processing effort as cost of changing beliefs

or: when unpredictable doesn't mean difficult

Jacob Hoover Vigly

11 March 2025, at Harvard LangCog

sentence processing

how do we understand what a sentence means?

meaning $z =$ 

utterance $\mathbf{u} =$ Near that bank there is an otter

- sentence unfolds word by word: $\mathbf{u} = u_1, u_2, \ldots$
- with each word, refine guess about meaning, z

sentence processing

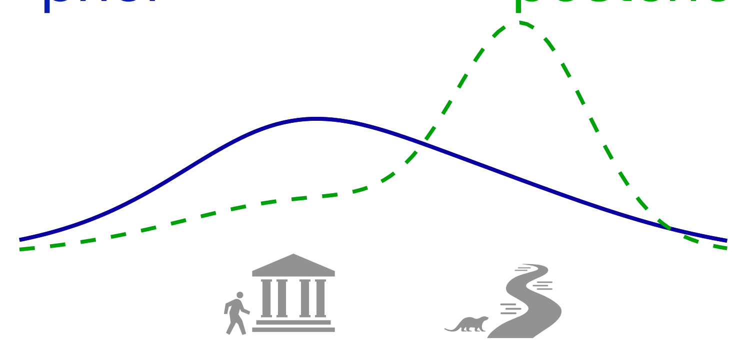
iterative inference problem

$z =$ 

$\mathbf{u} =$ Near that bank htere is an otter …

- observe otterance word by word: $\mathbf{u} = u_1, u_2, \ldots$ in noisy environment
- with each word, **update beliefs** about meaning, z

u_i causes belief update $\underbrace{p(Z \mid u_{1\ldots i-1})}_{\text{prior}} \overset{u_i}{\rightsquigarrow} \underbrace{p(Z \mid u_{1\ldots i-1}, u_i)}_{\text{posterior}}$

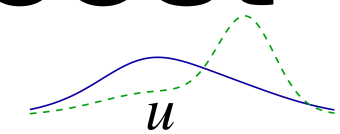


How? …with what processing algorithm?

- important clue: for humans, **unexpected words take more effort**.
- intuition: bigger update = more difficult

incremental processing cost

How? …with what processing algorithm?



$p_Z \overset{u}{\mapsto} p_{Z|u}$

- important clue: for humans, **unexpected words take more effort**.

has been formalized as:

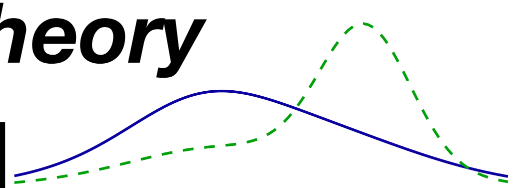
surprisal theory
(Hale '01, Levy '08)

$$\mathrm{cost}(u) \propto \overbrace{\log \frac{1}{p(u)}}^{\mathrm{surprisal}(u)}$$

precise description of phenomenon, … but how? what algorithm?

- refocus idea: difficult = big update (resource allocation cost)

update-size *theory*

$$\mathrm{cost}(u) = f(|$$  $$|)$$

size of belief update

hypothesis that cost measured as **<u>bits of information gained</u>** about Z

surprisal theory is special case, by two assumptions:

(a) that $\mathrm{D}(p_{Z|u} \| p_Z) = \mathrm{surprisal}$ (extra term is zero) ⟵ will focus on this later

(b) that f is linear

incremental processing cost

How? …with what processing algorithm? $p_Z \overset{u}{\mapsto} p_{Z|u}$

- important clue: for humans, **unexpected words take more effort**.
- intuition: bigger update = more difficult

many candidate algorithms don't have this property

- parsing algorithms (Z ranges over trees)
  - non-probabilistic algorithms
  - probabilistic enumerative algorithms
  - neural-parametrized parsing algorithms

- language model inference (e.g. n-gram, RNN, Transformer LLMs …)

… amount of work done during inference **doesn't depend on probabilistic properties at all**

(so, they don't directly explain this human behavior)

incremental processing cost

How? …with what processing algorithm? $p_Z \overset{u}{\mapsto} p_{Z|u}$

- important clue: for humans, **unexpected words take more effort**.
- intuition: bigger update = more difficult

what kind of algorithms *do* have this property?

those that somehow prioritize more probable hypotheses:

- sampling algorithms

➡ e.g. *rejection sampling* - guess-and-check until success

$$\begin{aligned}\mathbb{E}\,\#\text{samples} &= 1 \big/ \Pr(\text{success}) \\ &= 1 \big/ \textstyle\sum_z p(z) p(u \mid z) = 1 \big/ p(u) \\ &= e^{-\log p(u)} = e^{\text{surprisal}(u)}\end{aligned}$$

incremental processing cost

How? …with what processing algorithm? $p_Z \overset{u}{\mapsto} p_{Z|u}$

- important clue: for humans, **unexpected words take more effort**.
- intuition: bigger update = more difficult

what kind of algorithms *do* have this property?

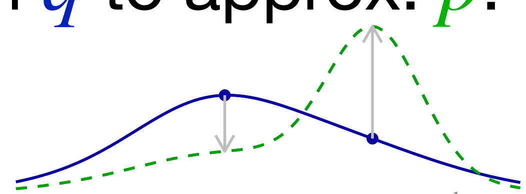
those that somehow prioritize more probable hypotheses:

- sampling algorithms

➡ *importance sampling* complexity scales in **divergence**:

sampling from q to approx. p: req #samples $\approx e^{D_{\mathrm{KL}}(p\|q)}$ Chatterjee & Diaconis 2018, …

$\approx D_{\chi^2}(p\|q)$ Agapiou et al. 2017, Sanz-Alonso 2018, …



importance weight $w(z) \propto \frac{\mathrm{d}p}{\mathrm{d}q}(z)$

$$\mathrm{cost}(u) = f\left(D(p_{Z|u}\|p_Z) \right)$$

The Plausibility of Sampling as an Algorithmic Theory of Sentence Processing

Jacob Louis Hoover[1,2], Morgan Sonderegger[1], Steven T. Piantadosi[3], and Timothy J. O'Donnell[1,2,4]

The Plausibility of Sampling as an Algorithmic Theory of Sentence Processing

Jacob Louis Hoover[1,2], **Morgan Sonderegger**[1], **Steven T. Piantadosi**[3], and **Timothy J. O'Donnell**[1,2,4]

What kinds of mechanisms prioritize high-probability hypotheses?

