The network is able to identify the class of that particular pattern because of the information it has extracted from the training data. Pattern recognition performed by a neural network is statistical in nature, with the patterns being represented by points in a multidimensional *decision space.* The decision space is divided into regions, each one of which is associated with a class. The decision boundaries are determined by the training process. The construction of these boundaries is made statistical by the inherent variability that exists within and between classes.

In generic terms, pattern-recognition machines using neural networks may take one of two forms:

- The machine is split into two parts, an unsupervised network for *feature extraction* and a supervised network for *classification*, as shown in the hybridized system of Fig. 28a. Such a method follows the traditional approach to statistical pattern recognition (Fukunaga, 1990; Duda et al., 2001; Theodoridis and Koutroumbas, 2003). In conceptual terms, a pattern is represented by a set of m observables, which may be viewed as a point $\mathbf{x}$ in an m-dimensional *observation (data) space.*

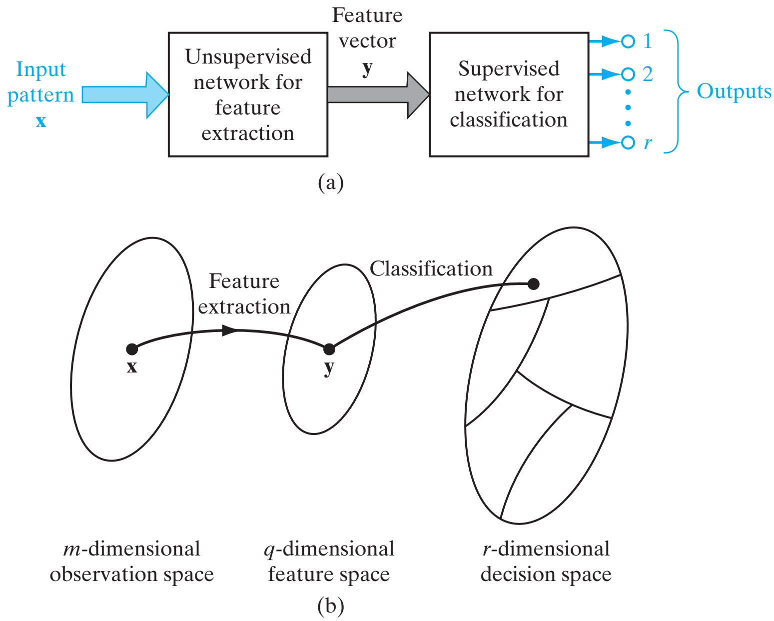


FIGURE 28 Illustration of the classical approach to pattern classification.

Feature extraction is described by a transformation that maps the point $\mathbf{x}$ into an intermediate point $\mathbf{y}$ in a q-dimensional *feature space* with $q < m$, as indicated in Fig. 28b. This transformation may be viewed as one of dimensionality reduction (i.e., data compression), the use of which is justified on the grounds that it simplifies the task of classification. The classification is itself described as a transformation that maps the intermediate point $\mathbf{y}$ into one of the classes in an r-dimensional decision space, where r is the number of classes to be distinguished.

- The machine is designed as a feedforward network using a supervised learning algorithm. In this second approach, the task of feature extraction is performed by the computational units in the hidden layer(s) of the network.

Function Approximation

The third learning task of interest is that of function approximation. Consider a nonlinear input–output mapping described by the functional relationship

$$\mathbf{d} = \mathbf{f}(\mathbf{x}) \tag{33}$$

where the vector $\mathbf{x}$ is the input and the vector $\mathbf{d}$ is the output. The vector-valued function $\mathbf{f}(\cdot)$ is assumed to be unknown. To make up for the lack of knowledge about the function $\mathbf{f}(\cdot)$, we are given the set of labeled examples:

