Data Science, 2022
Tut 5: Evaluation and Measurement- Hypothesis Testing

Make Assumptions about values when it is necessary in consistent manner. Refer necessary table from following link when necessary.

https://www.sheffield.ac.uk/polopoly_fs/1.43999!/file/tutorial-10-reading-tables.pdf

Testing a Proportion of small samples

1. H_0: $p = p_0$

2. One of the alternatives H\: $p < p_{n,}$ $p > p_{o,}$ or $p \neq p_o$

3. Choose a level of significance equal to a.

4. Test statistic: Binomial variable *X* with $p = p_0$.

5. Computations: Find x, the number of successes, and compute the appropriate P-value.

6. Decision: Draw appropriate conclusions based on the P-value

Ex. 1

A builder claims that air-conditions are installed in 70% of all homes being constructed today in the city of Mumbai. Would you agree with this claim
if a random survey of new homes in this city shows that 8 out of 15 had air-conditions installed? Use a 0.10 level of significance

DS Tutorial 5 (Ex 1)

According to the question,
consider a binomial variable x
having $P = 0.7$ and $n = 15$

$x = 8, n = 15 \Rightarrow$ 8 out 15 had ACs

given significance has level 0.1

$H_0 : P = 0.7$

$H_1 : P \neq 0.7$

$\therefore np_0 = (15)(0.7) = 10.5$

Substituting $np_0 = 10.5$ and $x = 8$

$P = 2P(X \leq 8, \text{ when } p = 0.7)$

[Applying $P = 2P(X \leq x$ when $p = p_0]$

$= 2 \sum_{x=0}^{8} (0.7)^x (0.3)^{15-x}$

$= 2 \times 0.1312$

$= 0.2624$

Here $P = 0.2624 > 0.1$

$\therefore$ We do not reject the claim [cant reject H_0]

<u>Ex.2</u>

A commonly prescribed drug for relieving nervous tension is believed to be only 60% effective. Experimental results with a new drug administered to a random sample of 100 adults who were suffering from nervous tension show that 70 received relief. Is this sufficient evidence to conclude that the new drug is superior to the one commonly prescribed? Use a 0.05 level of significance.

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DS Tutorial 5 (Ex 2)

According to the question,

We have a binomial variable X with $p = 0.6$

and level of significance $\alpha = 0.05$

$\therefore$ The critical value of $z = 1.645$

For new drug, $x = 70$, $n = 100$.

$\therefore p^* = 0.7$

$$z = \frac{0.7 - 0.6}{\sqrt{(0.6)(0.4)/100}} = 2.04$$

$P = P(z > 2.04) < 0.0207 < 0.05 (\alpha)$

$\therefore$ We reject H_0

$\therefore$ The new drug is superior

Ex.3

A vote is to be taken among the residents of a Mumbai and the surrounding area to determine whether a proposed Nuclear plant should be constructed. The construction site is within the Mumbai limits, and for this reason many voters in the surrounding area feel that the proposal will pass because of the large proportion of Mumbai voters who favor the construction. To determine if there is a significant difference in the proportion of Mumbai voters and surrounding area voters favoring the proposal, a poll is taken. If 120 of 200 Mumbai voters favor the proposal and 240 of 500 surrounding area residents favor it, would you agree that the proportion of Mumbai voters favoring the proposal is higher than the proportion of surrounding area voters? Use an a = 0.05 level of significance.

DS Tutorial5(Ex 3)

Let. p_1 be Mumbai voters favouring the proposal.
p_2 be surrounding area voters favouring
$\hat{p}_1$ be sample Mumbai voters favouring
$\hat{p}_2$ be sample surrounding area voters

Mumbai	Surrounding Area
$n_1 = 200$	$n_2 = 500$, $x_2 = 240$
$x_1 = 120$	$\hat{p}_2 = \frac{240}{500} = 0.48$
$\hat{p}_1 = \frac{x_1}{n_1} = 0.6$	$\hat{q}_2 = 1 - 0.48 = 0.52$
$\hat{q}_1 = 1 - 0.6 = 0.4$	

∴ Pooled estimate

$$\hat{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{120+240}{200+500} = 0.51$$

$$\hat{q} = 1 - 0.51 = 0.49$$

∴ $H_0: p_1 \le p_2$

$H_1: p_1 > p_2$

$$z = \frac{(\hat{p}_1 - \hat{p}_2)}{\sqrt{\hat{p}\cdot\hat{q}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} = \frac{0.6 - 0.48}{\sqrt{0.51 \times 0.49 \times \left(\frac{1}{200}+\frac{1}{500}\right)}}$$

$= 2.869$

$P = P(Z > 2.869)$

$= 0.044 < 0.05(\alpha)$

∴ we reject H_0

∴ $p_1 > p_2$

The proportion of Mumbai voters favouring the proposal is higher than the surrounding area voters

Ex.4

State the null and alternative hypotheses to be used in testing the following claims, and determine generally where the critical region is located:

(a) At most, 20% of next year's wheat, crop will be exported to the Russia..