- **sampling** according to probability
  - simple rejection **sampling**
  - rejection **sampling** w/o replacement
  - importance **sampling**
- **searching** in order of probability

$$\begin{aligned} \mathrm{cost}(u) &= f\left(\ D(p_{Z|u} \| p_Z)\ \right) \\ &= f(\mathrm{surprisal}(u)) \quad \text{assumption (a)} \longleftarrow \text{for this paper, we assumed this} \\ &\propto \ \mathrm{surprisal}(u) \quad \text{assumption (b)} \longleftarrow \text{focused on this} \end{aligned}$$

sampling algorithms predict:

⇒ cost increases **super-linearly**

⇒ with **increasing variance**

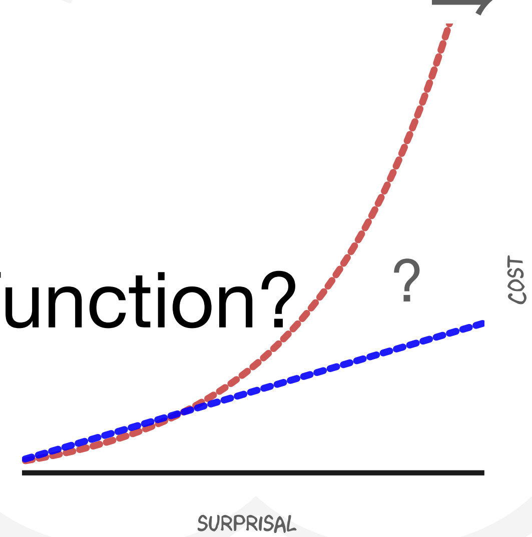
but surprisal theory proposes

⇒ cost increases **linearly**

⇒ (says nothing about variance)

Empirical question:

- what shape is the linking function?



The Plausibility of Sampling as an Algorithmic Theory of Sentence Processing

Jacob Louis Hoover[1,2], Morgan Sonderegger[1], Steven T. Piantadosi[3], and Timothy J. O'Donnell[1,2,4]

linking function: empirical study

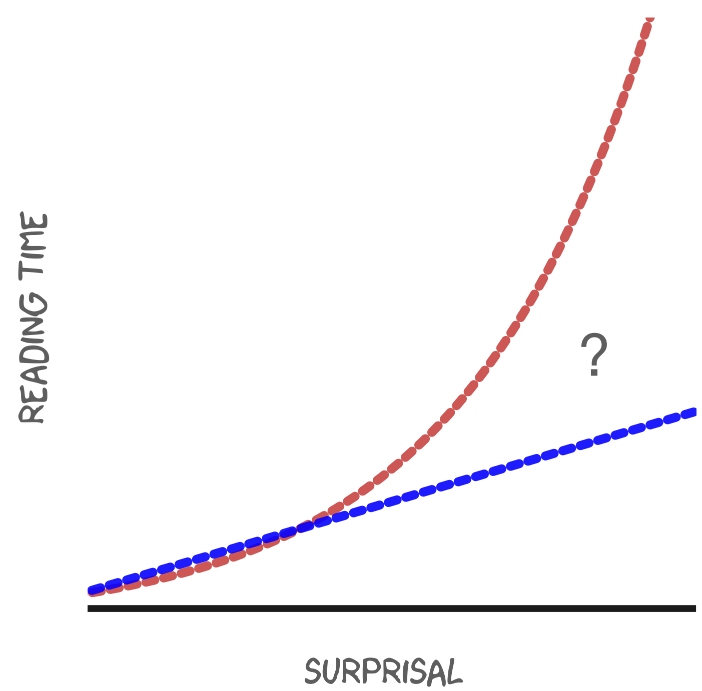
is the mean superlinear? does variance increase?

fit **location-scale Generalized Additive Model (GAMs)**

- potential **nonlinear effect of surprisal** on RT
- likewise on **variance** in RT

predictor of interest: surprisal

- estimate with **pretrained LLMs**



response: processing time

- self-paced reading time
- used Natural Stories data set
  - 10 stories, ~1000 words each
  - RTs from avg 84 participants
  - containing rare constructions (wide range of surprisals helpful to **distinguish linking function**)