$$\mathcal{T} = \{(\mathbf{x}_i, \mathbf{d}_i)\}_{i=1}^{N} \tag{34}$$

The requirement is to design a neural network that approximates the unknown function $\mathbf{f}(\cdot)$ such that the function $\mathbf{F}(\cdot)$ describing the input–output mapping actually realized by the network, is close enough to $\mathbf{f}(\cdot)$ in a Euclidean sense over all inputs, as shown by

$$\|\mathbf{F}(\mathbf{x}) - \mathbf{f}(\mathbf{x})\| < \varepsilon \quad \text{for all } \mathbf{x} \tag{35}$$

where ε is a small positive number. Provided that the size N of the training sample $\mathcal{T}$ is large enough and the network is equipped with an adequate number of free parameters, then the approximation error ε can be made small enough for the task.

The approximation problem described here is a perfect candidate for supervised learning, with $\mathbf{x}_i$ playing the role of input vector and $\mathbf{d}_i$ serving the role of desired response. We may turn this issue around and view supervised learning as an approximation problem.

The ability of a neural network to approximate an unknown input–output mapping may be exploited in two important ways:

(i) *System identification.* Let Eq. (33) describe the input–output relation of an unknown memoryless *multiple input–multiple output (MIMO) system*; by a "memoryless" system, we mean a system that is time invariant. We may then use the set of labeled examples in Eq. (34) to train a neural network as a model of the system. Let the vector $\mathbf{y}_i$ denote the actual output of the neural network produced in response to an input vector $\mathbf{x}_i$. The difference between $\mathbf{d}_i$ (associated with $\mathbf{x}_i$) and the network output $\mathbf{y}_i$ provides the error signal vector $\mathbf{e}_i$, as depicted in Fig. 29. This error signal is, in turn, used to adjust the free parameters of the network to minimize the squared difference between the outputs of the unknown system and the neural network in a statistical sense, and is computed over the entire training sample $\mathcal{T}$.

(ii) *Inverse modeling.* Suppose next we are given a known memoryless MIMO system whose input–output relation is described by Eq. (33). The requirement in this case is to construct an *inverse model* that produces the vector $\mathbf{x}$ in response to the vector $\mathbf{d}$. The inverse system may thus be described by

$$\mathbf{x} = \mathbf{f}^{-1}(\mathbf{d}) \tag{36}$$

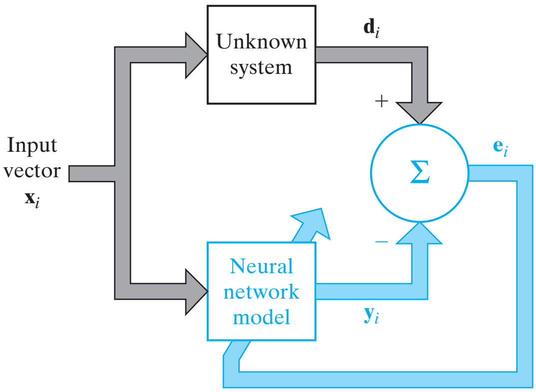


FIGURE 29 Block diagram of system identification: The neural network, doing the identification, is part of the feedback loop.

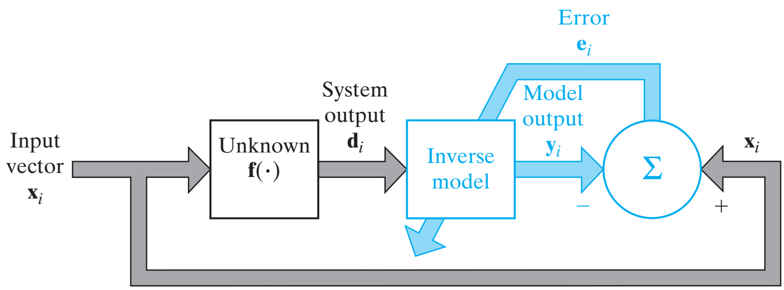


FIGURE 30 Block diagram of inverse system modeling. The neural network, acting as the inverse model, is part of the feedback loop.

