Proof of the Twin Primes Conjecture

Javier Horacio Pérez Ricárdez

March 18, 2022

Abstract. In this paper, a proof of the Prime Number Conjecture is presented, using set theory.

1. Introduction

The Twin Primes Conjecture says: There are infinitely many pairs of twin primes. Twin primes are those that are separated by two units. Namely,
{ (3, 5), (5, 7), (11, 13), (17, 19), (29, 31), ...}

Proof

Let W be a set, defined as

$$W = \{2n | n \in \mathbb{N} \ \ and \ \ n \geq 4\} \tag{1}$$

To have the entire set W, 5 cases are presented in the interval [n -1, n + 1] with n $\geq$ 4

i) (p, p + 2) where p is a prime number

ii) (p, t) where t is an odd number not prime

iii) (t, p)

iv) (s, s + 2) where s is an even number

v) (t, t + 2)

Derived from the previous cases, let us consider the following sets

$$\begin{aligned}
A_1 &= \{(p, p+2) | p + (p+2) \in W\} && (2)\\
A_2 &= \{(p, t) | p + t \in W\} && (3)\\
A_3 &= \{(t, p) | t + p \in W\} && (4)\\
A_4 &= \{(s, s+2) | s + (s+2) \in W\} && (5)\\
A_5 &= \{(t, t+2) | t + (t+2) \in W\} && (6)
\end{aligned}$$

From the above, we have

$$W = (A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5) \tag{7}$$

$$two\ things\ are\ fulfilled$$

$$W \subset (A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5) \tag{8}$$

$$and$$

$$A_1 \subset W,\ \ A_2 \subset W,\ \ A_3 \subset W,\ \ A_4 \subset W\ \ \ y\ \ A_5 \subset W \tag{9}$$

$$to\ prove\ \ |A_1| = |W|,\ \ |A_2| = |W|,\ \ |A_3| = |W|,\ \ |A_4| = |W|\ \ \ y\ \ |A_5| = |W| \tag{10}$$

But as the A_4 set

$$\begin{aligned} A_4 &= \{10, 14, 18, 22, 26, 30, ...\} = \{4(n-1) + 10 | n \in \mathbb{N}\ \ and\ \ n \geq 1\} && (11) \\ \Rightarrow |A_4| &= \infty && (12) \end{aligned}$$

So we will only take

$$(A_1 \cup A_2 \cup A_3 \cup A_5) \tag{13}$$

and, we must prove

$$\begin{aligned} |A_1| &= \infty && (14) \\ |A_2| &= \infty && (15) \\ |A_3| &= \infty && (16) \\ |A_5| &= \infty && (17) \end{aligned}$$

Let B be the following set

$$\begin{aligned}
B &= \{(2n+1, 2n+3) | (2n+1) + (2n+3) \in W \;\; with \;\; n \in \mathbb{N} \;\; and \;\; n \geq 1\} && (18)\\
\Rightarrow |B| &= \infty && (19)
\end{aligned}$$

Also, note that

$$B = (A_1 \cup A_2 \cup A_3 \cup A_5) \tag{20}$$

Let us take the following differences between sets

$$\begin{aligned}
B - (A_2 \cup A_3 \cup A_5) &= A_1 && (21)\\
B - (A_1 \cup A_3 \cup A_5) &= A_2 && (22)\\
B - (A_1 \cup A_2 \cup A_5) &= A_3 && (23)\\
B - (A_1 \cup A_2 \cup A_3) &= A_5 && (24)
\end{aligned}$$

later

$$\begin{aligned}
|B - (A_2 \cup A_3 \cup A_5)| &= |A_1| && (25)\\
|B - (A_1 \cup A_3 \cup A_5)| &= |A_2| && (26)\\
|B - (A_1 \cup A_2 \cup A_5)| &= |A_3| && (27)\\
|B - (A_1 \cup A_2 \cup A_3)| &= |A_5| && (28)
\end{aligned}$$

now, suppose that

$$\begin{aligned}
|B - (A_2 \cup A_3 \cup A_5)| &= |A_1| < \infty && (29)\\
\Rightarrow |B| &< \infty && (30)\\
and &\\
|(A_2 \cup A_3 \cup A_5)| &< \infty && (31)
\end{aligned}$$

But by equation (19)

$$\begin{aligned}
|B| &= \infty && (32)\\
\Rightarrow \;\; than \;\; equation \;\; (29) \;\; |A_1| &= \infty && (33)
\end{aligned}$$

$$
\begin{aligned}
|B-(A_1\cup A_3\cup A_5)| &= |A_2|<\infty && (34)\\
\Rightarrow |B| &< \infty && (35)\\
and & \\
|(A_1\cup A_3\cup A_5)| &< \infty && (36)
\end{aligned}
$$

But by equation (19)

$$
\begin{aligned}
|B| &= \infty && (37)\\
\Rightarrow \ than\ equation\ (34)\ \ |A_2| &= \infty && (38)\\
and\ by\ equation\ (33),\ equation\ (36)\ leaves\ us & \\
|(A_1\cup A_3\cup A_5)| &= \infty && (39)
\end{aligned}
$$

$$
\begin{aligned}
|B-(A_1\cup A_2\cup A_5)| &= |A_3|<\infty && (40)\\
\Rightarrow |B| &< \infty && (41)\\
and & \\
|(A_1\cup A_2\cup A_5)| &< \infty && (42)
\end{aligned}
$$

But by equation (19)

$$
\begin{aligned}
|B| &= \infty && (43)\\
\Rightarrow \ than\ equation\ (40)\ \ |A_3| &= \infty && (44)\\
and\ by\ equations\ (33)\ and\ (38),\ equation\ (42)\ leaves\ us & \\
|(A_1\cup A_2\cup A_5)| &= \infty && (45)
\end{aligned}
$$

$$
\begin{aligned}
|B-(A_1\cup A_2\cup A_3)| &= |A_5|<\infty && (46)\\
\Rightarrow |B| &< \infty && (47)\\
and & \\
|(A_1\cup A_2\cup A_3)| &< \infty && (48)
\end{aligned}
$$

But by equation (19)

$$
\begin{aligned}
|B| &= \infty && (49)\\
\Rightarrow \ than\ equation\ (46)\ \ |A_5| &= \infty && (50)\\
and\ by\ equations\ (33),\ (38),\ and (44),\ equation\ (48)\ leaves\ us & \\
|(A_1\cup A_2\cup A_3)| &= \infty && (51)
\end{aligned}
$$

Then by the equations (38), (44) and (50), we are left with the equation (31)

$$|(A_2 \cup A_3 \cup A_5)| \quad = \quad \infty \tag{52}$$

then, from equations (33), (38), (44) and (50) we conclude

$$|A_1 \cup A_2 \cup A_3 \cup A_5| \quad = \quad \infty \quad = \quad |A_1| + |A_2| + |A_3| + |A_5| \tag{53}$$

of equations (19), (20) and (53)

$$|B| \quad = \quad \infty \quad = \quad |A_1 \cup A_2 \cup A_3 \cup A_5| \tag{54}$$

and from equations (2) and (33), we have infinite pairs of twin primes

References

[1] Apostolos Doxiadis. Uncle Petros and the Goldbach's conjecture. Editions B. Barcelona, 2000

[2] Irving Kaplansky. Set theory and metric spaces. (Ams Chelsea Publishing), 2020

Javier Horacio Pérez Ricárdez

independient investigator

Email address: jahoperi@gmail.com