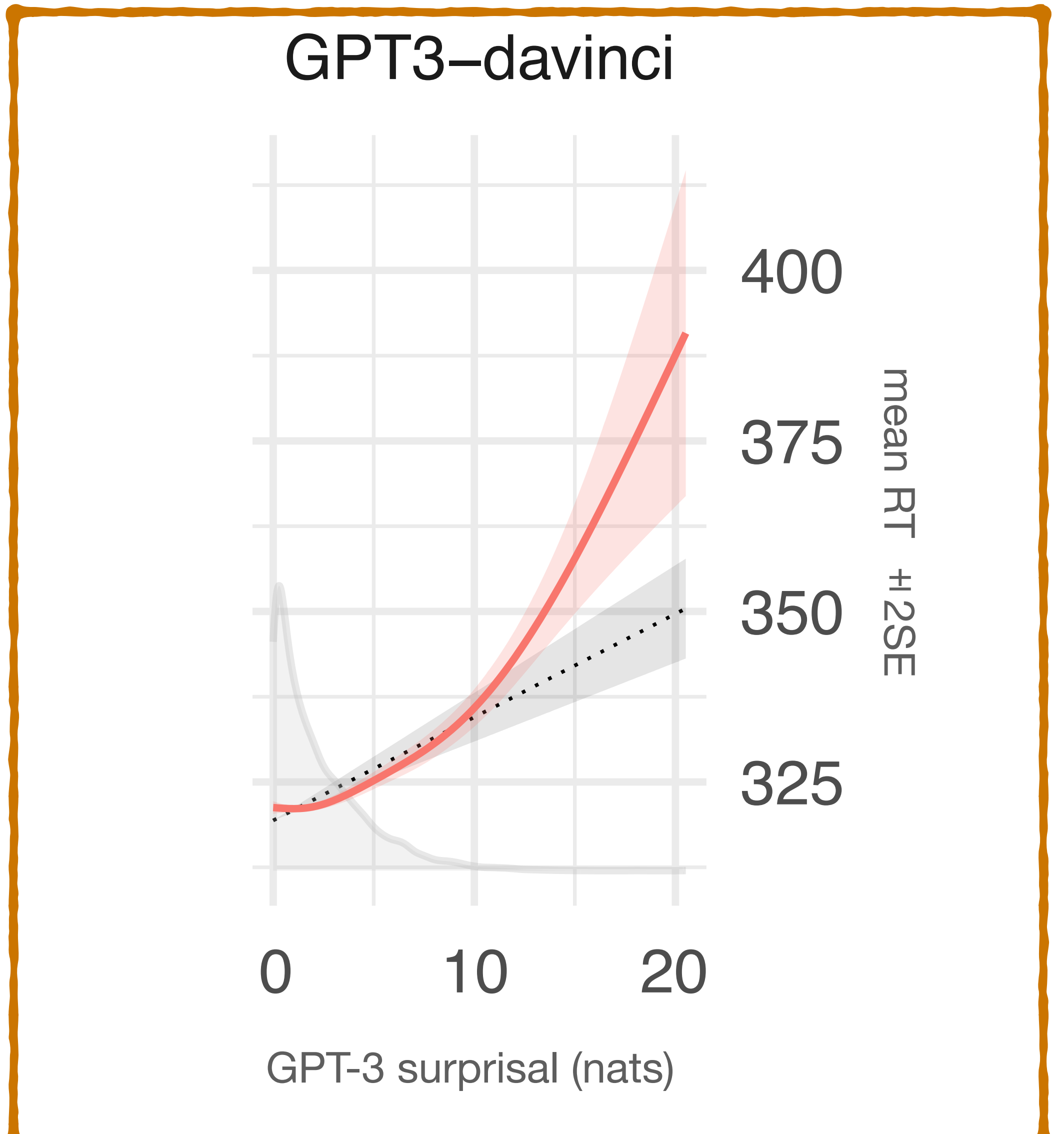
The Plausibility of Sampling as an Algorithmic Theory of Sentence Processing

Jacob Louis Hoover[1,2], Morgan Sonderegger[1], Steven T. Piantadosi[3], and Timothy J. O'Donnell[1,2,4]

linking function: empirical study

is the mean superlinear? does variance increase?

Yes



The Plausibility of Sampling as an Algorithmic Theory of Sentence Processing

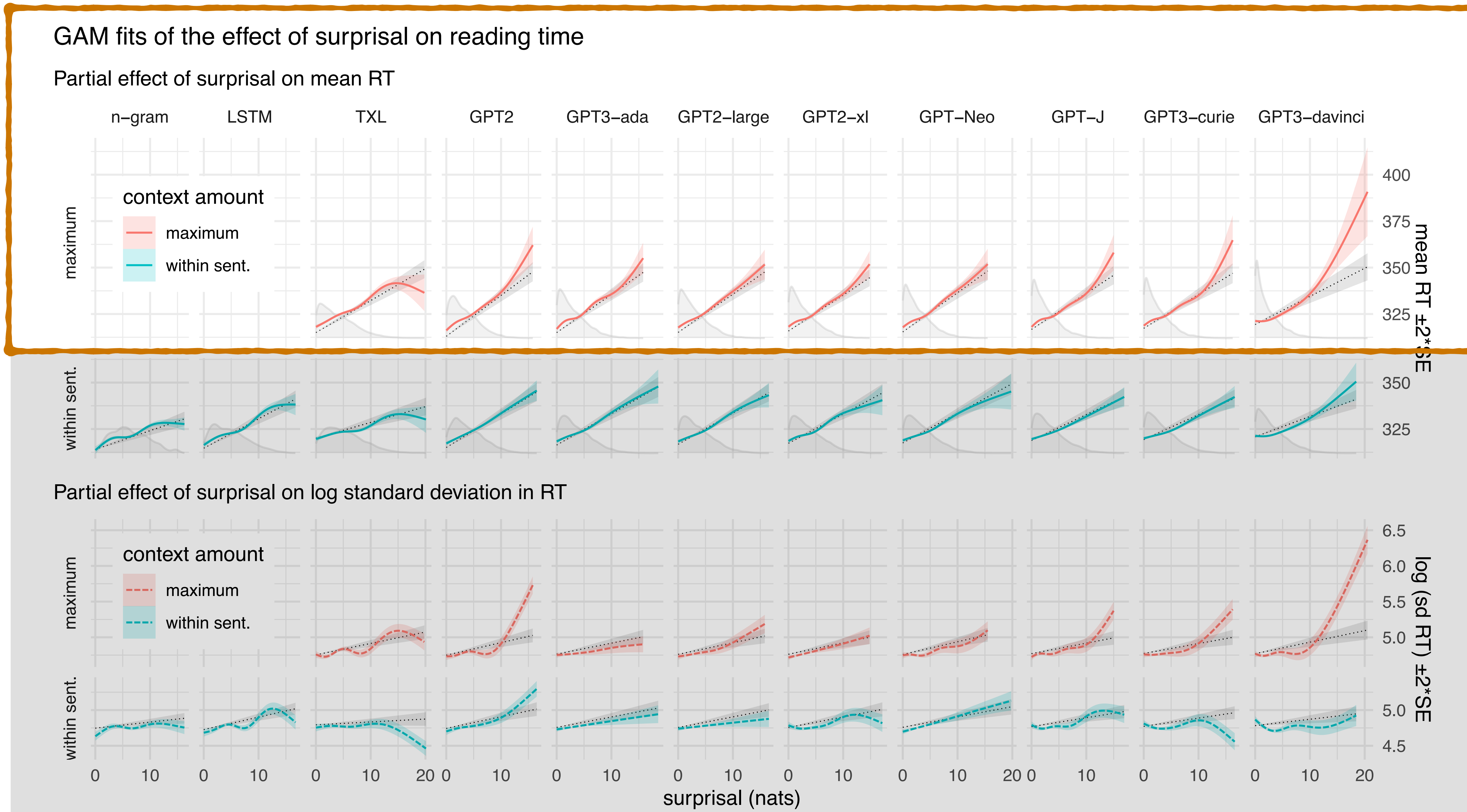
Jacob Louis Hoover[1,2], Morgan Sonderegger[1], Steven T. Piantadosi[3], and Timothy J. O'Donnell[1,2,4]

linking function: empirical study

is the mean superlinear? does variance increase?