where the vector-valued function $\mathbf{f}^{-1}(\cdot)$ denotes the inverse of $\mathbf{f}(\cdot)$. Note, however, that $\mathbf{f}^{-1}(\cdot)$ is not the reciprocal of $\mathbf{f}(\cdot)$; rather, the use of superscript -1 is merely a flag to indicate an inverse. In many situations encountered in practice, the vector-valued function $\mathbf{f}(\cdot)$ is much too complex and inhibits a straightforward formulation of the inverse function $\mathbf{f}^{-1}(\cdot)$. Given the set of labeled examples in Eq. (34), we may construct a neural network approximation of $\mathbf{f}^{-1}(\cdot)$ by using the scheme shown in Fig. 30. In the situation described here, the roles of $\mathbf{x}_i$ and $\mathbf{d}_i$ are interchanged: The vector $\mathbf{d}_i$ is used as the input, and $\mathbf{x}_i$ is treated as the desired response. Let the error signal vector $\mathbf{e}_i$ denote the difference between $\mathbf{x}_i$ and the actual output $\mathbf{y}_i$ of the neural network produced in response to $\mathbf{d}_i$. As with the system identification problem, this error signal vector is used to adjust the free parameters of the neural network to minimize the squared difference between the outputs of the unknown inverse system and the neural network in a statistical sense, and is computed over the complete training set $\mathcal{T}$. Typically, inverse modeling is a more difficult learning task than system identification, as there may not be a unique solution for it.

Control

The control of a *plant* is another learning task that is well suited for neural networks; by a "plant" we mean a process or critical part of a system that is to be maintained in a controlled condition. The relevance of learning to control should not be surprising because, after all, the human brain is a computer (i.e., information processor), the outputs of which as a whole system are *actions*. In the context of control, the brain is living proof that it is possible to build a generalized controller that takes full advantage of parallel distributed hardware, can control many thousands of actuators (muscle fibers) in parallel, can handle nonlinearity and noise, and can optimize over a long-range planning horizon (Werbos, 1992).

Consider the *feedback control system* shown in Fig. 31. The system involves the use of unity feedback around a plant to be controlled; that is, the plant output is fed back directly to the input. Thus, the plant output $\mathbf{y}$ is subtracted from a *reference signal* $\mathbf{d}$ supplied from an external source. The error signal $\mathbf{e}$ so produced is applied to a neural *controller* for the purpose of adjusting its free parameters. The primary objective of the controller is to supply appropriate inputs to the plant to make its output $\mathbf{y}$ track the

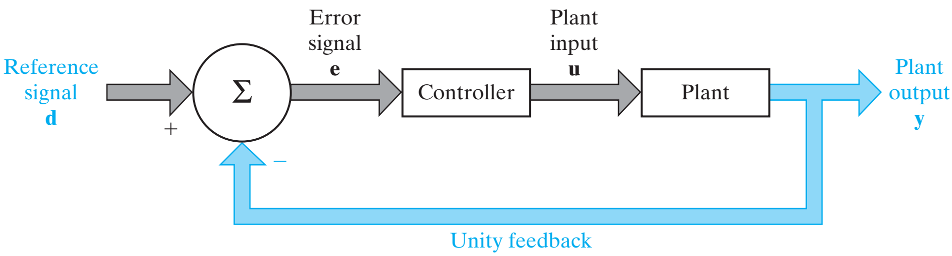


FIGURE 31 Block diagram of feedback control system.

reference signal **d**. In other words, the controller has to invert the plant's input–output behavior.

We note that in Fig. 31, the error signal **e** has to propagate through the neural controller before reaching the plant. Consequently, to perform adjustments on the free parameters of the plant in accordance with an error-correction learning algorithm, we need to know the *Jacobian,* made up of a matrix of partial derivatives as shown by

$$\mathbf{J} = \left\{ \frac{\partial y_k}{\partial u_j} \right\}_{j,k} \tag{37}$$

where y_k is an element of the plant output $\mathbf{y}$ and u_j is an element of the plant input $\mathbf{u}$. Unfortunately, the partial derivatives $\partial y_k / \partial u_j$ for the various k and j depend on the operating point of the plant and are therefore not known. We may use one of two approaches to account for them:

(i) *Indirect learning.* Using actual input–output measurements on the plant, we first construct a neural model to produce a copy of it. This model is, in turn, used to provide an estimate of the Jacobian **J**. The partial derivatives constituting this Jacobian are subsequently used in the error-correction learning algorithm for computing the adjustments to the free parameters of the neural controller (Nguyen and Widrow, 1989; Suykens et al., 1996; Widrow and Walach, 1996).