(b) On the average, Indian homemakers drink 3 cups of tea per day.

(c) The proportion of graduates in engineering this year majoring in the computer sciences is at least. 0.15.

(d) The average donation to the Indian Autism Association is no more than 500 INR.

(e) Residents in suburban Mumbai commute, on the average, 15 kilometers to their place of employment.

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DS Tutorial 5 (Ex 4)

(a) Null hypothesis : $H_0 : p = 0.20$
Alternative hypothesis : $H_1 = p > 0.20$
Critical region : The right tail

(b) Null hypothesis : $H_0 : \mu = 3$
Alternate hypothesis : $H_1 : \mu \neq 3$
Critical region :- Two tailed

c) Null Hypothesis : $H_0 : p = 0.15$
Alternate hypothesis : $H_1 : p < 0.15$
Critical region : left tail

d) Null Hypothesis : $H_0 : \mu = 500$
Alternate Hypothesis : $H_1 : \mu > 500$
Critical region : Right tail

e) Null Hypothesis : $H_0 : \mu = 15$
Alternate Hypothesis : $H_1 : \mu \neq 15$
Critical region : Both Tails

Ex.5

In a study conducted by the Department of computer Engineering and analyzed by the Statistics Consulting Center at SPIT the laptops supplied by two different companies were compared. Ten sample laptops were made out of the Intel chips supplied by each company and the "robustness" was studied. The data are as follows:

Company A: 9.3 8.8 6.8, 8.7 8.5 6.7 8.0 6.5 9.2 7.0

Company B: 11.0 9.8 9.9 10.2,10.1 9.7 11.0 11.1 10.2 9.6

Can you conclude that there is virtually no difference in means between the laptops supplied by the two companies? Use a P-value to reach your conclusion. Should variances be pooled here?

DS Tutorial 5 (Ex 5)

Null hypothesis $H_0 : \mu_1 = \mu_2$

Alternate hypothesis $H_1 : \mu_1 \neq \mu_2$

Let's assume $\alpha = 0.05$

$\bar{X}_1 = \frac{\sum x_i}{n_i} = \frac{79.5}{10}$

$= 7.95$

$\bar{X}_2 = \frac{102.6}{10} = 10.26$

Standard Deviation $SD_1^2 = \frac{1}{n_1 - 1}\left[\sum x_1^2 - n_1 \bar{x}_1^2\right]$

$= \frac{10.865}{9} = 1.2072$

$SD_2^2 = \frac{1}{n_2 - 1}\left[\sum x_2^2 - n_2 \bar{x}_2^2\right]$

$= \frac{2.924}{9} = 0.3248$

DS Tutorial 5 (Ex 5) continued

Degree of freedom $v = \dfrac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_{12}}\right)^2}{\left(\frac{1}{n_1-1}\right)\left(\frac{SD_1^2}{n_1}\right)^2 + \frac{1}{n_2-1}\left(\frac{s_2^2}{n_2}\right)^2}$

$= \dfrac{\left(\frac{1.2072}{10} + \frac{0.3248}{10}\right)^2}{\frac{1}{9}\left(\frac{1.2072}{10}\right)^2 + \frac{1}{9}\left(\frac{0.3248}{10}\right)^2}$

$= 10.30$

$\therefore$ 10 degrees of freedom

Test statistices $T = \dfrac{\bar{x}_1 - \bar{x}_2 - (\mu_1 - \mu_2)}{\sqrt{\frac{SD_1^2}{n_1} + \frac{SD_2^2}{n_2}}}$

Null hypothesis $\mu_1 - \mu_2 = 0 = \dfrac{7.95 - 10.26}{\sqrt{\frac{1.2072}{10} + \frac{0.3248}{10}}} = -5.902$

Two sided tail $|t| = |-5.902|$

$= 5.902$

p value $= 2P(T \geq |t|)$

$= 2P(T \geq 5.9)$

$t_{0.0005}(10) = 4.587$

$|t| = 5.9$

$P(T \geq 5.9) < 0.0005$

$\therefore$ Pvalue < 0.001

$p < \alpha$ $\therefore$ Null hypothesis is rejected

$\therefore$ Mean robustness is not same