Yes

- better LM ⇒ more superlinear



linking function: empirical study

is the mean superlinear?

Yes

- better LM ⇒ more superlinear

linear surprisal theory
(Hale '01, Levy '08)

$$\text{cost}(u) \propto \text{surprisal}(u)$$

***general** surprisal theory*
(Levy '05, Meister '21, Xu '23)

$$\text{cost}(u) = f(\text{surprisal}(u))$$

does variance increase?

Yes

- across LMs

consistent with sampling algorithms' predictions

$\Longrightarrow$ **motivation:** sampling mechanisms for processing

when surprisal ≠ divergence

now, let's revisit the other assumption: that surprisal = divergence

surprisal theory

$$\mathrm{cost}(u) = f(\mathrm{surprisal}(u))$$

belief-update theory

$$\mathrm{cost}(u) = f(\mathrm{D}_{\mathrm{KL}}(p_{Z|u} \| p_Z))$$

recall motivation: surprisal as measure of size of belief update

$$\underbrace{\mathrm{D}_{\mathrm{KL}}(p_{Z|u} \| p_Z)}_{\mathop{\mathbb{E}}_{p_{Z|u}}\left[\log \frac{p(z \mid u)}{p(z)}\right]} = \underbrace{\mathrm{surprisal}(u)}_{\log \frac{1}{p(u)}} - \underbrace{R(u)}_{\mathop{\mathbb{E}}_{p_{Z|u}}\left[\log \frac{1}{p(u \mid z)}\right]}$$

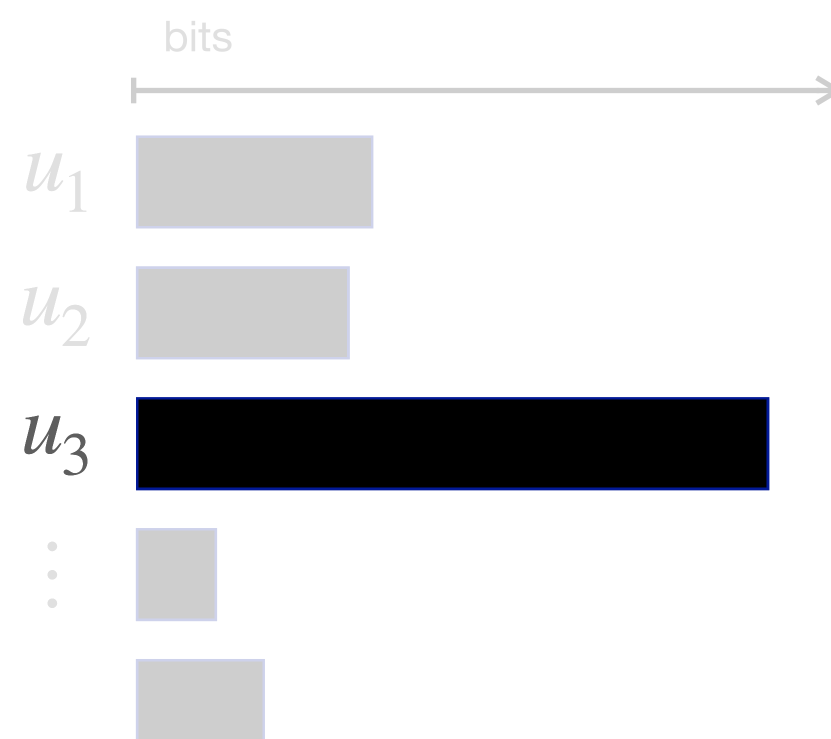
when surprisal > KL divergence

raw amount of info contained in u

size of belief update caused by observing u

reconstruction information: 'extra' bits that don't contribute to update

$$\text{surprisal} = \mathrm{D}_{\mathrm{KL}} + R$$

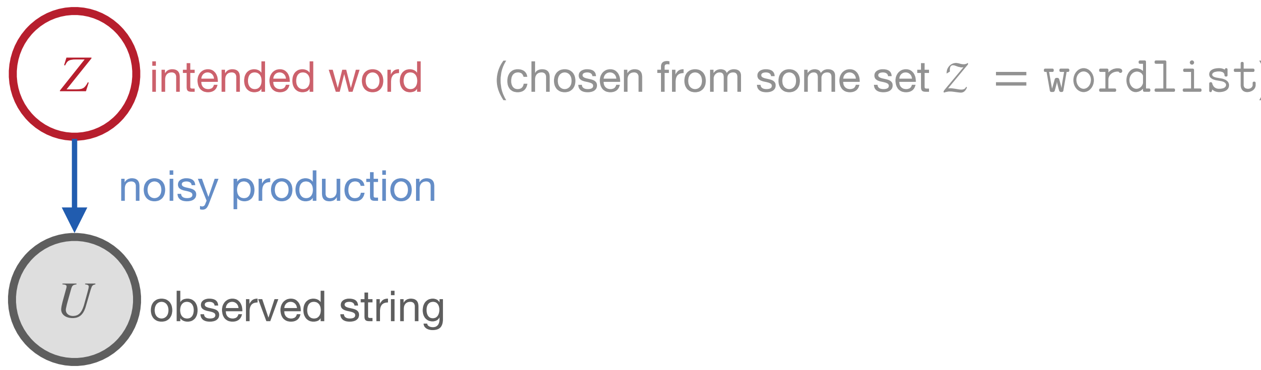


When unpredictable does not mean difficult
(WIP with Peng Qian, Morgan Sonderegger, Tim O'Donnell)

typos as a case study

typos as a case study

surprisal = $\mathrm{D}_{\mathrm{KL}} + R$



For now,

let latent Z (meaning) range over strings, representing **intended word**

- easy to model prior and likelihood
- narrow application where we might expect **LM surprisal of the observed string is intuitively inadequate** as measure of human processing cost
- (Note: I'm interested in broader applications to follow!)

bit.ly/KLdemo
interactive

typos as a case study

surprisal = $D_{KL} + R$

Example:

- *After tripping on the rug and falling in front of everyone, I felt deeply ______*

condition	target word	surprisal	divergence
1. expected	***embarrassed***	LOW	LOW 😐
2. unexpected	***innovative***	HIGH	HIGH 🤯
3. expected (typo)	***embarrsased***	HIGH	LOW 😕
4. unexpected (typo)	***innovaitve***	HIGH	HIGH