(ii) *Direct learning.* The signs of the partial derivatives $\partial y_k / \partial u_j$ are generally known and usually remain constant over the dynamic range of the plant. This suggests that we may approximate these partial derivatives by their individual signs. Their absolute values are given a distributed representation in the free parameters of the neural controller (Saerens and Soquet, 1991; Schiffman and Geffers, 1993). The neural controller is thereby enabled to learn the adjustments to its free parameters directly from the plant.

Beamforming

Beamforming is used to distinguish between the spatial properties of a target signal and background noise. The device used to do the beamforming is called a *beamformer.*

The task of beamforming is compatible, for example, with feature mapping in the cortical layers of auditory systems of echolocating bats (Suga, 1990a; Simmons et al., 1992). The echolocating bat illuminates the surrounding environment by broadcasting short-duration frequency-modulated (FM) sonar signals and then uses its auditory system (including a pair of ears) to focus attention on its prey (e.g., flying insect). The ears provide the bat with a beamforming capability that is exploited by the auditory system to produce *attentional selectivity*.

Beamforming is commonly used in radar and sonar systems where the primary task is to detect and track a target of interest in the combined presence of receiver noise and interfering signals (e.g., jammers). This task is complicated by two factors:

- the target signal originates from an unknown direction, and
- there is no *prior* information available on the interfering signals.

One way of coping with situations of this kind is to use a *generalized sidelobe canceller* (GSLC), the block diagram of which is shown in Fig. 32. The system consists of the following components (Griffiths and Jim, 1982; Haykin, 2002):

- An *array of antenna elements*, which provides a means of sampling the observation-space signal at discrete points in space.
- A *linear combiner* defined by a set of fixed weights $\{w_i\}_{i=1}^{m}$, the output of which performs the role of a desired response. This linear combiner acts like a "spatial filter," characterized by a radiation pattern (i.e., a polar plot of the amplitude of the antenna output versus the incidence angle of an incoming signal). The mainlobe of this radiation pattern is pointed along a prescribed direction, for which the GSLC is *constrained* to produce a distortionless response. The output of the linear combiner, denoted by $d(n)$, provides a desired response for the beamformer.
- A *signal-blocking matrix* $\mathbf{C}_a$, the function of which is to cancel interference that leaks through the sidelobes of the radiation pattern of the spatial filter representing the linear combiner.

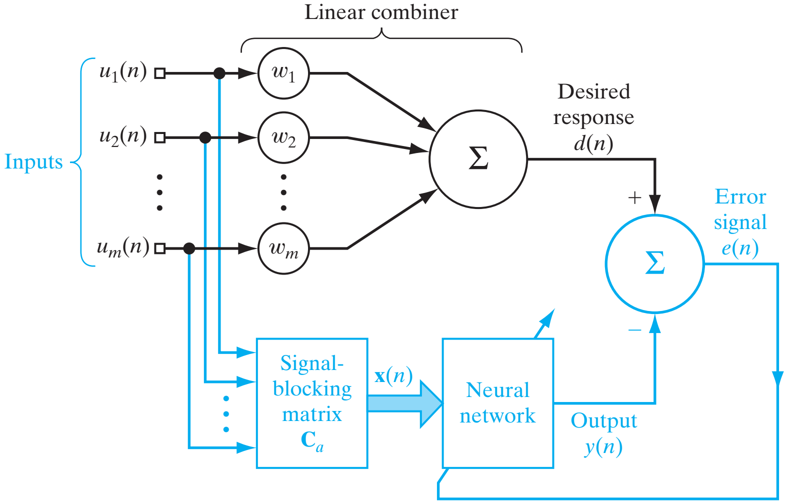


FIGURE 32 Block diagram of generalized sidelobe canceller.