(with *any* plausible noise model)

typos as a case study

$$\text{surprisal} = D_{KL} + R$$

Example:

- *After tripping on the rug and falling in front of everyone, I felt deeply* ______

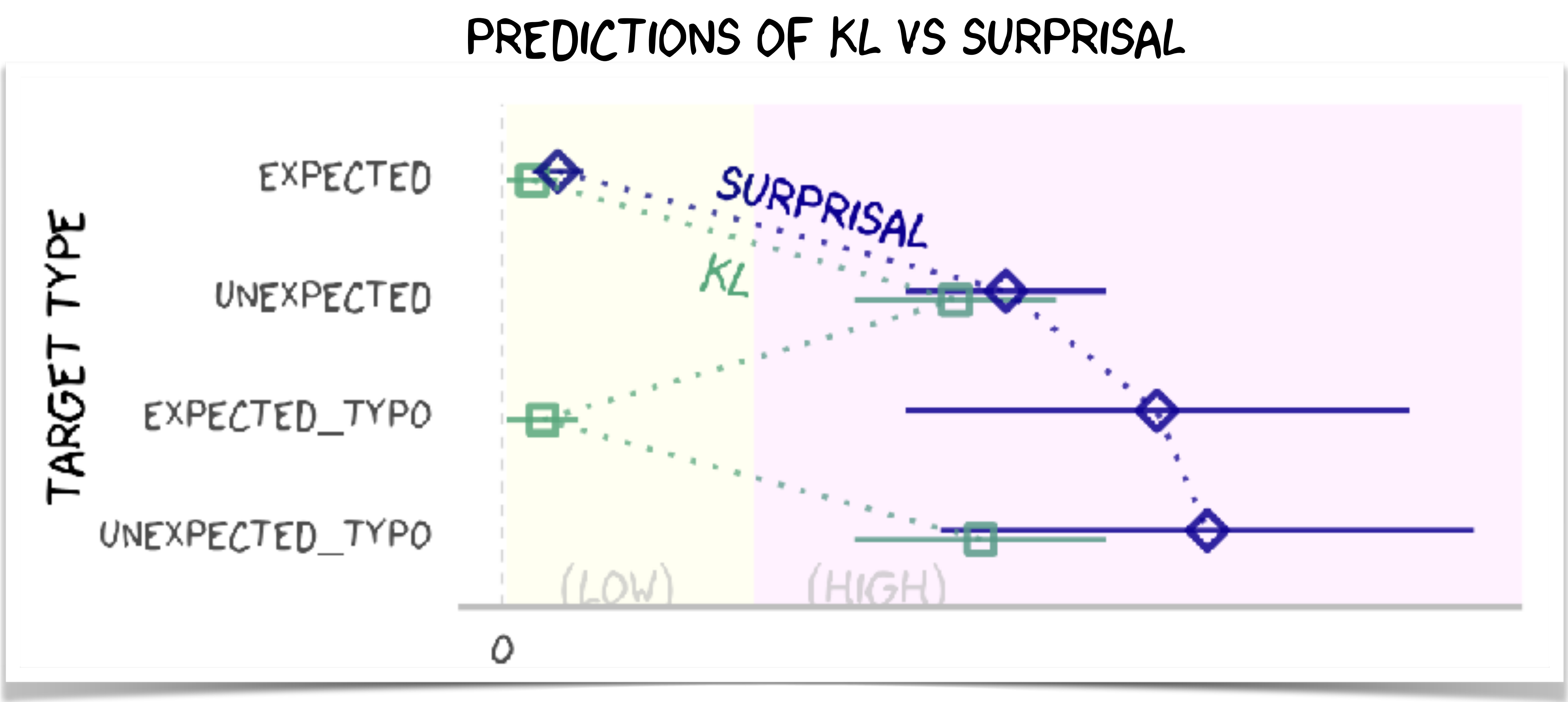
1.	expected	*embarrassed*
2.	unexpected	*innovative*
3.	expected (typo)	*embarrsased*
4.	unexpected (typo)	*innovaitve*

Self-paced reading time study:

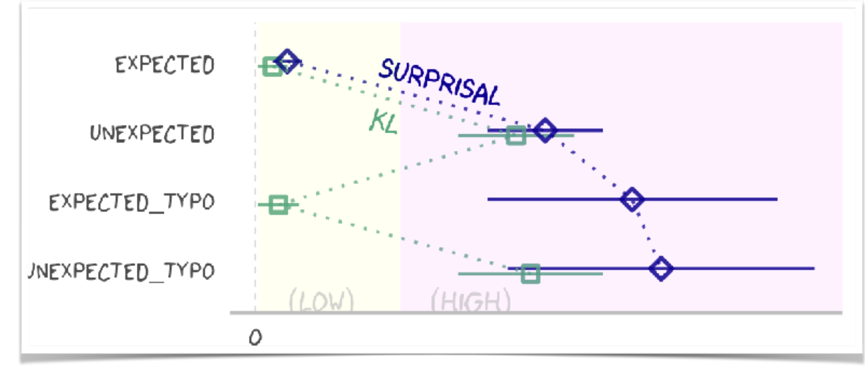
- 51 sentences x 4 conditions = 204 unique targets of interest.
- 104 participants on Prolific (post exclusions)

Fit mixed-effect regression models:

- predict human RT
- predict LLM surprisal (separately)
  - surprisals from collection of LLMs



typos as a case study



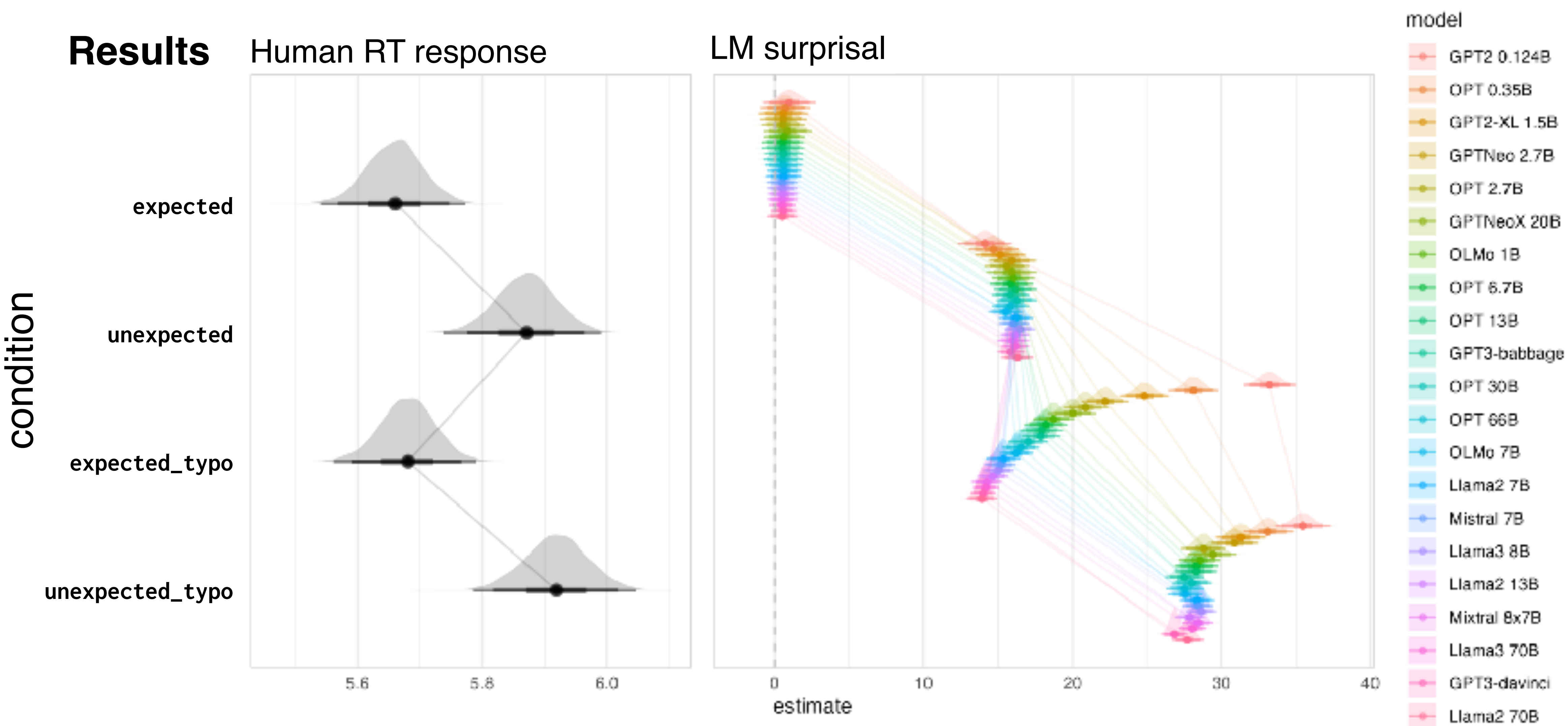
Does surprisal pattern as expected?

Yes. Surprisal is low in `expected` condition, but high in others.

Does human RT pattern like surprisal or divergence?

RTs zig-zag, **as divergence should predict**, contra surprisal.