- A *neural network* with adjustable parameters, which is designed to accommodate statistical variations in the interfering signals.

The adjustments to the free parameters of the neural network are performed by an error-correcting learning algorithm that operates on the error signal $e(n)$, defined as the difference between the linear combiner output $d(n)$ and the actual output $y(n)$ of the neural network. Thus the GSLC operates under the supervision of the linear combiner that assumes the role of a "teacher." As with ordinary supervised learning, notice that the linear combiner is outside the feedback loop acting on the neural network. A beamformer that uses a neural network for learning is called a *neuro-beamformer*. This class of learning machines comes under the general heading of *attentional neurocomputers* (Hecht-Nielsen, 1990).

10 CONCLUDING REMARKS

In the material covered in this introductory chapter, we have focused attention on neural networks, the study of which is motivated by the human brain. The one important property of neural networks that stands out is that of *learning*, which is categorized as follows:

(i) *supervised learning*, which requires the availability of a target or desired response for the realization of a specific input–output mapping by minimizing a cost function of interest;

(ii) *unsupervised learning*, the implementation of which relies on the provision of a task-independent measure of the quality of representation that the network is required to learn in a self-organized manner;

(iii) *reinforcement learning*, in which input–output mapping is performed through the continued interaction of a learning system with its environment so as to minimize a scalar index of performance.

Supervised learning relies on the availability of a training sample of *labeled examples*, with each example consisting of an input signal (stimulus) and the corresponding desired (target) response. In practice, we find that the collection of labeled examples is a time-consuming and expensive task, especially when we are dealing with large-scale learning problems; typically, we therefore find that labeled examples are in short supply. On the other hand, unsupervised learning relies solely on unlabeled examples, consisting simply of a set of input signals or stimuli, for which there is usually a plentiful supply. In light of these realities, there is a great deal of interest in another category of learning: *semisupervised learning*, which employs a training sample that consists of labeled as well as unlabeled examples. The challenge in semisupervised learning, discussed in a subsequent chapter, is to design a learning system that scales reasonably well for its implementation to be practically feasible when dealing with large-scale pattern-classification problems.

Reinforcement learning lies between supervised learning and unsupervised learning. It operates through continuing interactions between a learning system (agent) and the environment. The learning system performs an action and learns from the response of the environment to that action. In effect, the role of the teacher in supervised learning is replaced by a critic, for example, that is integrated into the learning machinery.

NOTES AND REFERENCES

1. This definition of a neural network is adapted from Aleksander and Morton (1990).
2. For a readable account of computational aspects of the brain, see Churchland and Sejnowski (1992). For more detailed descriptions, see Kandel et al. (1991), Shepherd (1990), Kuffler et al. (1984), and Freeman (1975).
3. For detailed treatment of spikes and spiking neurons, see Rieke et al. (1997). For a biophysical perspective of computation and information-processing capability of single neurons, see Koch (1999).
4. For a thorough account of sigmoid functions and related issues, see Mennon et al. (1996).
5. The logistic function, or more precisely, the *logistic distribution function*, derives its name from a transcendental "law of logistic growth" that has a huge literature. Measured in appropriate units, all growth processes are supposed to be represented by the logistic distribution function

$$F(t) = \frac{1}{1 + e^{\alpha t - \beta}}$$

   where t represents time, and α and β are constants.
6. According to Kuffler et al. (1984), the term "receptive field" was coined originally by Sherrington (1906) and reintroduced by Hartline (1940). In the context of a visual system, the receptive field of a neuron refers to the restricted area on the retinal surface, which influences the discharges of that neuron due to light.
7. The weight-sharing technique was originally described in Rumelhart et al. (1986b).