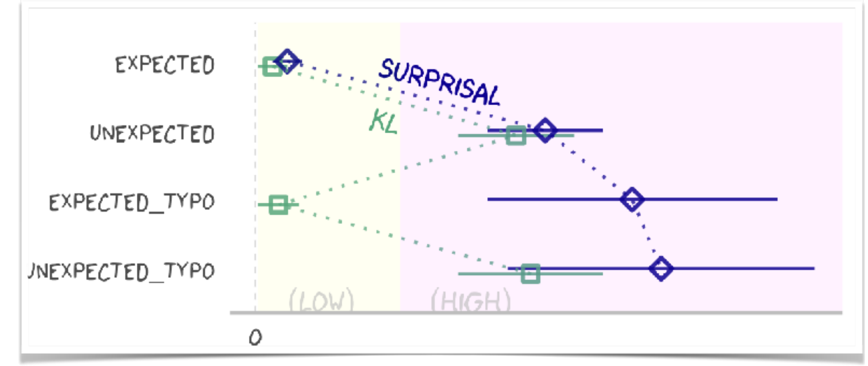
Results



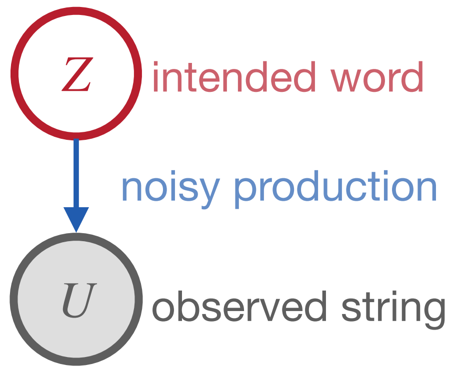
typos as a case study

estimating KL and surprisal

in noisy channel

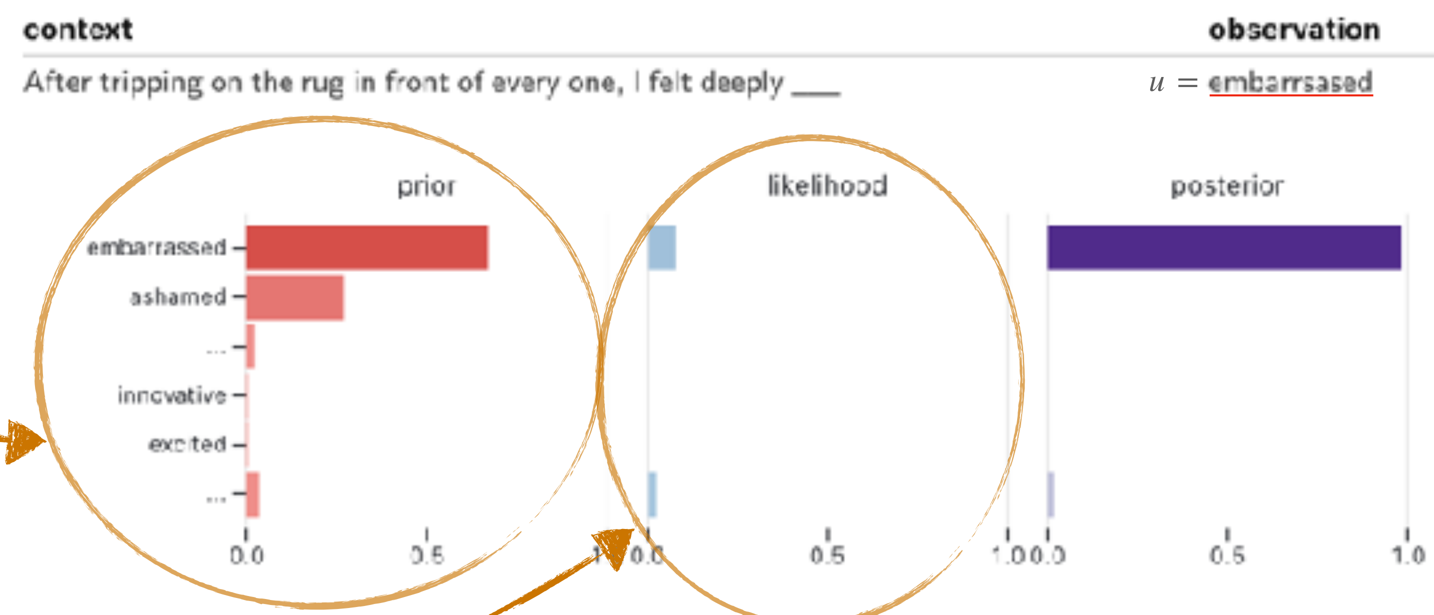


generative model:



(chosen from some set Z = wordlist)

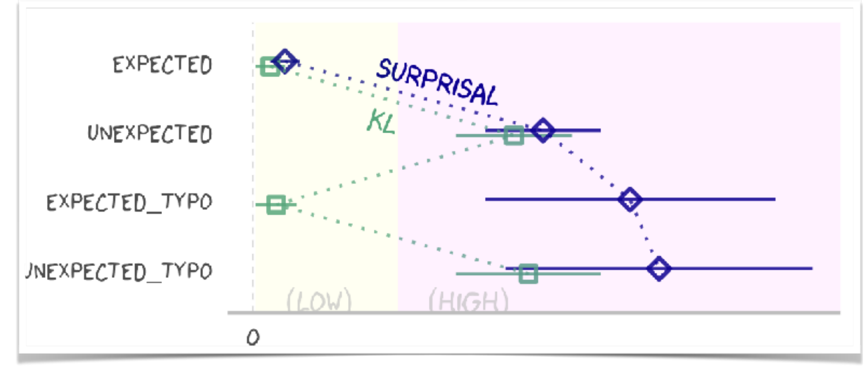
- **prior** over intended words
  $p(z\,|\,\text{context})$
  = LLM next-seq distribution
  constrained to wordlist
  $\propto p_{\text{LM}}(\text{context}) \odot \mathbf{1}_{\texttt{wordlist}}$



- **likelihood** of observed string:
  $p(u\mid z)$
  = string-edit distance model
  $p(D_{\text{Lev}}\,|\,z)\cdot p(u\,|\,D_{\text{Lev}}, z)$

typos as a case study

estimating KL and surprisal



context

After tripping over the rug in front of everyone at the party, she quickly got up, but her cheeks turned red and she felt deeply

	z	prior
_embarrassed	6.5668e-01	
_ashamed	2.6608e-01	
_guilty	1.6075e-02	
_uncomfortable	1.0753e-02	
_shy	7.0945e-03	
...		

observation $w =$ embarrassed (expected)

z	prior		likelihood		posterior	
_embarrassed	6.5668e-01		8.9583e-01		1.0000e+00	
_embraced	3.6091e-06		9.4480e-16		5.7964e-21	
_impressed	6.4865e-05		1.6926e-19		1.8663e-23	
_arrested	1.8016e-06		1.6229e-19		4.9701e-25	
...						

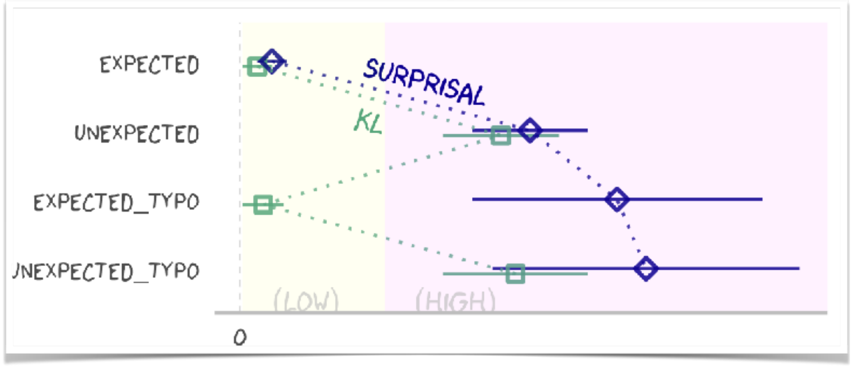
prior over intended words
$p(z)$ = LM

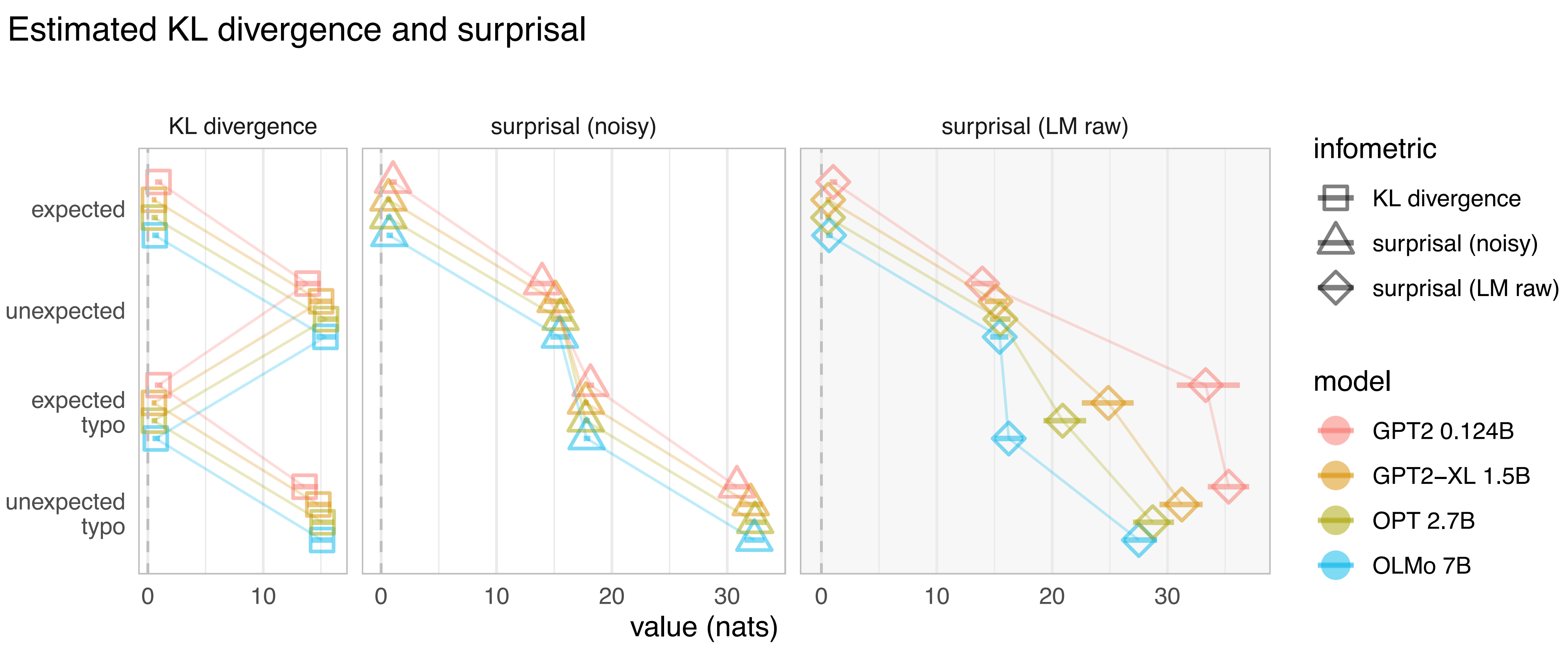
likelihood of observed string:
$p(u \mid z)$ = noisy string model

When unpredictable does not mean difficult
(WIP with Peng Qian, Morgan Sonderegger, Tim O'Donnell)

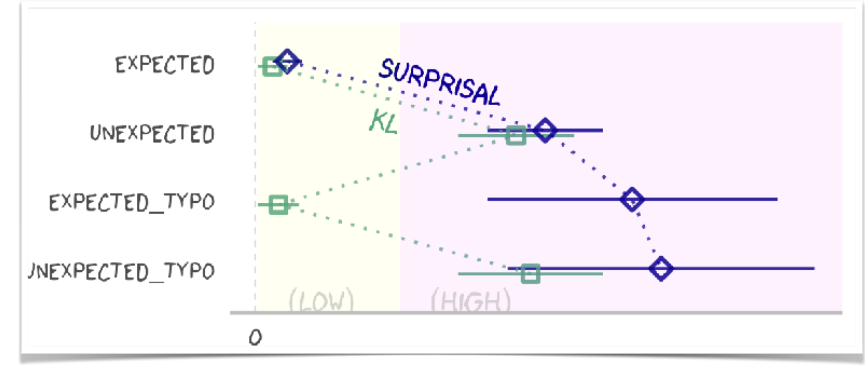
typos as a case study

estimating KL and surprisal





typos as a case study



Does surprisal pattern as expected?

Yes. Surprisal is low in **`expected`** condition, but high in others.

Does human RT pattern like surprisal or divergence?

RTs zig-zag, **as update-size predicts**, contra surprisal.

as estimated in our noisy channel model

surprisal theory
(Levy '08)

$$\mathrm{cost}(u) = f(\mathrm{surprisal}(u))$$

update-size theory

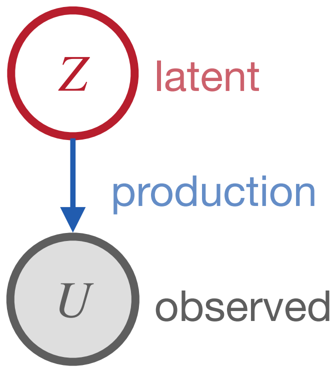
$$\mathrm{cost}(u) = f(\mathrm{D}_{\mathrm{KL}}(p_{Z|u} \| p_Z))$$

divergence (information gain)
connected to sampling complexity

$\Longrightarrow$ **motivates** sampling-based inference algorithms for processing

next steps - better estimates

- for typos
  - more realistic models of typos (using typing statistics)
  - broad-coverage model of KL (not just our materials)
- use **character level LMs** for prior and likelihood models
  - Giulianelli et al. 2024, Vieira et al. 2024
- more broadly: researcher must answer “**what is** Z?”
  - unlike surprisal, requires different models depending on **task**
    - infer intended **words**? **referent**? **sentiment**? etc. (model *task effects*)



thanks

- to you!
- to collaborators: Tim O'Donnell, Peng Qian, Morgan Sonderegger, Steve Piantadosi
- to NSF for SBE postdoc fellowship grant (SMA-2404644)

next steps - applications beyond typos

other places where we think surprisal $\gg D_{\mathrm{KL}}$ (that is, $R \gg 0$):

any (more interesting) constructions where some target region is processed without difficulty despite being very unpredictable

unexpected ways of communicating expected information (thanks to ideas from Alec Marantz)

- synonyms: *This living-room furniture set consists of a table, chair, and couch.* (vs *sofa*)
- epithets: *Boy do I hate that guy John. From the moment the bastard came in the room ....*

grammatical illusions (see e.g., Zhang et al. 2023, 2024)

- Moses illusions: *In the biblical story of the Ark, how many animals of each kind did Moses take with him?*
- agreement attraction: *The key to all the cabinets are on the table.*
- NPI illusions: *The bills that no senator voted for will ever become law.*
- depth-charge illusions: *No head injury is too trivial to ignore.*

malapropisms

- *Sure, if I reprehend (apprehend) anything in this world it is the use of my oracular (vernacular) tongue, and a nice derangement (arrangement) of epitaphs (epithets)!* (Sheridan, 1775)

multilingual codeswitching

- *"Veux-tu rentrer dans ma bubble?"*